A Multiple Items EPQ/EOQ Model for a Vendor and Multiple Buyers System with Considering Continuous and Discrete Demand Simultaneously

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Abstract. This paper proposed a mathematical model for multiple items Economic Production and Order Quantity (EPQ/EOQ) with considering continuous and discrete demand simultaneously in a system consisting of a vendor and multiple buyers. This model is used to investigate the optimal production lot size of the vendor and the number of shipments policy of orders to multiple buyers. The model considers the multiple buyers' holding cost as well as transportation cost, which minimize the total production and inventory costs of the system. The continuous demand from any other customers can be fulfilled anytime by the vendor while the discrete demand from multiple buyers can be fulfilled by the vendor using the multiple delivery policy with a number of shipments of items in the production cycle time. A mathematical model is developed to illustrate the system based on EPQ and EOQ model. Solution procedures are proposed to solve the model using a Mixed Integer Non Linear Programming (MINLP) and algorithm methods. Then, the numerical example is provided to illustrate the system and results are discussed.
Keywords: EPQ, EOQ, simultaneously, vendor, multiple buyers

1. Introduction

Inventory management is one of the vital competitive factors for industries [1,2,3]. It used inventory model as decision-making tool to control the inventory [4] and have received a lot attention since the last past century till today [5]. Many of inventory models based on Economic Order Quantity (EOQ) or Economic Production Quantity (EPQ) due to their simplicity and robustness [6, 7, 8]. The EOQ model is used to determine an optimal order size by balancing the setup cost and inventory holding cost [9] and EPQ model is used to determine the optimal replenishment lot size to produce [5]. However, due to their own assumptions and conditions, these models are bound to their applicability in real-world situations [1] so that it is extended by several academicians and researchers [10].

The basic EPQ model was proposed five years later after the basic EOQ model was presented and it used the same assumptions with EOQ model [11]. These assumptions sometimes considered as unrealistic situations, such as the continuous replenishment policy for satisfying the demand and the produces items are in perfect quality [5, 12, 13]. In the real life, the discontinuous or multiple delivery policies are commonly adapted [9, 14] and generation of defective items during production process is inevitable [5, 9, 15, 12, 14]. Therefore, there are many researchers or academicians relaxing the

assumptions. Tai [16] extended the EPQ model with considering imperfect/deteriorating items with rework process. Taleizadeh *et al* [12] proposed an EPQ model with backordering, scrap products, rework and interruption in manufacturing process. Sarkar *et al* [3] developed an EPQ model for production with random defective rate, rework process and backorders. Jaber *et al* [17] gives options for EPQ model with imperfect items, whether to buy the new ones or repair the imperfect items, Pasandideh *et al* [10] also considering imperfect production in EPQ model and they extended it with warehouse construction cost . Based on the best knowledge of authors, just few researches consider continuous and discrete demand simultaneously in a system.

In fact, many companies today meet the demand for both continuous and discrete situation. The companies fulfill the demand every time or every day and at the same time they also fulfill the demand every certain period. Therefore, this research proposed a multiple items EPQ/EOQ model with considering continuous and discrete demand simultaneously in a vendor-multiple buyers system. The continuous demand can be fulfilled by vendor anytime, otherwise the discrete demand will be fulfilled by using multiple delivery system [19, 20] and an integrated vendor-buyer inventory model has better performance than the non-integrated inventory model [2]. This research adopt Rahman *et al* [19] inventory model that also considering two-types demands (continuous and discrete demand) simultaneously. The objective of this research is to investigate the optimal production lot size and number of shipments for multiple items or products with considering both continuous and discrete demand simultaneously in the vendor-multiple buyers system. In the following sections, mathematical modeling is given in Section 2. Section 3 is numerical example and discussion. Lastly, the conclusion of this research is presented in section 4.

2. Mathematical modeling

2.1. System characteristics

Figure 1 shows that there are three type of items with two buyers with discrete demand. The vendor will be produce product for buyers (continuous and discrete demands) and the level inventory of products for continuous demand will continuous depleted until the next production cycle time. On the other hand, the discrete demand will be wait for several time until all types of product have finished and the discrete demands ready to deliver to the buyers. In the Figure 1, we can see that the number of shipments for the first buyer is three times deliveries and for the second buyer is two times deliveries in one cycle time.

