

14 1. Introduction

15 With a possible resource of up to 2.1 TW of power worldwide [1], wave
16 energy can become an important future energy resource, thus decreasing so-
17 ciety's greenhouse gas emissions. At the moment, the wave energy industry
18 is not mature yet: numerous wave energy converter (WEC) devices have
19 been developed, but none has been established as the best design yet. Ref-
20 erence [2] provides a thorough review of some of the most promising recent
21 technologies. Point absorbers are an established type of offshore WECs [2].
22 They comprise of a floating body, whose dimensions are small relative to the
23 characteristic wavelength, excited by ocean waves that drive a power take-off
24 (PTO) system, which absorbs energy. WECs are envisioned to be installed
25 in groups, i.e. wave farms, so as to reap the benefits of economies of scale
26 [3]. However, for simplicity we analyse a single, axisymmetric unit subject
27 to motions in heave.

28 Over the years, various control schemes have been proposed for the max-
29 imization of energy absorption of WECs, with [4] and [5] presenting compre-
30 hensive reviews of the initial and recent studies in the field. In theory, optimal
31 power generation can be obtained through complex-conjugate control, since
32 it regulates the system so as to achieve resonance with the incoming waves
33 [4]. Nevertheless, this is impractical in reality due to the associated large mo-
34 tions of, and loads on, the machine in extreme seas. Thus, alternative control
35 strategies have been implemented, which consider physical constraints on the
36 motions, forces and power rating of the WEC [3].

37 Latching, model-predictive and simple-but-effective control are real-time
38 techniques for the control of WECs. With latching control, first developed by
39 [6], there is an alternation over a wave cycle of stages when the device is lin-
40 early damped and locked in place by the PTO system. Resonance is achieved
41 by regulating the duration of each phase [7]. Model predictive control com-
42 putes at each time step the force that maximizes energy absorption during a
43 future time horizon [8, 9]. Simple-but-effective control applies a force that is
44 calculated by fitting a narrow-banded function to the wave excitation force
45 [10]. While the scaling of latching control to wave farms poses serious prob-
46 lems, [11] have applied model predictive control to multi-body WECs, and
47 [12, 13] to an array of three point absorbers. Although these methods include
48 limits on the response and loading of WECs, their behaviour is strongly influ-
49 enced by the quality of the forecast wave excitation force and of the model of
50 the device dynamics [5]. In addition, model predictive control presents a very

51 high computational cost associated with the real-time optimization. Simple-
52 but-effective control results in similar power generation to model predictive
53 control, but presents a simpler implementation [5].

54 An alternative type of control strategies relies on time-averaged sea states,
55 thus assuming stationary wave conditions over a prescribed time [3]. With
56 reactive control, simulations are run to calculate the combination of PTO
57 damping and stiffness coefficients that maximise the generated energy in
58 each sea state. Resistive control represents a specific case, where the stiffness
59 term is zero. Force and displacement constraints can be included within the
60 numerical model and cost function, respectively. While this technique may be
61 associated with lower energy extraction than on-line control strategies [13],
62 it is less computationally intensive and presents a simple implementation.
63 Furthermore, the control scheme can be easily extended to the treatment of
64 wave farms, as considered by [3].

65 All aforementioned methods are strongly affected by the accuracy of the
66 model of the body dynamics they use. For this reason, modelling errors can
67 result in a drop in the generated power. Additionally, the control strategies
68 cannot adapt to changes in the response of the WEC caused by its ageing,
69 with marine biofouling playing a major role. Therefore, in a previous arti-
70 cle the authors have developed an algorithm for resistive control based on
71 reinforcement learning that learns the optimal PTO damping coefficient in
72 every sea state directly from experience [14]. This work has been extended
73 to the reactive control of a point absorber with a reaction plate in [15]. In
74 contrast to resistive control, reactive control can lead to much higher efficien-
75 cies but requires an extension of the search space to two variables, namely
76 the PTO damping and stiffness coefficients. For this reason, learning time
77 in each sea state can become very long depending on the refinement of the
78 discretization of the PTO coefficients. Furthermore, continuous values of the
79 control parameters could result in higher efficiencies. Artificial Neural Net-
80 works (ANNs) represent an alternative set of machine learning algorithms
81 which are popular in the computer science industry. They can yield smooth,
82 non-linear function approximations [16] and therefore provide an elegant so-
83 lution to the above two problems with reinforcement learning. ANNs have
84 been used to provide real-time system identification for WEC dynamics by
85 [17] and [18]. Furthermore, [17] have successfully applied the ANNs model
86 to the control of the AWS Archimedes Swing WEC.

87 Here, ANNs will be applied for the first time to the reactive control of a
88 point absorber. Hence, they are employed to map the sea state conditions

89 averaged over a time interval and the applied PTO coefficients to the mean
 90 power and maximum displacement that occur over the duration of the time
 91 interval. The resulting mapping will be used to select optimal PTO damping
 92 and stiffness coefficient at the start of each time interval, once learning has
 93 been completed. Numerical simulations are run in both regular and irregular
 94 waves to test the efficiency and convergence properties of the proposed control
 95 algorithm.

96 2. Reactive Control of a Point Absorber

97 2.1. System Description

98 A point-absorber with an electromechanical PTO is considered, as for ex-
 99 ample analysed by [19] or proposed by [20]. Removing the hydraulic stage in
 100 the power conversion process results in an increase in efficiency [20]. Further-
 101 more, as opposed to direct-drive PTO, the use of smaller, cheaper rotating
 102 generators is still possible [20].

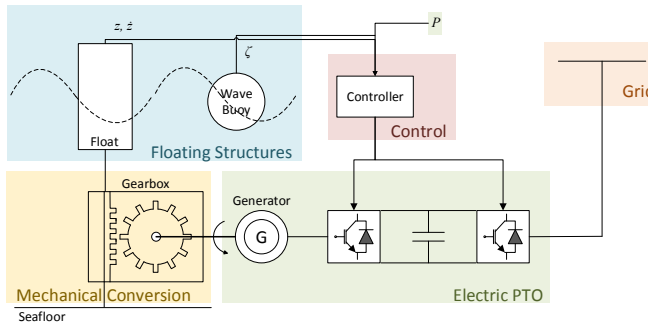


Figure 1: Schematic diagram of the WEC with its electromechanical PTO.

103 As shown in Figure 1, the movement of the float is converted into ro-
 104 tational motion through a mechanical stage. This mechanism drives a gen-
 105 erator, which can be of a permanent magnet design as proposed by [20].
 106 A variable-frequency converter delivers the generated power to the electri-
 107 cal grid at the requested frequency. The controller controls the generator
 108 through the machine-side converter in order to maximise energy absorption.
 109 The grid-side converter keeps a constant DC-link voltage and controls the
 110 active and reactive power transmitted to the network [21].

111 In order to select optimal control actions, the controller requires the heav-
 112 ing body displacement, z , and velocity, \dot{z} , as well as the wave elevation, ζ .

