

# Bayesian operational modal analysis of Jiangyin Yangtze River Bridge

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## 36 **Abstract**

37 Vibration testing of long span bridges is becoming a commissioning requirement, yet such exercises  
38 represent the extreme of experimental capability, with challenges for instrumentation (due to frequency  
39 range, resolution and km-order separation of sensor) and system identification (because of the extreme low  
40 frequencies).

41 The challenge with instrumentation for modal analysis is managing synchronous data acquisition from  
42 sensors distributed widely apart inside and outside the structure. The ideal solution is precisely synchronised  
43 autonomous recorders that do not need cables, GPS or wireless communication.

44 The challenge with system identification is to maximise the reliability of modal parameters through  
45 experimental design and subsequently to identify the parameters in terms of mean values and standard errors.  
46 The challenge is particularly severe for modes with low frequency and damping typical of long span bridges.  
47 One solution is to apply 'third generation' operational modal analysis procedures using Bayesian approaches  
48 in both the planning and analysis stages.

49 The paper presents an exercise on the Jiangyin Yangtze River Bridge, a suspension bridge with a 1,385m  
50 main span. The exercise comprised planning of a test campaign to optimise the reliability of operational  
51 modal analysis, the deployment of a set of independent data acquisition units synchronised using precision  
52 oven controlled crystal oscillators and the subsequent identification of a set of modal parameters in terms of  
53 mean and variance errors.

54 Although the bridge has had structural health monitoring technology installed since it was completed, this  
55 was the first full modal survey, aimed at identifying important features of the modal behaviour rather than  
56 providing fine resolution of mode shapes through the whole structure. Therefore, measurements were made  
57 in only the (south) tower, while torsional behaviour was identified by a single measurement using a pair of  
58 recorders across the carriageway. The modal survey revealed a first lateral symmetric mode with natural  
59 frequency 0.0536 Hz with standard error  $\pm 3.6\%$  and damping ratio 4.4% with standard error  $\pm 88\%$ . First  
60 vertical mode is antisymmetric with frequency 0.11 Hz  $\pm 1.2\%$  and damping ratio 4.9%  $\pm 41\%$ .

61 A significant and novel element of the exercise was planning of the measurement setups and their necessary  
62 duration linked to prior estimation of the precision of the frequency and damping estimates. The second  
63 novelty is the use of the multi-sensor precision synchronised acquisition without external time reference on a  
64 structure of this scale. The challenges of ambient vibration testing and modal identification in a complex  
65 environment are addressed leveraging on advances in practical implementation and scientific understanding  
66 of the problem.

## 67 **Highlights**

68 Ambient vibration test on 1385 m Jiangyin Yangtze River Bridge, by western academics.

69 Application of uncertainty laws of Bayesian operational modal analysis for test planning with and without  
70 preliminary data.

71 Bayesian operational modal analysis of acceleration data, including challenging lateral vibration modes

72 Design and use in the field testing of precisely synchronised loggers not relying on GPS or wireless.

73

74 **Keywords:** Bayesian operational modal analysis suspension bridge uncertainty synchronisation

## 75 **1 Introduction**

76 Ambient vibration testing (AVT) is becoming an indispensable tool for managing the performance of large  
77 scale civil infrastructure where forced vibration testing is logistically infeasible. However there are specific  
78 challenges to AVT in particular for long span and tall structures in both the experimental art and the  
79 subsequent analysis of response time series. This paper describes advances in both technologies aimed at  
80 improving the effectiveness of experimental AVT campaigns to provide the most reliable estimates of modal  
81 parameters (MP) with the least experimental cost and which are applied together on an exemplar study,  
82 Jiangyin Yangtze River Bridge ('Jiangyin Bridge'), a suspension bridge with a 1,385 m main span in Jiangsu  
83 Province, China.

## 84 1.1 Motivation for managing uncertainty of modal parameter (MP) estimates

85 Knowledge of structural modal parameters or MP (for both singular and plural form) representing modal  
86 frequencies, damping ratios, shapes and masses is a fundamental requirement for design against dynamic  
87 loads (wind, earthquake, human excitation). MP estimation is central to finite element model updating and  
88 the dependent performance simulations. It has long been recognised as an effective means for uncertainty  
89 mitigation in structural dynamics [1] and is a vital prerequisite for:

90 - Vibration control devices [2]: Design and maintenance of effective tuned mass dampers depends on precise  
91 knowledge of modal damping and frequency;

92 - Retrofits [3]: If the retrofit aims to improve vibration serviceability, both design and evaluation of the  
93 retrofit depend on reliable MP estimates;

94 - Structural Health Monitoring (SHM) systems [4]: MP (e.g. frequencies) are classic indicators of structural  
95 state so the ability to identify and statistically qualify small changes will enhance SHM system utility for  
96 tracking structure condition through shifts and drifts in MP.

97 Computer models of tall buildings often grossly misrepresent the as-built characteristics, which is evidenced  
98 by measured MPs significantly differing from predictions [1]. A specific example is the 280 m Republic  
99 Plaza building in Singapore where  $>0.19$  Hz fundamental mode frequencies measured by AVT indicated  
100 almost doubled as-built stiffness compared to a design estimate of 0.14 Hz [5]. Such conservatism and over-  
101 design is not sustainable and needs to be tempered by experience from AVT.

102 For damping, other than for discrete damping elements [6,7] available models are all empirical. Values used  
103 in design are based on historic databases [6,8] distilled in codes of practice [9], and usually derived from  
104 response using '1<sup>st</sup> generation' operational modal analysis (OMA) techniques (e.g. random decrement, half-  
105 power bandwidth) [10] having questionable reliability. Building occupants pay a premium for high-rise  
106 comfort and to be sure of avoiding poor vibration serviceability and loss of revenue amidst uncertainty in  
107 inherent structural damping, vibration control devices may be specified. These are expensive and use  
108 premium space. For long span bridges flutter speeds increase with damping. Unlike computational  
109 underestimation of natural frequency, experimental estimates of damping are usually high so that safety  
110 margin is also overestimated.

111 MP identification using forced vibration testing (FVT) with controlled input (force) [11] is believed to  
112 produce the most reliable MP estimates. However, FVT is only feasible for automotive/aerospace  
113 applications (such as ground vibration testing) and relatively small civil structures such as footbridges and  
114 floors having modal masses with maximum order  $10^3$  tonnes and natural frequencies usually above 1 Hz. For  
115 long span and tall structures having modal frequencies of the order 0.1 Hz and modal masses usually  
116 upwards of  $10^4$  tonnes, artificial forcing is logistically infeasible. Therefore, AVTs providing response data  
117 for OMA have become the standard investigation tool.

118 A trend in AVT for structural commissioning tests (e.g. Burj Khalifa and Queensferry Crossing) to  
119 corroborate design assumptions has accelerated with sophisticated '2<sup>nd</sup> generation' OMA techniques such as  
120 stochastic subspace identification (SSI) and frequency domain decomposition (FDD) [12,13]. However, the  
121 problem remains that absence of input loading data in AVT leads to fundamentally much higher MP estimate  
122 variability and lower repeatability compared to FVT. This uncertainty is well recognised [8,14] but there has  
123 been no quantitative account for its origin, nor any quantitative guideline for test configuration (sensor  
124 location, load intensity, signal/noise ratio (SNR) and measurement duration) to achieve a desired  
125 identification precision. Recent ASCE guidance [1] states that prediction of structural (dynamic) properties is  
126 more of an art than a science; there are few reliable measurements of structural damping and values used in  
127 performance predictions are generally 'crude and simplified estimates'.

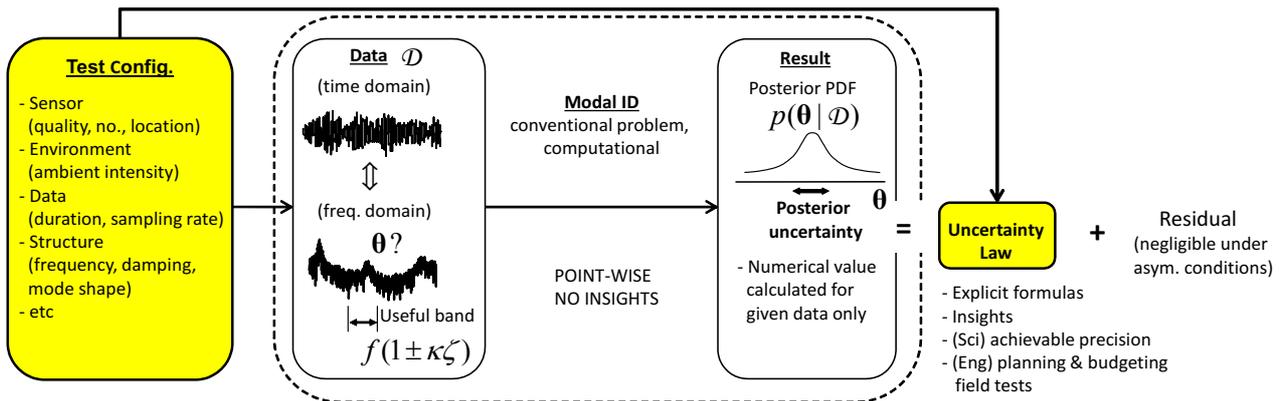
128 With aero-elasticity a major factor in long span bridge performance at ultimate limit state, designers rely  
129 heavily on damping estimation from existing prototypes via AVTs that have had poor uncertainty  
130 management. Existing databases [6,8] of field-identified tall building damping values used for design show  
131 great scatter with no information on their reliability [15,16] and there is still no commonly accepted  
132 amplitude-dependent damping model for even the simplest building configuration. For the tallest buildings  
133 vibration serviceability, which depends critically on first mode frequency and damping, governs design, and  
134 reliable performance data are both scarce and critically important. For long span bridges, collections of  
135 damping data [17][18] are rarer and older.

136 **1.2 Uncertainty laws for Bayesian OMA**

137 Due to the accepted reliability of MP estimation using forced vibration testing, there are few studies on  
 138 accuracy [19], while there are presently several techniques dealing with reliability of OMA, particularly for  
 139 SSI [20–23], including a commercial implementation. Uncertainty quantification is central to ‘3<sup>rd</sup>  
 140 generation’ Bayesian OMA methods [24] that adopt a radically different philosophy to 1<sup>st</sup> and 2<sup>nd</sup> generation  
 141 OMA methods by posing the question ‘what is known about MP based on the given data?’. Answers are  
 142 expressed via the *posterior* probability density function (PDF) of MP ( $\theta$ ) for given response data ( $D$ ) implied  
 143 fundamentally using Bayes’ theorem [25] i.e.  $P(\theta|D)$ . The covariance matrix associated with the posterior  
 144 PDF then naturally quantifies the identification uncertainty of MP for the given data set and modelling  
 145 assumptions. This is in contrast to non-Bayesian methods where the uncertainty refers to the estimates of the  
 146 MP rather than the MP themselves, and is ‘inherent’ in nature, i.e. does not depend on data.

147 Even so, algorithms for calculating uncertainty, be they Bayesian or non-Bayesian, only allow MP  
 148 uncertainty to be calculated in a ‘point-wise’ manner (i.e., ‘what’) for given data, which yields little or no  
 149 understanding about uncertainty (i.e., ‘why’). Therefore, the main aim of the 'BAYOMALAW' research  
 150 project [26] funded by the Engineering and Physical Sciences Research Council has been to deliver  
 151 ‘uncertainty laws’ using Bayesian inference to express MP uncertainties explicitly as functions of test  
 152 configuration (measurement noise, environmental load intensity and sensor configuration) so that MP  
 153 uncertainty can be prescribed and managed [27]. The project has aimed at providing a quantitative account  
 154 for the origin of uncertainty along with quantitative guideline for test configuration to achieve a desired  
 155 identification precision. The exercise reported here is an early application.

156 Figure 1 provides an outline of the research. (Conventional) efforts on Bayesian OMA algorithm  
 157 development belong to the central (unshaded) part of the figure. Uncertainty law research is concerned with  
 158 the shaded parts that address the challenge of understanding identification uncertainty and mastering its  
 159 relationship with test configuration. Previous experience in the ambient modal identification research  
 160 community has led to advice on best practice for reliable MP identification [28]. Following a Bayesian  
 161 approach, this has been formalised [29][30] in the form of 'zeroth order laws'. These are closed form  
 162 expressions for MP uncertainty (variance) derived assuming sufficiently long data with high SNR, analogous  
 163 to the Laws of Large Numbers in classical statistics. Accounting only for zeroth order effects, they link MP  
 164 identification precision with data duration and frequency domain resolution of spectral peaks in the case of  
 165 well-separated modes identified using data from a single configuration of sensor location and type (or setup).  
 166 The research was later extended [27] to include 'first order effects', where test configuration starts making  
 167 influence through the ‘modal SNR’ (see equation (2) later). The zeroth and first order laws are presented in  
 168 section 4.



