

Bayesian operational modal analysis of Jiangyin Yangtze River Bridge

James Mark William Brownjohn

Professor of Structural Dynamics, University of Exeter, UK

Siu-Kui Au

Professor, Chair of Uncertainty, Reliability and Risk, University of Liverpool, UK

Yichen Zhu

PhD Student, University of Liverpool, UK

Zhen Sun

State Key Laboratory of Safety and Health for In-Service Long Span Bridges, Jiangsu
Transportation Institute, Nanjing, China

Binbin Li

Research Associate, University of Liverpool, UK

James Bassitt

Experimental Officer, University of Exeter, UK

Emma Hudson

Research Fellow, University of Exeter, UK

Hongbin Sun

Jiangsu Yangtze Bridge Co. Ltd. Nanjing, China

Contact author: Professor James Brownjohn
CEMPS
University of Exeter
Harrison Building
Exeter EX4 4QF
E-mail: J.Brownjohn@Exeter.ac.uk
Tel: 01392-723698

36 **Abstract**

37 Vibration testing of long span bridges is becoming a commissioning requirement, yet such exercises
38 represent the extreme of experimental capability, with challenges for instrumentation (due to frequency
39 range, resolution and km-order separation of sensor) and system identification (because of the extreme low
40 frequencies).

41 The challenge with instrumentation for modal analysis is managing synchronous data acquisition from
42 sensors distributed widely apart inside and outside the structure. The ideal solution is precisely synchronised
43 autonomous recorders that do not need cables, GPS or wireless communication.

44 The challenge with system identification is to maximise the reliability of modal parameters through
45 experimental design and subsequently to identify the parameters in terms of mean values and standard errors.
46 The challenge is particularly severe for modes with low frequency and damping typical of long span bridges.
47 One solution is to apply 'third generation' operational modal analysis procedures using Bayesian approaches
48 in both the planning and analysis stages.

49 The paper presents an exercise on the Jiangyin Yangtze River Bridge, a suspension bridge with a 1,385m
50 main span. The exercise comprised planning of a test campaign to optimise the reliability of operational
51 modal analysis, the deployment of a set of independent data acquisition units synchronised using precision
52 oven controlled crystal oscillators and the subsequent identification of a set of modal parameters in terms of
53 mean and variance errors.

54 Although the bridge has had structural health monitoring technology installed since it was completed, this
55 was the first full modal survey, aimed at identifying important features of the modal behaviour rather than
56 providing fine resolution of mode shapes through the whole structure. Therefore, measurements were made
57 in only the (south) tower, while torsional behaviour was identified by a single measurement using a pair of
58 recorders across the carriageway. The modal survey revealed a first lateral symmetric mode with natural
59 frequency 0.0536 Hz with standard error $\pm 3.6\%$ and damping ratio 4.4% with standard error $\pm 88\%$. First
60 vertical mode is antisymmetric with frequency 0.11 Hz $\pm 1.2\%$ and damping ratio 4.9% $\pm 41\%$.

61 A significant and novel element of the exercise was planning of the measurement setups and their necessary
62 duration linked to prior estimation of the precision of the frequency and damping estimates. The second
63 novelty is the use of the multi-sensor precision synchronised acquisition without external time reference on a
64 structure of this scale. The challenges of ambient vibration testing and modal identification in a complex
65 environment are addressed leveraging on advances in practical implementation and scientific understanding
66 of the problem.

67 **Highlights**

68 Ambient vibration test on 1385 m Jiangyin Yangtze River Bridge, by western academics.

69 Application of uncertainty laws of Bayesian operational modal analysis for test planning with and without
70 preliminary data.

71 Bayesian operational modal analysis of acceleration data, including challenging lateral vibration modes

72 Design and use in the field testing of precisely synchronised loggers not relying on GPS or wireless.

73

74 **Keywords:** Bayesian operational modal analysis suspension bridge uncertainty synchronisation

75 **1 Introduction**

76 Ambient vibration testing (AVT) is becoming an indispensable tool for managing the performance of large
77 scale civil infrastructure where forced vibration testing is logistically infeasible. However there are specific
78 challenges to AVT in particular for long span and tall structures in both the experimental art and the
79 subsequent analysis of response time series. This paper describes advances in both technologies aimed at
80 improving the effectiveness of experimental AVT campaigns to provide the most reliable estimates of modal
81 parameters (MP) with the least experimental cost and which are applied together on an exemplar study,
82 Jiangyin Yangtze River Bridge ('Jiangyin Bridge'), a suspension bridge with a 1,385 m main span in Jiangsu
83 Province, China.

84 1.1 Motivation for managing uncertainty of modal parameter (MP) estimates

85 Knowledge of structural modal parameters or MP (for both singular and plural form) representing modal
86 frequencies, damping ratios, shapes and masses is a fundamental requirement for design against dynamic
87 loads (wind, earthquake, human excitation). MP estimation is central to finite element model updating and
88 the dependent performance simulations. It has long been recognised as an effective means for uncertainty
89 mitigation in structural dynamics [1] and is a vital prerequisite for:

90 - Vibration control devices [2]: Design and maintenance of effective tuned mass dampers depends on precise
91 knowledge of modal damping and frequency;

92 - Retrofits [3]: If the retrofit aims to improve vibration serviceability, both design and evaluation of the
93 retrofit depend on reliable MP estimates;

94 - Structural Health Monitoring (SHM) systems [4]: MP (e.g. frequencies) are classic indicators of structural
95 state so the ability to identify and statistically qualify small changes will enhance SHM system utility for
96 tracking structure condition through shifts and drifts in MP.

97 Computer models of tall buildings often grossly misrepresent the as-built characteristics, which is evidenced
98 by measured MPs significantly differing from predictions [1]. A specific example is the 280 m Republic
99 Plaza building in Singapore where >0.19 Hz fundamental mode frequencies measured by AVT indicated
100 almost doubled as-built stiffness compared to a design estimate of 0.14 Hz [5]. Such conservatism and over-
101 design is not sustainable and needs to be tempered by experience from AVT.

102 For damping, other than for discrete damping elements [6,7] available models are all empirical. Values used
103 in design are based on historic databases [6,8] distilled in codes of practice [9], and usually derived from
104 response using '1st generation' operational modal analysis (OMA) techniques (e.g. random decrement, half-
105 power bandwidth) [10] having questionable reliability. Building occupants pay a premium for high-rise
106 comfort and to be sure of avoiding poor vibration serviceability and loss of revenue amidst uncertainty in
107 inherent structural damping, vibration control devices may be specified. These are expensive and use
108 premium space. For long span bridges flutter speeds increase with damping. Unlike computational
109 underestimation of natural frequency, experimental estimates of damping are usually high so that safety
110 margin is also overestimated.

111 MP identification using forced vibration testing (FVT) with controlled input (force) [11] is believed to
112 produce the most reliable MP estimates. However, FVT is only feasible for automotive/aerospace
113 applications (such as ground vibration testing) and relatively small civil structures such as footbridges and
114 floors having modal masses with maximum order 10^3 tonnes and natural frequencies usually above 1 Hz. For
115 long span and tall structures having modal frequencies of the order 0.1 Hz and modal masses usually
116 upwards of 10^4 tonnes, artificial forcing is logistically infeasible. Therefore, AVTs providing response data
117 for OMA have become the standard investigation tool.

118 A trend in AVT for structural commissioning tests (e.g. Burj Khalifa and Queensferry Crossing) to
119 corroborate design assumptions has accelerated with sophisticated '2nd generation' OMA techniques such as
120 stochastic subspace identification (SSI) and frequency domain decomposition (FDD) [12,13]. However, the
121 problem remains that absence of input loading data in AVT leads to fundamentally much higher MP estimate
122 variability and lower repeatability compared to FVT. This uncertainty is well recognised [8,14] but there has
123 been no quantitative account for its origin, nor any quantitative guideline for test configuration (sensor
124 location, load intensity, signal/noise ratio (SNR) and measurement duration) to achieve a desired
125 identification precision. Recent ASCE guidance [1] states that prediction of structural (dynamic) properties is
126 more of an art than a science; there are few reliable measurements of structural damping and values used in
127 performance predictions are generally 'crude and simplified estimates'.

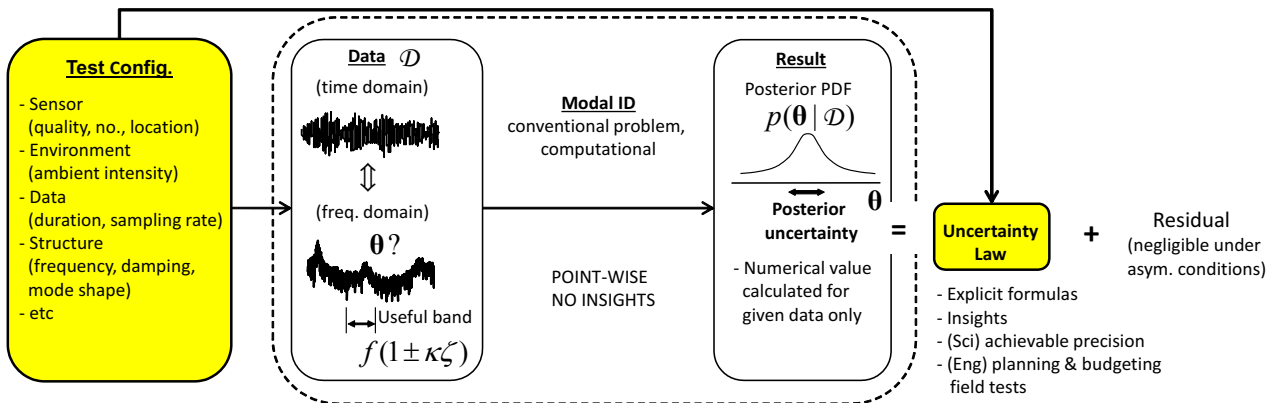
128 With aero-elasticity a major factor in long span bridge performance at ultimate limit state, designers rely
129 heavily on damping estimation from existing prototypes via AVTs that have had poor uncertainty
130 management. Existing databases [6,8] of field-identified tall building damping values used for design show
131 great scatter with no information on their reliability [15,16] and there is still no commonly accepted
132 amplitude-dependent damping model for even the simplest building configuration. For the tallest buildings
133 vibration serviceability, which depends critically on first mode frequency and damping, governs design, and
134 reliable performance data are both scarce and critically important. For long span bridges, collections of
135 damping data [17][18] are rarer and older.

136 **1.2 Uncertainty laws for Bayesian OMA**

137 Due to the accepted reliability of MP estimation using forced vibration testing, there are few studies on
 138 accuracy [19], while there are presently several techniques dealing with reliability of OMA, particularly for
 139 SSI [20–23], including a commercial implementation. Uncertainty quantification is central to ‘3rd
 140 generation’ Bayesian OMA methods [24] that adopt a radically different philosophy to 1st and 2nd generation
 141 OMA methods by posing the question ‘what is known about MP based on the given data?’. Answers are
 142 expressed via the *posterior* probability density function (PDF) of MP (θ) for given response data (D) implied
 143 fundamentally using Bayes’ theorem [25] i.e. $P(\theta|D)$. The covariance matrix associated with the posterior
 144 PDF then naturally quantifies the identification uncertainty of MP for the given data set and modelling
 145 assumptions. This is in contrast to non-Bayesian methods where the uncertainty refers to the estimates of the
 146 MP rather than the MP themselves, and is ‘inherent’ in nature, i.e. does not depend on data.

147 Even so, algorithms for calculating uncertainty, be they Bayesian or non-Bayesian, only allow MP
 148 uncertainty to be calculated in a ‘point-wise’ manner (i.e., ‘what’) for given data, which yields little or no
 149 understanding about uncertainty (i.e., ‘why’). Therefore, the main aim of the 'BAYOMALAW' research
 150 project [26] funded by the Engineering and Physical Sciences Research Council has been to deliver
 151 ‘uncertainty laws’ using Bayesian inference to express MP uncertainties explicitly as functions of test
 152 configuration (measurement noise, environmental load intensity and sensor configuration) so that MP
 153 uncertainty can be prescribed and managed [27]. The project has aimed at providing a quantitative account
 154 for the origin of uncertainty along with quantitative guideline for test configuration to achieve a desired
 155 identification precision. The exercise reported here is an early application.

