Online Appendix A: Analytical model

As described in the main text, the optimal level of female investment u_f^* can be found by solving the equation

$$\frac{\partial W}{\partial u_f} = \frac{\partial b_t(u_f,q_m)}{\partial u_f} + \alpha \frac{\partial c(u_f,q_m)}{\partial u_f} \frac{\partial \bar{b}_{t+1}(\tilde{u}_f, c(u_f,q_m))}{\partial c} = 0, \tag{A1}$$

where the bar in $\bar{b}'_{t+1}$ denotes the average over future male qualities $\tilde{q}_{m,i}$, weighted by the probability that the female pairs with each quality male, p_i. Qualities of future males do not depend on female investment in our model and thus can be treated as constants when calculating a female's best response to *current* male quality q_m.

A1: Absence or presence of differential allocation

We see that DA is expected when eq. (A1) holds and $\frac{\partial^2 W}{\partial q_m \partial u_f} \neq 0$. In the **Benefit**-scenarios, the current male's quality does not affect the female's future fecundity, i.e. $c(u_f,q_m) = c(u_f)$, and neither does the female's investment in the current time step affect her future mating opportunities. Therefore, we are only interested in whether the first term in the optimization equation (A1), $\frac{\partial b_t(u_f,q_m)}{\partial u_f}$, depends on q_m.

In the **Benefit-Elevation** scenario (figure 1 *A*), male quality only contributes additively to female current fecundity, we can write $b_t(u_f,q_m) = b_t(u_f) + g(q_m)$, where g is a positive function that reflects the strength with which the current male contributes to the female's offspring. Then, $\frac{\partial b_t(u_f,q_m)}{\partial u_f} = b'_t(u_f)$, which does not depend on q_m, and we do not expect DA to arise in a Benefit-Elevation scenario.

In the **Benefit-Slope** scenario (figure 1 *B*), female investment and male quality determine offspring fitness in a multiplicative fashion, so we write $b_t(u_f,q_m) = b_t(u_f) \times g(q_m)$ so that increasing male quality effectively accelerates the returns on female investment. Consequently, the $\frac{\partial b_t(u_f,q_m)}{\partial u_f}$ remains a function of both u_f and q_m, and DA should therefore occur.

The **Benefit-Position** scenario (figure 1 *C*) has the same expression for finding u_f^* as Benefit-Slope, the only mathematical difference between the scenarios being that Benefit-Position assumes decelerating returns as male quality increases, $\frac{\partial b'_t(u_f,q_m)}{\partial q_m} < 0$. Therefore, we expect DA here as well (but see section 2: Direction of DA).

In the **Cost**-scenarios, the current male's quality does not affect the female's current offspring fitness benefits, $b_t(u_f,q_m) = b_t(u_f)$, so only the dependence of the second term in eq. (A1) on q_m is relevant.

For the **Cost-Elevation** scenario, male quality affects $c(u_f,q_m) = c(u_f) - k(q_m)$, where the positive function $k(q_m)$ reflects how strongly male quality reduces the female cost function, as a current male of high quality is assumed to reduce future costs to females. Whether or not such a scenario produces DA depends on the shape of the future benefit function, since future fecundity b_{t+1} is affected by q_m through $k(q_m)$, and its derivative may also do so. If it depends linearly on $k(q_m)$, for example

$$b_{t+1}\left(\tilde{u}_f, c\left(u_f\right) - k(q_m), \tilde{q}_{m,i}\right) = \tilde{u}_f - c\left(u_f\right) + k(q_m) + \tilde{q}_{m,i},$$

then the derivative with respect to u_f becomes

$$\bar{b}'_{t+1}\left(\tilde{u}_f, c\left(u_f\right) - k(q_m), \tilde{q}_{m,i}\right) = -c'(u_f)$$

and DA does not occur. However, for a nonlinear benefit function, such as the four-parameter logistic function we use below, the derivative $\bar{b}'_{t+1}$ is a function of both u_f and q_m, so Cost-Elevation results in DA, despite the fact that the marginal costs of current reproduction $c'\left(u_f, q_m\right) = c'\left(u_f\right)$ are independent of the quality q_m of the current male. Rather, the non-linearity in the benefit function will allow any absolute change in costs to affect the marginal benefits of future reproduction $\bar{b}'_{t+1}$, and DA is thus favored.

