

# Offshore Wind Farm Layout Optimization Using Particle Swarm Optimization

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1 **Abstract** This article explores the application of a wind farm layout optimization  
2 framework using a particle swarm optimizer to three benchmark test cases. The de-  
3 veloped framework introduces an increased level of detail characterizing the impact  
4 that the wind farm layout can have on the levelized cost of energy by modelling the  
5 wind farm's electrical infrastructure, annual energy production, and cost as functions  
6 of the wind farm layout. Using this framework, this paper explores the application of  
7 a particle swarm optimizer to the wind farm layout optimization problem considering  
8 three different levels of wind farm constraint faced by modern wind farm developers.

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9 The particle swarm optimizer is found to yield improvements in the layout with respect  
10 to the levelized cost of energy for the three benchmark cases when compared to two  
11 past studies. This highlights both applicability of the particle swarm optimizer to the  
12 problem and the ways in which a wind farm developer could make use of the present  
13 framework in the development and design of future wind farms.

14 **Keywords** offshore wind · layout optimization · particle swarm optimization · wind  
15 farm design

## 16 1 Introduction

17 As the world transitions to a more sustainable energy sector, wind energy and in  
18 particular offshore wind farms represent a significant means for reducing the greenhouse  
19 gas emissions of electricity generation. As the offshore wind energy industry has grown,  
20 both the size of wind farms and the size of individual turbines have grown significantly.  
21 Wind farms now represent much larger projects both in terms of the area they cover  
22 and their generational capacity than the early projects of last decade. With many  
23 projects currently in development, it has become increasingly important to ensure that  
24 these wind farms are designed in a sophisticated manner making use of the available  
25 area as efficiently as possible.

26 To meet this need, tools have been developed exploring the optimal placement of  
27 wind turbines, offshore substations, and intra-array cables within an offshore wind farm.  
28 The original work in wind farm layout optimization done by Mosetti et al (1994) laid  
29 the ground work for this field introducing a general approach that following work has  
30 continued to utilize. This approach includes the assessment of both the energy produced  
31 by a wind farm and the cost of the wind farm over the lifetime of the project. More  
32 recent work in the field of offshore wind farm layout optimization has explored the  
33 applicability of different optimization algorithms as well as the inclusion of additional  
34 constraints and more detailed cost functions that a developer may face. The most  
35 frequent optimization algorithm applied to the wind farm layout optimization problem  
36 has been the genetic algorithm with several studies exploring its applicability to the

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37 problem as posed by Mosetti et al (1994) and to more complex extensions (Chen et al,  
38 2013; Couto et al, 2013; Elkinton, 2007; Elkinton et al, 2008; Geem and Hong, 2013;  
39 Grady et al, 2005; Huang, 2009; Mittal, 2010; Shakoor et al, 2016; Zhang et al, 2014). In  
40 a similar vein, recent studies have also explored optimization algorithms such as viral  
41 based optimization (Ituarte-Villarreal and Espiritu, 2011), pattern search (DuPont  
42 and Cagan, 2012), mixed-integer linear programming (Fagerfjäll, 2010), Monte Carlo  
43 method (Marmidis et al, 2008), and random search (Feng and Shen, 2015) applied to  
44 the wind farm layout optimization problem.

45 An optimization algorithm that has emerged as relevant to this problem and has  
46 frequently been deployed for variations on this problem is the particle swarm optimizer  
47 (PSO) (Chowdhury et al, 2012, 2013; Hou et al, 2017; Pookpant and Ongsakul, 2013;  
48 Wan et al, 2010a,b). These existing studies have included various considerations be-  
49 yond the problem originally defined in the seminal work in the field by Mosetti et al  
50 (1994) such as hub height variations, turbine capacity variations, and intra-array cable  
51 routing (Chowdhury et al, 2013; Feng et al, 2016; Hou et al, 2017). However, these  
52 have still not considered several elements that would be important to a real wind farm  
53 developer.

54 The present work, therefore, builds on the standard paradigm in wind farm layout  
55 optimization by considering not only the impact the wind farm layout has on the  
56 energy produced by the wind farm, but also the impact of layout design and turbine  
57 placement on the electrical infrastructure and the wind farm's lifetime costs. Extending  
58 the previous work in this field as well as that of the authors (Pillai et al, 2016b), the  
59 present work presents this optimization problem with the inclusion of three constraint  
60 sets of interest to wind farm developers and applies these to a series of benchmark  
61 cases in which the levelized cost of energy (LCOE), a single metric that considers the  
62 wind farm energy output and costs over the wind farm's lifetime, is used to compare  
63 layouts.

64 This paper introduces increased detail in the evaluation a wind farm layout as well  
65 as additional constraint levels that a developer will face in the design of a real offshore

66 wind farm, thereby striving to capture the impacts the wind farm layout can have  
67 on the LCOE and explores the optimization of wind farm layouts using a cooperative  
68 population based metaheuristic optimization approach<sup>1</sup>, particle swarm optimization.  
69 This therefore involves returning to the key reference work by Mosetti et al (1994)  
70 and Grady et al (2005) and demonstrating that with the increased level of detail in the  
71 evaluation function and the three different constraint sets, a particle swarm optimizer is  
72 not only a relevant optimization algorithm, but is capable of identifying improvements  
73 to the layouts regardless of the size of wind farm.

74 Section 2 introduces the approach of the wind farm layout optimization framework  
75 describing the components and the optimization algorithm deployed. Section 3 intro-  
76 duces the specific cases explored in this paper with the results presented in Section 4.  
77 Section 5 analyses these results before the conclusions of this study are summarized in  
78 Section 6.

## 79 **2 Approach**

80 In general, wind farm layout optimization requires two principal components, one for  
81 assessing the quality of a given wind farm layout and a second for altering the layouts  
82 in an effort to improve them. The standard paradigm for the optimization of wind farm  
83 layouts makes use of the LCOE for assessing the quality of the layout, integrating wind  
84 farm wake models and cost models in order to ascertain the LCOE for a given layout.  
85 In this application, lower LCOE values represent better layouts. The present method-  
86 ology expands on the standard paradigm by including the electrical infrastructure as  
87 the initial step in the determination of the LCOE. The location of the offshore substa-  
88 tions and the design of the intra-array cable network impacts both the annual energy  
89 production (AEP) and the costs and is therefore an important step in assessing the  
90 impact of changes to the turbine layout. The modular design of the approach, shown in  
91 fig. 1, has allowed different wake, cost, and optimization algorithms to be implemented

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<sup>1</sup> A *metaheuristic optimization approach* is a general strategy that is applicable to a wide range of optimization problems by making few or no assumptions about the problem (Burke and Kendall, 2013).

92 as part of the development of the tool. Prior to integration through the optimization  
 93 algorithm, each of the components of the evaluation function have been independently  
 94 validated (Pillai et al, 2016a, 2014, 2015). The optimization algorithm, the PSO in the  
 95 present work, then makes use of the LCOE values in order to advise the next iteration  
 96 of proposed layouts.

