Three Essays on Loss Aversion and Reference-Dependent Preferences

Submitted by Mingjuan Gao to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Economics in April 2017.

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Signature: ............ Mingjuan Gao. ..................................................
Abstract

This thesis studies loss aversion and reference-dependent preferences. The second chapter and the fourth chapter analyze the price strategy for the monopolist with a loss-averse consumer following the reference-dependent model of Kőszegi and Rabin (2006). The second chapter takes into account the happiness of not paying at the highest price and the disappointment of not paying at the lowest price and finds that this happiness has a positive effect on the monopolist’s revenue and this disappointment has a negative effect on the monopolist’s revenue. The fourth chapter proposes a two-period pricing model and shows that the monopolist could make use of two-price strategy to earn a revenue that is greater than the product value. The revenue of the two-period model is higher than one-period model when the weight of gain-loss utility is big enough. The third chapter studies the winner’s regret with bidders when they have reference-dependent preferences in the sealed-bid first-price auction, second-price auction and all-pay auction and shows that the optimal bid is smaller with regret than without regret for loss-averse bidders, is greater for gain-seeking bidders and is the same for risk-neutral bidders.
Acknowledgments

First of all I would like to express my sincere gratitude to my first supervisor, Todd R. Kaplan, for his continuous support throughout my PhD study. Without his patience, encouragement and immense knowledge, this thesis would have never been accomplished. I would also like to thank my second supervisor, Robin Mason, for all the valuable guidance and insightful comments he has provided in the last four years.

I wish to extend my sincere thanks to all the faculty members of the Business School at the University of Exeter for all the help and support I received. My thanks also goes to ESRC for providing me with the financial support during my doctoral study.

Finally, my special thanks to my parents, Fuqing and Peiyu, for their constant encouragement and love throughout my whole life. Without their moral support I would never have succeeded in completing my PhD.
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The First Chapter

1 Introduction

This thesis consists of three chapters that study loss aversion and reference-dependent preferences in auction and how monopolist respond to loss-averse consumers.

The second chapter studies the behavior of the loss-averse consumer and the price strategy of the profit-maximizing monopolist in the market. This chapter builds upon the reference-dependent model and takes into account the happiness of not paying at the highest price and the disappointment of not paying at the lowest price. Our results show that the happiness of not paying the highest price has a positive effect on the monopolist’s revenue and a non-monotonic effect on the consumer’s welfare while the disappointment of not paying the lowest price has a negative effect on the monopolist’s profit and a non-monotonic effect on the consumer’s welfare.

The third chapter models the winner’s regret with bidders when they have reference-dependent preferences in the sealed-bid first-price, second-price and all-pay auction. They have an endogenous expectation about the highest bid of other bidders and have a sense of loss or gain when the actual highest bid of other bidders is different with their expectation. Considering these gains and losses as regret, we find that the optimal bid is smaller than without regret for loss-averse bidders, greater for gain seeking bidders and the same for gain-loss neutral bidders.

The fourth chapter studies the price strategy for the monopolist with a loss-averse consumer in a two-period model and finds that the monopolist could make use of
two-price strategy to earn a revenue higher than the product value which hurts the consumer making his utility negative. The revenue of two-period model is higher than one-period model when the weight of gain-loss utility is big enough. And the time factor has a non-monotonic effect on the monopolist’s revenue and the consumer’s welfare in the two-period model.
The Second Chapter

2 Price Strategy with Loss-Averse Consumers

2.1 Introduction

Firms often offer deals to attract consumers. This means that two different consumers can buy the same product at different prices. Take for example, on a flight, a hypothetical passenger asks another passenger in the adjacent seat about his ticket price and gets the answer of £150. The answer may affect his utility. He will be very happy if his own ticket was purchased at £100, however, he will be disappointed if he bought it at £200. It’s natural to ask about the effect of such happiness and disappointment. If we incorporate such utility and disutility into the consumer’s preference, what is the optimal price strategy for the monopolist?

There are several papers focusing on the optimal price strategy of monopolist facing consumers with behavioral biases. Loss aversion is an important explanation of sales and regular price strategy. The theory, first introduced in 1979 by Kahneman and Tversky, refers to the tendency for individuals to prefer avoiding to losses over acquiring gains. Kőszegi and Rabin (2006) develop a reference-dependent model in which the loss-averse consumer use their rational expectations about the purchase as the reference point. This chapter builds upon the expectation-based reference-dependent preferences by adding the happiness of not paying the highest price and the disappointment of not paying the lowest price to consumer’s utility function.
This chapter develops a model that takes into account the happiness when not paying at the highest price and the disappointment when not paying at the lowest price and tries to find the optimal price strategy for the monopolist. Our results show that the happiness of not paying the highest price has a positive effect on the monopolist’s profit and an uncertain effect on the consumer’s welfare while the disappointment of not paying the lowest price has a negative effect on the monopolist’s profit and an uncertain effect on the consumer’s welfare.

This chapter proceeds as follows. Section 2 is the literature review. Section 3 describes the timeline and the expectation-based reference-dependent model. Section 4 analyzes consumer’s purchase plans and conditions for these plans to be the personal equilibrium. Section 5 presents the optimal price strategy and the effect of the happiness of not paying the highest price and the disappointment of not paying the lowest price. Section 6 illustrates an example to show the optimal price strategy and the effect of the happiness and disappointment in detail. Finally, I conclude in Section 7.

2.2 Literature review

In this section I review several studies about loss aversion and reference-dependent preferences.

Bernoulli (1738) proposes the expected utility hypothesis on decision making under risk which points out that people’s choices are based on their expected utility, but not on the expected value of outcomes. Based on this theory, von Neumann and Morgenstern (1944) introduces the von Neumann-Morgenstern utility function in their book “Theory of Games and Economic Behavior” which is used as the basis of rational decision making. Expected utility theory states that individual’s preferences are a function not only of the expected value of outcomes, but also of the attitude to risk on the final states of wealth. According to diminishing marginal utility, a
first £1000 is more worth to a person than the second £1000 when he is already a millionaire. This theory can help to explain risk aversion, but still assumes that gains and losses of the same size should have same utility for people.

Markowitz (1952) introduces a modified utility function to expected utility theory which distinguish between gains and losses. Later, Kahaneman and Tversky (1979) propose a full-fledged theory, Prospect Theory, on individual's decision making under risk. It is a powerful alternative theory to expected utility theory based on theoretical and experimental evidence which points out that value function is generally steeper for a loss than a similar gain. The central idea in the theory are reference-dependence preferences and loss aversion, that people are more sensitive to losses than to the same magnitude gains. Over decades, loss aversion has played an important role in a wide variety of economic domains. Benartzi and Thaler (1995) use loss aversion to explain the return gap between stocks and bonds which is known as equity premium puzzle. Camerer et al. (1997) presents the tendency of New York taxi drivers to work longer hours on bad days to avoid the loss from their daily income target. Hardie et al. (1993) use loss aversion to explain the asymmetries in consumer behavior to changes in prices and purchase of consumer goods. Bowman, Minehart and Rabin (1999) explain workers behavior of the insensitivity of consumption to bad news about income by loss aversion and reference-dependent utility.

One difficulty in applying prospect theory is how to define the reference point to identify a gain or a loss which Kahneman and Tversky give less guidance. Kőszegi and Rabin (2006) develop a reference-dependent model which assumes people’s rational expectations about recent outcomes as the reference point to identify gains and losses. In this model, an individual derives gains and losses from differences between consumptions and expected consumptions and also derives utility from consumption levels. They apply the model to explain the consumer behavior and cabdrivers behavior. Crawford and Meng (2011) follow Kőszegi and Rabin’s model
to analyze the New York City cab drivers' working behavior with both time and income target and yield a useful model on labor supply.

2.3 Model

This section introduces a pricing model with a loss-averse consumer. There is a risk-neutral profit-maximizing monopolist that supplies a product at zero cost to a representative loss-averse consumer. The timeline of the model is as follows. In time 0, the firm commits to choosing a price from a price cumulative distribution \( \Pi \) for its good. The consumer learns the distribution and forms expectations about his purchase decision: a purchase plan. In time 1, a price \( p \) is drawn from the price distribution, the consumer observes it and decides whether or not to buy the item. We assume that a consumer that is indifferent between buying and not buying will buy the product.

The consumer has expectation based reference-dependent preferences and his utility consists three parts: (i) consumption utility, buying the product at price \( p \) gives the consumer an intrinsic value \( v \) and a disutility for paying the price \( p \). (ii) gain-loss utility, as proposed in Köszegi and Rabin (2006), the consumer also derives utility from the difference between his actual consumption and his purchase plan \( (\hat{p}, \hat{q}) \), where \( \hat{q} \) is the quantity the consumer expected to consume (equivalent to the probability of consuming) and \( \hat{p} \) is the average price he expected to pay when making a purchase. For instance, if the consumer expected to buy the product at £9 with one-third probability, buy at £6 with one-third probability, and not buy with one-third probability, then paying £8 is a gain of £1 relative to the one-third probability of paying £9, a loss of £2 relative to the one-third probability of paying £6 and a loss of £8 relative to the one-third probability of paying £0 (not buy). In this purchase plan \( \hat{p} \) equals 7.5 and \( \hat{q} \) equals two-thirds. (iii) the consumer experiences a loss (denoted by \( \gamma \)) if paying a price that is not the lowest price in the price
distribution when he expected to buy with positive probability ($\tilde{q} \neq 0$) and a gain (denoted by $\gamma_h$) if paying a price that is not the highest price when he is willing to buy at the highest price ($\tilde{q} = 1$). For the above consumption outcome with the expectations, the consumer’s utility can be expressed as:

$$u(v, \Pi, p, \tilde{p}, q, \tilde{q}) = (v - p)q + \eta \cdot \mu(q - \tilde{q})v - \eta(pq - \tilde{p}\tilde{q}),$$

$$-\gamma_l \cdot 1_{p > \min(\text{supp}(\Pi))}, q = 1 \& \tilde{q} \neq 0$$

$$+\gamma_h \cdot 1_{p < \max(\text{supp}(\Pi))}, q = 1 \& \tilde{q} = 1$$

where

$$\mu(x) = \begin{cases} 
  x & \text{if } x \geq 0 \\
  \lambda x & \text{if } x < 0 
\end{cases}.$$  

The quantity $q \in \{0, 1\}$ is the amount that the consumer buys. The parameter $\eta$ measures the weight attached to gain-loss utility and $\lambda > 1$ is the coefficient of loss aversion. Note that we assume that loss aversion is only with goods. \(^1\)

In the expression, the consumer evaluates gains and losses over goods and money separately. For instance, if the consumer expected to receive the product and pay for it, then he treats not receiving the good and paying nothing as a loss in the good dimension and a gain in the money dimension rather than a single gain or loss depending on the total consumption outcome relative to the reference point. The consumer evaluating these separately allows the monopolist to earn more than the intrinsic value of the product by manipulating the consumer’s reference points. This is consistent with experimental results of loss aversion, such as Kahneman, Knetsch and Thaler (1991).

To complete the reference-dependent preferences model, the consumer’s purchase plan should be time consistent, he makes the best plan based on the correct anticipations in time 0 and knows that he will follow the plan in time 1. For time

\(^1\)This is consistent with Kőszegi and Rabin (2009) and Novemsky and Kahneman (2005) who suggest that loss aversion is weaker in the money dimension than the good dimension.
consistent expectations, if a consumer does not want to follow a plan in time 1, then this plan cannot be formulated in time 0. Hence, a credible plan must be optimal given the expectations it generates.

**Theorem 1** Given the expectations \((\tilde{p}, \tilde{q})\), if it is optimal for the consumer to buy at a price \(p'\), then it is optimal for him to buy at price \(p < p'\) for all \(p \in \text{supp}(\Pi)\).

**Proof.** When the consumer buys at price \(p'\), the utility of buying is greater or equal to the utility of not buying, \(u(v, \Pi, p', \tilde{p}, 1, \tilde{q}) \geq u(v, \Pi, p', \tilde{p}, 0, \tilde{q})\). The utility of buying at price \(p\) is higher than buying at price \(p'\), \(u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) > u(v, \Pi, p', \tilde{p}, 1, \tilde{q})\). Paying less increases the consumption utility and creates a gain in the money dimension while the reference points do not change. If \(p'\) is the highest price, paying a lower price \(p\) instead creates an additional happiness of \(\gamma_h\) and if \(p\) is the lowest price, paying \(p\) avoids the disappointment of \(\gamma_l\), otherwise, the third part of the utility function does not change. The utility of not buying at price \(p\) is equal to the utility of not buying at price \(p'\), \(u(v, \Pi, p, \tilde{p}, 0, \tilde{q}) = u(v, \Pi, p', \tilde{p}, 0, \tilde{q})\): decreasing the realized price does not affect utility function when the consumer does not buy. Therefore, the utility of buying is greater than not buying, \(u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) > u(v, \Pi, p, \tilde{p}, 0, \tilde{q})\) by transitivity, and the consumer will buy at price \(p\). ■

**Theorem 2** Given the expectations \((\tilde{p}, \tilde{q})\), if it is optimal for the consumer not to buy at price \(p''\), then it is optimal for him not to buy at price \(p > p''\) for all \(p \in \text{supp}(\Pi)\).

**Proof.** If the consumer does not buy at price \(p''\), the utility of buying is smaller than the utility of not buying \(u(v, \Pi, p'', \tilde{p}, 1, \tilde{q}) < u(v, \Pi, p'', \tilde{p}, 0, \tilde{q})\). The utility of buying at price \(p\) is lower than buying at price \(p''\), \(u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) < u(v, \Pi, p'', \tilde{p}, 1, \tilde{q})\). Paying more decreases the consumption utility and creates a loss in the money dimension while the reference points do not change. If \(p''\) is the lowest price, paying a higher price \(p\) instead creates an additional disappointment of \(\gamma_l\) and if \(p\) is the highest price, paying \(p\) loses the happiness of \(\gamma_h\), otherwise, the third part of the utility
function does not change. The utility of not buying at price $p$ is equal to the utility of not buying at price $p''$, $u(v, \Pi, p, \tilde{p}, 0, \tilde{q}) = u(v, \Pi, p'', \tilde{p}, 0, \tilde{q})$: increasing the realized price does not affect utility function when the consumer does not buy. Therefore, the utility of buying is smaller than not buying, $u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) < u(v, \Pi, p, \tilde{p}, 0, \tilde{q})$ by transitivity, and the consumer will not buy at price $p$. ■

Hence, no matter what the consumer has expected in time 0, he will buy at prices up to and including some cutoff in time 1 (which could be 0). And any credible plan should have this cutoff structure. Suppose a credible plan up to buy at price $p^*$, then this plan has an expectation of receiving the consumption value $v$ and paying an average price $\tilde{p} = \frac{\int_0^{p^*} pd\Pi(p)}{\Pi(p^*)}$ with probability $\tilde{q} = \Pi(p^*)$. Following Kőszegi and Rabin (2006), we call such a credible plan a personal equilibrium (PE).

**Definition 2.1** The cutoff price $p^*$ is a personal equilibrium (PE) for price distribution $\Pi$ if for the induced expectations $(\tilde{p}, \tilde{q})$ we have

\[
    u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) \geq u(v, \Pi, p, \tilde{p}, 0, \tilde{q}) \text{ for any } p \leq p^* \text{ and } p \in \text{supp}(\Pi)
\]

\[
    u(v, \Pi, p, \tilde{p}, 1, \tilde{q}) < u(v, \Pi, p, \tilde{p}, 0, \tilde{q}) \text{ for any } p > p^* \text{ and } p \in \text{supp}(\Pi)
\]

In time 0, the consumer chooses the PE plan that maximizes his expected utility:

**Definition 2.2** The cutoff price $p^*$ with induced expectations $(\tilde{p}, \tilde{q})$ is a preferred personal equilibrium (PPE) for price distribution $\Pi$ if it is a PE, and for any PE cutoff price $p^*$ and its induced expectations $(\tilde{p}', \tilde{q}')$, we have

\[
    E[u(v, \Pi, p, \tilde{p}, q(p, p^*), \tilde{q})|\Pi, p^*] \geq E[u(v, \Pi, p, \tilde{p}', q(p, p^*'), \tilde{q}')|\Pi, p^*'].
\]
2.4 Consumer’s demand

Suppose the monopolist charges two prices, a low price $p$ with probability $q$ and a high price $p + \Delta$ with probability $1 - q$. Then, a consumer has three possible plans: plan N (never buy), $p^* < p$; plan L (buy only at low price), $p < p^* < p + \Delta$; and plan B (always buy: independent of the price being low or high), $p + \Delta \leq p^*$. The plan of only buying at a high price is ruled out by Theorems 1 & 2. In the following analysis, I will determine the conditions for when plans N, L and B are PEs and the expected utility for each plan.

