Application of the modal superposition technique combined with analytical elastoplastic approaches to assess the fatigue crack initiation on structural components

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Abstract

Local fatigue approaches, such as, the stress-life, strain-life or energetic approaches defines a framework to estimate the fatigue crack initiation from notches of structural details. Various engineering structures, such as, bridges, wind towers, among others, are subjected to cyclic dynamic loadings which may substantially reduce the strength of these structures. Nowadays, the structural systems tend to be more complex being necessary to find computationally efficient solutions to perform their fatigue analysis, accounting for dynamic actions corresponding to long complex loading events (e.g. diversity of trains crossing a bridge), mainly if local approaches are envisaged. Thus, this paper aims at presenting and validating a generalization of a methodology based on modal superposition technique, for fatigue damage parameters evaluation, which can be applied in fatigue analysis using local approaches. This technique was applied recently in the context of fatigue crack propagation based on fracture mechanics, although it can be extended to compute the history of local notch stresses and strains at notches. A very important conclusion is that the technique can be explored for the case of local confined plasticity at notches whenever the global elastic behaviour of the component prevails. Local submodelling can be explored with this technique to avoid the necessity of large computational models. Local models are only needed to be run under linear elastic conditions for the selected modal shapes of the structure, being the local time history of fatigue damage variable computed by modal superposition for each loading event. That time history may be further post-processed for elastoplastic conditions using Neuber or Glinka’s analyses. Comparisons with direct integration elastoplastic dynamic analysis confirmed the feasibility of the proposed approach.

Keywords: *Fatigue local models; Modal superposition; Dynamic analysis; Cyclic elastoplastic analysis; Structural notched components.*

Nomenclature

a	half of the crack length (crack length in the case of a lateral crack)
b	cyclic fatigue strength exponent
c	fatigue ductility exponent
C	geometry-dependent factor of the stress intensity factor
$\underline{C}$	damping matrix
D	fatigue damage
E	Young modulus
$\underline{F}$	nodal forces vector dependent of the dynamic load
f	nodal forces vector dependent of the dynamic load of the i^{th} mode of vibration
k_t	stress concentration factor
k_i	modal stiffness of the i^{th} mode of vibration
K	stress intensity factor
K'	cyclic strain hardening coefficient
$\underline{K}$	stiffness matrix
K_{dyn}	stress intensity factor due to the dynamic loading
K_i	stress intensity factor related with the i^{th} mode of vibration
K_{stat}	stress intensity factor due to the static loading
$\underline{M}$	mass matrix
m_i	modal mass of the i^{th} mode of vibration
n'	cyclic strain hardening exponent
N_f	number of cycles to the crack initiation
n_f	number of cycles to failure related with a certain $\Delta\sigma_{nom}$

p	magnitude of the loading
t	time
v	load velocity
Y	geometry-dependent stress intensity magnification factor
Y_i	modal coordinate of the ith mode of vibration
α	Rayleigh law damping coefficient
β	Rayleigh law damping coefficient
Δt	time step increment
$\Delta\sigma$	local stress range
$\Delta\sigma_{nom}$	nominal stress range
$\Delta\sigma^{E}$	local elastic stress range
$\Delta\sigma^{EP}$	local elastoplastic stress range
$\Delta\varepsilon^{E}$	local elastic stress range
$\Delta\varepsilon^{EP}$	local elastoplastic strain range
$\Delta\varepsilon^{\,p}$	plastic strain range
$\Delta\varepsilon$	local elastoplastic strain range
ε_f'	fatigue ductility coefficient
ξ_i	damping coefficient associated to the i^{th} mode of vibration
ρ	material density
σ_{stat}	nominal static stress
σ_{dyn}	nominal dynamic stress
σ_i	modal stress related with the i^{th} mode of vibration
σ_f'	cyclic fatigue strength coefficient
σ_m	mean stress
ϕ_i	mode shape of the i^{th} mode of vibration

ϕ_{stat} static deformed shape

w_i natural frequency of the i^{th} mode of vibration

1. Introduction

The local and global collapse of large structures due to progressive fatigue damage is a problem that, from a structural point of view, has been gaining importance in design, rehabilitation and maintenance of these structural systems. The development of fatigue damage can be divided in two different steps: firstly, a crack initiation phase takes place, which is followed by a crack propagation phase that is developed until an instability condition may occur, making unsafe the operation of the structure.

Fatigue damage can be assessed using different methods, namely global S-N approaches, local stress, strain and energetic approaches and Fracture Mechanics based approaches. The global S-N approach has been proposed to establish a relation between the nominal or geometric stress range applied to the structural detail and the whole fatigue life of the detail, being the one that is most considered in design codes, including Eurocode 3, Part 1-9 [1]. This approach has some important limitations; among them the fact of being applicable only to a limited number of structural details and simple loading conditions as anticipated in the codes and also not accounting for the material influence since S-N curves are generally applicable for a broad range of materials. The local and the Fracture Mechanics approaches can be used as more precise alternatives to the S-N methodology. In fact, in the study of large metallic structures, the applicability of the local approaches has been gaining importance to evaluate the fatigue issues [2–5].

The number of cycles required for the fatigue crack initiation may be computed using a local notch approach which, considering the localized nature of the early stage fatigue damage, proposes a correlation between a local parameter (e.g. strain, energy) and the required cycles to initiate a macroscopy crack. The most well-known relations in this area derive from proposals by Basquin [6], Coffin [7], Manson [8] and Morrow [9,10]. Very often the application of these relations requires elastoplastic analyses since plasticity may develop at notch roots and their vicinities. With this respect, the approximate analytical tools, such as the ones provided by the combination of the

Ramberg-Osgood [11], Neuber [11] and Glinka [13–15] approaches, can be applied to establish the relation between the local assumed elastic stress/strain histories and the actual elastoplastic stress/strain histories.

The Fracture Mechanics can be applied to study the fatigue crack propagation problem, complementing the local approaches or allowing the calculation of the residual fatigue life of a structural detail with an initial crack. This approach is supported by fatigue crack propagation laws, being the Paris' law the most important one [16,17].

The feasibility and accuracy of the determination of the stress ranges or other local fatigue damage variables, is related to the quality of the modelling. The increment of the complexity of the structural system leads, naturally, to significant numerical modelling challenges which appear to be, in most of the cases, associated to complex geometries and to difficulties in defining completely the dynamic loading. Algorithms for solving the dynamic numerical problems, as Newmark [18] or Hilber-Hughes-Taylor (HHT) [19], often require the calculation of thousands of load steps, leading to a process with excessive computation time that hinders refined analyses aiming at computing the local fatigue damage. Having in mind the referred limitations, it was proposed by Albuquerque et al. [20,21] the modal superposition technique for the computation of stress intensity factors for a propagating crack, assuming a linear global behaviour, and combining structural global and local submodels, a fact that allows the global model of the structure to be simplified without neglecting the correct numerical representation of the structural behaviour [2,3,22–24].