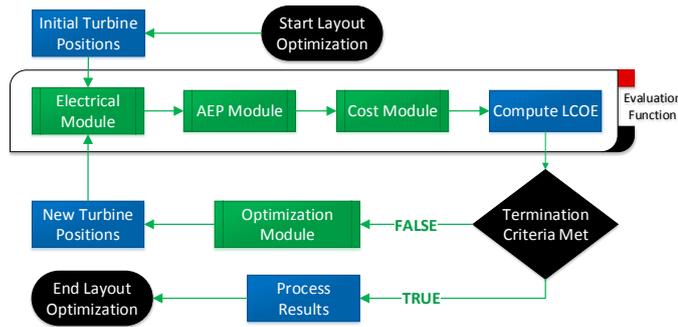


Fig. 1: Modular approach to wind farm layout optimization.

97 Existing offshore wind farms have generally been designed using simple spacing  
 98 rules with turbines laid out along regular grids. Though this is the preferred approach  
 99 from the perspective of search and rescue practitioners and helps to maintain naviga-  
 100 tional routes through the wind farm, it does limit the designs that a developer could  
 101 deploy (NOREL Group, 2014). In order to explore the different levels of constraints  
 102 under which wind farms are currently being designed, allowing greater flexibility to the  
 103 wind farm developer, three constraint sets are implemented each requiring a different  
 104 optimization problem to be implemented. Under these constraints, the wind turbine  
 105 positions are either on a fixed grid defined by the optimizer, one of a set of pre-defined  
 106 allowable turbine positions, or anywhere within the wind farm area that satisfies the  
 107 seabed constraints. These varying degrees of constraint on the wind farm design repre-  
 108 sent the different approaches taken by European regulators in order to offer flexibility  
 109 to the wind farm developers while still accounting for the interests and concerns of  
 110 other marine stakeholders.

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## 111 2.1 Evaluation of LCOE

112 As described, the wind farm layout optimization tool compares layouts on a basis of  
 113 LCOE as this is a single metric which represents the cost effectiveness of a layout.  
 114 The LCOE, measured in energy generation per unit cost, takes into account both the  
 115 lifetime energy production of the wind farm and the lifetime costs of the project and  
 116 is a common metric used by project developers to compare designs and competing  
 117 projects. The energy production and costs are both discounted in order to represent  
 118 the total lifetime energy production and lifetime costs in present value terms. In this  
 119 way, the LCOE represents the ratio of the present value of the inputs to the present  
 120 value of the outputs of the wind farm (Tegen et al, 2012, 2013).

$$LCOE = \frac{\sum_{t=1}^n \frac{C_t}{(1+r)^t}}{\sum_{t=1}^n \frac{AEP}{(1+r)^t}} \quad (1)$$

121 where  $C_t$  is the total costs incurred in year  $t$ ,  $n$  is the project lifetime,  $AEP_t$ , is  
 122 the annual energy production in year  $t$ , and  $r$  is the discount rate of the project.

### 123 2.1.1 Electrical Infrastructure Design

124 The first step in the evaluation of a layout as shown in fig. 1 is the design of the  
 125 necessary electrical infrastructure to support the given layout considering any seabed  
 126 restrictions which may be present at the site. As the electrical infrastructure impacts  
 127 the energy produced by the wind farm due to losses through the electrical system, and  
 128 changes to the electrical infrastructure can impact the project costs, the inclusion of  
 129 this step helps quantify the impact on the LCOE of changes to the wind farm lay-  
 130 out. The methodology for this is described in greater detail by the authors in Pillai  
 131 et al (2015). The majority of existing wind farm layout optimization tools have not  
 132 considered the impact of the turbine layout on either the intra-array cable collection  
 133 networks or substation positions and the impact that these changes will have on the  
 134 LCOE (Chen et al, 2013; Chowdhury et al, 2013; Couto et al, 2013; DuPont and Cagan,

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135 2012; Elkinton, 2007; Elkinton et al, 2008; Geem and Hong, 2013; Grady et al, 2005;  
136 Huang, 2009; Ituarte-Villarreal and Espiritu, 2011; Marmidis et al, 2008; Mosetti et al,  
137 1994; Réthoré et al, 2011; Shakoor et al, 2016; Zhang, 2013; Zhang et al, 2014). The  
138 existing tools that have included this step in the optimization of a wind farm layout,  
139 have, however, omitted bathymetric constraints which a real-world developer would  
140 face (Feng et al, 2016; Hou et al, 2017). Furthermore, existing standalone tools have  
141 explored the optimization of the intra-array cable network for an offshore wind farm  
142 as an independent problem. These approaches have similarly, also not considered the  
143 irregular seabed exclusion areas for intra-array cables which arise from both bathymet-  
144 ric and regulatory constraints that the developer may face at sites. As these exclusion  
145 areas are often non-convex polygons in shape, their accurate inclusion in previous work  
146 has proven challenging (Bauer and Lysgaard, 2015; Dutta and Overbye, 2013; Lindahl  
147 et al, 2013; Rodrigues et al, 2016).

148 The optimization of the electrical infrastructure as developed in Pillai et al (2015)  
149 uses of a series of heuristics and is therefore not guaranteed to identify the proven  
150 optimal solution, however, it has been found to identify good quality solutions in an  
151 acceptable runtime thereby representing a pragmatic approach to this real-world prob-  
152 lem. This optimization process identifies not only the substation positions, and cable  
153 paths given the bathymetric constraints, but also the conductor sizes for each electri-  
154 cal cable in the network. This methodology to optimize the electrical infrastructure is  
155 shown in algorithm 1.

156 The first step in this process is the determination of the substation positions by  
157 clustering the turbine positions. By making use of a modified clustering algorithm  
158 based on *k-means++* (Arthur and Vassilvitskii, 2007), the clustering process is capable  
159 of generating substation positions which adhere to the seabed constraints and their own  
160 capacity constraints while still minimizing the distance to the turbines. From here, a  
161 pathfinding algorithm is executed to generate the fully connected set of cable paths for  
162 the given turbine and substation positions. The pathfinding algorithm is used in order  
163 to consider the seabed obstacles which define where the cables cannot be placed. Using

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**Algorithm 1** Offshore Wind Farm Intra-Array Cable Optimization
 

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**Require:** The turbine positions, the GIS obstacles, and the number of substations

- 1: Given the number of substations assign each turbine to a substation and compute the substation positions using the *Capacitated kmeans++ Clustering*
  - 2: **for** all substations **do**
  - 3:   **for** all turbines assigned to substation **do**
  - 4:     Identify the 10 closest turbines
  - 5:     Identify the constrained shortest path between the turbine and substation using *Delaunay Triangulation Based Navigational Mesh Pathfinding*.
  - 6:     **for** 10 closest turbines **do**
  - 7:       Identify the constrained shortest path between turbine pair using *Delaunay Triangulation Based Navigational Mesh Pathfinding*.
  - 8:     **end for**
  - 9:   **end for**
  - 10: Formulate mixed-integer linear program for substation and its assigned turbines given the 11 possible arcs for each turbine computed above
  - 11: **repeat**
  - 12:   Solve *mixed-integer linear program*
  - 13:   **if** any cables in mixed-integer linear program solution cross **then**
  - 14:     Add individual crossing constraints
  - 15:   **end if**
  - 16: **until** No cables cross
  - 17: **end for**
  - 18: **return** substation positions, cable paths, cable flows, and cable types
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164 the accurate lengths of cables determined by the pathfinding algorithm, a capacitated  
 165 minimum spanning tree (CMST) problem is formulated and solved using a commercial  
 166 MILP solver, Gurobi (Gurobi Optimization Inc., 2015). The solution to the CMST  
 167 identifies which of the cables should be deployed in the final network. In this way, the  
 168 pathfinding step defines all the possible cables to consider and their accurate lengths,  
 169 while the construction of the CMST selects which of these cables should be used to  
 170 minimize the cost of the infrastructure. Following this, the pathfinding algorithm is  
 171 again deployed to determine the export cable path from each substation now consider-  
 172 ing the intra-array network as constraint regions to ensure that the export cable does  
 173 not cross any of the intra-array cables.