156 Figure 1 provides an outline of the research. (Conventional) efforts on Bayesian OMA algorithm
 157 development belong to the central (unshaded) part of the figure. Uncertainty law research is concerned with
 158 the shaded parts that address the challenge of understanding identification uncertainty and mastering its
 159 relationship with test configuration. Previous experience in the ambient modal identification research
 160 community has led to advice on best practice for reliable MP identification [28]. Following a Bayesian
 161 approach, this has been formalised [29][30] in the form of 'zeroth order laws'. These are closed form
 162 expressions for MP uncertainty (variance) derived assuming sufficiently long data with high SNR, analogous
 163 to the Laws of Large Numbers in classical statistics. Accounting only for zeroth order effects, they link MP
 164 identification precision with data duration and frequency domain resolution of spectral peaks in the case of
 165 well-separated modes identified using data from a single configuration of sensor location and type (or setup).
 166 The research was later extended [27] to include 'first order effects', where test configuration starts making
 167 influence through the ‘modal SNR’ (see equation (2) later). The zeroth and first order laws are presented in
 168 section 4.



169 Figure 1: Objective of BAYOMALAW project

170 A secondary aim of the project, in support of the uncertainty management, is the creation of a bespoke
 171 instrumentation system that simplifies the experimental work by removing the requirement to use wires,
 172 radio transmission or GPS for synchronising data among accelerometers separated by distances of the order
 173 of 1 km. This is achieved using high precision acquisition clocks and robust data acquisition architecture.

174 The project combines theoretical development with experimental trials and verification in a number of steps,
175 beginning with the effects of modal SNR. Modal SNR is a key parameter that quantifies test configurations
176 such as measurement noise, environmental load intensities and the measured degrees of freedom.

177 The large-scale experimental application and test of BAYOMALAW methodology focused on the 1,385 m
178 span Jiangyin Bridge, with phases of preparation, on-site measurement and then post-processing. The phase
179 of preparation included

- 180 • development of precision synchronised autonomous acceleration recorders developed as bespoke
181 improvement on the technology used for ambient vibration testing of the Humber Bridge in 2008
182 [31] and
- 183 • obtaining sample data from the structural health monitoring system operated by Jiangsu
184 Transportation Institute (JSTI). The data were used to evaluate expected SNR and the strategy of
185 reference and rover positions, as well as for defining measurement setup duration.

186 The testing phase involved three days of vibration measurements carried out by a team from JSTI, the
187 University of Liverpool and the University of Exeter. Finally, the post-processing phase to recover MP and
188 their uncertainties was carried out by researchers from the University of Liverpool.

189 Hence the paper first describes the bridge, then development of the autonomous loggers. Next the sample
190 data and test planning are described. The field measurements are then summarised and the MP estimates
191 presented. Finally, the implications for future studies and next steps are defined.

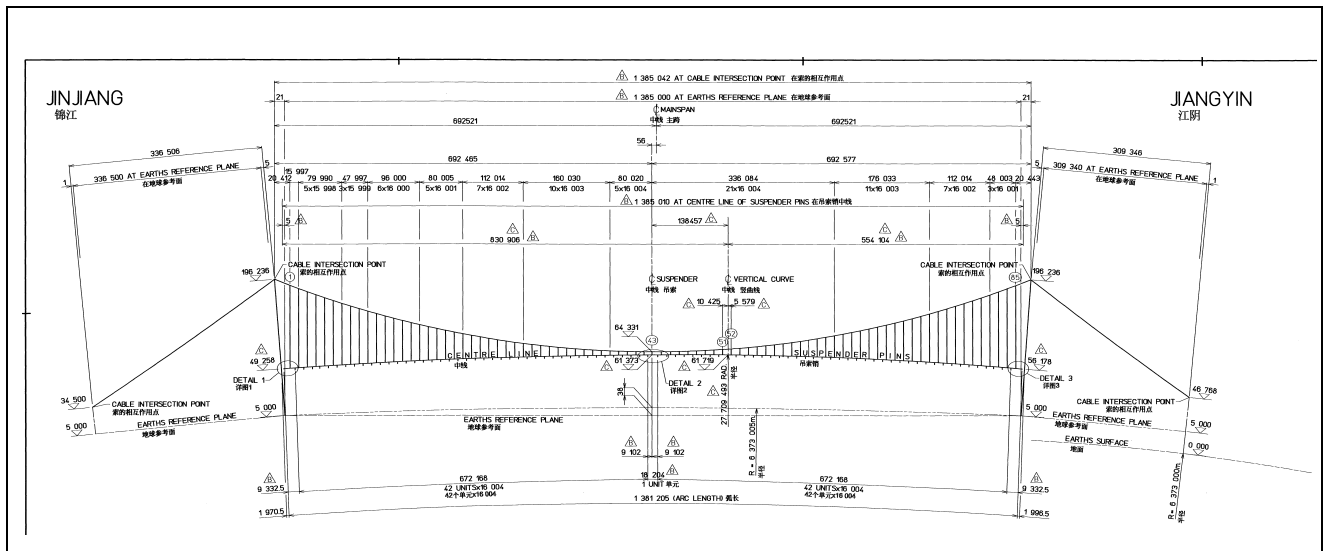
192 2 Jiangyin Yantze River Bridge

193 Jiangyin Bridge (Figure 2) with a single span of 1,385 m carries the G2 expressway across the Yangtze River
194 in Jiangsu province, China, approximately 160 km northeast of Shanghai. It was the world's fourth longest
195 span when completed in 1999. The 0.876 m diameter main cables each support the deck via 85 pairs of
196 vertical hangers and are formed by prefabricated parallel wire strands, each constructed from 127×5.35 mm
197 diameter galvanised steel wires with 1,600 MPa tensile strength. Back stays attached to large gravity
198 anchorages carry no deck load and have a diameter of 0.897 m. Towers with a height of 190 m are portal
199 frames created by two hollow tower columns of reinforced concrete and three transverse beams. The main
200 girder is a streamlined box section with a width of 36.9 m and a height of 3 m carrying a 29.5 m wide deck
201 with three lanes of cars, buses and trucks in each direction. There is a 1.8 m walkway each side with limited
202 access, for maintenance purposes.



203 Figure 2: Jiangyin Bridge

204 Information on the design and construction is provided in [32] and an elevation view is provided in Figure 3.



205 Figure 3: Jiangyin Bridge elevation

206 A structural health monitoring (SHM) system was installed at the time of construction but was upgraded in
 207 2005 [32] to comprise 116 fibre Bragg gating sensors for strain and temperature measurement, 35 uniaxial
 208 accelerometers, nine GPS receivers and four displacement sensors. Relevant for the present study, 15 of the
 209 uniaxial accelerometers are located in the girder, at the 1/8, 1/4, 3/8 1/2 and 3/4-span distances from the
 210 north tower and GPS receivers are mounted on rigid posts at 1/4, 1/2 and 3/4-span locations on the east and
 211 west sides of the roadway.

212 While SHM system data have been extensively analysed [32,33], there is no information available on modal
 213 properties other than a Chinese language report [34] referring to GPS and acceleration data from the SHM
 214 system. For lateral vibration a symmetric mode at 0.0546 Hz and an antisymmetric mode at 0.1314 Hz were
 215 identified from accelerometer data, while for vertical vibrations an antisymmetric mode at 0.0891 Hz and a
 216 symmetric mode at 0.1258 Hz were identified. The identification algorithm is not known.

217 One specific concern with Jiangyin Bridge is the behaviour of the expansion joints located at each end of the
 218 main span. These were replaced in 2007 due to damage, and movement has been controlled using large
 219 viscous dampers [35] installed at the same time, so it is possible that these may affect the dynamic behaviour
 220 of the bridge.

221 3 Logging systems for long span bridge ambient vibration testing

222 3.1 Previous instrumentation deployments for long span suspension bridges

223 Previous experience in ambient vibration tests (AVTs) on long span bridges had considerable bearing on the
 224 design of bespoke instrumentation for Jiangyin Bridge AVT and subsequent deployments.

225 In the 1980s, AVTs on:

- 226 • Humber Bridge [36], with main span 1,410 m and suspended sidespans of 280 m and 530 m.
- 227 • 15 July Martyrs Bridge, also known as First Bosphorus Bridge, and referred to here as B1 [37] and
- 228 • Fatih Sultan Mehmet Bridge, also known as Second Bosphorus Bridge and referred to here as B2 [38]

229 used small wired arrays comprising a fixed reference accelerometer and no more than four roving
 230 accelerometers and used 1st generation methods for OMA. Up to eight signal cables (250 m each) were laid
 231 inside the box girder and daisy-chained to reach accelerometers that could be up to 1 km from the location of
 232 the reference sensor and acquisition system. A four-channel analog tape recorder with poor signal to noise
 233 ratio was used to record acceleration data that were post-processed by replaying signals through a two-
 234 channel spectrum analyser, one pair at a time. These AVTs took up to 10 working days covering a sufficient
 235 number of degrees of freedom using available cables and sensors while trying to obtain recordings of
 236 approximately one hour per measurement. B1 (1,074 m main span) and B2 (1,090 m main span) do not have
 237 suspended side spans which simplified the measurements, but for each of the three bridges considerable
 238 trouble was taken to measure at several locations in both pylons of both towers.

239 An AVT of the Tamar Bridge, which has a 335 m main span was carried in 2006 [39]. Being a much shorter
240 bridge, it was possible, thanks to good weather and strong support from the bridge management team, to do
241 the test in a single day, but with some limitations. First, the bridge has a truss girder which is very difficult to
242 negotiate so it was possible only to work outside the bridge, placing accelerometers on flat spaces between
243 the walkway and south traffic lanes. It was also possible (with some difficulty) to carry two signal cables
244 under the road deck, through the truss to the north side to be used as reference sensors opposite two sensors
245 on the south side. Second, a limited number of 100 m signal cables was available.

246 To compensate for restrictions, best practice available for planning measurements to be used for OMA (now
247 formally outlined in [28]) was followed. This defines the minimum number of cycles of the vibration mode
248 to be identified as $10/\zeta$, so that assuming the same damping, the higher modal frequencies (shorter periods)
249 allowed shorter recordings, as little as 15 minutes. Also a high specification 24-channel data acquisition
250 system with a full set of accelerometers allowed for the simultaneous recording of 16 acceleration signals.
251 Accelerometers were roved only along the south side of the bridge and only at the upper portal of the north
252 pylons, allowing for a much more efficient measurement campaign. A 2nd generation OMA was used,
253 resulting in a rapid evaluation of the MP, and helping to identify modes that were subsequently tracked in
254 real time by a permanent SHM system [40] using automated and optimised OMA.

255 The Humber Bridge was retested in 2008 [31] in collaboration with researchers from the Faculty of
256 Engineering of the University of Porto (FEUP) and from City University of Hong Kong. The campaign took
257 advantage of the FEUP AVT technology that used GeoSIG autonomous recorders with GPS-synchronised
258 acquisition. Combining resources provided a set of ten recorders each running a triaxial accelerometer set.
259 Two pairs of recorders acted as rovers (two span-wise locations, one recorder each side of the deck on the
260 walkway) leaving three pairs to rove all other span-wise locations, as well as all pier and portal levels of both
261 towers' accelerometers channels (76 locations), doubling the level of detail in the 1985 test. Apart from
262 measurements at mid-portals levels in the towers with no exterior access, all measurements used recorders on
263 or outside the bridge, ensuring good reception by GPS antennae and enabling faster relocation of roving
264 accelerometers between measurements. With this level of instrumentation it was possible to finish the AVT
265 within five days which was followed by OMA using a range of 2nd generation procedures. One problem that
266 surfaced during the OMA was the imperfect synchronisation of recorders. Errors of up to 7 ms were
267 identified for units with GPS line of sight, more so for recorders inside the tower without the GPS satellite
268 visibility.

269 **3.2 Instrumentation for Jiangyin Bridge AVT: the OCXO loggers**

270 The experience described above clearly indicated that the most efficient and effective data acquisition system
271 for an AVT of a long span bridge would comprise autonomous recorders such as the GeoSIG recorder units.
272 Concerns about synchronisation accuracy and the need to operate inside or outside a structure without any
273 requirement for wired or wireless communication led to a specification for autonomous recorders with highly
274 accurate clocks that would maintain synchronisation to better than 1 ms during a day of recording. There are
275 two problems to address, the first being how to start a recording across a set of data *loggers* using a common
276 timebase, the second being how to minimise drift due to acquisition timing variations.