The crack initiation mechanisms from notches may involve the development of a localized plastic zone around the stress concentrator apex. However in most practical structural applications it can be said that local elastoplastic response does not interfere with the global behaviour of the structure which is still expected to be linear. Also even with local crack initiation the structural system, globally, behaves as linear. Such conditions should allow the application of the modal superposition which can lead to significant gains in terms of computational times. Computational costs could be further optimized, in the study of large structures, adopting submodelling techniques [25].

Considering the above mentioned, the aim of this paper is to propose the use of the modal superposition technique to determine the dynamic structural response, in particular to compute the notch elastoplastic stress/strain histories including the stress and strain ranges, in order to allow evaluating the fatigue crack initiation phase, using the local fatigue approaches. Two different routes, involving the suggested modal superposition methodology for fatigue crack initiation assessment and the consideration or not of submodelling technique, are proposed. Moreover, the efficiency of suggested technique is evaluated using a case study of a simple supported beam submitted to dynamic load events, being the structural behaviour analysed through the proposed modal superposition technique and compared with the results provided by the application of the HHT algorithm [2], the latter considered as reference values.

2. Theoretical background

The theoretical background underlying to the proposed methodology in this paper, linking the concepts of the modal superposition method, local approaches, either to perform elastoplastic analysis or to compute the necessary number of cycles of a dynamic loading for crack initiation and the possibility of using submodelling techniques is presented in this section.

2.1. Analysis of dynamic structural behaviour using modal superposition

A cyclic loading acting on a structural system gives origin to a dynamic behaviour highly dependent of the characteristics of the structure and of the loading history. If this loading is known and well characterized, the dynamic behaviour of the structure can be simulated using a numerical finite element model, which allows computing the nodal forces for each time step. These values, related to the loading on a certain time, added to the knowledge of the structural properties like mass, stiffness and damping allows characterizing the structural system and its dynamic behaviour through the consideration of a direct time integration method or using the modal superposition technique. The dynamic behaviour of the structural system can be defined by the system of equations (1):

$$\underline{M}.\ddot{u}(t)+\underline{C}.\dot{u}(t)+\underline{K}.u(t)=\underline{F}(t) \tag{1}$$

where $\underline{M}$ is the mass matrix, $\underline{C}$ the damping matrix, $\underline{K}$ the stifness matrix, each with a dimension of $N{\times}N$, $\underline{F}$ the nodal forces vector, $N{\times}1$, for a certain time step, $u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$, respectively, the vectors of displacement, velocities and acceleration whose terms are associated to the N degrees of freedom. The computation of equation (1) can be done using a direct time integration method for each time step, although it is easily understood that for structural systems with a large number of degrees of freedom the computational costs starts to be very significant or even unsustainable.

The modal superposition technique is computationally more efficient than the direct time integration once the global dynamic behavior of the structure can be properly reproduced considering the superposition of a limited number of vibration modes, being this possible if the structure has a global elastic behavior and has invariant properties along the time. Using the modal superposition method, the system of NxN simultaneous equations is converted in N uncoupled equations that can be solved independently [26]:

$$\ddot{Y}_i(t)+2w_i.\xi_i.\dot{Y}_i(t)+{w_i}^2.Y_i(t)=f_i(t) \tag{2}$$

As already referred, equation (2) is the decoupled equation related to the vibration mode i, where $Y_i(t)$ is the modal coordinate vector, w_i the natural frequency, ξ_i the damping coefficient, and $f_i(t)$ the vector of nodal forces associated to the N degrees of freedom for the vibration mode i. Besides the decoupling of the vibration modes, and subsequent transformation of the N simultaneous equations system into i decoupled equations, the efficiency of the modal superposition is further increased by the fact of the number of modes being in general much smaller than the number N of the degrees of freedom.

Albuquerque *et al.* [20,21] proposed the use of modal superposition technique to study the dynamic behavior of a structure with an initial elliptical crack aiming at computing the stress intensity factor histories, $K(t)$, being such information essential to predict the fatigue crack propagation through fatigue crack propagation laws, such as the Paris' law. The stress intensity factor can be computed by the following equation:

$$K(t) = C.\sigma(t).\sqrt{\pi a} \tag{3}$$

where C is a parameter that depends on the geometry of the structure and on the crack dimensions, σ(t) is the nominal stress history acting on the detail and a the crack dimension. Taking into account that the loading acting on a structure can be composed by static and dynamic components, the stress intensity factor can result from the sum of two different values, K_{stat} and $K_{dyn}(t)$, one that depends on the static stress level, σ_{stat}, and another that depends on the dynamic stress, $\sigma_{dyn}(t)$:

$$K(t) = K_{stat} + K_{dyn}(t) \tag{4}$$

$$K_{stat} = C.\sigma_{stat}.\sqrt{\pi a} \tag{5}$$

$$K_{dyn}(t) = C.\sigma_{dyn}(t).\sqrt{\pi a} \tag{6}$$

As already referred, the modal superposition method can be applied to structures with a global linear behavior which means that it is only applicable to assess the crack initiation or crack propagation due to fatigue when the local plasticity phenomenon or the non-linear contact between crack faces do not influence the linearity of global behavior. Thus, if these assumptions are verified, the dynamic stress can be determined by:

$$\sigma_{dyn}(t) = \sum_i \sigma_i \,.\, Y_i\,(t) \tag{7}$$

where σ_i is the nominal stress associated to the i^{th} mode shape and $Y_i\ (t)$ is, as already referred, the modal coordinate of the i^{th} mode of vibration. Considering equations (6) and (7), the determination of the stress intensity factor is performed according to:

$$K_{dyn}(t) = C.\sum_i \sigma_i\,.\,Y_i\ (t).\sqrt{\pi a} = \sum_i K_i\,.\,Y_i\ (t) \tag{8}$$

K_i can be defined as the stress intensity factor obtained for the mode shape of the i^{th} mode of vibration, which means the modal stress intensity factor. Thus, the total stress intensity factor can be computed by:

$$K(t) = K_{stat} + \sum_i K_i\,.\,Y_i\ (t) \tag{9}$$

The logic underlying the concept of the modal stress intensity factors can be extended to other local structural quantities as stresses, strains or energetic parameters. Thus equations (4) and (8) can be written in the following general form:

$$\psi(t) = \psi_{stat} + \sum_i \psi_i\,.\,Y_i\ (t) \tag{10}$$

where ψ can be a generic fatigue damage quantity (e.g. stress, strain, energy, stress intensity, J-Integral, COD), being ψ_{stat} the part of this quantity that depends on the static loading and ψ_i the modal value determined considering the mode shape of the i^{th} mode of vibration. Taking into account equation (10) it is easily understandable that the modal superposition can be extended to compute local quantities required to assess the crack initiation due to fatigue phenomenon.