113 While the former two variables are inferred from on-board accelerometers,
 114 the latter is usually provided by an separate wave buoy for the whole wave
 115 farm. Furthermore, the generated power P is obtained from the electric PTO
 116 system.

117 2.2. Hydrodynamics Modelling

118 The hydrodynamic model has been obtained as in [14]. With the assump-
 119 tions of small body motions and linear wave theory, it is possible to express
 120 the response of the point absorber through the superposition of inertial, hy-
 121 drostatic, radiation, excitation and control forces [22]. Therefore, modelling
 122 the radiation force according to Cummins [23], it is possible to obtain the
 123 following time-domain equation of motion of the WEC [24]:

$$(M + A_{3,3}(\infty)) \ddot{z}(t) + \int_0^t K_{3,3}(t-\tau) \dot{z}(\tau) d\tau + C_{3,3} z(t) = F_3(t) + F_{\text{PTO}}(t), \quad (1)$$

124 with the index 3 expressing heaving motions. M is the float mass, $C_{3,3}$
 125 the hydrostatic stiffness coefficient, $A_{3,3}(\infty)$ the added mass at infinite wave
 126 frequency, and $K_{3,3}(t)$ the radiation impulse response function. The panel-
 127 code WAMIT has been used for their determination. The right-hand side of
 128 (1) comprises of the sum of the PTO force, F_{PTO} , and the wave excitation
 129 force, F_3 . The derivation of (1), as well as a more thorough explanation can
 130 be found in [25].

131 Equation (1) is shown graphically in Figure 2. The radiation force is
 132 approximated through a state-space system in order to speed up the simula-
 133 tions. The state-space matrices have been computed as described in [24].

134 2.3. Reactive Control

135 As can be seen in Figure 2, with reactive control the sum of a damping
 136 and a stiffness term yields the PTO force [3]:

$$F_{\text{PTO}}(t) = B_{\text{PTO}} \dot{z}(t) + C_{\text{PTO}} z(t). \quad (2)$$

137 In electromechanical PTO units, variations in the generator excitation cur-
 138 rent or the power converter conduction angle result in changes in the PTO
 139 damping and stiffness coefficients [5], B_{PTO} and C_{PTO} respectively. As we
 140 deal with B_{PTO} and C_{PTO} directly in this article for simplicity, the method
 141 may in fact be applied to hydraulic or direct-drive PTO systems as well.

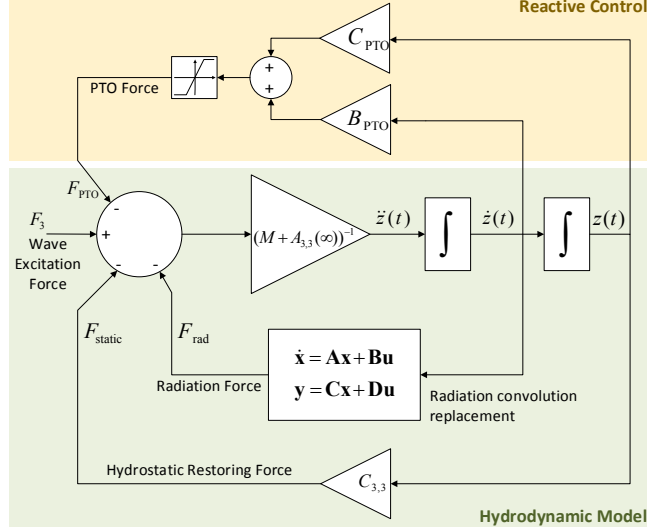


Figure 2: Block diagram employed in the computation of the float dynamics.

142 The control force is actually clipped at $\pm F_{\text{Max}}$ owing to the physical lim-
 143 its of the PTO system. In Figure 2, this is represented through a saturation
 144 block. The calculation of the generated power with reactive control is stan-
 145 dard [25], and given by [3]:

$$P(t) = \begin{cases} \eta F_{\text{PTO}}(t) \dot{z}(t) & \text{if } F_{\text{PTO}}(t) \dot{z}(t) > 0 \\ F_{\text{PTO}}(t) \dot{z}(t) / \eta & \text{if } F_{\text{PTO}}(t) \dot{z}(t) \leq 0 \end{cases}, \quad (3)$$

146 with η being the PTO efficiency.

147 Including the effects of force saturation, it is possible to maximize the en-
 148 ergy extraction through the selection of suitable PTO damping and stiffness
 149 coefficients, which depend on the wave energy period, T_e , and the significant
 150 wave height, H_s . In regular waves, these correspond to the wave period and
 151 height respectively. Furthermore, the float displacement is to be bounded to
 152 $|z| < z_{\text{Max}}$ so as to avoid structural damage in highly energetic waves. This
 153 means that the buoy should be prevented from reaching the end stops of the
 154 mechanical system, with the limits derived during the design stage. Hence,
 155 the optimal PTO damping and stiffness coefficients, $B_{\text{PTO,opt}}$ and $C_{\text{PTO,opt}}$
 156 respectively, need to be chosen so that they maximise the generated power,
 157 while abiding by the displacement constraint.

158 At the moment, the state-of-the-art approach for reactive control is to
159 pre-calculate $B_{\text{PTO,opt}}$ and $C_{\text{PTO,opt}}$ for a set of discrete sea states, generating
160 a matrix. The simulations are run using a time-domain model similar to the
161 one described here in order to account for the force saturation. Once the
162 point absorber is at sea, the controller meets the PTO coefficients associated
163 with the encountered sea state through the power electronics. However, this
164 technique suffers from modelling errors, and it cannot recognized changes in
165 the WEC dynamics caused by its ageing.

166 3. ANN-based Reactive Control of WECs

167 In order to obtain a model-free control with a continuous search space, the
168 authors propose to use ANNs to learn from experience the mapping between
169 the mean absorbed power and the maximum PTO displacement, and the
170 sea state and the PTO damping and stiffness coefficients. This corresponds
171 to system identification. However, rather than being on-line as in [17, 18],
172 due to their statistical nature these parameters are computed from the data
173 collected over a number of wave cycles based on the observation that the
174 energy content of waves changes with wave groups [26]. In particular, the
175 length of the time interval or time horizon is selected to be long enough to
176 ensure the full decay of transient effects associated with a change in PTO
177 coefficients. As a result, the coefficients from previous time intervals do not
178 greatly affect the data in the current interval so that simpler feedforward
179 ANNs can be used instead of autoregressive and local recurrent ANNs [16].

180 Hence, in order to train the ANNs, values of H_s , T_e , B_{PTO} , C_{PTO} , the
181 mean absorbed power, P_{avg} , and $\max z$ are collected for each time horizon
182 throughout the operation of the device, as entries of the training vector.
183 The estimates for P_{avg} and $\max |z|$ can be expressed through the functions
184 $f(H_s, T_e, B_{\text{PTO}}, C_{\text{PTO}})$ and $g(H_s, T_e, B_{\text{PTO}}, C_{\text{PTO}})$ respectively. The trained
185 ANNs will then be fed to optimization functions in order to find the optimal
186 PTO damping and stiffness coefficients for every new time horizon based on
187 the forecast sea state conditions.