277 **3.2.1 Logger design**

278 In principle, a common timebase could be set using GPS [41], but standard crystal oscillators controlling
279 data acquisition hardware such as the popular National Instruments NI 9234 have quoted internal master
280 timebase accuracy 50 parts per million (ppm) maximum. This means that a pair of initially synchronised but
281 separate units could, in the worst case scenario, have timebases different by 0.36 s after just one hour of
282 operation.

283 The best precision quartz oscillators available quote an accuracy of no better than 5 ppm, while oven
284 controlled crystal oscillators (OCXOs) offer precision better than 1 ppm. Oscillator quartz crystal
285 frequencies vary because of stiffness dependence on temperature, and OCXOs precisely control the oscillator
286 temperature using a small oven. The technique of using OCXOs to maintain synchronisation became
287 available in proprietary systems e.g. [42] but a system was required with complete flexibility e.g. to work
288 with available triaxial or uniaxial accelerometers. Therefore, a set of autonomous loggers was built around
289 National Instruments (NI) compact acquisition units controlled by 10 MHz OCXOs.

290 Each data logger, in a waterproof case, comprises power supply modules, switching and connectors as well
291 as a 4-slot CompactRIO cRIO-9064 embedded controller. Each cRIO-9064 controls a 24-bit four channel NI
292 9234 vibration input module set to acquire $\pm 5V$. To achieve the sub-1 ppm synchronisation a blank NI 9977
293 C-series module is used to house the OCXO while a NI-9402 high speed digital IO module monitors the
294 'local' OCXO clock ticks at 120 MHz. The cRIO-9064's integrated FPGA (programmed in LabVIEW
295 environment) drives the acquisition module at its maximum 51.2kHz, then decimating the acquired signals
296 based on the local OCXO clock ticks, after appropriate filtering to avoid aliasing issues.

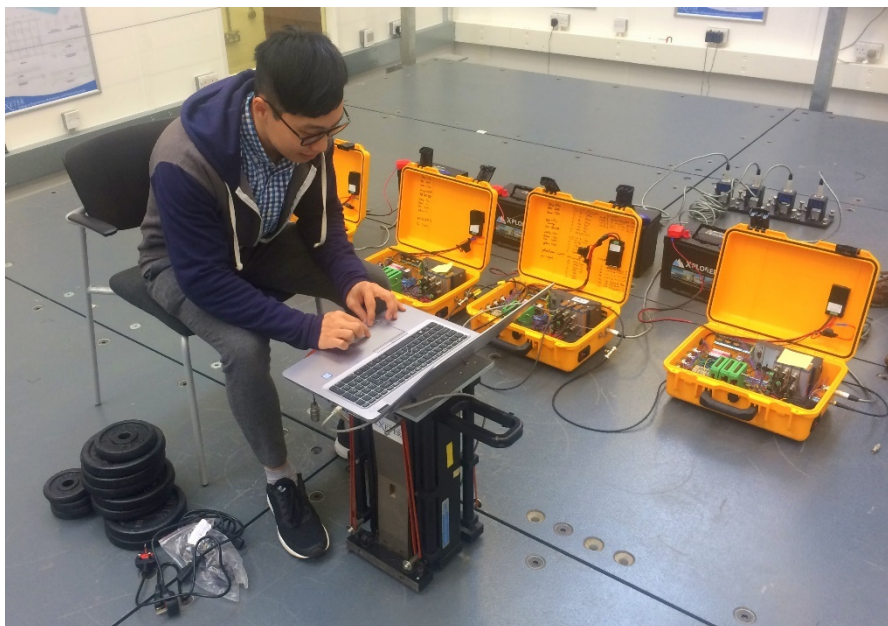
297 One logger is programmed as a master, the remaining loggers as slaves. To start synchronisation, the master
298 unit is first powered up (using a battery or mains supply), followed by each of the slave loggers. Once the
299 master is running and acquiring data (indicated by an LED flash sequence) the clock signal containing timing
300 information from its master OCXO is briefly connected to the OCXO inside each of the slave units by BNC
301 cable. The slave OCXO updates its own timing information so that the two clocks are synchronised. When
302 the master OCXO has been connected to each slave (the process takes a few seconds and is confirmed by a
303 flashing LED sequence) all loggers are then operating with the same timebase.

304 Using high-speed direct memory access, all acquired vibration/acceleration data are streamed to technical
305 data management (TDMS) files stored on a high-capacity USB drive in the cRIO-9064. The data can be
306 accessed during measurement via Ethernet to check data streams and to retrieve stored data using FTP.
307 Alternatively each USB drive can be removed to copy files directly to the analysis PC.

308 When powered by batteries the loggers can then be spatially distributed. Each logger is set to work with
309 either a set of Honeywell QA-750 uniaxial accelerometers or a single Japanese Aerospace JA-70SA triaxial
310 accelerometer; Figure 4 shows a set of four synchronised loggers using battery power and acquiring from
311 JA-70SA accelerometers. The LabVIEW application to check data acquisition during operation, is being
312 used on the laptop in the setup shown in Figure 4.

313 The measurement duration depends on the test plan, logistics and ultimately battery capacity. NI hardware
314 and OCXO together draw 7 W so planned battery capacity has to be more than adequate for a day of site
315 measurement that can potentially last up to 12 hours. In fact a more compact but lower capacity battery can
316 be installed in the yellow box, but could only be used if the boxes are shipped to site by road or rail, due to
317 civil aviation restrictions on carrying batteries which work against any battery operated system.

318 When all measurements are complete, the loggers are turned off, USB drives removed and data transferred to
319 a prepared set of directories on the PC. A MATLAB script is used to assemble data into a common time-
320 synchronised set covering the common period of logger operation and the data file is chopped into pieces
321 representing individual measurements. This part of the operation is low-tech, requiring written records of
322 time and location for each logger.



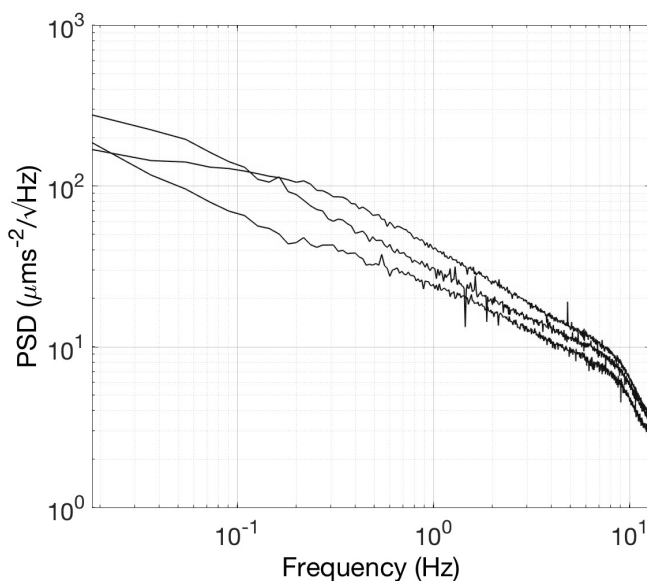
324 Figure 4: Using a set of four OCXO-controlled loggers with JA-70SA triaxial accelerometers.

325 3.2.2 Logger performance: noise floor and synchronisation drift

326 The logger system was function-tested using a field test on a 100 m cable-stayed footbridge in Exeter [43]
327 shortly before the equipment was shipped to Shanghai for the Jiangyin Bridge AVT. In addition the noise
328 floor of the system was evaluated using a 'huddle test' [44] which is a common recording of collocated
329 sensors and loggers that isolates sensor noise by removing the common true structural vibration. Figure 4
330 shows preparation for the huddle test using the JA-70SA accelerometers planned to be used at Jiangyin
331 Bridge. Due to an operational problem with the JAs (subsequently resolved) QA-750 accelerometers were in
332 fact used, and the noise floor was separately evaluated in set up similar to Figure 4.

333 The huddle test is based on the discovery [45] that no more than three channels of data from three identical
334 and collocated sensors can be used to identify their noise floor. For a single sensor, noise floor evaluation
335 would require either an extremely quiet environment (e.g. in a salt mine) or some form of isolation [45] but
336 using the minimum of three sensors allows redundant information about the mechanical excitation of the
337 collocated sensors to be removed from their signals.

338 Figure 5 shows the procedure applied to 14 hours of vertical acceleration data from QA-750 accelerometers
339 connected to three (separate) loggers. The logarithmic axes emphasise the relatively high noise at low
340 frequencies common to all types of accelerometer. Based on the plot, a conservative estimate of self-noise of
341 sensor and analog to digital converter at 0.05 Hz (where the first lateral mode is expected) is $30 \mu\text{ms}^{-2}/\sqrt{\text{Hz}}$
342 i.e. $3 \mu\text{g}/\sqrt{\text{Hz}}$. This noise floor was used in the test planning described in 4.2.



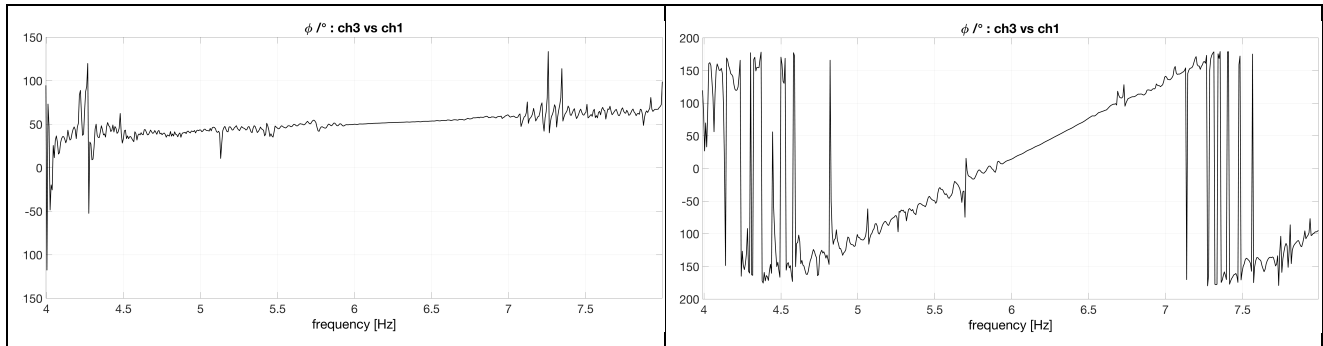
343
344 Figure 5: Power spectral density (PSD) of QA-750 noise floor from Huddle test of Figure 4, using [45].

345 A check in synchronisation drift was required because the resulting phase angle shift between channels could
346 compromise MP identification. The four loggers were sat on a laboratory floor, synchronised and left for 42
347 hours to record ambient vibrations including weak response of the floor to footfalls. Data samples were taken
348 from the recording after three hours and 42 hours and cross-power spectra evaluated. Transfer functions were
349 constructed for three slave loggers with respect to the master, and Figure 6 shows phase angle for one of the
350 slave loggers (relative to the master logger) for around 6 Hz, which is the first mode frequency of the floor.

351 Consider that the common signal in a recording sample should result in the same Fourier amplitude
352 $A \sin(\omega t + \phi)$ at a given frequency ω having a common phase angle ϕ that depends on the starting angle of
353 the sinusoid at $t = 0$. Any difference in phase angle among signals recorded by different loggers must be due
354 to a different local value t at the start time of the sample so that synchronisation drift $\Delta t = \phi/\omega$.

355 The slope at 3 hours is approximately $10^\circ/\text{Hz}$ i.e. 0.1745 radian per 2π radian/s. This is $\Delta t = 0.028$ s drift in
356 10,800 s i.e. 2.6 ppm. After 42 hours the phase shift is $126^\circ/\text{Hz}$ corresponding to $\Delta t = 0.35$ s i.e. 2.3 ppm.
357 This indicates a drift that is approximately linear with time, and this varies between loggers. The implication
358 is that the phase shift for 1 Hz mode after 10 hours of operation would be at a maximum of 30° .