2.2. Crack initiation assessment

The fatigue crack initiation can be analyzed considering local approaches which require the computation of local fatigue damage parameters in order to establish a relation between these local parameters and the number of cycles required to the crack initiation. The most well-known relations in this area are the Basquin [6], equation (11), Coffin [7] and Manson [8], equation (12), Basquin-Coffin-Manson [5], equation (13), and Morrow [10], equation (14):

$$\frac{\Delta\sigma}{2} = \sigma_f'\left(2N_f\right)^b \quad (11)$$

$$\frac{\Delta\varepsilon^P}{2} = \varepsilon_f'\left(2N_f\right)^c \quad (12)$$

$$\frac{\Delta\varepsilon^{EP}}{2} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \quad (13)$$

$$\frac{\Delta\varepsilon^{EP}}{2} = \frac{\sigma_f' - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \quad (14)$$

where $\Delta\sigma$ is the local stress range, $\Delta\varepsilon^p$ the plastic stress range, $\Delta\varepsilon^{EP}$ the local elastoplastic strain range, σ_f' and *b*, respectively, the cyclic fatigue strength coefficient and exponent, ε_f' and *c*, respectively, the fatigue ductility coefficient and exponent, σ_m the mean stress, N_f the number of cycles to the crack initiation, and *E* the Young modulus. Taking into account these quantities, and in order to use the stress/strain results after a linear elastic finite element analysis, a relation between the nominal elastic stress and the local notch elastoplastic stress/strain range can be established using the Neuber [12], equation (15), or Glinka [13–15], equation (16), and the Ramberg-Osgood [11], equation (17), relations.

$$\frac{(k_t\Delta\sigma_{nom})^2}{E} = \frac{\Delta\sigma^2}{E} + 2\Delta\sigma\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (15)$$

$$\frac{(k_t\Delta\sigma_{nom})^2}{E} = \frac{\Delta\sigma^2}{E} + \frac{4\Delta\sigma}{n'+1}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (16)$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (17)$$

In equations (15)-(17), k_t is the elastic stress concentration factor, K' are n' are, respectively, the cyclic strain hardening coefficient and exponent and $\Delta\sigma_{nom}$ the nominal elastic stress range, computed near the notch. Analyzing the expressions presented above, equations (11)-(17), it is clear that besides the material constants/parameters, k_t and $\Delta\sigma_{nom}$ are the only unknowns. If the value of k_t can be eventually determined through a static analysis using the finite element model, the nominal stress range can only be obtained after the dynamic analysis of the structural system, which means that $\Delta\sigma_{nom}$ can be computed solving equation (1) applying a direct time integration method or, more efficiently, the modal superposition technique.

In the case of complex structures, the elastic stress concentration factor, k_t, is not easy to compute since the definition of the nominal stress is generally not clear. The application of the Neuber [12] and Glinka [13-15] approaches need to be performed without the stress concentration factor formulation, relating directly the local elastic stress/strain field with the local elastoplastic stress/strain field:

$$\Delta\sigma^E.\Delta\varepsilon^E = \frac{\Delta\sigma^{EP^2}}{E} + 2\Delta\sigma^{EP}\left(\frac{\Delta\sigma^{EP}}{2K\prime}\right)^{1/n'} \quad (18)$$

$$\Delta\sigma^E.\Delta\varepsilon^E = \frac{\Delta\sigma^{EP^2}}{E} + \frac{4\Delta\sigma^{EP}}{n'+1}\left(\frac{\Delta\sigma^{EP}}{2K\prime}\right)^{1/n'} \quad (19)$$

where $\Delta\sigma^E$ and $\Delta\varepsilon^E$ are, respectively, the local elastic stress and strain ranges at the notch, and $\Delta\sigma^{EP}$ and $\Delta\varepsilon^{EP}$ the local elastoplastic stress and strain ranges at the same point. In simple cases the value of $\Delta\sigma^E$ is indirectly calculated through $k_t.\Delta\sigma_{nom}$, something that is not expected when the structural geometry and loading are complex. Also, in the simpler version of Neuber/Glinka relations $\Delta\varepsilon^E$ is computed from $\Delta\sigma^E$/E but this is an approximation only valid for near uniaxial stress conditions. For multiaxial stress states, the numerical model will provide a better approximation for the elastic strain and consequently the energy term, $\Delta\sigma^E.\ \Delta\varepsilon^E$.

2.3. Submodelling

Generally, the global numerical model of a large structure, even when built with beam or shell elements discretized with a coarse mesh, is able to properly reproduce the dynamic global behaviour [2,3,24]. However, despite the accuracy of these results, a local fatigue analysis demands a much more refined model, typically built with shell or brick elements, which tend to be hard to handle using a global model because it increases significantly the computational costs.

An alternative approach, lighter in terms of computational costs, consists in the analysis of the global model and the subsequent imposition of the obtained displacement field to a refined local model. Hence, the utilization of submodelling techniques, such as beam-to-shell, shell-to-shell, shell-to-solid, beam-to-solid, are particularly useful since the displacement fields from the global model is applied to the local model using shape functions which means that there are not any constraints to the global or local modelling.