188 3.1. Application of ANNs to the Reactive Control of WECs

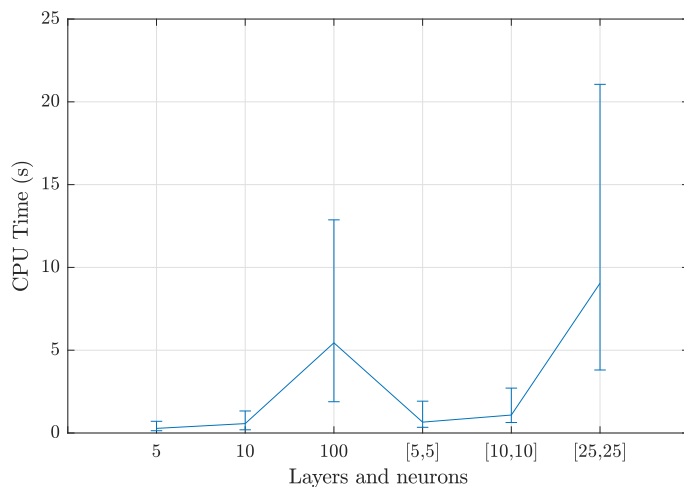
189 ANNs are a class of supervised learning algorithms [16]. Taking inspira-
190 tion from their biological equivalent, they present a network of interconnected
191 nodes, or neurons. Each neuron is a computational unit that maps input to
192 output values. By combining multiple neurons in a number of layer, so that

193 the output of the neurons in one layer becomes the input to the neurons in
194 the next layer, ANNs can be used to fit non-linear functions with a large
195 number of input values.

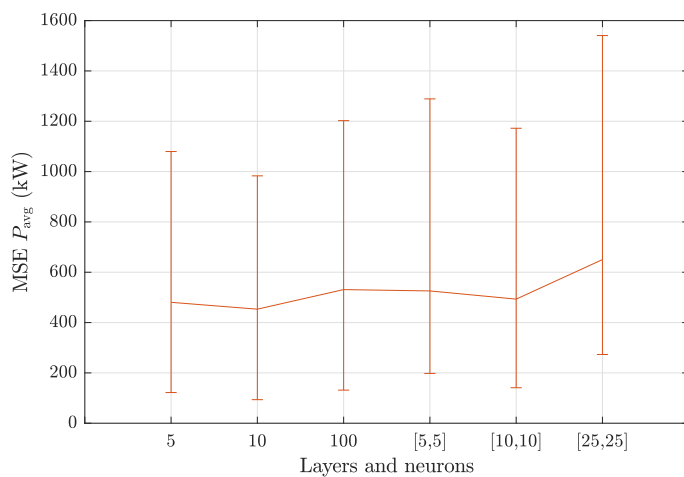
196 As aforementioned, in this work ANNs are employed in order to map the
197 mean generated power and the maximum displacement at the PTO to H_s , T_e ,
198 B_{PTO} and C_{PTO} . This is achieved through a multi-layer, feedforward ANN
199 with two output variables: P_{avg} and $\max |z|$.

200 In order to select a suitable size for the ANN, a preliminary study was
201 conducted to assess the performance of possible network configurations in es-
202 timating the mean absorbed power (hence, ignoring $\max |z|$ and reducing the
203 number of output variables to one). In particular, a single hidden layer with
204 5, 10, and 100 neurons, and two hidden layers with 5, 10 and 25 neurons each
205 have been considered. For each configuration, 25 cases have been generated
206 as the combination of 5 different random initializations of the weight matri-
207 ces [27] and 5 training and test datasets. In fact, a single training dataset
208 has been sampled from simulations in irregular waves for the sea states in
209 Table 1, which has also been used to pre-initialize the ANN-based control
210 in Section 4.3. According to standard practice with ANN training [27], the
211 whole set has been subdivided into the five distinct training and test sets
212 by randomly reordering it, and each time selecting the first 250 points for
213 the test set (about 10%) and the remaining 2239 samples for the training
214 set (approximately 90%). For each case, the ANN has been trained using
215 the training samples, and then used to estimate P_{avg} for the test set. The
216 mean square error between the prediction and the actual mean generated
217 power value has been calculated, as well as the overall computing time re-
218 quired for the ANN implementation described below. Afterwards, the mean
219 and standard deviation of these values have been computed for each network
220 configuration, and plotted in Figure 3.

221 From Figure 3, it is clear that the decision on the size of the ANN should
222 be based on a compromise between performance and accuracy. On the one
223 hand, denser networks result in greater memory requirements and computa-
224 tional cost, as shown in Figure 3a. In particular, it is interesting to notice
225 that the configuration with two hidden layers with 10 neurons each, which
226 contains a total of 100 connections between the two hidden layers, presents a
227 much lower computational cost than a single layer with 100 neurons, mainly
228 due to implementation reasons. On the other hand, the deeper the network,
229 the greater the number of features that can be matched from the original
230 function; similarly, the greater the number of neurons, the more complex



(a)



(b)

Figure 3: Mean central processing unit (CPU) time (a) and mean square error (MSE) (b) associated with the prediction of the mean generated power for different ANN configurations in terms of hidden layers and neurons for 5 weight initializations and 5 training and test sets. The upper bar corresponds to the sum of the mean value and half the standard deviation, while the lower bar to the minimum value of all cases in order to prevent negative values.

231 the fitted function shape [16]. An example is the lower mean square error
 232 associated with the configurations with 10 neurons as compared with those
 233 with 5 in Figure 3b. Nevertheless, an excessive number of neurons can result

234 in overfitting the input data [27], i.e. fitting the random noise in addition
 235 to the underlying relationship, which is highly undesirable since the ANN is
 236 expected to generalise the shape of the P_{avg} and $\max |z|$ curves. In Figure
 237 3b, this evidently occurs for a single hidden layer with 100 neurons and two
 238 hidden layers with 25 neurons each. Although a single hidden layer seems
 239 to perform best, this preliminary study has been carried out on a relatively
 240 small dataset, considering only a limited number of sea states. Therefore, it
 241 has been preferred to use a configuration with two hidden layers each with
 242 10 neurons in order to represent the possible extra features associated with
 243 the additional sea states. Additionally, this results in only a minor increase
 244 in computational time. Similar results are obtained from the mapping of the
 245 maximum displacement.