In the **Cost-Slope** and **Cost-Position** scenarios, female investment and male quality affect the female's cost function multiplicatively, so we write $c(u_f,u_m) = c(u_f) \times k(q_m)$. The marginal cost c' then depends on both u_f and q_m, so DA will always be expected, whether the benefit function is linear or nonlinear. The only mathematical difference between the two scenarios is the extra assumption in Cost-Position that better male quality pushes the female down on her cost function, and costs increase less steeply further down the cost function, so we have $\frac{\partial c'(u_f,q_m)}{\partial q_m} < 0$.

A2: Direction of DA

We use second-order conditions to investigate direction of DA (positive or negative), calculating female investment u_f as the female's best response $r(q_m)$ to current male quality q_m, after which we substitute for $u_f \equiv r(q_m)$ in equation A1 and take the derivative of $\left(\partial W / \partial u_f\right)\big|_{u_f = r(q_m)}$ with respect to q_m, which will allow us to solve for $r'(q_m)$. We then obtain

$$\frac{\partial}{\partial q_m}\left[\left.\frac{\partial W}{\partial u_f}\right|_{u_f=r(q_m)}\right] = \frac{\partial}{\partial q_m}\left[\frac{\partial b_t(r(q_m),q_m)}{\partial u_f} + \alpha\frac{\partial c(r(q_m),q_m)}{\partial u_f}\frac{\partial \overline{b}_{t+1}(\tilde{u}_f,c(r(q_m),q_m))}{\partial c}\right]$$

$$= r'(q_m)\frac{\partial^2 b_t(r(q_m),q_m)}{\partial r\,\partial u_f} + \frac{\partial^2 b_t(r(q_m),q_m)}{\partial q_m\,\partial u_f}$$

$$+ \quad \alpha\left[r'(q_m)\frac{\partial^2 c(r(q_m),q_m)}{\partial r\,\partial u_f} + \frac{\partial^2 c(r(q_m),q_m)}{\partial q_m\,\partial u_f}\right]\frac{\partial \overline{b}_{t+1}(\tilde{u}_f,c(r(q_m),q_m))}{\partial c}$$

$$+ \quad \alpha\frac{\partial c(r(q_m),q_m)}{\partial u_f}\left[r'(q_m)\frac{\partial c(r(q_m),q_m)}{\partial r} + \frac{\partial c(r(q_m),q_m)}{\partial q_m}\right]\frac{\partial^2 \overline{b}_{t+1}(\tilde{u}_f,c(r(q_m),q_m))}{\partial c^2} = 0$$

(A2)

Solving for $r'(q_m)$ and dropping other obvious notation, we obtain

$$r'(q_m) = \left(\frac{\partial^2 b_t}{\partial q_m\,\partial u_f} + \alpha\frac{\partial^2 c}{\partial q_m\,\partial u_f}\frac{\partial \overline{b}_{t+1}}{\partial c} + \alpha\frac{\partial c}{\partial u_f}\frac{\partial c}{\partial q_m}\frac{\partial^2 \overline{b}_{t+1}}{\partial c^2}\right)$$
$$/\left(-\frac{\partial^2 b_t}{\partial r\,\partial u_f} - \alpha\frac{\partial^2 c}{\partial r\,\partial u_f}\frac{\partial \overline{b}_{t+1}}{\partial c} - \frac{\partial c}{\partial u_f}\frac{\partial c}{\partial r}\frac{\partial^2 \overline{b}_{t+1}}{\partial c^2}\right),$$

(A3)

where the survival probability α is a positive constant and hence does not affect the sign of the expression above. We show below that the denominator is always positive (see section A2b). The sign of the numerator (and hence the direction of DA) depends, however, on the specific scenario.

A2a. The sign of the numerator of eq. A3.

