Effectiveness of Monitoring, Managerial Entrenchment, and Corporate Liquidity*

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Abstract

We present a continuous-time model of partially delegated cash management, where effectiveness of monitoring and managerial entrenchment are explicitly accounted for. Shareholders trade off the wedge in cash flows between the manager-run firm and their outside option with the tunneling activities of the manager. Our framework results in realistic equity issuance proceeds even in the absence of marginal costs of issuance. We demonstrate that while both more effective monitoring and higher managerial entrenchment lead to higher cash holdings, their relations with the value of cash are U-shaped and strictly negative, respectively. A higher value of cash is therefore associated with enhanced external corporate control mechanisms, but not necessarily with tighter internal monitoring procedures. Although the risk management policy largely depends on the allocation of the respective control rights, our numerical implementation reveals a substantial range of cash levels in which both parties benefit from risk-reducing actions.

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1 Introduction

Managers of corporations are on the top of a firm's decision-making hierarchy. Their job consists of everyday choices intended, in theory, to increase shareholder value. However, they have the discretion to use the firm's resources to achieve their own targets which can, depending on the level of oversight that shareholders have on their actions, substantially deviate from the intended goal. Nevertheless, they are themselves employees. As such, in order to maintain their position they must be viewed by their employers, the firm's shareholders, as at least as good an alternative as any manager who can be hired in the labor market. Thus, they have incentives to and can, to an extent depending on their relative power over shareholders, take measures to protect themselves against their replacement. Shareholders' oversight over managerial decisions as well as the easiness of replacement of higher management are core aspects of what is generally understood by the term "corporate governance".

In this paper, we isolate these two facets of corporate governance, namely monitoring and managerial entrenchment, and examine their distinct effects on levels and values of corporate cash holdings, a significant part of a firm's assets. Cash holdings of listed firms have increased sharply over the last 25 years. This liquidity boom has attracted an increasing interest among finance academics. Bates, Kahle, and Stulz (2009) point out that the average cash holdings as a percentage of firms' total assets has more than doubled during the last quarter of a century, increasing from 10.5% in 1980 to 24% in 2004. Corporate cash holdings have become over time a significant component of a firm's balance sheet, and thus, its valuation is of growing importance in ultimately determining firm value.

Since Jensen and Meckling's (1976) seminal work, significant research has been done towards the determination of the effect of the separation of ownership and control on various aspects of a firm's operation, and consequently the estimation of agency costs that stakeholders of a corporation incur due to the "complex contracting relationships that govern it". Hart (1995) argues that corporate governance is only meaningful in the presence of incomplete contracts and defines it as the allocation of "residual control rights over the firm's nonhuman capital". Interestingly, despite Jensen's (1986) notable free cash flow hypothesis about agency conflicts being an essential determinant of the (mis-)use of internal funds, the emanating extension on the relation between corporate governance measures and the *level* of cash reserves is empirically far from clear. Opler, Pinkowitz, Stulz, and Williamson (1999), Mikkelson and Partch (2003) and later Bates et al. (2009), fail to prove a significant relation between agency costs and corporate liquidity. In a cross-country study, Dittmar, Mahrt-Smith, and Servaes (2003) find that firms in countries with strong shareholders' protection hold less cash than their counterparts in countries where shareholders' rights are not well protected. On the other hand, Harford, Mansi, and Maxwell (2008) report that the opposite is true for US firms, where poorly governed firms have lower cash holdings.

In order to highlight the ambiguous effect that the quality of corporate governance has on liquidity policy and to examine its impact on the value of cash, we construct an infinite-horizon continuous-time model which is able to capture two key attributes of the quality of corporate governance. In the model, shareholders delegate the firm's liquidity management to a manager. Extending Jensen's (1986) free cash flow hypothesis, the manager is able to extract more perquisites from the firm's cash flow *when* the level of accumulated cash holdings is higher. Her hoarding propensity is mitigated by the fact that shareholders hold a right to dismiss her at any time they wish to do so. The manager exercises such a liquidity policy that guarantees her job security — a solution in line with Faleye's (2004) observation that there are only so few proxy contests recorded, despite them being such a powerful mechanism of corporate control.

Managerial entrenchment is captured by a wedge between the expected cash flows of the firm under current management and those of the shareholders' outside option, representing in fact the manager's bargaining power vis-a-vis her employers, the firm's shareholders. We restrict financing to costly equity issuance, the timing and proceeds of which are chosen by shareholders. Thus, in our model, shareholders are actively involved in the firm's cash management as they control the firm's refinancing. Put differently, the model's solution yields a Nash equilibrium where each of the two parties involved controls separate decisions affecting the firm's cash policy.

We examine the effects of the model's parameters on cash policy and the marginal values of cash and obtain interesting insights and novel results. First, the marginal value of cash can be lower than one when we incorporate managerial entrenchment, matching results of empirical studies on the value of cash (Pinkowitz, Stulz, and Williamson, 2006; Dittmar and Mahrt-Smith, 2007) more closely than models that do not incorporate agency considerations (e.g. Bolton, Chen, and Wang, 2011). A second, related, result is that the conflict of interest between managers and shareholders creates a structural wedge between target refinancing and the payout threshold. In the absence of this conflict, this wedge is usually generated through a marginal cost of refinancing, which is though not needed in our setup. This feature highlights the informativeness of equity issuance proceeds and, by relieving the model from a hardly interpretable proxy of adverse selection costs, can potentially benefit structural modeling researchers to quantify the particularly unobservable notions of monitoring and entrenchment.

Turning to the effects of monitoring and entrenchment on cash policy, numerical results confirm that both effective monitoring and high managerial entrenchment lead to higher cash holdings. More interestingly, the model produces a novel result regarding the impact of these two parameters on the value of cash. The recursiveness of the continuous-time model yields a U-shaped relation between the effectiveness of monitoring and the value of cash. This is due to two conflicting effects of stricter monitoring: a direct, positive one whereby the manager's tunneling activity is restricted, and an indirect, negative one stemming from the consequential increase in the manager's bargaining power. Considering that the value of cash is strictly decreasing with managerial entrenchment, this result highlights the *superiority* of external corporate control mechanisms over internal monitoring procedures in cash value creation.

Lastly, the model produces interesting implications on risk management policies. Although the value functions of shareholders and the manager have different shapes, we find a substantial range of cash levels for which both parties would benefit from risk-reducing actions. Allocating the control rights of the risk management strategy in a way that the firm would engage in hedging activities only if both shareholders and the manager agree to them generates a novel hump-shaped relation between hedging and liquidity. It also predicts that corporate hedging is negatively related to both the effectiveness of monitoring managerial activities and managerial entrenchment. Since these two aspects of corporate governance are typically negatively related, the overall effect of aggregate measures of shareholders' rights is unclear, as reflected in relevant empirical studies (e.g. Haushalter, 2000; Lel, 2012).

The model developed in this paper belongs to the wider family of cash accumulation models pioneered by Baumol (1952), Tobin (1956), and later Miller and Orr (1966) who applied inventory management models to cash. The latest developments in this area (Bolton et al., 2011; Anderson and Carverhill, 2012; Hugonnier, Malamud, and Morellec, 2014) are related to the examination of the joint investment-financing decision. Our model bears similarities with Nikolov and Whited (2014), who also focus on the relation of agency conflicts and corporate cash holdings. In their model, the manager trades off the opportunity to tunnel some of the firm's cash to his own benefit at a given point in time against investing and benefiting from higher cash flows (and thus higher accumulated cash holdings) at a future date. However, their model does not explicitly account for the shareholders' option for collective action against management, which is how the upper boundary of the distribution of cash holdings is determined in our framework. In their case, managerial alignment to shareholders' interests is achieved through a compensation package.

Our model is most closely related to Décamps, Mariotti, Rochet, and Villeneuve (2011), which can be considered as a special case of our framework. Several of our results reaffirm their findings, while others are contradicted. We show that our model preserves heteroskedasticity of stock returns and the asymmetric volatility phenomenon examined in Décamps et al. (2011). But, the absence of the manager-shareholders' conflict leads them to a monotonic relation between the cost of carrying cash (the arithmetic inverse of what we refer to as monitoring effectiveness) and the value of cash, which, as already highlighted, is not the case in our model.

The remainder of this paper is structured as follows. Section 2 develops the setup of the dynamic continuous-time model and describes the differences between the afore-mentioned cases. In Section 3, we characterize the function of shareholders' value and describe the resulting liquidity policy for each case. With the help of a numerical implementation of the model, in Section 4, we illustrate the effects of the different assumptions on shareholders' value, on the marginal value of cash, and on stock returns. In the same section, we also describe the effects of the model's parameters on the levels and marginal values of cash. Section 6 concludes.

2 Setup

Consider a firm, the cumulative operating cash flows (Y_t) of which evolve according to an arithmetic Brownian Motion, such that

$$dY_t = \mu\, dt + \sigma\, dW_t \tag{1}$$

where μ represents the expected cash flows per period of time, $\sigma > 0$ the standard deviation of these cash flows, and dW_t the increment of a standard Wiener process. The firm has no growth opportunities and both the mean and standard deviation of cash flows are constant over time. The firm has no access to debt markets — or debt issuance is prohibitively expensive — and it is all-equity financed.[1]

The firm may pay out its cash flow as dividends or retain them as cash reserves; at every point in time t, the cumulative dividends and cash reserves are denoted by U_t and C_t

[1]We abstain from incorporating debt policy in order to preserve recursiveness of the model and isolate the desired effects on cash policy. A more comprehensive model including a dynamic financing policy on a similar setup with tax considerations could yield interesting findings and is left to future research.

respectively. Holding cash in the firm is costly for shareholders, as the instantaneous return on cash reserves is $\theta\, dt$ lower than the risk-free return $r\, dt$ that one unit of cash can otherwise earn. This cost-of-carry is intended to capture the degree of managerial discretion over the use of the firm's cash reserves, and thus, paraphrasing Jensen (1986), it can be interpreted as the (*direct*) agency costs of cash *stock*. This friction can alternatively be representing the effectiveness of monitoring mechanisms, since all else equal it is considered less costly for shareholders of a better monitored firm to keep a portion of their wealth in the form of corporate cash.

Given the inaccessibility of debt markets, the firm can be refinanced solely with equity which is issued whenever deemed necessary. Denoting by I_t the cumulative equity issuance process, the corporate cash inventory evolves according to

$$dC_t = dY_t + (r - \theta)\, C_t dt + dI_t - dU_t \tag{2}$$

which is simply the sum of the operating cash flow and the interest generated by existing cash net of the cost-of-carry in a time interval dt, plus the amount of external financing obtained, less the payout to shareholders that occurred during the same time interval.

Similar to safety stock (*s,S*) models, as described in, e.g., Dixit (1993), the corporate liquidity policy consists of four decisions: a) when should the firm pay out cash to equity-holders, b) how much cash should the firm pay out, c) when should the firm ask for external financing, and d) how much external financing should the firm get. The liquidity policy is thus summarized by a two barrier policy, the payout barrier, and the external financing barrier. When the level of cash, C_t, reaches the upper threshold, $\overline{C}$, the firm pays out an amount of cash, equal to ν, to equityholders; and the level of cash jumps from $\overline{C}$ to $\overline{C} - \nu$. Similarly, when the level of cash drops to a lower threshold, $\underline{C}$, equity is issued, and an amount of cash, equal to m, flows into the firm, and the level of cash instantaneously jumps from $\underline{C}$ to $\underline{C} + m$. Note that the equity issuance threshold $\underline{C}$ is naturally bounded by zero, as inaccessibility to debt markets would contradict the presence of a credit line.

In order to examine the effects of the effectiveness of monitoring and managerial entrenchment separately, but also jointly, we analyze four cases by alternating the presence of two frictions on top of the inherent cost-of-carry: equity issuance costs[2] and managerial entrenchment. These are sequentially developed below.

Issuance costs One of the frictions in the model is a positive fixed cost of external funding, denoted by ϕ. These cases are indexed by S. Algebraically, at any time t, the total cost of issuance dJ_t born by shareholders is equal to

$$dJ_t = dI_t + \phi\, 1_{dI_t>0} \tag{3}$$

where $1_{dI_t>0}$ is a indicator taking a value of 1 if shareholders inject funds in the firm and 0 otherwise.

Notice that the issuance cost function (3) does not include proportional (marginal) costs of issuance. These would normally represent the proportional component of a brokerage fee contract, but can also be interpreted as a reduced-form equivalent of adverse selection costs (Hennessy and Whited 2007). Technically, proportional costs are particularly useful in optimal control models to introduce a wedge between critical values (thresholds) of the state variable. For instance, removing the marginal issuance cost from the model of Décamps et al. (2011) would yield a limiting case, which is in fact case S of this paper, where shareholders inject cash into the firm up to the payout threshold, i.e. $\underline{C} + m \equiv \overline{C}$. The absence of the marginal issuance cost would imply a very high probability of a firm paying out dividends immediately after an equity issuance, which would deviate from a typical pattern documented in empirical research (Leary and Roberts, 2005).

Their absence is deliberate here and serves to highlight one of the main intuitions of this model. That is, deviation from optimal liquidity policy by introducing managerial

[2] In line with Décamps et al. (2011), we show below that, under optimal liquidity policy, the cost of carrying cash (θ) matters only in the presence of issuance costs.

entrenchment in the existing framework (discussed in detail below) creates a structural wedge between issuance proceeds and the payout threshold. In other words, and more importantly, our model allows the marginal value of cash to drop below one, i.e. the cost of injecting an additional dollar of cash in the firm can be higher than its benefit, even in a framework without proportional issuance costs.

Managerial entrenchment Another friction introduced in our framework is managerial entrenchment. The firm's liquidity policy is delegated to a self-serving manager who is able to tunnel the cost of carrying cash, $\theta\, C_t\, dt$, to her own benefit. Still, shareholders maintain the right to replace her whenever they see it fit. Upon managerial dismissal, i.e. at time τ^L, the firm is liquidated for an equivalent of $L\left(C_{\tau^L}\right)$. The liquidation function $L\left(\cdot\right)$ represents the supremum value among the subset of alternatives that shareholders have regarding their option value on the firm's assets. In other words, $L\left(\cdot\right)$ is the value of shareholders' best outside option.