**Conditions for plan N to be a PE.** Suppose the consumer expected never to buy the product, in this case, the reference point is to consume nothing and pay nothing. When the realized price is the low price $p$, if the consumer sticks to his plan and does not buy the item, then his realized utility is

$$u(v, \Pi, p, 0, 0, 0) = 0.$$  

The consumption utility is zero and the gain-loss utility is also zero since he acts as he expected. Instead, if the consumer deviates from his plan and buys the product, his utility is

$$u(v, \Pi, p, 0, 1, 0) = v - p + \eta v - \eta p,$$

where $v - p$ is the intrinsic consumption utility from buying the product and paying the price, $\eta v$ captures the gain he experiences from getting the product when he was expected to consume nothing, and $-\eta p$ is the loss he experiences from paying the price $p$ when he was expected to pay nothing, and no additional happiness $\gamma_h$ and disappointment $\gamma_l$ since he expected never to buy ($\tilde{q} = 0$). If plan N is an equilibrium, then

$$(v, \Pi, p, 0, 0, 0) > u(v, \Pi, p, 0, 1, 0) \implies p > v.$$
Similarly, when the realized price is the high price \( p + \Delta \), if the consumer follows his plan and does not buy the item, his utility is

\[ u(v, \Pi, p + \Delta, 0, 0, 0) = 0. \]

Instead, if the consumer deviates from his plan and buys the product, his utility is

\[ u(v, \Pi, p + \Delta, 0, 1, 0) = v - (p + \Delta) + \eta v - \eta(p + \Delta). \]

If plan N is an equilibrium, then

\[ (v, \Pi, p + \Delta, 0, 0, 0) > u(v, \Pi, p + \Delta, 0, 1, 0) \implies p + \Delta > v. \]

Hence, plan N (never buy) is a PE if and only if \( p > v \), otherwise the consumer will violate his plan and buy the product. Therefore, if the low price \( p \leq v \), plan N cannot be a PE.

The expected utility of this plan is

\[ q \cdot u(v, \Pi, p, 0, 0, 0) + (1 - q) \cdot u(v, \Pi, p + \Delta, 0, 0, 0) = 0. \]

**Lemma 1** (i) Plan N is a PE if and only if the low price \( p \) is strictly bigger than the product value. (ii) The expected utility of plan N is zero.

This lemma shows that even if the consumer expected not to buy the product, he will buy at a low price equal to or smaller than \( v \). In other words, setting a low price equal to or smaller than \( v \) will make the consumer buy with positive probability.

**Conditions for plan L to be a PE.** Suppose the consumer only buy at the low price, in the good dimension, the reference point is receiving the product with probability \( q \) and receiving nothing with probability \( 1 - q \); in the money dimension,
the reference point is paying price $p$ with probability $q$ and paying nothing with probability $1 - q$. When the realized price is $p$, following the plan, the consumer buys the product and his realized utility is

$$u(v, \Pi, p, p, 1, q) = v - p + \eta(1 - q)v - \eta(p - pq).$$

The intrinsic consumption utility $v - p$ derives from buying the product and paying the price, $\eta(1 - q)v$ is the gain he experiences from buying the product when he expected to receiving nothing with probability $1 - q$, $-\eta(p - pq)$ captures the utility derives from paying the realized price $p$ when he expected to pay price $p$ with probability $q$, and no additional happiness $\gamma_h$ and disappointment $\gamma_l$ since he only expected to buy at low price ($\tilde{q} \neq 1$).

Instead, if the consumer violates his exceptions and does not buy the product, the overall utility is

$$u(v, \Pi, p, p, 0, q) = 0 - \eta \lambda qv + \eta pq,$$

where the consumption utility is zero, $-\eta \lambda qv$ captures the loss he experiences when he expected to get the product with probability $q$ and $\eta pq$ is the gain he experiences when he expected to pay the low price $p$ with probability $q$.

If plan $L$ is an equilibrium, then

$$u(v, \Pi, p, p, 1, q) \geq u(v, \Pi, p, p, 0, q) \implies p \leq v + \frac{\eta q(\lambda - 1)v}{1 + \eta}.$$

Similarly, when the realized price is $p + \Delta$, if the consumer follows his plan and does not buy the item, his utility is

$$u(v, \Pi, p + \Delta, p, 0, q) = 0 - \eta \lambda qv + \eta pq.$$
Instead, if the consumer deviates from his plan and buys the product, his utility is
\[
u(v, \Pi, p + \Delta, p, 1, q) = v - (p + \Delta) + \eta(1 - q)v - \eta(p + \Delta - pq).
\]

If plan L is an equilibrium, then
\[
u(v, \Pi, p + \Delta, p, 0, q) > u(v, \Pi, p + \Delta, p, 1, q) \implies p + \Delta > v + \frac{\eta q(\lambda - 1)v}{1 + \eta}.
\]

Hence, plan L (buy only at low price) is a PE if and only if \(v - \Delta + \frac{\eta q(\lambda - 1)v}{1 + \eta} < p \leq v + \frac{\eta q(\lambda - 1)v}{1 + \eta}\), otherwise the consumer will violate his plan. Therefore, if the low price \(p > v + \frac{\eta q(\lambda - 1)v}{1 + \eta}\) or the high price \(p + \Delta \leq v + \frac{\eta q(\lambda - 1)v}{1 + \eta}\), plan L cannot be a PE.

The expected utility of this plan is
\[
q \cdot u(v, \Pi, p, p, 1, q) + (1 - q) \cdot u(v, \Pi, p + \Delta, p, 0, q) = q(v - p) - \eta q(1 - q)(\lambda - 1)v.
\]

**Lemma 2** (i) Plan L is a PE if and only if the low price \(p\) is smaller or equal to \(v + \frac{\eta q(\lambda - 1)v}{1 + \eta}\) and the high price \(p + \Delta\) is strictly bigger than \(v + \frac{\eta q(\lambda - 1)v}{1 + \eta}\). (ii) The expected utility of plan L is \(q(v - p) - \eta q(1 - q)(\lambda - 1)v\).

This lemma shows that when the consumer expected to buy with positive probability, he will be willing to accept a low price that is even strictly bigger than the product value. And the higher the probability of purchase is, the higher the acceptable low price is for the consumer. On the other hand, to make the consumer follow his plan and not buy at the high price, the high price should be strictly bigger than an amount. If not, the consumer will also buy at the high price and the probability of purchase will equal to one.

**Conditions for plan B to be a PE** Suppose the consumer expected to buy the item no matter the price is low or high, in this case, the reference point in the good dimension is enjoying the product value \(v\) with probability one while in the money
dimension, the reference point is paying low price $p$ with probability $q$ and paying high price $p + \Delta$ with probability $1 - q$. When the realized price is $p$, if the consumer sticks to the plan and buys the product, his overall utility is

$$u(v, \Pi, p, p + (1 - q)\Delta, 1, 1) = v - p - \eta[p - pq - (1 - q)(p + \Delta)] + \gamma_h.$$  

The intrinsic consumption utility $v - p$ derives from buying the product and paying the price, $-\eta[p - pq - (1 - q)(p + \Delta)]$ captures the utility from paying the realized price $p$ when he expected to pay price $p$ with probability $q$ and price $p + \Delta$ with probability $1 - q$, and $\gamma_h$ captures the happiness that he does not pay at high price. Instead, if the consumer deviates from the plan and does not buy the product, his realized utility is

$$u(v, \Pi, p, p + (1 - q)\Delta, 0, 1) = 0 - \eta \lambda v - \eta[0 - pq - (1 - q)(p + \Delta)],$$

where zero is the consumption utility since he consumes nothing and pays nothing, $-\eta \lambda v$ is the loss he experiences from consuming nothing when he expected to consume the product, $-\eta[0 - pq - (1 - q)(p + \Delta)]$ is the gain he experiences from paying nothing when he expected to pay price $p$ with probability $q$ and price $p + \Delta$ with probability $1 - q$.

If plan B is an equilibrium, then

$$u(v, \Pi, p, p + (1 - q)\Delta, 1, 1) \geq u(v, \Pi, p, p + (1 - q)\Delta, 0, 1) \implies p \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta} + \frac{\gamma_h}{1 + \eta}.$$  

Similarly, when price is $p + \Delta$, if the consumer follows his plan and buys the item, his utility is

$$u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 1, 1) = v - (p + \Delta) - \eta[p + \Delta - pq - (1 - q)(p + \Delta)] - \eta.$$
where $-\gamma_l$ captures the disappointment that he does not pay at the low price.

Instead, if the consumer deviates from his plan and does not buy the product, his utility is

$$u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 0, 1) = 0 - \eta \lambda v - \eta[0 - pq - (1 - q)(p + \Delta)].$$

If plan B is an equilibrium, then

$$u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 1, 1) \geq u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 0, 1) \implies p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \frac{\gamma_l}{1 + \eta}.$$

Hence, plan B (always buy) is a PE if and only if $p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \frac{\gamma_l}{1 + \eta}$, otherwise the consumer will violate his plan. The consumer prefers to plan B (always buy) as it is essentially insuring the consumer against extreme fluctuations in the consumption outcome.

The expected utility of this plan is

$$q \cdot u(v, \Pi, p, p + (1 - q)\Delta, 1, 1) + (1 - q) \cdot u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 1, 1) = v - p - \Delta - (1 - q)\gamma_l + q\gamma_h.$$

**Lemma 3** (i) Plan B is a PE if and only if the high price is less or equal to $v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \frac{\gamma_l}{1 + \eta}$. (ii) The expected utility of Plan B is $v - p - \Delta - (1 - q)\gamma_l + q\gamma_h$.

To avoid the disappointment of not getting the product, the consumer will accept a high price under an amount strictly bigger than $v$. Here, the disappointment of not paying the low price $\gamma_l$ has a negative effect on this amount as it appears when the realized price is the high price. However, the happiness $\gamma_h$ has no effect on the condition for plan B to be a PE since it is related to the low price. Both $\gamma_l$ and $\gamma_h$ affect the consumer’s utility, hence have effect on whether plan B could be the PPE.
2.5 Optimal pricing

The monopolist is risk-neutral and tries to maximize its expected profit given the consumer’s behavior. The consumer has three purchase plans, hence the PPE could be plan N, L or B.

2.5.1 Plan B is the PPE

If plan B is the PPE, the monopolist maximizes the revenue $R_B = p + (1 - q)\Delta$. To make plan B be a PPE, there are four cases as follow: Case 1, plan N, L are not PEs, only plan B is a PE; Case 2, plan N is not a PE, plan L, B are PEs and $u_L \leq u_B$; Case 3, plan L is not a PE, plan N, B are PEs and $u_N \leq u_B$; Case 4, plan N, L and B are all PEs, and $u_N \leq u_B, u_L \leq u_B$.

**Case 1** The monopolist maximizes the revenue subject to the constraints that plan N and plan L are not PEs, only plan B is PE. Solving the maximization problem,

$$
\begin{align*}
\text{Max}_{p,q,\Delta} \quad R_B &= p + (1 - q)\Delta \\
\text{s.t.} \quad p &\leq v \\
&\quad p + \Delta \leq v + \frac{\eta(q(\lambda - 1)v}{1 + \eta} \quad \text{or} \quad v + \frac{\eta(q(\lambda - 1)v}{1 + \eta} < p \\
&\quad p + \Delta \leq v + \frac{\eta(q(\lambda - 1)v}{1 + \eta} - \frac{\gamma}{1 + \eta} \\
\end{align*}
$$

We have two regions of the solution.

When $0 \leq \gamma \leq \frac{\eta(\lambda - 1)v}{2}$,

$$
\begin{align*}
p &= v \\
q &= \frac{1}{2} \\
\Delta &= \frac{\eta(\lambda - 1)v}{2(1 + \eta)} \\
R_1 &= v + \frac{\eta(\lambda - 1)v}{4(1 + \eta)}
\end{align*}
$$
And when $\frac{\eta(\lambda-1)v}{2} \leq \gamma_l < \eta(\lambda - 1)v,$

\[ p = v \]
\[ q = 1 - \frac{\gamma_l}{\eta (\lambda - 1)v} \]
\[ \Delta = \frac{\eta(\lambda-1)v - \gamma_l}{1+\eta} \]
\[ R_2 = v + \frac{\gamma_l[\eta(\lambda-1)v - \gamma_l]}{(1+\eta)\eta(\lambda-1)v} \]

**Case 2** The monopolist maximizes the revenue subject to the constraints that plan N is not a PE, plan L, B are PEs and $u_L \leq u_B$. Solving the maximization problem,

\[
\begin{align*}
\text{Max} \quad & R_B = p + (1 - q)\Delta \\
\text{s.t.} \quad & p \leq v \\
& v - \Delta + \frac{\eta(\lambda-1)v}{1+\eta} < p \leq v + \frac{\eta(\lambda-1)v}{1+\eta} \\
& p + \Delta \leq v + \frac{\eta(\lambda-1)v}{1+\eta} - \frac{\gamma_l}{1+\eta} \\
& p + \Delta \leq v + \eta q (\lambda - 1)v + \frac{q}{1-q} \gamma_h - \gamma_l
\end{align*}
\]

We have two regions of the solution.

Under the conditions

\[
\frac{(1 - \eta)[\eta(\lambda - 1)v - \gamma_l]}{2(1+\eta)} + \frac{\gamma_h + \gamma_l}{2(1+\eta)} - \gamma_l > 0,
\]

\[ p = v \]
\[ q = \frac{1}{2} + \frac{\gamma_h + \gamma_l}{2(1+\eta)} \]
\[ \Delta = \frac{1}{2} \eta(\lambda - 1)v + \frac{\gamma_h - \gamma_l}{2} + \frac{\eta(\lambda-1)v + \gamma_h + \gamma_l}{\eta(\lambda - 1)v - (\gamma_h + \gamma_l)} \gamma_h \\
\]
\[ R_3 = v + \left[ \frac{1}{2} - \frac{\gamma_h + \gamma_l}{2(\lambda - 1)v} \right] \left[ \frac{1}{2} \eta(\lambda - 1)v + \frac{\gamma_h - \gamma_l}{2} + \frac{\eta(\lambda-1)v + \gamma_h + \gamma_l}{\eta(\lambda - 1)v - (\gamma_h + \gamma_l)} \gamma_h \right] \]

And under condition $\gamma_l < \frac{\eta^2(\lambda - 1)v^2 - [\eta(\gamma_h + \gamma_l)\gamma_h] + \sqrt{D}}{2(1+\eta)}$, where $D = [\eta^2(\lambda - 1)v]^2 + 2(2 +
\( \eta \eta (\lambda - 1)v(1 + \eta)\gamma_h - 2\eta^3(\lambda - 1)v\gamma_l + [\eta\gamma_l + (1 + \eta)\gamma_h]^2, \)

\[
\begin{align*}
p &= v \\
q &= \frac{[2 + \eta \eta (\lambda - 1)v + \eta \eta (1 + \eta)\gamma_h] - \sqrt{D}}{2(1 + \eta)\eta (\lambda - 1)v} \\
\Delta &= \frac{\eta (\lambda - 1)v - \gamma_l}{1 + \eta} \\
R_4 &= v + \frac{\eta^2 (\lambda - 1)v - [\eta \gamma_l + (1 + \eta)\gamma_h] + \sqrt{D}}{2(1 + \eta)\eta (\lambda - 1)v} \\
R_5 &= \frac{\eta (\lambda - 1)v - \gamma_l}{1 + \eta}
\end{align*}
\]

**Case 3** The monopolist maximizes the revenue subject to the constraints that plan L is not a PE, plan N, B are PEs and \( u_N \leq u_B \). Solving the maximization problem,

\[
\begin{align*}
\max_{p,q,\Delta} R_B &= p + (1 - q)\Delta \\
s.t. & \quad v < p \\
& \quad p + \Delta \leq v + \frac{\eta (\lambda - 1)v}{1 + \eta} \quad \text{or} \quad p + \frac{\eta (\lambda - 1)v}{1 + \eta} < p \\
& \quad p + \Delta \leq v + \frac{\eta (\lambda - 1)v}{1 + \eta} - \frac{\gamma_l}{1 + \eta} \\
& \quad p \leq v - (1 - q)\Delta + q\gamma_h - (1 - q)\gamma_l
\end{align*}
\]

In this case, the optimal strategy does not exist. When \( q \) equals to 1 and \( \Delta \) equals to 0, we have the biggest revenue equals to \( \min(\gamma_h, \frac{10 - 2\gamma_l}{3}) \). However, the monopolist using a two-price strategy, \( q \) and \( \Delta \) cannot be zero. Here, I justify it with epsilon equilibrium\(^2\) which one does not want to change a price that would increase the revenue by epsilon or less.

We have two regions of the solution.