169 Figure 1: Objective of BAYOMALAW project

170 A secondary aim of the project, in support of the uncertainty management, is the creation of a bespoke  
 171 instrumentation system that simplifies the experimental work by removing the requirement to use wires,  
 172 radio transmission or GPS for synchronising data among accelerometers separated by distances of the order  
 173 of 1 km. This is achieved using high precision acquisition clocks and robust data acquisition architecture.

174 The project combines theoretical development with experimental trials and verification in a number of steps,  
175 beginning with the effects of modal SNR. Modal SNR is a key parameter that quantifies test configurations  
176 such as measurement noise, environmental load intensities and the measured degrees of freedom.

177 The large-scale experimental application and test of BAYOMALAW methodology focused on the 1,385 m  
178 span Jiangyin Bridge, with phases of preparation, on-site measurement and then post-processing. The phase  
179 of preparation included

- 180 • development of precision synchronised autonomous acceleration recorders developed as bespoke  
181 improvement on the technology used for ambient vibration testing of the Humber Bridge in 2008  
182 [31] and
- 183 • obtaining sample data from the structural health monitoring system operated by Jiangsu  
184 Transportation Institute (JSTI). The data were used to evaluate expected SNR and the strategy of  
185 reference and rover positions, as well as for defining measurement setup duration.

186 The testing phase involved three days of vibration measurements carried out by a team from JSTI, the  
187 University of Liverpool and the University of Exeter. Finally, the post-processing phase to recover MP and  
188 their uncertainties was carried out by researchers from the University of Liverpool.

189 Hence the paper first describes the bridge, then development of the autonomous loggers. Next the sample  
190 data and test planning are described. The field measurements are then summarised and the MP estimates  
191 presented. Finally, the implications for future studies and next steps are defined.

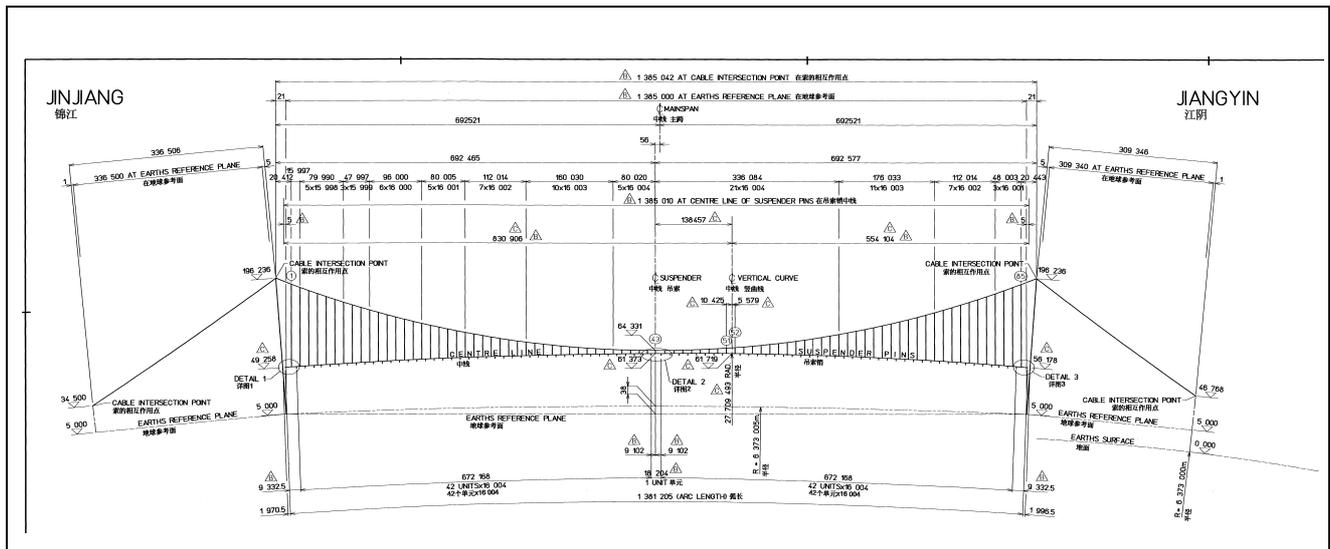
## 192 2 Jiangyin Yantze River Bridge

193 Jiangyin Bridge (Figure 2) with a single span of 1,385 m carries the G2 expressway across the Yangtze River  
194 in Jiangsu province, China, approximately 160 km northeast of Shanghai. It was the world's fourth longest  
195 span when completed in 1999. The 0.876 m diameter main cables each support the deck via 85 pairs of  
196 vertical hangers and are formed by prefabricated parallel wire strands, each constructed from  $127 \times 5.35$  mm  
197 diameter galvanised steel wires with 1,600 MPa tensile strength. Back stays attached to large gravity  
198 anchorages carry no deck load and have a diameter of 0.897 m. Towers with a height of 190 m are portal  
199 frames created by two hollow tower columns of reinforced concrete and three transverse beams. The main  
200 girder is a streamlined box section with a width of 36.9 m and a height of 3 m carrying a 29.5 m wide deck  
201 with three lanes of cars, buses and trucks in each direction. There is a 1.8 m walkway each side with limited  
202 access, for maintenance purposes.



203 Figure 2: Jiangyin Bridge

204 Information on the design and construction is provided in [32] and an elevation view is provided in Figure 3.



205 Figure 3: Jiangyin Bridge elevation

206 A structural health monitoring (SHM) system was installed at the time of construction but was upgraded in  
 207 2005 [32] to comprise 116 fibre Bragg gating sensors for strain and temperature measurement, 35 uniaxial  
 208 accelerometers, nine GPS receivers and four displacement sensors. Relevant for the present study, 15 of the  
 209 uniaxial accelerometers are located in the girder, at the 1/8, 1/4, 3/8 1/2 and 3/4-span distances from the  
 210 north tower and GPS receivers are mounted on rigid posts at 1/4, 1/2 and 3/4-span locations on the east and  
 211 west sides of the roadway.

212 While SHM system data have been extensively analysed [32,33], there is no information available on modal  
 213 properties other than a Chinese language report [34] referring to GPS and acceleration data from the SHM  
 214 system. For lateral vibration a symmetric mode at 0.0546 Hz and an antisymmetric mode at 0.1314 Hz were  
 215 identified from accelerometer data, while for vertical vibrations an antisymmetric mode at 0.0891 Hz and a  
 216 symmetric mode at 0.1258 Hz were identified. The identification algorithm is not known.

217 One specific concern with Jiangyin Bridge is the behaviour of the expansion joints located at each end of the  
 218 main span. These were replaced in 2007 due to damage, and movement has been controlled using large  
 219 viscous dampers [35] installed at the same time, so it is possible that these may affect the dynamic behaviour  
 220 of the bridge.

### 221 3 Logging systems for long span bridge ambient vibration testing

#### 222 3.1 Previous instrumentation deployments for long span suspension bridges

223 Previous experience in ambient vibration tests (AVTs) on long span bridges had considerable bearing on the  
 224 design of bespoke instrumentation for Jiangyin Bridge AVT and subsequent deployments.

225 In the 1980s, AVTs on:

- 226 • Humber Bridge [36], with main span 1,410 m and suspended side spans of 280 m and 530 m.
- 227 • 15 July Martyrs Bridge, also known as First Bosphorus Bridge, and referred to here as B1 [37] and
- 228 • Fatih Sultan Mehmet Bridge, also known as Second Bosphorus Bridge and referred to here as B2 [38]

229 used small wired arrays comprising a fixed reference accelerometer and no more than four roving  
 230 accelerometers and used 1<sup>st</sup> generation methods for OMA. Up to eight signal cables (250 m each) were laid  
 231 inside the box girder and daisy-chained to reach accelerometers that could be up to 1 km from the location of  
 232 the reference sensor and acquisition system. A four-channel analog tape recorder with poor signal to noise  
 233 ratio was used to record acceleration data that were post-processed by replaying signals through a two-  
 234 channel spectrum analyser, one pair at a time. These AVTs took up to 10 working days covering a sufficient  
 235 number of degrees of freedom using available cables and sensors while trying to obtain recordings of  
 236 approximately one hour per measurement. B1 (1,074 m main span) and B2 (1,090 m main span) do not have  
 237 suspended side spans which simplified the measurements, but for each of the three bridges considerable  
 238 trouble was taken to measure at several locations in both pylons of both towers.

239 An AVT of the Tamar Bridge, which has a 335 m main span was carried in 2006 [39]. Being a much shorter  
240 bridge, it was possible, thanks to good weather and strong support from the bridge management team, to do  
241 the test in a single day, but with some limitations. First, the bridge has a truss girder which is very difficult to  
242 negotiate so it was possible only to work outside the bridge, placing accelerometers on flat spaces between  
243 the walkway and south traffic lanes. It was also possible (with some difficulty) to carry two signal cables  
244 under the road deck, through the truss to the north side to be used as reference sensors opposite two sensors  
245 on the south side. Second, a limited number of 100 m signal cables was available.

246 To compensate for restrictions, best practice available for planning measurements to be used for OMA (now  
247 formally outlined in [28]) was followed. This defines the minimum number of cycles of the vibration mode  
248 to be identified as  $10/\zeta$ , so that assuming the same damping, the higher modal frequencies (shorter periods)  
249 allowed shorter recordings, as little as 15 minutes. Also a high specification 24-channel data acquisition  
250 system with a full set of accelerometers allowed for the simultaneous recording of 16 acceleration signals.  
251 Accelerometers were roved only along the south side of the bridge and only at the upper portal of the north  
252 pylons, allowing for a much more efficient measurement campaign. A 2<sup>nd</sup> generation OMA was used,  
253 resulting in a rapid evaluation of the MP, and helping to identify modes that were subsequently tracked in  
254 real time by a permanent SHM system [40] using automated and optimised OMA.

255 The Humber Bridge was retested in 2008 [31] in collaboration with researchers from the Faculty of  
256 Engineering of the University of Porto (FEUP) and from City University of Hong Kong. The campaign took  
257 advantage of the FEUP AVT technology that used GeoSIG autonomous recorders with GPS-synchronised  
258 acquisition. Combining resources provided a set of ten recorders each running a triaxial accelerometer set.  
259 Two pairs of recorders acted as rovers (two span-wise locations, one recorder each side of the deck on the  
260 walkway) leaving three pairs to rove all other span-wise locations, as well as all pier and portal levels of both  
261 towers' accelerometers channels (76 locations), doubling the level of detail in the 1985 test. Apart from  
262 measurements at mid-portals levels in the towers with no exterior access, all measurements used recorders on  
263 or outside the bridge, ensuring good reception by GPS antennae and enabling faster relocation of roving  
264 accelerometers between measurements. With this level of instrumentation it was possible to finish the AVT  
265 within five days which was followed by OMA using a range of 2<sup>nd</sup> generation procedures. One problem that  
266 surfaced during the OMA was the imperfect synchronisation of recorders. Errors of up to 7 ms were  
267 identified for units with GPS line of sight, more so for recorders inside the tower without the GPS satellite  
268 visibility.

## 269 **3.2 Instrumentation for Jiangyin Bridge AVT: the OCXO loggers**

270 The experience described above clearly indicated that the most efficient and effective data acquisition system  
271 for an AVT of a long span bridge would comprise autonomous recorders such as the GeoSIG recorder units.  
272 Concerns about synchronisation accuracy and the need to operate inside or outside a structure without any  
273 requirement for wired or wireless communication led to a specification for autonomous recorders with highly  
274 accurate clocks that would maintain synchronisation to better than 1 ms during a day of recording. There are  
275 two problems to address, the first being how to start a recording across a set of data *loggers* using a common  
276 timebase, the second being how to minimise drift due to acquisition timing variations.

### 277 **3.2.1 Logger design**

278 In principle, a common timebase could be set using GPS [41], but standard crystal oscillators controlling  
279 data acquisition hardware such as the popular National Instruments NI 9234 have quoted internal master  
280 timebase accuracy 50 parts per million (ppm) maximum. This means that a pair of initially synchronised but  
281 separate units could, in the worst case scenario, have timebases different by 0.36 s after just one hour of  
282 operation.