Upon liquidation, shareholders receive the value of an equivalent firm, the expected cash flows of which may be lower than the respective expected cash flows of the firm under the current management ($\mu^L \leq \mu$). The managerial entrenchment parameter δ represents the gap in the firm's expected cash flows between the shareholders' two alternatives, such that $\mu^L = \mu - \delta$, where $0 < \delta < \mu$. Alternatively, this can be interpreted as a cost of managerial dismissal equal to the present value of an expected additional cash flow of δ. Additionally, the value of the firm under shareholders' next best alternative may suffer from a lower cost of carrying cash, i.e. $0 \leq \theta^L \leq \theta$. Without loss of generality, the volatility of cash flows and the refinancing costs, where applicable, remain the same under both alternatives. That is, $\sigma^L = \sigma$ and $\phi^L = \phi$.

To maintain the tractability of the model, we set the cost-of-carry of shareholders' alternative option to zero.[3] The interpretation of this assumption is that, upon dismissal,

[3]Or an infinitesimal quantity; see Appendix A.1

shareholders operate the firm themselves. Under their control, cash mismanagement due to managerial discretion over the firm's decisions disappears, i.e. $\theta^L = 0$.

Overall, managerial entrenchment results in the partial transfer of decision rights to a self-serving manager leading to liquidity policies that do not necessarily maximize shareholder's value. The allocation of decision rights stems from the implicit contract between the two parties. The manager holds control rights over the firm's cash with the shareholders maintaining the respective residual rights. This means that shareholders cannot explicitly force the manager to pay out dividends, but can only threaten to dismiss her. Symmetry of information being assumed, the manager does not pay out cash unless she knows that shareholders can materialize this threat, i.e. they have at least an equally valuable outside option. Equivalently, the manager cannot force refinancing nor has any control on the amount of cash to be injected in the firm when refinancing occurs. The implicit assumption is that the manager does not have more valuable alternatives for her human capital and thus cannot credibly threaten to terminate her contract with the shareholders. Given her lack of outside options, her optimal strategy simplifies to ensuring that she remains the shareholders' best option.

The manager is able to continuously extract a portion of cash reserves as private benefit until shareholders decide to replace her. After her removal, she is assumed to remain unemployed ever after. Normalizing her wage to zero, the managerial objective function can be expressed as

$$\max_{\overline{C},\,\nu} \mathbb{E}\left[\int_0^{\tau^L} \theta\, C_t\, e^{-rt} \mathrm{d}t\right], \tag{4}$$

where C_t is the regulated cash process described by (2).

Shareholders wish to maximize the present value of the total payout they expect to receive as dividends from the firm net of the present value of the total issuance costs they expect to bear plus the present value of their outside option. The total issuance cost is equal to the sum of the equity they will have to inject into the firm and the costs they will incur anytime they do so, as long as the manager keeps her position. The shareholders' objective function can thus be expressed as

$$\max_{\underline{C}, m, \tau^L} \mathbb{E}\left[\int_0^{\tau^L} (dU_t - dJ_t)\, e^{-rt} + L(C_{\tau^L})e^{-r\tau^L}\right]. \tag{5}$$

The liquidation function $L(C_t)$ is derived in the Appendix A.1.

In the sections that follow, we initially solve for the effect of these frictions on firm value and the distribution of cash reserves, and subsequently investigate more subtle effects on the value of cash and the volatility of stock returns.

3 Solution

In this section, we develop the solution for the four cases mentioned in Section 2 above. Initially, we expose the general framework that applies to all four cases and, subsequently, we elaborate on the conditions and solution of each particular case in a separate subsection.

For all cases, we distinguish three regions depending on the level of the state variable C_t: the equity issuance region, the inaction region, and the payout region. Suppose that the level of the firm's cash holdings is such that the firm is in the inaction region. If in the next time period Δt the firm remains in the inaction region, as shareholders realize only capital gains, the value to shareholders, $V^i(\cdot)$, [4] satisfies

$$V^i(C_t) = e^{-r\,\Delta t}\, \mathbb{E}_t\left[V^i(C_{t+\Delta t})\right] \tag{6}$$

Letting Δt decrease to an infinitesimal time increment dt yields

$$V^i(C_t) = (1 - rdt)\left[V^i(C_t) + \frac{\partial V^i(C_t)}{\partial C}\, dC + \frac{1}{2}\frac{\partial^2 V^i(C_t)}{\partial C^2}\,(dC)^2 + \ldots\right] \tag{7}$$

[4] Where $i = \{FB, S, M, SM\}$ is a case indicator

Expanding and substituting (2) into (7) obtains the following ordinary differential equation

$$\frac{1}{2}\,\sigma^2\,V^i_{CC} + \left[\mu + (r-\theta)\,C\right]\,V^i_C - r\,V^i = 0 \tag{8}$$

that defines the value to shareholders in the inaction region.[5]

The ODE (8) and positiveness of both r and σ result in the following lemma which is central to the determination of monotonicity and concavity of each case's value function. The proof of Lemma 1 is provided in the Appendix.

Lemma 1. *For any smooth function $V^i(x)$ satisfying the differential equation* (8)*, its second derivative V^i_{xx} has at most one root in the interval $\left[0, \overline{C}^i\right]$.*

Given the absence of a lump sum cost of paying out cash, we show below that the upper barrier, $\overline{C}$, is in fact a reflecting barrier, i.e. $\nu = 0$. That is, the firm pays out to equityholders anything above $\overline{C}$, as soon as the threshold is hit by the sum of the cumulated cash reserves and operating cash flow. The issuance threshold $\underline{\mathrm{C}}$ is zero, such that the firm never issues equity before it runs out of cash. Thus, the liquidity policy of the firm is in fact reduced to two decisions: a) the payout timing (threshold), and b) the amount of external financing (issuance proceeds).

3.1 First-best value

Before discussing the solution for the cases where issuance costs and/or managerial entrenchment apply, we present the solution for the frictionless first-best firm value. Given

[5]The interested reader can cross-check that the general solution to this ODE can be expressed as

$$V^i\,(C) = e^{-\left[\nu(C)^2 - \nu(0)^2\right]}\left[H_{-1-\frac{r}{r-\theta}}\left(\nu\,(C)\right)A_1 + {}_1F_1\left(\frac{1}{2}\left(1+\frac{r}{r-\theta}\right);\frac{1}{2};\nu\,(C)^2\right)A_2\right] \tag{9}$$

where $\nu\,(x) = \dfrac{\mu + (r-\theta)\,x}{\sigma\sqrt{r-\theta}}$, $H_n(x)$ is the n^{th} Hermite polynomial of x, ${}_1F_1(a;b;z)$ is the Kummer confluent hypergeometric function, and A_1 and A_2 are constants that need to be determined with the help of the respective case's boundary conditions.

that carrying cash is costly, i.e. $\theta > 0$,[6] the optimal shareholders' behavior would be to keep the minimum acceptable cash stock in the firm. In conjunction with the absence of issuance costs, the optimal solution for shareholders is to hold zero cash reserves, i.e. to distribute all positive cash flows and to match negative cash flows by issuing equal amounts of equity. In other words, in the first-best case, the inaction region is in fact eliminated as the issuance threshold coincides with the payout threshold. At time 0, i.e. for some random non-negative initial cash endowment of C, the value of the firm satisfies

$$V^{FB}(C) = \mathbb{E}_0 \left[\int_0^{+\infty} dC_t \right] + C = \frac{\mu}{r} + C \tag{10}$$

where dC_t is given by (2), and the value is simply the sum of a perpetuity of instantaneous returns of $\mu\, dt$ and the initial cash reserves which are instantaneously paid out to shareholders.

3.2 Case S: firm value for positive issuance costs

In this subsection, we examine the effect of fixed issuance costs by setting $\phi > 0$. In this case, hoarding cash increases firm value as the eventuality of incurring external issuance costs decreases with the size of the cash buffer. Therefore, in order to increase the time until reincurrence of fixed costs, the optimal cash policy consists of issuance proceeds being lumpy rather than infinitesimal amounts. On the other hand, the model does not involve any fixed costs to be paid every time that the firm decides to distribute cash to its shareholders, and thus the optimal payout policy consists of setting a payout threshold $\overline{C}^S$, above which all

[6] Notice from (10) that the cost-of-carry does not in fact affect the firm value.

subsequent cash flows are paid out to shareholders.[7] This condition can be expressed as

$$V_C^S\left(\overline{C}^S\right) = 1 \tag{13}$$

and be interpreted as the value to shareholders of an additional dollar of cash at the payout threshold is equal to one dollar, as this is the amount that would be paid out as dividend. In the absence of a self-serving manager, the optimal threshold is chosen optimally and thus satisfies the super-contact condition as, e.g., in Dixit and Pindyck (1994)

$$V_{CC}^S\left(\overline{C}^S\right) = 0 \tag{14}$$

Turning back to the optimal issuance policy, shareholders would delay equity issuance until the firm runs out of cash, i.e. the issuance threshold occurs at any time t where $C_t = 0$. At this point, shareholders replenish the firm's cash reserves up to $\tilde{C}$ and the value function satisfies

$$V^S(0) = V^S\left(\tilde{C}^S\right) - \tilde{C}^S - \phi \tag{15}$$

which simply indicates that the shareholders' value when the firm's cash has run out is equal to their value after the firm has replenished their cash inventory minus the amount paid (cash injected plus refinancing costs). Optimal issuance policy involves the smooth-pasting condition

$$V_C^S\left(\tilde{C}^S\right) = 1 \tag{16}$$

[7] The proof of the optimality of this policy goes by contradicting the optimality of any other payout policy: suppose instead that, whenever cash hit some resetting payout threshold l, the firm paid lumpy amounts of $l - k$ out to shareholders, such that the cash stock left after the payout is equal to k. According to this policy, the value function should satisfy

$$V^S(l) = V^S(k) + (l-k) \iff \frac{V^S(l) - V^S(k)}{l-k} = 1 \tag{11}$$

and the respective optimality conditions for this policy are

$$V_C^S(l) = V_C^S(k) = 1 \tag{12}$$

According to Lemma 1 and for C^* being the value of C for which $V_C(C)$ is minimized, this would mean that $k < C^* < l$ and $V_C^S(C) < 1$ for every $C \in (k, l)$, contradicting thus (11).

Grouping conditions (13)-(16) with (8) yields the following proposition, the proof of which is deferred to the Appendix A.2.2.

Proposition 1. *For positive costs of issuance and carrying cash, i.e. $\phi > 0$ and $\theta > 0$, it holds that*

1. *The value of the firm, $V^S(C)$, is an increasing and concave function of its cash stock C.*

2. *The marginal value of cash, $V^S_C(C)$, is strictly higher than 1 in the entire interval $\left[0, \overline{C}^S\right)$, and equal to 1 for $C = \overline{C}^S$.*

3. *Payout occurs when the accumulated cash reaches the payout threshold $\overline{C}^S$ which acts as a reflecting barrier, i.e. any positive cash flow after the barrier is reached is paid out to shareholders so that cash stock remains equal to $\overline{C}^S$.*

4. *Equity issuance occurs whenever the firm runs out of cash, at which point shareholders replenish the cash reserves up to the payout threshold $\overline{C}^S$, i.e. $\tilde{C} = \overline{C}$.*

3.3 Case M: firm value for delegated payout

In this subsection, we examine the case where there are no issuance costs, but the cost of carrying cash is a consequence of the firm being run by a self-serving manager. In particular, the manager sets payout threshold $\overline{C}^M$ and parameter θ can in this case be interpreted as the proportion of cash the manager can divert to her own interests.

The intuition of the model is that shareholders will dispose of the manager as soon as firm value under the next best alternative exceeds firm value under the current status. Given the absence of an outside option for herself, the manager will behave in such a way that ensures this will never occur. In terms of her objective function (4), she chooses $\overline{C}^M$ in such a way that τ^L becomes infinite. Algebraically, the manager will pay out dividends to shareholders at the first instance where the firm value under her management equals the value of shareholders' outside option, i.e.

$$V^M\left(\overline{C}^M\right) = L^M\left(\overline{C}^M\right) = \frac{\mu}{r} + \overline{C}^M \tag{17}$$

$$V^M(C) > L^M(C) \qquad \forall\, C \in \left[0, \overline{C}^M\right) \tag{18}$$

$\overline{C}^M$ being the highest possible cash stock, the amount that the manager can tunnel to her own benefit is at its maximum. Hence, paying a lump-sum dividend to shareholders would only decrease her perks in the next time increment. Therefore, as in the previous case, dividends would be paid out incrementally upon reaching $\overline{C}^M$, i.e.

$$V_C^M\left(\overline{C}^M\right) = 1 \tag{19}$$

As before, the value of one additional dollar of cash at the payout threshold increases the value of shareholders by exactly one dollar. Regarding the firm's issuance policy, shareholders unilaterally decide how much cash to inject in the firm. The issuance proceeds $\tilde{C}^M$ will be optimally chosen, such that they maximize shareholders value. Hence,

$$V^M(0) = V^M\left(\tilde{C}^M\right) - \tilde{C}^M \tag{20}$$

$$V_C^M\left(\tilde{C}^M\right) = 1 \tag{21}$$

In the Appendix A.2.3, we show that these last two conditions in fact coincide, and combined with the remaining conditions (17)-(19) yield the following findings compiled in the proposition below.

Proposition 2. *For a firm run by an entrenched self-serving manager, but in the absence of issuance costs, it holds that*

1. *The value of the firm, $V^M(C)$, is an strictly increasing function of its cash stock C, concave for $C \in \left[0, C^{*M}\right)$ and convex for $C \in \left(C^{*M}, \overline{C}^M\right]$, where $0 < C^{*M} < \overline{C}^M$.*

2. *The marginal value of cash, $V_C^M(C)$, is strictly lower than one in the entire interval $\left(0, \overline{C}^M\right)$, and equal to one for $C = 0$ and $C = \overline{C}^M$.*

3. *Payout occurs when the accumulated cash reaches the payout threshold $\overline{C}^M$ which acts as a reflecting barrier, i.e. any positive cash flow after the barrier is reached is paid out to shareholders so that cash stock remains equal to $\overline{C}^M$.*

4. *Equity issuance occurs whenever the firm runs out of cash at which point shareholders replenish the cash reserves just enough to avoid inefficient liquidation, i.e. $C = 0$ also acts a reflecting barrier.*

3.4 Case SM: Firm value for both positive issuance costs and delegated payout

In this subsection, we switch on both the issuance costs and delegation of payout to a self-serving manager, and examine their joint effect on firm value. Recall that the manager chooses to pay out dividends to shareholders as soon as the value difference between the manager-run firm and its alternative, the equivalent "shareholder-run" firm, reaches zero. Similarly, dividends are paid out incrementally whenever the sum of the cash stock and the inflow from operations exceed the payout threshold, $\overline{C}^{SM}$. This yields

$$V^{SM}\left(\overline{C}^{SM}\right) = L^{SM}\left(\overline{C}^{SM}\right) \tag{22}$$

$$V^{SM}(C) > L^{SM}(C) \qquad \forall\, C \in \left[0, \overline{C}^{SM}\right) \tag{23}$$

$$V_C^{SM}\left(\overline{C}^{SM}\right) = 1 \tag{24}$$

where $L^{SM}(C)$ is defined in Appendix A.1.