When \( \gamma_l \leq \eta (\lambda - 1)v - (1 + \eta)\gamma_h \),

\[
\begin{align*}
p^* &= v + \gamma_h \\
q^* &= 1 \\
\Delta^* &= 0 \\
R_5^* &= v + \gamma_h
\end{align*}
\]

\(^2\)See Baye and Morgan (2004)
And when \( \gamma_l \geq \eta(\lambda - 1)v - (1 + \eta)\gamma_h \),

\[
\begin{align*}
    p' &= v + \frac{\eta(\lambda - 1)v - \gamma_l}{1 + \eta} \\
    q' &= 1 \\
    \Delta' &= 0 \\
    R_6' &= v + \frac{\eta(\lambda - 1)v - \gamma_l}{1 + \eta}
\end{align*}
\]

**Case 4** The monopolist maximizes the revenue subject to the constraints that plan N, L and B are all PEs, and \( u_N \leq u_B \), \( u_L \leq u_B \). The maximization problem is

\[
\begin{align*}
    \max_{p,q,\Delta} \quad & R_B = p + (1 - q)\Delta \\
    \text{s.t.} \quad & v < p \quad (N \text{ is a PE}) \\
                      & v - \Delta + \frac{\eta(\lambda - 1)v}{1 + \eta} < p \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta} \quad (L \text{ is a PE}) \\
                      & p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \frac{\gamma_l}{1 + \eta} \quad (B \text{ is a PE}) \\
                      & p \leq v - (1 - q)\Delta + q\gamma_h - (1 - q)\gamma_l \quad (u_N \leq u_B) \\
                      & p + \Delta \leq v + \eta q(\lambda - 1)v + \frac{\gamma_l}{1 - q}\gamma_h - \gamma_l \quad (u_L \leq u_B)
\end{align*}
\]

We find that the revenue of case 4 is always smaller than case 3 from the following analysis. Since \( u_N = 0 \) and \( u_L = q(v - p) - \eta q(1 - q)(\lambda - 1)v \) which is always negative when \( v < p \), we have \( u_L < u_N \). Then we can eliminate the constraint \( u_L \leq u_B \) and then case 4 only has four constraints, N is a PE, L is a PE, B is a PE, and \( u_N \leq u_B \), since if the price strategy satisfies the constraint \( u_N \leq u_B \), it will also satisfies \( u_L \leq u_B \). In case 3, there are also four constraints, N is a PE, L is not PE, B is a PE, and \( u_N \leq u_B \), three of them are the same as constraints in case 4 and only the constraint about plan L is different. Under the three same constraints, the revenue achieves the highest value when \( q \) goes to 1. However, \( q \) approaching 1, satisfies the constraint L is not PE (in case 3) but not satisfies the constraint L is a PE (in case 4). Therefore, the revenue of case 4 is always smaller than case 3 and the firm will not set price strategies as case 4.
Summary  There are six possible price strategies when $\gamma_l$ and $\gamma_h$ have different values. The first two strategies make only plan B a PE, the middle two make plan L and B PEs, but $u_L \leq u_B$ and the last two make plan N and B PEs, but $u_N \leq u_B$. Plan B is PPE for all the six price strategies, and all of the six revenues are greater than the product value $v$.

2.5.2 Plan L is the PPE

If plan L is the PPE, the monopolist maximizes the revenue $R_L = pq$. To make plan L be a PPE, there are four cases as follow: Case 1, plan N, B are not PEs, only plan L is a PE; Case 2, plan N is not a PE, plan L, B are PEs and $u_B \leq u_L$; Case 3, plan B is not a PE, plan N, L are PEs and $u_N \leq u_L$; Case 4, plan N, L and B are all PEs, and $u_N \leq u_L$, $u_B \leq u_L$.

Both case 1 and 2 have constraint N is not a PE ($p \leq v$), then the revenue $R_L = pq$ is smaller than the product value $v$ since $q$ is smaller than 1. Remember that when plan B is a PPE, the monopolist can always earn a revenue bigger than $v$, the firm will not use the price strategies in these two cases. Both case 3 and 4 have constraints N is PE ($v < p$) and $u_N \leq u_L$ ($p \leq v - \eta(1 - q)(\lambda - 1)v$) which are contradictory. Hence, case 3 and case 4 cannot hold. Therefore, the monopolist will not choose a price strategy that make plan L a PPE.

2.5.3 Plan N is the PPE

If plan N is the PPE, the revenue is zero, remember that when plan B is PPE, the monopolist can always earn a revenue bigger than $v$, the firm will not use the price strategies to make plan N a PPE.
2.5.4 Summary

From the above analysis, there are six strategies may be optimal for the monopolist. Given the value of the parameters $\eta$ and $\lambda$, we could work out which is the optimal one (see 2.6 Example).

With different conditions about $\gamma_l$ and $\gamma_h$, the six expressions of the price strategies are as follows,

Under condition $0 \leq \gamma_l \leq \eta(\lambda-1)v$, Price strategy 1 is $p = v, q = \frac{1}{2}, \Delta = \eta(\lambda-1)v \frac{v}{2(1+\eta)}$, $R_1 = v + \eta(\lambda-1)v 4(1+\eta)$, $u_1 = -\eta(\lambda-1)v 4(1+\eta)$ + $\frac{\gamma_h}{2}$.

Under condition $\eta(\lambda-1)v \leq \gamma_l < \eta(\lambda-1)v$, Price strategy 2 is $p = v, q = 1$ - $\frac{2\gamma_h}{\eta(\lambda-1)v}, \Delta = \frac{\eta(\lambda-1)v - \gamma_l}{2(1+\eta)}, R_2 = v + \frac{\gamma_l}{\eta(\lambda-1)v} \frac{\gamma_l}{1+\eta} + \gamma_h], u_2 = -\frac{\gamma_l}{\eta(\lambda-1)v} \frac{\gamma_l}{1+\eta} + \gamma_h + \gamma_h$.

Under conditions $\gamma_l < \gamma_l < \eta(\lambda-1)v$, Price strategy 3 is $p = v, q = \frac{1}{2} + \frac{\gamma_h - \gamma_l}{2\eta(\lambda-1)v}, \Delta = \frac{\gamma_l}{2(1+\eta)} \frac{\gamma_l}{2(1+\eta)} + \frac{\gamma_h - \gamma_l}{\eta(\lambda-1)v} + \eta(\lambda-1)v \frac{v - \gamma_l}{2(1+\eta)} + \gamma_h], u_3 = -\frac{\gamma_l}{2(1+\eta)} \frac{\gamma_l}{2(1+\eta)} + \frac{\gamma_h - \gamma_l}{\eta(\lambda-1)v} + \eta(\lambda-1)v \frac{v - \gamma_l}{2(1+\eta)} + \gamma_h$.

Under condition $\gamma_l < \eta^2(\lambda-1)v - |(\gamma_l + (1+\eta)\gamma_h) + \sqrt{D}], \Delta = \eta(\lambda-1)v - \gamma_l, R_4 = v + \eta^2(\lambda-1)v - \gamma_l + \sqrt{D}] \frac{\eta(\lambda-1)v - \gamma_l}{2(1+\eta)\eta(\lambda-1)v}, R_4 = v + \eta^2(\lambda-1)v - \gamma_l + \sqrt{D}] \frac{\eta(\lambda-1)v - \gamma_l}{2(1+\eta)\eta(\lambda-1)v}, u_4 = -\frac{\gamma_l}{\eta^2(\lambda-1)v - \gamma_l + \sqrt{D}] \frac{\eta(\lambda-1)v - \gamma_l}{2(1+\eta)\eta(\lambda-1)v} + \gamma_h] + \gamma_h, where D = \eta^2(\lambda - 1)v^2 + 2(2 + \eta)(\lambda - 1)v(1 + \eta)\gamma_h - 2\eta^3(\lambda - 1)v\gamma_l + [\eta\gamma_l + (1 + \eta)\gamma_h]^2$.

Under condition $\gamma_l \leq \eta(\lambda-1)v - (1 + \eta)\gamma_h$, Price strategy 5 is $p^e = v + \gamma_h, q^e = 1, \Delta^e = 0, R_5^e = v + \gamma_h, u_5^e = 0$.

Under condition $\gamma_l \geq \eta(\lambda-1)v - (1 + \eta)\gamma_h$, Price strategy 6 is $p^e = v + \eta(\lambda-1)v - \gamma_l, q^e = 1, \Delta^e = 0, R_6^e = v + \eta(\lambda-1)v - \gamma_l, u_6^e = -\frac{\gamma_l}{\eta(1+\eta)} + \gamma_h$.

**Proposition 1** The monopolist’s revenue is decreasing in $\gamma_l$, the disutility of not paying the lowest price.
From the above expression, we have $\frac{\partial R}{\partial \gamma_l} < 0$, hence $\gamma_l$ has a negative effect on the revenue. The consumer faces uncertain prices before arriving at the store and a higher price associated as a loss which reduces the consumer’s willingness to pay and hence has a negative effect on the firm’s revenue.

**Proposition 2** The monopolist’s revenue is increasing in $\gamma_h$, the utility of not paying the highest price.

From the above expression, we have $\frac{\partial R}{\partial \gamma_h} > 0$, hence $\gamma_h$ has a positive effect on the revenue. The consumer faces uncertain prices before arriving at the store and a lower price associated as a gain which increases the consumer’s willingness to pay and hence has a positive effect on the firm’s revenue.

**Proposition 3** The consumer’s utility is non-monotonic in the disutility of not paying the lowest price $\gamma_l$ (but is monotonic in several regions when a particular price strategy is used).

From the above expression, we have $\frac{\partial u_1}{\partial \gamma_l} < 0$, $\frac{\partial u_2}{\partial \gamma_l} < 0$, $\frac{\partial u_5}{\partial \gamma_l} = 0$ and $\frac{\partial u_6}{\partial \gamma_l} > 0$. The consumer’s utility is decreasing in $\gamma_l$ when price strategy 1 or 2 is used; is increasing in $\gamma_l$ when price strategy 6 is used, and is keeping the same when price strategy 5 is used. For price strategy 3 and 4, the effect of $\gamma_l$ various depending on the value of $\gamma_l$ and $\gamma_h$.

**Proposition 4** The consumer’s utility is non-monotonic in the utility of not paying the highest price $\gamma_h$ (but is monotonic in several regions when a particular price strategy is used).

From the above expression, we have $\frac{\partial u_1}{\partial \gamma_h} > 0$, $\frac{\partial u_2}{\partial \gamma_h} > 0$, $\frac{\partial u_5}{\partial \gamma_h} = 0$ and $\frac{\partial u_6}{\partial \gamma_h} > 0$. The consumer’s utility is increasing in $\gamma_h$ when price strategy 1, 2 or 6 is used and is keeping the same when price strategy 5 is used. For price strategy 3 and 4, the effect of $\gamma_h$ various depending on the value of $\gamma_l$ and $\gamma_h$. 
2.6 Example

The following is an example to analyze the optimal price strategies and the effect of $\gamma_l$ and $\gamma_h$ in detail. Let $v = 10, \eta = 1/2, \lambda = 2$, then the monopolist sets the price strategy as follows.

\[
v = 10, \eta = \frac{1}{2}, \lambda = 2, \text{ then the monopolist sets the price strategy as follows.}
\]

\[
\begin{align*}
p &= 10, \\
q &= \frac{1}{2}, \\
\Delta &= \frac{5}{3}, \\
R_1 &= 10 + \frac{5}{6}, \\
u_1 &= -\frac{5}{6} + \frac{7\gamma - \gamma_l}{2}.
\end{align*}
\]

In region 2, the optimal price strategy is, $p = 10, q = 1 - \frac{\gamma_l}{5}, \Delta = \frac{10 - 2\gamma_l}{3}, R_2 = 10 + \frac{2\gamma_l(5 - \gamma_l)}{15}, u_2 = \frac{-10\gamma_l - \gamma_l^2 + 15\gamma_h - 3\gamma_l\gamma_h}{15}.$

In the above two regions, the happiness of not paying the high price $\gamma_h$ is small enough, the monopolist charges a low price equal to the product value and a suitable high price to make plan B the only PE.
In region 3, the optimal price strategy is, \( p = 10 \), \( q = \frac{1}{2} + \frac{\gamma_l + \gamma_h}{10}, \Delta = \frac{5}{2} + \frac{\gamma_l - \gamma_h}{2} + \frac{5 + (\gamma_l + \gamma_h)}{5 - (\gamma_l + \gamma_h)} \gamma_h \), \( R_3 = 10 + \frac{5}{2} + \frac{10(\gamma_h - \gamma_l) + (\gamma_l + \gamma_h)^2}{20}, u_3 = -(\frac{1}{2} - \frac{\gamma_l + \gamma_h}{10}) \cdot \left[ \frac{5}{2} + \frac{3 \gamma_h + \gamma_l}{2} + \frac{5 + (\gamma_l + \gamma_h)}{5 - (\gamma_l + \gamma_h)} \gamma_h \right] + \gamma_h \).

In region 4, the optimal price strategy is, \( p = 10 \), \( q = \frac{25 + 4 \gamma_l}{30} - \sqrt{25 + 150 \gamma_h - 10 \gamma_l + (3 \gamma_h + \gamma_l)^2}, \Delta = \frac{10 - 2 \gamma_l}{3}, R_4 = 10 + \frac{(5 - 3 \gamma_h - \gamma_l) + \sqrt{25 + 150 \gamma_h - 10 \gamma_l + (3 \gamma_h + \gamma_l)^2}}{30} \cdot \frac{10 - 2 \gamma_l}{3}, u_4 = -(\frac{10}{3} + \frac{\gamma_l}{3} + \gamma_h) \cdot \left[ \frac{5 - 3 \gamma_h - \gamma_l}{30} - \sqrt{25 + 150 \gamma_h - 10 \gamma_l + (3 \gamma_h + \gamma_l)^2} + \gamma_h \right] \).

In the above two regions, the happiness \( \gamma_h \) and the disappointment \( \gamma_l \) are both small enough, the monopolist charges a low price equal to the product value and a suitable high price to make plan L and B the only PEs.

In region 5, the optimal price strategy is, \( q^\epsilon = 1 \), \( \Delta^\epsilon = 0 \), \( p^\epsilon = 10 + \gamma_h, R_5^\epsilon = 10 + \gamma_h, u_5^\epsilon = 0 \).

In region 6, the optimal price strategy is, \( q^\epsilon = 1 \), \( \Delta^\epsilon = 0 \), \( p^\epsilon = 10 + \frac{10 - 2 \gamma_l}{3}, R_6^\epsilon = 10 + \frac{10 - 2 \gamma_l}{3}, u_6^\epsilon = -\frac{10 - 2 \gamma_l}{3} + \gamma_h \).

In the above two regions, the happiness of not paying the high price \( \gamma_h \) is big enough, the monopolist charges a low price strictly greater than the product value with a high probability approaching one to make plans N and B the only PEs.

2.6.1 The effect of \( \gamma_l \)

The following figures show the effect of \( \gamma_l \), the disappointment of not paying the lowest price on the monopolist’s revenue and the consumer’s welfare when \( \gamma_h \) keep constant (\( \gamma_h = 0.4 \)). We can see that the revenue is decreasing in \( \gamma_l \) for all six price strategies.

However, it’s effect on the consumer’s utility are different when different price strategies are used. We can see that the consumer’s utility is increasing when \( \gamma_l \) goes from 0 to 0.44 (price strategy 3 is used), from 0.44 to 1.59 (price strategy 4 is used) and
from 4.4 to 5 (price strategy 6 is used); the utility is decreasing when \( \gamma_l \) goes from 1.59 to 2.5 (price strategy 1 is used) and from 2.5 to 4.3 (price strategy 2 is used); and the utility does not change when \( \gamma_l \) goes from 4.3 to 4.4 (price strategy 5 is used).

![Figure 2.2. The effect of \( \gamma_l \) on the monopolist’s revenue and the consumer’s utility when \( \gamma_h = 0.4 \).](image)

2.6.2 The effect of \( \gamma_h \)

The following figures show the effect of \( \gamma_h \), the happiness of not paying the highest price on the monopolist’s revenue and the consumer’s utility when \( \gamma_l \) keep constant (\( \gamma_l = 1.2 \) or \( \gamma_l = 3 \)). We can see that the revenue is increasing in \( \gamma_h \) for all the six price strategies.

However, it’s effect on consumer’s utility are different when different price strategies are used. When \( \gamma_l = 1.2 \), we can see that the consumer’s utility is increasing when \( \gamma_h \) goes from 0 to 0.18 (price strategy 1 is used), from 0.18 to 0.27 (price strategy 3 is used), and from 2.53 to 5 (price strategy 6 is used); the utility is decreasing when \( \gamma_h \) goes from 0.27 to 1.28 (price strategy 4 is used); and the utility does not change when \( \gamma_h \) goes from 1.28 to 2.53 (price strategy 5 is used). When \( \gamma_l = 3 \), we can see that the consumer’s utility is increasing when \( \gamma_h \) goes from 0 to 0.8 (price
strategy 2 is used) and from 1.33 to 5 (price strategy 6 is used); and the utility does not change when $\gamma_h$ goes from 0.8 to 1.33 (price strategy 5 is used).