246 A schematic diagram of the feedforward ANN can be seen in Figure 4.
 247 The network presents an input layer with 4 neurons (one for each input
 248 variable), two hidden layers with $m = 10$ and $n = 10$ neurons each, and an
 249 output layer with two output variables. Furthermore, it is possible to see that
 250 the input and hidden layers have an additional bias term, which is required
 251 to find the intercept of the fitted functions at each stage in the ANN [27].
 252 Each layer l presents input and output variables, which are expressed as \mathbf{x}_l
 253 and \mathbf{y}_l respectively in vector notation. The input variables correspond to \mathbf{y}_1 ,
 254 while the output to \mathbf{y}_4 . The signal between each two matrices is multiplied
 255 by weight matrices \mathbf{W}_i , with $i = 1, 2, 3$. The weight matrices for the bias
 256 terms are represented as \mathbf{b}_i .

257 Given the input data for one training example, \mathbf{y}_1 , it is possible to obtain
 258 \mathbf{y}_4 , the ANN estimate for the output by propagating the signal from one layer
 259 to the next one. Using this technique, known as forward propagation, the
 260 input and output vectors of each layer $l = 2, 3, 4$ can be computed for each
 261 training point in matrix notation, which is convenient from a programming
 262 perspective, as follows [27]:

$$\mathbf{x}_l = \mathbf{W}_{l-1}\mathbf{y}_{l-1} + \mathbf{b}_{l-1}, \quad (4)$$

$$\mathbf{y}_l = e_l(\mathbf{x}_l). \quad (5)$$

263 In (5), e_l denotes the activation function of the neurons in each layer. Figure 4
 264 shows that the two hidden layers use the tanh activation function, while the
 265 output layer presents a linear activation function. The hyperbolic tangent is
 266 a standard smooth, non-linear activation function, which is superior to the
 267 sigmoid function, as its output is zero-centred [16]. As described in [16], for a

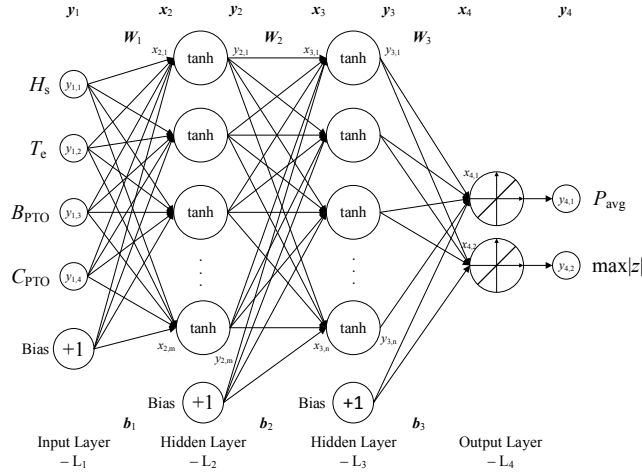


Figure 4: Schematic diagram of the feedforward ANN for the approximation of the mean generated power or maximum PTO displacement.

268 small number of layers, as in this case, tanh is preferred over rectified linear
 269 units, which are standard in deep learning. The linear activation function is
 270 employed in the output layer in order to return a real value, not bounded
 271 within ± 1 as would be the case if tanh had been used instead.

272 The mapping between input and output is dictated by the weights of the
 273 ANN [16]. Hence, learning can occur by tuning these parameters based on
 274 the training data so as to minimize an objective function, which is a measure
 275 of the error between actual and predicted output. In order to update the
 276 weight matrices, it is necessary to calculate a gradient matrix that indicates
 277 the change in the error due to a change in each weight. The gradient matrices
 278 are computed by propagating the error signal, or sensitivity, \mathbf{s}_l , from the
 279 output layer to the input layer in a process known as backpropagation [27]:

$$\begin{aligned} \mathbf{s}_4 &= -(\mathbf{y}_{\text{tr}} - \mathbf{y}_4) \odot \dot{e}_4(\mathbf{x}_4) \\ \mathbf{s}_l &= (\mathbf{W}_l^T \mathbf{s}_{l+1}) \odot \dot{e}_l(\mathbf{x}_l), \end{aligned} \quad (6)$$

280 where \odot indicates the Hadamard, or element-wise, product, and \dot{e}_l the first
 281 derivative of the activation function of each layer. \mathbf{y}_{tr} indicates the exact out-
 282 put of each training sample, i.e. the variables the ANNs should fit. Therefore,

283 $\mathbf{y}_{\text{tr}} = [P_{\text{avg}} \quad \max |z|]^T$. The change in the weight matrices is given by [27]:

$$\Delta \mathbf{W}_l = \mathbf{s}_{l+1} \mathbf{y}_l^T, \quad (7)$$

$$\Delta \mathbf{b}_l = \mathbf{y}_l. \quad (8)$$

284 The equations above are used if the ANNs are trained using one training
 285 sample at a time, such as when the simple gradient descent scheme is applied
 286 [27]. Nevertheless, batch-mode training, i.e. employing multiple training
 287 samples at a time, is much more efficient. For this reason, the highly efficient
 288 Levenberg-Marquardt backpropagation training algorithm has been adopted
 289 instead [27]. A detailed explanation of the method, including the necessary
 290 extra equations, can be found in [28, 27] in matrix notation. The implemen-
 291 tation within the Mathworks neural networks toolbox has been used in this
 292 work, with the default settings.

293 It is important to notice that the input variables, i.e. H_s , T_e , B_{PTO} and
 294 C_{PTO} , need to be normalized through their mean and standard deviation
 295 before being fed to the ANNs for training. Furthermore, the mean power
 296 values have also been normalized with respect to the maximum (for positive
 297 values) and minimum (for negative values). This has been necessary because
 298 the points lying on the $B_{\text{PTO}} = 0$ boundary of the search space presented
 299 excessively high negative power values that seriously affected the quality of
 300 the function fit.

301 3.2. Multistart Optimization

302 At the start of every new time horizon, the controller should select the
 303 PTO damping and stiffness coefficients that will result in maximum energy
 304 extraction for the predicted sea state during the horizon, in compliance with
 305 the constraint on the PTO displacement. This is clearly a non-linear opti-
 306 mization problem, since both P_{avg} and $\max |z|$ are non-linear functions of H_s ,
 307 T_e , B_{PTO} and C_{PTO} . In addition, the values of the PTO damping and stiff-
 308 ness coefficients must be bounded within sensible values, so that the problem
 309 is constrained as well.

310 By removing the dependence on the significant wave height and wave
 311 energy period from functions f and g due to space limitations for display
 312 purposes, the cost function can be expressed at the start of each new time
 313 horizon h as follows:

$$c(h) = \begin{cases} -f(B_{\text{PTO}}, C_{\text{PTO}}) & \text{if } |g(B_{\text{PTO}}, C_{\text{PTO}})| \leq z_{\text{Max}} \\ +1 & \text{if } |g(B_{\text{PTO}}, C_{\text{PTO}})| > z_{\text{Max}} \end{cases} \quad (9)$$

subject to:

$$B_{\min} \leq B_{\text{PTO}} \leq B_{\text{Max}}, C_{\min} \leq C_{\text{PTO}} \leq C_{\text{Max}}.$$

314 The values of the maximum and minimum allowable PTO damping and stiff-
315 ness coefficients can be derived using accurate, non-linear models during the
316 design stage in order to prevent damage to the generator in the most energetic
317 sea states likely to be encountered, where the buoy velocity and displacement
318 are highest.