Shareholders unilaterally decide the firm's issuance policy and incur issuance costs every time they decide to replenish the firm's cash stock. Finally, the issuance proceeds $\tilde{C}^{SM}$ will be optimally chosen to maximize shareholders' value. Hence,

$$V^{SM}(0) = V^{SM}\left(\tilde{C}^{SM}\right) - \tilde{C}^{SM} - \phi \tag{25}$$

$$V_C^{SM}\left(\tilde{C}^{SM}\right) = 1 \tag{26}$$

In the Appendix A.2.4, we show that $\tilde{C}^{SM}$ is the lowest root of $V_C^{SM}\left(\tilde{C}^{SM}\right) - 1 = 0$ in the interval $\left[0, \overline{C}^{SM}\right]$ for the last condition to satisfy optimality. The results satisfying all conditions (22)-(26) are summarized in the proposition below.

Proposition 3. *For a firm run by an entrenched self-serving manager and facing issuance costs, it holds that*

1. *The value of the firm, $V^{SM}(C)$, is an increasing function of its cash stock C, concave for $C \in \left[0, C^{*SM}\right)$ and convex for $C \in \left(C^{*SM}, \overline{C}^{S}M\right]$, where $0 < C^{*SM} < \overline{C}^{M}$.*

2. *The marginal value of cash, $V_C^{SM}(C)$, is strictly higher than one in the interval $\left[0, \tilde{C}^{SM}\right)$, strictly lower than one in the interval $\left(\tilde{C}^{SM}, \overline{C}^{SM}\right)$ and equal to one for $C = \tilde{C}^{SM}$ and $C = \overline{C}^{SM}$.*

3. *Payout occurs when the accumulated cash reaches the payout threshold $\overline{C}^{SM}$ which acts as a reflecting barrier.*

4. *Equity issuance occurs whenever the firm runs out of cash, i.e. at $C = 0$, at which point shareholders replenish the cash reserves up to $\tilde{C}^{SM} < \overline{C}^{SM}$.*

4 Numerical implementation

In this section, we illustrate the propositions above and investigate the effect of the parameters on the liquidity policy of a firm using a numerical implementation. First, we set up a base case of SM the firm value and marginal value of cash of which we compare with their counterparts of cases FB, S, and M. In a second step, we focus our attention on how the model's parameters affect the liquidity policy and the marginal value of cash.

4.1 Base case parameter values

We set the base case values of the expected operating cash flow and its standard deviation parameters to $\mu = \sigma = 0.18$, suggesting a probability of a negative cash flow occurring of approximately $\Phi(-1) \approx 15.9\%$ per year.[8] The risk-free rate r is set to 6%, while the cost of carrying cash θ is set to 2%. The parameter capturing managerial entrenchment is set to $\delta = 0.9\%$, such that the expected cash flow of the shareholders' outside option $(\mu - \delta)$ is 95% of the expected cash flow under the incumbent management. Finally, we choose $\phi = 0.01$ to match the predictions of related studies (Bolton et al., 2011; Décamps et al., 2011) for the level of equity issuance $(\tilde{C})$.

4.2 Firm value and marginal value of cash

We start the numerical analysis by comparing the values of the four cases developed above. Figure 1 plots the four values as a fraction of the first-best case, V^{FB}. Solid lines represent the firm value in the inaction region, while dotted lines represent the value to shareholders of each case i given some initial cash endowment higher than the payout threshold (for all cases, $C_0 > \overline{C}^i$ triggers immediate dividend payments of $C_0 - \overline{C}^i$). The first-best value function, V^{FB} (blue line), is dotted throughout $\mathbb{R}_+$, as there are only costs associated with holding cash, and hence optimal cash reserves are zero.

[Please insert Figure 1 about here]

[8] In a related study, Décamps et al. (2011) set $\mu = 0.18$ and $\sigma = 0.09$, which implies that the respective probability of a negative cash flow is a modest 2.3% per year. We choose $\mu = \sigma$ to match better empirical observation: indicatively, the frequency of negative cash flows per firm with at least 10 consecutive annual observations in Compustat database is around 15%. In any case, our choice of $\sigma = 0.18$ has a mere expositional, but not qualitative, effect on the results exposed below.

The value function of case S (purple line) falls approximately 3.2% short of the first-best case for $C = 0$. As argued by Décamps et al. (2011), the presence of issuance costs decreases the firm value as it gives rise to positive cash reserves which are costly to maintain. In this model, we introduce a different friction, managerial entrenchment, which, reflecting the conflict of interest between shareholders and managers, also produces (costly) positive cash reserves.[9] Looking at the value of case M (yellow line), setting the cash flow wedge to $\delta = 0.05\mu$ results in a drop in firm value of approximately 4.7% for $C = 0$. Remember that this drop is purely due to agency costs as both firms, FB and M, have identically distributed operating cash flows, given by (1). The value that the manager extracts is less than $\frac{\delta}{\mu}$, with the difference representing the value of shareholders' option to replace her whenever they see fit. The solution for case M yields that cash reserves deviate from their otherwise optimal level of zero and can reach up to 13.4% of firm value (at $\overline{C}^M$).

Combining the two frictions, $\phi > 0$ and $\delta > 0$, yields case SM, illustrated by the light green line in Figure 1. Comparing initially to case S, the introduction of entrenchment decreases firm value through the suboptimal delay of dividend payments.[10] Specifically, the cash ratio at which payout takes place $\left(\frac{\overline{C}^i}{V^i(\overline{C}^i)}\right)$ increases from roughly 7% for case S to approximately 13.3% for case SM. Comparing next to case M, observe the convergence of the two firm values as the level of cash increases and the proximity of the two payout thresholds. Recalling that the difference between these two cases is the presence of issuance costs, Figure 1 yields a couple of interesting inferences. First, in the presence of managerial entrenchment, issuance costs have a small impact on the payout threshold; a result that is also confirmed in Subsection 4.4 below. Second, comparing the 3.2% change in value between FB and S with the respective 0.5% between M and SM, it can be argued that the (negative) effect of issuance costs on firm value is significantly mitigated by managerial entrenchment. Similarly, comparing the 4.7% change in value between FB and M with the

[9] Note that managerial entrenchment δ results in a deviation from the optimality conditions of cases FB and S.

[10] As explained in Section 2, case S can be thought of as a special case of SM where there is a infinite pool of equally good managers ready to take over the firm's cash management.

respective 2% between S and SM indicates that the value extraction due to managerial entrenchment shrinks with issuance costs.

[Please insert Figure 2 about here]

Finally, turning to the convexity of the functions, as solved for in Section 3, V^S is a strictly concave function of C, while both V^M and V^{SM} are concave at low values of C and convex at higher values of C. As the choice of base case parameters fails to sufficiently highlight these differences in convexity in Figure 1, we plot the first derivatives of the four functions with respect to C in Figure 2. The (non-)monotonicity of V_C^i for each case confirms the afore-mentioned solutions. Additionally, and maybe more importantly, note that the plotted V_C^i represent the marginal value of cash. The graph shows that optimality assumptions (case S) result in marginal values of cash strictly higher than one as, e.g., in Bolton et al. (2011) or Décamps et al. (2011). Introducing managerial entrenchment causes deviations from shareholders' optimality conditions at the payout threshold and allows marginal values of cash to drop below one. This implication seems to be a better fit to empirical evidence where marginal values of cash have been estimated at levels lower than one dollar (Pinkowitz et al., 2006; Dittmar and Mahrt-Smith, 2007).

4.3 Stock returns

We now turn to the examination of the model's implications about stock prices and returns. Given that shares are issued only when the firm runs out of cash, in the inaction region the instantaneous return satisfies

$$\frac{dS_t^i}{S_t^i} = \frac{dV_t^i}{V_t^i} \tag{27}$$

where S_t^i represents the stock price of each case i at time t. Applying Itō's lemma on the numerator of the right hand side, using (2), yields

$$dV_t^i = \left\{ \left[\mu + (r-\theta)\, C_t\right] \frac{\partial V_t^i}{\partial C_t} + \frac{1}{2}\,\sigma^2\, \frac{\partial^2 V_t^i}{\partial C_t^2} \right\} dt + \sigma \frac{\partial V_t^i}{\partial C_t^i} dW_t \tag{28}$$

Simplifying the term into brackets using (8) yields

$$dV_t^i = r\, V_t^i\, dt + \sigma\, \frac{\partial V_t^i}{\partial C_t}\, dW_t \tag{29}$$

Dividing both sides by V_t^i, substituting (27) and dropping the time subscript yields the following relation for instantaneous returns:

$$\frac{dS^i}{S^i} = r\, dt + \sigma\, \frac{V_C^i}{V^i}\, dW_t \tag{30}$$

where $\frac{V_C^i}{V^i}$ is the semi-elasticity of the firm value with respect to cash holdings. In the absence of frictions and given that the marginal value of one unit of cash is equal to one for FB, substituting (10) in (30) yields

$$\frac{dS^{FB}}{S^{FB}} = r\, dt + \frac{r\, \sigma}{\mu}\, dW_t \tag{31}$$

Equation (31) reveals that the volatility of returns for the first-best case is constant and equal to the volatility of operating cash flows times a factor $\frac{r}{\mu}$. The (non-)monotonicity of the volatility of returns for the remaining cases being challenging to determine analytically, we use the values of the base case parameters to plot the volatility of returns $\sigma_{\frac{dS^i}{S^i}} = \sigma\, \frac{V_C^i}{V^i}$ as a function of cash reserves in Figure 3.

[Please insert Figure 3 about here]

The graph confirms that Décamps' et al. (2011) result on the returns of case S being heteroscedastic also holds for both cases M and SM. Specifically, the volatility of stock returns decreases with the levels of cash. As $V^i(C)$ and hence $S^i(C)$ are both strictly increasing functions of C, the latter result implies that the volatility of stock returns decreases with the stock price, a phenomenon known as *asymmetric volatility* (Black, 1976; Christie, 1982). Complementing Décamps' et al. (2011) novel explanation for the phenomenon,[11] according to which asymmetric volatility can be attributed to costs of external financing, the monotonicity of the volatility of returns of case M indicates that this explanation can be potentially generalized to frictions giving rise to positive cash reserves, such as the conflict of interest between the manager and shareholders.

Having discussed the implications of each case's assumptions on firm and cash values, we turn next to the effect of the model's parameters on the liquidity policy and the marginal values of cash.

4.4 Issuance and payout policies

Figures 4 and 5 depict the comparative statics of key features of the firm's liquidity policy with respect to the model's parameters. In particular, the left panels illustrate the effect, in monetary units, on the cash policy by plotting the payout threshold, $\overline{C}$ (blue line), the equity issuance proceeds, $\tilde{C}$ (purple line), and the average cash, $\bar{c}$ (yellow line), captured by the mean of the ergodic stationary distribution of cash given the barrier $\overline{C}$ and $\tilde{C}$ and the cash dynamics as expressed in (2).[12] In the right panels, we extend the results to testable implications by expressing the same features as a ratio of firm value, a proxy of which (cash over book value) is typically used in related empirical research. We plot respectively the payout threshold as a function of the value of the firm at payout $\left(\frac{\overline{C}}{V(\overline{C})}\right)$ (blue line), the

[11] Prevailing theories for the causes of the asymmetric volatility phenomenon are the leverage hypothesis (Black, 1976; Christie, 1982) and the volatility feedback hypothesis (French, Schwert, Stambaugh, 1987; Campbell and Hentschel, 1992). The former attributes increases in volatility in market downturns to an increase in financial/operating leverage which results in higher risk for equityholders. The latter treats increases in volatility as exogenous shocks which in turn decrease stock prices.

[12] The ergodic distribution, and subsequently the average cash reserves are explicitly derived in the Appendix A.3.

equity issuance proceeds as a function of firm value at issuance $\left(\frac{\tilde{C}}{V(0)}\right)$ (purple line), and the cash-firm value ratio at the average level of cash $\left(\frac{\bar{c}}{V(\bar{c})}\right)$.

[Please insert Figure 4 about here]

The impact of changes in the main variables of interest, i.e. the cost-of-carry θ and managerial entrenchment δ, is illustrated in Figure 4. The top row shows the effect of the cost-of-carrying cash parameter, θ, on cash policy: a higher cost-of-carry results in lower payout thresholds (earlier payouts), higher cash injections when the firm issues new equity, and, on average, lower cash reserves. Intuitively, the larger the shortfall on cash, the lower the benefit that shareholders enjoy from the manager's presence. A lower contribution to firm value makes her tenure more insecure. In order to maintain her position, she commits to restricting her tunneling activity by paying out dividends to shareholders earlier. In the model's notation, this results in lower $\overline{C}$. On the other hand, a more intense tunneling activity increases the probability of generating losses and subsequently the probability for shareholders to incur issuance costs. To hedge for this risk, they issue a larger cash buffer when cash is needed; hence a higher $\tilde{C}$. Combining the above with a slower cash accumulation rate (2) results in lower average levels of cash, as captured by the decrease of $\bar{c}$.

The bottom row of Figure 4 depicts the changes in cash policy when varying the managerial entrenchment parameter δ. As the value of the shareholder's outside option decreases, the manager becomes more irreplaceable and can extract more rents from her decision-making position. In our model, this translates into a delayed dividend payout (higher $\overline{C}$), which can be alternatively interpreted as overinvestment in negative NPV projects. In the next subsection, we illustrate that this leads to lower marginal values of cash, i.e. the value of an additional dollar injected in the firm decreases, and hence proceeds from equity issuances are poorer. The effect being much more pronounced for the payout threshold as illustrated in Figure 4, higher managerial entrenchment leads to higher average cash reserves.