283 The best precision quartz oscillators available quote an accuracy of no better than 5 ppm, while oven  
284 controlled crystal oscillators (OCXOs) offer precision better than 1 ppm. Oscillator quartz crystal  
285 frequencies vary because of stiffness dependence on temperature, and OCXOs precisely control the oscillator  
286 temperature using a small oven. The technique of using OCXOs to maintain synchronisation became  
287 available in proprietary systems e.g. [42] but a system was required with complete flexibility e.g. to work  
288 with available triaxial or uniaxial accelerometers. Therefore, a set of autonomous loggers was built around  
289 National Instruments (NI) compact acquisition units controlled by 10 MHz OCXOs.

290 Each data logger, in a waterproof case, comprises power supply modules, switching and connectors as well  
291 as a 4-slot CompactRIO cRIO-9064 embedded controller. Each cRIO-9064 controls a 24-bit four channel NI  
292 9234 vibration input module set to acquire  $\pm 5V$ . To achieve the sub-1 ppm synchronisation a blank NI 9977  
293 C-series module is used to house the OCXO while a NI-9402 high speed digital IO module monitors the  
294 'local' OCXO clock ticks at 120 MHz. The cRIO-9064's integrated FPGA (programmed in LabVIEW  
295 environment) drives the acquisition module at its maximum 51.2kHz, then decimating the acquired signals  
296 based on the local OCXO clock ticks, after appropriate filtering to avoid aliasing issues.

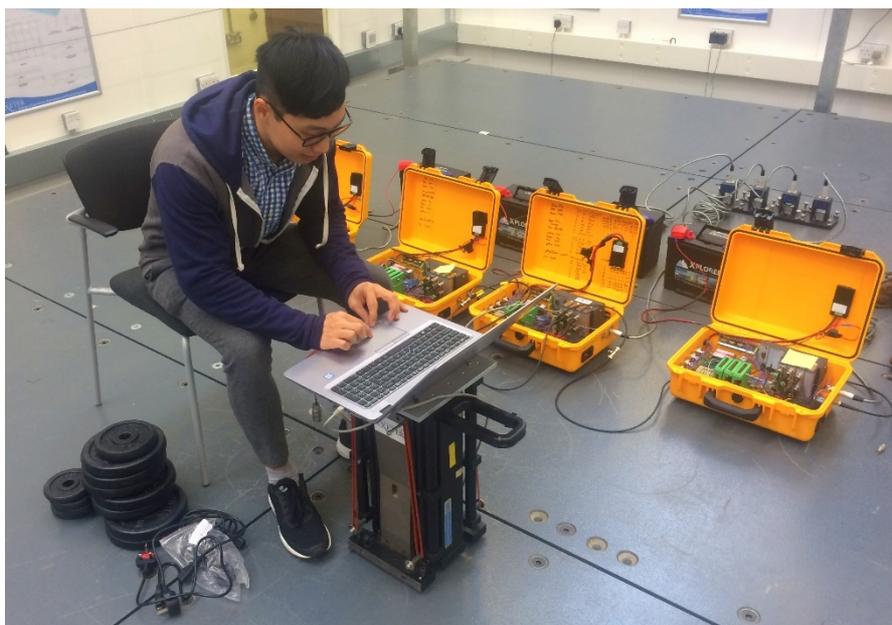
297 One logger is programmed as a master, the remaining loggers as slaves. To start synchronisation, the master  
298 unit is first powered up (using a battery or mains supply), followed by each of the slave loggers. Once the  
299 master is running and acquiring data (indicated by an LED flash sequence) the clock signal containing timing  
300 information from its master OCXO is briefly connected to the OCXO inside each of the slave units by BNC  
301 cable. The slave OCXO updates its own timing information so that the two clocks are synchronised. When  
302 the master OCXO has been connected to each slave (the process takes a few seconds and is confirmed by a  
303 flashing LED sequence) all loggers are then operating with the same timebase.

304 Using high-speed direct memory access, all acquired vibration/acceleration data are streamed to technical  
305 data management (TDMS) files stored on a high-capacity USB drive in the cRIO-9064. The data can be  
306 accessed during measurement via Ethernet to check data streams and to retrieve stored data using FTP.  
307 Alternatively each USB drive can be removed to copy files directly to the analysis PC.

308 When powered by batteries the loggers can then be spatially distributed. Each logger is set to work with  
309 either a set of Honeywell QA-750 uniaxial accelerometers or a single Japanese Aerospace JA-70SA triaxial  
310 accelerometer; Figure 4 shows a set of four synchronised loggers using battery power and acquiring from  
311 JA-70SA accelerometers. The LabVIEW application to check data acquisition during operation, is being  
312 used on the laptop in the setup shown in Figure 4.

313 The measurement duration depends on the test plan, logistics and ultimately battery capacity. NI hardware  
314 and OCXO together draw 7 W so planned battery capacity has to be more than adequate for a day of site  
315 measurement that can potentially last up to 12 hours. In fact a more compact but lower capacity battery can  
316 be installed in the yellow box, but could only be used if the boxes are shipped to site by road or rail, due to  
317 civil aviation restrictions on carrying batteries which work against any battery operated system.

318 When all measurements are complete, the loggers are turned off, USB drives removed and data transferred to  
319 a prepared set of directories on the PC. A MATLAB script is used to assemble data into a common time-  
320 synchronised set covering the common period of logger operation and the data file is chopped into pieces  
321 representing individual measurements. This part of the operation is low-tech, requiring written records of  
322 time and location for each logger.



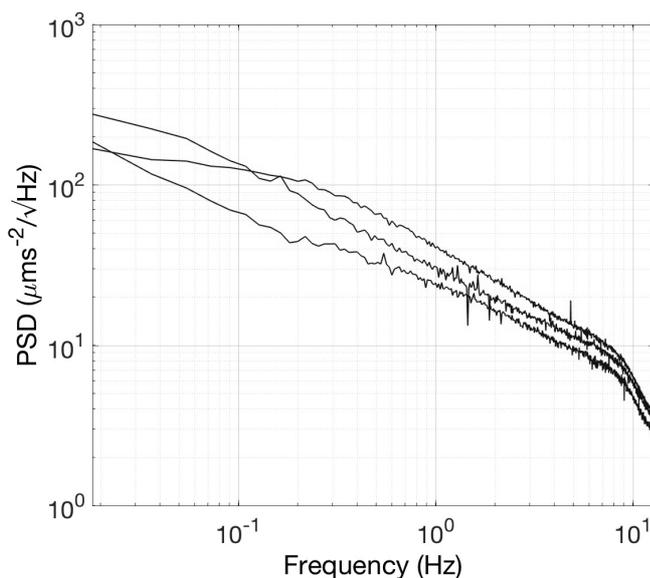
324 Figure 4: Using a set of four OCXO-controlled loggers with JA-70SA triaxial accelerometers.

### 325 3.2.2 Logger performance: noise floor and synchronisation drift

326 The logger system was function-tested using a field test on a 100 m cable-stayed footbridge in Exeter [43]  
327 shortly before the equipment was shipped to Shanghai for the Jiangyin Bridge AVT. In addition the noise  
328 floor of the system was evaluated using a 'huddle test' [44] which is a common recording of collocated  
329 sensors and loggers that isolates sensor noise by removing the common true structural vibration. Figure 4  
330 shows preparation for the huddle test using the JA-70SA accelerometers planned to be used at Jiangyin  
331 Bridge. Due to an operational problem with the JAs (subsequently resolved) QA-750 accelerometers were in  
332 fact used, and the noise floor was separately evaluated in set up similar to Figure 4.

333 The huddle test is based on the discovery [45] that no more than three channels of data from three identical  
334 and collocated sensors can be used to identify their noise floor. For a single sensor, noise floor evaluation  
335 would require either an extremely quiet environment (e.g. in a salt mine) or some form of isolation [45] but  
336 using the minimum of three sensors allows redundant information about the mechanical excitation of the  
337 collocated sensors to be removed from their signals.

338 Figure 5 shows the procedure applied to 14 hours of vertical acceleration data from QA-750 accelerometers  
339 connected to three (separate) loggers. The logarithmic axes emphasise the relatively high noise at low  
340 frequencies common to all types of accelerometer. Based on the plot, a conservative estimate of self-noise of  
341 sensor and analog to digital converter at 0.05 Hz (where the first lateral mode is expected) is  $30 \mu\text{ms}^{-2}/\sqrt{\text{Hz}}$   
342 i.e.  $3 \mu\text{g}/\sqrt{\text{Hz}}$ . This noise floor was used in the test planning described in 4.2.



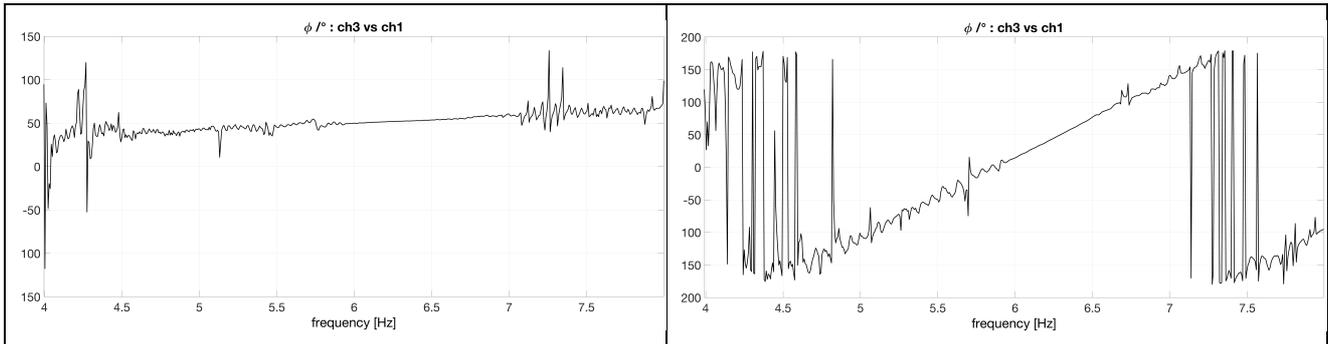
343  
344 Figure 5: Power spectral density (PSD) of QA-750 noise floor from Huddle test of Figure 4, using [45].

345 A check in synchronisation drift was required because the resulting phase angle shift between channels could  
346 compromise MP identification. The four loggers were sat on a laboratory floor, synchronised and left for 42  
347 hours to record ambient vibrations including weak response of the floor to footfalls. Data samples were taken  
348 from the recording after three hours and 42 hours and cross-power spectra evaluated. Transfer functions were  
349 constructed for three slave loggers with respect to the master, and Figure 6 shows phase angle for one of the  
350 slave loggers (relative to the master logger) for around 6 Hz, which is the first mode frequency of the floor.

351 Consider that the common signal in a recording sample should result in the same Fourier amplitude  
352  $A \sin(\omega t + \phi)$  at a given frequency  $\omega$  having a common phase angle  $\phi$  that depends on the starting angle of  
353 the sinusoid at  $t = 0$ . Any difference in phase angle among signals recorded by different loggers must be due  
354 to a different local value  $t$  at the start time of the sample so that synchronisation drift  $\Delta t = \phi/\omega$ .

355 The slope at 3 hours is approximately  $10^\circ/\text{Hz}$  i.e. 0.1745 radian per  $2\pi$  radian/s. This is  $\Delta t = 0.028$  s drift in  
356 10,800 s i.e. 2.6 ppm. After 42 hours the phase shift is  $126^\circ/\text{Hz}$  corresponding to  $\Delta t = 0.35$  s i.e. 2.3 ppm.  
357 This indicates a drift that is approximately linear with time, and this varies between loggers. The implication  
358 is that the phase shift for 1 Hz mode after 10 hours of operation would be at a maximum of  $30^\circ$ .