Figure 2.3. The effect of $\gamma_h$ on the monopolist’s revenue and the consumer’s utility when $\gamma_l = 1.2$

Figure 2.4. The effect of $\gamma_h$ on the monopolist’s revenue and the consumer’s utility when $\gamma_l = 3$

2.7 Conclusion

This chapter models the optimal price strategy for loss-averse consumers which takes into account the happiness, $\gamma_h$, not paying at the highest price and the disappointment, $\gamma_l$, not paying at the lowest price. Our results show that with different conditions about $\gamma_l$ and $\gamma_h$, there are six possible price strategies could be optimal.
and all of them make the revenue greater than the product value. The happiness \( \gamma_h \) has a positive effect on the monopolist’s revenue but has a non-monotonic effect on the consumer’s utility while the disappointment \( \gamma_l \) has a negative effect on the monopolist’s revenue but has a non-monotonic effect on the consumer’s utility.
References


The Third Chapter

3 Bid Strategy with Reference-Dependent Preferences

3.1 Introduction

Traditional models of auction assume that bidders’ utility for an outcome only depend on their own profit, that is, the difference between their value and bid. Later, Engebrecth-Wiffans (1989) introduces regret into the bidder’s utility function. Winner regret exists if the winner will regret when he could bid lower and still win the auction and the loser regret exists if the loser will regret when he could outbid the winner at a bid below his reservation value.

This chapter tries to model the bidder’s preferences including loss-averse and gain seeking bidders. In a sealed-bid auction, not only the winner’s own profit affects his utility, but also the highest bid of other competitors will affect the winner’s utility. He may have an endogenous expectation about the highest bid of other competitors, and uses to decide his own bid.

For example, you arrive at a tube station and expect the tube to arrive in 5 minutes. If the tube arrives in 1 minute, you will be happy while if the tube arrives in 9 minutes, you will be disappointed. The happiness and the disappointment will affect my utility and have different effects. Similarly, in auctions, the gains and losses derive from the difference between the expectation of the highest bid of other
competitors and the actual second highest bid will also affect the winner’s utility. In the sealed-bid first-price auction and all-pay auction, the winner will feel a sense of loss if the second highest bid is lower than expected and feel lucky (a sense of gain) if the second highest bid is higher than expected. For the sealed-bid second-price auction, the winner will feel a sense of gain if the second highest bid is lower than expected and feel a sense of loss if the second highest bid is higher than expected. Therefore, we form the winner’s utility by a linear combination of profit and the gain and loss.

Our results show that in the first-price auction, second-price auction and all-pay auction, for loss-averse bidders, the optimal bid for the winner is smaller with regret than without regret, while for gain seeking bidders, the optimal bid for the winner is greater with regret than without regret, and for gain-loss neutral bidders, the optimal bid is the same with regret as without regret.

The outline of this chapter is as follows. Section 2 is the literature review. Section 3 describes the bidding model, defines the regret and provides the results and illustrative examples. There are three subsections that model the first-price auction, second-price auction and all-pay auction separately and find that the optimal winning bid is smaller with regret than without regret for loss-averse bidders, greater for gain seeking bidders, and the same for gain-loss neutral bidders. Section 4 is the comparison of revenues for the three types of auctions, and finds that the revenue of first-price auction and all-pay auction equal to each other, but the revenue of second-price auction is not. Finally, I conclude in Section 5.

3.2 Literature review

In this section I review several studies about bidding behavior of regret and loss aversion.
Engelbrecht-Wiggans (1989) studies the effect of regret in bidding behavior. He considers the winner’s regret if he could bid lower and still win the bid and the loser’s regret if he could outbid the winner with a bid less than his own value. He finds that for risk-neutral bidders the optimal bid is the same with and without regret if they weigh two forms of regret equally. If the weight of winner’s regret is heavy, the equilibrium bid is lower than without regret and if the weight of loser’s regret is heavy, the optimal bid is greater than without regret. Filiz-Ozbay and Ozbay (2007) run a first-price auction experiment with three regret conditions, no regret, only loser’s regret and only winner’s regret and find that the optimal bid is lower with only winner’s regret and is higher with only loser’s regret. Engelbrecht-Wiggans and Katok (2008) also test the two types of regret in the sealed-bid first-price auction and present the similar result that sensitive to winner’s regret lower the bid and sensitive to loser’s regret yield to a higher bid.

Maskin and Riley (1984), Matthews (1987) and Fibich, Gavious, and Sela (2006) study implications of first-price auction, second-price auction and all-pay auction for risk averse bidders. Lange and Ratan (2010) study the first-price auction and the second-price auction with loss-averse bidders and find that the former has a higher expected revenue. This chapter studies the effect of regret for optimal bidding with loss-averse consumers in the first-price auction, second-price auction and all-pay auction.

3.3 Model and results

This section models independent private auctions in the presence of bidders who have reference-dependent preferences in the sealed-bid first-price auction, second-price auction and all-pay auction separately and then establishes the main results and shows with examples and figures.
3.3.1 First-price auction

There are \( n \) bidders with independent private values that submit sealed bids for a single object, the bidder with the highest bid wins the item and pays an amount equal to his bid. The winner’s utility consists three parts: (i) the expected profit, that is, the probability of winning times the difference between his private value \( v \) and his bid \( b \), (ii) the loss when the second highest bid is lower than what he expected, (iii) the gain when the second highest bid is higher than his expectation. Then, the winner’s utility can be expressed as,

\[
 u(v, b) = G(b) \cdot (v - b) - \alpha_L \int_0^\hat{b} (\hat{b} - B)dG(B) + \alpha_G \int_\hat{b}^b (B - \hat{b})dG(B),
\]

where \( B \) is the maximum bid of other bidders and \( \hat{b} \) is the expectation of the highest bid of other bidders, defined by \( \hat{b} = \frac{\int_0^b b dG(B)}{G(b)} \). \( G(\cdot) \) is the cumulative distribution function of the second highest bid, \( \alpha_L \) is the parameter for unhappiness when the actual bid is lower than the expectation and \( \alpha_G \) is the parameter for happiness when the actual bid is higher than the expectation. Simplifying the above expression, we have

\[
 G(b) \cdot (v - b) - \alpha_L \int_0^\hat{b} (\hat{b} - B)dG(B) + \alpha_G \int_\hat{b}^b (B - \hat{b})dG(B)
 = G(b) \cdot (v - b) - (\alpha_L - \alpha_G) \int_0^\hat{b} (\hat{b} - B)dG(B) - \alpha_G \int_0^\hat{b} (\hat{b} - B)dG(B)
 = G(b) \cdot (v - b) - (\alpha_L - \alpha_G) \left[ \int_0^\hat{b} b dG(B) - B G(B) \right] \int_0^\hat{b} G(B) dB
 = G(b)(v - b) - (\alpha_L - \alpha_G) \int_0^b G(B) dB
\]

Suppose \( b(v) \) is a symmetric equilibrium in increasing differentiable strategies. Then in the equilibrium, the winner’s payoff with value \( v \) is

\[
 \pi(v) = \max_b u(v, b) = u(v, b(v)).
\]
Applying the Envelope Theorem, we have

\[
\frac{d\pi(v)}{dv} = u_v + u_b\big|_{b=b(v)} \frac{db(v)}{dv} = G(b(v)) = F^{n-1}(v),
\]

where \( F \) is the cumulative function of private values and \( u_b\big|_{b=b(v)} \) is zero in the equilibrium. Also, we have

\[
\pi(v) = \pi(0) + \int_0^v F(\tilde{v})^{n-1}d\tilde{v}.
\]

Assume that the bidder with value zero will never win the auction \( \pi(0) = 0 \). Hence, we can solve for the equilibrium bid

\[
b^*(v) = v - \int_0^v F(\tilde{v})^{n-1}d\tilde{v} + (\alpha_L - \alpha_G) \int_0^v G(B)dB.
\]

Remember that without regret, the bidder’s utility function is just \( u(v, b) = G(b) \cdot (v - b) \), by the Envelope Theorem, we have \( \frac{d\pi(v)}{dv} = F^{n-1}(v) \) and \( \pi(v) = \int_0^v F(\tilde{v})^{n-1}d\tilde{v} \) which is the same as the case with regret. Therefore the equilibrium bid without regret is \( b(v) = v - \frac{\int_0^v F(\tilde{v})^{n-1}d\tilde{v}}{F(v)^{n-1}} \).

**Proposition 1** The equilibrium bid is smaller with regret than without regret for loss-averse bidders in a first-price auction.

For loss-averse bidders, \( \alpha_L > \alpha_G \), they are more sensitive to losses than to gains, then \( b^* < b \), the optimal bid is smaller than the bid without regret. For gain-loss neutral bidders, \( \alpha_L = \alpha_G \), they treat gains and losses equally, then \( b^* = b \), the optimal bid equals to the bid without regret. And for gain seeking bidders, \( \alpha_L < \alpha_G \), they are more sensitive to gains than to losses, then \( b^* > b \), the optimal bid is bigger than without regret.

When the private values are uniformly distributed over \([0,1]\), the optimal bid with regret is \( \frac{n-1}{n + (n-1)\alpha_L - \alpha_G} \cdot v \) while the optimal bid without regret is \( \frac{n-1}{n} \cdot v \), the former is smaller than the latter with loss-averse bidders, greater with gain seeking.
bidders and the same with gain-loss neutral bidders. When $n = 2$, with regret $b = \frac{v}{2 + \frac{1}{4}(\alpha_L - \alpha_G)}$ and without regret $b = \frac{v}{2}$, the following figure shows the relation between the optimal bid $b$ and value $v$ when $\alpha_L - \alpha_G = -0.5, 0, 0.5$.

Figure 3.1. The relation between optimal bid $b$ and value $v$

We can see that the green line for gain-loss neutral bidders ($\alpha_L - \alpha_G = 0$) is higher than the red line is for loss-averse bidders ($\alpha_L - \alpha_G = 0.5$), is lower than the blue line for gain seeking bidders ($\alpha_L - \alpha_G = -0.5$).

### 3.3.2 Second-price auction

There are $n$ bidders with independent private values that submit sealed bids for a single object, the bidder with the highest bid wins the item and pays an amount equal to the second highest bid. The winner’s utility consists three parts: (i) the expected profit, that is, the probability of winning times the difference between his private value $v$ and the expectation of the second highest bid $\tilde{b}$, (ii) the gain when the second highest bid is lower than what he expected, (iii) the loss when the second highest bid is higher than his expectation. Then, the winner’s utility can be
expressed as,

\[ u(v, b) = G(b)(v - \tilde{b}) + \alpha_G \int_0^{\tilde{b}} (\tilde{b} - B)dG(B) - \alpha_L \int_{\tilde{b}}^b (B - \tilde{b})dG(B), \]

where \( B \) is the maximum bid of other bidders and \( \tilde{b} \) is the expectation of the highest bid of other bidders, define by \( \tilde{b} = \int_0^b \frac{BdG(B)}{G(b)} \), \( G(\cdot) \) is the cumulative distribution function of the second highest bid, \( \alpha_G \) is the parameter for happiness when the actual bid is lower than the expectation and \( \alpha_L \) is the parameter for unhappiness when the actual bid is higher than the expectation. Simplifying the above expression, we have

\[
G(b)(v - \tilde{b}) + \alpha_G \int_0^{\tilde{b}} (\tilde{b} - B)dG(B) - \alpha_L \int_{\tilde{b}}^b (B - \tilde{b})dG(B) \\
= G(b) \cdot (v - \tilde{b}) + (\alpha_G - \alpha_L) \int_0^{\tilde{b}} BdG(B) - \alpha_L \int_0^{\tilde{b}} BdG(B) \\
= G(b) \cdot (v - \tilde{b}) + (\alpha_G - \alpha_L) \int_0^{\tilde{b}} \tilde{b}dG(B) - BG(B)|_0^{\tilde{b}} + \int_0^{\tilde{b}} G(B)dB \\
= G(b)(v - \tilde{b}) + (\alpha_G - \alpha_L) \int_0^{\tilde{b}} G(B)dB
\]

Suppose \( b(v) \) is a symmetric equilibrium in increasing differentiable strategies. Then in the equilibrium, the winner’s payoff with value \( v \) is

\[ \pi(v) = \max_b u(v, b) = u(v, b(v)). \]

Applying the Envelope Theorem, we have

\[
\frac{d\pi(v)}{dv} = u_v + u_b|_{b=b(v)} \frac{db(v)}{dv} = G(b(v)) = F^{n-1}(v),
\]

where \( F \) is the cumulative function of private values and \( u_b|_{b=b(v)} \) is zero in the equilibrium. Also, we have

\[ \pi(v) = \pi(0) + \int_0^v F(\tilde{v})^{n-1}d\tilde{v}. \]
Assume that the bidder with value zero will never win the auction \( \pi(0) = 0 \). Hence, we can solve for the equilibrium bid

\[
b'(v) = v - \frac{\int_v^\infty F(\tilde{v})^{n-1}d\tilde{v} - (\alpha_G - \alpha_L) \int_0^b G(B)dB - \int_0^b G(B)dB}{F(v)^{n-1}}.
\]

Remember that without regret, the bidder’s utility function is just \( u(v, b) = G(b) \cdot (v - \tilde{b}) \), by the Envelope Theorem, we have \( \frac{d\pi(v)}{dv} = F^{n-1}(v) \) and \( \pi(v) = \int_0^v F(\tilde{v})^{n-1}d\tilde{v} \) which is the same as the case with regret. Therefore the equilibrium bid without regret is \( b(v) = v - \frac{\int_v^\infty F(\tilde{v})^{n-1}d\tilde{v} - \int_0^b G(B)dB}{F(v)^{n-1}}. \)

**Proposition 2** The equilibrium bid is smaller with regret than without regret for loss-averse bidders in a second-price auction.

For loss-averse bidders, \( \alpha_L > \alpha_G \), they are more sensitive to losses than to gains, then \( b' < b \), the optimal bid is smaller than the bid without regret. For gain-loss neutral bidders, \( \alpha_L = \alpha_G \), they treat gains and losses equally, then \( b' = b \), the optimal bid equals to the bid without regret. And for gain seeking bidders, \( \alpha_L < \alpha_G \), they are more sensitive to gains than to losses, then \( b' > b \), the optimal bid is bigger than without regret.

When the private values are uniformly distributed over \([0,1]\), the optimal bid with regret is \( \frac{n-1}{(n-1)+\frac{n-1}{n}(\alpha_L-\alpha_G)} \cdot v \) while the optimal bid without regret is \( v \), the former is smaller than the latter with loss-averse bidders, greater with gain seeking bidders and the same with gain-loss neutral bidders. When \( n = 2 \), with regret \( b = \frac{v}{1+\frac{1}{4}(\alpha_L-\alpha_G)} \) and without regret \( b = v \), the following figure shows the relation between the optimal bid \( b \) and value \( v \) when \( \alpha_L - \alpha_G = -0.5, 0, 0.5 \).
We can see that the green line for gain-loss neutral bidders \((\alpha_L - \alpha_G = 0)\) is higher than the red line is for loss-averse bidders \((\alpha_L - \alpha_G = 0.5)\), is lower than the blue line for gain seeking bidders \((\alpha_L - \alpha_G = -0.5)\).

### 3.3.3 All-pay auction

There are \(n\) bidders with independent private values that submit sealed bids for a single object, all bidders forfeit their bids and the high bidder receives the item. The winner’s utility consists three parts: (i) the expected profit, that is, the probability of winning times his private value \(v\) minus his bid \(b\), (ii) the loss when the second highest bid is lower than what he expected, (iii) the gain when the second highest bid is higher than his expectation. Then, the winner’s utility can be expressed as,

\[
u(v, b) = vG(b) - b - \alpha_L \int_0^b (\tilde{b} - B)dG(B) + \alpha_G \int_b^b (B - \tilde{b})dG(B),
\]

where \(B\) is the maximum bid of other bidders and \(\tilde{b}\) is the expectation of the highest bid of other bidders, define by \(\tilde{b} = \frac{bB_{\tilde{b}}G(B)}{G(b)}\), \(G(\cdot)\) is the cumulative distribution
function of the second highest bid, $\alpha_L$ is the parameter for unhappiness when the actual bid is lower than the expectation and $\alpha_G$ is the parameter for happiness when the actual bid is higher than the expectation. Simplifying the above expression, we have

$$vG(b) - b - \alpha_L \int_0^b (\tilde{b} - B) dG(B) + \alpha_G \int_b^\tilde{b} (B - \tilde{b}) dG(B)$$

$$= vG(b) - b - (\alpha_L - \alpha_G) \int_0^b (\tilde{b} - B) dG(B) - \alpha_G \int_0^b (\tilde{b} - B) dG(B)$$

$$= vG(b) - b - (\alpha_L - \alpha_G) [\int_0^b b dG(B) - BG(B)]^\tilde{b} + \int_0^b G(B) dB$$

$$= \pi(v) = \max_b u(v, b) = u(v, b(v))$$

Applying the Envelope Theorem, we have

$$\frac{d\pi(v)}{dv} = u_v + u_b|_{b=b(v)} \frac{db(v)}{dv} = G(b(v)) = F^{n-1}(v),$$

where $F$ is the cumulative function of private values and $u_b|_{b=b(v)}$ is zero in the equilibrium. Also, we have

$$\pi(v) = \pi(0) + \int_0^v F(\tilde{v})^{n-1} d\tilde{v}.$$ 

Assume that the bidder with value zero will never win the auction $\pi(0) = 0$. Hence, we can solve for the equilibrium bid

$$b^*(v) = vF(v)^{n-1} - \int_0^v F(\tilde{v})^{n-1} d\tilde{v} - (\alpha_L - \alpha_G) \int_0^\tilde{b} G(B) dB.$$ 

Remember that without regret, the bidder’s utility function is just $u(v, b) = G(b) \cdot (v - b)$, by the Envelope Theorem, we have $\frac{d\pi(v)}{dv} = F^{n-1}(v)$ and $\pi(v) = \int_0^v F(\tilde{v})^{n-1} d\tilde{v}$.
which is the same as the case with regret. Therefore the equilibrium bid without regret is 
\[ b(v) = vF(v) - \int_0^v F(\tilde{v}) d\tilde{v}. \]

**Proposition 3** The equilibrium bid is smaller with regret than without regret for loss-averse bidders in an all-pay auction.