[Please insert Figure 5 about here]

The effects of the remaining parameters on the corporate liquidity policy are depicted in Figure 5. The top row illustrates the effect of varying the expected operating cash flows of the firm (μ). As the firm's profitability increases, the probability of the firm running out of cash decreases. This implies that a lower level of cash reserves provides the same insurance against incurring refinancing costs and hence less cash is injected in the firm at every issuance date. At the other end of the cash stock distribution, an increase in μ results in a drop of the payout threshold as well. In terms of the model, this is a consequence of holding δ constant,[13] which results in an erosion of the manager's relative bargaining position. The lower the manager's relative contribution to value, the lower the shareholders' tolerance towards the manager's perquisite extraction. This forces the manager to distribute cash to shareholders earlier, i.e. lower payout thresholds. Although both $\overline{C}$ and $\tilde{C}$ decrease, their effect on average cash is outweighed by the higher rate of cash accumulation resulting in an increase in average cash. The top right panel of Figure 5 indicates that the increase in firm value stemming from higher expected cash flows inverses this relation for average cash ratios.[14]

The second row depicts the results for changes in volatility of operating cash flows (σ). Higher volatility increases the frequency of refinancing and hence shareholders need a higher cash buffer to hedge against issuance costs, leading to a higher target barrier $\tilde{C}$. Although the latter consistently increases with volatility, observe that the payout threshold, $\overline{C}$, has a non-monotonic relation to this parameter. As volatility increases from low levels, payout occurs at higher levels of cash because shareholders are more willing to allow the manager to keep more cash into the firm, i.e. the shareholders' outside option decreases more quickly in value than the value of the "managed" firm. However, as volatility increases further, so

[13] Modeling entrenchment in monetary units rather than a proportion of expected operating cash flows matches common perception that managers make a stronger impact in less profitable firms. Alternatively, this can be interpreted as higher competition among managers in more profitable industries.

[14] Algebraically, the elasticity of firm value with respect to μ is higher than the respective elasticity of $\bar{c}$.

does the amount of equity that shareholders are willing to inject into the firm in order to avoid incurring financing costs in the near future. This results in higher costs due to the managerial expropriation of cash, making shareholders less tolerant towards this behavior, and triggers earlier payouts. Lastly, all else equal, the overall effect of volatility on average cash holdings is negative as the divergence of the mean of the distribution from the payout outweighs the increase in the payout threshold.

The latter result intriguingly contradicts existing empirical evidence (Opler et al., 1999; Bates et al., 2009). Deviating from a *ceteris paribus* environment, a potential explanation is that increases in volatility might be associated with increases in the discount rate[15] which in turn lead to higher cash ratios (discussed below). Furthermore, in the real world, managerial behavior can deviate from the assumptions binding this model by occasionally tunneling less than θ (leading to higher cash ratios)! An extended version of the model that allows the manager to freely choose her tunneling activity could potentially yield yet more realistic predictions and is left for further research.

Varying the issuance costs (ϕ) yields the results illustrated by the third row of Figure 5. Costlier equity issuance leads to larger blocks of equity being issued at refinancing, earlier dividend payouts, and lower average cash ratios. The interpretation of these results closely follows the rationale for high levels of volatility discussed above: higher costs of issuance induce larger cash injections to decrease the probability of (costlier) refinancing; optimal refinancing being bounded by the manager's tunneling activity leads to lower firm values and induces the latter to compensate shareholders by paying out cash earlier.

Lastly, the bottom row depicts the results for changes in the risk-free rate (r). An increase in the risk-free rate increases the speed of cash accumulation and therefore the amount of perks that the manager can extract from his position of power. This increases the cost of retaining the manager who is thus forced to distribute dividends at lower levels of cash

[15]This could be the case if the increase in volatility is related to an increase in systematic risk. As here agents are assumed to be risk-neutral, this effect cannot be captured by this version of the model. Future research could match this prediction by breaking down cash flow volatility in market and idiosyncratic components.

reserves to hold her position. The effect on the equity issuance proceeds being negligible, the higher speed of accumulation seem to counterbalance the decrease of the payout threshold as average cash remains constant. Notice that the empirically observable effect (depicted on the right panel) is predicted to be substantially positive for all three features of the cash stock distribution since an increase in r results in a considerable drop in firm value.

4.5 The value of cash

In this section, we examine how the parameters of our model affect the value of cash. For each parameter, we initially discuss how changes in its value impact on the marginal value of cash over the range $\left[0, \overline{C}\right]$. To this end, we let them vary by 50% on either side from their base case value and comment below on the results which are illustrated in the left panels of Figures 6 and 7. As both the range and the shape of the cash holdings' distribution vary themselves with parameter values (see Section 4.4), this exercise alone does not suffice to make clear predictions about the overall effect on the value of cash. Hence, we also examine the effect of the model's parameters on the *average* marginal value of cash. To do so, we use the probability density function of cash stock, $f^{SM}(C)$, derived in the Appendix A.3, to weigh the marginal value of cash at each level within the range $\left[0, \overline{C}\right]$. Integrating over the entire range yields the average marginal value of a unit of cash:

$$\overline{V_C}^{SM} = \int_0^{\overline{C}^{SM}} f^{SM}(C) V_C^{SM}(C) \, \mathrm{d}C \tag{32}$$

The results of this exercise are plotted in the right panels of Figures 6 and 7.

Similar to the previous subsection, we start by discussing the effect of the two main parameters of this study, θ and δ, on the value of cash, as illustrated in Figure 6. As the graphs of the first row reveal, the dynamic aspect of our model yields a novel result on the relation between the manager's tunneling activity and the value of a unit of cash: incorporating agency considerations, even in a parsimonious way, unveils a dual effect of

θ on the value of cash. Naturally, as in cases considering optimal liquidity policy,[16] higher tunneling activity reduces the value of cash altogether. This *direct* effect simulates the moral hazard problem in information asymmetry environments à la Jensen (1986). Nevertheless, in a model allowing for deviations from payout optimality, higher tunneling activity also decreases the marginal contribution of the manager to firm value, forcing earlier payouts. This *indirect* effect yields higher values of cash not only close to the upper threshold, but over the entire support of cash reserves: given that keeping the manager in place is shareholders' best option in this study, the slower accumulation of cash stock due to higher tunneling activity makes every marginal unit of cash more valuable than otherwise; enhancing its value even for low cash reserves, i.e. close to refinancing.

[Please insert Figure 6 about here]

This twofold effect of θ is better illustrated by incorporating the changes in the levels of cash, as displayed in the top right panel of Figure 6. Scaling the marginal value for each cash level by the probability of observing this level results in a U-shaped relation between θ and the average marginal value of cash $\overline{V_C}^{SM}$. Increasing θ from low to intermediate levels leads to a decrease in the value of cash, i.e. the negative (*direct*) effect of the manager's tunneling activity dominates. A further increase though from intermediate to high levels of θ considerably tightens the time until dividend distributions and the value stemming from the decrease of the manager's bargaining power (*indirect* effect) overweighs the negative effect, leading to higher values of cash on average.

This result is further documented by isolating the effect of the manager's bargaining power, i.e. the entrenchment parameter δ. The left panel of Figure 6 illustrates the decrease in the value of cash for all cash stock levels. As δ does not affect the cash accumulation process (2), the only impact on the value of cash stems from the deferral of payouts, allowing

[16] Décamps et al. (2011) —alike to untabulated results of case S above— predict that an increase in the cost of carrying cash leads to lower marginal values of cash.

the manager to extract higher perks in the long-run. This leads to a unit of cash contributing less to meeting the payout threshold, thus lowering its marginal value and subsequently the proceeds from equity issuance $\tilde{C}$. The bottom right panel depicts the overall negative effect of entrenchment on the value of cash, reflecting its detrimental effect on firm value.

Combining the two results above hints to interesting implications for corporate governance policies. Specifically, the non-monotonicity of the relation between the cost of carrying cash and the value of doing so highlights the superiority of external corporate control mechanisms over internal monitoring procedures in cash value creation. Tightening the monitoring of managerial actions (i.e. lowering θ) would result in less corporate resources being wasted on unprofitable projects. Although one extra dollar of cash reserves is more valuable to shareholders in this case, it may contribute less on average to bridging the gap until payout as the manager would still try to cash in as much of her comparative advantage by hoarding more resources within her reach ($\frac{\partial \overline{C}}{\partial \theta} < 0$). On the other hand, adding credibility to an irreversible replacement threat (i.e. decreasing δ) acts as a disciplining mechanism forcing the manager to self-restrict her tunneling activity[17] through earlier payouts ($\frac{\partial \overline{C}}{\partial \delta} > 0$). Consequently, the relative contribution of an extra dollar of cash to the reduction of the time until payout, and hence its value, increases.

[Please insert Figure 7 about here]

The effects of the productivity parameters on the value of cash are depicted in Figure 7. The graphs on the first row illustrate the results for varying the expected cash flows of the firm μ. As an increase in μ results in a higher cash accumulation speed, the contribution of an additional unit of cash to the prevention of incurring issuance costs decreases; and hence so does its value closer to the refinancing threshold. On the other hand, a *relative* decrease in managerial entrenchment leads to earlier payouts, and the value of one unit of cash increases

[17] In a way similar to a manager's choice of debt issuance as a voluntary self-constraint in Zwiebel's (1996) capital structure model.

in the proximity of the payout threshold. The top right panel of Figure 7 indicates that the latter effect dominates overall, reflecting the beneficial effect of μ to firm value.

The second row of Figure 7 treats the cash value implications of a change in cash flow volatility. Higher variability of cash flows increases the risk of incurring issuance costs in the near future, increasing thus the marginal value of one unit of cash close to the refinancing threshold. On the other hand, as shown in Section 4.4, the payout threshold has an inverse U-shaped relation to volatility, which leads to a negative relation between the marginal value of cash and volatility towards the higher end of the cash stock range. As depicted on the right panel of the second row, the aggregation of these forces yields an overall U-shaped relation between σ and the average marginal value of cash.

Varying ϕ returns a straightforward relation to values, as costlier issuances increase the value of a unit of cash kept inside the firm. Lastly, the bottom row of Figure 7 reveals that, in line with predictions regarding the issuance and payout policies, an increase in the risk-free rate r has an minimal positive effect on the value of cash, as it induces earlier payouts and, equivalently, restricts the tunneling activity of the manager.

5 Risk management

In this section, we conjecture how the different claimants of the firm's cash flows (shareholders and the manager) would behave if faced with the opportunity to reduce (hedge) or amplify (gamble on) their risk exposure. Although this section's results are based on somehow heuristic arguments, they follow in spirit and hence are comparable to ones stemming from a more technical approach, such as the one implemented in Bolton et al.(2013) or Hugonnier et al. (2014).

As pointed out in Proposition 3, the value to shareholders is concave when cash reserves are low (from 0 to C_S^*)[18] and convex when cash reserves are high (from C_S^* to $\overline{C}$). This implies

[18]For ease of exposition, C^{*SM} is renamed to C_S^* for the remainder of this section.

that shareholders would prefer to adopt risk-averse strategies in low states and risk-loving strategies in high states of the cash variable. Hence, if a frictionless futures contract whose price is a Brownian Motion uncorrelated to the one driving the firm's cash flows (W) were available, they would like the firm to enter a short position in $(0, C_S^*)$ and a long position in $\left(C_S^*, \overline{C}\right)$. Intuitively, shareholders would like to reduce cash flow volatility when incurring issuance costs is more likely and the cost of holding cash $(\theta\, C)$ is low, i.e. for cash levels close to zero. However, as cash increases and the probability of running out of cash decreases, the cost from the manager's tunneling activities grows and considerably delays payout. Hence, at high levels of cash, it can be profitable for shareholders to increase cash flow volatility: a positive outcome would lead to earlier payout, while a negative outcome would be mitigated by a reduction of the cash return shortfall from tunneling.

The strategy described above would be implemented only if the control rights of the firm's risk management policy lay with shareholders. If, however, the latter was also delegated to the manager together with the payout policy, the risk management strategy of the firm would differ. In order to determine the strategy that the manager would choose, one needs to characterize her value function. Letting $M\,(C)$ denote the value function of the firm's manager, it should satisfy

$$M\,(C_t) = \theta\, C_t\, \Delta t + e^{-r\,\Delta t}\, \mathbb{E}_t\left[M\,(C_{t+\Delta t})\right] \tag{33}$$

which, similar to the procedure used in Section 3, results in the ordinary differential equation

$$\frac{1}{2}\,\sigma^2\, M_{CC} + \left[\mu + (r-\theta)\, C\right]\, M_C - r\, M + \theta\, C = 0 \tag{34}$$

This ODE is subject to two conditions stemming from our setup. The first one reflects that payout occurs at $\overline{C}$; at this point, adding a unit of cash returns no additional value to the manager as the entire unit is paid out as dividend to shareholders. Hence,

$$M_C\left(\overline{C}\right)=0$$

The second condition relates to refinancing. As soon as the firm runs out of cash, the reserves are replenished up to $\tilde{C}$ and the manager's value function satisfies

$$M(0)=M\left(\tilde{C}\right)$$

Combining these two conditions with the ODE (34) yields the following proposition, the proof of which can be found in the Appendix.

Proposition 4. *For a firm facing equity issuance costs and run by an entrenched self-serving manager, it holds that*

1. *The value to the firm's manager, $M(C)$, is U-shaped with respect to cash stock C, i.e. decreasing in $\left[0,\hat{C}\right)$ and increasing in $\left(\hat{C},\overline{C}\right)$*
2. *$M(C)$ is convex for $C\in[0,C_M^*)$ and concave for $C\in\left(C_M^*,\overline{C}\right]$, where $\hat{C}<C_M^*<\overline{C}$.*

Proposition 4 reveals the risk management preferences of the manager with respect to the level of cash holdings. For low levels of cash, the manager would benefit from a (temporary) increase in the volatility of cash flows as hitting the downward bound of cash reserves results in refinancing and, hence, an upward jump in her payoffs. For high levels of cash, the manager's return from tunneling activities approaches its upward bound (the payout threshold) and increases her willingness to reduce the probability of low states reoccuring. Hence, in the presence of the same frictionless futures contract as above, the manager would like the firm to hold a long position in $[0,C_M^*)$ and a short position in $\left(C_M^*,\overline{C}\right]$. Hence, in case the risk management strategy is also delegated to the firm's manager, she would be more likely to gamble when cash reserves are low and hedge when these are high.