174 Using this sub-tool, the electrical constraints of the cables and substations are not  
 175 only taken into account, but seabed features dictating where this equipment cannot  
 176 be placed are also considered. As intra-array cables can exceed £500,000 per kilometre  
 177 installed, it is important that the impact the wind farm layout has on the amount

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178 of cable needed is included in the assessment of the layout's cost (Gaillard, 2015).  
179 Furthermore it is not uncommon for large offshore wind farms to be characterized  
180 by a number of constraint regions which can significantly impact the design of the  
181 intra-array collection network (Pillai et al, 2015).

### 182 *2.1.2 AEP Estimation*

183 It is well understood that any device extracting energy from a natural flux has some  
184 impact on that flux. Wind turbines are no different, and directly behind an operating  
185 wind turbine, the air flow is affected due to the extraction of energy. In this region,  
186 known as the wake, the wind is characterized by reduced speeds and increased levels  
187 of turbulence compared to the conditions upstream of the turbine (Barthelmie et al,  
188 2006, 2009; Makridis and Chick, 2013; Renkema, 2007). The layout of a wind farm can  
189 therefore have a major impact on the wind speeds that each individual wind turbine  
190 within the wind farm experiences and thereby the energy production of the farm as a  
191 whole. As a result of this, it is important for the wind turbine wakes to be accounted for  
192 both when estimating wind farm production figures and the LCOE of a given layout.

193 The calculation of the AEP within this tool is done using an industry standard  
194 analytic approach in which the wake losses are accounted for using the Larsen wake  
195 model (Larsen, 1988). This model has been selected as validation using site data demon-  
196 strated that it represented a good compromise between computational intensity and  
197 accuracy (Gaumond et al, 2012; Pillai et al, 2014). The Jensen wake model used in pre-  
198 vious layout optimization work has been found in validation studies to under-estimate  
199 the AEP and is therefore not as well suited for this work as the Larsen model (Gaumond  
200 et al, 2012).

201 To compute the AEP, each wind speed and direction combination are stepped  
202 through in turn. For each free wind speed and wind direction the analytic wake model  
203 is used to update each turbine's experienced wind speed based on the performance  
204 of all upwind turbines. From this, the wind turbine power curve is used to convert  
205 the incident wind speed to the energy generated under the given conditions. For each

206 wind speed and direction combination, the energy losses through the electrical cable  
 207 network are then computed based on each turbine’s individual contribution to the AEP  
 208 and the total wind farm contribution to AEP under the given free-stream wind speed  
 209 and direction is updated. This total production for each wind speed and direction  
 210 combination is then scaled by the probability of occurrence of this combination for the  
 211 site in question before being added to the AEP.

$$AEP = 8766 \times \sum_{d_i} \sum_{v_i} P(d_i, v_i) \times [E(d_i, v_i) - L(E(d_i, v_i))] \quad (2)$$

212 where  $d_i$  is the wind direction;  $v_i$  is the wind speed;  $P(d_i, v_i)$  is the joint probability  
 213 of  $d_i$  and  $v_i$ ;  $E(d_i, v_i)$  is the energy production for the wind farm for the combination of  
 214 incident wind speed and direction considering the wake losses; and  $L(E(d_i, v_i))$  is the  
 215 electrical losses associated with the wind speed and direction as a result of the intra-  
 216 array cable network. These electrical losses are assessed using an IEC loss calculation  
 217 based on IEC 60228 and IEC 60287 (IEC, 2006a,b). This methodology is similar to  
 218 that used by commercial tools such as WindFarmer and WindPRO which include both  
 219 the losses due to wakes and within the intra-array cable network (DNV GL - Energy,  
 220 2014; Thøgersen, 2005).

### 221 2.1.3 Cost Assessment

222 The final step in the evaluation of the LCOE as shown in fig. 1 is the estimation of  
 223 the costs over the lifetime of the project. Where previous tools have assumed a cost  
 224 which scales with the number of turbines, the approach used in this tool seeks to more  
 225 accurately capture the impact that the wind farm layout has on the lifetime costs.  
 226 Layouts with the same number of turbines may therefore have different costs using  
 227 this model as opposed to the cost model frequently deployed in layout optimization  
 228 which represents the cost as a function of only the number of turbines.

229 The project costs are divided into eight principal cost centres with varying degrees  
 230 of dependency to the wind farm layout as shown in table 1. The capital expenditure  
 231 (CAPEX) elements are incurred either in the construction stage of the project or in

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232 the case of decommissioning at the end of the project life and discounted appropriately  
233 while the operational expenditure (OPEX) elements are incurred in each year of oper-  
234 ation following the construction period and prior to the decommissioning period. The  
235 decommissioning costs are categorized as decommissioning expenditure (DECEX) and  
236 are incurred at the end of life during the decommissioning period during which there  
237 is no OPEX incurred.

238 Each of these cost elements considers not only the turbine positions relative to  
239 one another, but also the turbine positions relative to the construction and O&M  
240 ports, as well as the depth at each individual turbine's position. Relevant cost centres  
241 also consider the vessel parameters, cable parameters, and design parameters of the  
242 substations. The specific cost relationships have been developed in discussions with  
243 wind farm developers and suppliers in order to ensure that the costs are representative  
244 of the costs to be incurred by future projects in European waters and accurately capture  
245 the impact that the turbine layout can have on these costs.

246 *Turbine Supply* The cost associated with the supply of the turbines is based entirely on  
247 a price per turbine supplied by turbine manufactures. This cost is therefore independent  
248 of the layout of the wind farm and factor only of the number of turbines or installed  
249 capacity of the wind farm.

250 *Turbine installation* Using market values for vessel costs and their capacities, the tur-  
251 bine installation costs are modelled by assessing the total amount of time required  
252 to install the turbines at their specific locations within the wind farm. This therefore  
253 includes the calculation of the time required for each installation operation, the travel  
254 time between turbines, and the travel time to and from the construction port. In order  
255 to determine the optimal vessel installation route, the turbines are clustered based on  
256 the capacity of the installation vessel, and for each cluster a shortest path is computed  
257 between the port, each turbine in the cluster, and the port again. This approach there-  
258 fore accurately computes the distance that the vessel must travel over the installation  
259 process. From this, the total time is computed based on assumed weather availability

260 and the costs computed based on the vessel and equipment day rates. The turbine  
261 layout, therefore, has a direct impact on the time needed to travel between turbine  
262 positions as well as to and from the port.