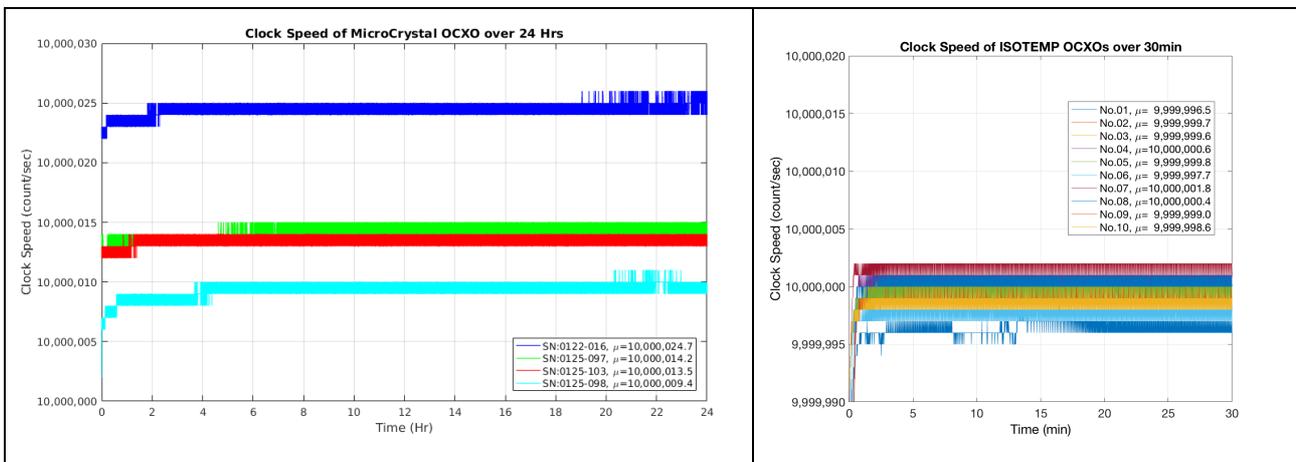
359 To avoid compromising OMA procedures that are sensitive to phase angle this drift be fixed by resampling  
 360 signals in post-processing if the drift is known.  
 361



362 Figure 6: Effect of synchronisation drift on phase angle after a) 3 hours and b) 42 hours.

363 While the OCXO clocks are expected to have minimal drift, an explanation of the observed drift is that  
 364 actual clock speeds could apparently differ slightly from the nominal 120 MHz. To check this, the loggers  
 365 OCXOs were transplanted into GPS-synchronised wireless sensor nodes [41] to test the clock speed. Due to  
 366 lack of time between logger construction and deployment in China, both this study and the 42 h recording  
 367 followed the field test, and the actual clock speeds for the four deployed OCXOs are shown in Figure 7. The  
 368 plot indicates that clocks need a short warming-up period period to reach a stable frequency which is  
 369 maintained to within 0.2 ppm for a long duration. Even though individual clock speeds are stable, the set of  
 370 four speeds shows significant variance.

371 The OCXOs have been replaced with units having tighter clock speed tolerances and shorter times to warm  
 372 up. The replacement OCXOs which will be used in a larger set of ten loggers reach the stable frequency  
 373 faster and limit aggregate drift to 0.4 ppm.



374 Figure 7: OCXO clock speeds for two types of OCXO (left) for four units used at Jiangyin Bridge and (right)  
 375 for ten units used subsequently.

## 376 4 Planning using BAYOMALAW

377 Planning for the Jiangyin Bridge modal test was based on a combination of prior experience with other  
 378 bridges and preliminary data for Jiangyin Bridge obtained by JSTI.

379 In a Bayesian context, the identification uncertainty is quantified in terms of its variance given information  
 380 of the data set, but the value itself (for the given set of data) reveals little insight about how it depends on the  
 381 quality or statistical characteristics of data. Beyond the ability to *calculate*, planning ambient vibration tests  
 382 requires the ability to *understand* identification uncertainty and master its relationship with test  
 383 configuration. When the data indeed follow the distribution assumed in the identification, i.e., stochastic  
 384 stationary and classically damped, it can be shown that the variance follows a statistical law analogous to the  
 385 Laws of Large Numbers in classical statistics. It has a deterministic part and a random part. The random part  
 386 depends on specific details of the data and is asymptotically negligible as data length increases. The

387 deterministic part depends on the ‘information content’ of the data that is related to test configuration and is  
 388 what affects test-planning decisions. This deterministic part is referred to as uncertainty laws in recent  
 389 studies [27,29].

390 The OMA uncertainty laws are closed form asymptotic expressions and they are the primary scientific target  
 391 of the BAYOMALAW project. The derivation involves asymptotics and leverages on the particular  
 392 mathematical structure of the OMA problem in specific contexts such as well-separated modes, closely-  
 393 spaced modes, multiple setups, but the findings to date show that the results can be remarkably simple [29].  
 394 The findings can be used to guide test planning for different circumstances of prior information.

#### 395 **4.1 BAYOMALAW for planning with preliminary data**

396 When the preliminary data are available, their quality can be assessed using their power spectral density  
 397 (PSD), and the identification uncertainty of the MP can then be quantified using 'zeroth order' relationships  
 398 with e.g. data duration [29][30] corrected for 'first order' effects [27] due to test configuration. The  
 399 uncertainty laws are developed for modal frequency, damping and shape.

400 Of these, damping ratio is the parameter whose uncertainty is simultaneously the most important and hardest  
 401 to control. For well-separated modes, small damping and long data in a single measurement, or setup using  
 402 one or more response data channels the posterior CoV.  $\delta$  (coefficient of variation = standard deviation/mean)  
 403 for damping ratio is asymptotically given by

$$404 \quad \delta \sim \delta_1 = \delta_0 \sqrt{1 + \frac{a(\kappa)}{\gamma}}. \quad (1)$$

405 ‘ $\sim$ ’ means ‘asymptotic to’ and denotes that CoV.  $\delta$  tends to  $\delta_1$  asymptotically as  $\zeta \rightarrow 0$ , and data length  $\rightarrow \infty$ .  
 406 Here,  $\delta_0$  and  $\delta_1$  represent the CoVs, respectively, given by ‘zeroth order’ and ‘first order’ uncertainty laws.

407 The parameter of particular relevance is the modal SNR  $\gamma$ , whose definition is motivated from the  
 408 uncertainty law theory. Defining  $S$  as the 'modal force PSD', the modal SNR is the ratio of modal PSD  $S/4\zeta^2$   
 409 to the noise PSD  $S_e$  at the natural frequency  $f$  i.e.

$$410 \quad \gamma = \frac{S}{4S_e\zeta^2} \quad (2)$$

411 while the zeroth order law that gives  $\delta$  when the modal SNR is infinite [29,30] is given by

$$412 \quad \delta_0 = \frac{1}{\sqrt{2\pi\zeta N_c B(\kappa)}} \quad (3)$$

413  $N_c$  is the ‘effective data length’ expressing the data duration  $T$  as a multiple of the natural period  $1/f$ , i.e.

$$414 \quad N_c = Tf \quad (4)$$

415 and is conventionally understood to be a key parameter controlling reliability of MP obtained from OMA.

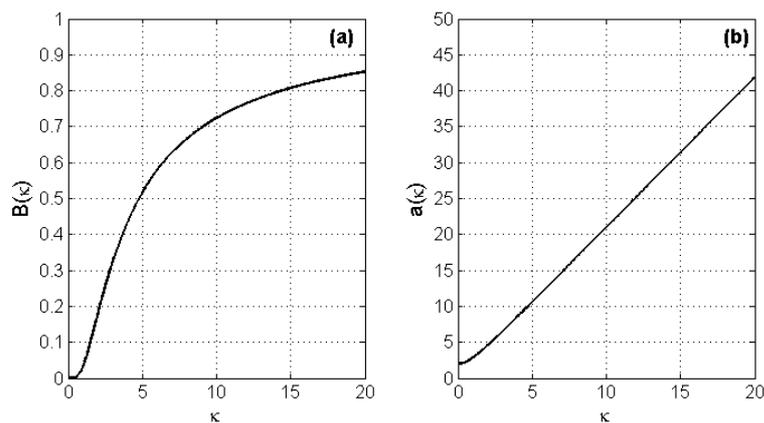
416  $\kappa$  is a dimensionless ‘bandwidth factor’ that reflects usable bandwidth  $f(1 \pm \kappa\zeta)$  around the natural  
 417 frequency  $f$  and on which depend parameters  $a(\kappa)$  and  $B(\kappa)$ :

$$418 \quad B(\kappa) = \frac{2}{\pi} \left[ \tan^{-1}(\kappa) + \frac{\kappa}{\kappa^2+1} - \frac{2}{\kappa} (\tan^{-1}\kappa)^2 \right] \quad (5)$$

$$419 \quad a(\kappa) = \frac{4(\kappa^2+1)(3\tan^{-1}\kappa - 3\kappa + \kappa^2 \tan^{-1}\kappa) \tan^{-1}\kappa}{3[(\kappa^2+1)(\kappa - 2\tan^{-1}\kappa) \tan^{-1}\kappa + \kappa^2]}. \quad (6)$$

420 Figure 8 plots  $B(\kappa)$  and  $a(\kappa)$ , which are both increasing functions of  $\kappa$ .

421  $\kappa = 1$  represents the classical ‘half power bandwidth’ containing  $2\zeta N_c$  FFT points [18] and  $T$  can be  
 422 considered to be long when the number of FFT points  $N_f = 2\kappa\zeta N_c$  is large compared to 1.

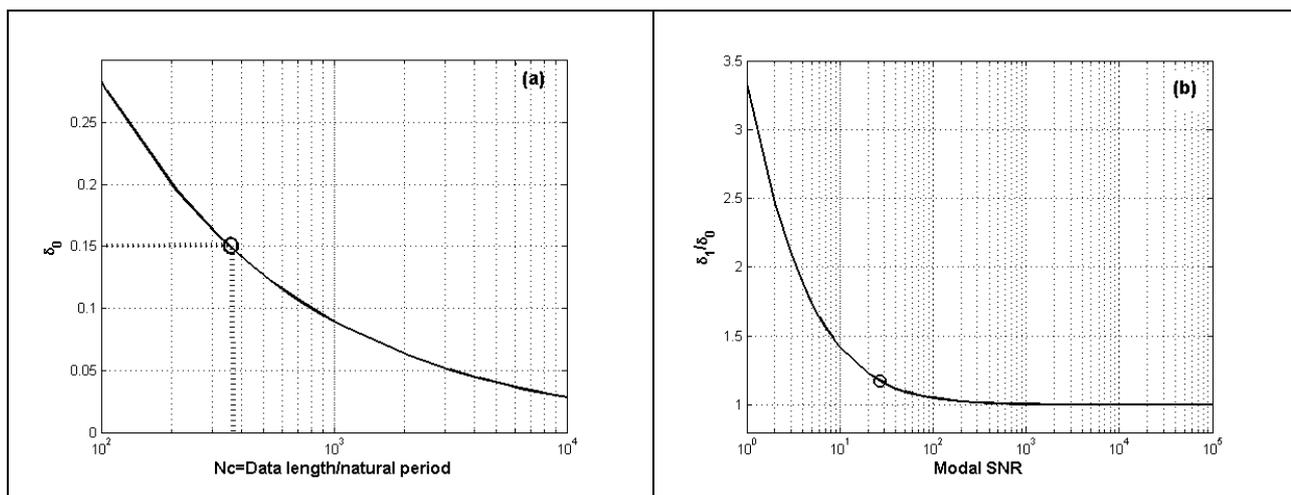


423 Figure 8: Plot of  $B(\kappa)$  (a) and  $a(\kappa)$  (b) with bandwidth factor  $\kappa$ .

424 The zeroth order uncertainty law  $\delta_0$  gives the achievable identification precision because it is the lower limit  
 425 of uncertainty for ideal situation with noiseless equipment ( $\gamma \rightarrow \infty$ ). This limit is not zero because the (input)  
 426 ambient excitation is unknown. The zeroth order law is less informative in guiding the ambient vibration test  
 427 because the effect of modal SNR  $\gamma$  has not been explicitly captured. The first order uncertainty law  $\delta_1$   
 428 depends on the modal SNR, thus providing a key quantity for planning the ambient vibration test.

429 A sample of data from the Jiangyin Bridge SHM system was provided by JSTI but proved to be problematic  
 430 in terms of the synchronisation and orientation of the sensors. However the data were sufficient to indicate a  
 431 vertical mode at 0.106 Hz with a damping ratio of 4 % and a usable identification band of [0.07 0.11] Hz that  
 432 gives a bandwidth factor 4.78. Figure 9a shows that for 15% CoV.,  $N_e=360$  natural periods are required,  
 433 corresponding to one hour for the given mode.

434 From the preliminary data, it is estimated  $S=9 \times 10^{-8} \text{g}^2/\text{Hz}$  and  $S_e=5.3 \times 10^{-7} \text{g}^2/\text{Hz}$ . Using the 4% damping  
 435 estimate for the first vertical mode provides an estimated  $\text{SNR}=27$  using equation (2). This SNR leads to a  
 436 20% increase (compared to ideal noiseless situation) using the first order correction (Figure 9b). The ambient  
 437 excitation  $S$  cannot be directly controlled but would be expected to have a similar level of spectral density in  
 438 the planned test compared to the sample data. By optimising the sensor locations and using sensors with a  
 439 lower noise level in the main test, it would be expected that a larger SNR would be achievable.



440 Figure 9: Zeroth and first order CoV. for damping ratio given parameters for Jiangyin Bridge first vertical  
 441 mode. (a) zeroth order without effect of SNR, (b) first order correction depending on SNR.