Summarizing the analysis above, shareholders would be willing to increase the firm's hedging position when cash holdings lie in $[0, C_S^*)$ and lower it for cash levels in $\left(C_S^*, \overline{C}\right]$; while managers would prefer to reduce the firm's hedging activities when cash holdings lie in $[0, C_M^*)$ and conversely for cash levels in $\left(C_M^*, \overline{C}\right]$. Hence, there are conflicting preferences among the two parties towards the boundaries of the no-payout region and different risk management strategies could be adopted depending on the allocation of control rights. Interestingly, this is not necessarily the case towards the center of the no payout region as there might be an alignment of the risk preferences of both parties. Depending on the sign of the difference $(C_S^* - C_M^*)$, hedging (positive) or gambling (negative) may be benefiting both parties; in the case where $C_S^* = C_M^*$, there is disagreement on the risk management policies across the entire range $\left[0, \overline{C}\right]$.

Applying the above to our numerical base-case scenario, we find that $C_S^* = 0.283$ or a cash ratio of 9.04%, while $C_M^* = 0.209$, i.e. a cash ratio of 6.84%. Hence, even if cooperation between both parties is needed so that a temporary change of cash flow volatility is implemented,[19] there is a range of cash ratios where both claimants of our base-case firm would benefit from hedging. Extended numerical tests reveal that this result (i.e. $C_M^* < C_S^*$) remains throughout the whole range of parameter values.

[Please insert Figure 8 about here]

In Figure 8, we plot the hedging thresholds for shareholders (C_S^*) and management (C_M^*) against the parameters of main interest, θ and δ, in order to examine their effect on the

[19] A more formal way to evaluate the volatility switching points would involve allocating the control of one switch to each party, i.e. the upper threshold of the hedging region is chosen by shareholders while the lower threshold by management. The firm values under the two regimes (hedging/no hedging) can be calculated using a position in a futures contract as in Bolton et al. (2011) and Hugonnier et al. (2014) or a choice among two predetermined volatility levels as in Leland (1998). This implementation could yield more accurate results on the determination of the switching points and is left for future research. The more heuristic approach presented in this section is adequate to highlight the existence of a region where hedging is beneficial for both shareholders and management.

hedging propensity of firms where both parties have to jointly agree on the risk management strategy. Given that shareholders would prefer to hedge for cash holdings less than C_S^* and gamble otherwise, and that management would gain from hedging for cash levels above C_M^* and increase risk otherwise, the shaded area in both sides of the panel indicates the range of cash levels for which a firm would benefit from hedging. We scale both thresholds by the payout threshold $\overline{C}$ in order to illustrate the portion of the ergodic distribution of cash that the hedging range occupies. The left panel of Figure 8 shows that both thresholds approach the payout threshold as θ increases, but at different rates such that the range of values for which hedging would occur is wider. Conversely, the right panel of Figure 8 reveals that, as δ increases, the two thresholds not only approach zero, but also converge, i.e. the portion of the cash distribution for which the firm would hedge is narrower.[20]

The results of the analysis above are best summarized in the form of empirically testable implications. First, *all else equal*, risk management (hedging) activities have a hump-shaped relation to cash reserves; that is, the firm engages in risk-reducing strategies for moderate levels of cash but not in the proximity of refinancing- or payout-triggering levels. This prediction deviates from the traditional view of substitutability between liquidity and risk management (e.g. Bolton et al., 2011) as in our model these policies result from an interplay between the manager's and shareholders' preferences. Although the no-hedging strategy close to payout is straightforward and consistent with the literature, the one close to refinancing can be explained as follows. Contrary to the common assumption of managerial risk-aversion (Stulz, 1984; Smith and Stulz, 1985), the manager in our model is willing to engage in risk-increasing activities at low values of cash. Similar to the argument in Morellec and Smith (2007) regarding the relation of hedging and leverage, if left uncontrolled she would in fact gamble as her opportunity cost is quite low for cash levels close to refinancing, i.e. there are scarce resources for her to extract value from. As long as the risk management policy

[20]Note that although the distribution of cash is not uniform, it is negatively skewed for all parameter values examined in this study with the mode being at the truncation (payout) threshold. Hence, the probability density function increases with the level of cash.

needs to be collectively decided between shareholders and the manager, this risk-shifting propensity will not materialize leading to the hump-shaped relation described above.

The model also yields predictions regarding the relation between risk management policies and agency considerations. In particular, hedging activities are decreasing in the effectiveness of monitoring managerial activities, or more generally, and consistently with previous literature (e.g. DeMarzo and Duffie, 1991; Breeden and Viswanathan, 1998; DaDalt, Gay, and Nam, 2002), increasing in information asymmetry. This corporate policy implication can be better explained through the results of Section 4.5. Remember that the marginal value of a dollar of cash decreases with monitoring effectiveness. This reflects the fact that holding cash becomes more costly for shareholders, which naturally reduces their incentives for risk management. Furthermore, the model predicts that firms operated by more entrenched managers will engage less often in hedging activities. Results of Section 4.5 reveal that the marginal value of cash decreases with managerial entrenchment. Similar to the argumentation above, this increases the risk tolerance of shareholders and reduces their propensity to hedge.

We conclude this section on risk management with some remarks relative to the empirical testing of these predictions. Similar to results in Section 4.4 regarding cash levels, notice that two manifestations of poor (good) corporate governance, ineffective (effective) monitoring and high (low) managerial entrenchment, have conflicting effects on the firm's propensity to hedge. Hence, outcomes of empirical tests involving aggregate measures of corporate governance can very well be mixed. In this light, it comes as little surprise that empirical evidence on the effects of corporate governance on hedging is inconclusive.[21] Moreover, these two aspects of corporate governance can be significantly correlated to each other and both are shown here to affect hedging preferences; hence, unless both are adequately controlled for in an empirical specification, results are prone to omitted variable biases. Finally, note that an omission of either monitoring effectiveness and managerial entrenchment would violate the *ceteris paribus* clause of the hump-shaped relation between hedging and cash levels. Given

[21]See Aretz and Bartram (2010) for a comprehensive review on the current standing of empirical literature.

that these are driving both cash levels and hedging propensities, such an omission could yield a biased empirical result of complementarity between liquidity and risk management policies.

6 Conclusion

In this paper, we design a model of cash management where the payout decision is delegated to an entrenched manager. Shareholders retain the refinancing decision and hold a perpetual option to dismiss the manager whenever they see fit. The trade-off between the manager's self-serving tunneling activities and the shortfall in expected cash flows that her dismissal ensues yields novel empirical predictions about liquidity and risk management policies.

Two conflicting notions of corporate governance, the effectiveness of monitoring of managerial actions and managerial entrenchment, have a positive effect on corporate cash reserves. Since these negatively correlated facets of corporate governance affect cash levels in the same direction, empirical tests involving aggregate measures of corporate governance can be expected to generate mixed results. Our numerical implementation reveals that the relation between the effectiveness of monitoring and the value of cash is U-shaped indicating that restraining the manager's leeway does not necessarily increase the contribution of a dollar of cash to total firm value. Precisely, more effective monitoring leads to later payout thresholds, a delay which can overweigh the benefit from limiting the managerial tunneling activity. Between managerial entrenchment and the value of cash, the relation is strictly negative. Hence, in terms of cash value creation, external corporate control mechanisms, such as takeover threats, trump internal monitoring procedures.

Dividing control rights on risk management policies between shareholders and managers produces new testable predictions. The model yields a novel hump-shaped relation between liquidity and hedging activities. This result stems from the conflicting risk preferences of shareholders and the manager; the risk tolerance of the former increases with cash levels, while the latter's decreases. Regarding the effect of corporate governance, our results reveal that hedging is negatively related to both the effectiveness of monitoring managerial activities and managerial entrenchment. Similar to the empirical implications regarding the effect of corporate governance measures on cash, colliding evidence on the governance-hedging relation can be anticipated.

Interestingly, the introduction of managerial entrenchment and the separation of control rights yields a substantial range of cash levels where the marginal value of cash drops below one, a result that matches related findings of the empirical literature more closely than standard cash accumulation models. Hence, our framework returns realistic levels of equity issuance proceeds which could not otherwise be generated in a model without marginal costs of refinancing. This feature highlights the informativeness of equity issuance proceeds and, by relieving the model from a hardly interpretable proxy of adverse selection costs, can benefit structural modeling research to quantify the particularly unobservable notions of monitoring and entrenchment.

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A Appendix

A.1 Liquidation functions

In the absence of both costs of issuance and carrying cash, shareholders are indifferent between keeping cash in or out of the firm. For costly external funding, an agency cost free firm would have no reason to pay out cash, as every additional dollar of cash reduces the probability of incurring refinancing costs. Nevertheless, this results in a conceptual paradox as, if payout never occurs, the present value of expected dividends, and thus the value to shareholders, is zero. This issue can be overcome by setting the cost-of-carry to an infinitesimal amount $\epsilon \leq dt$. Hence, the cash inventory of a shareholder-run firm evolves according to

$$dC_t = [\mu - \delta + (r - \epsilon) C_t]\, dt + \sigma dW_t + dI_t \tag{A.35}$$

$L^M(C_t)$: liquidation function of case M As with case FB, any positive cost of carrying cash results in both the issuance and payout threshold to be set to zero and the liquidation function in the absence of issuance costs is simply

$$L^M(C) = \frac{\mu - \delta}{r} + C \tag{A.36}$$

$L^{SM}(C_t)$: liquidation function of case SM For ease of exposition, we heuristically determine the payout threshold of the shareholder-run firm. As cash increases the probability of incurring the costs of external funding decreases. Eventually, the marginal benefit of holding one unit of cash drops to ϵ and the firm pays out dividends to shareholders. As $\epsilon \to 0$, $\overline{C}^L \to +\infty$. On the other hand, shareholders would delay paying the costs of external funding as much as possible and $\underline{C}^L = 0$. In the inaction region, the value of the shareholder-run firm's equity, $L^{SM}(\cdot)$, obeys at time t

$$L(C_t) = e^{-r\,\Delta t}\, L(C_{t+\Delta t}) \tag{A.37}$$

Letting Δt approach zero and expanding the right-hand side as a Taylor's series obtains

$$L = (1 - rdt)\left[L + \frac{\partial L}{\partial C} dC + \frac{1}{2}\frac{\partial^2 L}{\partial C^2}(dC)^2 + \dots\right] \tag{A.38}$$

where the subscripts denote partial derivatives. Substituting (A.35) into (A.38) and letting terms lower than dt^2 go to zero obtains

$$\frac{1}{2}\sigma^2 L_{CC} + [\mu - \delta + rC_t]\; L_C - r\,L = 0 \tag{A.39}$$

The value of the liquidation function is determined using two conditions. The first stipulates that dividends are paid out to shareholders incrementally once the payout threshold has been reached, i.e.

$$\lim_{\overline{C}^L \to \infty} L_C\left(\overline{C}^L\right) = 1. \tag{A.40}$$

When the firm runs out of cash, shareholders need to replenish its cash reserves by an amount m. Since there is no marginal cost of issuing equity, shareholders will replenish the cash stock up to the payout threshold. The second condition is thus

$$L^{SM}(0) = \lim_{m \to \infty} \left[L^{SM}(m) - m\right] - \phi. \tag{A.41}$$

Combining (A.39) with (A.40) and (A.41) yields

$$L(C) = \frac{\mu - \delta}{r} + C - \phi \frac{M\left[E\left(C\right)\right]}{M\left[E\left(0\right)\right]} \tag{A.42}$$

where

$$E\left(x\right) = \frac{\mu - \delta + rx}{\sigma\sqrt{r}}$$
$$M\left(x\right) = e^{-x^2} - x\sqrt{\pi}\left[1 - \operatorname{erf}\left(x\right)\right]$$

A.2 Proofs

A.2.1 Proof of Lemma 1

Differentiating (8) by C yields

$$\frac{1}{2}\sigma^2 V_{CCC} + \left[\mu + \left(r - \theta\right) C\right] V_{CC} - \theta V_C = 0 \tag{A.43}$$

where the case identifier has been dropped for convenience. Assume V_{CC} has at least one root, and let the first one in $\left[0, \overline{C}\right]$ being denoted by C^{*j}. Following (A.43), it holds that

$$\frac{1}{2}\sigma^2 V_{CCC}\left(C^{*j}\right) = \theta V_C\left(C^{*j}\right) \Longrightarrow \operatorname{sgn}\left[V_{CCC}\left(C^{*j}\right)\right] = \operatorname{sgn}\left[V_C\left(C^{*j}\right)\right] \tag{A.44}$$

Hence,

- If $V_C\left(C^{*j}\right) > 0$, then C^{*j} is a minimum, and
- If $V_C\left(C^{*j}\right) < 0$, then C^{*j} is a maximum.