Benefit-Slope: positive DA – Because costs c only depend on u_f and not on q_m, all terms in the numerator are 0, except the first. We have $\partial^2 b_t/\partial q_m\,\partial u_f > 0$, as increasing male quality increases the slope of b_t with respect to female investment (see figure 1 *B*). Consequently, we have $\partial r(q_m)/\partial q_m > 0$ (positive DA).

Benefit-Position: negative DA – Again, because costs c only depend on u_f and not on q_m, all terms in the numerator are 0, except the first. We have $\partial^2 b_t/\partial q_m\,\partial u_f < 0$, as increasing male quality now decreases the slope of b_t with respect to female investment (see figure 1 *C*). Consequently, we have $\partial r(q_m)/\partial q_m < 0$ (negative DA).

All cost scenarios: positive DA – Regarding the first term in the numerator, we have $\partial^2 b_t/\partial q_m\,\partial u_f = 0$, because male quality has no effect on b_t for all cost scenarios (see Figure 1 *D-F*). Regarding the second term, we have $\partial \overline{b}_{t+1}/\partial \mathrm{c} < 0$ as increasing costs reduce future fecundity. Next, we have $\partial^2 c/\partial q_m\,\partial u_f \leq 0$, as costs are either linear in u_f for the cost elevation and cost slope scenario, in which case $\partial^2 c/\partial q_m\,\partial u_f = 0$, while $\partial^2 c/\partial q_m\,\partial u_f < 0$ holds for the cost position scenario, as increasing male quality reduces the marginal increase of female costs with increased female investment. Hence, the second term is positive. Regarding the third term, we have $\partial^2 \bar{b}_{t+1}/\partial c^2 \leq 0$, as future female benefits either decrease linearly or in an accelerating fashion with increasing costs. In addition, $\partial c/\partial u_f > 0$ as costs increase with increasing female investment, and $\partial c/\partial q_m < 0$ as costs decrease with increasing male quality. Hence, also the third term is positive. Consequently, as we have already shown in section A1 that the numerator is nonzero for the cost scenarios, the numerator is thus strictly positive. Hence, all cost scenarios result in $\partial r(q_m)/\partial q_m > 0$ (positive DA).

A2b. The denominator of eq. A3 is positive

We have $\partial^2 b_t/\partial r\,\partial u_f \leq 0$, because current fecundity is either linear in u_f, or fecundity is a decelerating function of u_f (and hence r). Regarding the second term in the numerator, we have $\partial^2 c/\partial r\,\partial u_f \geq 0$ because costs either increase linearly or acceleratingly with female investment. In addition, we have $\partial \bar{b}_{t+1}/\partial c < 0$ because increasing costs reduce future fecundity. With regards to the third term, we have $\partial c/\partial u_f > 0$ and $\partial c/\partial r > 0$ because costs increase with increasing levels of female investment, while we have $\partial^2 \bar{b}_{t+1}/\partial c^2 \leq 0$, because future female benefits either decrease linearly or in an accelerating fashion with increasing costs. Ignoring the degenerate case where the denominator is 0 (in which case $\partial r(q_m)/\partial q_m$ has no solution), the denominator is thus positive.

A3. Cost of current reproduction affects female survival instead of future fecundity

So far, we have assumed that costs of current reproduction $c(u_f,q_m)$ reduce a female's amount of energy that can be allocated to future reproduction. Here we consider an alternative scenario in which costs of current reproduction affect a female's survival to the next time step $\alpha \equiv \alpha\left[c\left(u_f, q_m\right)\right]$ instead, and where future fecundity $b_{t+1} \equiv b_{t+1}\left(\tilde{u}_f, \tilde{q}_{m,i}\right)$ is now only affected by a female's future investment $\tilde{u}_f$ and future male quality $\tilde{q}_{m,i}$. We then have

$$\frac{\partial W}{\partial u_f} = \frac{\partial b_t(u_f,q_m)}{\partial u_f} + \frac{\partial c(u_f,q_m)}{\partial u_f}\frac{\partial \alpha\left[c\left(u_f,q_m\right)\right]}{\partial c}\bar{b}_{t+1}\left(\tilde{u}_f\right) = 0, \tag{A4}$$