## 442 4.2 Planning without preliminary data

443 It is reasonable to expect that MPs for Jiangyin Bridge would be similar to those obtained at Humber Bridge  
 444 [31] and Fatih Mehmet Sultan Bridge [38] (B2, Turkey). The critical modes for each of these bridges are the  
 445 fundamental lateral and vertical modes L1 and V1 and could be used in the absence of the JSTI sample data.

446 For B2, MP identified by 1<sup>st</sup> generation OMA are V1 ( $f=0.125$  Hz,  $\zeta=1.33\%$ ), L1 ( $f=0.077$  Hz,  $\zeta<14.4\%$ ), for  
 447 the British bridge using 2<sup>nd</sup> generation OMA V1 ( $f=0.116$  Hz,  $\zeta=3.1\%$ ), L1 ( $f=0.055$  Hz,  $\zeta=4\%$ ).

448 Practical guidelines for planning ambient vibration tests for Bayesian OMA are given in [27]. Without  
 449 preliminary data, MP for the similar bridges (i.e. Humber and B2) could be used to guide planning, although  
 450 in this case natural frequency estimates are available, which are quite close to values for Humber and B2.  
 451 Without other information on data quality the bandwidth factor  $\kappa$  is not known. To separate the effect of data  
 452 duration and SNR, the first order posterior CoV. represented in equations (1) and (3) is rearranged into two  
 453 factors:

454 
$$\delta_1 = A_1 A_2$$

455 where

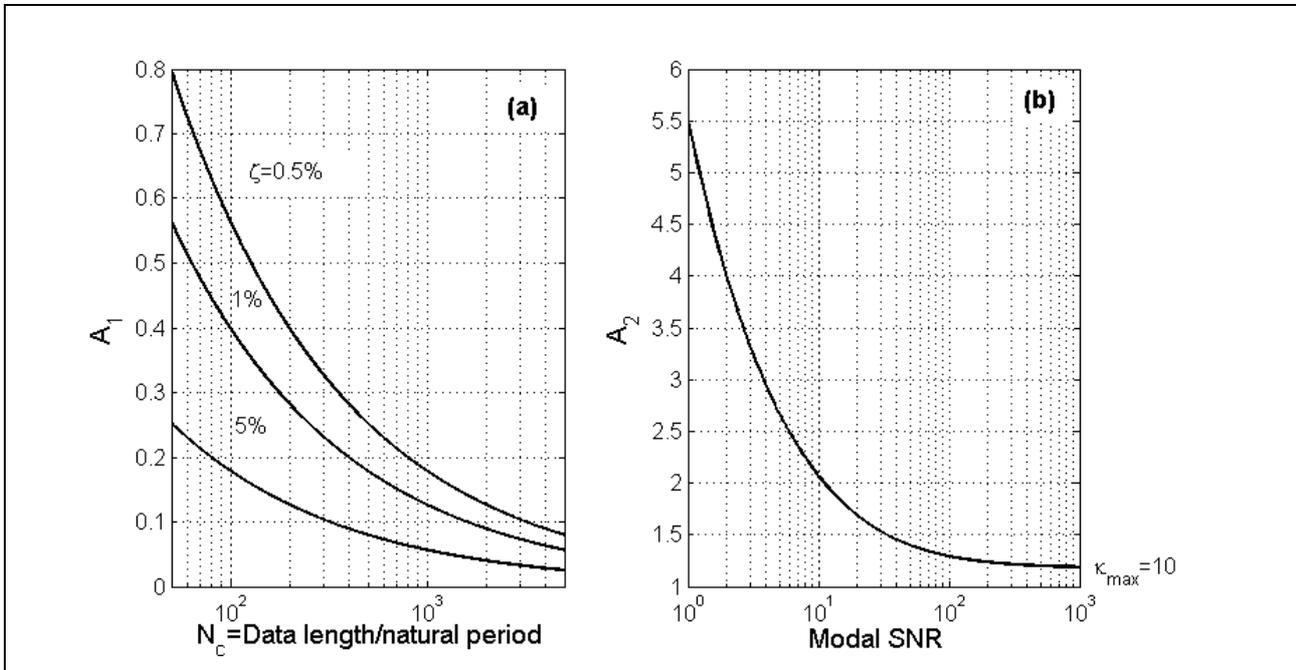
456 
$$A_1 = \frac{1}{\sqrt{2\pi\zeta N_c}}$$

457 
$$A_2 = \sqrt{\frac{1 + a(\kappa)/\gamma}{B(\kappa)}}$$

458 and  $B(\kappa)$  and  $a(\kappa)$  are given in equations (5) and (6).

459 Figure 10 shows  $A_1$ , which accounts for the duration (in terms of number of cycles) and  $A_2$ , which separately  
 460 accounts for the SNR. At the planning stage the bandwidth factor  $\kappa$  is not known, but it can be taken to  
 461 depend on the SNR [27] since a more noisy signal buries the spectral peak, reducing the useful frequency  
 462 range of information. A reasonable choice consistent with common practice is the minimum of  $2\sqrt{\gamma}$  and  
 463 some  $\kappa_{\max}$  that controls the risk of modelling error (e.g. unaccounted mode). Again, these considerations  
 464 on the bandwidth factor are only relevant at the planning stage because its value directly results from the  
 465 choice of the band for identification when the data is available.

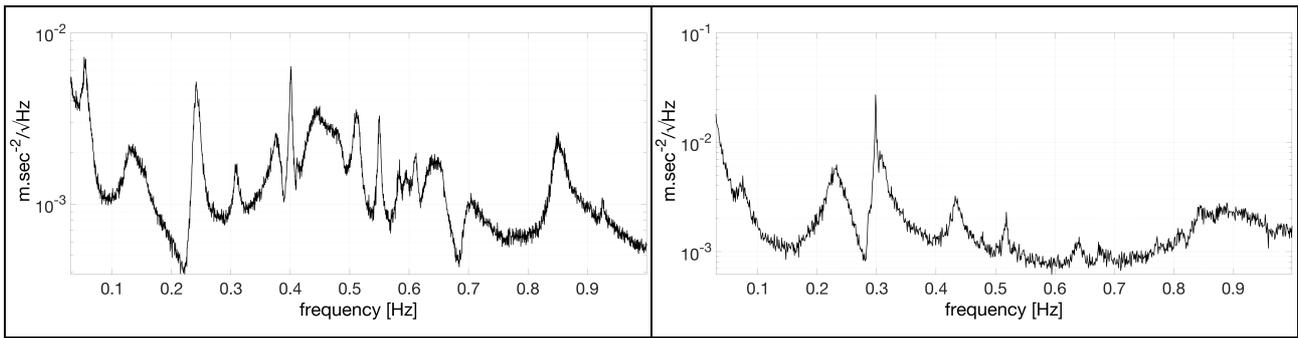
466



467 Figure 10: Assessing test configuration in terms of a) data duration and b) modal SNR.

468 Although a value of SNR=27 is available from preliminary data, a typical value could also be obtained using  
 469 the accelerometer noise floor and the experience from other bridges. From the OCXO laboratory huddle test,  
 470 the sensor and acquisition system background noise is  $\sqrt{S_e}=30 \mu\text{ms}^{-2}/\sqrt{\text{Hz}}$ , while the square root of modal  
 471 acceleration PSD for the most challenging first lateral mode would be somewhere between the values

472 obtained from the Humber Bridge,  $\sqrt{S}/2\zeta=0.007 \text{ ms}^{-2}/\sqrt{\text{Hz}}$ , and B2,  $\sqrt{S}/2\zeta=0.005 \text{ ms}^{-2}/\sqrt{\text{Hz}}$  (Figure 11),  
 473 giving a worst case  $\text{SNR}=3\times 10^4$ , much better than from the preliminary data.



474 Figure 11: Square root PSD for Humber Bridge (left) and Second Bosphorus Bridge (right) lateral  
 475 acceleration.

### 476 4.3 Factors controlling modal SNR

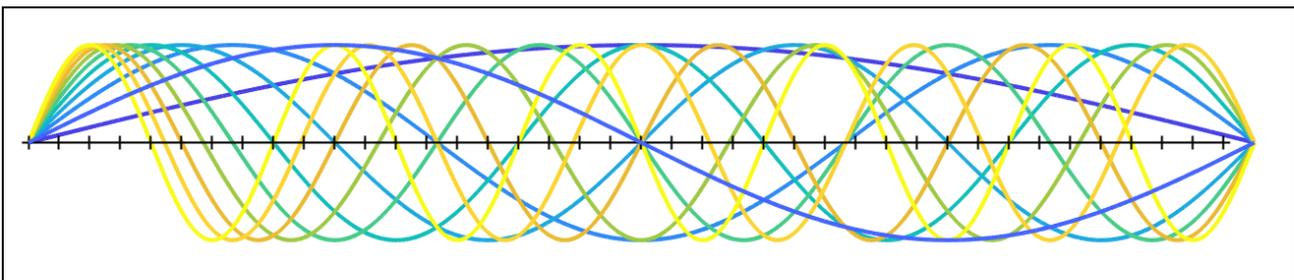
477 Modal SNR derives from two factors. The quality of the sensor and data acquisition hardware control the  
 478 noise PSD  $S_e$  while the number and location of sensors control modal force PSD  $S$ , which is proportional to  
 479 the square of the mode shape values at the measured degrees of freedom (DOFs). For identifying a single  
 480 mode with multiple sensors, the obviously strategy is to maximise the sum of squares of mode shape values.

481 Sensor locations on a structure (test points or TPs) will have up to three DOFs, and their locations will be  
 482 chosen logically according to convenient structural features - for example, the hanger (suspender) attachment  
 483 points to the deck. For all the referenced long span bridge AVTs tests there were more DOFs than available  
 484 single-axis sensors. Reference TPs and DOFs are selected common to all setups so that mode shape pieces  
 485 can be glued or assembled [46]. Therefore, for the purposes of planning a modal test, the questions to be  
 486 answered are:

- 487 1. How many reference locations should be measured?
- 488 2. Where should the reference sensors be placed?
- 489 3. How should the remaining sensors be roved in how many setups?
- 490 4. How long should the data (acquisition) of each setup be?

491 Although there are possibilities, such as daisy chaining references, to simplify planning we assume that the  
 492 reference location or locations are fixed in all setups. Reference sensors should be located so that the target  
 493 modes, or as many modes as possible, can be identified in all setups, avoiding modes of interest while  
 494 maximising modal responses (via mode shape ordinates) for expected mode shapes, which can be based on  
 495 experience or numerical simulations.

496 So where are the best reference locations? Figure 12 shows a set of ten mode shapes for a simply supported  
 497 beam with 41 TPs (indicated by vertical bars, including support locations). These mode shapes might  
 498 resemble vertical mode shapes of a suspension bridge and siting a reference anywhere from the 5<sup>th</sup> to the 37<sup>th</sup>  
 499 TPs will find at least one mode having modal ordinate at or close to zero. However, the first mode, which has  
 500 the smallest modal ordinates in the TP ranges 2-4 and 38-40, is likely to be the most important to identify  
 501 reliably. One possible strategy is to use multiple references, but this reduces the number of ‘roving’ sensors  
 502 available for remaining test points, which will require more setups each using (for a fixed available test  
 503 campaign duration) reduced data duration  $T$ .

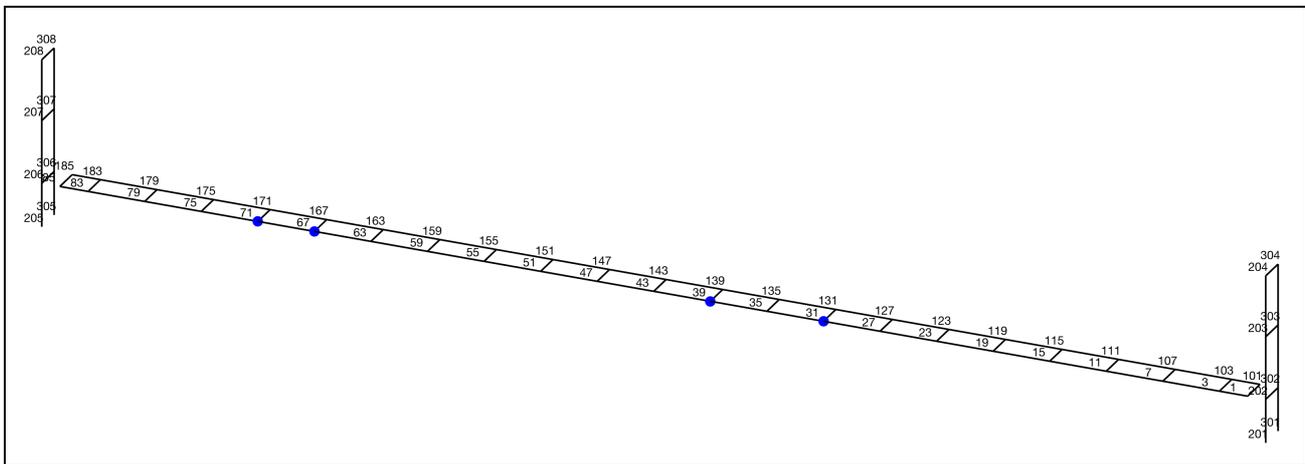


504 Figure 12: Ten mode shapes for a simply supported beam with 41 TPs.

505 For a single mode where the reference DOF ordinate is counted only once, maximising the SNR for the same  
506 modal excitation and sensor type means maximising the sum of squares of mode shape values. For the usual  
507 case of measuring multiple modes where the SNR is proportional to at least the square of the modal ordinate,  
508 there is as yet no metric to maximise.