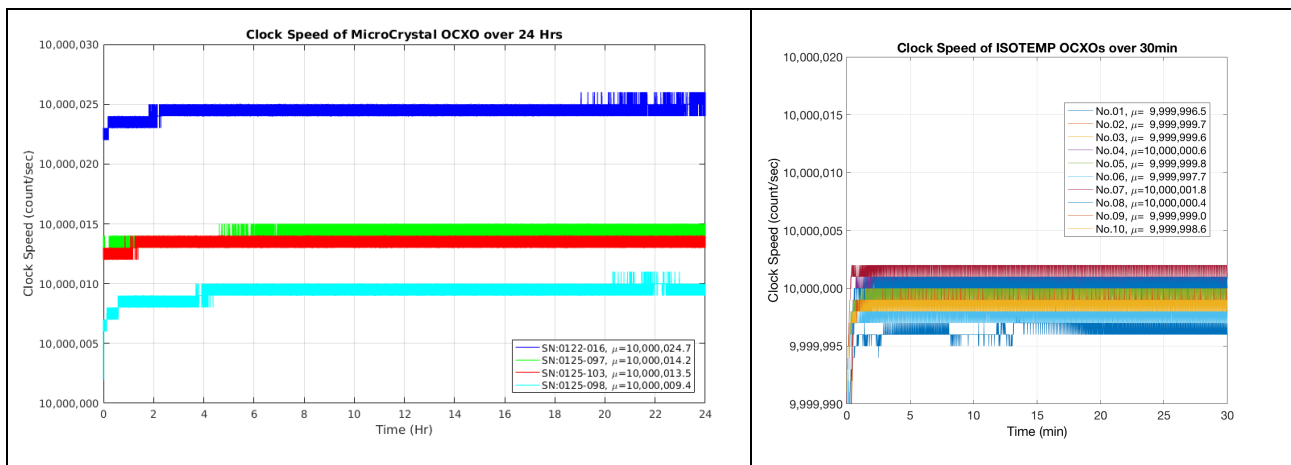
359 To avoid compromising OMA procedures that are sensitive to phase angle this drift be fixed by resampling
 360 signals in post-processing if the drift is known.
 361



362 Figure 6: Effect of synchronisation drift on phase angle after a) 3 hours and b) 42 hours.

363 While the OCXO clocks are expected to have minimal drift, an explanation of the observed drift is that
 364 actual clock speeds could apparently differ slightly from the nominal 120 MHz. To check this, the loggers
 365 OCXOs were transplanted into GPS-synchronised wireless sensor nodes [41] to test the clock speed. Due to
 366 lack of time between logger construction and deployment in China, both this study and the 42 h recording
 367 followed the field test, and the actual clock speeds for the four deployed OCXOs are shown in Figure 7. The
 368 plot indicates that clocks need a short warming-up period period to reach a stable frequency which is
 369 maintained to within 0.2 ppm for a long duration. Even though individual clock speeds are stable, the set of
 370 four speeds shows significant variance.

371 The OCXOs have been replaced with units having tighter clock speed tolerances and shorter times to warm
 372 up. The replacement OCXOs which will be used in a larger set of ten loggers reach the stable frequency
 373 faster and limit aggregate drift to 0.4 ppm.



374 Figure 7: OCXO clock speeds for two types of OCXO (left) for four units used at Jiangyin Bridge and (right)
 375 for ten units used subsequently.

376 4 Planning using BAYOMALAW

377 Planning for the Jiangyin Bridge modal test was based on a combination of prior experience with other
 378 bridges and preliminary data for Jiangyin Bridge obtained by JSTI.

379 In a Bayesian context, the identification uncertainty is quantified in terms of its variance given information
 380 of the data set, but the value itself (for the given set of data) reveals little insight about how it depends on the
 381 quality or statistical characteristics of data. Beyond the ability to *calculate*, planning ambient vibration tests
 382 requires the ability to *understand* identification uncertainty and master its relationship with test
 383 configuration. When the data indeed follow the distribution assumed in the identification, i.e., stochastic
 384 stationary and classically damped, it can be shown that the variance follows a statistical law analogous to the
 385 Laws of Large Numbers in classical statistics. It has a deterministic part and a random part. The random part
 386 depends on specific details of the data and is asymptotically negligible as data length increases. The

387 deterministic part depends on the ‘information content’ of the data that is related to test configuration and is
 388 what affects test-planning decisions. This deterministic part is referred to as uncertainty laws in recent
 389 studies [27,29].

390 The OMA uncertainty laws are closed form asymptotic expressions and they are the primary scientific target
 391 of the BAYOMALAW project. The derivation involves asymptotics and leverages on the particular
 392 mathematical structure of the OMA problem in specific contexts such as well-separated modes, closely-
 393 spaced modes, multiple setups, but the findings to date show that the results can be remarkably simple [29].
 394 The findings can be used to guide test planning for different circumstances of prior information.

395 **4.1 BAYOMALAW for planning with preliminary data**

396 When the preliminary data are available, their quality can be assessed using their power spectral density
 397 (PSD), and the identification uncertainty of the MP can then be quantified using 'zeroth order' relationships
 398 with e.g. data duration [29][30] corrected for 'first order' effects [27] due to test configuration. The
 399 uncertainty laws are developed for modal frequency, damping and shape.

400 Of these, damping ratio is the parameter whose uncertainty is simultaneously the most important and hardest
 401 to control. For well-separated modes, small damping and long data in a single measurement, or setup using
 402 one or more response data channels the posterior CoV. δ (coefficient of variation = standard deviation/mean)
 403 for damping ratio is asymptotically given by

$$404 \quad \delta \sim \delta_1 = \delta_0 \sqrt{1 + \frac{a(\kappa)}{\gamma}}. \quad (1)$$

405 ‘ \sim ’ means ‘asymptotic to’ and denotes that CoV. δ tends to δ_1 asymptotically as $\zeta \rightarrow 0$, and data length $\rightarrow \infty$.
 406 Here, δ_0 and δ_1 represent the CoVs, respectively, given by ‘zeroth order’ and ‘first order’ uncertainty laws.

407 The parameter of particular relevance is the modal SNR γ , whose definition is motivated from the
 408 uncertainty law theory. Defining S as the 'modal force PSD', the modal SNR is the ratio of modal PSD $S/4\zeta^2$
 409 to the noise PSD S_e at the natural frequency f i.e.

$$410 \quad \gamma = \frac{S}{4S_e\zeta^2} \quad (2)$$

411 while the zeroth order law that gives δ when the modal SNR is infinite [29,30] is given by

$$412 \quad \delta_0 = \frac{1}{\sqrt{2\pi\zeta N_c B(\kappa)}} \quad (3)$$

413 N_c is the ‘effective data length’ expressing the data duration T as a multiple of the natural period $1/f$, i.e.

$$414 \quad N_c = Tf \quad (4)$$

415 and is conventionally understood to be a key parameter controlling reliability of MP obtained from OMA.

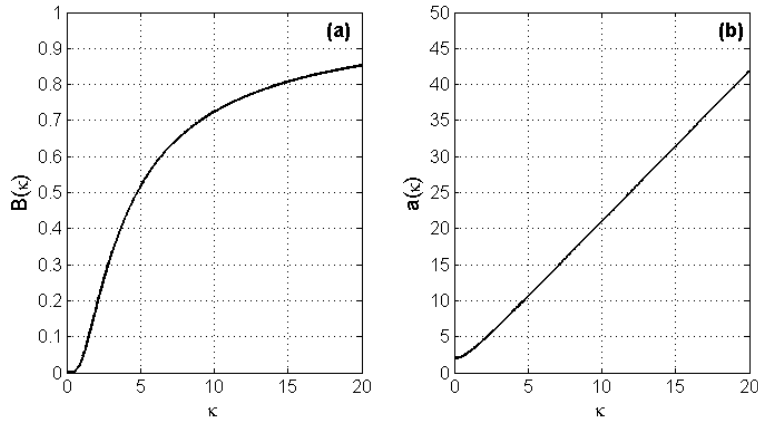
416 κ is a dimensionless ‘bandwidth factor’ that reflects usable bandwidth $f(1 \pm \kappa\zeta)$ around the natural
 417 frequency f and on which depend parameters $a(\kappa)$ and $B(\kappa)$:

$$418 \quad B(\kappa) = \frac{2}{\pi} \left[\tan^{-1}(\kappa) + \frac{\kappa}{\kappa^2+1} - \frac{2}{\kappa} (\tan^{-1}\kappa)^2 \right] \quad (5)$$

$$419 \quad a(\kappa) = \frac{4(\kappa^2+1)(3\tan^{-1}\kappa - 3\kappa + \kappa^2 \tan^{-1}\kappa) \tan^{-1}\kappa}{3[(\kappa^2+1)(\kappa - 2\tan^{-1}\kappa) \tan^{-1}\kappa + \kappa^2]}. \quad (6)$$

420 Figure 8 plots $B(\kappa)$ and $a(\kappa)$, which are both increasing functions of κ .

421 $\kappa = 1$ represents the classical ‘half power bandwidth’ containing $2\zeta N_c$ FFT points [18] and T can be
 422 considered to be long when the number of FFT points $N_f = 2\kappa\zeta N_c$ is large compared to 1.

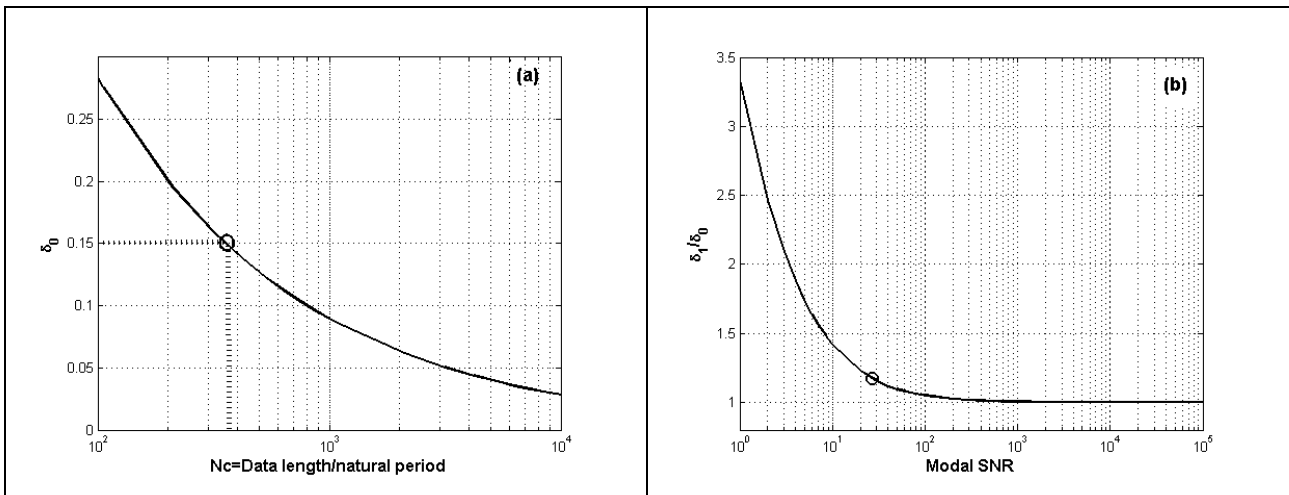


423 Figure 8: Plot of $B(\kappa)$ (a) and $a(\kappa)$ (b) with bandwidth factor κ .

424 The zeroth order uncertainty law δ_0 gives the achievable identification precision because it is the lower limit
 425 of uncertainty for ideal situation with noiseless equipment ($\gamma \rightarrow \infty$). This limit is not zero because the (input)
 426 ambient excitation is unknown. The zeroth order law is less informative in guiding the ambient vibration test
 427 because the effect of modal SNR γ has not been explicitly captured. The first order uncertainty law δ_1
 428 depends on the modal SNR, thus providing a key quantity for planning the ambient vibration test.

429 A sample of data from the Jiangyin Bridge SHM system was provided by JSTI but proved to be problematic
 430 in terms of the synchronisation and orientation of the sensors. However the data were sufficient to indicate a
 431 vertical mode at 0.106 Hz with a damping ratio of 4 % and a usable identification band of [0.07 0.11] Hz that
 432 gives a bandwidth factor 4.78. Figure 9a shows that for 15% CoV., $N_e=360$ natural periods are required,
 433 corresponding to one hour for the given mode.

434 From the preliminary data, it is estimated $S=9 \times 10^{-8} \text{g}^2/\text{Hz}$ and $S_e=5.3 \times 10^{-7} \text{g}^2/\text{Hz}$. Using the 4% damping
 435 estimate for the first vertical mode provides an estimated $\text{SNR}=27$ using equation (2). This SNR leads to a
 436 20% increase (compared to ideal noiseless situation) using the first order correction (Figure 9b). The ambient
 437 excitation S cannot be directly controlled but would be expected to have a similar level of spectral density in
 438 the planned test compared to the sample data. By optimising the sensor locations and using sensors with a
 439 lower noise level in the main test, it would be expected that a larger SNR would be achievable.