263 *Foundation supply* Foundation costs are found to be highly dependent on the site  
264 conditions where the foundation is to be installed. To account for this dependence,  
265 previous cost models have attempted a bottom up approach based on the soil char-  
266 acteristics at the installation site to model the costs. Unfortunately this approach has  
267 proven difficult to validate for all foundation types (Elkinton, 2007). For the present  
268 tool therefore, a depth dependency has been developed from discussions with manufac-  
269 turers and the specific soil conditions are not included. Detailed bathymetry of a site is  
270 therefore necessary in order to accurately estimate the variation in foundation supply  
271 costs as a function of the turbine layout. As the original cases defined by Mosetti et al  
272 (1994) did not include bathymetric data, a constant depth has been assumed across  
273 the site.

274 *Foundation installation* The foundation installation process like the turbine installa-  
275 tion module is based on estimating the time needed to complete the operations and  
276 converting this time to a cost. Unlike the turbine installation though, this is modelled  
277 as three distinct phases which each uses a different vessel to complete.

278       Regardless of the foundation type (gravity-based, monopile, or jacket), some seabed  
279 preparation is necessary. For a gravity-based foundation this might be the necessary  
280 dredging and levelling of the seabed, while for monopiles and jackets this would more  
281 likely be pre-piling works including surveying and drilling. After this step, the foun-  
282 dations will be installed as a separate operation following which some kind of scour  
283 protection will often be added. The installation of scour protection is again modelled as  
284 a separate step involving a different vessel from either the site preparation or foundation  
285 installation processes. In some conditions, the scour protection will not be necessary,  
286 however, for the time being the present model assumes that all turbines will require  
287 scour protection.

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288 *Intra-array cable costs* The total horizontal length of intra-array cables required is  
289 computed from the intra-array cable optimization tool described earlier. This tool is  
290 described in detail in previous work by the authors (Pillai et al. 2015). This tool has the  
291 support for optimizing the layout for different cable cross-section sizes and therefore can  
292 output not only the total length of cable, but the horizontal lengths required for each  
293 segment and the required cross-section. From this, the intra-array cable cost module  
294 computes the necessary vertical cable and the necessary spare cable before computing  
295 the costs.

296 Following the calculation of the supply cost, the installation cost is computed in a  
297 similar manner to the turbine and foundation installation modules. This is done based  
298 on data available for cable trenching vessels and therefore assumes that all cables are  
299 trenched and buried.

300 *Operations and Maintenance* The operations and maintenance costs are based on a  
301 tool developed by EDF Energy R&D UK Centre which models the anticipated oper-  
302 ations and maintenance cost of a project to vary with the projects distance from the  
303 operations and maintenance port and the capacity of the project. As this term is af-  
304 fected by distance of the wind farm to the operations and maintenance port, this too is  
305 affected by the layout. The operations and maintenance costs are classed as operational  
306 expenditure (OPEX) as these are incurred each year of operation as opposed to the  
307 preceding cost elements which are only incurred during the construction period and  
308 are therefore classed as CAPEX elements.

309 *Decommissioning* The decommissioning costs include the removal of the turbines and  
310 foundations. At the moment, it is unclear what will happen to the transmission and  
311 export cables at the end of a wind farm's life. The model therefore assumes that  
312 these cables are not removed at the time of decommissioning, but simply cut at the  
313 turbines and substation, leaving the buried lengths as they are. The decommissioning  
314 costs are therefore modelled similar to the installation processes with the time each  
315 vessel is required first computed before this is converted to a cost. Like the installation

316 processes it is assumed that the vessels have some finite capacity and must return to  
 317 the decommissioning port during the overall operation. The turbines and foundations  
 318 are assumed to be decommissioned in separate steps requiring separate vessels. Like  
 319 the installation phases, this term is therefore dependent on the turbine positions and  
 320 is affected by the proposed layout.

321 *Offshore Transmission Assets* The final cost element of this cost model is the inclu-  
 322 sion of the offshore transmission asset transfer fees. In the UK, the offshore substation,  
 323 export cables, and onshore substation must be owned and operated by a separate com-  
 324 pany from the wind farm operator. Practically, therefore, most wind farm developers  
 325 build these assets, and then transfer them to a transmission operator before commis-  
 326 sioning the wind farm. As a result, only some of the CAPEX is incurred by the project,  
 327 and the rest is incurred as a component of the transmission fee along with regionally  
 328 based costs set by the network operator, in the UK this is National Grid. Both the  
 329 CAPEX and OPEX components of the Offshore Transmission Owners assets have been  
 330 computed in discussion with National Grid and equipment manufacturers based on the  
 331 capacity of the assets.

Table 1: Cost Contribution to CAPEX and OPEX

Cost Element	CAPEX	OPEX	DECEX	Inclusion of Layout
Turbine Supply	✓	-	-	Low
Turbine Installation	✓	-	-	Medium
Foundation Supply	✓	-	-	Medium
Foundation Installation	✓	-	-	Medium
Intra-Array Cables	✓	-	-	High
Operations and Maintenance (O&M)	-	✓	-	Medium
Decommissioning	-	-	✓	Medium
Offshore Transmission Assets	✓	✓	-	Low

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## 332 2.2 Particle Swarm Optimization

333 The particle swarm optimization algorithm is a population based metaheuristic based  
 334 on the behaviour of flocking birds or shoaling fish (Eberhart and Kennedy, 1995;  
 335 Kennedy and Eberhart, 1995). In this respect, the algorithm treats the candidate so-  
 336 lutions as particles within a swarm which are exploring the search space cooperatively.  
 337 Each particle (solution) changes its position in the search space between iterations  
 338 based on a velocity vector defined by the knowledge of both the swarm's past position  
 339 and the individual particle's historical positions within the search space. For iteration  
 340  $i$  of the process, this velocity,  $v$ , for a given particle is given by:

$$v_i = C_1 \times v_{i-1} + C_2 \times r_1 (p - x_i) + C_3 \times r_2 (g - x_i) \quad (3)$$

341 where  $C_1$ ,  $C_2$ , and  $C_3$  are coefficients representing the weighting of each of the  
 342 contributors determined by tuning the PSO;  $p$  is the best position that the particle has  
 343 historically occupied within the search space;  $g$  is the best position that any individual  
 344 within the swarm as a whole has ever occupied;  $x$  is the solution under consideration;  
 345 and  $r_1$  and  $r_2$  are two random numbers between 0 and 1 selected using a uniform  
 346 distribution. With this velocity the particle's position the next iteration is given by:

$$x_{i+1} = x_i + v_i \quad (4)$$

347 Once each particle's position is updated, the evaluation function is used to de-  
 348 termine the corresponding LCOE for each of the proposed layouts. Each particle's  
 349 historical best position  $p$  is then updated if needed, and the best  $p$  value is used to  
 350 define  $g$ . These updated  $p$  and  $g$  values are needed in the determination of the updated  
 351 particle velocities for the next iteration of the process.