## 509 **5 Ambient vibration test (AVT)**

510 The AVT was scheduled for Tuesday 25th to Thursday 27th April 2017. Equipment comprising four loggers,  
511 12 QA-750 accelerometers, 240 m of colour-coded signal cable in 10 m and 20 m lengths and a laptop were  
512 flown from the UK as checked baggage, with NI hardware removed from the loggers and packed securely. A  
513 set of four high-capacity batteries were provided by JSTI and local transport was provided by JSTI. Weather  
514 was mostly fine, usually low visibility with smog clearing towards the end of the test, minimal rain and weak  
515 to moderate winds. The south end of the east walkway of the bridge was accessed by driving from the south  
516 and parking at the tower at the edge of the traffic lanes, then climbing onto the walkway from the approach  
517 span side. Hanger attachment points are clearly numbered from north to south, and the four channels of the  
518 master OCXO box were used as references with lateral (transverse direction) and vertical accelerometers at  
519 TP67 and TP71, aiming to identify mode shape pieces using three roving OCXO boxes, each typically  
520 measuring at a single TP. Figure 13 shows Jiangyin Bridge TPs, corresponding to hanger attachment points  
521 and tower portals, as well as indicating reference locations and TPs for a typical measurement setup.



522 Figure 13: Jiangyin Bridge test points and sensor locations for a measurement setup.

523 A detailed method statement, including a risk assessment, was prepared for the scheduled three days  
524 comprising a sequence of one-hour measurements, with an option to use a fourth day in the event of bad  
525 weather. The plan covered every fourth TP on the east walkway, two TPs on the west walkway opposite the  
526 reference TPs, as many as possible TPs on the south tower, and if time permitted, the equivalent locations on  
527 the north tower. Figure 14 shows the loggers being synchronised before a huddle test at the start of the  
528 sequence and the pair of accelerometers at TP67. The bridge is aligned approximately in north-south axis  
529 with a coordinate system of X=longitudinal, positive to the north, Y=lateral, positive to the west and  
530 Z=vertical, positive upwards.

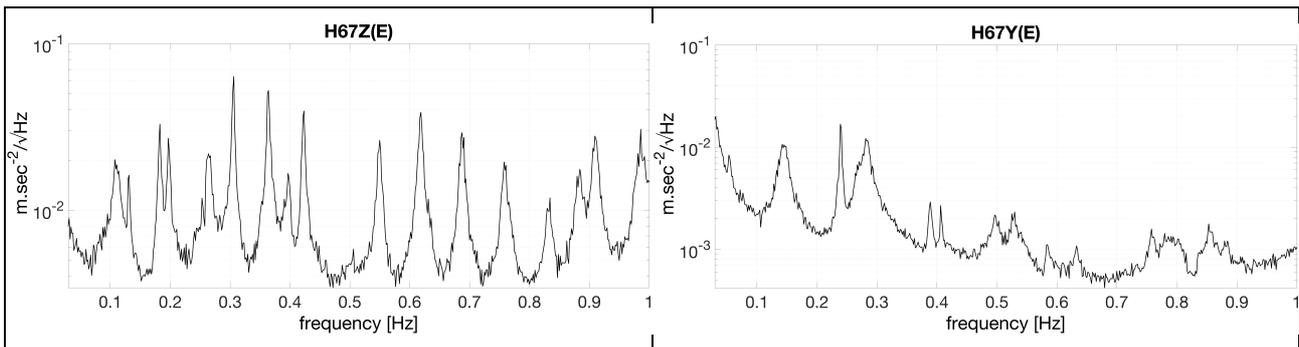
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532 Figure 14: Setting up acquisition for initial huddle test and reference accelerometers at H67.

533 The reference accelerometer signals were recorded by the master logger continuously during the day of  
 534 measurement while the three slave loggers were roved to remaining TPs. The moves were coordinated by  
 535 mobile phone text messages and timed so as to maintain a minimum one hour recording for all loggers.  
 536 Square root PSD plots for the first day of vertical and lateral acceleration data at TP67 are shown in Figure  
 537 15.

538



539 Figure 15: Sample square root PSD for TP67 vertical (Z) and lateral (Y) acceleration.

540 Table 1 Summarises the measurement sequence over three days.

541 Table 1: Measurement setup using two to four acquisition channels (ADs) of master and slave loggers.

setup	day	start	end	M				S1			S3		
				DA1	DA2	DA3	DA4	DA1	DA2	DA3	DA1	DA2	DA3
1	26	11:14	12:17	H67EZ	H67EY	H71EZ	H71EY	H83EZ	H83EY	-	H75EZ	H75EY	-
2	26	12:30	13:34	H67EZ	H67EY	H71EZ	H71EY	H63EZ	H63EY	-	H55EZ	H55EY	-
3	26	13:41	14:45	H67EZ	H67EY	H71EZ	H71EY	H51EZ	H51EY	-	H43EZ	H43EY	-
4	26	14:54	16:00	H67EZ	H67EY	H71EZ	H71EY	H39EZ	H39EY	-	H31EZ	H31EY	-
5	26	16:08	17:10	H67EZ	H67EY	H71EZ	H71EY	H27EZ	H27EY	-	H19EZ	H19EY	-
6A	27	10:19	11:20	H67EZ	H67EY	H71EZ	H71EY	H23EZ	H23EY	-	H15EZ	H15EY	-
6B	27	11:26	12:30	H67EZ	H67EY	H71EZ	H71EY	H35EZ	H35EY	-	H11EZ	H11EY	-
6C	27	12:36	13:38	H67EZ	H67EY	H71EZ	H71EY	H47EZ	H47EY	-	H7EZ	H7EY	-
6D	27	13:53	14:55	H67EZ	H67EY	H71EZ	H71EY	H59EZ	H59EY	-	H3EZ	H3EY	-
6E	27	15:02	16:05	H67EZ	H67EY	H71EZ	H71EY	H79EZ	H79EY	-	-	-	-
6F	27	16:12	17:13	H67EZ	H67EY	H71EZ	H71EY	H85EZ	H85EY	-	STLEX	STLEY	-
7	28	10:16	11:22	H67EZ	H67EY	H71EZ	H71EY	H71WZ	H71WY	-	H67WZ	H67WY	-
8	28	12:30	13:30	H67EZ	H67EY	H71EZ	H71EY	STUEX	STUEY	STUWX	STMEX	STMUY	STMWX

542 E/W are east/west side of bridge; L/M/U are lower/mid/upper portal levels of tower

543 Logger S2 was used in the first day of measurements but when data were merged it was found that  
544 synchronisation had not been correctly initiated. This was due to allowing too short a time for the  
545 synchronisation, requiring only a minor change in the initialisation procedure. An inelegant method to guard  
546 against this would be to generate a similar and obvious signal on all sensors while collocated at the start of  
547 the sequence after initialisation. An example would be a sequence of ‘heel drops’ on the bridge deck,  
548 providing obvious signals that could be subsequently aligned using correlations between data channels.

549 In this case, being the first major exercise, no chances were taken and measurements covered by S2 were  
550 repeated in setups 6A-6F, while S2 was not used until it could be thoroughly checked on return to the UK.  
551 Setup 6F included X and Y measurement of the south tower (ST) lower (L) portal, deck level east side  
552 (Figure 16 left) and setup 7 was designed to identify torsional modes.

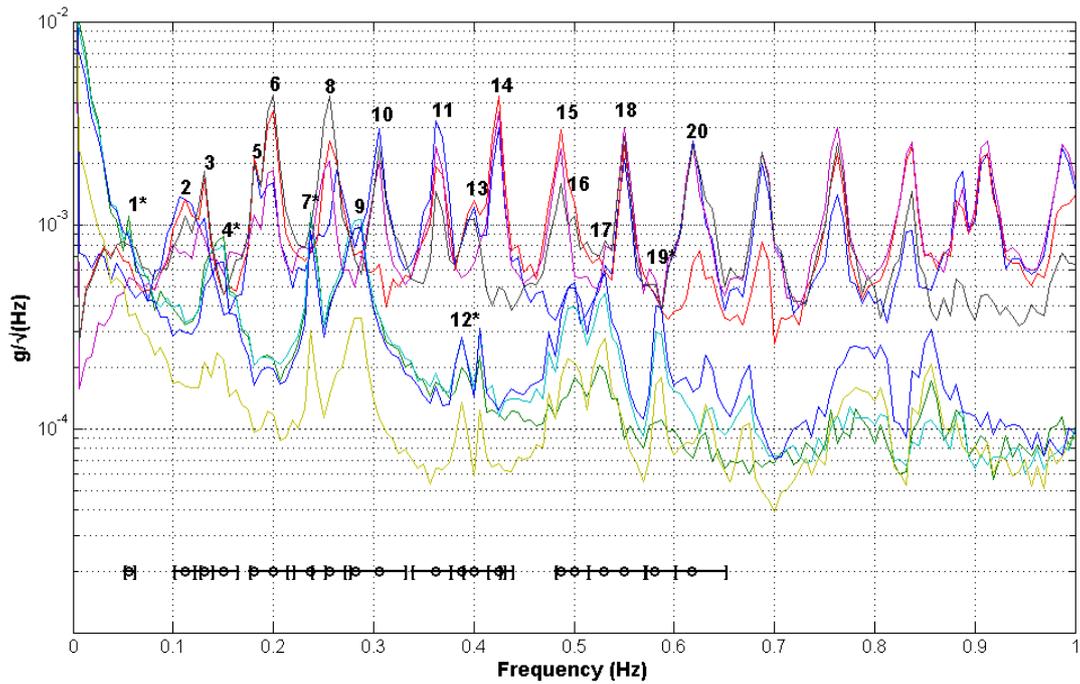
553 Setup 8, with measurements in the middle and upper portal, was designed to characterise the tower  
554 component of deck girder modes as well as modes only involving the tower. Jiangsu Yangtze Bridge Co.  
555 provided access to the south tower via the door visible in Figure 16 (left), and the middle portal (interior  
556 shown in Figure 16 right reflecting the depth visible in Figure 2) was accessed by a lift. Access to the upper  
557 portal was via transfer to a second lift at a mezzanine level and to the top of the portal for the view of Figure  
558 2 by staircase. The value of autonomous loggers was proven by the lack of GPS or wireless signal reception  
559 inside the middle portal, and it was more convenient to measure the inside the upper portal.



560 Figure 16: Measurements at lower (left) and middle (right) portal levels

## 561 **6 Modal parameter estimates using Bayesian operational modal analysis**

562 To visualise the modes of interest and the relevant frequency bands, the (root) PSD spectrum of Setup 1 (as a  
563 typical case) is shown in Figure 17. It can be seen that the lines plotted in the PSD spectrum are generally  
564 separated into two levels, indicating two different PSD levels of modal responses among the data channels.  
565 The higher levels are those measuring vertical directions and the lower ones are those measuring lateral  
566 directions. This is a typical case for AVT of long-span bridges as the spectral density of modal forces in the  
567 vertical direction (mainly due to traffic) is normally much larger than those in the lateral direction (mainly  
568 due to wind). With this situation, it is easier to detect potential modes based on the PSD spectrum than on the  
569 singular value (SV) spectrum. This issue will also cause problems in identifying the lateral modes, which  
570 will be discussed later.