440 Figure 9: Zeroth and first order CoV. for damping ratio given parameters for Jiangyin Bridge first vertical
 441 mode. (a) zeroth order without effect of SNR, (b) first order correction depending on SNR.

442 4.2 Planning without preliminary data

443 It is reasonable to expect that MPs for Jiangyin Bridge would be similar to those obtained at Humber Bridge
 444 [31] and Fatih Mehmet Sultan Bridge [38] (B2, Turkey). The critical modes for each of these bridges are the
 445 fundamental lateral and vertical modes L1 and V1 and could be used in the absence of the JSTI sample data.

446 For B2, MP identified by 1st generation OMA are V1 ($f=0.125$ Hz, $\zeta=1.33\%$), L1 ($f=0.077$ Hz, $\zeta<14.4\%$), for
 447 the British bridge using 2nd generation OMA V1 ($f=0.116$ Hz, $\zeta=3.1\%$), L1 ($f=0.055$ Hz, $\zeta=4\%$).

448 Practical guidelines for planning ambient vibration tests for Bayesian OMA are given in [27]. Without
 449 preliminary data, MP for the similar bridges (i.e. Humber and B2) could be used to guide planning, although
 450 in this case natural frequency estimates are available, which are quite close to values for Humber and B2.
 451 Without other information on data quality the bandwidth factor κ is not known. To separate the effect of data
 452 duration and SNR, the first order posterior CoV. represented in equations (1) and (3) is rearranged into two
 453 factors:

454
$$\delta_1 = A_1 A_2$$

455 where

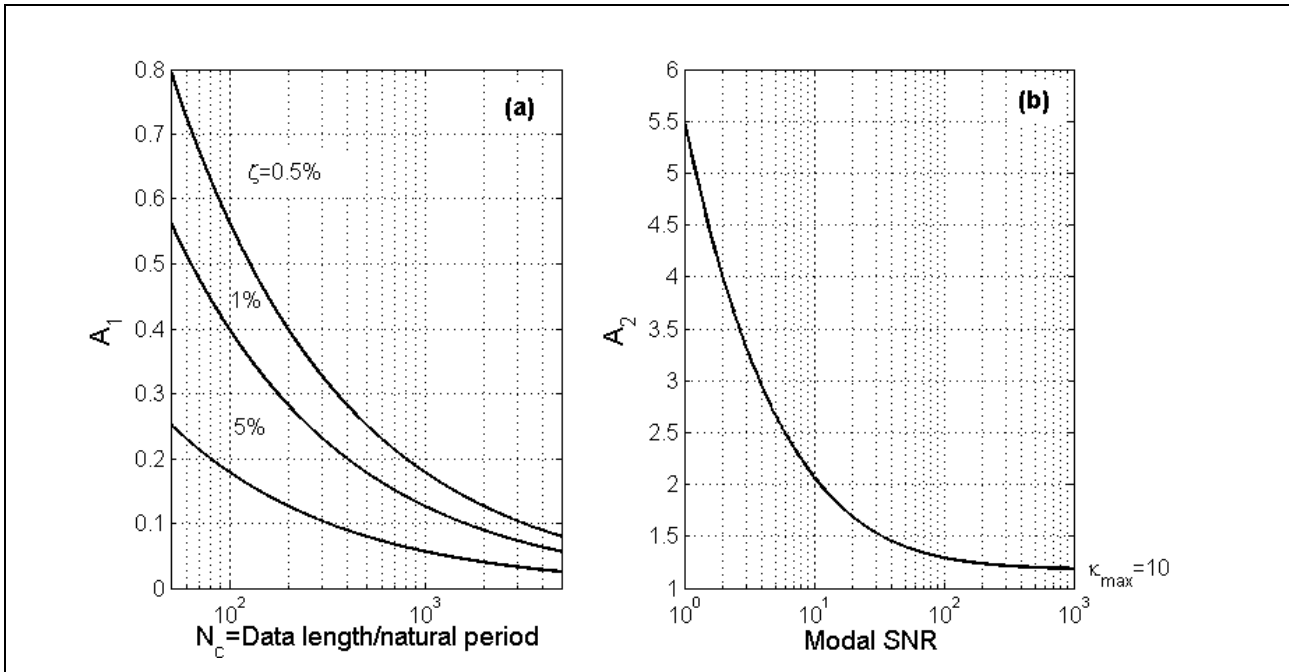
456
$$A_1 = \frac{1}{\sqrt{2\pi\zeta N_c}}$$

457
$$A_2 = \sqrt{\frac{1 + a(\kappa)/\gamma}{B(\kappa)}}$$

458 and $B(\kappa)$ and $a(\kappa)$ are given in equations (5) and (6).

459 Figure 10 shows A_1 , which accounts for the duration (in terms of number of cycles) and A_2 , which separately
 460 accounts for the SNR. At the planning stage the bandwidth factor κ is not known, but it can be taken to
 461 depend on the SNR [27] since a more noisy signal buries the spectral peak, reducing the useful frequency
 462 range of information. A reasonable choice consistent with common practice is the minimum of $2\sqrt{\gamma}$ and
 463 some κ_{\max} that controls the risk of modelling error (e.g. unaccounted mode). Again, these considerations
 464 on the bandwidth factor are only relevant at the planning stage because its value directly results from the
 465 choice of the band for identification when the data is available.

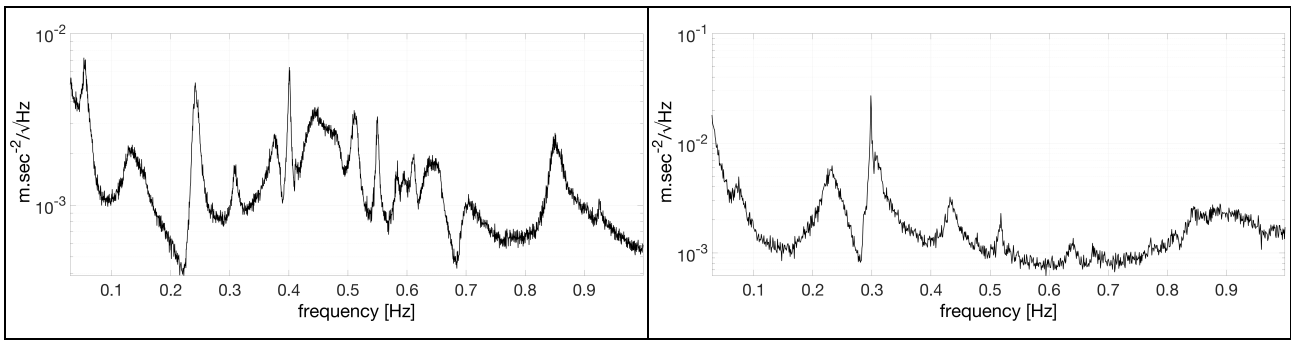
466



467 Figure 10: Assessing test configuration in terms of a) data duration and b) modal SNR.

468 Although a value of SNR=27 is available from preliminary data, a typical value could also be obtained using
 469 the accelerometer noise floor and the experience from other bridges. From the OCXO laboratory huddle test,
 470 the sensor and acquisition system background noise is $\sqrt{S_e}=30 \mu\text{ms}^{-2}/\sqrt{\text{Hz}}$, while the square root of modal
 471 acceleration PSD for the most challenging first lateral mode would be somewhere between the values

472 obtained from the Humber Bridge, $\sqrt{S}/2\zeta=0.007 \text{ ms}^{-2}/\sqrt{\text{Hz}}$, and B2, $\sqrt{S}/2\zeta=0.005 \text{ ms}^{-2}/\sqrt{\text{Hz}}$ (Figure 11),
 473 giving a worst case $\text{SNR}=3\times 10^4$, much better than from the preliminary data.



474 Figure 11: Square root PSD for Humber Bridge (left) and Second Bosphorus Bridge (right) lateral
 475 acceleration.

476 4.3 Factors controlling modal SNR

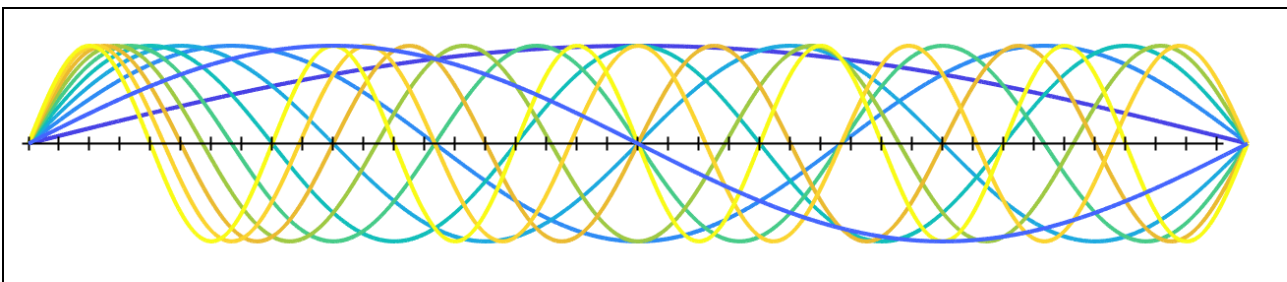
477 Modal SNR derives from two factors. The quality of the sensor and data acquisition hardware control the
 478 noise PSD S_e while the number and location of sensors control modal force PSD S , which is proportional to
 479 the square of the mode shape values at the measured degrees of freedom (DOFs). For identifying a single
 480 mode with multiple sensors, the obviously strategy is to maximise the sum of squares of mode shape values.

481 Sensor locations on a structure (test points or TPs) will have up to three DOFs, and their locations will be
 482 chosen logically according to convenient structural features - for example, the hanger (suspender) attachment
 483 points to the deck. For all the referenced long span bridge AVTs tests there were more DOFs than available
 484 single-axis sensors. Reference TPs and DOFs are selected common to all setups so that mode shape pieces
 485 can be glued or assembled [46]. Therefore, for the purposes of planning a modal test, the questions to be
 486 answered are:

- 487 1. How many reference locations should be measured?
- 488 2. Where should the reference sensors be placed?
- 489 3. How should the remaining sensors be roved in how many setups?
- 490 4. How long should the data (acquisition) of each setup be?

491 Although there are possibilities, such as daisy chaining references, to simplify planning we assume that the
 492 reference location or locations are fixed in all setups. Reference sensors should be located so that the target
 493 modes, or as many modes as possible, can be identified in all setups, avoiding modes of interest while
 494 maximising modal responses (via mode shape ordinates) for expected mode shapes, which can be based on
 495 experience or numerical simulations.

496 So where are the best reference locations? Figure 12 shows a set of ten mode shapes for a simply supported
 497 beam with 41 TPs (indicated by vertical bars, including support locations). These mode shapes might
 498 resemble vertical mode shapes of a suspension bridge and siting a reference anywhere from the 5th to the 37th
 499 TPs will find at least one mode having modal ordinate at or close to zero. However, the first mode, which has
 500 the smallest modal ordinates in the TP ranges 2-4 and 38-40, is likely to be the most important to identify
 501 reliably. One possible strategy is to use multiple references, but this reduces the number of ‘roving’ sensors
 502 available for remaining test points, which will require more setups each using (for a fixed available test
 503 campaign duration) reduced data duration T .