For loss-averse bidders, \( \alpha_L > \alpha_G \), they are more sensitive to losses than to gains, then \( \tilde{b} < b \), the optimal bid is smaller than the bid without regret. For gain-loss neutral bidders, \( \alpha_L = \alpha_G \), they treat gains and losses equally, then \( \tilde{b} = b \), the optimal bid equals to the bid without regret. And for gain seeking bidders, \( \alpha_L < \alpha_G \), they are more sensitive to gains than to losses, then \( \tilde{b} > b \), the optimal bid is bigger than without regret.

The logic of all-pay auction is similar like first-price auction, the winner will also feel lucky if the actual second highest bid is higher than expected. Also he will regret if the second highest bid is lower than expected such that he feels he may have won with a lower bid. The difference between the actual second highest bid and the expectation in both first-price auction and all-pay auction does not directly affect bidder’s utility through payment but indirectly though regret.

Both in first-price auction and all-pay auction the regret is not about how much one pays (either when one wins in first-price auction or overall in all-pay auction), since this known in advance. This is like the second-price auction where it is not known in advance. It is about whether or not one could have saved money while still winning.

The fact that equilibrium revenue are the same between both formats (first-price and all-pay) indicates to us using a consistent model (see 3.4 Revenue non equivalence).
3.4 Revenue non equivalence

This section is the comparison of the seller’s expected revenue from different auction formats. Recall that we have the optimal bid strategies for first-price auction, second-price auction and all-pay are as follow,

\[ b_F(v) = v - \int_v^0 F^{n-1}(\tilde{v})d\tilde{v} + (\alpha_L - \alpha_G) \int_0^\infty G(B)dB \]
\[ b_S(v) = v - \int_v^0 F^{n-1}(\tilde{v})d\tilde{v} + (\alpha_L - \alpha_G) \int_0^\infty G(B)dB - \int_0^v F^{n-1}(\tilde{v})d\tilde{v} \]
\[ b_A(v) = v - \int_v^{F^{-1}(v)} F^{n-1}(\tilde{v})d\tilde{v} - (\alpha_L - \alpha_G) \int_0^\infty G(B)dB \]

And we know the density function for first-price auction, second-price auction and all-pay auction are \(nf(v)F^{n-1}(v)\), \(n(n-1)f(v)(1 - F(v))F^{n-2}(v)\) and \(nf(v)\) respectively. Then the expected revenue of the three types of auctions will be the integral,

\[ R_F = \int b_F(v) \cdot nf(v)F^{n-1}(v)dv \]
\[ R_S = \int b_S(v) \cdot n(n-1)f(v)(1 - F(v))F^{n-2}(v)dv \]
\[ R_A = \int b_A(v) \cdot nf(v)dv \]

Note that \(b_F(v) \cdot F^{n-1}(v) = b_A(v)\), so \(R_F = \int b_F(v) \cdot nf(v)F^{n-1}(v)dv = \int b_A(v) \cdot nf(v) = R_A\). The revenue of first-price auction and all-pay auction equal to each other, but the revenue of second-price auction is not. Suppose that there are two bidders, with private values uniformly distributed over \([0,1]\), the revenue of first-price auction and all-pay auction are \(R_F = R_A = \frac{8}{24+3(\alpha_L - \alpha_G)}\) while the revenue of second-price auction is \(R_S = \frac{8}{24+6(\alpha_L - \alpha_G)}\).

In the first-price auction and all-pay auction, \(\alpha_G\) is associated with the gain when the actual second highest bid is higher than expected. The winner is a relieved to still win since his expectation was lower. He feels lucky to win the item in the same sense as just catching a train. And \(\alpha_L\) is associated with the loss when the actual second highest bid is lower than expected. The winner regrets since he may have won the auction with a slightly lower bid. However, gains and losses are different in
the second-price auction. The weight $\alpha_{G}$ calculates the gain when the actual second highest bid is lower than expected since he pays less while $\alpha_{L}$ calculates the loss when the actual second highest bid is higher than expected since he needs to pay more. These effects are totally different and hence the revenue equivalence does not hold in general.

3.5 Conclusion

This chapter models independent private auctions in the presence of bidders who have reference-dependent preferences. They have an endogenous expectation about the highest bid of other bidders and using to decide own bid. In the first-price auction and all-pay auction, the winner will have a sense of gain if the expectation is lower the actual second highest bid and have a sense of loss if the expectation is higher than the actual one while in the second-price auction, the winner will have a sense of gain if the expectation is higher than the actual one and feel loss if the expectation is lower than actual one. Consider these gains and losses as regret, we find that in the first-price, second-price and all-pay auction, the optimal bid is smaller than the bid without regret for loss-averse bidders, is greater than the bid without regret for gain seeking bidders and the same for gain-loss neutral bidders.
References


The Fourth Chapter

4 Price Strategy with Loss-Averse Consumers in the Two-period Model

4.1 Introduction

The economics behind sales is an important area of research. One explanation for sales is that they generate higher profits from individual’s loss aversion where they care more about losses than the same size gains. Using loss aversion, Kőszegi and Rabin (2006) propose a reference-dependent model that defines the reference point as consumer’s rational expectations of purchase. In this chapter, I extend the model of Kőszegi and Rabin to a two-period model to see whether the optimal price strategy of sales that is in one-period model also is optimal for a two-period model.

Our results show that in a two-period model, the monopolist could make use of two-price strategy to earn a revenue that is higher than the product value. For example, the price strategy could be a sale price equal to the product value and a regular price slightly higher than the product value. This strategy would then make the discounted revenue higher than the product value when the time factor is large enough since consumers would wait to buy. Also, the price strategy could be a sale price slightly lower than the product value to attract the consumer and a regular price higher than the product value. This strategy could make the revenue higher than the product value independent of the time factor, since consumers buy
immediately. The monopolist's revenue of two-period model could be greater than one-period model when the weight of gain-loss utility is big enough.

This chapter proceeds as follows. Section 2 is the literature review. Section 3 describes the two-period pricing model with loss-averse consumers. Section 4 discusses consumer's purchase plans and conditions for them to be equilibria. Section 5 establishes the main results and shows that in the two-period model the monopolist also could use a two-price strategy to earn a revenue higher than the product value and the revenue is greater than one-period model when the weight of gain-loss utility is large enough. Finally, I conclude in Section 6.

4.2 Literature review

In this section I review several studies about how firms respond to the loss-averse consumers.

Kőszegi and Rabin (2006) propose a expectation-based reference-dependent model to study consumer behavior and find that the expected prices and the expected probability of purchase have a positive effect on consumer’s willingness to pay for the product. Follow the reference-dependent model, Heidhues and Kőszegi (2014) consider a monopolist selling a single product to a loss-averse consumer. The optimal price strategy consists of the low sale price and the high regular price which will lead to a revenue higher than the product value. The sale price is low enough to ensure that the consumer will buy at this price and hence he will purchase with positive probability. To avoid the disappointment of not getting the item, the consumer will also choose to buy at a suitable regular high price. This price strategy helps the monopolist to earn a higher revenue but hurts the consumer’s welfare.

Carbajal and Ely (2012) study the monopolist’s optimal contract with consumer loss aversion and reference dependence. Karle (2013) studies the firm’s advertising strategy when consumers are loss aversion. Hahn, Kim, Kim and Lee (2010) consider the design of menus for monopolist when consumers have reference-dependent preferences and are loss aversion. This chapter extends the the reference-dependent model to two periods and studies implications of reference-dependent preferences and loss aversion in monopolistic setting.

4.3 Model

4.3.1 Simple example

This sub-section introduces a simple example to help explain the two-period pricing model. Suppose that we have a cake with value $v$ and we expect to eat today with probability $q$. If we do not eat eat today, the probability of eating tomorrow is also $q$. The parameter $\eta$ measures the weight attached to gain-loss utility, $\lambda > 1$ is the coefficient of loss aversion and $\beta$ is the time factor.

If we eat the cake today, the utility function is

$$v + \eta(-q + 1)v + \beta\eta[-(1 - q)q + 0]\lambda v$$

where $v$ is the consumption utility of eating the cake, $\eta(-q + 1)v$ is the gain we experience from eating the cake when we expected to eat with probability $1 - q$. The expression $\beta\eta[-(1 - q)q + 0]\lambda v$ is the loss that we experience from not eating tomorrow when we expected to eat tomorrow with probability $(1 - q)q$. 

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Instead, if we do not eat today, the utility function is

\[
0 + \eta(-q + 0)\lambda v + \beta\eta[-(1 - q)q + q]v
+ q\beta[v + \eta(-q + 1)v]
+(1 - q)\beta[0 + \eta(-q + 0)\lambda v]
\]

where zero is the consumption utility today, \(\eta(-q+0)v\) is the loss we experience from not eating today when we expected to eat with probability \(q\), and \(\beta\eta[-(1 - q)q + q]v\) is the gain-loss utility of eating tomorrow caused by its probability changing from \((1-q)q\) to \(q\). The second line is the expected consumption utility and gain-loss utility of tomorrow when we eat the cake tomorrow, while the third line is the expected consumption utility and gain-loss utility of tomorrow when we do not eat tomorrow.

From the example, we can see that the consumer has the ability to eat tomorrow which can offset not eating today. However, there can be loss aversion incurred from eating today since it lowers the chance of eating tomorrow. In this two-period model, it's natural to ask what is the optimal strategy for two-period model and compare this to one-period model and see if the monopolist earn more in the two-period model.

### 4.3.2 Two-period pricing model

This section introduces a two-period pricing model with a risk-neutral monopolist that supplies a nondurable good at zero cost to a representative loss-averse consumer. The interaction between the monopolist and the consumer is as follows. Before the two periods start, the firm commits to choosing a price from a price cumulative distribution \(\Pi\) for both periods. The consumer learns the distribution and forms expectations about his purchase decision for time 1 and 2. We call these expectations as a purchase plan. In time 1, a price \(p_1\) is drawn from the price distribution. The consumer observes it and decides whether or not to buy the product. If the consumer
buy the item, the model ends. If not, in time 2, a price $p_2$ drawn from the price distribution $\Pi$, the consumer observes it and decides whether or not to buy. The consumer only needs one product and does not buy at both time 1 and 2. We assume that a consumer that is indifferent between buying and not buying will buy the product.

The consumer has expectation-based reference-dependent preferences and his utility consists two parts. One is consumption utility, buying the product at price $p_i$ gives the consumer an intrinsic value $v$ and a disutility for paying the price $p_i$, where $p_i$ is the price in time $i$ ($i \in \{1, 2\}$). The other is gain-loss utility, as proposed in Kőszegi and Rabin (2006), that is, the consumer also derives utility from the difference between his actual consumption and his purchase plan $(\tilde{p}_i, \tilde{q}_i)$, where $\tilde{q}_i$ is the quantity the consumer expected to consume in time $i$ (equivalent to the probability of consuming) and $\tilde{p}_i$ is the average price in time $i$ that he expected to pay when making a purchase. Hence, the consumer’s utility in time 2 can be expressed as:

$$
u_2(v, \Pi, p_2, \tilde{p}_2, q_2, \tilde{q}_2) = q_2(v - p_2) + \eta \cdot \mu(-\tilde{q}_2 + q_2)v + \eta(-\tilde{q}_2\tilde{p}_2 + q_2p_2),$$

where

$$\mu(x) = \begin{cases} 
    x & \text{if } x \geq 0, \\
    \lambda x & \text{if } x < 0.
\end{cases}$$

The quantity $q_i \in \{0, 1\}$ is the amount that the consumer buys time $i$ and $\beta_c$ is the consumer's time factor. Note that we assume that loss aversion is only with goods.
The consumer’s utility in time 1 can be expressed as

\[ u_1(v, \Pi, p_1, \tilde{p}_1, p_2, \tilde{q}_1, \tilde{q}_2) = q_1(v - p_1) + \eta \cdot \mu(-\tilde{q}_1 + q_1)v + \eta(-\tilde{p}_1 \tilde{q}_1 + p_1 q_1) + \beta_c \eta \cdot \mu[-(1 - \tilde{q}_1)\tilde{q}_2 + (1 - q_1)\tilde{q}_2]v - \beta_c \eta[-(1 - \tilde{q}_1)\tilde{q}_2 \tilde{p}_2 + (1 - q_1)\tilde{q}_2 \tilde{p}_2] + (1 - q_1)\tilde{q}_2 \beta_c (v - \tilde{p}_2) + (1 - q_1)(\tilde{q}_2 \beta_c(\eta(-\tilde{q}_2 + 1)(v - \tilde{p}_2) - (1 - \tilde{q}_2)\beta_c \eta(-\tilde{q}_2 + 0)(\lambda v - \tilde{p}_2)). \]

The first line is the consumption utility in time 1, second line is the gain-loss utility from the probability changing of purchase behavior in time 1, the third line is the gain-loss utility from the probability changing of purchase behavior in time 2, the forth line is the expected consumption utility in time 2, and the fifth line is the expected gain-loss utility from the probability changing of purchase behavior in time 2.

To complete the reference-dependent preferences model, the consumer’s purchase plan should be time consistent, that is, he makes the best plan for time 1 and 2 based on correct anticipations before the two periods start and knows that he will follow the plan in time 1 and 2. For time consistent expectations, if a consumer does not want to follow a plan in time 1 and 2, then this plan cannot be formulated. Hence, a credible plan must be optimal given the expectations it generates.

Hence, no matter what the consumer has expected before the two periods start, he will buy at prices up to and including some cutoff (which could be 0). Any credible plan should have this cutoff structure. Suppose a credible plan is to buy up to price \( p_i^* \), then this plan has an expectation of receiving the consumption value \( v \) and paying an average price \( \tilde{p}_i = [\int_0^{p_i^*} p d\Pi(p)]/\Pi(p_i^*) \) with probability \( \tilde{q}_i = \Pi(p_i^*) \). Following Kőszegi and Rabin (2006), we call such a credible plan a personal equilibrium (PE).

**Definition 4.1** The cutoff price \( p_i^* \) is a personal equilibrium (PE) for price distri-

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bution $\Pi$ if for the induced expectations $(\tilde{p}_i, \tilde{q}_i)$ we have

$$u_1(v, \Pi, p^*_1, \tilde{p}_1, 1, \tilde{q}_1) = u_1(v, \Pi, p^*_1, \tilde{p}_1, 0, \tilde{q}_1) \text{ for any } p^*_i \in \text{supp}(\Pi),$$

$$u_2(v, \Pi, p^*_2, \tilde{p}_2, 1, \tilde{q}_2) = u_2(v, \Pi, p^*_2, \tilde{p}_2, 0, \tilde{q}_2) \text{ for any } p^*_i \in \text{supp}(\Pi).$$

Before the two periods start, the consumer chooses the PE plan that maximizes his expected utility.

**Definition 4.2** The cutoff price $p^*_i$ with induced expectations $(\tilde{p}_i, \tilde{q}_i)$ is a preferred personal equilibrium (PPE) for price distribution $\Pi$ if it is a PE, and for any PE cutoff price $p^{*'}_i$ and its induced expectations $(\tilde{p}^{*'}_i, \tilde{q}^{*'}_i)$, we have

$$E[u_i(v, \Pi, p^*_i, \tilde{p}_i, q_i, \tilde{q}_i)] \geq E[u_i(v, \Pi, p^{*'}_i, \tilde{p}^{*'}_i, q_i, \tilde{q}^{*'}_i)].$$

### 4.4 Consumer’s demand

Suppose the monopolist charges two prices, a low price $p$ with probability $q$ and a high price $p + \Delta$ with probability $1 - q$ in both time 1 and time 2. The consumer has three possible plans in each period: N, never buy; $p^* < p; L$, buy only at the low price, $p \leq p^* < p + \Delta$; and B, always buy: independent of the price being low or high, $p + \Delta \leq p^*$. There are two periods, each period has three choices, therefore, the consumer has nine plans in total. This section analyzes conditions to be a PE and expected utilities for all the nine plans.

#### 4.4.1 Time 2

We start the analysis with time 2’s plans since they will help us to understand time 1’s plans which is more complicated. If the consumer does not buy the product
in time 1, the model goes in time 2. There are three different expectations about purchase behavior in time 2: –N, never buy; –L, buy only at the low price; and –B, always buy (– is the plan of time 1).