2.4. Computational algorithm

Considering the main objectives of this paper, the simulation of fatigue crack initiation using local approaches and the modal superposition method to assess the local damage parameters, the following steps are proposed:

- Computation of nominal stress, σ_{stat}, and displacement field, ϕ_{stat}, due to the static loading;
- Modal analysis of the structure and consequent calculation, for each i^{th} vibration mode, of the modal frequencies, w_i, the modal mass, m_i, the modal stiffness, k_i, and the vibration mode shapes, Φ_i;
- Evaluation of the dynamic loading, $F(t)$, and determination of the nodal forces, $f_i(t)$;
- Calculation of the time histories of the modal coordinates, $Y_i(t)$;
- Computation of the nominal stress spectrum, $\sigma_{nom}(t)$, considering the static and dynamic parts:

$$\sigma_{nom}(t) = \sigma_{stat} + \sum_i \sigma_i \,.\, Y_i\,(t) \tag{20}$$

- Determination of the mean stress, σ_m;
- Selection of the local approach depending on whether the detail remains elastic or not;
- Calculation of the nominal stress range, $\Delta\sigma_{nom}$, and the corresponding number of cycles, n_f, applying the rainflow method to the nominal stress spectrum, $\sigma_{nom}(t)$;
- Determination of the required number of cycles to the crack initiation , N_f, applying a local approach;

- Computation of the linear accumulation damage, D, using the Miner's relation:

$$D = \sum \frac{n_f}{N_f} \tag{21}$$

Taking into account the presented computational algorithm, the steps sequence to assess the necessary number of cycles to the crack initiation can be summarized as shown in Fig. 1. The computational algorithm may include, or not, the consideration of the submodeling techniques. If it is not considered a submodel, the nominal stress spectrum is computed after the application of the modal superposition method to the global model (Fig. 2).

The proposed methodology, besides the already referred advantages, presents a major value in terms of the computation process when several calculations related with different loading scenarios are needed. More specifically, for each time step, the consideration of several dynamic events only requires the definition of $f_i(t)$ and $Y_i(t)$ for each loading and not the solution of the N×N system of simultaneous equations for each dynamic event.

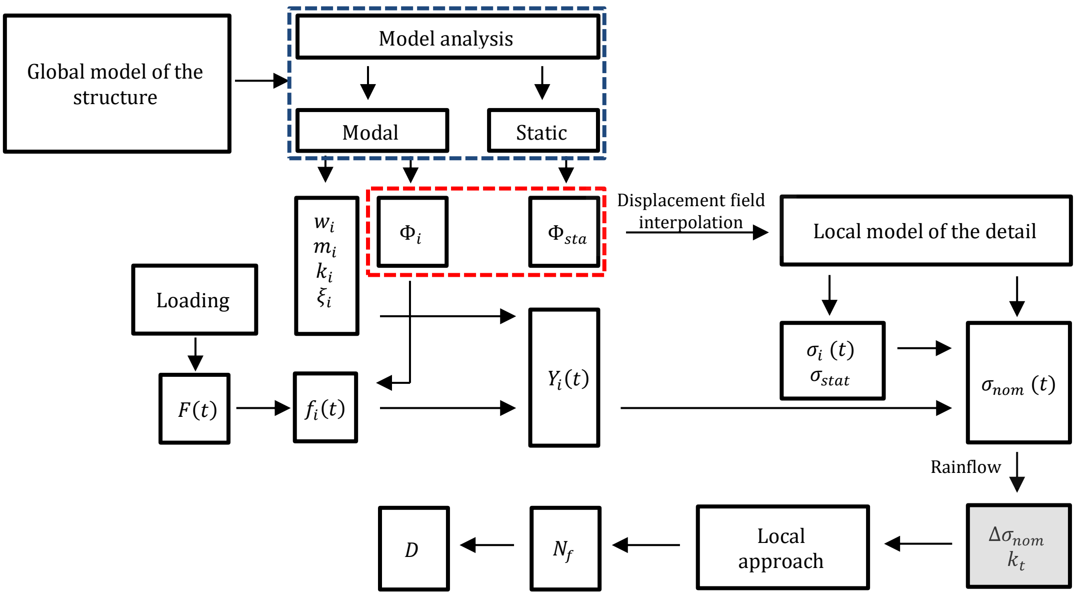


Fig. 1. Flow chart for the application of the proposed modal superposition methodology for fatigue crack initiation assessment (with submodeling).

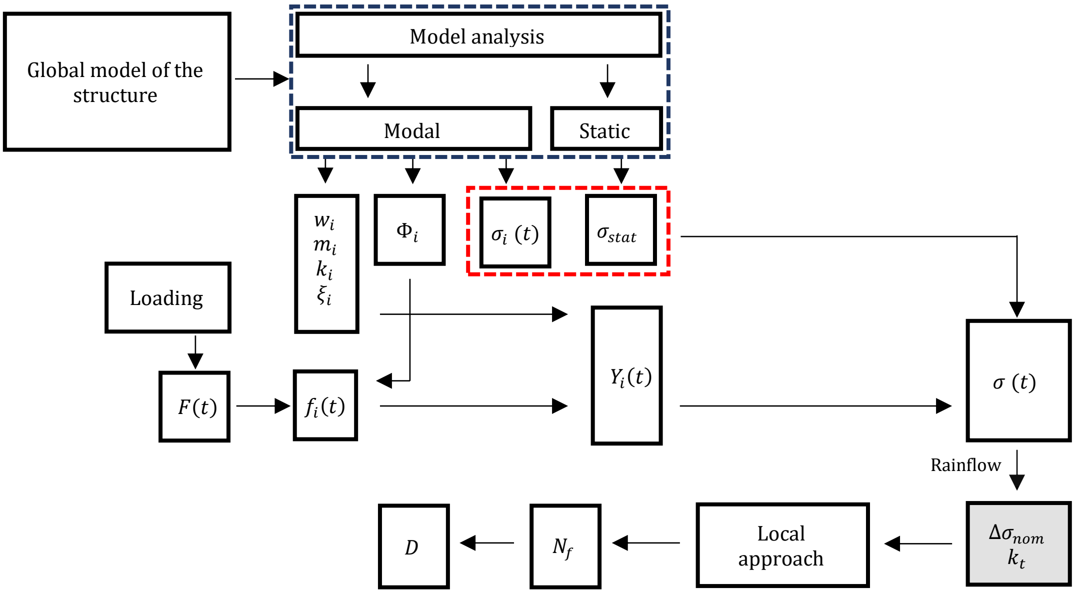


Fig. 2. Flow chart for the application of the proposed modal superposition methodology for fatigue crack initiation assessment (without submodeling).

For both of the presented computational algorithms pictured in the Fig. 1 and Fig. 2 it is necessary to know in advance the elastic stress concentration factor. If the structural engineer has to deal with a complex structure, as already mentioned, the value of k_t is not easily computed, being necessary working directly with the elastic notch stress/strains, σ^E and ε^E from the FE analysis. Also, after computing the local/notch elastic stress/strain histories, one may apply the Neuber or Glinka analyses to derive the local elastoplastic stress/strain histories, the fatigue cycles resulting directly from hysteresis loops counting and not from the rainflow analysis. Hysteresis cycles counting is more consistent with fatigue damage.