319 Genetic and other nature-inspired algorithms have been extensively used
320 recently for the solution of non-linear optimization problems that present
321 multiple minima, as in this case [29]. Nevertheless, in this work, a strong
322 emphasis is given to performance, since the optimization needs to be repeated
323 at the start of each new time interval. For this reason, it has been preferred
324 to use the Multistart algorithm [30]. This technique consists in generating
325 a number of start points, sampled randomly within the $B_{\text{PTO}}, C_{\text{PTO}}$ search
326 space. Although convergence is not assured, a large number of starting points
327 greatly increase the chances. A value of 100 starting points has been selected
328 for this reason. From each point, an optimization is run using a non-linear,
329 constrained programming solver. In particular, the Mathworks functions
330 *MultiStart* and *fmincon* have been used respectively. The main advantage of
331 this technique over alternative methods, such as global search, is its simple
332 parallel implementation, which can result in large savings in computational
333 time. For instance, one Multistart optimization using the cost function in (9)
334 takes 8.62 s on a quad-core, i7 computer with 16GB RAM, whereas a global
335 search takes 29.20 s. A greater number of cores and an implementation
336 in a lower-order language, such as C or Fortran, can result in even greater
337 computational savings.

338 3.3. Algorithm

339 Figure 5 shows the algorithm for the ANN-based reactive control of the
340 point absorber described in this article. As aforementioned, a time-averaged
341 approach is used, where new values of B_{PTO} and C_{PTO} are selected at the
342 start of every new time horizon h and applied throughout its duration $D(h)$.
343 On the one hand, a longer duration is preferable for the power averaging and
344 sea state statistical analysis so as to produce less noisy training data. On the
345 other hand, a shorter time span can result in faster training. Furthermore,
346 the controller would be able to track changes in the sea state on a smaller
347 time scale, thus moving towards real-time control and possibly higher energy

348 extraction. For these reasons, $D(h) = 20T_e(h)$ has been chosen in both
 349 regular and irregular waves.

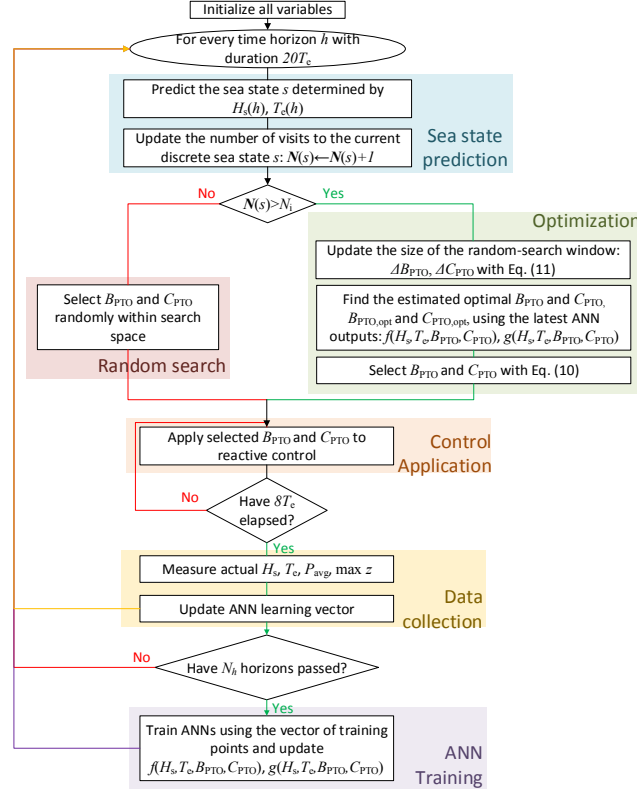


Figure 5: Flow chart of the ANN-based reactive control of a point absorber.

350 As can be seen from Figure 5, the first step in every time horizon is to
 351 predict the significant wave height and energy wave period during the time
 352 interval. Different approaches have been proposed for this problem, with
 353 example methods being Kalman filters, deterministic sea wave prediction
 354 [31], autoregressive models [32], and even ANNs [33]. Although these studies
 355 analyse the wave elevation, which is forecast with accuracy only 15 s into
 356 the future, it is assumed that similar strategies can be found for the forecast
 357 of the statistical wave conditions for one time horizon. For simplicity, in
 358 this initial work the actual values for H_s and T_e have been used, since the
 359 wave traces employed in the simulations are known in advance. $H_s(h)$ and
 360 $T_e(h)$ are then used to update the count of the number of observations in

361 the current discrete sea state, s . For this purpose a table, \mathbf{N} , is employed,
 362 with an entry for each discrete sea state with ranges of 1 m and 1 s for each
 363 dimension respectively.

364 During the first N_i visits to each discrete sea state, the values of the
 365 PTO damping and stiffness coefficients are selected randomly to ensure initial
 366 exploration. Once $\mathbf{N}(s) > N_i$, the Multistart optimization can be run using
 367 the cost function in (9) in order to find the optimal coefficients, $B_{\text{PTO,opt}}$ and
 368 $C_{\text{PTO,opt}}$, for the forecast significant wave height and energy wave period.
 369 However, the ANN estimates f and g can be very inaccurate initially. For
 370 this reason, B_{PTO} and C_{PTO} are in fact selected randomly within a region
 371 around the optimum that shrinks with the number of data points collected
 372 in the sea state:

$$B_{\text{PTO}} = B_{\text{PTO,opt}} + \Delta B_{\text{PTO}}, \quad (10)$$

373 where

$$\Delta B_{\text{PTO}} = (r - 0.5) \cdot \text{range}(B_{\text{PTO}}) \cdot 0.9^{\mathbf{N}(s) - N_i}, \quad (11)$$

374 with $r = [0, 1]$ a random number. The same applies to C_{PTO} . Upper and
 375 lower bounds are used to ensure the chosen values lie within the desired
 376 range. As more data points are collected in the optimal region, the accuracy
 377 of the ANN fit increases.

378 Once B_{PTO} and C_{PTO} are chosen and applied, measurements are em-
 379 ployed to compute the mean absorbed power, maximum PTO displacement
 380 and actual $H_s(h)$ and $T_e(h)$ during the time interval. These values are in fact
 381 calculated using the data only after an initial time of $8T_e(h)$ within the cur-
 382 rent horizon h in order to exclude the initial transient effects. This relatively
 383 long time also ensures that the time required for the Multistart optimization
 384 does not become an issue. Once the desired values are obtained, they are
 385 stored in memory as a data sample so that they can be used for training the
 386 ANN.