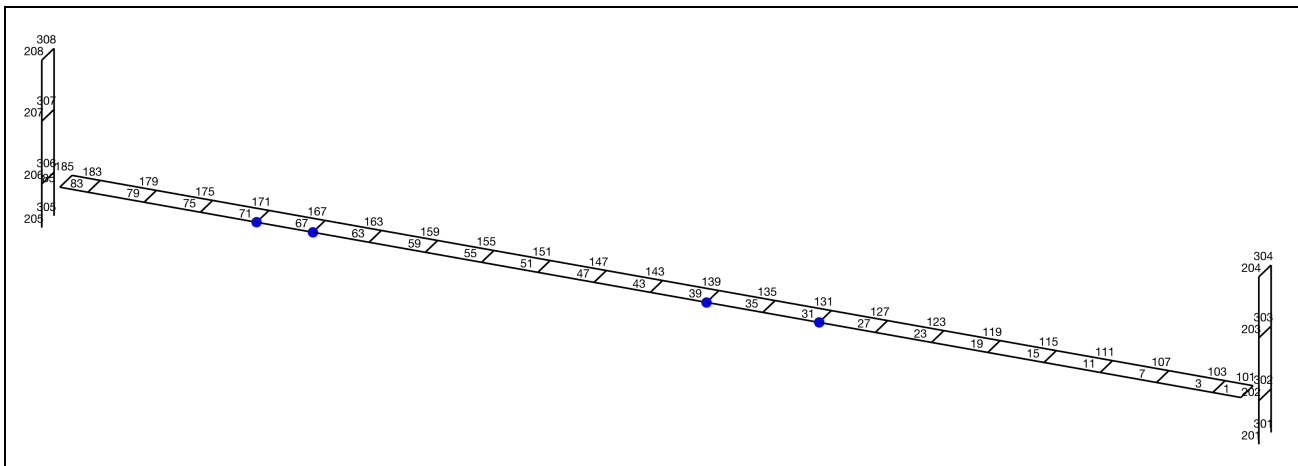


504 Figure 12: Ten mode shapes for a simply supported beam with 41 TPs.

505 For a single mode where the reference DOF ordinate is counted only once, maximising the SNR for the same
506 modal excitation and sensor type means maximising the sum of squares of mode shape values. For the usual
507 case of measuring multiple modes where the SNR is proportional to at least the square of the modal ordinate,
508 there is as yet no metric to maximise.

509 **5 Ambient vibration test (AVT)**

510 The AVT was scheduled for Tuesday 25th to Thursday 27th April 2017. Equipment comprising four loggers,
511 12 QA-750 accelerometers, 240 m of colour-coded signal cable in 10 m and 20 m lengths and a laptop were
512 flown from the UK as checked baggage, with NI hardware removed from the loggers and packed securely. A
513 set of four high-capacity batteries were provided by JSTI and local transport was provided by JSTI. Weather
514 was mostly fine, usually low visibility with smog clearing towards the end of the test, minimal rain and weak
515 to moderate winds. The south end of the east walkway of the bridge was accessed by driving from the south
516 and parking at the tower at the edge of the traffic lanes, then climbing onto the walkway from the approach
517 span side. Hanger attachment points are clearly numbered from north to south, and the four channels of the
518 master OCXO box were used as references with lateral (transverse direction) and vertical accelerometers at
519 TP67 and TP71, aiming to identify mode shape pieces using three roving OCXO boxes, each typically
520 measuring at a single TP. Figure 13 shows Jiangyin Bridge TPs, corresponding to hanger attachment points
521 and tower portals, as well as indicating reference locations and TPs for a typical measurement setup.



522 Figure 13: Jiangyin Bridge test points and sensor locations for a measurement setup.

523 A detailed method statement, including a risk assessment, was prepared for the scheduled three days
524 comprising a sequence of one-hour measurements, with an option to use a fourth day in the event of bad
525 weather. The plan covered every fourth TP on the east walkway, two TPs on the west walkway opposite the
526 reference TPs, as many as possible TPs on the south tower, and if time permitted, the equivalent locations on
527 the north tower. Figure 14 shows the loggers being synchronised before a huddle test at the start of the
528 sequence and the pair of accelerometers at TP67. The bridge is aligned approximately in north-south axis
529 with a coordinate system of X=longitudinal, positive to the north, Y=lateral, positive to the west and
530 Z=vertical, positive upwards.

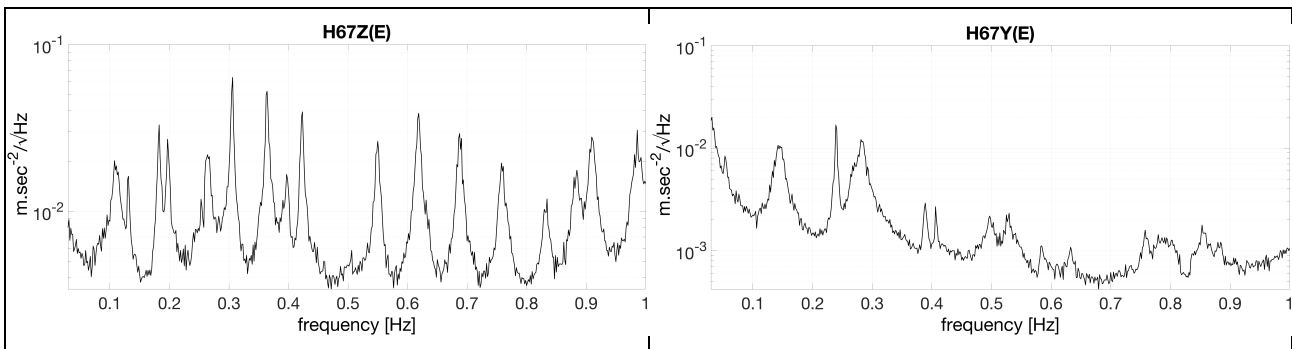
531



532 Figure 14: Setting up acquisition for initial huddle test and reference accelerometers at H67.

533 The reference accelerometer signals were recorded by the master logger continuously during the day of
 534 measurement while the three slave loggers were roved to remaining TPs. The moves were coordinated by
 535 mobile phone text messages and timed so as to maintain a minimum one hour recording for all loggers.
 536 Square root PSD plots for the first day of vertical and lateral acceleration data at TP67 are shown in Figure
 537 15.

538



539 Figure 15: Sample square root PSD for TP67 vertical (Z) and lateral (Y) acceleration.

540 Table 1 Summarises the measurement sequence over three days.

541 Table 1: Measurement setup using two to four acquisition channels (ADs) of master and slave loggers.

setup	day	start	end	M				S1			S3		
				DA1	DA2	DA3	DA4	DA1	DA2	DA3	DA1	DA2	DA3
1	26	11:14	12:17	H67EZ	H67EY	H71EZ	H71EY	H83EZ	H83EY	-	H75EZ	H75EY	-
2	26	12:30	13:34	H67EZ	H67EY	H71EZ	H71EY	H63EZ	H63EY	-	H55EZ	H55EY	-
3	26	13:41	14:45	H67EZ	H67EY	H71EZ	H71EY	H51EZ	H51EY	-	H43EZ	H43EY	-
4	26	14:54	16:00	H67EZ	H67EY	H71EZ	H71EY	H39EZ	H39EY	-	H31EZ	H31EY	-
5	26	16:08	17:10	H67EZ	H67EY	H71EZ	H71EY	H27EZ	H27EY	-	H19EZ	H19EY	-
6A	27	10:19	11:20	H67EZ	H67EY	H71EZ	H71EY	H23EZ	H23EY	-	H15EZ	H15EY	-
6B	27	11:26	12:30	H67EZ	H67EY	H71EZ	H71EY	H35EZ	H35EY	-	H11EZ	H11EY	-
6C	27	12:36	13:38	H67EZ	H67EY	H71EZ	H71EY	H47EZ	H47EY	-	H7EZ	H7EY	-
6D	27	13:53	14:55	H67EZ	H67EY	H71EZ	H71EY	H59EZ	H59EY	-	H3EZ	H3EY	-
6E	27	15:02	16:05	H67EZ	H67EY	H71EZ	H71EY	H79EZ	H79EY	-	-	-	-
6F	27	16:12	17:13	H67EZ	H67EY	H71EZ	H71EY	H85EZ	H85EY	-	STLEX	STLEY	-
7	28	10:16	11:22	H67EZ	H67EY	H71EZ	H71EY	H71WZ	H71WY	-	H67WZ	H67WY	-
8	28	12:30	13:30	H67EZ	H67EY	H71EZ	H71EY	STUEX	STUEY	STUWX	STMEX	STMUY	STMWX

542 E/W are east/west side of bridge; L/M/U are lower/mid/upper portal levels of tower

543 Logger S2 was used in the first day of measurements but when data were merged it was found that
544 synchronisation had not been correctly initiated. This was due to allowing too short a time for the
545 synchronisation, requiring only a minor change in the initialisation procedure. An inelegant method to guard
546 against this would be to generate a similar and obvious signal on all sensors while collocated at the start of
547 the sequence after initialisation. An example would be a sequence of ‘heel drops’ on the bridge deck,
548 providing obvious signals that could be subsequently aligned using correlations between data channels.

549 In this case, being the first major exercise, no chances were taken and measurements covered by S2 were
550 repeated in setups 6A-6F, while S2 was not used until it could be thoroughly checked on return to the UK.
551 Setup 6F included X and Y measurement of the south tower (ST) lower (L) portal, deck level east side
552 (Figure 16 left) and setup 7 was designed to identify torsional modes.

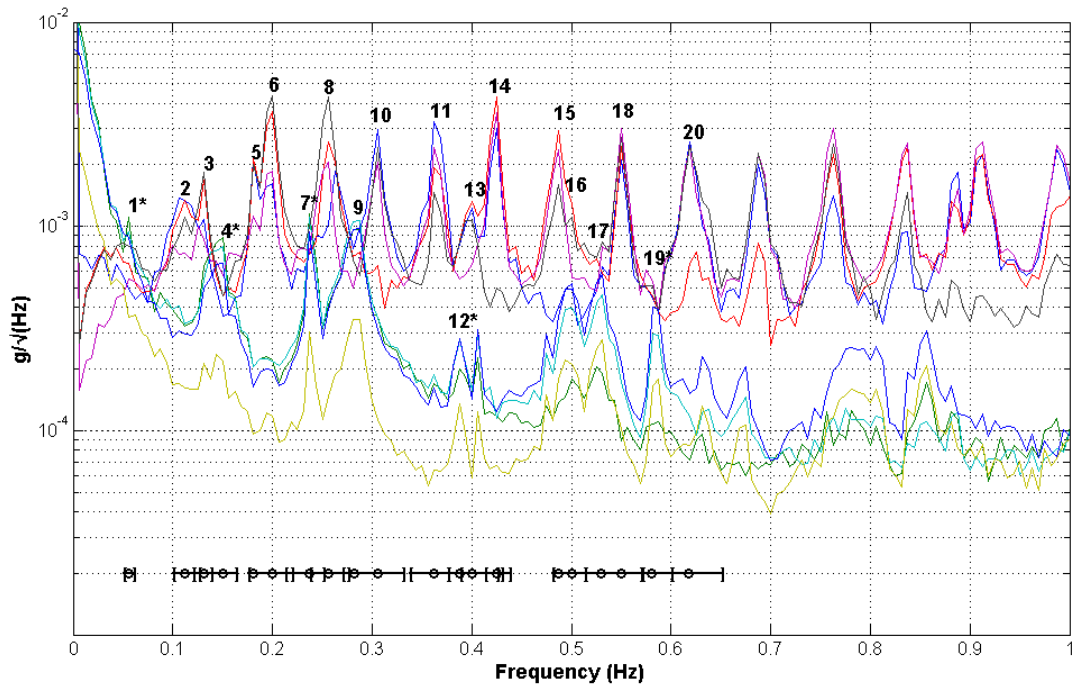
553 Setup 8, with measurements in the middle and upper portal, was designed to characterise the tower
554 component of deck girder modes as well as modes only involving the tower. Jiangsu Yangtze Bridge Co.
555 provided access to the south tower via the door visible in Figure 16 (left), and the middle portal (interior
556 shown in Figure 16 right reflecting the depth visible in Figure 2) was accessed by a lift. Access to the upper
557 portal was via transfer to a second lift at a mezzanine level and to the top of the portal for the view of Figure
558 2 by staircase. The value of autonomous loggers was proven by the lack of GPS or wireless signal reception
559 inside the middle portal, and it was more convenient to measure the inside the upper portal.



560 Figure 16: Measurements at lower (left) and middle (right) portal levels

561 **6 Modal parameter estimates using Bayesian operational modal analysis**

562 To visualise the modes of interest and the relevant frequency bands, the (root) PSD spectrum of Setup 1 (as a
563 typical case) is shown in Figure 17. It can be seen that the lines plotted in the PSD spectrum are generally
564 separated into two levels, indicating two different PSD levels of modal responses among the data channels.
565 The higher levels are those measuring vertical directions and the lower ones are those measuring lateral
566 directions. This is a typical case for AVT of long-span bridges as the spectral density of modal forces in the
567 vertical direction (mainly due to traffic) is normally much larger than those in the lateral direction (mainly
568 due to wind). With this situation, it is easier to detect potential modes based on the PSD spectrum than on the
569 singular value (SV) spectrum. This issue will also cause problems in identifying the lateral modes, which
570 will be discussed later.