The proposed modal superposition technique for the computation of local fatigue damage parameters was explored in previous works concerning the application of Facture Mechanics principles to the fatigue crack propagation analysis [2,3,20,21]. In the present paper a generalization and complementary approach is proposed for the computation of fatigue damage parameters aiming at simulating fatigue crack initiation. In both cases, crack closure effects (Fracture Mechanics) and local notch elastoplasticity (fatigue crack initiation) are expected non-linearities, but while they do not change the global linear behaviour of the structure, the modal superposition can be successfully applied. For large structures the fatigue crack initiation process does not influence the global behaviour of the structure, therefore modal analysis only needs to be performed one time. However, when fatigue crack propagation is performed, it is worthwhile to refer that the propagating fatigue crack may change the global behaviour of the structure which may require periodic updates of the structural vibration modes.

3. Crack initiation assessment – application to a notched structural member

In order to validate the applicability of the modal superposition methodology for fatigue crack initiation assessment, a numerical model of a notched simple structure was developed and duly characterized from a geometric and material point of view. According to the computational algorithm presented in the section 2.4, the flow chart pictured in Fig. 2 was implemented, i.e., the explicit submodelling was not applied. The option for a simple structure was taken in order to control all the parameters with direct or indirect influence in the structural dynamic behaviour.

The effectiveness of the proposed methodology is naturally dependent on the analytical elastoplastic approach considered, hence the referred influence was tested through the consideration of the Neuber [12] and Glinka [13-15] proposals. Instead of k_t-σ_{nom} based elastoplastic formulations, elastoplastic analyses formulated from the local elastic stress/strain computed directly from the finite element model, Equations (18)-(19), are used.

The finite element model was submitted to dynamic loadings with variable intensity obtained through the moving loads approach. The dynamic analyses were carried out using the proposed modal superposition methodology with post-processing elastoplastic analysis and HHT algorithm [18] coupled with elastoplastic material behaviour, being compared the obtained stress-strain diagrams. Additionally, the necessary number of cycles to the crack initiation, N_f, is computed using the Morrow approach [10]. The extension of local plastic volume is also evaluated and commented as regards the reliability of the simulated modal superposition methodology.

3.1. Structure description

A notched simply supported steel beam, with a 10m span and a HEB 700 cross section was idealised. At mid span a circular hole with 10mm of radius was admitted to simulate a notch/defect on the inferior flange. The numerical model was conceived using the ANSYS software [27] (Fig. 3).

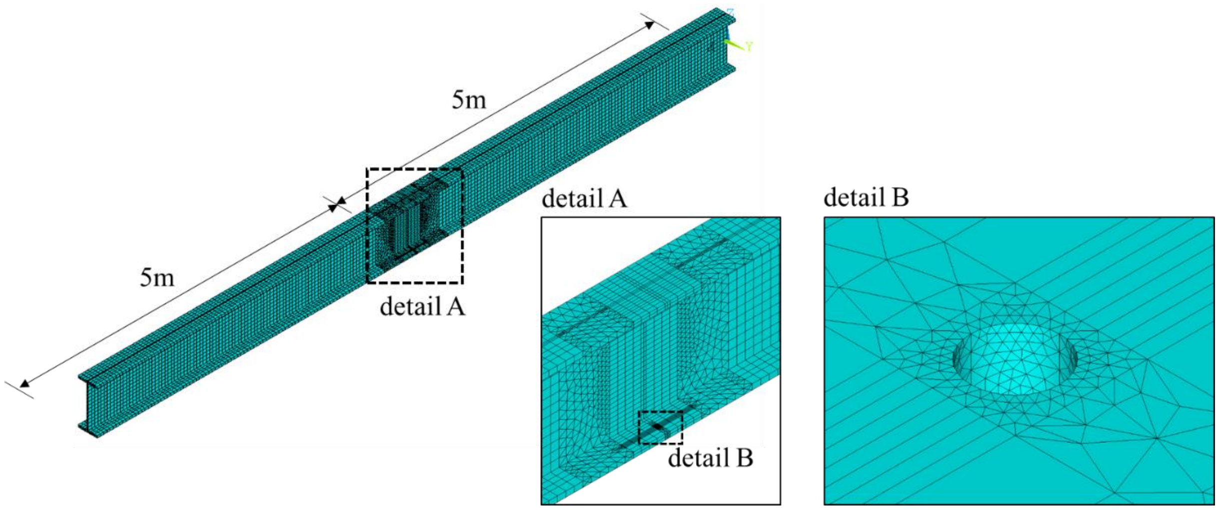
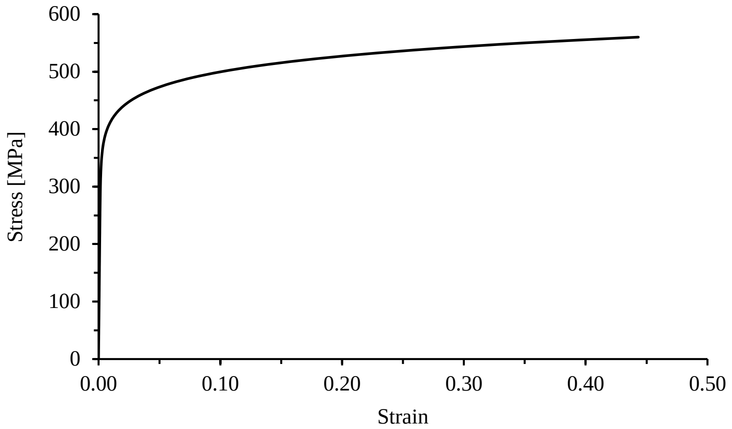


Fig. 3. Finite element model of a simple supported beam with a circular hole at the lower flange.

The finite element model was built with 20-node brick elements with mesh refinement on the notched zone to improve the description of the local stress and strain fields [28]. The beam is assumed of S355 steel which was characterized in terms of elastoplastic behaviour and fatigue by De Jesus et al. [29]. In the HHT analysis, the material is assumed elastoplastic in a central length of 4m of the bottom flange and web. The remaining of the structure was assumed linear elastic with the same density (ρ), Young modulus (E) and Poisson ratio (υ) of the elastoplastic material. These two materials option corresponds to a partial submodelling that helps numerical convergence and computation costs reduction. Fig. 4 represents the elastoplastic stress-strain cyclic curve of the S355 steel grade, adapted from De Jesus et al. [29].



- Density, ρ=7850kg/m^3
- Young modulus, E=209.4GPa
- Poisson's ratio, υ=0.3

- Cyclic curve parameters:
  k'=595.85MPa
  n'=0.757
  $\sigma f'$=952.2MPa
  $\varepsilon f'$=0.737MPa
  b=-0.089
  c=-0.664

Fig. 4. Cyclic stress-strain curve of the S355 grade (adapted from [29]).