352 Compared to the genetic algorithm or alternate metaheuristics which have been  
 353 applied to the wind farm layout optimization problem, the PSO is of interest as in op-  
 354 timization benchmarking studies it has been found to find high quality solutions in less  
 355 time than a similar genetic algorithm (Eberhart et al, 2001; Hassan et al, 2005). Given

the complexity of future wind farms, this is of interest to wind farm developers as the PSO could therefore identify better solutions than the industry standard approaches using commercial software tools thereby leading to more efficient wind farm layouts. Furthermore, where the genetic algorithm is seen as a competitive metaheuristic in which individual solutions compete for survival, the PSO fosters a cooperative environment where the individual solutions directly impact one another. In this way, all members of the swarm are made aware of the improvements found by each individual particle, using this information to inform their future movements within the search space.

The parameters of the present PSO are given in table 2. Due to the available computational power, this study used a constant swarm size of 100 particles. In order to ensure that the velocity vector does not take a particle outside of the search space, a dynamic velocity clamping approach was used in which velocity limits are imposed in each direction based on the location of the particle. This is similar to the trajectory constriction approach described by Clerc and Kennedy (2002); Van Den Bergh and Engelbrecht (2006). For the binary constraints described below, a binary implementation of the PSO in which all decision variables are binary variables is necessary. As the velocity in the binary implementation must correspond to a specific decision variable being either a 1 or a 0, a velocity transfer function is required to convert the velocity for each decision variable into a probability that the decision variable should be a 1.

Table 2: Particle Swarm Parameters

Parameter	Description
Swarm Size	100
Velocity Clamping	Dynamic
Velocity Transfer Function (Binary Encoding)	$T(x) = \left  \frac{2}{\pi} \times \arctan \left( x \cdot \frac{\pi}{2} \right) \right $
Neighbourhood Topology	Global (gBest)
Stop Criteria	Diversity <10%
	Maximum generations reached
	No improvement over 50 generations

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376 In the original study by Mosetti et al (1994), the wind farm area was discretized  
377 into 100 allowable turbine positions. The optimizer is therefore tasked with the selec-  
378 tion of which of these positions to use for the deployed wind turbines. This therefore  
379 represents a constraint on the turbine placement and it would be expected that better  
380 layouts could be achieved if this constraint was relaxed. To explore this, three different  
381 constraints on the turbine placement are used in the present study:

- 382 1. **Array constraints** - The turbine positions are constrained to being on a regular  
383 grid with constant downwind and crosswind spacings. The decision variables of  
384 the optimization problem define the spacing and orientation of the regular grid of  
385 turbine positions with constant downwind and crosswind spacing throughout the  
386 site.
- 387 2. **Binary constraints** - The turbine positions are limited to being one of a predefined  
388 set of allowable turbine positions. For the present study, the wind farm area is  
389 discretized into 100 allowable turbine positions as defined Mosetti et al (1994) and  
390 the decision variables of the optimization problem are binary variables representing  
391 the presence of a turbine in a particular cell. This represents the case in which the  
392 wind farm developer, regulator, and stakeholders define a set of acceptable turbine  
393 positions and the wind farm is designed by selecting turbine positions from this  
394 set.
- 395 3. **Continuous constraints** - The turbine positions can be anywhere within the wind  
396 farm boundary that is technically feasible. The decision variables directly define the  
397 turbine coordinates and may therefore occupy any value within the wind farm area.  
398 This represents a situation in which the wind farm developer is free to design the  
399 wind farm as they see best limited only by the technical constraints of the site.

400 The three approaches represent different ways in which the problem can be defined  
401 all of which are used by wind farm developers to design and explore the available  
402 options in the design of an offshore wind farm. The array and binary constraint sets are  
403 of interest to a wind farm developer in regions where the regulator imposes some degree  
404 of symmetry as a result of navigational and search and rescue safety concerns (NOREL

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405 Group, 2014). As the three constraint sets have fundamentally different degrees of  
406 complexity and represent different design spaces the optimizers were tuned individually  
407 for each of the problems in an attempt to maximize the performance though the same  
408 swarm size was used for all cases. Regardless of the placement constraints used, the  
409 technical seabed constraints such as the position of wrecks, unexploded ordnance, and  
410 the seabed slope are considered. For all three constraint sets, a minimum separation  
411 constraint is applied to ensure that turbines do not risk colliding and the wind farm  
412 boundary explicitly defines the limits of the wind farm.

### 413 **3 Definition of Cases**

414 In the development of layout optimization tools three case studies have been defined by  
415 Mosetti et al (1994). These three cases have been commonly used in order to evaluate  
416 the performance and demonstrate the capabilities of wind farm layout optimization  
417 tools. In order to demonstrate the capabilities of the present framework, which makes  
418 use of a more detailed layout evaluation function, the three cases are approached us-  
419 ing the original constraints as well as under two different sets of relaxed constraints.  
420 Through this, the capabilities of the present framework using a PSO are highlighted.

421 The three cases all consider a 2 km by 2 km area in which turbines must be placed,  
422 however, they differ with regards to the wind resource. Case one considers a case  
423 of *constant wind speed and constant wind direction* in which the wind is constantly  
424  $12 \text{ m s}^{-1}$  and from the  $10^\circ$  sector centred on  $0^\circ$ . The second case is described as the  
425 case of *constant wind speed and variable direction* in which the wind is again constantly  
426  $12 \text{ m s}^{-1}$ , but now has an equal probability of blowing from any of the 36 discrete  
427 wind directions. Finally, the third case, the case of *variable wind speed and direction*,  
428 describes a case in which both the wind speed and wind direction are variable and  
429 most closely resembles a true wind farm. All three cases describe the resource using  
430 36 discrete wind directions which are each used in the calculation of the AEP and  
431 the modelling of the wakes in the evaluation function. Validation studies of analytic  
432 wake models have found that these models are not necessarily more accurate when

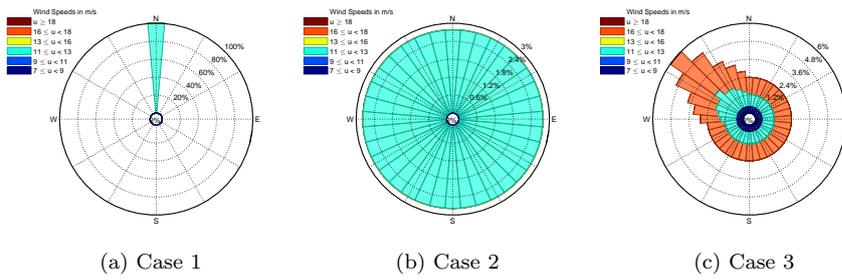


Fig. 2: Wind roses for the three different resource cases.

433 using narrower wind direction sectors, and discrete wind sectors of  $10^\circ$  to  $30^\circ$  in size  
 434 should be used when deploying analytic wake models such as the Jensen or Larsen  
 435 models (Gaumond et al, 2013; Pillai et al, 2014).

436 The original cases do not define the water depth nor are the locations of the relevant  
 437 ports defined. In order to allow comparison with existing results for these case studies,  
 438 the water depth has been assumed constant across the site and the ports have been  
 439 placed far away relative to the size of the wind farm.