**Plan –N** When the consumer expected never to buy in time 2, the reference point is to consume nothing and pay nothing in time 2. Assume that the realized price is \( p_2^* \), if the consumer sticks to his plan and does not buy the product, his utility is

\[
 u_2(v, \Pi, p_2^*, 0, 0, 0) = 0.
\]

The consumption utility is zero and the gain-loss utility is also zero since he acts according to his plan. Instead, if the consumer deviates from his plan and buys the product, his utility is

\[
 u_2(v, \Pi, p_2^*, 0, 1, 0) = v - p_2^* + \eta v - \eta p_2^*,
\]

where \( v - p_2^* \) is the intrinsic consumption utility from buying the product and paying the price, \( \eta v \) captures the gain of the unexpected product value, and \(-\eta p_2^*\) is the loss of the unexpected payment \( p_2^* \).

When the utility of buying equals the utility of not buying, we have

\[
 v - p_2^* + \eta v - \eta p_2^* = 0.
\] (1)

Equation (1) implies that the cutoff price \( p_2^* = v \), together with the condition \( p^* < p \), we have the condition \( p > v \). Thus, plan –N is a PE if and only if \( p > v \), otherwise the consumer will violate his plan and buy the product as the left of equation (1) is strictly decreasing in \( p_2^* \). Therefore, if the low price \( p \leq v \), plan –N cannot be a PE.

The expected utility of this plan consists of two parts. When realized price is \( p \), we
have
\[ u_2(v, \Pi, p, 0, 0, 0) = 0, \]
where both the consumption utility and the gain-loss utility is zero, and it is the same when the realized price is \( p + \Delta, \)
\[ u_2(v, \Pi, p + \Delta, 0, 0, 0) = 0. \]

Therefore, the expected utility is
\[
q \cdot u_2(v, \Pi, p, 0, 0, 0) + (1 - q) \cdot u_2(v, \Pi, p + \Delta, 0, 0, 0) = 0.
\]

**Plan –L**  When the consumer expected to buy only at the low price in time 2, the reference point in the good dimension is receiving the product with probability \( q \) and receiving nothing with probability \( 1 - q \) and in the money dimension is paying price \( p \) with probability \( q \) and paying nothing with probability \( 1 - q \) in time 2. If the consumer buys the product, his utility is
\[
u(v, \Pi, p_2^*, p, 1, q) = v - p_2^* + \eta(1 - q)v - \eta(p_2^* - pq).
\]
The consumption utility \( v - p_2^* \) derives from receiving the product and paying the price, \( \eta(1 - q)v \) is the gain he experiences from receiving the product when he expected to receiving nothing with probability \( 1 - q, \eta(p_2^* - pq) \) captures the utility derives from paying the realized price \( p_2^* \) when he expected to pay price \( p \) with probability \( q \).

Instead, if the consumer does not buy the item, his utility is
\[
u(v, \Pi, p_2^*, p, 0, q) = 0 - \eta\lambdaqv + \eta pq,
\]
where zero is the consumption utility, \( -\eta\lambdaqv \) captures the loss he experiences from
receiving nothing when he expected to receive the product with probability $q$ and $\eta pq$ is the gain he experiences from paying nothing when he expected to pay the low price $p$ with probability $q$.

When the utility of buying equals the utility of not buying, we have

$$v - p_2^* + \eta(1 - q)v - \eta(p_2^* - pq) = 0 - \eta \lambda q v + \eta pq.$$  (2)

Equation (2) implies the cutoff price $p_2^* = v + \frac{\eta(\lambda - 1)v}{1 + \eta}$, together with the condition $p \leq p^* < p + \Delta$, we have $v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \Delta < p \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta}$. Thus, plan $-L$ is a PE if and only if $v + \frac{\eta(\lambda - 1)v}{1 + \eta} - \Delta < p < v + \frac{\eta(\lambda - 1)v}{1 + \eta}$, if the high price $p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta}$, the consumer will buy the item even if the realized price is high while if the low price $p > v + \frac{\eta(\lambda - 1)v}{1 + \eta}$, the consumer will not buy even if the realized price is low. Therefore, if the high price $p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta}$ or the low price $p > v + \frac{\eta(\lambda - 1)v}{1 + \eta}$, plan $-L$ cannot be a PE.

The expected utility of this plan consists of two parts. When realized price is $p$, we have

$$u_2(v, \Pi, p, p, 1, q) = v - p + \eta(1 - q)v - \eta(p - pq),$$

where $v - p$ is the consumption utility, $\eta(1 - q)v$ is the gain in the good dimension while $-\eta(p - pq)$ is the loss in the money dimension. When the realized price is $p + \Delta$, we have

$$u_2(v, \Pi, p + \Delta, p, 0, q) = 0 - \eta \lambda q v + \eta pq,$$

where zero is the consumption utility, $-\eta \lambda q v$ is the loss in the good dimension and $\eta pq$ is the gain in the money dimension. Therefore, the expected utility is

$$q \cdot u(v, \Pi, p, p, 1, q) + (1 - q) \cdot u(v, \Pi, p + \Delta, p, 0, q) = q(v - p) - \eta q(1 - q)(\lambda - 1)v.$$
Plan -B  When the consumer expected always to buy the item in time 2 no matter the price is low or high, the reference point in the good dimension is enjoying the product value $v$ with probability one while in the money dimension, the reference point is paying the low price $p$ with probability $q$ and paying the high price $p + \Delta$ with probability $1 - q$ in time 2. If the consumer sticks to the plan and buys the product, his utility is

$$u(v, \Pi, p^*_2, p + (1 - q)\Delta, 1, 1) = v - p^*_2 - \eta[p^*_2 - pq - (1 - q)(p + \Delta)].$$

The intrinsic consumption utility is $v - p^*_2$ and $-\eta[p^*_2 - pq - (1 - q)(p + \Delta)]$ captures the utility from paying the realized price $p^*_2$ when he expected to pay price $p$ with probability $q$ and price $p + \Delta$ with probability $1 - q$.

Instead, if the consumer deviates from the plan and does not buy the product, his utility is

$$u(v, \Pi, p^*_2, p + (1 - q)\Delta, 0, 1) = 0 - \eta\lambda v - \eta[0 - pq - (1 - q)(p + \Delta)],$$

where zero is the consumption utility, $-\eta v$ is the loss he experiences from consuming nothing when he expected to consume the product and $-\eta[0 - pq - (1 - q)(p + \Delta)]$ is the gain he experiences from paying nothing when he expected to pay price $p$ with probability $q$ and price $p + \Delta$ with probability $1 - q$.

When the utility of buying equals the utility of not buying, we have

$$v - p^*_2 - \eta[p^*_2 - pq - (1 - q)(p + \Delta)] = 0 - \eta\lambda v - \eta[0 - pq - (1 - q)(p + \Delta)].$$

Equation (3) implies the cutoff price $p^*_2 = \frac{1 + \eta \lambda}{1 + \eta} v$, together with $p + \Delta \leq p^*$, we have the condition $p + \Delta \leq \frac{1 + \eta \lambda}{1 + \eta} v$. Hence, plan -B is a PE if and only if $p \leq \frac{1 + \eta \lambda}{1 + \eta} v - \Delta$.

The expected utility of this plan consist of two parts. When realized price is $p$, we
have
\[ u_2(v, \Pi, p, p + (1 - q)\Delta, 1, 1) = v - p + \eta(1 - q)\Delta, \]

where \( v - p \) is the consumption utility and \( \eta(1 - q)\Delta \) is the gain in the money dimension. When the realized price is \( p + \Delta \)
\[ u_2(v, \Pi, p + \Delta, p + (1 - q)\Delta, 1, 1) = v - p - \Delta - \eta q \Delta, \]

where \( v - p - \Delta \) is the consumption utility and \( -\eta q \Delta \) is the loss in the money dimension. Therefore, the expected utility is
\[ q \cdot u(v, \Pi, p + (1 - q)\Delta, 1, 1) + (1 - q) \cdot u(v, \Pi, p + \Delta, p + (1 - q)\Delta, 1, 1) = v - p - \Delta(1 - q). \]

These conditions for plan -N, -L and -B to be the PE are the same as conditions of plan N, L, B in one-period model, which state that if the low price equals to or smaller than product value \( v \), the consumer will buy with positive probability (buy at this low price) with any plan. And with positive probability of buying, the consumer is willing to accept a suitable high price which is strictly higher than \( v \). Under the two-price strategy, the consumer will buy with probability one and the monopolist will earn a revenue higher than the product value.

4.4.2 Time 1

In time 0, the consumer learns the price distribution of the product, the low price \( p \) with probability \( q \) and the high price \( p + \Delta \) with probability \( 1 - q \) and then he makes his purchase plan for time 1 and time 2. There are nine possibilities: plan NN, NL, NB, LN, LL, LB, BN, BL, BB. For example, plan LL means that the consumer expected to buy at the low price in time 1, if there is a high price in time 1 then buy only at the low price in time 2.
Plan NN  The consumer expected never to buy the product both in time 1 and 2. In time 1, the realized price is $p_1^\ast$, if the consumer sticks to his plan and does not buy in time 1, his utility is

$$u_1(v, \Pi, p_1^\ast, 0, 0, 0, 0, 0) = 0,$$

where the consumption utility is zero and the gain-loss utility is zero since he acts as his expectation. Instead, if he deviates from his plan and buys in time 1, his utility is

$$u_1(v, \Pi, p_1^\ast, 0, 0, 1, 0, 0) = v - p_1^\ast + \eta v - \eta p_1^\ast,$$

where $v - p_1^\ast$ is the consumption utility derives from receiving the product and pay the realized price, $\eta v$ is the gain he experiences from reviving the product where he expected to receive nothing and $-\eta p_1^\ast$ is the loss he experiences from paying the realized price where he expected to pay nothing.

When the utility of buying equals the utility of not buying, we have

$$v - p_1^\ast + \eta v - \eta p_1^\ast = 0. \quad (4)$$

Equation (4) implies that the cutoff price $p_1^\ast = v$, together with the condition $p_1^\ast < p$, we have $v < p$. Hence, plan NN is a PE if and only if $v < p$, otherwise plan NN cannot be a PE.

This condition is the same as the condition of plan N in one-period model, which states that the product value $v$ is a price that the consumer cannot reject even when the consumer expected not buy.

The expected utility of plan NN is

$$q \cdot u_1(v, \Pi, p, 0, 0, 0, 0, 0) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, 0, 0, 0, 0, 0),$$
where the first expression is the utility of not buying when price is low with probability $q$ which equals zero and the second expression is the utility of not buying when the price is high with probability $1 - q$ which equals zero, too. Therefore, the total expected utility is zero.

**Plan NL** The consumer expected never to buy in time 1 and buy only at the low price in time 2. If the consumer sticks to his plan and does not buy in time 1, his utility is

$$u_1(v, \Pi, p_1^*, 0, p, 0, 0, q) = 0$$

$$+ \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c (1 - q)[0 + \eta q(-\lambda v + p)],$$

where the first line is the consumption utility and gain-loss utility in time 1 which is the same as plan NN. The second line is the expected utility of time 2, $\beta_c q[v - p + \eta(1 - q)(v - p)]$ is the consumption utility and gain-loss utility of buying the item in time 2 when the realized price is low and $\beta_c (1 - q)[0 + \eta q(-\lambda v + p)]$ is the utility of not buying the item when the realized price is high.

Instead, if the consumer violates his plan and buys the product in time 1, his utility is

$$u_1(v, \Pi, p_1^*, 0, p, 1, 0, q) = v - p_1^* + \eta v - \eta p_1^*$$

$$+ \beta_c \eta(-q + 0)(\lambda v - p),$$

where the first line is consumption utility and gain-loss utility in time 1 which is the same as plan NN. The second line is the expected utility of time 2, buying in time 1 means lost the opportunity to buy in time 2, $\beta_c \eta(-q + 0)(\lambda v - p)$ is the the loss of the product and the gain of the money with probability $q$ since originally he expected to buy in time 2 with probability $q$ and then the probability of buying in time 2 goes to zero if he buys in time 1.
When the utility of buying equals the utility of not buying, we have

\[ v - p^*_1 + \eta(v - p^*_1) + \beta_c \eta(-q + 0)(\lambda v - p) \]
\[ = 0 + \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c (1 - q)[0 + \eta q(-\lambda v + p)]. \tag{5} \]

Equation (5) implies that the cutoff price \( p^*_1 = (1 + \eta) v - \frac{\eta(\lambda - 1) v}{1 + \eta} \cdot \frac{\beta_c q}{(1 - \beta_c q)} \), together with the condition \( p^*_1 < p \), we have \( v - \frac{\eta(\lambda - 1) v}{1 + \eta} < p \). Hence, this is a necessary condition for plan NL to buy a PE, otherwise plan NL cannot be a PE.

Remember that the condition for plan N in one-period model to be a PE is \( v < p \), which tells us that \( v \) is a price that the consumer cannot reject. Here, in the two period model, the consumer has more choices, hence the price that the consumer cannot reject becomes lower which should be strictly smaller than the product value. And with plan NL, the bigger the consumer’s time factor \( \beta_c \) is, the smaller the acceptable low price is for the consumer.

The expected utility of plan NL is

\[ q \cdot u_1(v, \Pi, p, 0, p, 1, 0, q) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, 0, 0, 0), \]

where the first expression is the utility of not buying when price is low with probability \( q \) which equals \( 0 + \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c (1 - q)[0 + \eta q(-\lambda v + p)] \) and the second expression is the utility of not buying when the price is high with probability \( 1 - q \) which equals \( 0 + \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c (1 - q)[0 + \eta q(-\lambda v + p)] \).

Therefore, the total expected utility is \( \beta_c q(v - p) - \beta_c \eta q(1 - q)(\lambda - 1)v \).

**Plan NB** The consumer expected never to buy in time 1 and always buy in time 2. If the consumer follows his plan and does not buy in time 1, his utility is

\[ u_1(v, \Pi, p^*_1, 0, p + (1 - q)\Delta, 0, 0, 1) = 0 \]
\[ + \beta_c q[v - p + \eta(1 - q)\Delta] + \beta_c (1 - q)[v - p - \Delta - \eta q\Delta], \]

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where the first line is the consumption utility and the gain-loss utility in time 1 which is the same as plan NN and NL. The second line is the expected utility of time 2, \( \beta_c q[v - p + \eta(1 - q)\Delta] \) is the utility of buying when the price is low with probability \( q \) and \( \beta_c (1 - q)[v - p - \Delta - \eta q \Delta] \) is the utility of buying when the price is high with probability \( 1 - q \).

Instead, if the consumer violates his plan and buys the item in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, 0, p+(1-q)\Delta, 1, 0, 1) = v - p_1^* + \eta v - \eta p_1^* + \beta_c \eta(-q + 0) (\lambda v - p) + \beta_c \eta[-(1-q) + 0] (\lambda v - p - \Delta),
\]

where the first line is the consumption utility and gain-loss utility in time 1 which is the same as plan NN and NL. The second line is the gain-loss utility of time 2, originally, the consumer expected always to buy in time 2 and then he cannot buy in time 2 since he buys in time 1, \( \beta_c \eta(-q + 0) (\lambda v - p) \) is the loss of the product and the gain of the money when time 2’s price is low with probability \( q \) and \( \beta_c \eta[-(1-q) + 0] (\lambda v - p - \Delta) \) is the loss of the product and the gain of the money when the price is high with probability \( 1 - q \).

When the utility of buying equals the utility of not buying, we have

\[
v - p_1^* + \eta v - \eta p_1^* + \beta_c \eta(-q + 0) (\lambda v - p) + \beta_c \eta[-(1-q) + 0] (\lambda v - p - \Delta) = 0 + \beta_c q[v - p + \eta(1 - q)\Delta] + \beta_c (1-q)[v - p - \Delta - \eta q \Delta].
\]

Equation (6) implies the cutoff price \( p_1^* = \frac{(1+\eta)v - \beta(1+\eta)u + \beta_c (1+\eta)\eta + \beta_c (1+\eta)(1-q)\Delta}{1+\eta} \), together with the condition \( p_1^* < p \), we have the condition \( v - \frac{\beta_c}{(1-\beta_c)} \frac{\eta(\lambda-1)v}{(1+\eta)(1-q)\Delta} < p \). Hence, this is a necessary condition for plan NB to be a PE, otherwise plan NB cannot be a PE.