In Fig. 4., the material parameters required to perform a cyclic elastoplastic analysis taking into account the Neuber [12], Eqs. (15) or (18), or Glinka [13-15], Eqs. (16) or (19), approaches and, subsequently, to compute the necessary number of cycles to the crack initiation, N_f, according to Morrow [10], eq. (14), are presented.

3.2. Dynamic analysis assumptions

The structural mass was calibrated in order the main natural frequencies being close to available data in the literature [30], seeking to approximate the idealized structure of a practical case. The stiffness was determined for the described simple supported beam considering the defined cross section, although it should be noted that the out of plane degrees of freedom were also restrained in order to eliminate purely local modes, without contributions to the global dynamic behavior, and whose existence is only possible due to the absence of transverse bracing.

In the modal superposition analysis, a constant modal damping ratio, ξ, of 0.5% was assumed for all the considered modes; concerning the calculation considering the HHT algorithm [19] it was assumed a Rayleigh damping law characterized with the coefficients α equal to 0.682414 and β to 0.000024, values computed considering the first and third modes of vibration [26]. In the analyses, a time step increment, Δt, equal to 0.0001s and a residual free vibration period of 0.1s were admitted.

Taking into account the simplicity of the structure, it was expected a small number of modes to contributed significantly to the total dynamic response. A sensitivity analysis allowed to prove such assumption, being considered only the first 5 vibration modes to capture accurately the total structural response.

3.3. Dynamic loading

Several dynamic loadings were considered to evaluate the efficiency of the proposed methodology and the influence of the plasticity phenomenon at the notched bottom flange. In a given time instant, t, the nodal loads were computed with base on their position along the beam and on the node coordinates. Each loading, characterized by a given magnitude, p, and by a certain velocity, v, applies only one cycle to the structure, n_f. The considered dynamic loads are summarized in Table 1.

Table 1. Considered dynamic loading: moving concentrated loadings.

Loading Id	(1)	(2)	(3)	(4)	(5)	(6)
p (kN)	800	800	900	900	1000	1000
v (km/h)	100	200	100	200	100	200

As shown in Table 1, six different loadings were admitted which gave origin to eighteen dynamic analyses. Twelve performed according the proposed modal superposition methodology, six considering the Neuber approach [12], eq. (15) or (18), and other six the one proposed by Glinka [13-15], eq. (16) or (19), according to the flow chart presented in Fig. 2, and the last six applying the HHT algorithm [18] in order to allow a validation process. To avoid local singularities or unexpected resonant effects that would cause problems to the numerical convergence of the results, the loading of the structure was made considering linear loads, perpendicular to the longitudinal direction of the beam, with 0.085m of development. The resultants of the linear loads were equal to the magnitude, p.

The implementation of the Neuber and Glinka approaches were made using the *Fatigue Life Prediction* –FLP software developed by Silva [31].

3.4. Results of the dynamic analysis

Taking into account the established fundamentals, the aimed dynamic analyses were carried out. According to the eq. (10) the generic fatigue damage quantity, ψ, was defined as stresses and strains along the longitudinal direction, allowing to obtain the stress-strains diagrams at the notch apex.