571

572 Figure 17: Square root PSD spectrum of Setup 1

573 There are many modes within the 0-1 Hz band, including several closely-spaced modes, which increases the
 574 identification difficulties and the associated uncertainty. Modal identification here focuses on the first twenty
 575 modes indicated in the PSD spectrum. The initial guesses of natural frequency and the selected frequency
 576 bands for modes of interest are indicated in the figure with symbols 'o' and '['', respectively. The symbol
 577 '*' (as the superscript) denotes that the mode is identified using data in the lateral direction only. Modal
 578 parameters are identified in individual setups based on the Bayesian FFT method [47,48] and the overall
 579 mode shapes are assembled using the global least square method [46]. In this procedure, the real parts (or
 580 projections on to real axes) of the mode shape ordinates are used, so the small phase angle shifts suggested
 581 earlier would have no consequence.

582 Table 2 summarises the representative values of the identified modal parameters and associate uncertainties,
 583 where the MPV (most probable value) in the table denotes the sample mean of MPVs among the setups and
 584 the CoV. CoV is calculated based on the representative variance (i.e., sum of sample variance among the
 585 setups and the sample mean of the posterior variance among the setups).

586

587

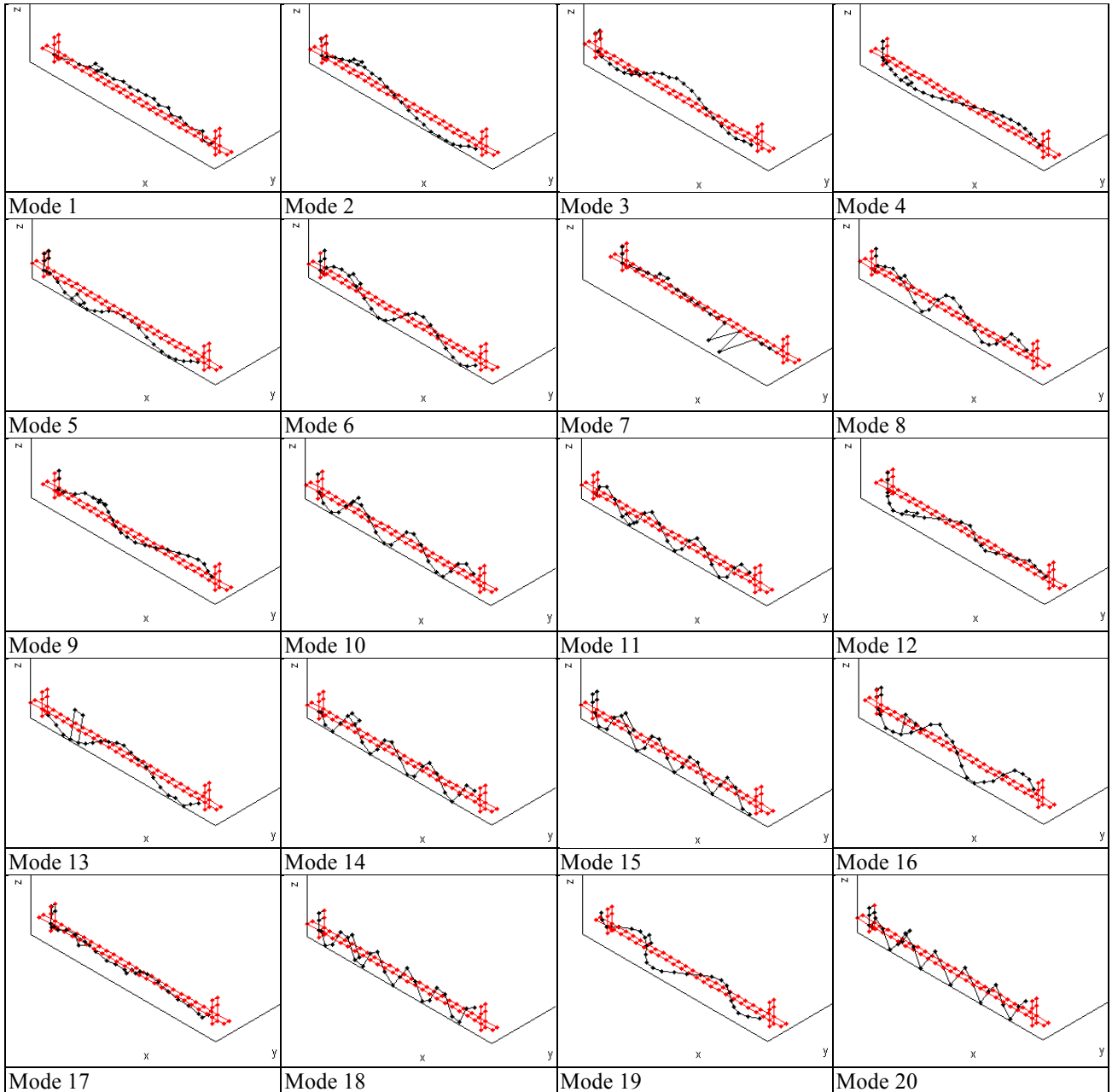
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589

Table 2: Identification results (L: lateral; V: vertical; T: torsional; S: symmetric; A: antisymmetric; U: unsymmetrical)

Mode	Characteristic	Natural Frequency		Damping Ratio	
		MPV (Hz)	CoV (%)	MPV (%)	CoV (%)
1	L-1-S	0.0536	3.56	4.32	88.2
2	V-1-A	0.1095	1.17	4.88	40.6
3	V-2-S	0.1304	0.54	1.41	53.6
4	L-2-A	0.1434	0.68	3.78	28.1
5	V-3-S	0.1828	0.48	1.50	68.2
6	V-4-A	0.1971	0.32	0.91	34.3
7	L-3-U	0.2393	0.61	0.49	75.0
8	V-5-S	0.2560	1.03	0.98	45.8
9	T-1-S	0.2811	0.50	1.61	25.0
10	V-6-A	0.3044	0.31	0.97	48.2
11	V-7-S	0.3638	0.16	0.61	25.5
12	L-4-A	0.3896	0.34	1.63	57.3
13	V-8-S	0.3974	0.45	0.83	29.6
14	V-9-A	0.4227	0.11	0.41	37.8
15	V-10-S	0.4854	0.17	0.50	29.2
16	T-2-A	0.4995	0.45	1.29	27.7
17	T-3-A	0.5236	0.69	1.67	34.2
18	V-11-A	0.5499	0.16	0.54	31.2
19	L-5-A	0.5865	0.21	0.64	35.4
20	V-12-S	0.6185	0.18	0.50	28.3

590
591

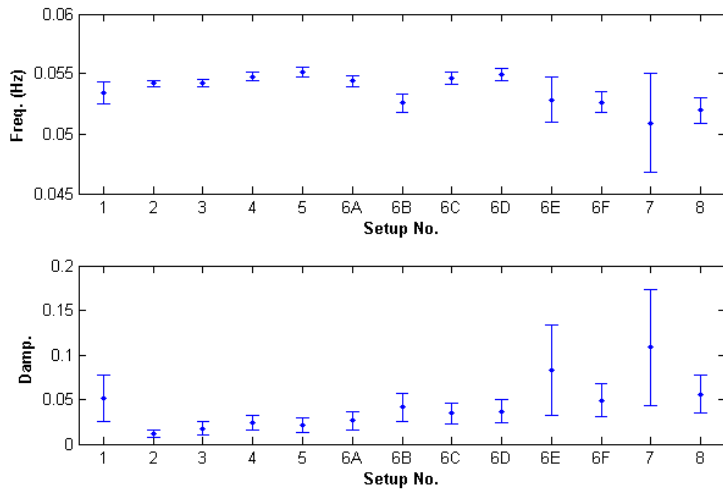
592 Figure 18 shows the assembled global mode shapes of the modes identified in Table 2.



593 Figure 18: Identified global mode shapes

594 The first mode is a symmetric lateral mode with a natural frequency of 0.0536Hz, which is quite close to the
595 value reported in the Chinese paper [34]. However, this mode cannot be identified based on the preliminary
596 data (modal SNR is no better than unity), due to the high measurement noise in the low frequency range,
597 meaning that the spectral peaks for this mode are not significant even in this AVT. It can be seen that there is
598 an increase of the PSD in the frequency range lower than 0.1 Hz which derives from the measurement noise
599 of the sensors as well as from the quasi-static rotation. The rotation appears as a DC offset of the signal and,
600 along with the noise, overlays the modal contribution of the mode.

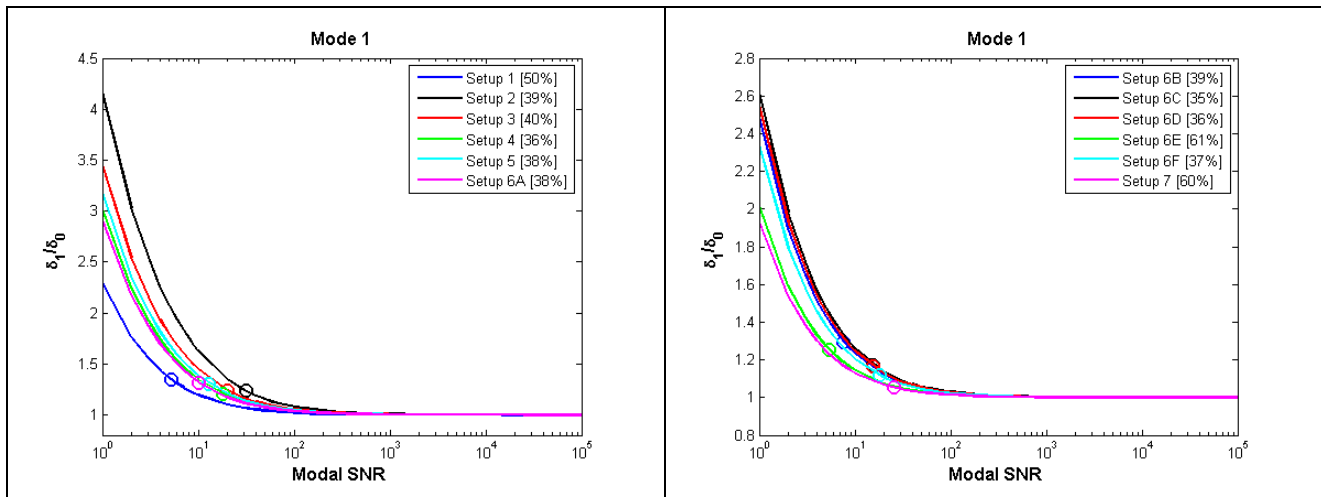
601 Spectral peaks cannot even be detected in some of the data channels in the vertical direction, which cause
602 convergence problems in the modal identification procedure when all the data channels are involved for
603 inference. In view of this, only the measured data in the lateral direction are used for Mode 1 identification
604 and the overall mode shape is assembled assuming zero mode shape values in the vertical direction. Figure
605 19 shows the identified natural frequencies and damping ratios of Mode 1 among the setups, where the error
606 bar represents \pm posterior standard deviation of the identified MP. Large variations in the identified damping
607 ratio can be found for Setup 6E and 7, which raise the overall identification uncertainty.