387 The ANN is trained every $N_h = 20$ time horizons, employing 90% of
 388 training points. The remaining 10% of the samples is used for validation and
 389 hence to check the quality of the fit. Each sample presents H_s , T_e , B_{PTO} ,
 390 C_{PTO} as input, and P_{avg} and $\max |z|$ as output. The larger the number
 391 of training points, the less the risk of overfitting the data and the more
 392 accurate the estimates of the ANN. However, this will also cause an increase
 393 in training time and, more importantly, it may result in an excessive memory
 394 requirement. Therefore, for a practical application, it is expected that the
 395 number of training points will be limited to a large number, say 10^6 . Care

396 will be needed in order to ensure that a similar number of data points is kept
397 for each discrete sea state when overriding old data with new readings, as
398 well as to explore a broad range of B_{PTO} and C_{PTO} values so as to aid the
399 training of the ANN.

400 4. Simulation Results

401 4.1. Simulation System

402 The proposed algorithm has been tested using the same point absorber
403 as in [22] and [14]: a floating vertical cylinder with 5-m radius and 8-m
404 draught. Deep water is assumed in the determination of the hydrodynamic
405 coefficients, with the radiation approximation state-space vector presenting
406 five entries as in [14]. Similarly, the hydrodynamic model in Fig. 2 has been
407 arranged in a state-space system and discretized using a first-order accurate
408 Euler scheme, with a sampling time of 0.1 s. The same PTO force saturation
409 and float displacement limits of 1 MN and ± 5 m respectively have also been
410 adopted, as well as a PTO efficiency of 75%.

411 Figure 6 shows graphically the the program used for the simulation of the
412 WEC. Instead of sensors installed on a wave buoy, in the simulations a wave
413 model provides the wave elevation record as in [14], as can be seen in Figure 1.
414 For irregular waves, the wave elevation is computed as the superposition of
415 multiple individual wave components, whose amplitude is derived from the
416 specified wave spectrum [26]. A value of 0.005 rad/s has been selected for
417 the circular wave frequency step, since this value is smaller than the Nyquist
418 frequency for a 15-minute window so as to prevent a repetition of the wave
419 trace [34]. Therefore, each trace of irregular waves is generated as the com-
420 bination of 15-minute-long time series, where the random number generator
421 is initialized with a different seed for each component. In order to smooth
422 the connection between the separate traces, a 20-point filter is employed over
423 the last and first of each consecutive time series. The wave elevation time
424 series has a dual purpose: on the one hand, it is used to establish H_s and T_e
425 in each time horizon [26]; on the other hand, the convolution integral of the
426 wave elevation and diffraction impulse response function produces the wave
427 excitation force [35].

428 The search space has been limited to within $B_{\text{min}} = 0$ and $B_{\text{Max}} =$
429 2 MNs/m, and $C_{\text{min}} = -1$ MN/m and $C_{\text{Max}} = 0$ for the PTO damping
430 and stiffness coefficients respectively. A wider search space has been selected
431 for the PTO damping coefficient in order to prevent damage in large waves,

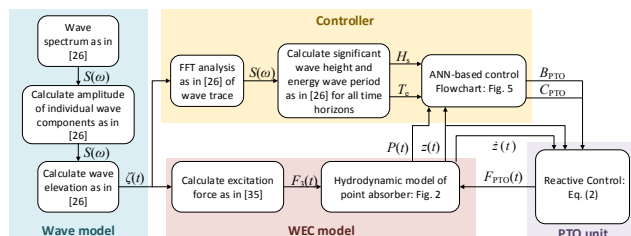


Figure 6: Flow chart of the program used in the simulations of the point absorber.

432 when greater damping and no stiffness are required. Nevertheless, the larger
 433 the search space, the longer the learning time; hence, an excessive search
 434 space needs to be avoided.

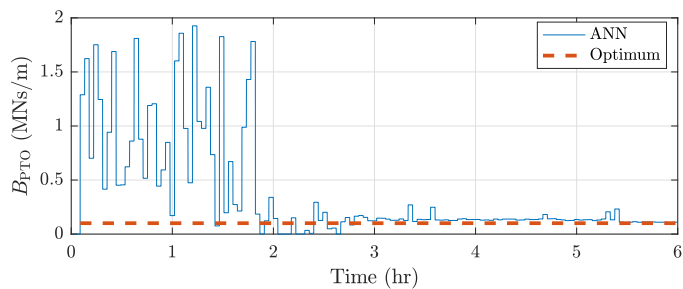
435 For the first 15 minutes of the simulations, no control force is applied in
 436 order to let the system dynamics settle. For this reason, all wave traces are
 437 in fact generated with an extra 15-minute interval at the start.

438 4.2. Results in Regular Waves

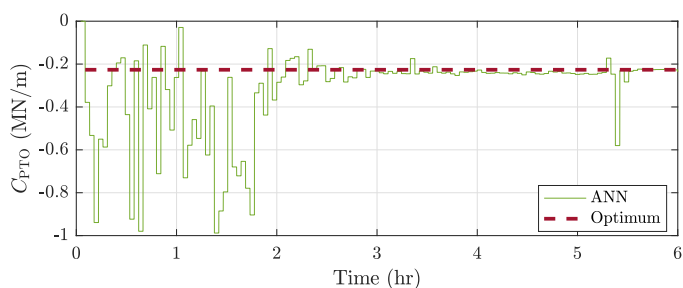
439 In regular waves, a 6-hour-long wave trace with unit amplitude and a
 440 wave period of 8 s has been analysed. As can be seen in Figure 7a and
 441 Figure 7b, the ANN-based algorithm learns successfully the optimal PTO
 442 damping and stiffness coefficients respectively. In particular, the optimal
 443 values (dotted lines) have been obtained with a Multistart optimization using
 444 a wave trace lasting 20 minutes and the analysed WEC model. Figure 7c
 445 shows the difference in the mean power generated with ANN-based control
 446 and state-of-the-art reactive control, where $P_{\text{avg,opt}} = 176.24$ kW. A value of
 447 $N_i = 40$ has been used.

448 4.3. Results in Irregular Waves

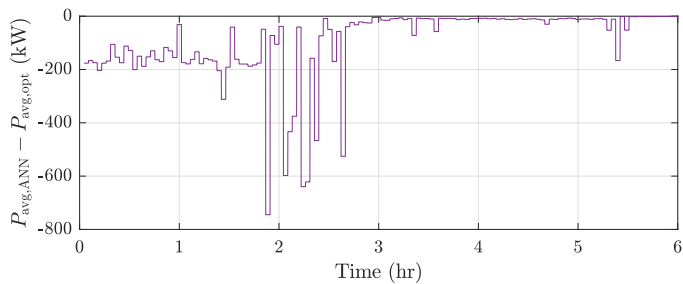
449 In irregular waves, even within a single sea state, the significant wave
 450 height and wave energy period do vary, if they are measured within a short
 451 time interval like $20T_e$. Since reinforcement learning in [14] and [15] em-
 452 ploys discrete states, it was possible to show the convergence behaviour of
 453 the algorithm in one sea state only. Conversely, the accuracy of ANNs is
 454 greatly improved and the effects of overfitting greatly reduced the wider the
 455 range of their samples [27] and thus the wider the range of sea conditions.
 456 For this reason, the proposed ANN-based reactive control algorithm is run
 457 for the 9 wave traces shown in Table 1. Each wave time series is generated



(a)



(b)



(c)

Figure 7: PTO damping (a) and stiffness (b) coefficients obtained from the ANN-based control as compared with the optimal value in regular waves with $H_s = 2$ m and $T_e = 8$ s. (c) shows the difference in the corresponding mean generated power.