2.2. Notations and assumptions

Notations that used in this model are list in Appendix A. The mathematical model have some assumptions, there are:

a. Infinite planning horizon.
b. Single facility for all types of products.
c. Shortages are not allowed.
d. There are no imperfect products during production.
e. The total of discrete and continuous demand must least or equal to production rate.

$$\sum_j^k D_{ij} + \sum_r^y C_{ir} \leq P_i \tag{1}$$

f. The total ratio of demand and production rate for all items are maximum 1.

$$\sum_{i=1}^{n} \left(\frac{\sum_{j}^{k} D_{ij} + \sum_{r}^{y} C_{ir}}{P_i} \right) \leq 1 \tag{2}$$

g. The length of total production time can not exceed the length of cycle time

$$TP \leq T \tag{3}$$

2.3. Model formulation

The objective of this mathematical model are determine the optimal production lot size and number of shipments for each buyers with considering both continuous and discrete demand simultaneously in a system of multiple items and multiple buyers. The mathematical model consist of production cost, setup cost, holding cost, and transportation cost.

2.3.1. Production cost. The production cost is calculated by considering total of discrete demands and continuous demands from buyers with production cost per item. The total production cost (TPC) is given by,

$$TPC = \sum_{i=1}^{n} B_i \left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right) \tag{4}$$

2.3.2. Setup cost. The total setup cost (TSC) is given by,

$$TSC = \frac{\sum_{i=1}^{n} S_i}{T} \tag{5}$$

2.3.3. Holding costs. The total holding cost (*THC*) are divided to five parts that can be seen in Figure 1. The holding cost that formulated in this model are holding cost during its production process (I), holding cost for discrete demand (II), holding cost for continuous demand (III), holding cost of discrete demand during delivery time (IV) and buyer's holding cost (V).
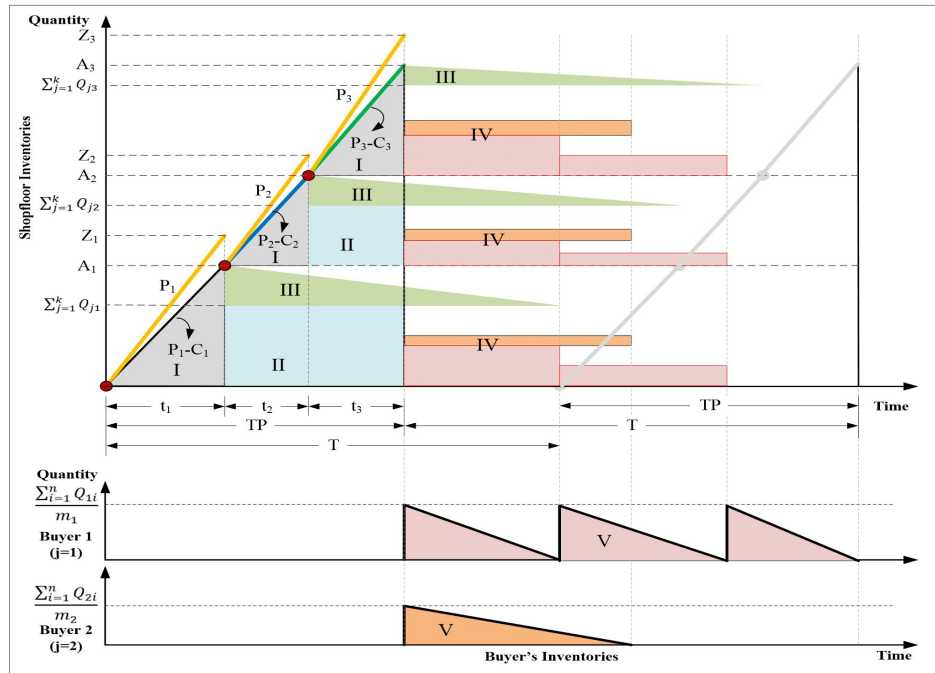

Figure 1. System characteristics with both demand and multiple buyers.

a. The holding cost of item i during its production time.

$$= \frac{1}{2} T \sum_{i=1}^{n} H_i \cdot \left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)^2 \frac{(P_i - \sum_{r=1}^{y} C_{ir})}{P_i^2} \qquad (6)$$

b. The holding cost of item i to fulfil its discrete demand during the production time of the other items.