This condition is similar as plan NL, just the price difference \( \Delta \) also has an effect here.
The expected utility of plan NB is

\[ q \cdot u_1(v, \Pi, p, 0, p + (1 - q)\Delta, 1, 0, 1) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, 0, p + (1 - q)\Delta, 1, 0, 1), \]

where the first expression is the utility of not buying when price is low with probability \( q \) which equals

\[ 0 + \beta_c q[v - p + \eta(1 - q)\Delta] + \beta_c(1 - q)(v - p - \Delta - \eta q \Delta) \]

and the second expression is the utility of not buying when the price is high with probability \( 1 - q \) which equals

\[ 0 + \beta_c q[v - p + \eta(1 - q)\Delta] + \beta_c(1 - q)(v - p - \Delta - \eta q \Delta). \]

Therefore, the total expected utility is

\[ \beta_c(v - p) - \beta_c(1 - q)\Delta. \]

**Plan LN**  
The consumer expected to buy only at the low price in time 1 and never buy in time 2. If he buys the item in time 1, his utility is

\[ u_1(v, \Pi, p^*_1, p, 0, 1, q, 0) = v - p^*_1 + \eta q(p - p^*_1) + \eta(1 - q)(v - p^*_1), \]

where \( v - p^*_1 \) is the consumption utility derives from receiving the product and paying the realized price, \( \eta q(p - p^*_1) \) is the gain-loss utility from paying the realized price where he expected to pay the low price with probability \( q \) and \( \eta(1 - q)(v - p^*_1) \) is the unexpected gain of the product and the unexpected loss of the money from buying where he expected not buy when price is high with probability \( 1 - q \).

Instead, if he does not buy in time 1, his utility is

\[ u_1(v, \Pi, p^*_1, p, 0, 0, q, 0) = 0 + \eta q(-\lambda v + p), \]

where zero is the consumption utility and \( \eta q(-\lambda v + p) \) is the gain-loss utility that he does not buy where he expected to buy when price is low with probability \( q \).

When the utility of buying equals the utility of not buying, we have

\[ v - p^*_1 + \eta q(p - p^*_1) + \eta(1 - q)(v - p^*_1) = 0 + \eta q(-\lambda v + p). \]

(7)
Equation (7) implies that the cutoff price $p^*_1 = \frac{v + \eta(1 - q)v + \eta\lambda v}{1 + \eta}$, together with the condition $p \leq p^*_1 < p + \Delta$, we have the condition $p \leq v + \frac{\eta(\lambda - 1)q v}{1 + \eta} < p + \Delta$. Hence, this is a necessary condition for plan LN to be a PE, otherwise, plan LN cannot be a PE.

This condition is the same as the condition of plan L in one-period model, which states that the consumer is willing to accept a low price that is strictly greater than $v$ when the consumer expected to buy with positive probability. And the high price should be strictly than an amount, otherwise the consumer will violate his expectation and buy at the high price in time 1.

The expected utility of plan LN is

$$q \cdot u_1(v, \Pi, p, p, 0, 1, q, 0) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, p, 0, 0, q, 0)$$

where the first expression is the utility of buying when price is low with probability $q$ which equals $v - p + \eta(1 - q)(v - p)$ and the second expression is the utility of not buying when price is high with probability $1 - q$ which equals $0 + \eta q(-\lambda v + p)$. Therefore, the total expected utility of plan LN is $q(v - p) - \eta q(1 - q)(\lambda - 1)v$.

**Plan LL** The consumer expected to buy only at the low price in time 1 and if there is a high price in time 1 then buy only at the low price in time 2. If he buys the product in time 1, his utility is

$$u_1(v, \Pi, p^*_1, p, p, 1, q, q) = v - p^*_1 + \eta q(p - p^*_1) + \eta(1 - q)(v - p^*_1)$$

$$+ \beta \eta[-(1 - q)q + 0](\lambda v - p),$$

where the first line is the consumption utility and gain-loss utility in time 1 which is the same as plan LN. The second line is the expected gain-loss utility of time 2, $\beta \eta[-(1 - q)q + 0](\lambda v - p)$ is the gain-loss utility of not buying in time 2 where originally he expected to buy in time 2 with probability $(1 - q)q$. 

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Instead, if he does not buy in time 1, his utility is

\[
\begin{align*}
    u_1(v, \Pi, p^*_1, p, p, 0, q, q) &= 0 + \eta q(-\lambda v + p) \\
    &+ \beta c \eta[-(1 - q)q + q](v - p) \\
    &+ \beta c q[v - p + \eta(1 - q)(v - p)] + \beta c (1 - q)[0 + \eta q(-\lambda v + p)],
\end{align*}
\]

where the first line is the consumption utility and gain-loss utility which is the same as plan LN. The second line is the expected gain-loss utility of buying in time 2, it’s probability changing from \((1 - q)q\) to \(q\) since the probability of not buying in time 1 changing from \((1 - q)\) to 1. For the third line, \(\beta c q[v - p + \eta(1 - q)(v - p)]\) is the utility of buying in time 2 when the price is low with probability \(q\) while \(\beta c (1 - q)[0 + \eta q(-\lambda v + p)]\) is the utility of not buying in time 2 when the price is high with probability \(1 - q\).

When the utility of buying equals the utility of not buying, we have

\[
\begin{align*}
    v - p^*_1 + \eta q(p - p^*_1) + \eta(1 - q)(v - p^*_1) + \beta c \eta[-(1 - q)q + 0](\lambda v - p) \\
    &= 0 + \eta q(-\lambda v + p) + \beta c \eta[-(1 - q)q + q](v - p) \\
    &+ \beta c q[v - p + \eta(1 - q)(v - p)] + \beta c (1 - q)[0 + \eta q(-\lambda v + p)].
\end{align*}
\]

Equation (8) implies that the cutoff price \(p^*_1 = \frac{v + \eta(1 - q)v + \eta \lambda q v - \beta c (1 - q)v + \beta c (1 + q)q p}{1 + \eta}\), together with the condition \(p \leq p^*_1 < p + \Delta\), we have \(p \leq v + \frac{1}{(1 - \beta c q)} \cdot \frac{\eta q(1 - 1)v}{(1 + \eta)} < p + \frac{\Delta}{(1 - \beta c q)}\). Hence, this is a necessary condition for plan LL to be a PE, otherwise plan LL cannot be a PE.

This condition is similar like the condition for plan LN. Comparing with plan LN, the acceptable low price for plan LL could be slightly bigger. And the higher the consumer’s time factor is, the higher the acceptable low price could be.

The expected utility of plan LL is

\[
q \cdot u_1(v, \Pi, p, p, 1, q) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, p, 0, q),
\]
where the first expression is the utility of buying when price is low with probability \( q \) which equals \( v - p + \eta(1 - q)(v - p) + \beta_c \eta[-(1 - q)q + 0](\lambda v - p) \) and the second expression is the utility of not buying when the price is high with probability \( 1 - q \) which equals \( 0 + \eta q(-\lambda v + p) + \beta_c \eta[-(1 - q)q + q](v - p) + \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c(1 - q)[0 + \eta q(-\lambda v + p)] \). Therefore, the total expected utility of plan LL is
\[
q(v - p) - \eta q(1 - q)(\lambda - 1)v + \beta_c q(1 - q)(v - p) - \beta_c \eta q(1 - q)(\lambda - 1)v.
\]

**Plan LB** The consumer expected to buy only at the low price in time 1 and if there is a high price in time 1 then always buy in time 2. If the consumer sticks to his plan and buys in time 1, his utility is

\[
\begin{align*}
\text{u}_1(v, \Pi, p_1', p, p + (1 - q)\Delta, 1, q, 1) &= v - p_1' + \eta q(p - p_1') + \eta(1 - q)(v - p_1') + \\
&\quad + \beta_c \eta[-(1 - q)q + 0](\lambda v - p) \\
&\quad + \beta_c \eta[-(1 - q)(1 - q) + 0](\lambda v - p - \Delta)
\end{align*}
\]

where the first line is the consumption utility and the gain-loss utility in time 1 which is the same as plan LN and LL. The second and third lines are the expected utility of time 2, \( \beta_c \eta[-(1 - q)q + 0](\lambda v - p) \) is the gain-loss utility of not buying in time 2 where he expected to buy at the low price with probability \( (1 - q)q \) while \( \beta_c \eta[-(1 - q)(1 - q) + 0](\lambda v - p - \Delta) \) is the gain-loss utility of not buying in time 2 where he expected to buy at the high price with probability \( (1 - q)(1 - q) \).

Instead, if he deviates from his plan and does not buy in time 1, his utility is

\[
\begin{align*}
\text{u}_1(v, \Pi, p_1', p, p + (1 - q)\Delta, 0, q, 1) &= 0 + \eta q(-\lambda v + p) \\
&\quad + \beta_c \eta[-(1 - q)q + q](v - p) \\
&\quad + \beta_c \eta[-(1 - q)(1 - q) + 1 - q](v - p - \Delta) \\
&\quad + \beta_c q[v - p + \eta(1 - q)\Delta] + \beta_c(1 - q)(v - p - \Delta - \eta q\Delta)
\end{align*}
\]

where the first line is the consumption utility and gain-loss utility in time 1 which is the same as LN and LL. The second and third lines are the expected utility of
time 2, $\beta_c \eta[-(1 - q)q + q](v - p)$ is the gain-loss utility of buying at the low price in time 2 where the probability changing from $(1 - q)q$ to $q$ since he does not buy in time 1 while $\beta_c \eta[-(1 - q)(1 - q) + 1 - q](v - p - \Delta)$ is the gain-loss utility of buying at the high price in time 2 where its probability changing from $(1 - q)(1 - q)$ to $1 - q$ since he does not buy in time 1. For the fourth line, $\beta_c q[v - p + \eta(1 - q)\Delta]$ is the consumption utility and the gain-loss utility of buying at the low price in time 2 with probability $q$ and $\beta_c(1 - q)(v - p - \Delta - \eta q \Delta)$ is the utility of buying at the high price with probability $1 - q$.

When the utility of buying equals the utility of not buying, we have

$$v - p^*_1 + \eta q (p - p^*_1) + \eta (1 - q)(v - p^*_1) + \beta_c \eta[-(1 - q)q + 0](\lambda v - p) + \beta_c \eta[-(1 - q)(1 - q) + 0](\lambda v - p - \Delta)$$

$$= 0 + \eta q(-\lambda v + p) + \beta_c \eta[-(1 - q)q + q](v - p) + \beta_c \eta[-(1 - q)(1 - q) + 1 - q](v - p - \Delta) + \beta c q[v - p + \eta(1 - q)\Delta] + \beta c(1 - q)(v - p - \Delta - \eta q \Delta).$$

Equation (9) implies that the cutoff price $p^*_1 = \frac{v + \eta(1 - q)v + \eta q v - \beta_c [1 + \eta q + \eta(1 - q)v v]}{1 + \eta}$, together with the condition $p \leq p^*_1 < p + \Delta$, we have $p - \frac{(1 - q)\Delta}{1 - \beta_c} \leq v + \eta q(\lambda - 1)v - \beta_c(\lambda - 1)v + \beta_c q(\lambda - 1)v v < p + \frac{1 - \beta_c(1 - q)\Delta}{1 - \beta_c}$. Hence, this is the condition of plan LB to be a PE, otherwise plan LB cannot be a PE.

This condition is similar like the condition for plan LN, just the amount which low price should be bigger than and the amount which the high price should smaller than changed. This change has close relate with the consumer’s time factor and the probability the consumer expected to buy.

The expected utility of plan LB is

$$q \cdot u_1(v, \Pi, p, p, p + (1 - q)\Delta, 1, q, 1) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, p, p + (1 - q)\Delta, 0, q, 1),$$

where the first expression is the utility of buying when price is low with probability $q$ which equals $v - p + \eta(1 - q)(v - p) + \beta_c \eta[-(1 - q)q + 0](\lambda v - p) + \beta c \eta[-(1 -
\[ q(1-q)+0](\lambda v-p-\Delta) \] and the second expression is the utility of not buying when the price is high with probability \(1-q\) which equals \(0 + \eta q(-\lambda v + p) + \beta c\eta[-(1-q)q + q](v-p) + \beta c\eta[-(1-q)(1-q) + 1-q](v-p-\Delta) + \beta c[qv-p + \eta(1-q)\Delta] + \beta(1-q)(v-p-\Delta - \eta q\Delta). \] Therefore, the total expected utility of plan LL is \(q(v-p) - \eta q(1-q)(\lambda - 1)v + \beta c(1-q)(v-p) - \beta c(1-q)^2\Delta + \beta c\eta q(1-q)(\lambda - 1)v.\)

**Plan BN**  The consumer expected always to buy in time 1 and never buy in time 2. If the consumer follows his plan and buys in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, p + (1-q)\Delta, 0, 1, 1, 0) = v - p_1^* + \eta q(p - p_1^*) + \eta(1-q)(p + \Delta - p_1^*),
\]

where \(v - p_1^*\) is the consumption utility derives from receiving the product and paying the price, \(\eta q(p - p_1^*)\) is the gain-loss utility of paying the realized price \(p_1^*\) where he expected to pay the low price with probability \(q\) and \(\eta(1-q)(p + \Delta - p_1^*)\) is the gain-loss utility of paying the realized price \(p_1^*\) where he expected to pay the high price with probability \(1-q\).

Instead, if he violates his plan and does not buy in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, p + (1-q)\Delta, 0, 0, 0, 1, 0) = 0 + \eta q(-\lambda v + p) + \eta(1-q)(-\lambda v + p + \Delta),
\]

where zero is the consumption utility, \(\eta q(-\lambda v + p)\) is the loss of the product and the gain of the money where he expected to buy at the low price with probability \(q\) and \(\eta(1-q)(-\lambda v + p + \Delta)\) is the loss of the product and the gain of the money where he expected to buy at the high price with probability \(1-q\).

When the utility of buying equals the utility of not buying, we have

\[v - p_1^* + \eta q(p - p_1^*) + \eta(1-q)(p + \Delta - p_1^*) = 0 + \eta q(-\lambda v + p) + \eta(1-q)(-\lambda v + p + \Delta). \] (10)

Equation (10) implies that the cutoff price \(p_1^* = \frac{\nu + n\lambda v}{1+n}\), together with the condition


\[ p + \Delta \leq p_1^*, \text{ we have } p + \Delta \leq v + \frac{\eta(\lambda - 1)v}{1 + \eta}. \] 

Hence, this is a necessary condition for plan BN to be a PE, otherwise plan BN cannot be a PE.

This condition is the same as the condition of plan B in one-period model, which states that even when the consumer expected to buy with probability one, the high price cannot be too big, otherwise, the consumer will violate his expectation and not buy at the high price.

The expected utility of plan BN is

\[
q \cdot u_1(v, \Pi, p, p + (1 - q)\Delta, 0, 1, 1, 0) + (1 - q) \cdot u_1(v, \Pi, p + \Delta, p + (1 - q)\Delta, 0, 1, 1, 0),
\]

where the first expression is the utility of buying at the low price with probability \( q \) which equals \( v - p + \eta(1 - q)\Delta \) and the second expression is the utility of buying at the high price with probability \( 1 - q \) which equals \( v - p - \Delta - \eta q \Delta \). Therefore, the total expected utility of plan BN is \( v - p - (1 - q)\Delta \).

**Plan BL** The consumer expected always to buy in time 1 and if off the equilibrium plan and does not buy in time 1 then buy only at the low price in time 2. If the consumer follows his plan and buys in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, p + (1 - q)\Delta, p, 1, 1, q) = v - p_1^* + \eta q(p - p_1^*) + \eta(1 - q)(p + \Delta - p_1^*),
\]

where \( v - p_1^* \) is the consumption utility and \( \eta q(p - p_1^*) + \eta(1 - q)(p + \Delta - p_1^*) \) is the gain-loss utility which is the same as plan BN.

Instead, if he violates his plan and does not buy in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, p + (1 - q)\Delta, p, 0, 1, q) = 0 + \eta q(-\lambda v + p) + \eta(1 - q)(-\lambda v + p + \Delta)
+ \beta c\eta(-0 + q)(v - p)
+ \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c(1 - q)[0 + \eta q(-\lambda v + p)],
\]

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where the first line is the consumption utility and the gain-loss utility in time 1 which is the same as plan BN, the second line is the gain-loss utility of buying in time 2 where the probability changing from zero to \(q\) since he does not buy in time 1, the third line is the expected utility of time 2, \(\beta_c q[v - p + \eta(1-q)(v-p)]\) is the utility of buying in time 2 at the low price with probability \(q\) and \(\beta_c(1-q)[0 + \eta q(\lambda v + p)]\) is the utility of not buying at the high price with probability \(1 - q\).

When the utility of buying equals the utility of not buying, we have

\[
v - p_1^* + \eta q(p - p_1^*) + \eta(1 - q)(p + \Delta - p_1^*) = 0 + \eta q(-\lambda v + p) + \eta(1 - q)(-\lambda v + p + \Delta) + \beta_c \eta(-0 + q)(v - p) + \beta_c q[v - p + \eta(1 - q)(v - p)] + \beta_c(1 - q)[0 + \eta q(-\lambda v + p)].
\]

Equation (11) implies that the cutoff price \(p_1^* = \frac{v + \eta \lambda v - \beta_c(1+\eta)(1-q)(\lambda-1)q v + \beta_c(1+\eta)q p}{1+\eta (1-\beta_c q)}\), together with the condition \(p + \Delta \leq p_1^*\), we have 

\[
p + \frac{\Delta}{1-\beta_c q} \leq v + \frac{\eta \lambda(1-q) v + \beta_c \eta(1-q)(\lambda-1)q v}{(1+\eta)(1-\beta_c q)}.
\]

Hence, this is a necessary condition for plan BL to be a PE, otherwise plan BL cannot be a PE.