Suppose that $V_C(C^{*j}) > 0$ and thus $V_{CCC}(C^{*j}) > 0$. Then, for some $\epsilon > 0$, it holds that $V_{CC}(C^{*j} + \epsilon) > 0$, precluding $V_{CC}(C)$ being constant at zero thereafter.[22] Denoting by C^{*k} the next root of V_{CC} in $\left[0, \overline{C}\right]$, it holds that $V_{CC}(C) > 0$ in $\left(C^{*j}, C^{*k}\right)$ and thus $V_C\left(C^{*k}\right) > V_C(C^{*j}) > 0$. But, for C^{*j} to be the next root, $V_{CCC}\left(C^{*k}\right) \leq 0$ has to hold, and therefore $V_{CCC}\left(C^{*k}\right) \leq 0 < V_C\left(C^{*k}\right)$ which contradicts (A.44). The proof for $V_C(C^{*j}) < 0$ is similar and therefore omitted. ■

A.2.2 Proof of Proposition 1

Given conditions (13) and (14) and following Lemma 1, $\overline{C}^S$ is the minimum of $V_C^S(C)$, i.e. $\overline{C}^S \equiv C^{*S}$. It follows that $V_C^S C(C) < 0$, and thus $V_C^S(C) > 1$ for $C \in \left[0, \overline{C}^S\right)$. Hence, condition (16) can be satisfied if and only if $\tilde{C}^S \equiv \overline{C}^S$. This in turn implies that $V^S(C)$ is strictly increasing and concave in the entire interval $\left[0, \overline{C}^S\right]$. ■

A.2.3 Proof of Proposition 2

Assume that $\tilde{C}^M$ is located within the range $\left(0, \overline{C}^M\right)$ and thus condition (21) holds. Given Lemma 1 and the fact that $V_C^M\left(\overline{C}^M\right) = V_C^M\left(\tilde{C}^M\right) = 1$, then $\tilde{C}^M < C^{*M} < \overline{C}^M$ and $V_C^M(C) > 1$ in $\left[0, \tilde{C}^M\right)$. Integrating the last inequality over the range $\left[0, \tilde{C}^M\right]$ yields

$$\int_0^{\tilde{C}^M} V_C^M(C)\,\mathrm{d}C > \int_0^{\tilde{C}^M} \mathrm{d}C \Longrightarrow V_C^M\left(\tilde{C}^M\right) - V_C^M(0) > \tilde{C}^M$$

which contradicts condition (20). Hence, $\tilde{C}^M \in \left\{0, \overline{C}^M\right\}$. The optimal issuance policy is chosen by shareholders whenever they are about to replenish the cash reserves. As this happens at $C = 0$, shareholders want to maximize $V^M(0)$ with respect to their choice variable, $\tilde{C}^M$. Thus, in this case, their objective function (5) can be expressed as

$$\max_{\tilde{C}^M \in \left\{0, \overline{C}^M\right\}} V^M(0)$$

Noting that the manager's choice of $\overline{C}^M$ changes with the choice of $\tilde{C}^M$, we denote by $\overline{C}_1^M$ the payout threshold if $\tilde{C}^M \equiv \overline{C}^M$ and by $\overline{C}_2^M$ if $\tilde{C}^M \equiv 0$. Following (20), the former choice yields $V^M(0) = V^M\left(\overline{C}_1^M\right) - \overline{C}_1^M$. Substituting (17) obtains $V^M(0) = L^M(0)$.

If instead shareholders choose to replenish enough equity to keep cash reserves non-negative, (20) yields $V_C^M(0) = 1$ and, following Lemma 1, $V_C^M(C) < 1$ in $\left(0, \overline{C}_2^M\right)$. Integrat-

[22]Note that the special case where $V_{CCC}\left(C^{*j}\right) = V_{CC}\left(C^{*j}\right) = 0$ would imply, based on (A.43) and (8), that also $V_C\left(C^{*j}\right) = V\left(C^{*j}\right) = 0$, which does not apply to our setup.

ing the latter inequality over the range $\left[0, \overline{C}_2^M\right]$ obtains $V^M(0) > V^M\left(\overline{C}_2^M\right) - \overline{C}_2^M = L^M(0)$, which makes $\tilde{C}^M \equiv 0$ the optimal equity issuance.

A.2.4 Proof of Proposition 3

Condition (25) implies $0 < \tilde{C}^{SM} \leq \overline{C}^{SM}$ and, in conjunction with Lemma 1, $V_C^{SM}(C)$ is either strictly decreasing or U-shaped in $\left[0, \overline{C}^{SM}\right]$. If $\tilde{C}^{SM} \equiv \overline{C}^{SM}$, (25) yields

$$V^{SM}(0) = V^{SM}\left(\overline{C}^{SM}\right) - \overline{C}^{SM} - \phi$$

Substituting (22) and simplifying yields

$$V^{SM}(0) = L^{SM}(0) - \phi\, \frac{M\left[E\left(\overline{C}^{SM}\right)\right]}{M\left[E\left(\overline{C}^{SM}\right)\right]} < L^{SM}(0)$$

which contradicts (23).

Hence, $\tilde{C}^{SM} < C^{*SM} < \overline{C}^{SM}$ and $V^{SM}(C)$ is concave in $\left[0, C^{*SM}\right]$ and convex in $\left[C^{*SM}, \overline{C}^{SM}\right]$. Moreover, $V^{SM}(C) > 1$ in $\left[0, \tilde{C}^{SM}\right)$, while $V^{SM}(C) < 1$ in $\left(\tilde{C}^{SM}, \overline{C}^{SM}\right]$. Considering that $V^{SM}\left(\tilde{C}^{SM}\right) = V^{SM}\left(\overline{C}^{SM}\right) = 1$, integrating the last inequality yields

$$V^{SM}\left(\overline{C}^{SM}\right) - V^{SM}\left(\tilde{C}^{SM}\right) < \overline{C}^{SM} - \tilde{C}^{SM}$$

Combining with (25) and (22) yields

$$V^{SM}(0) - L^{SM}(0) > -\phi \frac{E\left(\overline{C}^{SM}\right)}{E\left(\overline{C}^{SM}\right)}$$

which is a necessary condition for (23).

In fact, plugging (25) and (26) in the ODE (8) for $C = \tilde{C}^{SM}$ and simplifying yields

$$V^{SM}(0) - L^{SM}(0) = \frac{\delta - \theta \tilde{C}^{SM}}{r} + \frac{\sigma^2}{2r} V_{CC}^{SM}\left(\tilde{C}^{SM}\right)$$

Moreover, applying (22) and (24) in the same ODE for $C = \overline{C}^{SM}$ obtains

$$\frac{\sigma^2}{2} V_{CC}^{SM}\left(\overline{C}^{SM}\right) = \theta \overline{C}^{SM} - \delta - \phi\, \frac{E\left(\overline{C}^{SM}\right)}{E\left(\overline{C}^{SM}\right)}$$

Given that a feasible solution should necessarily satisfy $V_{CC}^{SM}\left(\tilde{C}^{SM}\right) < 0 < V_{CC}^{SM}\left(\overline{C}^{SM}\right)$, the two equations above yield the following necessary condition

$$\tilde{C}^{SM} < \frac{\delta}{\theta} < \frac{\delta}{\theta} + \frac{\phi}{\theta}\frac{E\left(\overline{C}^{SM}\right)}{E\left(0\right)} < \overline{C}^{SM}$$

A.3 Average cash

In this section of the Appendix, we initially derive the distribution of cash stock (following standard methodology as in, e.g., Bertola and Caballero, 1990) which we subsequently use to calculate the average cash holdings.

Similar to standard textbook approach, we use a binomial tree to approximate the Itō process (2) that the accumulated cash reserves follow Cox and Miller (1965). We start by dividing the entire space $(0;\overline{C})$ into equal intervals of $\Delta c = \sigma\sqrt{\Delta t}$. Within the space $[\Delta c;\overline{C}-\Delta c]$ and for every time t, cash stock can move either up or down by Δc and the probabilities of an upward and a downward movement are respectively

$$\begin{aligned} p(c) &= \tfrac{1}{2}\left[1+\eta(c)\tfrac{\Delta t}{\Delta c}\right] \\ q(c) &= \tfrac{1}{2}\left[1-\eta(c)\tfrac{\Delta t}{\Delta c}\right] \end{aligned} \tag{A.45}$$

where $\eta(x) = \mu + (r-\theta)\,x$.[23] To illustrate this representation, denoting by c_0 the initial state of cash stock at $t=0$, cash stock at time Δt can be either $(c_0+\Delta c)$ with probability $p(c_0)$ or $(c_0-\Delta c)$ with probability $q(c_0)$; at time $2\Delta t$, $c_{2\Delta t}$ may be equal to $c_0+2\Delta c$, c_0 or $(c_0-2\Delta c)$ with probabilities $[p(c_0)\,p(c0+\Delta c)]$, $[p(c_0)\,q(c_0+\Delta c)+q(c_0)p(c_0-\Delta c)]$ and $[q(c_0)\,q(c_0-\Delta c)]$ respectively; and so on. As Δt (and thus Δc) approach to zero, this approximation converges to an Itō process with a drift of $\eta(x)$ and a diffusion coefficient of σ.

As time passes, the firm's cash reserves oscillate between 0 and $\overline{C}$ and they settle to a long-run stationary distribution, no longer depending on the initial value of cash reserves or time. Hence, after a long period of time T, the cash stock can be at any point c in the $(0;\overline{C}]$ interval. It may have moved to c following either an upward movement from $c-\Delta c$ or a downward movement from $c+\Delta c$. Thus, denoting by $f(c)$ the density function of the long-run distribution, in each of the intervals $(0,\tilde{C})$ and $(\tilde{C},\overline{C})$, it holds that

$$f(c) = p(c-\Delta c)\,f(c-\Delta c) + q(c+\Delta c)\,f(c+\Delta c) \tag{A.46}$$

[23] Evidently, these probabilities need to be bounded between 0 and 1, for which choosing Δc such that $\Delta c < \dfrac{\sigma^2}{\max_c\left(|\eta(c)|\right)}$ is a sufficient condition.

Subtracting $\frac{1}{2}[f(c+\Delta c)+f(c-\Delta c)]$ from both sides and rearranging yields

$$\begin{aligned}[f(c+\Delta c)-f(c)]-[f(c)-f(c-\Delta c)] &= [\eta(c+\Delta c)\,f(c+\Delta c)-\eta(c-\Delta c)\,f(c-\Delta c)]\,\tfrac{\Delta t}{\Delta c}\\ \iff \Delta f(c)-\Delta f(c-\Delta c) &= (\Delta\,[\eta(c)\,f(c)]+\Delta\,[\eta(c-\Delta c)\,f(c-\Delta c)])\,\tfrac{\Delta t}{\Delta c} \qquad \text{(A.47)}\end{aligned}$$

Dividing both sides by $(\Delta c)^2$ and taking to the limit yields

$$\frac{1}{2}\sigma^2 f''(c)-\eta(c)f'(c)-\eta'(c)f(c)=0 \tag{A.48}$$

which is in fact equivalent to the respective forward Kolmogorov (or Fokker-Planck) equation

$$\frac{1}{2}\frac{\partial^2}{\partial c^2}\left[\sigma^2\,\psi(c,t)\right]-\frac{\partial}{\partial c}\left[\eta(c)\,\psi(c,t)\right]=\frac{\partial}{\partial t}\psi(c,t) \tag{A.49}$$

where the probability density function $\psi(c,T)=f(c)$. For $r>\theta$,[24][25] the general solution to this ordinary differential equation is

$$f(c)=e^{A(c)^2}\left(\frac{\operatorname{erf}\left[A(c)\right]}{\sqrt{r-\theta}}\,Y+Z\right) \tag{A.50}$$

where $\operatorname{erf}(x)=\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x e^{-y^2}\,\mathrm{d}y$ is the (Gauss) error function, $A(c)=\frac{\eta(c)}{\sigma\sqrt{r-\theta}}$, while Y and Z are constants of integration.

All cases examined in this chapter involve a reflecting barrier (payout threshold) at some level $\overline{C}$. First, we comprehensively expose the derivation of the density function for the case SM where there is a resetting barrier (issuance threshold) at 0 which, when hit, resets the

[24] Irrespectively of the sign of $(r-\theta)$, the general solution can be expressed as

$$f(c)=e^{\left(\frac{-i}{\sqrt{-(r-\theta)}}\frac{\eta(c)}{\sigma}\right)^2}\left(\frac{-i}{\sqrt{-(r-\theta)}}\operatorname{erf}\left[\frac{-i}{\sqrt{-(r-\theta)}}\frac{\eta(c)}{\sigma}\right]Y+Z\right)$$

where i is the imaginary unit. Noting that $\left(\frac{-i}{\sqrt{-(r-\theta)}}\right)^2=\frac{1}{r-\theta}$, the coefficient of constant Y is

$$\frac{-i}{\sqrt{-(r-\theta)}}\operatorname{erf}\left[\frac{-i}{\sqrt{-(r-\theta)}}\frac{\eta(c)}{\sigma}\right]=\begin{cases}\frac{\operatorname{erf}[A(c)]}{\sqrt{r-\theta}} & \text{if } r>\theta\\ \frac{2}{\sqrt{\pi}}F\left[-\frac{\eta(c)}{\sigma\sqrt{\theta-r}}\right]e^{-A(c)^2} & \text{if } r<\theta\end{cases}$$

where $F(x)=e^{-x^2}\displaystyle\int_0^x e^{y^2}\,\mathrm{d}y$ is the Dawson integral. In the relatively simpler special case where $r=\theta$, the general solution is

$$f_z(c)=e^{\frac{2\mu}{\sigma^2}c}\,Y_z+Z_z$$

where Y_z and Z_z are the respective constants of integration.

[25] Although the remainder of this section treats only cases for $r-\theta>0$, results for $r-\theta\le 0$ can be easily replicated following the procedure exposed below.

cash stock at $\tilde{C}^{SM}$ (issuance proceeds). In the remaining subsections of this section, we show that the resulting density function in fact nests the ones stemming from the remaining two cases, i.e. the one where the issuance proceeds are equal to the payout threshold (case S for which $\tilde{C}^S = \overline{C}^S$) and the one where the issuance threshold is a reflecting barrier (case M, $\tilde{C}^M \to 0$).[26]

A.3.1 Case SM

Beyond an upward or downward movement from its neighboring $\left(\tilde{C} - \Delta c\right)$ and $\left(\tilde{C} + \Delta c\right)$, $\tilde{C}$ can also been reached after a reset from 0 (i.e. a downward movement from node Δc). This means that although continuous, the density function is not continuously differentiable at $\tilde{C}$. This can be expressed as

$$f(c) = \begin{cases} e^{A(c)^2}\left(\frac{\operatorname{erf}[A(c)]}{\sqrt{r-\theta}}\, Y_1 + Z_1\right) & \text{if } c \leq \tilde{C} \\ e^{A(c)^2}\left(\frac{\operatorname{erf}[A(c)]}{\sqrt{r-\theta}}\, Y_2 + Z_2\right) & \text{if } c \geq \tilde{C} \end{cases} \tag{A.51}$$

The constants Y_1, Y_2, Z_1, and Z_2 are determined by the boundary conditions of the density function. A first condition stems from the resetting barrier at 0, i.e. the node Δc can be reached only from a downward movement from node $2\Delta c$:

$$f(\Delta c) = q(2\Delta c)\, f(2\Delta c) \tag{A.52}$$

Using Taylor's expansion for $[q\,(2\Delta c)\; f\,(2\Delta c)]$, rearranging and taking to the limit yields

$$\lim_{\Delta c \to 0} \Big([1 - q(\Delta c)]\, f(\Delta c) \Big) = 0 \tag{A.53}$$

Given that $q(\Delta c) < 1$, the first condition can be summarized as

$$\lim_{c \to 0} f(c) = 0 \tag{A.54}$$

implying continuity at 0 since by construction $f(0) = 0$. The second condition stems from continuity at $\tilde{C}$, yielding

$$\lim_{c \to \tilde{C}^-} f(c) = \lim_{c \to \tilde{C}^+} f(c) \tag{A.55}$$

The third condition represents the discontinuity of $f'(c)$ at $\tilde{C}$, i.e. following the three ways by which $\tilde{C}$ can be reached, it holds that

$$f(\tilde{C}) = p(\tilde{C} - \Delta c)\, f(\tilde{C} - \Delta c) + q(\tilde{C} + \Delta c)\, f(\tilde{C} + \Delta c) + q(\Delta c)\, f(\Delta c) \tag{A.56}$$

[26] As each case is examined separately, the subscripts indexing the case are dropped unnecessary and thus dropped.