571

572 Figure 17: Square root PSD spectrum of Setup 1

573 There are many modes within the 0-1 Hz band, including several closely-spaced modes, which increases the  
 574 identification difficulties and the associated uncertainty. Modal identification here focuses on the first twenty  
 575 modes indicated in the PSD spectrum. The initial guesses of natural frequency and the selected frequency  
 576 bands for modes of interest are indicated in the figure with symbols 'o' and '[-]', respectively. The symbol  
 577 '\*' (as the superscript) denotes that the mode is identified using data in the lateral direction only. Modal  
 578 parameters are identified in individual setups based on the Bayesian FFT method [47,48] and the overall  
 579 mode shapes are assembled using the global least square method [46]. In this procedure, the real parts (or  
 580 projections on to real axes) of the mode shape ordinates are used, so the small phase angle shifts suggested  
 581 earlier would have no consequence.

582 Table 2 summarises the representative values of the identified modal parameters and associate uncertainties,  
 583 where the MPV (most probable value) in the table denotes the sample mean of MPVs among the setups and  
 584 the CoV. CoV is calculated based on the representative variance (i.e., sum of sample variance among the  
 585 setups and the sample mean of the posterior variance among the setups).

586

587

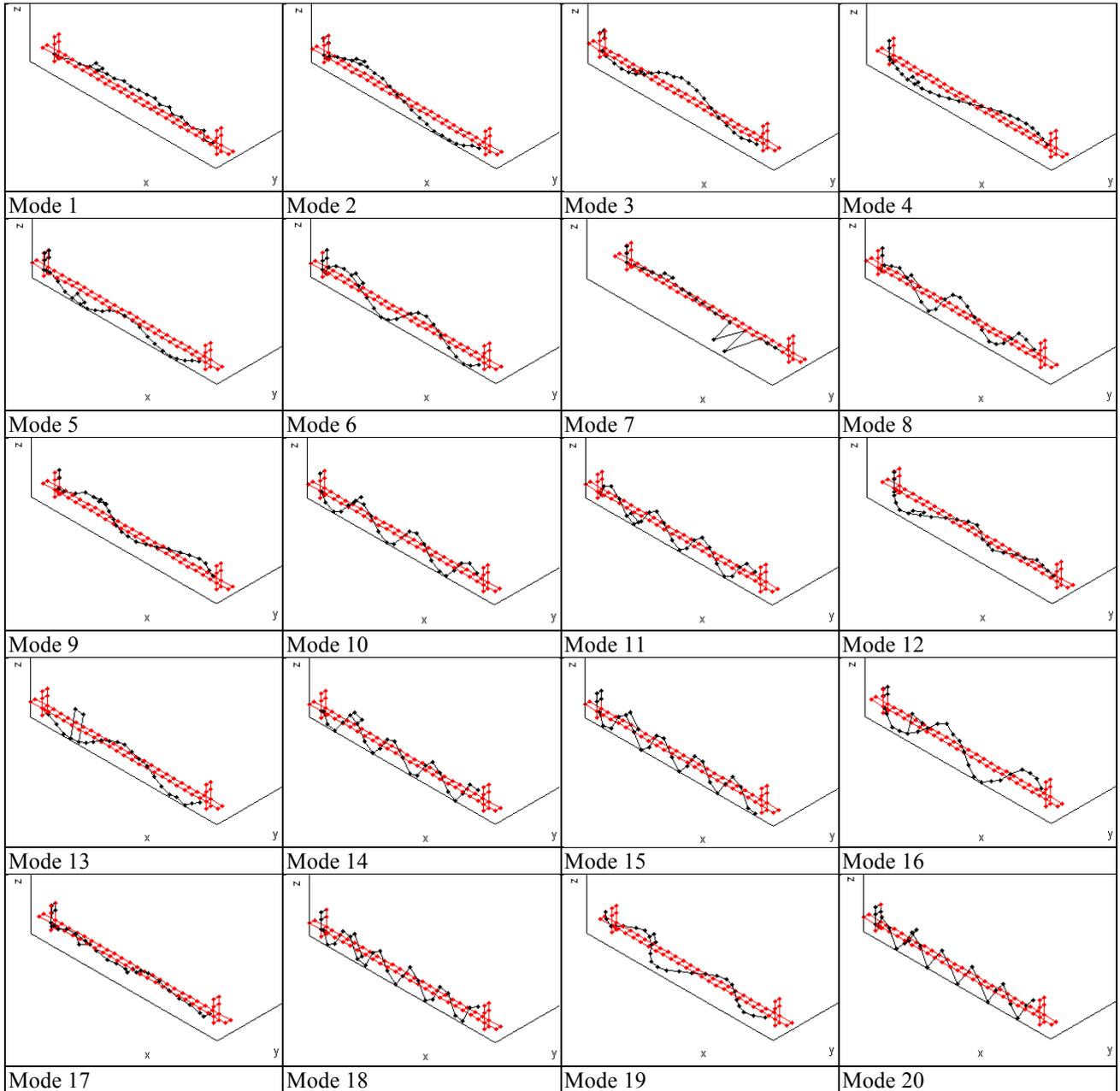
588  
589

Table 2: Identification results (L: lateral; V: vertical; T: torsional; S: symmetric; A: antisymmetric; U: unsymmetrical)

Mode	Characteristic	Natural Frequency		Damping Ratio	
		MPV (Hz)	CoV (%)	MPV (%)	CoV (%)
1	L-1-S	0.0536	3.56	4.32	88.2
2	V-1-A	0.1095	1.17	4.88	40.6
3	V-2-S	0.1304	0.54	1.41	53.6
4	L-2-A	0.1434	0.68	3.78	28.1
5	V-3-S	0.1828	0.48	1.50	68.2
6	V-4-A	0.1971	0.32	0.91	34.3
7	L-3-U	0.2393	0.61	0.49	75.0
8	V-5-S	0.2560	1.03	0.98	45.8
9	T-1-S	0.2811	0.50	1.61	25.0
10	V-6-A	0.3044	0.31	0.97	48.2
11	V-7-S	0.3638	0.16	0.61	25.5
12	L-4-A	0.3896	0.34	1.63	57.3
13	V-8-S	0.3974	0.45	0.83	29.6
14	V-9-A	0.4227	0.11	0.41	37.8
15	V-10-S	0.4854	0.17	0.50	29.2
16	T-2-A	0.4995	0.45	1.29	27.7
17	T-3-A	0.5236	0.69	1.67	34.2
18	V-11-A	0.5499	0.16	0.54	31.2
19	L-5-A	0.5865	0.21	0.64	35.4
20	V-12-S	0.6185	0.18	0.50	28.3

590  
591

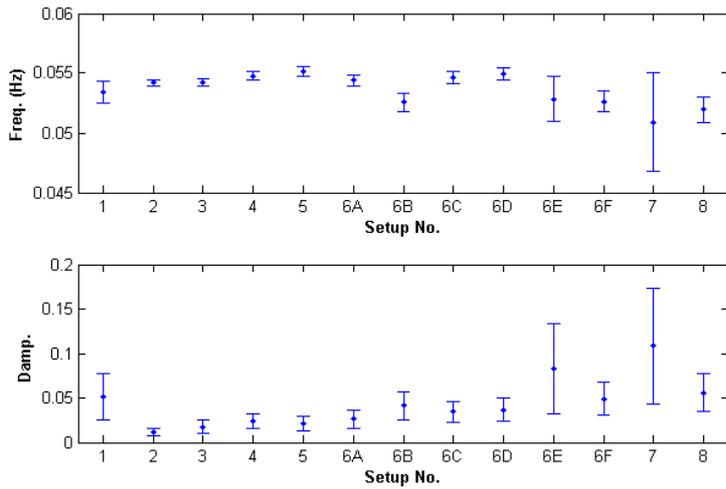
592 Figure 18 shows the assembled global mode shapes of the modes identified in Table 2.



593 Figure 18: Identified global mode shapes

594 The first mode is a symmetric lateral mode with a natural frequency of 0.0536Hz, which is quite close to the  
 595 value reported in the Chinese paper [34]. However, this mode cannot be identified based on the preliminary  
 596 data (modal SNR is no better than unity), due to the high measurement noise in the low frequency range,  
 597 meaning that the spectral peaks for this mode are not significant even in this AVT. It can be seen that there is  
 598 an increase of the PSD in the frequency range lower than 0.1 Hz which derives from the measurement noise  
 599 of the sensors as well as from the quasi-static rotation. The rotation appears as a DC offset of the signal and,  
 600 along with the noise, overlays the modal contribution of the mode.

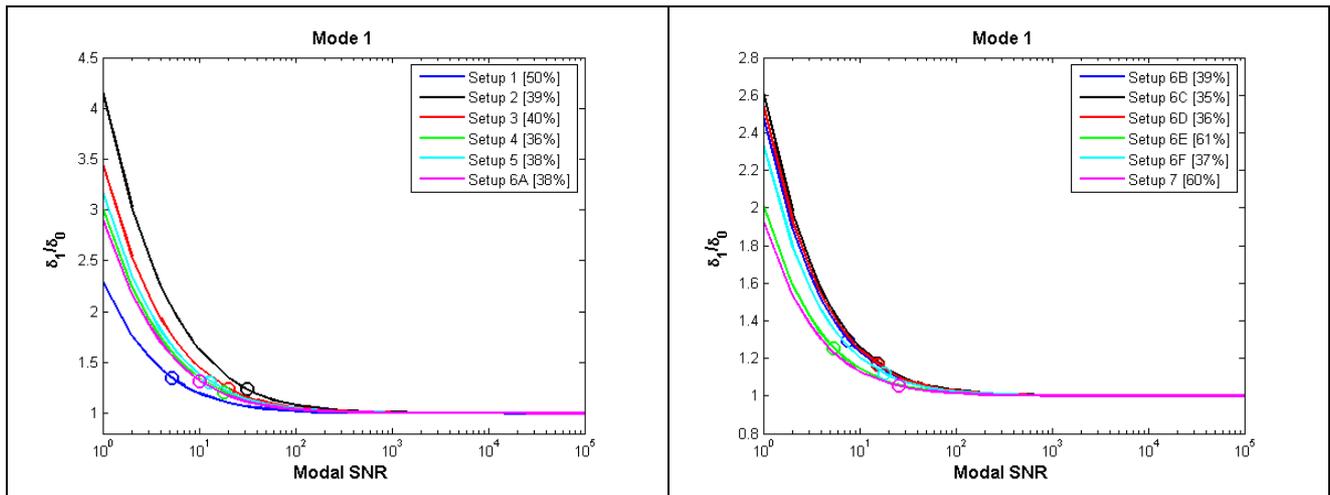
601 Spectral peaks cannot even be detected in some of the data channels in the vertical direction, which cause  
 602 convergence problems in the modal identification procedure when all the data channels are involved for  
 603 inference. In view of this, only the measured data in the lateral direction are used for Mode 1 identification  
 604 and the overall mode shape is assembled assuming zero mode shape values in the vertical direction. Figure  
 605 19 shows the identified natural frequencies and damping ratios of Mode 1 among the setups, where the error  
 606 bar represents  $\pm$  posterior standard deviation of the identified MP. Large variations in the identified damping  
 607 ratio can be found for Setup 6E and 7, which raise the overall identification uncertainty.



608

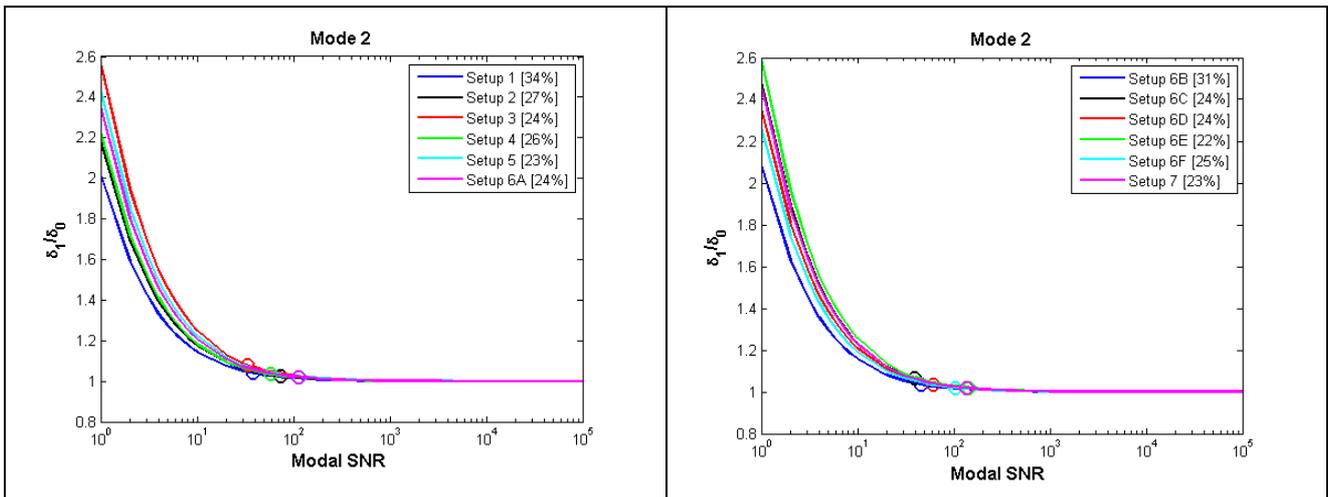
609 Figure 19: Identified natural frequency and damping ratio among the setups, Mode 1. (dot: MPV; errorbar:  $\pm$   
610 posterior standard deviation)

611 To illustrate the effect of modal SNR, Figure 20 shows the CoV. ratio  $\delta_1/\delta_0$  for the posterior damping ratio  
612 of Mode 1. The circles reflect the identification uncertainty based on the current test configuration and the  
613 curves denote the value for each setup predicted by uncertainty laws. The values in the legend denote the  
614 CoV. of damping ratio quantified based on the measured data. It can be seen that all the setups lie in the low  
615 SNR region, where further increases in SNR can reduce the identification uncertainty by 30%.