This condition is similar like the condition for plan BN, which states that the high price should be smaller than an amount to make the consumer follow his original plan. Here the amount is higher than plan BN.

The expected utility of plan BL is \(v - p - (1 - q)\Delta\) which the same as plan BN.

**Plan BB** The consumer expected always to buy in time 1 and if off the equilibrium plan and does not buy in time 1 then the always buy in time 2. If the consumer follows his plan and buys in time 1, his utility is

\[
u_1(v, \Pi, p_1^*, p+(1-q)\Delta, p+(1-q)\Delta, 1, 1, 1) = v - p_1^* + \eta q(p - p_1^*) + \eta(1-q)(p + \Delta - p_1^*),
\]

where \(v - p_1^*\) is the consumption utility and \(\eta q(p - p_1^*) + \eta(1-q)(p + \Delta - p_1^*)\) is the gain-loss utility which is the same as plan BN and BL.
Instead, if he violates his plan and does not buy in time 1, his utility is
\[
u_1(v, \Pi, p_1^*, p + (1-q)\Delta, p + (1-q)\Delta, 0, 1, 1) = 0 + \eta q(-\lambda v + p) + \eta(1-q)(-\lambda v + p + \Delta) \\
+ \beta_c \eta(-0 + q)(v - p) + \beta_c \eta(-0 + 1 - q)(v - p - \Delta) \\
+ \beta_c q[v - p + \eta(1-q)\Delta] + \beta_c(1-q)(v - p - \Delta - \eta q\Delta),
\]

where the first line is the consumption utility and the gain-loss utility in time 1 which is the same as plan BN and BL, in the second line, \(\beta_c \eta(-0 + q)(v - p)\) is the gain-loss utility of buying at the low price in time 2 where it’s probability changing from zero to \(q\) since he does not buy in time 1 and \(\beta_c \eta(-0 + 1 - q)(v - p - \Delta)\) is the gain-loss utility of buying at the high price where the probability changing from zero to \(1 - q\), the third line is the expected utility of time 2, \(\beta_c q[v - p + \eta(1-q)\Delta]\) is the consumption utility and gain-loss utility of buying in time 2 at the low price with probability \(q\) and \(\beta_c(1-q)(v - p - \Delta - \eta q\Delta)\) is the utility of buying at the high price with probability \(1 - q\).

When the utility of buying equals the utility of not buying, we have
\[
v - p_1^* + \eta q(p - p_1^*) + \eta(1-q)(p + \Delta - p_1^*) \\
= 0 + \eta q(-\lambda v + p) + \eta(1-q)(-\lambda v + p + \Delta) + \beta_c \eta(-0 + q)(v - p) \\
+ \beta_c \eta(-0 + 1 - q)(v - p - \Delta) + \beta_c q[v - p + \eta(1-q)\Delta] + \beta_c(1-q)(v - p - \Delta - \eta q\Delta). \tag{12}
\]

Equation (12) implies that the cutoff price \(p_1^* = \frac{v + \eta \lambda v - \beta_c(1+\eta)v + \beta_c(1+\eta)p + \beta_c(1+\eta)(1-q)\Delta}{1+\eta}\), together with the condition \(p + \Delta \leq p_1^*\), we have \(p + \frac{1-\beta_c(1-q)}{1-\beta_c} \Delta \leq v + \frac{\eta(\lambda-1)v}{(1+\eta)(1-\beta_c)}\). Hence, this is a necessary condition for plan BB to be a PE, otherwise plan BB cannot be a PE.

This condition is similar like the condition for plan BN, which states that the high price should be smaller than an amount to make the consumer follow his original plan. Here the amount is more flexible than plan BN.

The expected utility of plan BB is \(v - p - (1-q)\Delta\) which the same as plan BN and
4.5 Main result

The monopolist is risk-neutral and tries to maximize the expected profit given the consumer’s behavior. The consumer has nine plans and the following is the analysis of the revenues for the nine plans. The monopolist’s time factor is denoted by $\beta_m$.

The revenue of plan NN is zero and the consumer’s utility of plan NN is greater than all the other eight plans which means that if plan NN is a PE, it is the PPE. To make the revenue greater than the product value $v$, plan NN cannot be a PE and we have the condition $p \leq v$.

The revenue of plan NL is $\beta_m qp$ which is smaller than $v$ with condition $p \leq v$.

The revenue of plan NB is $\beta_m p + \beta_m (1-q) \Delta$ which may be greater than the product value $v$.

The revenue of plan LN is $qp$ which is smaller than $v$ with condition $p \leq v$.

The revenue of plan LL is $qp + \beta_m (1-q) qp$ which is smaller than $v$ with condition $p \leq v$.

The revenue of plan LB is $qp + \beta_m (1-q) (p + (1-q) \Delta)$ which may be greater than the product value $v$.

For plan BN, the choice of time 2 ($-N$) is not a PE with with condition $p \leq v$ and plan LB cannot be the PPE.

The revenue of plan BL is $p + (1-q) \Delta$ may be greater than the product value $v$.

The revenue of plan BB is $p + (1-q) \Delta$ may be greater than the product value $v$.

There are four plans NB, LB, BL and BB may help the monopolist earn a revenue that is greater than the product value $v$. We have the optimal price strategy for
each case by maximizing the four revenues subject to each own conditions. The revenue of the optimal price strategy for plan LB is smaller than plan BB and the revenue of the optimal price strategy for plan BB is smaller than plan BL. Then, we left two choice, buy at time 1 no matter the price is low or high (plan BL) and wait to buy at time 2 no matter the price is low or high (plan NB).

4.5.1 Wait to buy at time 2 (plan NB)

For plan NB, the consumer always wait and buy the item with probability 1 at time 2. Maximizing the revenue \( R_{NB} = \beta_m p + \beta_m (1 - q) \Delta \) subject to the conditions that plan NB is the PPE, we have the optimal price strategy for this plan.

When \( \eta \leq 1 \),

\[
\begin{align*}
p &= v, \\
q &= \frac{1}{2}, \\
\Delta &= \frac{\eta(\lambda-1)v}{2}, \\
R_{NB} &= \beta_m v + \beta_m \eta(\lambda-1)v, \text{ and} \\
u_{NB} &= -\frac{\beta_m \eta(\lambda-1)v}{4}.
\end{align*}
\]

And when \( \eta > 1 \),

\[
\begin{align*}
p &= v, \\
q &= \frac{1}{1+\eta}, \\
\Delta &= \frac{\eta(\lambda-1)v}{1+\eta}, \\
R_{NB} &= \beta_m v + \frac{\beta_m \eta^2(\lambda-1)v}{(1+\eta)^2}, \text{ and} \\
u_{NB} &= -\frac{\beta_m \eta^2(\lambda-1)v}{(1+\eta)^2}.
\end{align*}
\]

**Proposition 1** The above price strategy helps the monopolist earn a revenue greater than the product value \( v \) only when the time factor is big enough. The revenue is increasing in the monopolist’s time factor \( \beta_m \) and the consumer’s utility is decreasing in the consumer’s time factor \( \beta_c \).

From the above expression, we have \( \frac{\partial R_{NB}}{\partial \beta_m} > 0 \), hence the monopolist’s time factor
has a positive effect on the revenue. Also we have $\frac{\partial u_{NB}}{\partial \beta} < 0$, hence the consumer’s the factor has a negative effect on the consumer’s welfare.

![Graph showing the relation between $\beta_m$ and the revenue](image)

Figure 4.1. The relation between $\beta_m$ and the revenue

We can see that the revenue $R_{NB}$ is increasing in the monopolist’s time factor $\beta_m$ and it is greater than the product value $v$ only when $\beta_m$ is big enough (If $\eta \leq 1$ then $\beta_m \geq \frac{4}{5+\eta}$ and if $1 < \eta$ then $\beta_m \geq \frac{1+\eta}{1+2\eta}$). By the above price strategy, the consumer will wait and buy the product at time 2.

Remember that this strategy is also the optimal price strategy for the one-period model which make the consumer always buy the product even when the realized price is high. Under the above price distribution, the consumer will buy the product with probability one in the one-period model while the consumer will wait to buy at time 2 in the two-period model. The revenue for plan NB $R_{NB} = \beta_m \cdot R_{One}$ which is always smaller than one-period model.

### 4.5.2 Buy at time 1 (plan BL)

For plan BL, the consumer always buy the product at time 1 even when the realized price is high. Maximizing the revenue $R_{BL} = p + (1 - q)\Delta$ subject to the conditions
that plan BL is the PPE, we have the optimal price strategy for this plan.

The domain of the price strategy is as following, the x-axis the consumer’s time factor $\beta_c$ and the y-axis is the weight of gain-loss utility $\eta$.

For area 1, we have

$$p = v - \frac{\beta_c q^2}{1-\beta_c q}, \quad q = \sqrt{\frac{\eta(\lambda-1)w}{1+\eta}(1+\beta_c) - \frac{\beta_c q^2}{1-\beta_c q}},$$

$$\Delta = (1 + \beta_c q) \cdot \frac{\eta(\lambda-1)w}{1+\eta},$$

$$R_{BL} = v + [(1 - q)(1 + \beta_c q) - \frac{\beta_c q^2}{1-\beta_c q}] \cdot \frac{\eta(\lambda-1)w}{1+\eta}, \quad \text{and}$$

$$u_{BL} = -[(1 - q)(1 + \beta_c q) - \frac{\beta_c q^2}{1-\beta_c q}] \cdot \frac{\eta(\lambda-1)w}{1+\eta}.$$

For area 2, we have

$$p = v - \frac{\beta_c q^2}{1-\beta_c q}, \quad q = \sqrt{\frac{\eta(\lambda-1)w}{1+\eta}(1+\beta_c) + 4\beta_c - [(1+\eta)(1+\beta_c) - \beta_c]},$$

$$\Delta = (1 + \beta_c q) \cdot \frac{\eta(\lambda-1)w}{1+\eta},$$

$$R_{BL} = v + [(1 - q)(1 + \beta_c q) - \frac{\beta_c q^2}{1-\beta_c q}] \cdot \frac{\eta(\lambda-1)w}{1+\eta}, \quad \text{and}$$

$$u_{BL} = -[(1 - q)(1 + \beta_c q) - \frac{\beta_c q^2}{1-\beta_c q}] \cdot \frac{\eta(\lambda-1)w}{1+\eta}.$$
The probability of the low price, $q$, is the root of the function $2\beta_c^2 q^3 - \beta_c (3 + 2\beta_c)q^2 - 2(1 + \eta)(1 + \beta_c)q + (1 + \eta)(1 + \beta_c) = 0$ which is always between zero and one.

For the area 3, the optimal strategy does not exists. I justify it with epsilon equilibrium which one does not want to change a price that would increase the revenue by epsilon or less. Here, the limit is that at the boundary plan NB is the PPE but not plan BL.

$$p^\epsilon = v - \frac{\beta_c q^2}{1 - \beta_c} \cdot \frac{\eta(\lambda-1)v}{1+\eta},$$

$$\Delta^\epsilon = (1 + \frac{\beta_c q^2}{1 - \beta_c}) \cdot \frac{\eta(\lambda-1)v}{1+\eta},$$

$$R^\epsilon_{BL} = v + (1 - q - \frac{\beta_c q^2}{1 - \beta_c}) \cdot \frac{\eta(\lambda-1)v}{1+\eta},$$

$$u^\epsilon_{BL} = -(1 - q - \frac{\beta_c q^2}{1 - \beta_c}) \cdot \frac{\eta(\lambda-1)v}{1+\eta}.$$ 

The probability of the low price, $q^\epsilon$, is the root of the function $\beta_c^2 q^3 + \beta_c (1 + \eta)(1 + \beta_c)q^2 - [(1 + \eta)(1 + \beta_c) + \beta_c]q + 1 = 0$ which is always between zero and one.

For area 4, we have

$$p = v - \frac{(1-2q)(1+\beta_c)\eta(\lambda-1)v}{1+\beta_c-2\beta_c q},$$

$$q = \frac{2((1+\eta)(1+\beta_c) - \beta_c^2)\eta(\lambda-1)v}{2\beta_c(1+\eta)(1+\beta_c)},$$

$$\Delta = (1-q)(1+\beta_c)\eta(\lambda-1)v + \frac{2\beta_c(1+\eta)(1+\beta_c)}{1+\beta_c-2\beta_c q},$$

$$R^\epsilon_{BB} = v + \frac{q^2(1+\beta_c)\eta(\lambda-1)v}{1+\beta_c-2\beta_c q},$$

$$u^\epsilon_{BB} = -\frac{q^2(1+\beta_c)\eta(\lambda-1)v}{1+\beta_c-2\beta_c q}.$$ 

For the area 5, the optimal strategy does not exists. I justify it with epsilon equilibrium which one does not want to change a price that would increase the revenue by epsilon or less. Here, the limit is that at the boundary plan NB is the PPE but not plan BL.

$$p^\epsilon = v$$

$$q^\epsilon = \frac{1}{\beta_c(1+\eta)},$$

$$\Delta^\epsilon = \frac{\eta(\lambda-1)v}{1+\eta},$$

$$R^\epsilon_{BL} = v + \frac{\beta_c(1+\eta)-1}{\beta_c(1+\eta)} \cdot \frac{\eta(\lambda-1)v}{1+\eta},$$

$$u^\epsilon_{BL} = -\frac{\beta_c(1+\eta)-1}{\beta_c(1+\eta)} \cdot \frac{\eta(\lambda-1)v}{1+\eta}.$$ 

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By charging the above price strategy, the monopolist could manipulate the consumer to buy at time 1 with probability one and earn a revenue which is greater than the product value \( v \) in the whole domain. Here monopolist’s time factor affects nothing since it does not appear in the price distribution and the consumer is willing to buy at time 1.

**Proposition 2** The above price strategy helps the monopolist earn a revenue greater than the product value \( v \) and hurts the consumer by making consumer utility negative.

From above expression, we can see that the revenue equals the product value plus an additional positive value and the consumer’s utility equals minus the additional value. In this price strategy, the sale price is low enough to ensure the consumer buy at the low price and hence his purchase probability is positive. Then, to avoid the disappointment of not getting the product, the consumer will also buy at the suitable high price. This price strategy manipulates the loss-averse consumer buy with probability one which helps the revenue and hurts the consumer welfare.

**Proposition 3** The monopolist’s revenue is increasing in the loss aversion parameter \( \lambda \) and the consumer’s utility is decreasing in the loss aversion parameter \( \lambda \).

From the above expression, we have \( \frac{\partial R}{\partial \lambda} > 0 \) and \( \frac{\partial u}{\partial \lambda} < 0 \). Hence, the loss aversion parameter help the revenue and hurts the consumer’s welfare.

**The effect of \( \beta_c \) on the revenue** This subsection shows the effect of the consumer’s time factor \( \beta_c \) on the monopolist’s revenue when the weight of gain-loss utility is constant.
Figure 4.3. The relation between $\beta_c$ and the revenue when $\eta = 0.7$

Figure 4.3 shows that the revenue is increasing in the consumer’s time factor $\beta_c$ when $\eta = 0.7$ (the weight of gain-loss utility is big enough).

Figure 4.4. The relation between $\beta_c$ and the revenue when $\eta = 0.2$

Figure 4.4 shows that the revenue first decreasing and then increasing in the consumer’s time factor $\beta_c$ when $\eta = 0.2$, the consumer’s time factor $\beta_c$ has an non-monotonic effect on the revenue when $\eta$ is small.

**Proposition 4** The monopolist’s revenue is non-monotonic in the consumer’s time factor $\beta_c$, and the same as the consumer’s utility.
When the consumer's time factor $\beta_c$ is big enough, it has a positive effect on the monopolist's revenue and a negative effect on the consumer's welfare. However, when $\beta_c$ is small, its effect is non-monotonic for the revenue and consumer's welfare.

**Comparing with one-period model**

![Figure 4.5. Domain of comparison](image)

Comparing the revenue of plan BL with the one-period model, we find that the revenue of plan BL is greater than the one-period in the domain that above the curve while the revenue of BL is smaller in the domain below the curve. Here, when the weight of the gain-loss utility is big enough, the two-period model could lead to a higher revenue than one-period model.

**Proposition 5** The monopolist's revenue of two-period model is greater than one-period model when the weight of the gain-loss utility is big enough.

From the above figure, we can see that revenue of two-period model may be greater than one-period model which is depending on the weight of gain-loss utility $\eta$ and the consumer's time factor $\beta_c$. For example, if the consumer treats the consumption utility and the gain-loss utility equally, then the monopolist's revenue of two-period model is greater than one-period model.
4.6 Conclusion

This chapter studies the two-period pricing model with loss-averse consumers, and finds that the optimal price strategy could earn a higher revenue greater than the product value \( v \) and hurts the consumer making consumer utility negative. Comparing with one-period model, the revenue of two-period model is greater when the weight of gain-loss utility is big enough. The loss aversion parameter \( \lambda \) has a positive effect on the monopolist’s revenue and a negative effect on the consumer’s welfare. While the consumer’s time factor has a non-monotonic effect on the monopolist’s revenue and the consumer’s welfare.
References