Substituting (A.45), using $f(0) = 0$ and rearranging yields

$$\left[f\left(\tilde{C}\right) - f\left(\tilde{C} - \Delta c\right)\right] = \left[f\left(\tilde{C} + \Delta c\right) - f\left(\tilde{C}\right)\right] + \left[f\left(\Delta c\right) - f\left(0\right)\right] - W\left(\Delta c\right) \tag{A.57}$$

where

$$W\left(\Delta c\right) = \frac{\Delta t}{\Delta c}\left[\eta\left(\tilde{C} + \Delta c\right)\, f\left(\tilde{C} + \Delta c\right) - \eta\left(\tilde{C} - \Delta c\right)\, f\left(\tilde{C} - \Delta c\right) + \eta\left(\Delta c\right)\, f\left(\Delta c\right)\right]$$

Noticing that $\lim\limits_{\Delta c \to 0} \dfrac{W\left(\Delta c\right)}{\Delta c} = 0$, dividing (A.57) by Δc and taking to the limit reduces to

$$\lim_{c \to C^{I-}} f'(c) = \lim_{c \to C^{I+}} f'(c) + \lim_{c \to 0} f'(c) \tag{A.58}$$

The final condition is the sum of probabilities constraint

$$\int_0^{\overline{C}} f(c)\,\mathrm{d}c = \int_0^{\tilde{C}} f(c)\,\mathrm{d}c + \int_{\tilde{C}}^{\overline{C}} f(c)\,\mathrm{d}c = 1 \tag{A.59}$$

Solving for the above conditions gives the density function[27]

$$f^{SM}(c) = \frac{\sqrt{2\left(r - \theta\right)}}{\sigma}\, \frac{e^{A(c)^2}}{B\left[A\left(\overline{C}\right)\right] - \frac{D\left[A\left(\tilde{C}\right)\right] - D[A(0)]}{\mathrm{erf}\left[A\left(\tilde{C}\right)\right] - \mathrm{erf}[A(0)]}} * \begin{cases} \frac{\mathrm{erf}[A(c)] - \mathrm{erf}[A(0)]}{\mathrm{erf}\left[A\left(\tilde{C}\right)\right] - \mathrm{erf}[A(0)]} & \text{if } c \leq \tilde{C} \\ 1 & \text{if } c \geq \tilde{C} \end{cases} \tag{A.61}$$

where

$$B\left(x\right) = \sqrt{2}\, e^{x^2}\, F\left(x\right)$$
$$D\left(x\right) = \mathrm{erf}\left(x\right)\, B\left(x\right) - \sqrt{\frac{2}{\pi}}\, x^2\, {}_2F_2\left(1, 1; \frac{3}{2}, 2; x^2\right)$$

and ${}_2F_2\left(a, b; c, d; z\right)$ is the generalized hypergeometric function of order $(2, 2)$.

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[27] The validity of the function can be confirmed by checking the (superfluous) condition at the reflecting barrier $\overline{C}$, i.e. the node $\overline{C}$ can be reached by either of two upward movements, from $\left(\overline{C} - \Delta c\right)$ or $\overline{C}$:

$$f\left(\overline{C}\right) = p\left(\overline{C} - \Delta c\right)\, f\left(\overline{C} - \Delta c\right) + p\left(\overline{C}\right)\, f\left(\overline{C}\right) \iff$$
$$\left[f\left(\overline{C}\right) - f\left(\overline{C} - \Delta c\right)\right] = \left[\eta\left(\overline{C}\right)\, f\left(\overline{C}\right) + \eta\left(\overline{C} - \Delta c\right)\, f\left(\overline{C} - \Delta c\right)\right] \frac{\Delta t}{\Delta c}$$

Dividing by Δc and taking to the limit yields

$$\lim_{c \to C^P} f'(c) = \frac{2\,\eta\left(\overline{C}\right)}{\sigma^2}\, f\left(\overline{C}\right) \tag{A.60}$$

Given the density function, finding the long-run average cash stock is simply

$$\overline{c} = \int_0^{\overline{C}} c\, f(c)\, \mathrm{d}c = \int_0^{\tilde{C}} c\, f(c)\, \mathrm{d}c + \int_{\tilde{C}}^{\overline{C}} c\, f(c)\, \mathrm{d}c \tag{A.62}$$

Substituting and simplifying yields

$$\overline{c}^{SM} = \frac{\frac{\sigma}{\sqrt{2(r-\theta)}}\, e^{A\left(\overline{C}\right)^2} - \sqrt{\frac{2}{\pi}}\, \frac{\tilde{C}}{\mathrm{erf}\left[A\left(\tilde{C}\right)\right]-\mathrm{erf}[A(0)]}}{B\left[A\left(\overline{C}\right)\right] - \frac{D\left[A\left(\tilde{C}\right)\right]-D[A(0)]}{\mathrm{erf}\left[A\left(\tilde{C}\right)\right]-\mathrm{erf}[A(0)]}} - \frac{\mu}{r-\theta} \tag{A.63}$$

A.3.2 Case S

In the absence of a manager, payout is chosen optimally to maximize value to shareholders. As proven in Proposition 1 above, the issuance proceeds are such that cash stock jumps directly to the payout threshold, i.e. $\tilde{C} = \overline{C}$. Given the general solution (A.50) and in the absence of the breaking point $\tilde{C}$, there are two constants that need to be determined by an equal amount of conditions.

The two conditions needed in this case are: the equivalent of condition (A.52) for $f(c)$, indicating continuity of $f(c)$ at $c = 0$; and the sum of probabilities constraint (equivalent of (A.59)). The resulting density function is given by[28]

$$f_S(c) = \frac{\sqrt{2\left(r-\theta\right)}}{\sigma}\, \frac{e^{A(c)^2}}{B\left[A\left(\overline{C}\right)\right] - \frac{D\left[A\left(\overline{C}\right)\right]-D[A(0)]}{\mathrm{erf}\left[A\left(\overline{C}\right)\right]-\mathrm{erf}[A(0)]}}\, \frac{\mathrm{erf}\left[A\left(c\right)\right] - \mathrm{erf}\left[A\left(0\right)\right]}{\mathrm{erf}\left[A\left(\overline{C}\right)\right] - \mathrm{erf}\left[A\left(0\right)\right]} = \lim_{\tilde{C}\to\overline{C}} f^{SM}(c) \tag{A.65}$$

[28] As with the previous case, the validity of the density function can be cross-checked against the remaining condition, i.e. the movements leading the cash stock to the payout threshold node $\overline{C}$. Cash stock can either remain on the payout threshold (after an upward movement from the payout threshold itself) or can reach the payout threshold by either an upward movement from $\left(\overline{C}-\Delta c\right)$ or a downward movement from Δc. This yields

$$f\left(\overline{C}\right) = p\left(\overline{C}\right)\, f\left(\overline{C}\right) + p\left(\overline{C}-\Delta c\right)\, f\left(\overline{C}-\Delta c\right) + q\left(\Delta c\right)\, f\left(\Delta c\right) \iff$$

$$\left[f\left(\overline{C}\right) - f\left(\overline{C}-\Delta c\right)\right] - \left[f\left(\Delta c\right) - f\left(0\right)\right] = \left[\eta\left(\overline{C}\right)\, f\left(\overline{C}\right) + \eta\left(\overline{C}-\Delta c\right)\, f\left(\overline{C}-\Delta c\right) - \eta\left(\Delta c\right)\, f\left(\Delta c\right)\right] \frac{\Delta t}{\Delta c}$$

Dividing by Δc and taking to the limit yields

$$\lim_{c\to C^P} f'(c) - \lim_{c\to 0} f'(c) = \frac{2\,\eta\left(\overline{C}\right)}{\sigma^2}\, f\left(\overline{C}\right) \tag{A.64}$$

The average cash in this case is then

$$\overline{c}_S = \frac{\frac{\sigma}{\sqrt{2(r-\theta)}}\, e^{A(\overline{C})^2} - \sqrt{\frac{2}{\pi}}\, \frac{\overline{C}}{\mathrm{erf}\left[A\left(\overline{C}\right)\right]-\mathrm{erf}[A(0)]}}{B\left[A\left(\overline{C}\right)\right] - \frac{D\left[A\left(\overline{C}\right)\right]-D[A(0)]}{\mathrm{erf}\left[A\left(\overline{C}\right)\right]-\mathrm{erf}[A(0)]}} - \frac{\mu}{r-\theta} = \lim_{\tilde{C}\to\overline{C}} \overline{c}^{SM} \tag{A.66}$$

A.3.3 Case M

In the absence of issuance costs, the issuance proceeds are just covering for the cash deficit, i.e. $c = 0$ becomes a reflecting barrier. Still, as long as cash management is delegated to a self-serving manager, the payout threshold $\overline{C}$ is substantially different than 0. Using the general solution (A.50), the two conditions relevant for this case are: the equivalent of the reflecting barrier condition (A.60) for $f(c)$ and the sum of probabilities constraint (equivalent of (A.59)).

The resulting density function is given by[29]

$$f^M(c) = \frac{\sqrt{2\,(r-\theta)}}{\sigma}\, \frac{e^{A(c)^2}}{B\left[A\left(\overline{C}\right)\right] - B\left[A\left(0\right)\right]} = \lim_{\tilde{C}\to 0} f_{SM}(c) \tag{A.68}$$

where the last equality results from

$$\lim_{\tilde{C}\to 0}\left[\frac{D\left[A\left(\tilde{C}\right)\right] - D\left[A\left(0\right)\right]}{\mathrm{erf}\left[A\left(\tilde{C}\right)\right] - \mathrm{erf}\left[A\left(0\right)\right]}\right] = \frac{\frac{\partial}{\partial c}\Big|_{c=0} D\left[A\left(c\right)\right]}{\frac{\partial}{\partial c}\Big|_{c=0} \mathrm{erf}\left[A\left(c\right)\right]} = B\left[A\left(0\right)\right]$$

The average cash in this case is then

$$\overline{c}^S = \frac{\sigma}{\sqrt{2\,(r-\theta)}}\, \frac{e^{A(\overline{C})^2} - e^{A(0)^2}}{B\left[A\left(\overline{C}\right)\right] - B\left[A\left(0\right)\right]} - \frac{\mu}{r-\theta} = \lim_{\tilde{C}\to 0} \overline{c}^{SM} \tag{A.69}$$

[29]The validity of the density function can be cross-checked against the reflecting barrier condition at $c = 0$. Similar argumentation to the one preceding (A.60) yields

$$f_M\left(0\right) = q\left(\Delta c\right)\, f_M\left(\Delta c\right) + q\left(0\right)\, f_M\left(0\right) \iff$$
$$\left[f_M\left(\Delta c\right) - f_M\left(0\right)\right] = \left[\eta\left(0\right)\, f_M\left(0\right) + \eta\left(\Delta c\right)\, f_M\left(\Delta c\right)\right]\frac{\Delta t}{\Delta c}$$

Dividing by Δc and taking to the limit yields

$$\lim_{c\to 0} f'(c) = \frac{2\,\eta\left(0\right)}{\sigma^2}\, f\left(0\right) \tag{A.67}$$

where the last equality results from

$$\lim_{\tilde{C}\to 0}\left[\frac{\tilde{C}}{\operatorname{erf}\left[A\left(\tilde{C}\right)\right]-\operatorname{erf}\left[A\left(0\right)\right]}\right]=\frac{\frac{\partial}{\partial c}\big|_{c=0}\,c}{\frac{\partial}{\partial c}\big|_{c=0}\operatorname{erf}\left[A\left(c\right)\right]}=\sqrt{\frac{\pi}{2}}\,\frac{\sigma}{\sqrt{2\left(r-\theta\right)}}\,e^{A(0)^2}$$

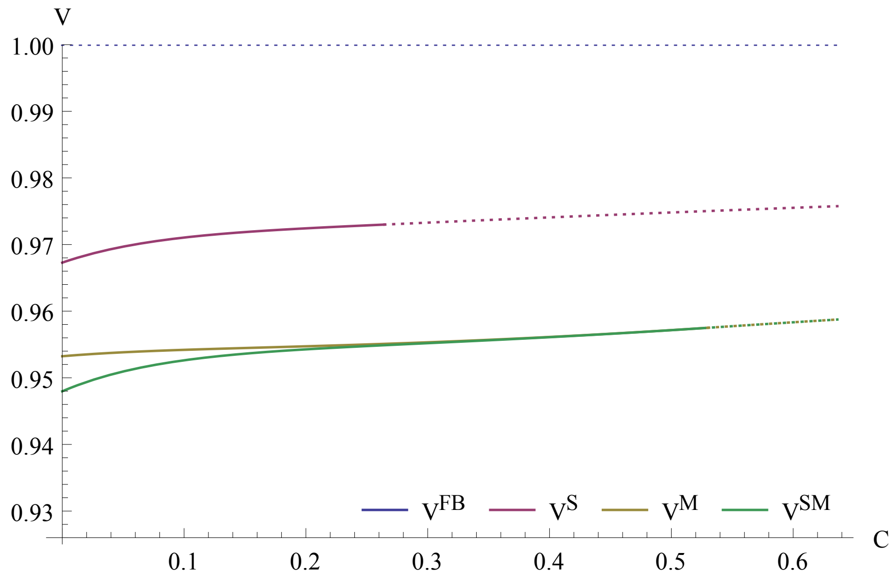

Figure 1: Firm value.

The graph plots the firm value $V^i(C)$ for each special case $i = \{FB, S, M, SM\}$, scaled by $V^{FB}(C)$, for different levels of cash reserves C. The blue line plots $V^{FB}(C)$, the purple $V^S(C)$, the yellow $V^M(C)$, and the green $V^{SM}(C)$, scaled by $V^{FB}(C)$. Solid lines represent the firm value in the inaction region, while dotted lines represent the value to shareholders of each case i given some initial cash endowment higher than the payout threshold $\overline{C}^i$.