608

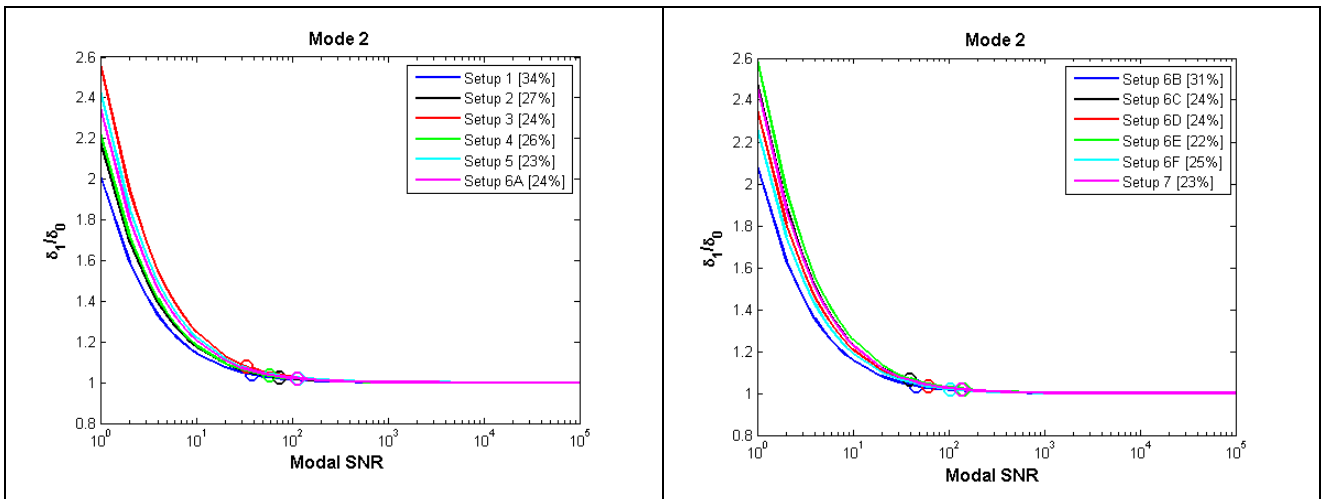
609 Figure 19: Identified natural frequency and damping ratio among the setups, Mode 1. (dot: MPV; errorbar: \pm
610 posterior standard deviation)

611 To illustrate the effect of modal SNR, Figure 20 shows the CoV. ratio δ_1/δ_0 for the posterior damping ratio
612 of Mode 1. The circles reflect the identification uncertainty based on the current test configuration and the
613 curves denote the value for each setup predicted by uncertainty laws. The values in the legend denote the
614 CoV. of damping ratio quantified based on the measured data. It can be seen that all the setups lie in the low
615 SNR region, where further increases in SNR can reduce the identification uncertainty by 30%.



616 Figure 20: SNR effect with damping CoV., Mode 1 (left: Setup 1 to Setup 6A; right: Setup 6B to Setup 7)

617 The first vertical mode (Mode 2) is antisymmetric, with a natural frequency of 0.1095Hz. This is the mode
618 corresponding to the one identified in the preliminary data for test planning. Figure 21 shows the CoV. ratio
619 δ_1/δ_0 for posterior damping ratio of this mode. The posterior CoVs of damping ratio (as shown in the
620 legend) are higher than the targeted value (i.e. 15%) in test planning. This is reasonable as the zeroth order
621 uncertainty law used for the measurement duration plan assumes infinite SNR. The SNR of this mode varies
622 among the setups, all are higher than the ~unity value for preliminary data, and they are close to the flat
623 region of the predicted δ_1/δ_0 curve. Further increasing SNR will provide significant help in reducing the
624 identification uncertainty, confirming the effectiveness of BAYOMALAW for planning.

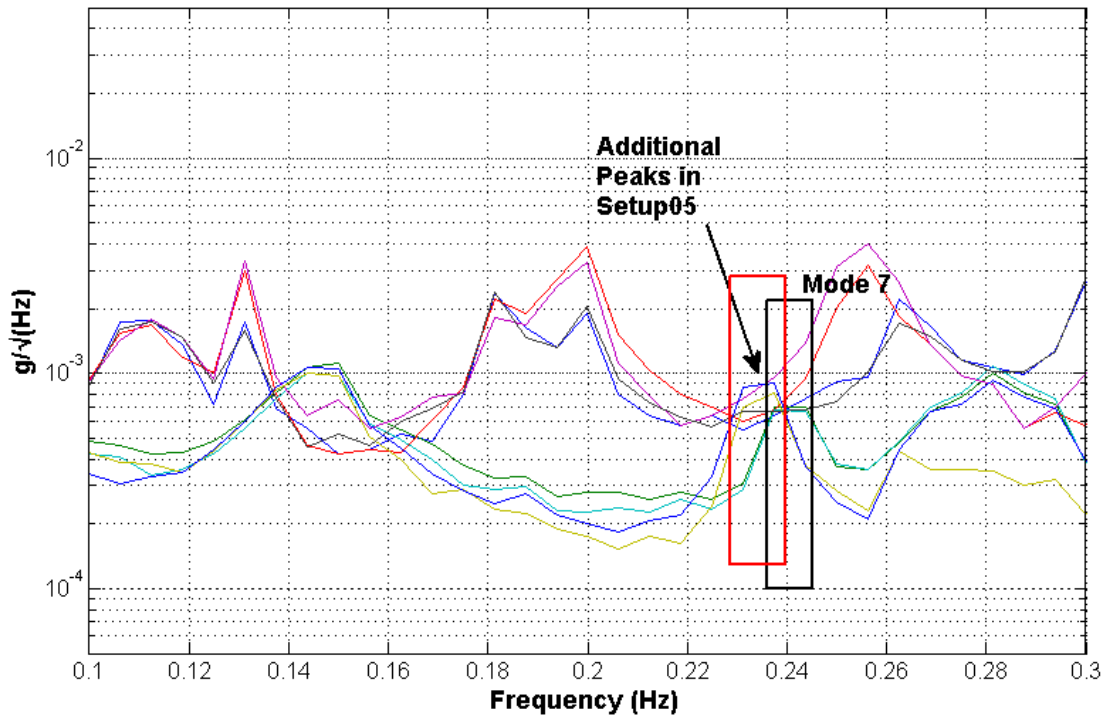


625 Figure 21: SNR effect with damping CoV., Mode 2 (left: Setup 1 to Setup 6A; right: Setup 6B to Setup 7)

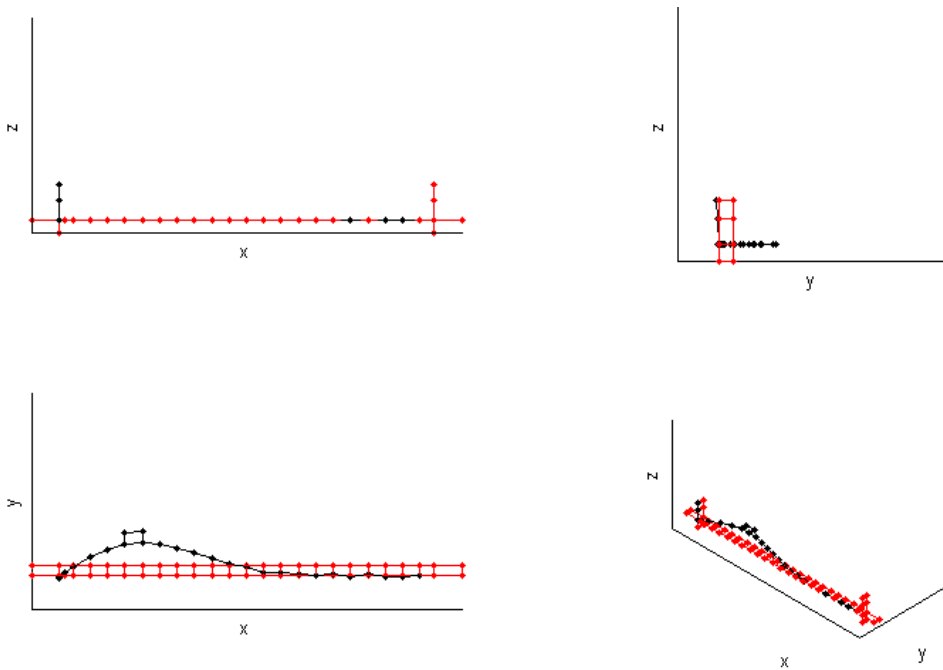
626 Within the resonance band of Mode 4 (lateral mode), large contributions are found from Modes 2 and 3 (both
 627 are vertical modes). Similar situations can be found for Mode 7, 12 and 19. Identifying these modes within
 628 their resonance frequency bands by assuming a single mode is not feasible, as the modal contribution of
 629 other modes in the bands are not negligible and sometimes even larger than the mode to be identified. On the
 630 other hand, selecting a wide band involving the nearby modes and identifying them simultaneously will
 631 increase the modelling error risk due to non-structural components within the band, as these modes may not
 632 be closely spaced. It turns out that the identification procedure cannot converge or provide a reasonable
 633 estimation on the modal parameters of these modes. In view of this, these modes (i.e., Modes 4, 7, 12 and 19)
 634 are identified within their resonance frequency band using data channels in the lateral direction only to
 635 eliminate the effect of nearby modes. The mode shape values in the vertical direction are assumed to be zero
 636 when assembling the overall mode shapes.

637 The identified mode shape of Mode 7 in Setup 5 is peculiar because of two anomalous large ordinates. The
 638 cause is evident from Figure 22, showing the PSD spectrum of Setup 5 where there are additional peaks in
 639 the selected frequency band for Mode 7. The mode shape values are erroneously identified due to these
 640 peaks, which may not refer to a global mode as they cannot be found in other setups. To avoid their effect,
 641 modal identification has been conducted for mode 7 with a narrower frequency band where the additional
 642 peaks are not involved. Figure 23 shows the identified mode shapes, which now look more reasonable.

643



644
645 Figure 22: Square root PSD spectrum of Setup 5.



646
647 Figure 23: Overall mode shape of Mode 7 identified based on the narrower band

648 Thanks to the setup on the east side of the deck (i.e., Setup 7), torsional modes can be detected and
 649 identified. Specifically, Mode 9 is identified as a symmetric torsional mode and Modes 16 and 17 are
 650 antisymmetric torsional modes. Some torsional modes may be erroneously counted as lateral modes in this
 651 test as some modes are identified using data channels in the lateral direction only where the mode shape
 652 values in the vertical direction are not identified.

653

654 **7 Discussion and conclusions**

655 Modal tests of long span (suspension) bridges are among the most challenging of all types of field
656 measurements due to their scale and to (as far as the authors know) the lowest frequencies observed in any
657 man-made structure. Reliable MP identification, particularly of fundamental vertical and lateral mode
658 damping ratios has been compromised by both bias errors introduced by early (first generation) OMA
659 procedures and the large variance errors that appear to reduce only by using extremely long measurement
660 periods, trading off stationarity properties of signals and environmental influence on structural properties.
661 The question has always been how long a data record is necessary, what quantity and quality of equipment is
662 adequate, and what level of accuracy can be achieved. Now it is possible to define a level of accuracy and
663 determine the best measurement configuration in terms of sensor type, location and measurement duration in
664 order to achieve it.

665 Then, if mode shapes are required to completely define the mode, how should they be best constructed using
666 resources available and how to get around the difficulty of synchronising acquisition at locations separated
667 by distances of the order of 1 km without constraint of cables, or the need to receive radio signals from
668 satellites (GPS) or other loggers (by radio) for synchronisation?

669 These challenges have been addressed in a single exercise, with the bonus of a rare collaboration with
670 Chinese researchers allowing a foreign team access to test a Chinese bridge.

671 The decision to use an hour of data for each measurement appeared optimal from the point of view of MP
672 uncertainty and allowing for sufficient setups to define the global modal characteristics. It was not believed
673 to be necessary to measure both sides of the bridge completely due to the single measurement on the west
674 side and symmetry about the longitudinal axis.

675 However, the bridge may not be symmetric about the mid span, as indicated by lateral modes 7 and 12, so it
676 cannot be assumed that lateral mode participation of the north tower would be identical to that of the south
677 tower. Experience with suspension bridge towers e.g. [38] has shown that in addition to participating in deck
678 modes driven by the cables, they have their own set of modes consistent with a cantilever having fixity at the
679 freer end depending on the cable detail and back span arrangement.

680 The combination of sensor and data acquisition systems gives rise to background noise S_e , which is adequate
681 for the purpose, judging by the SNR of the acquired data (Figure 20, Figure 21). In fact, good enough SNR
682 would be obtained for fundamental modes even with reference sensors much closer to the tower, where the
683 mode shape ordinates are much reduced. This would mean less chance of locating at the nodal point of a
684 high frequency mode and possibly negating the need for a backup reference location. Other sensor options
685 would be to use JA-70SA or Guralp CMG5T units, both low-noise tri-axial devices, but while single axis
686 sensors might be more cumbersome and require more local wiring, they offer more flexibility.

687 Based on the experience, to obtain the most useful planning data, the authors would recommend a
688 preliminary measurement using a single logger and small set of uniaxial accelerometers laid out close to the
689 tower of a long span bridge, aligned first vertically then laterally. This would provide enough information to
690 characterise the signal to noise ratio of the fundamental modes and determine an appropriate setup duration.
691 It would also qualify the use of a reference location close to a tower.

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699

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