#### 440 4 Results

441 In order to demonstrate the capabilities of the present framework using a PSO, the  
 442 final layouts from the original study by Mosetti et al (1994) and the final layouts from  
 443 a subsequent study by Grady et al (2005) are evaluated using the present evaluation  
 444 function in order to offer a fair comparison to the new layouts proposed. These two  
 445 studies used different numbers of turbines for each resource case and therefore cannot  
 446 be directly compared to one another. Likewise, much of the literature has also allowed  
 447 the number of turbines to vary thereby making direct comparisons challenging. In the  
 448 present framework, the number of turbines is fixed thereby allowing a direct comparison  
 449 on the same number of turbines against both the reference case study and the different  
 450 constraint sets.

451 The original layouts produced in the studies by Mosetti et al (1994) and Grady  
 452 et al (2005) for all three resource cases are shown in fig. 3. The studies performed by

453 Mosetti et al (1994) and Grady et al (2005) both allowed the number of turbines to  
 454 vary and therefore for each of the resource cases, the two studies present different wind  
 455 farm sizes. In the present study, each wind farm resource is executed with all three sets  
 456 of constraints and at same the wind farm sizes as reported in the two past studies in  
 457 order to fairly compare to the reference studies. The binary constraint set, represents  
 458 the most similar case to the problem originally defined by Mosetti et al (1994), however,  
 459 the present tool uses a fixed number of turbines, while the original studies allowed this  
 460 to change. Each of the presented optimization results represents the converged results  
 461 after a maximum of 100 generations. In general, less than 60 generations were required  
 462 to reach the converged results presented.

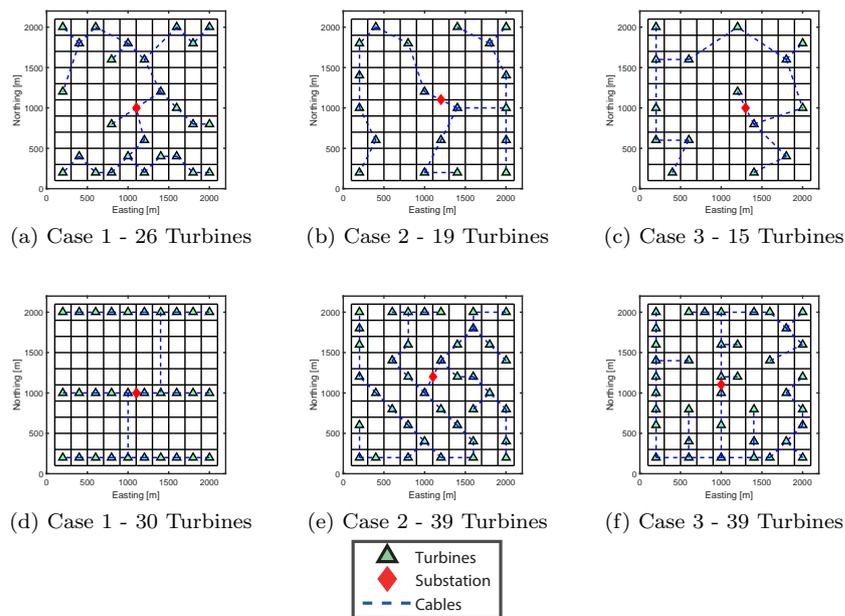


Fig. 3: Original optimized layouts proposed by Mosetti et al (1994) on the top row and Grady et al (2005) on the bottom row for the three resource cases.

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463 4.1 Case 1: Constant Wind Speed, Constant Direction

464 The results presented in table 3 shows the outputs from re-evaluating the original  
465 layouts proposed in the previous studies (Grady et al, 2005; Mosetti et al, 1994) as  
466 well as the outputs from the execution of the PSO for this case. As the developed  
467 method uses the number of turbines as an input to the optimization process, it was  
468 necessary to execute the optimizer for two different wind farm sizes corresponding  
469 to the studies originally performed by Mosetti et al (1994) and Grady et al (2005)  
470 respectively, allowing the results to be directly compared to these past studies (shown  
471 in figs. 3a and 3d). As described above, each of the wind farm sizes was run with three  
472 different types of constraints corresponding to different requirements on the placement  
473 of the turbines.

Table 3: Layout Optimization Results: Constant Wind Speed, Constant Direction

Study	Number of Turbines	Lifetime Cost [£]	AEP [MWh]	LCOE [£/MWh]
Mosetti et al (1994)	26	$4.42 \times 10^8$	$9.90 \times 10^4$	522.87
Array Constraints	26	$4.39 \times 10^8$	$1.18 \times 10^5$	<b>434.87</b>
Binary Constraints	26	$4.41 \times 10^8$	$1.01 \times 10^5$	510.46
Continuous Constraints	26	$4.42 \times 10^8$	$1.16 \times 10^5$	447.18
Grady et al (2005)	30	$4.77 \times 10^8$	$1.13 \times 10^5$	496.29
Array Constraints	30	$4.76 \times 10^8$	$1.33 \times 10^5$	<b>419.61</b>
Binary Constraints	30	$4.77 \times 10^8$	$1.13 \times 10^5$	496.29
Continuous Constraints	30	$4.78 \times 10^8$	$1.33 \times 10^5$	421.64

474 From the results presented in table 3 it can be observed that for both wind farm  
475 sizes, the PSO either finds improvements or the same solution proposed by the refer-  
476 ences cases regardless of which constraint set was used. Specifically, using the binary  
477 constraint set for the larger wind farm size resulted in the same layout presented by  
478 Grady et al (2005) whereas for each of the other five cases, improvements were high-  
479 lighted compared to the relevant reference case. As is highlighted in table 3, for both  
480 wind farm sizes, the variation in costs as a result of the changes in layout are very small  
481 as the micrositing within the  $4 \text{ km}^2$  wind farm area results in very minimal changes

482 in the installation costs. In fact, as the port position was not defined in the original  
 483 case, it was necessary to place the port very far away relative to the size of the wind  
 484 farm in order to remove any bias to the port's position. As a result of this, there are  
 485 relatively large transit times to the wind farm included in each installation cost which  
 486 are unaffected by the wind farm layout, but a function of the wind farm's distance  
 487 from the installation port.

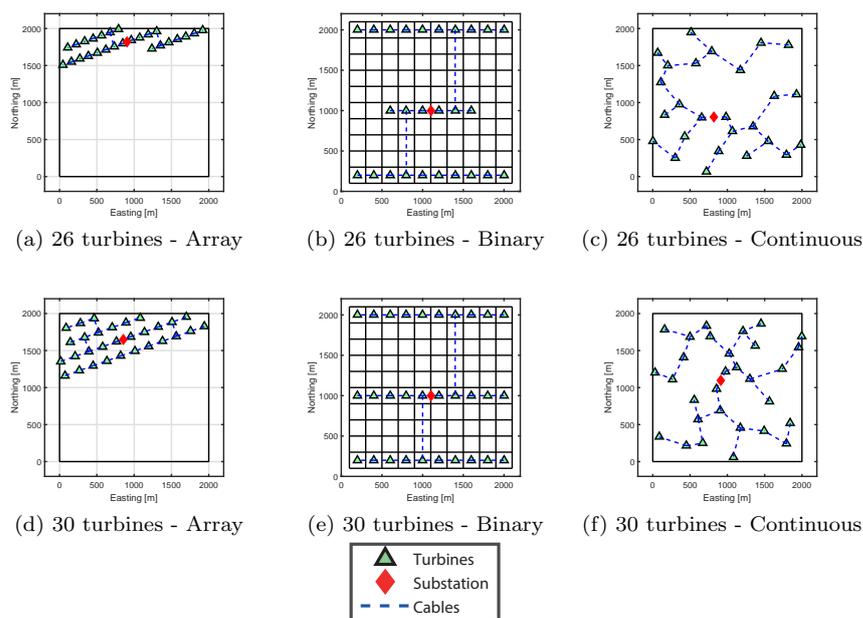


Fig. 4: Optimized layouts for the case of a constant wind speed and constant direction with 26 turbines (top row) and 30 turbines (bottom row) using both optimization algorithms and all three constraint sets.