458 with a Bretschneider spectrum (thus, broad-banded) [26] and lasts 3 hours.
 459 Although these wave traces have been simulated independently due to com-
 460 putational constraints, they should be treated as a continuous, time series
 461 where 9 independent sea states are observed in the order provide in Table 1,
 462 with a value of $N_i = 120$ being used. In particular, for each wave trace the list
 463 of samples is initialized with the values observed in the previous runs. The

464 series of a sea states is repeated another time but with a different seed num-
 465 ber to the random number generator for a total wave trace with an overall
 466 duration of 54 hours (excluding the 15 minutes required for the initialization
 467 of each wave trace).

Table 1: Significant wave height, energy wave period and duration of the wave traces used for the analysis of the ANN-based control in irregular waves.

| | | | | | | | | | |
|-----------------|---|---|---|----|---|----|----|---|---|
| [!h] H_s (m) | 2 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| T_e (s) | 8 | 8 | 9 | 10 | 9 | 10 | 10 | 9 | 8 |
| duration (hr) | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| no. repetitions | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

468 The learning behaviour of the proposed ANN-based reactive control algo-
 469 rithm in irregular waves is displayed in a compact way in Figure 8. The figure
 470 shows the controller performance for the first wave trace, i.e. $H_s = 2$ m and
 471 $T_e = 8$ s. In particular, the very first run (when the list of samples is empty at
 472 the start) is shown with dotted lines and labelled as "initial", since learning
 473 has just been initialized. The system is simulated in the same wave condi-
 474 tions again *after* the control has been applied for 54 hours in the wave traces
 475 shown in Table 1. The corresponding performance is shown with continu-
 476 ous lines in Figure 8 and labelled as "trained", since learning has completed
 477 by then with a large number of samples being available for the training of
 478 the ANN. Furthermore, in this case the exploration rate has almost fully
 479 decayed, as the discrete sea state has already been experienced for 6 hours.
 480 Additionally, the optimal value for the PTO coefficients and the correspond-
 481 ing absorbed energy is calculated running a MultiStart optimization of the
 482 WEC model in the same wave trace.

483 5. Discussion

484 5.1. Regular Waves

485 As shown in Figure 7, the ANN-based algorithm learns the optimal PTO
 486 damping and stiffness coefficients in regular waves within 4 hours after being
 487 randomly initialized. In the figures, it is possible to recognize three dis-
 488 tinct regions: an initial region where completely random actions are selected
 489 ($\mathbf{N}(s) \leq N_i$), a section where random actions are taken around the expected

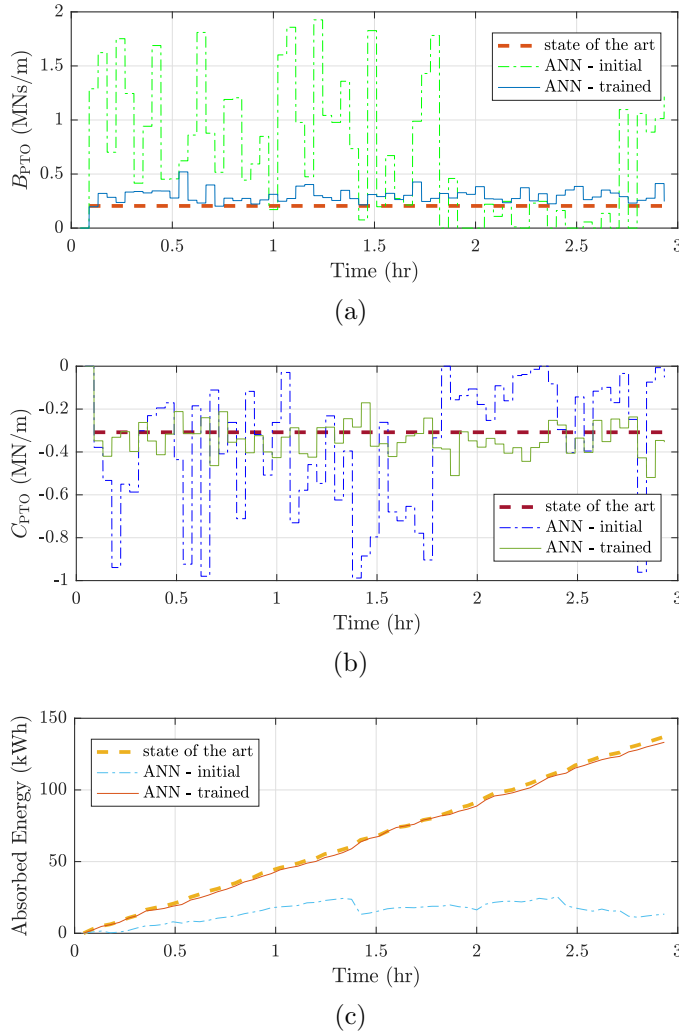


Figure 8: PTO damping (a) and stiffness (b) coefficients adopted by the ANN-based control at the start and after 54-hours of training in the wave conditions shown in Table 1 in irregular waves with $H_s = 2$ m and $T_e = 8$ s. Additionally, (a) and (b) display the results of state-of-the-art reactive control. The corresponding curves for the absorbed energy are plotted in (c).

490 optimum within a shrinking range (until $0.9^{N^{(s)}-N_i} \rightarrow 0$), and a final part
 491 where convergence has been reached. Within this last region, it is interesting
 492 to notice three random points (after approximately 5.5 hours). These are
 493 caused by the Multistart algorithm converging towards the wrong local opti-
 494 mum in the corresponding time horizons. This is a possibility that needs to

495 be taken into account when designing the control for an actual device, with
496 its probability decreasing with the number of starting points. Nevertheless,
497 the low computational cost means this optimization method is still preferred
498 over global search or genetic algorithms. Oddly, the three random points also
499 provide the ANNs with the missing training points for perfect convergence
500 to the optimal PTO coefficients.

501 It should be noted that the ANN-based algorithm presents faster learning
502 than reinforcement learning, which requires approximately 6 hours in regular
503 waves with resistive control and 8 hours with reactive control in [14] and [15],
504 respectively.

505 5.2. Irregular Waves

506 The convergence of the algorithm to the optimal PTO coefficients in ir-
507 regular waves is shown by the "trained" lines in Figure 8. Oscillations in
508 the values obtained with the ANN-based control are due to changes in wave
509 conditions over the smaller time scale of $20T_e$. The energy absorption is al-
510 most identical to state-of-the-art reactive control applied using the optimal
511 coefficients for the WEC model in this wave trace.