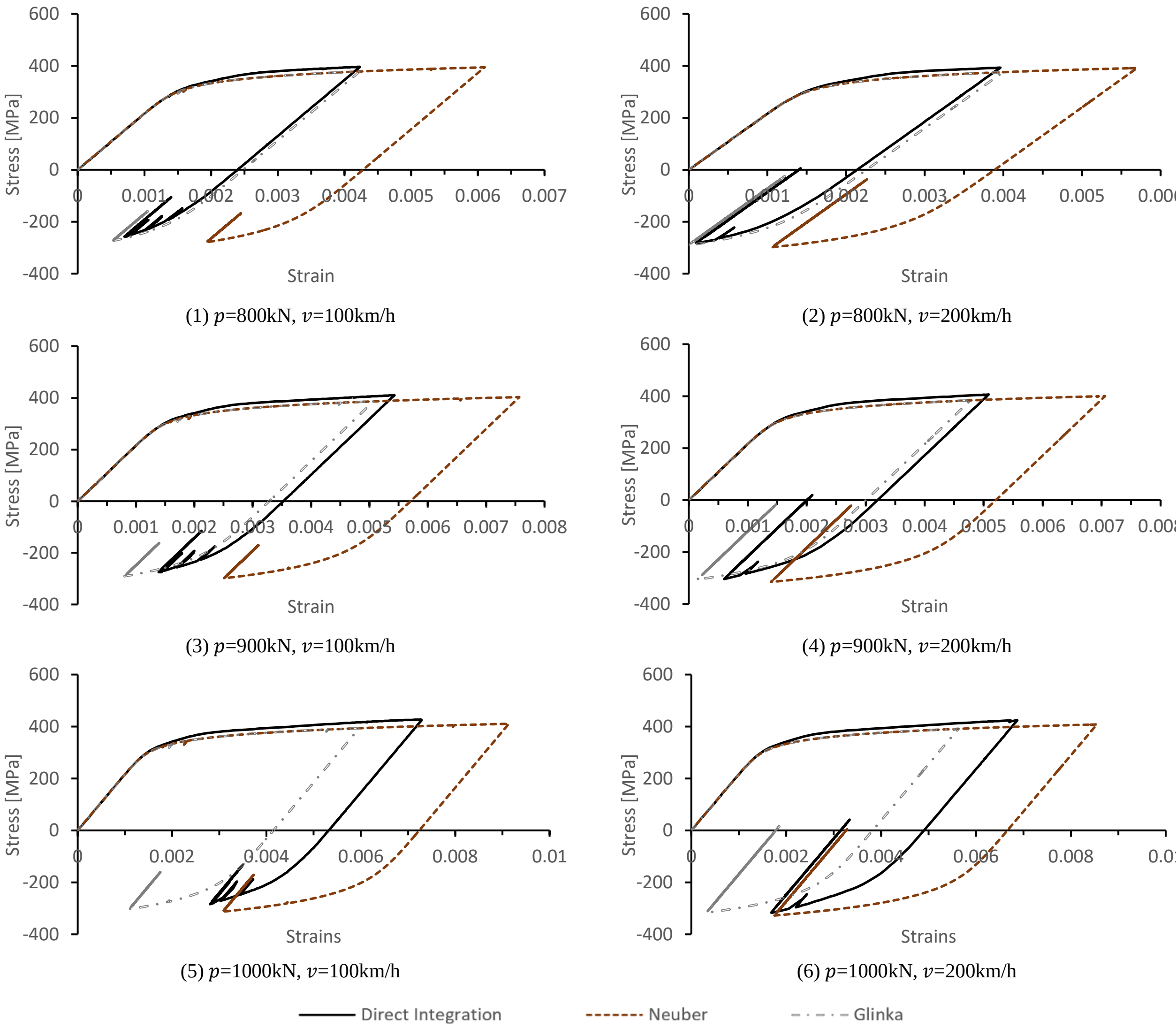


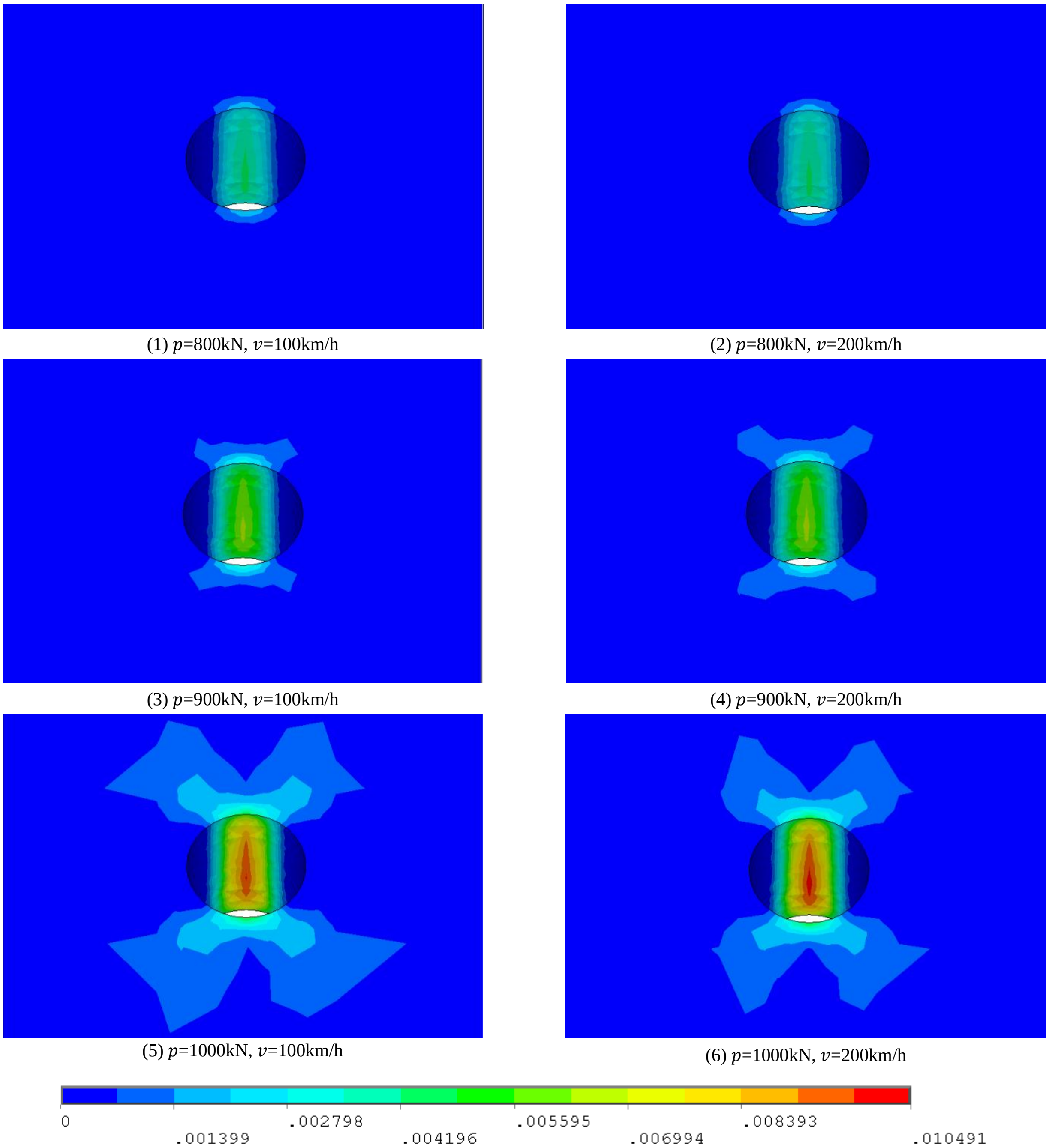
Fig. 5. Elastoplastic stress-strain diagrams under dynamic loading.

In Fig.5, it is possible to observe, for the loadings (1), (2), (3) and (4), a notorious agreement between the obtained stress-strain diagrams through the modal superposition combined with the Glinka approach [13-15] and the results of the direct time integrations using the HHT algorithm [19]. A discrepancy between the results was found when the modal superposition was use together with the Neuber approach [12], the Neuber approach resulting in higher average strains which is consistent with the conservatism usually associated to this rule. Therefore, despite the plasticity phenomenon, it was possible for the referred loadings to achieve satisfactory agreement in the stress-strain diagrams with the modal superposition analysis plus Glinka approach. In the cases of the loadings (5) and (6), the increment on the magnitude, p, resulted in the development of larger plastic zones, which explain the progressive discrepancy between the stress-strain diagrams from the two dynamic analyses.

Table 2 presents the relevant results obtained after the two dynamic analyses, including cycles information in terms of strains and stresses and life estimations using the Morrow model, eq. (14). Percentage differences relative to the results obtained with the HHT algorithm [19], assumed as reference, (actual value/reference value-1), are also presented. The data from the table confirms the conclusions pointed out after the observation of Fig. 5. The necessary number of cycles for the crack initiation computed using the Morrow approach, confirmed the Neuber proposal as considerably conservative. Also the deviations in the stress computations are significantly lower than in strain computations. The computations of the stress and strain variations are also much more precise than the minimum and maximum stress and strain values. These two considerations led to accurate life predictions using the modal superposition analysis plus simplified analytical elastoplastic analysis, with maximum deviations of 21% using the Glinka approach for the loading (5), which is the loading generating the highest residual plastic zone (see Fig. 6). While Neuber approach generated maximum deviations of 3% on stress ranges and 35% on strain ranges, the Glinka approach generated deviations of -4.4% on stress range and 8.5% on stress range.

Table 2. Relevant results of the dynamic and fatigue analysis.