#### 488 4.2 Case 2: Constant Wind Speed, Variable Direction

489 The results for each of the constraint sets and wind farm sizes are summarized in table 4  
 490 and the corresponding layouts are shown in fig. 5. The original layouts proposed by  
 491 the reference studies are shown in figs. 3b and 3e. From table 4, it can be seen that

492 similar to the results for Case 1, the newly developed layout optimization framework  
 493 for offshore wind farms is capable of identifying improvements using the PSO under  
 494 all three constraint sets for both wind farm sizes.

Table 4: Layout Optimization Results: Constant Wind Speed, Variable Direction

Study	Number of Turbines	Lifetime Cost [£]	AEP [MWh]	LCOE [£/MWh]
Mosetti et al (1994)	19	$3.77 \times 10^8$	$8.17 \times 10^4$	540.25
Array Constraints	19	$3.77 \times 10^8$	$8.32 \times 10^4$	<b>530.79</b>
Binary Constraints	19	$3.77 \times 10^8$	$8.21 \times 10^4$	537.49
Continuous Constraints	19	$3.77 \times 10^8$	$8.19 \times 10^4$	538.29
Grady et al (2005)	39	$5.62 \times 10^8$	$1.57 \times 10^5$	419.13
Array Constraints	39	$5.61 \times 10^8$	$1.61 \times 10^5$	<b>408.07</b>
Binary Constraints	39	$5.61 \times 10^8$	$1.59 \times 10^5$	413.00
Continuous Constraints	39	$5.62 \times 10^8$	$1.58 \times 10^5$	417.29

#### 495 4.3 Case 3: Variable Wind Speed, Variable Direction

496 The results of executing the current framework with the PSO are found in table 5  
 497 with the corresponding layouts plotted in fig. 6 and the original reference layouts in  
 498 figs. 3c and 3f. Similar to the previous cases, the PSO using any of the constraint sets  
 499 was capable of identifying improved layouts with regards to the LCOE. Similar to the  
 500 previous cases, the best results were found using the array constraints.

## 501 5 Discussion

502 Using the present tool, cost variations as a result of changes to the wind farm layout  
 503 are captured and included in the calculation of the layout's LCOE. For a small wind  
 504 farm such as those considered here, it is, however, the increase in AEP which drives  
 505 the decreases in LCOE, which is why for many cases an increase in lifetime cost is  
 506 observed, however, the corresponding increase in AEP is sufficiently large to still result  
 507 in a net reduction of the LCOE.

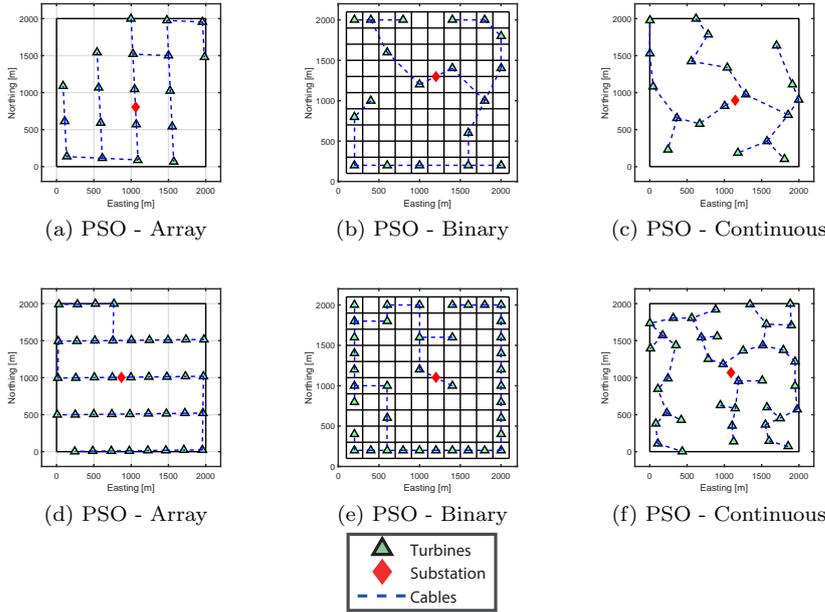


Fig. 5: Optimized layout for the case of a constant wind speed and variable direction with 19 and 39 turbines using both optimization algorithms and all three constraint sets.

Table 5: Layout Optimization Results: Variable Wind Speed, Variable Direction

Study	Number of Turbines	Lifetime Cost [£]	AEP [MWh]	LCOE [£/MWh]
Mosetti et al (1994)	15	$3.40 \times 10^8$	$6.89 \times 10^4$	576.94
Array Constraints	15	$3.39 \times 10^8$	$6.93 \times 10^4$	<b>571.51</b>
Binary Constraints	15	$3.39 \times 10^8$	$6.91 \times 10^4$	573.87
Continuous Constraints	15	$3.39 \times 10^8$	$6.91 \times 10^4$	574.22
Grady et al (2005)	39	$5.62 \times 10^8$	$1.74 \times 10^5$	377.14
Array Constraints	39	$5.63 \times 10^8$	$1.75 \times 10^5$	<b>375.50</b>
Binary Constraints	39	$5.62 \times 10^8$	$1.75 \times 10^5$	376.72
Continuous Constraints	39	$5.62 \times 10^8$	$1.75 \times 10^5$	376.72

508 As would be expected, relaxing the turbine positioning constraints by designing  
509 arrays within the boundary or by treating the wind farm area as a continuous do-  
510 main, results in significant improvements in the LCOE as the shape of the layout can  
511 be designed to best utilize the characteristics of the site. Somewhat surprisingly, the

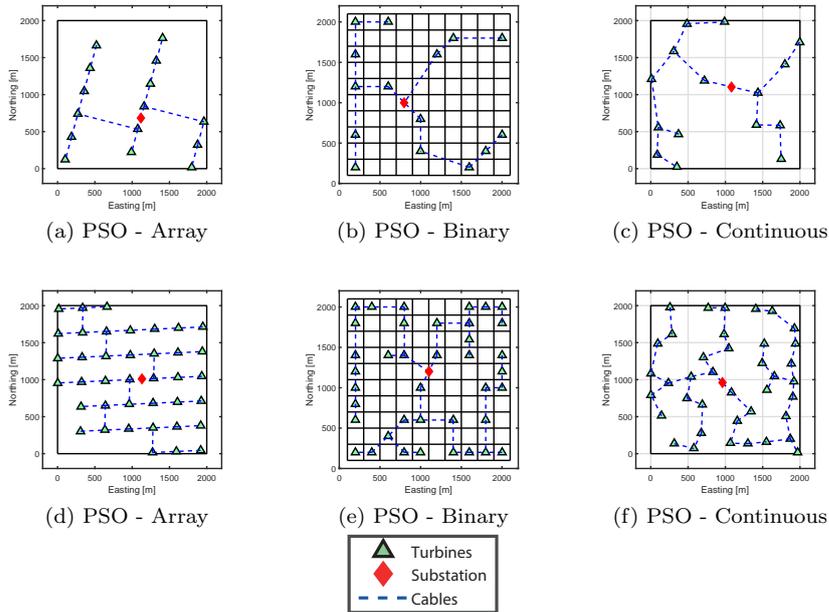


Fig. 6: Optimized layout for the case of a variable wind speed and variable direction with 15 and 39 turbines using both optimization algorithms and all three constraint sets.