512 In this case, the comparison in learning performance between reinforce-
513 ment learning and ANNs is harder to understand. At first sight, 54 hours
514 may seem like a very long learning time. However, this corresponds to 6
515 hours of learning time per discrete sea state, which is less than the 10 hours
516 required by reinforcement learning in a single sea state of irregular waves for
517 resistive control in [14]. Once a sufficient number of points is obtained, the
518 ANN can generalise the information to unseen sea states, thus further reduc-
519 ing the learning time as compared with reinforcement learning with discrete
520 states. In addition, the convergence time should be assessed in the context
521 of the lifetime of a WEC, which is expected to be 20 to 25 years long [25].

522 In this work, discrete sea states have been analysed, each lasting 3 hours
523 due to practical issues with the code implementation. In reality, the energy
524 content in waves changes uniformly in time (hence, not through discrete sea
525 states), with the duration of a typical sea state being 0.5 to 6 hours [26].
526 Since P_{avg} and $\max|z|$ can be considered to be purely dependent on the
527 values of B_{PTO} , C_{PTO} , H_s and T_e in the *current* time interval, the samples
528 of the ANN are independent of past data. Therefore, the algorithm can be
529 safely applied to realistic, continuously varying wave conditions. In fact,
530 the quality of the mapping provided by the ANN is expected to improve in
531 continuously varying sea states, which result in a broader range of samples

532 [27]. Furthermore, under realistic wave conditions, the ANN-based reactive
533 control is expected to result in higher energy absorption than state-of-the-
534 art reactive control, since the latter uses a look-up table with discrete sea
535 states, thus being less responsive to changes in wave energy over a shorter
536 time scale. Additionally, the ANN-based method can adapt to changes in
537 the device dynamics with time, e.g. due to marine growth.

538 5.3. Practical Considerations

539 Although ANNs are a supervised learning strategy, they are employed
540 here in an approach reminiscent of reinforcement learning, which is unsu-
541 pervised, and that entails exploration. This may result in damage to and
542 even failure of the device if explorative negative actions are selected at the
543 wrong time, e.g. a high PTO stiffness coefficient with low damping in highly
544 energetic waves. Strategies that rely on explorations suffer from this prob-
545 lem, but the ANN-based control is more affected than reinforcement learning
546 because:

- 547 • reinforcement learning [15] makes a step change in the PTO coefficients
548 at the start of each interval. Hence, it is difficult to encounter highly
549 negative situations, since the algorithm corrects the PTO coefficients
550 as soon as it starts receiving negative feedback on the actions it has
551 selected in that particular sea state. Conversely, the proposed ANN-
552 based method is able to explore the whole search space during the first
553 observations of a particular discrete sea state.
- 554 • the quality of the mapping produced by the ANN is improved for a
555 wider range of samples. Hence, in order to improve the training process,
556 the algorithm is incentivized to explore most combinations of the PTO
557 coefficients in each sea state.

558 In order to prevent failure or damage to the WEC, two practical possibilities
559 should be investigated:

- 560 • initializing the ANN with samples pre-generated using accurate, non-
561 linear models of the WEC. In particular, simulations should be run
562 using extreme values of the PTO coefficients so that once the algorithm
563 is applied on the real-device, the Multi-Start optimization should home
564 in onto the optimal conditions rather than risk selecting extreme control
565 settings.

566 • an alternative approach consists in initially applying state-of-the-art
567 reactive control with the look-up table approach and slowly changing
568 the PTO coefficients in each sea state. The collected data will be then
569 employed for the training of the ANN, and then the proposed algorithm
570 will be applied. This process is designed to remove the exploration stage
571 from the presented scheme, focusing only on the supervised nature of
572 ANN algorithms.

573 At the moment, exact knowledge of the values of H_s and T_e during the
574 following time horizon is assumed. In practice, errors will be associated with
575 the estimation method [31, 32, 33]. Nevertheless, including information on
576 the expected future wave excitation is a fundamental tool for the control
577 of WECs in order to try to achieve optimal performance [5]. This is a fur-
578 ther improvement over the reinforcement learning algorithm proposed in [15],
579 since the study assumed the wave height and period to be identical between
580 neighbouring horizons.

581 As compared with reinforcement learning, the selection of the control ac-
582 tion with the proposed method requires greater computational power. Never-
583 theless, an on-line implementation is completely feasible with modern hard-
584 ware and parallel computing. As described in Section 3.3, the control strategy
585 consists in two main stages. On the one hand, the weight of the ANNs are up-
586 dated every 20 algorithm steps, using all training points (at most, say, 10^6).
587 This process occurs off-line, with the older weights not being overwritten on
588 the memory until the new ones are ready, so that computing time is not an
589 issue. On the other hand, at every time step, a new training point is collected
590 (minimal computation effort), and a new action is selected through the Mul-
591 tistart optimization. This process is speeded up through parallel processing,
592 and possibly an implementation in a low-order computational language, e.g.
593 C. As described in Section 3.2, one optimization using the current simulation
594 in Matlab and a quad-core i7 computer with 16 GB RAM takes less than 9 s.
595 This period is less than 10% of the minimal expected time horizon duration,
596 namely 100 s for a 5-s wave energy period, which is the smallest encountered
597 in typical sea states [26]. Hence, computation efficiency is not critical in this
598 case, since the rate of change of the plant is much slower than that of the
599 control algorithm.

600 Real-time strategies are more efficient than time-averaged methods for the
601 control of WECs [5]. Hence, although machine learning schemes are interest-
602 ing due to their model-free approach of the WEC control problem, they will

603 need to be applied in real-time in order to compete with model-predictive
604 control. The application of real-time system identification with ANNs to
605 real-time strategies, such as model predictive control, will be investigated in
606 the future.

607 **6. Conclusion**

608 In this article, an on-line, model-free strategy has been developed for
609 the reactive control of WECs using ANNs. The aim is to maximise energy
610 absorption, whilst limiting the PTO displacement to prevent failure in ener-
611 getic sea conditions. A simple model of a point absorber has been employed
612 to analyse the behaviour of the algorithm. Firstly, regular waves show that
613 the strategy learns rapidly the optimal PTO damping and stiffness coeffi-
614 cients because of their periodicity. A longer convergence time is necessary in
615 irregular waves, since the ANNs require a greater number of training sam-
616 ples in order to learn the mapping between the mean absorbed power and
617 PTO displacement, and the significant wave height, wave energy period, and
618 the PTO damping and stiffness coefficients. Nevertheless, this ensures the
619 scheme can recognize variations in the wave conditions on a shorter time
620 scale than state-of-the-art reactive control, which uses discrete sea states.
621 Furthermore, implementation on a full-scale WEC is simple, as the tech-
622 nique is independent of models of the machine dynamics. More importantly,
623 this method is able to treat changes in the device response as the structure
624 is affected by marine biofouling.

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