$$= T \sum_{i=1}^{n-1} H_i \cdot \sum_{j=1}^{k} D_{ij} \cdot \sum_{i=i+1}^{n} \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} \qquad (7)$$

c. The holding cost of item i to fulfill its continuous demand since the production time of next other items until the next its production time.

$$= \frac{1}{2} T \sum_{i=1}^{n} H_i \cdot \sum_{r=1}^{y} C_{ir} \cdot \left[1 - \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} \right]^2 \qquad (8)$$

d. The holding cost of item i to fulfill its discrete demand during delivery periods.

$$= \frac{1}{2} T \sum_{j=1}^{k} \sum_{i=1}^{n} H_i \cdot D_{ij} - \frac{1}{2} T \sum_{j=1}^{k} \frac{\sum_{i=1}^{n} H_i \cdot D_{ij}}{m_j} \qquad (9)$$

e. The holding cost of the buyers.

$$= \frac{1}{2} \cdot T \cdot \sum_{j=1}^{k} \frac{\sum_{i=1}^{n} D_{ij} \cdot L_i}{m_j} \qquad (10)$$

2.3.4. Transportation cost. The transportation are consist of fixed transportation cost of each buyers and variable transportation cost. The total transportation cost (TTC) is given by,

$$TTC = \frac{\sum_{j=1}^{k} m_j \cdot F_j}{T} + \sum_{i=1}^{n} \left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right) \cdot V_i \qquad (11)$$

2.3.5. Total inventory-production cost. The total inventory-production cost (TIPC) are given by equation (12) and verified by using dimension test that can be seen in Appendix B. The TIPC is cumulative of TPC, TSC, THC and TTC.

$$TIPC = \sum_{i=1}^{n} (B_i + V_i) \left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right) + \frac{\sum_{i=1}^{n} S_i}{T} + \frac{1}{2} T \sum_{i=1}^{n} H_i \cdot \left(\sum_{j=1}^{k} D_{ij} + \right.$$

$$\sum_{r=1}^{y} C_{ir} \right)^2 \frac{(P_i - \sum_{r=1}^{y} C_{ir})}{P_i^2} + \frac{1}{2} T \sum_{i=1}^{n} H_i \cdot \sum_{r=1}^{y} C_{ir} \cdot \left[1 - \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} \right]^2 +$$

$$T \sum_{i=1}^{n-1} H_i \cdot \sum_{j=1}^{k} D_{ij} \cdot \sum_{i=i+1}^{n} \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} + \frac{1}{2} T \sum_{j=1}^{k} \sum_{i=1}^{n} H_i \cdot D_{ij} +$$

$$\frac{1}{2} T \sum_{j=1}^{k} \frac{\sum_{i=1}^{n} D_{ij} \, (L_i - H_i)}{m_j} + \frac{\sum_{j=1}^{k} m_j \cdot F_j}{T} \qquad (12)$$

The total inventory and production cost will be have optimum solution if Equation $(L_i - H_i > 0)$ and Equation (13) is satisfied. These equations are obtained by using second differential to Equation (12).

$$\left[\left(\frac{2 \sum_{i=1}^{n} S_i}{T^3} + \frac{2 \sum_{j=1}^{k} m_j \cdot F_j}{T^3} \right) \left(T \sum_{j=1}^{k} \frac{\sum_{i=1}^{n} D_{ij} \, (L_i - H_i)}{m_j^3} \right) \right] - \left[\frac{1}{2} \sum_{j=1}^{k} \frac{\sum_{i=1}^{n} D_{ij} \, (L_i - H_i)}{m_j^2} + \frac{\sum_{j=1}^{k} F_j}{T^2} \right]^2 > 0 \quad (13)$$

2.4. The solution procedures

Algorithm method solves the model step by step. It solves the problem by searching for optimal cycle time first and then determine the optimal number of shipments for each buyers based on the optimal cycle time and total inventory-production cost. There are steps by using algorithm method:

Step 0: Make sure the assumptions of the model are satisfied by doing calculation to equation (1) and equation (2). If the assumptions are satisfied then next to step 1.