616 Figure 20: SNR effect with damping CoV., Mode 1 (left: Setup 1 to Setup 6A; right: Setup 6B to Setup 7)

617 The first vertical mode (Mode 2) is antisymmetric, with a natural frequency of 0.1095Hz. This is the mode  
618 corresponding to the one identified in the preliminary data for test planning. Figure 21 shows the CoV. ratio  
619  $\delta_1/\delta_0$  for posterior damping ratio of this mode. The posterior CoVs of damping ratio (as shown in the  
620 legend) are higher than the targeted value (i.e. 15%) in test planning. This is reasonable as the zeroth order  
621 uncertainty law used for the measurement duration plan assumes infinite SNR. The SNR of this mode varies  
622 among the setups, all are higher than the ~unity value for preliminary data, and they are close to the flat  
623 region of the predicted  $\delta_1/\delta_0$  curve. Further increasing SNR will provide significant help in reducing the  
624 identification uncertainty, confirming the effectiveness of BAYOMALAW for planning.

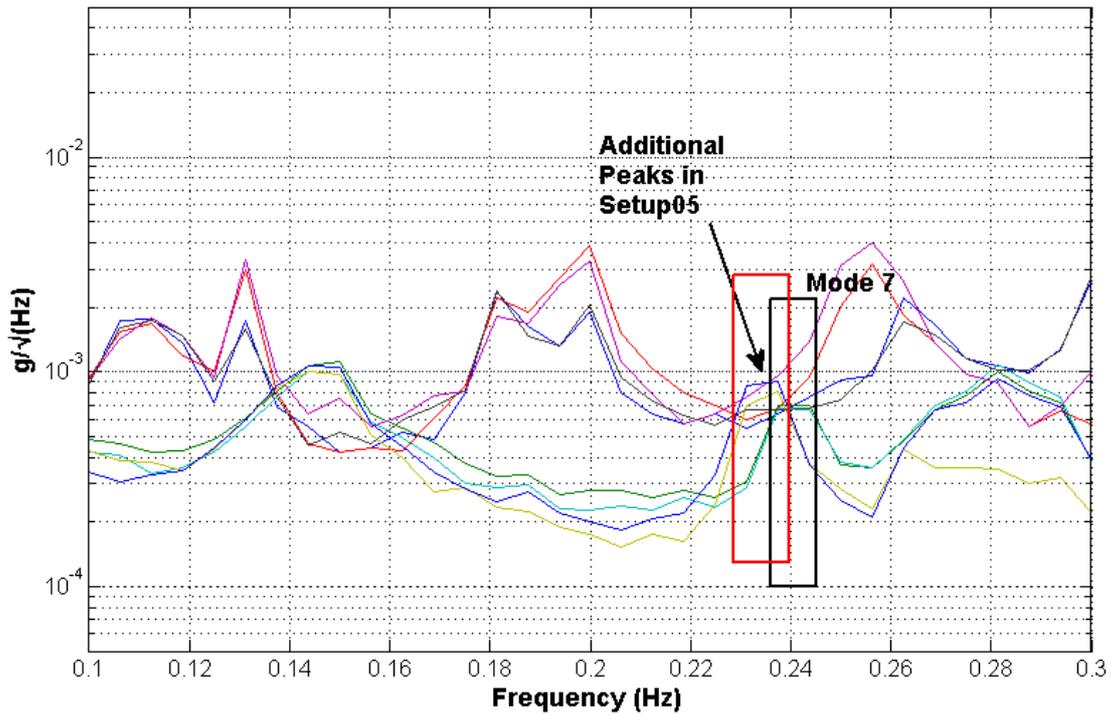


625 Figure 21: SNR effect with damping CoV., Mode 2 (left: Setup 1 to Setup 6A; right: Setup 6B to Setup 7)

626 Within the resonance band of Mode 4 (lateral mode), large contributions are found from Modes 2 and 3 (both  
 627 are vertical modes). Similar situations can be found for Mode 7, 12 and 19. Identifying these modes within  
 628 their resonance frequency bands by assuming a single mode is not feasible, as the modal contribution of  
 629 other modes in the bands are not negligible and sometimes even larger than the mode to be identified. On the  
 630 other hand, selecting a wide band involving the nearby modes and identifying them simultaneously will  
 631 increase the modelling error risk due to non-structural components within the band, as these modes may not  
 632 be closely spaced. It turns out that the identification procedure cannot converge or provide a reasonable  
 633 estimation on the modal parameters of these modes. In view of this, these modes (i.e., Modes 4, 7, 12 and 19)  
 634 are identified within their resonance frequency band using data channels in the lateral direction only to  
 635 eliminate the effect of nearby modes. The mode shape values in the vertical direction are assumed to be zero  
 636 when assembling the overall mode shapes.

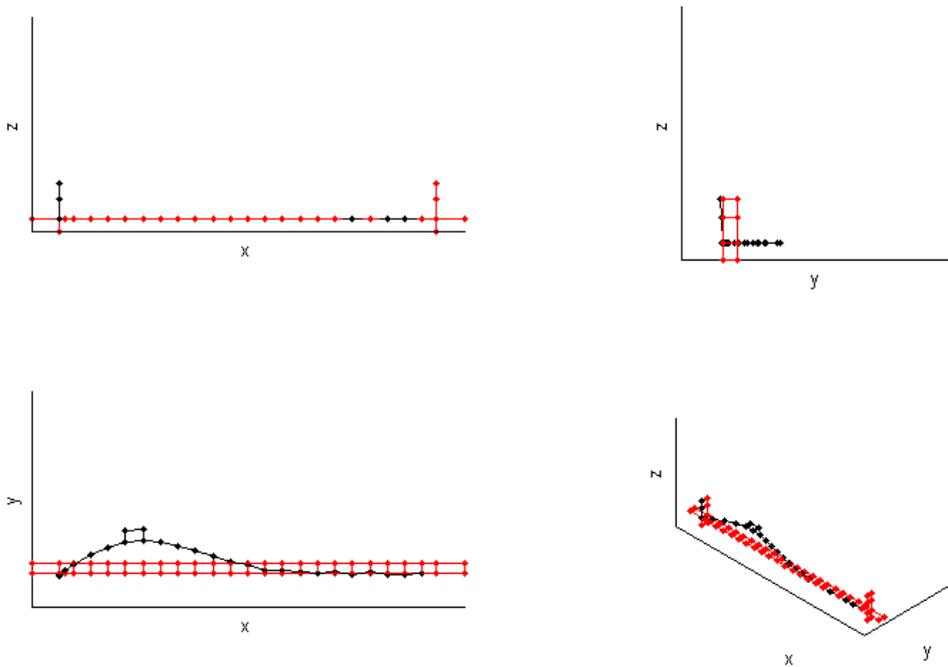
637 The identified mode shape of Mode 7 in Setup 5 is peculiar because of two anomalous large ordinates. The  
 638 cause is evident from Figure 22, showing the PSD spectrum of Setup 5 where there are additional peaks in  
 639 the selected frequency band for Mode 7. The mode shape values are erroneously identified due to these  
 640 peaks, which may not refer to a global mode as they cannot be found in other setups. To avoid their effect,  
 641 modal identification has been conducted for mode 7 with a narrower frequency band where the additional  
 642 peaks are not involved. Figure 23 shows the identified mode shapes, which now look more reasonable.

643



644

645 Figure 22: Square root PSD spectrum of Setup 5.



646

647 Figure 23: Overall mode shape of Mode 7 identified based on the narrower band

648 Thanks to the setup on the east side of the deck (i.e., Setup 7), torsional modes can be detected and  
 649 identified. Specifically, Mode 9 is identified as a symmetric torsional mode and Modes 16 and 17 are  
 650 antisymmetric torsional modes. Some torsional modes may be erroneously counted as lateral modes in this  
 651 test as some modes are identified using data channels in the lateral direction only where the mode shape  
 652 values in the vertical direction are not identified.

653

## 654 **7 Discussion and conclusions**

655 Modal tests of long span (suspension) bridges are among the most challenging of all types of field  
656 measurements due to their scale and to (as far as the authors know) the lowest frequencies observed in any  
657 man-made structure. Reliable MP identification, particularly of fundamental vertical and lateral mode  
658 damping ratios has been compromised by both bias errors introduced by early (first generation) OMA  
659 procedures and the large variance errors that appear to reduce only by using extremely long measurement  
660 periods, trading off stationarity properties of signals and environmental influence on structural properties.  
661 The question has always been how long a data record is necessary, what quantity and quality of equipment is  
662 adequate, and what level of accuracy can be achieved. Now it is possible to define a level of accuracy and  
663 determine the best measurement configuration in terms of sensor type, location and measurement duration in  
664 order to achieve it.

665 Then, if mode shapes are required to completely define the mode, how should they be best constructed using  
666 resources available and how to get around the difficulty of synchronising acquisition at locations separated  
667 by distances of the order of 1 km without constraint of cables, or the need to receive radio signals from  
668 satellites (GPS) or other loggers (by radio) for synchronisation?

669 These challenges have been addressed in a single exercise, with the bonus of a rare collaboration with  
670 Chinese researchers allowing a foreign team access to test a Chinese bridge.

671 The decision to use an hour of data for each measurement appeared optimal from the point of view of MP  
672 uncertainty and allowing for sufficient setups to define the global modal characteristics. It was not believed  
673 to be necessary to measure both sides of the bridge completely due to the single measurement on the west  
674 side and symmetry about the longitudinal axis.

675 However, the bridge may not be symmetric about the mid span, as indicated by lateral modes 7 and 12, so it  
676 cannot be assumed that lateral mode participation of the north tower would be identical to that of the south  
677 tower. Experience with suspension bridge towers e.g. [38] has shown that in addition to participating in deck  
678 modes driven by the cables, they have their own set of modes consistent with a cantilever having fixity at the  
679 freer end depending on the cable detail and back span arrangement.

680 The combination of sensor and data acquisition systems gives rise to background noise  $S_e$ , which is adequate  
681 for the purpose, judging by the SNR of the acquired data (Figure 20, Figure 21). In fact, good enough SNR  
682 would be obtained for fundamental modes even with reference sensors much closer to the tower, where the  
683 mode shape ordinates are much reduced. This would mean less chance of locating at the nodal point of a  
684 high frequency mode and possibly negating the need for a backup reference location. Other sensor options  
685 would be to use JA-70SA or Guralp CMG5T units, both low-noise tri-axial devices, but while single axis  
686 sensors might be more cumbersome and require more local wiring, they offer more flexibility.

687 Based on the experience, to obtain the most useful planning data, the authors would recommend a  
688 preliminary measurement using a single logger and small set of uniaxial accelerometers laid out close to the  
689 tower of a long span bridge, aligned first vertically then laterally. This would provide enough information to  
690 characterise the signal to noise ratio of the fundamental modes and determine an appropriate setup duration.  
691 It would also qualify the use of a reference location close to a tower.

## 692 **8 Acknowledgements**

693 The research was funded by the Engineering and Physical Sciences Research Council (grant EP/N017897/1  
694 and EP/N017803).

695 We are very grateful to: Prof. Jun Chen of Tongji University for logistical assistance; to Dr Vincent Ao for  
696 help designing the OCXO logger system; to Dr Ki Koo for evaluating the OCXO clock performance; to Ms  
697 Karen Faulkner for her participation in the site activities; to the JSTI driver who ferried the UK AVT team  
698 (Bassitt/Brownjohn/Faulkner/Zhu) around Jiangsu Province.

699

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