512 continuous optimizer which represents the most unconstrained case was unable to con-  
 513 sistently find improvements over the array optimizer. However, both were consistently  
 514 able to find improvements compared to the binary optimizer which made use of the  
 515 discretized wind farm area. Interestingly, the array optimizer appears more capable  
 516 than the others to adjust the shape of the wind farm layout to take advantage of the  
 517 wind resource.

518 As the array optimizer and continuous optimizer did not identify similar solutions  
 519 it suggests that further tuning of the PSO is necessary in order to ensure that the  
 520 optimizers are not prematurely converging to a local solution. Furthermore, given the  
 521 results it indicates that moving from the binary or array optimizers to the continuous  
 522 optimizer increases the size of the problem quite significantly. In the present case,  
 523 all three constraint sets were solved using the same size of swarm, however, it might  
 524 be more prudent for the swarm size to change depending on which constraint set is

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525 used thereby allowing the more complex problem to be solved with a larger swarm  
526 in order to avoid premature convergence. With a sufficiently large swarm, it should  
527 be possible for the PSO to converge to a higher quality solution closer to that of  
528 the global optimum. It should be noted, however, that metaheuristic algorithms like  
529 the PSO cannot guarantee, especially for a complex objective function such as the  
530 LCOE, that the optimization process will converge to the global optimum. Given the  
531 computational power allocated for this study, however, it was not possible to execute  
532 the optimizers with larger swarms. With swarms of 100 individuals as used in this  
533 study, each optimization took between one and three days to execute depending on the  
534 wind farm size and the selected constraints when executed on a desktop computer with  
535 an Intel Xeon 8-CPU processor rated at 3.3 GHz. As the three different constraint sets  
536 lead to three different instances of the problem with different decision variables, the  
537 design spaces are not directly comparable and each of the three optimizers should be  
538 tuned independently in order to ensure the best performance.

539 Looking at Case 1, it can be seen that both the binary and continuous optimizers use  
540 the majority of the available space, while the array optimizer is capable of identifying  
541 that it should sacrifice a close spacing in the direction perpendicular to the single  
542 wind direction. The binary optimizer is unable to find a similar solution due to the  
543 resolution of the discrete grid used in the binary optimization. This suggests that the  
544 discretization of the wind farm area should be done at a higher resolution to afford  
545 the optimizer a greater degree of flexibility. The present study used the 100 allowable  
546 turbine positions as this is what had been used in past studies. Increasing the number  
547 of allowable turbine positions through a higher resolution would, however, increase  
548 the size of the problem and potentially slow the rate of convergence. The continuous  
549 optimizer should, however, be capable of identifying a similar solution, and the fact  
550 that it does not highlights that further work remains to be done with this optimizer in  
551 order to ensure that high quality solutions are reached.

552 The results from Case 2, however, indicate that the binary optimizer is placing more  
553 turbines on the edge of the wind farm in order to take advantage of the symmetrical

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554 wind resource, especially in the larger wind farm case. For this resource case and the  
555 larger wind farm, compared to the full continuous optimizer the binary optimizer results  
556 in better AEP values, demonstrating that additional constraints on the problem can  
557 reduce the search space without sacrificing the quality of the ultimate layouts.

558 Limiting the turbine positions to 100 possible positions significantly constrained  
559 the search space such that the solutions had inferior fitness values compared to the  
560 more relaxed constraint sets. This indicates that moving to the binary constraints with  
561 a discretized set of turbine positions over-constrains the problem, eliminating high  
562 quality valid solutions. Considering the Mosetti cases, the impact of this on the LCOE  
563 varied from £1 per MWh to £70 per MWh increases, corresponding to 0-16% potential  
564 improvements in LCOE from relaxing the constraints. Given some of the assumptions,  
565 the percentage difference is smaller than it would be if this were a real site, as there  
566 are some fixed costs which are intentionally overestimated. As described earlier, the  
567 port location was defined as far away relative to the size of the wind farm in order to  
568 avoid the optimizer clustering turbines close to the installation port. The installation  
569 costs are therefore larger than they would be for a real case thereby increasing the  
570 LCOE. For these cases, it is therefore more valuable to analyse the absolute difference  
571 in LCOE rather than the percentage reduction.

572 Interestingly, Case 3 which represents the most realistic wind resource case finds  
573 very small variations in AEP across the three different constraint sets demonstrating  
574 that for a more varied wind speed and wind direction combinations all three constraint  
575 sets have merit and are capable of finding good solutions. The choice of which constraint  
576 set to use therefore becomes a function of what constraints are imposed on the site  
577 developer by consenting agencies or other stakeholders. The results from this case  
578 also demonstrate that there are several different layouts with similar AEP, cost and  
579 LCOE values showing the complexity of the search space. Given that there are different  
580 layouts which can result in similar solutions the tuning of the optimizer becomes more  
581 important and further work will need to further explore this in order to ensure that the

582 optimization process is not overlooking significant improvements and that the optimizer  
583 is operating in appropriate time scales.

## 584 **6 Conclusion**

585 This paper has presented the first results of an extended wind farm layout optimiza-  
586 tion framework making use of a more detailed LCOE evaluation function than existing  
587 layout optimization tools. This framework which makes use of a previously validated  
588 LCOE evaluation function has been applied to three different case studies using three  
589 different sets of placement constraints and two different wind farm sizes for each re-  
590 source case in order to highlight both the applicability of a PSO given the increased  
591 detail and the improvements that can be made relative to the reference studies. The  
592 PSO applied to these three benchmark case studies have presented layouts with im-  
593 proved LCOE compared to past studies using a genetic algorithm. Furthermore, the  
594 results shown here indicate that the PSO is of interest to this area of research as the  
595 results can be obtained at a lower computational cost compared to a genetic algorithm.

596 By using multiple constraint sets it is also shown that by limiting the optimizer to  
597 create gridded layouts does not result in poor solutions, though the observed trends  
598 highlight the need for further tuning of the PSO in order to insure that the optimizer  
599 does not prematurely converge. Further work should explore both using multiple runs  
600 rather than single runs in order to avoid any seeding bias as well as using additional  
601 computational power thereby allowing larger swarms to be tested.

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