and again we assess when $\frac{\partial^2 W}{\partial q_m \partial u_f} \neq 0$. In the **Benefit**-scenarios, we have $c(u_f,q_m) = c(u_f)$, so that again only $\frac{\partial b_t(u_f,q_m)}{\partial u_f}$ needs to be evaluated (which has already been done in section A1). Hence, regardless of whether costs affect future fecundity or survival, DA occurs in the **Benefit-slope** and **Benefit-position** scenarios. In the **Cost**-scenarios, we have $b_t(u_f,q_m) = b_t(u_f)$, so that only the second part in (A4) affects DA and $\frac{\partial^2 W}{\partial q_m \partial u_f} = \left(\frac{\partial^2 c}{\partial q_m \partial u_f}\frac{\partial \alpha}{\partial c} + \frac{\partial c}{\partial u_f}\frac{\partial^2 \alpha}{\partial c^2}\frac{\partial c}{\partial q_m}\right)\overline{b}_{t+1}$ (where we drop obvious notation). For the **Cost-Elevation** scenario, we have $c(u_f,q_m) = c(u_f) - k(q_m)$, so that $\frac{\partial^2 c}{\partial q_m \partial u_f} = 0$. Hence, DA only occurs when costs affect survival in a nonlinear fashion (i.e., $\partial^2\alpha/\partial^2 c \neq 0$), which is analogous to the finding in section A1 that the future fecundity function should be nonlinear. For the **Cost-Slope** and **Cost-Position** scenarios, this restriction does not apply and DA occurs also when costs affect survival in a linear fashion. Hence, we conclude that there are no differences between the survival cost and the future fecundity cost scenarios in terms of DA.

Regarding the direction of DA, we find

$$\begin{aligned} r'(q_m) \quad &= \left(\frac{\partial^2 b_t}{\partial q_m \partial u_f} + \frac{\partial^2 \alpha}{\partial c^2}\frac{\partial c}{\partial q_m}\frac{\partial c}{\partial u_f}\overline{b}_{t+1} + \frac{\partial \alpha}{\partial c}\frac{\partial^2 c}{\partial q_m \partial u_f}\overline{b}_{t+1}\right) \\ &\quad / \left(-\frac{\partial^2 b_t}{\partial r \partial u_f} - \frac{\partial^2 \alpha}{\partial c^2}\frac{\partial c}{\partial r}\frac{\partial c}{\partial u_f}\overline{b}_{t+1} - \frac{\partial \alpha}{\partial c}\frac{\partial^2 c}{\partial r \partial u_f}\overline{b}_{t+1}\right), \end{aligned} \tag{A5}$$

where again the denominator is positive (see below). For the **Benefit** scenarios, costs *c* only depend on u_f and not on q_m, so all terms in the numerator are 0, except the first. Hence, results collapse to those in section A2a, with positive DA in a **Benefit-slope** scenario and negative DA in a **Benefit position** scenario. For the **Cost** scenarios we again always find positive DA: as in A2a, we have $\partial^2 b_t / \partial q_m \partial u_f = 0$. Regarding the second term, we have $\partial^2\alpha/\partial c^2 \leq 0$ (costs either linearly or acceleratingly decrease survival), while $\partial c/\partial q_m < 0$ and $\partial c/\partial u_f > 0$ (see section A2a). Hence, the second term is always positive. Regarding the third term, we have $\partial\alpha/\partial c < 0$ (costs decrease survival) and $\partial^2 c/\partial q_m \partial u_f \leq 0$ (see section A2a), so that also the third term is positive. Note that $\overline{b}_{t+1} \geq 0$ since fecundity is a nonnegative number.

The denominator of A5 is positive: As set out in section A2b, we have $\partial^2 b_t/\partial r\, \partial u_f \leq 0$, $\partial c/\partial r > 0$ and $\partial c/\partial u_f > 0$ and $\partial^2 c/\partial r\, \partial u_f \geq 0$. In addition, $\partial^2\alpha/\partial c^2 \leq 0$ and $\partial\alpha/\partial c < 0$ (costs either linearly or acceleratingly decrease survival), and $\overline{b}_{t+1} \geq 0$, which together result in a positive denominator.