Loading	#	σ_{max} (MPa)	σ_{min} (MPa)	σ_m (MPa)	$\Delta\sigma$ (MPa)	ε_{max}	ε_{min}	ε_m	$\Delta\varepsilon$	N_f
(1)	HHT	396.25	-257.27	69.49	**653.52**	0.00423	0.00070	0.00247	**0.00353**	**57953**
	Neuber	394.31	-278.84	57.73	**673.15**	0.00610	0.00193	0.00402	**0.00417**	**34387**
p=800kN	Δ%	-0.5%	8.4%	-16.9%	**3.0%**	44.2%	175.7%	62.8%	**18.1%**	**-40.7%**
v=100km/h	Glinka	378.36	-273.66	52.35	**652.02**	0.00423	0.00052	0.00238	**0.00371**	**48937**
	Δ%	-4.5%	6.4%	-24.7%	**-0.2%**	0.0%	-25.7%	-3.6%	**5.1%**	**-15.6%**
(2)	HHT	392.94	-282.37	55.28	**675.31**	0.00396	0.00010	0.00203	**0.00387**	**39407**
	Neuber	391.47	-298.11	46.68	**689.58**	0.00570	0.00106	0.00338	**0.00464**	**21457**
p=800kN	Δ%	-0.4%	5.6%	-15.6%	**2.1%**	43.9%	960.0%	66.5%	**19.9%**	**-45.6%**
v=200km/h	Glinka	375.70	-290.74	42.48	**666.45**	0.00400	-0.00001	0.00199	**0.00401**	**35144**
	Δ%	-4.4%	3.0%	-23.2%	**-1.3%**	1.0%	-110.0%	-2.0%	**3.6%**	**-10.8%**
(3)	HHT	410.45	-274.78	67.83	**685.23**	0.00543	0.00139	0.00341	**0.00404**	**30661**
	Neuber	402.99	-298.41	52.29	**701.39**	0.00757	0.00251	0.00504	**0.00506**	**15250**
p=900kN	Δ%	-1.8%	8.6%	-22.9%	**2.4%**	39.4%	80.6%	47.8%	**25.2%**	**-50.3%**
v=100km/h	Glinka	386.47	-290.44	48.01	**676.91**	0.00507	0.00080	0.00294	**0.00427**	**26074**
	Δ%	-5.8%	5.7%	-29.2%	**-1.2%**	-6.6%	-42.4%	-13.8%	**5.7%**	**-15.0%**
(4)	HHT	406.20	-304.26	50.97	**710.45**	0.00508	0.00060	0.00284	**0.00448**	**21022**
	Neuber	400.21	-315.06	42.57	**715.27**	0.00706	0.00139	0.00423	**0.00566**	**10208**
p=800kN	Δ%	-1.5%	3.5%	-16.5%	**0.7%**	39.0%	131.7%	48.9%	**26.3%**	**-51.4%**
v=200km/h	Glinka	383.88	-305.45	39.21	**689.33**	0.00478	0.00014	0.00246	**0.00463**	**19169**
	Δ%	-5.5%	0.4%	-23.1%	**-3.0%**	-5.9%	-76.7%	-13.4%	**3.3%**	**-8.8%**
(5)	HHT	426.93	-284.07	71.43	**711.00**	0.00729	0.00280	0.00504	**0.00449**	**19658**
	Neuber	410.32	-312.74	48.79	**723.05**	0.00914	0.00308	0.00611	**0.00606**	**8157**
p=1000kN	Δ%	-3.9%	10.1%	-31.7%	**1.7%**	25.4%	10.0%	21.2%	**35.0%**	**-58.5%**
v=100km/h	Glinka	393.38	-302.97	45.20	**696.35**	0.00597	0.00109	0.00353	**0.00487**	**15531**
	Δ%	-7.9%	6.7%	-36.7%	**-2.1%**	-18.1%	-61.1%	-30.0%	**8.5%**	**-21.0%**
(6)	HHT	423.84	-316.37	53.73	**740.21**	0.00687	0.00168	0.00428	**0.00519**	**12114**
	Neuber	407.75	-327.64	40.05	**735.40**	0.00855	0.00175	0.00515	**0.00680**	**5735**
p=1000kN	Δ%	-3.8%	3.6%	-25.5%	**-0.6%**	24.5%	4.2%	20.3%	**31.0%**	**-52.7%**
v=200km/h	Glinka	390.96	-316.60	37.18	**707.56**	0.00563	0.00032	0.00297	**0.00531**	**11650**
	Δ%	-7.8%	0.1%	-30.8%	**-4.4%**	-18.0%	-81.0%	-30.6%	**2.3%**	**-3.8%**



(1) p=800kN, v=100km/h

(2) p=800kN, v=200km/h

(3) p=900kN, v=100km/h

(4) p=900kN, v=200km/h

(5) p=1000kN, v=100km/h

(6) p=1000kN, v=200km/h

Fig. 6. Residual equivalent plastic strain, evolution of the local plastic material volume.

Fig. 6 shows the residual equivalent plastic strain fields [32] after the load passage which gives a measure of the volume of material plastically deformed. The evolution of the plastic zone with the magnitude, p, of the loading is notorious, but less important when the loading speed doubles. Nevertheless for all cases the plastic zone region is still confined and therefore no significant effect on stress and strain ranges is verified as well as in the life calculations.

4. Concluding remarks

The fatigue damage is a structural problem that is fundamental to assess in order to extend the operational life of the existing structures. The scale of large structures puts significant problems in terms of numerical modelling and structural analysis, particularly in what refers to the application of local models to fatigue.

The obtained results, after the validation of the proposed modal superposition technique for fatigue crack initiation assessment, allow concluding that they are promising for the analysis of more complex structural problems. The proposed technique presented in the present paper, under the condition of localized plasticity, combined with the work of Albuquerque *et al.* [20,21] enables the structural engineer to fully assess the fatigue phenomenon using the modal superposition method.

The consideration of the approached technique permits to optimize the calculation process, making possible the dynamic structural analysis in an affordable computational time. Regarding the simple structure idealized and considering the assumption underlying the dynamic analysis, the calculation involving the proposed technique took 10 seconds to compute the stress and strain results, while the application of the HHT algorithm spent more than 5 hours, such difference giving a clear quantification of the potential of the proposed technique. Moreover, the use of this method conjugated with submodelling techniques can increase the accuracy and efficiency of refined analysis of complex notched details, which means the computation of the local stress and strain data required to assess the fatigue crack initiation at notches. The modal superposition analysis with the Glinka elastoplastic post-processing is more precise than using the Neuber post-processing.

Further studies for more complex structures are needed, in particularly, the evaluation of the accuracy of the results when contact non-linearities are present at the notched area (e.g. riveted joints).

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