Step 1: Determine the optimal cycle time (T^*) by doing calculation with equation (14) and then next to step 2. Equation (14) is given by doing first differential of T to Equation (12). The process to obtain the equation can be seen in Appendix D.

$$T^* = \left(2 \sum_{i=1}^{n} S_i / \left\{ \sum_{i=1}^{n} H_i \cdot \left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)^2 \frac{(P_i - \sum_{r=1}^{y} C_{ir})}{P_i^2} + \sum_{i=1}^{n} H_i \cdot \sum_{r=1}^{y} C_{ir} \cdot [1 - \right.\right.$$
$$\left. \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} \right]^2 + 2 \sum_{i=1}^{n-1} H_i \cdot \sum_{j=1}^{k} D_{ij} \cdot \sum_{i=i+1}^{n} \frac{\left(\sum_{j=1}^{k} D_{ij} + \sum_{r=1}^{y} C_{ir} \right)}{P_i} +$$
$$\left. \sum_{j=1}^{k} \sum_{i=1}^{n} H_i \cdot D_{ij} \right\} \right)^{1/2} \tag{14}$$

Step 2: Determine the optimal numbers of shipment (m_j^*) by doing calculation with equation (15). Equation (15) is given by doing first differential of m_j to Equation (12). If the calculation result is in integer for each buyers (j), then next to Step 3. If not, next to step 4.

$$m_j^* = T \sqrt{\frac{\sum_{i=1}^{n} D_{ij} \ (L_i - H_i)}{2 \ F_j}} \tag{15}$$

Step 3: Determine the total inventory-production cost by calculating equation (12).

Step 4: Determine the total inventory-production cost by calculating equation (12) for both roundup and round down of m_j^* from step 2 and for each buyer. Then choose the best combination of the optimal number of shipment for buyers based on minimum total inventory-production cost.

3. Numerical example and discussion

Numerical example is given to illustrate how the proposed mathematical model solve the problem. The data that used in this numerical example consists of six type of items, two buyers with continuous demand and three buyers with discrete demand. The detailed of input data can be seen in Table 1.

Table 1. Input Data

Input	Product (i)						
	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	
D_{i1}	809,500	186,025	1,389,000	432,688	3,218,760	13,868,762	Units / Year
D_{i2}	2,023,750	260,435	1,215,375	605,763	3,755,220	12,607,966	Units / Year
D_{i3}	1,214,250	297,640	868,125	692,300	3,755,220	15,549,824	Units / Year

C_{i1}		0	0	3,069,424	250,000	4,193,700	952,200	Units / Year
C_{i2}		0	0	4,604,136	250,000	2,795,800	1,163,800	Units / Year
P_i		90,720,000	108,864,000	90,720,000	108,864,000	90,720,000	108,864,000	Units / Year
B_i	Rp	3,000	2,300	3,000	2,300	3,000	2,300	/ Unit
S_i	Rp	20,000,000	20,000,000	20,000,000	20,000,000	20,000,000	20,000,000	/ Setup
H_i	Rp	440	440	440	440	440	440	/ Unit.Year
L_i	Rp	880	880	880	880	880	880	/ Unit.Year
V_i	Rp	100	100	100	100	100	100	/ Unit
F_1	Rp			3,0500,000				/ Shipment
F_2	Rp			3,000,000				/ Shipment
F_3	Rp			2,500,000				/ Shipment

Based on the data input, algorithm method gives solution for the problem with length of cycle time is 0.068 year and number of shipments are either 2 or 3 times for both buyer 1 and buyer 2 and either 3 or 4 times for buyer 3. By trying all alternative combinations for the number of shipments, the best solution is 3 times of shipments for buyer 1, buyer 2 and buyer 3 with total inventory-production is Rp 219,341,667,157.62 per year. This result is indifferent with MINLP method, with cycle time is 0.069 year, 3 times of shipments for buyer 1, buyer 2 and buyer 3, and total inventory-production cost is 219,341,200,000.00 per year. Although the differences of the result for both methods is small, the algorithm will take much more time and complexion in calculation, especially when the number of buyer with discrete demand are big, and must calculate the total inventory-production cost with all alternative combination of number of shipments.

4. Conclusions

This research proposes a mathematical model to investigate the optimal production lot size of the vendor and number of shipments policy of orders to multiple buyers with considering continuous and discrete demand simultaneously. The research also proposes two alternative solution model, algorithm method and Mixed Integer Non Linear Programming (MINLP) method. Although both solution model have small differences in problem solving results, the algorithm method will be more complicated to do when the number of buyers are big, since it is used all alternative combination of number of shipments for all buyer and choose the most economic ones. This model still have limitations that can be extended for further research, such as including safety stock, backorder policy or lost sale, considering imperfect product during production process and rework process, and considering delivery capacity for each buyers.

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