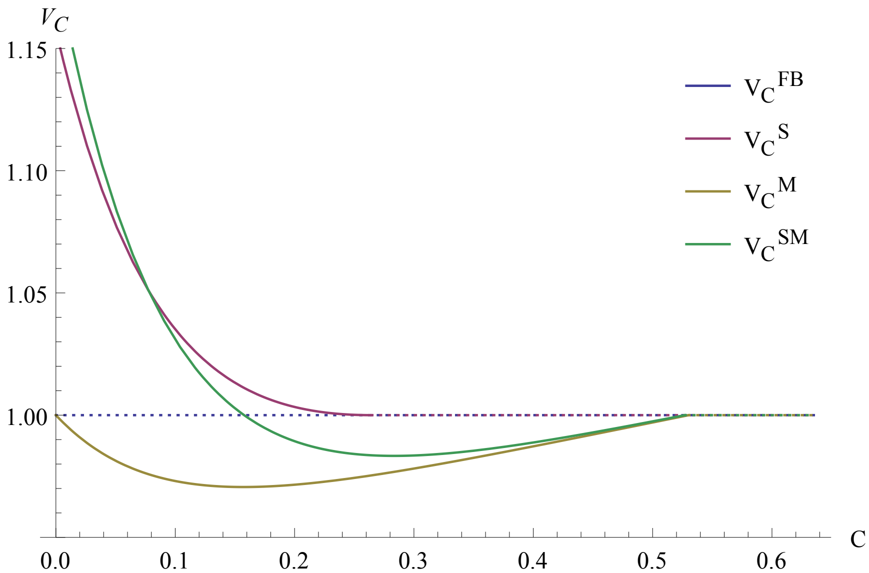

Figure 2: Marginal value of cash.

The graph plots the marginal value of cash for each special case $i = \{FB, S, M, SM\}$, $V_C^i(C)$ defined as the first derivative of firm value with respect to C, for different levels of cash reserves C. The blue line plots $V_C^{FB}(C)$, the purple $V_C^S(C)$, the yellow $V_C^M(C)$, and the green $V_C^{SM}(C)$. Solid lines represent the marginal value of cash in the inaction region, while dotted lines represent the marginal value of cash to shareholders of each case i given some initial cash endowment higher than the payout threshold $\overline{C}^i$.

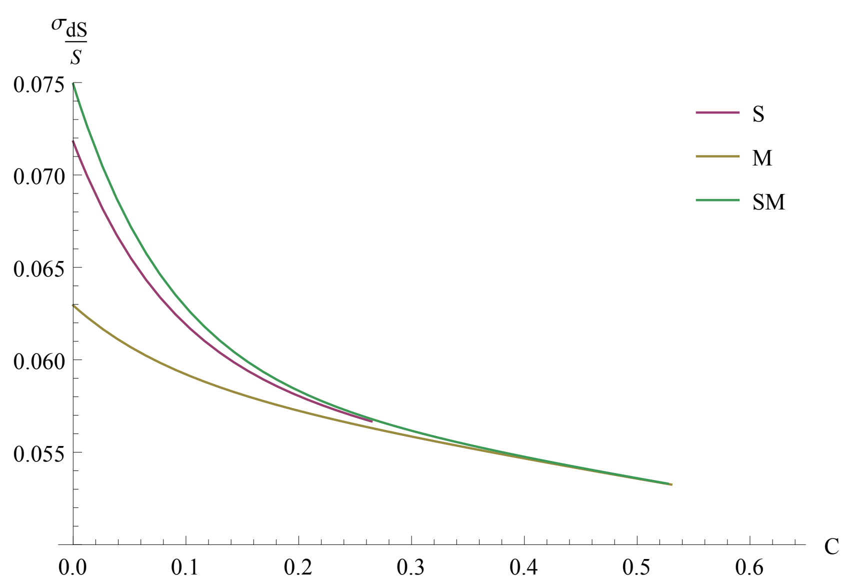

Figure 3: Volatility of stock returns.

The graph plots the volatility of stock returns, for each special case $i = \{FB, S, M, SM\}$ for different levels of cash reserves C. The purple $V_C^S(C)$, the yellow $V_C^M(C)$, and the green $V_C^{SM}(C)$.

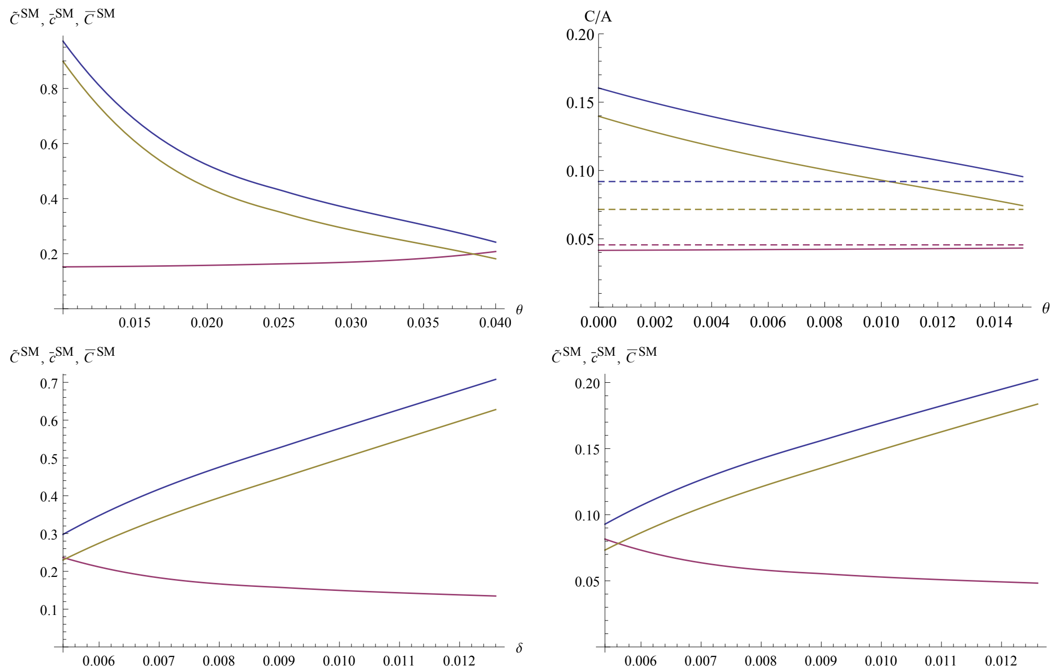

Figure 4: Effects of monitoring and entrenchment parameters on thresholds and average cash.

The graphs plot the relation between the main parameters of the model and corporate cash policy. For every panel, the blue line represents the upper threshold, $\overline{C}^{SM}$; the purple line represents the target barrier, $\tilde{C}^{SM}$, and the yellow line represents the average of the stationary distribution of cash holdings. The horizontal axis represents effectiveness of monitoring (θ) in the top row, and managerial entrenchment (δ) in the bottom row. The vertical axis is denominated in monetary units in the left panels and as a ratio of firm value in the right panels.

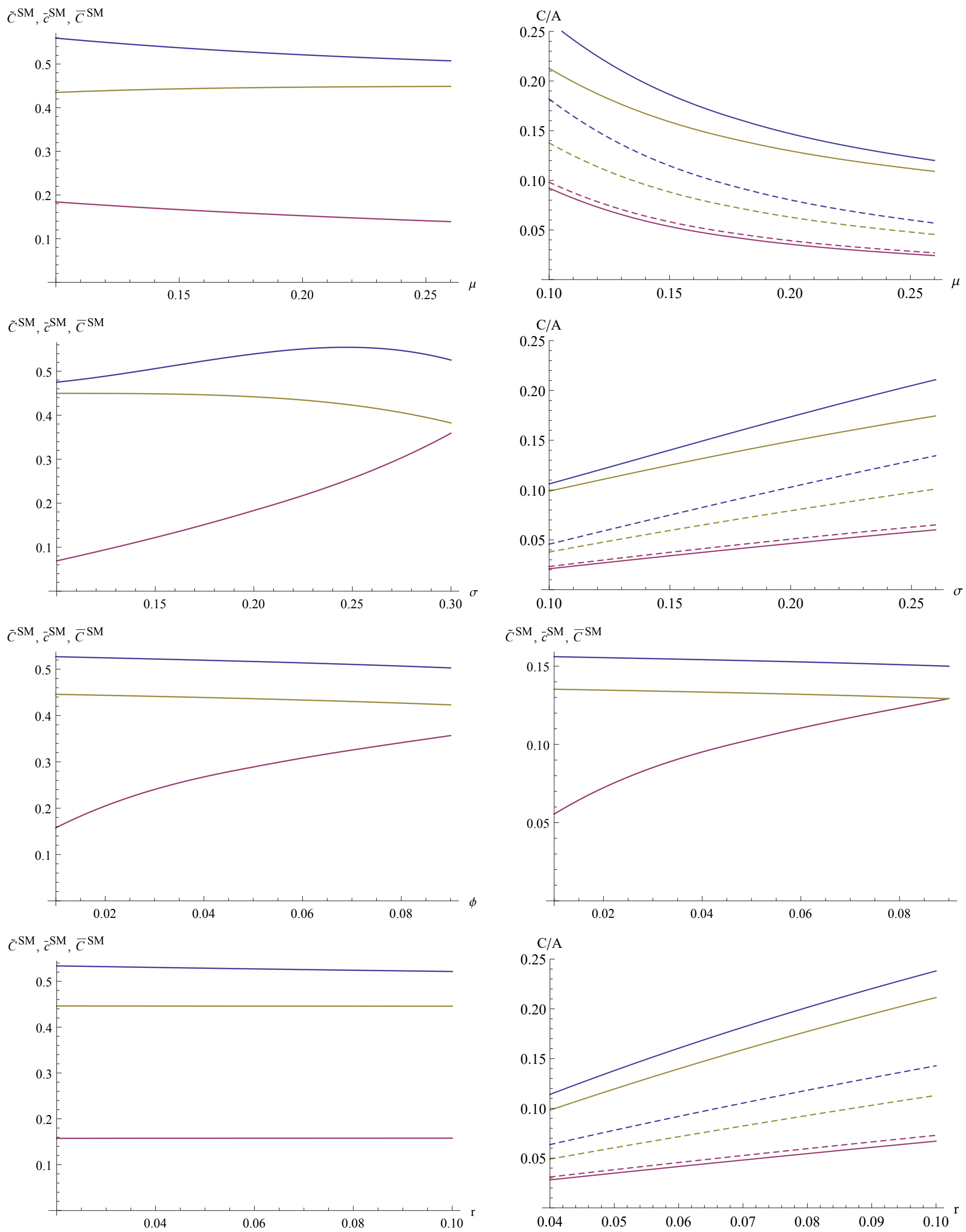

Figure 5: Effects of productivity and economy-wide parameters on thresholds and average cash.

The graphs plot the relation between the remaining parameters of the model and corporate cash policy. For every panel, the blue line represents the upper threshold, $\overline{C}^{SM}$; the purple line represents the target barrier, $\tilde{C}^{SM}$, and the yellow line represents the average of the stationary distribution of cash holdings $\bar{c}^{SM}$. The horizontal axis represents the expected cash flow (μ) in the first row, the volatility of cash flows (σ) in the second row, the costs of issuance (ϕ) in the third row, the risk-free rate (r) in the fourth row. The vertical axis is denominated in monetary units in the left panels and as a ratio of firm value in the right panels.

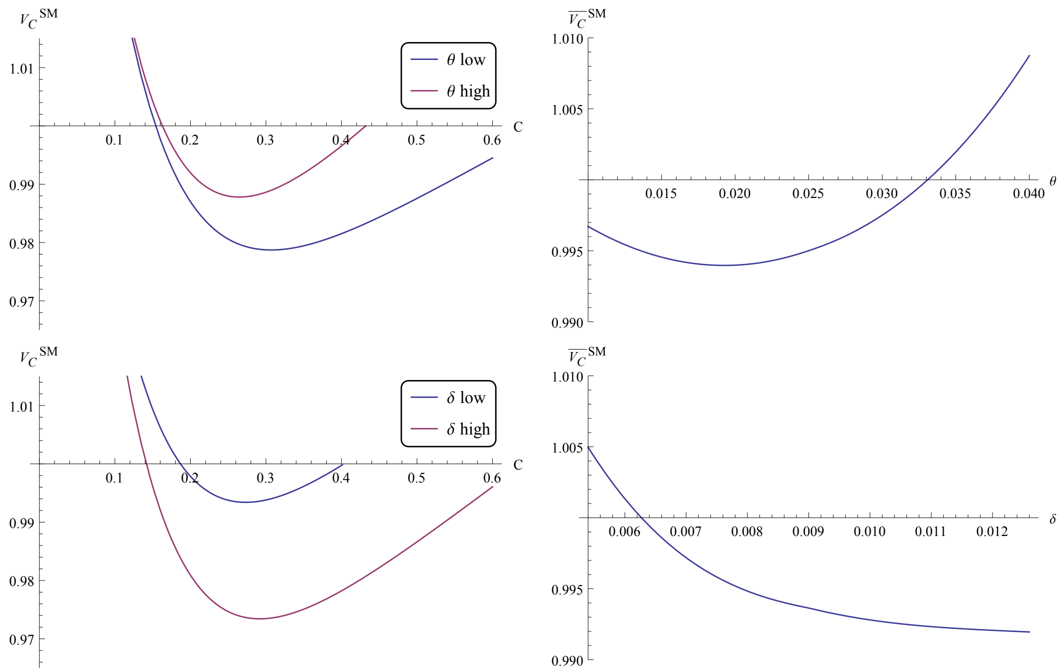

Figure 6: Effect of monitoring and entrenchment parameters on the marginal value of cash. The graphs plot the relation between the main parameters of the model and the value of cash. In the left panels, the value of the parameter is varied by 50% from its base case value and the marginal value of cash is plotted against cash levels; the blue line is for the lower value of the parameter, while the purple line is for the higher one. In the right panels, we plot the average marginal value of cash against a range of values of the parameter. The first row shows results for the effectiveness of monitoring (θ), while the second row for managerial entrenchment (δ).

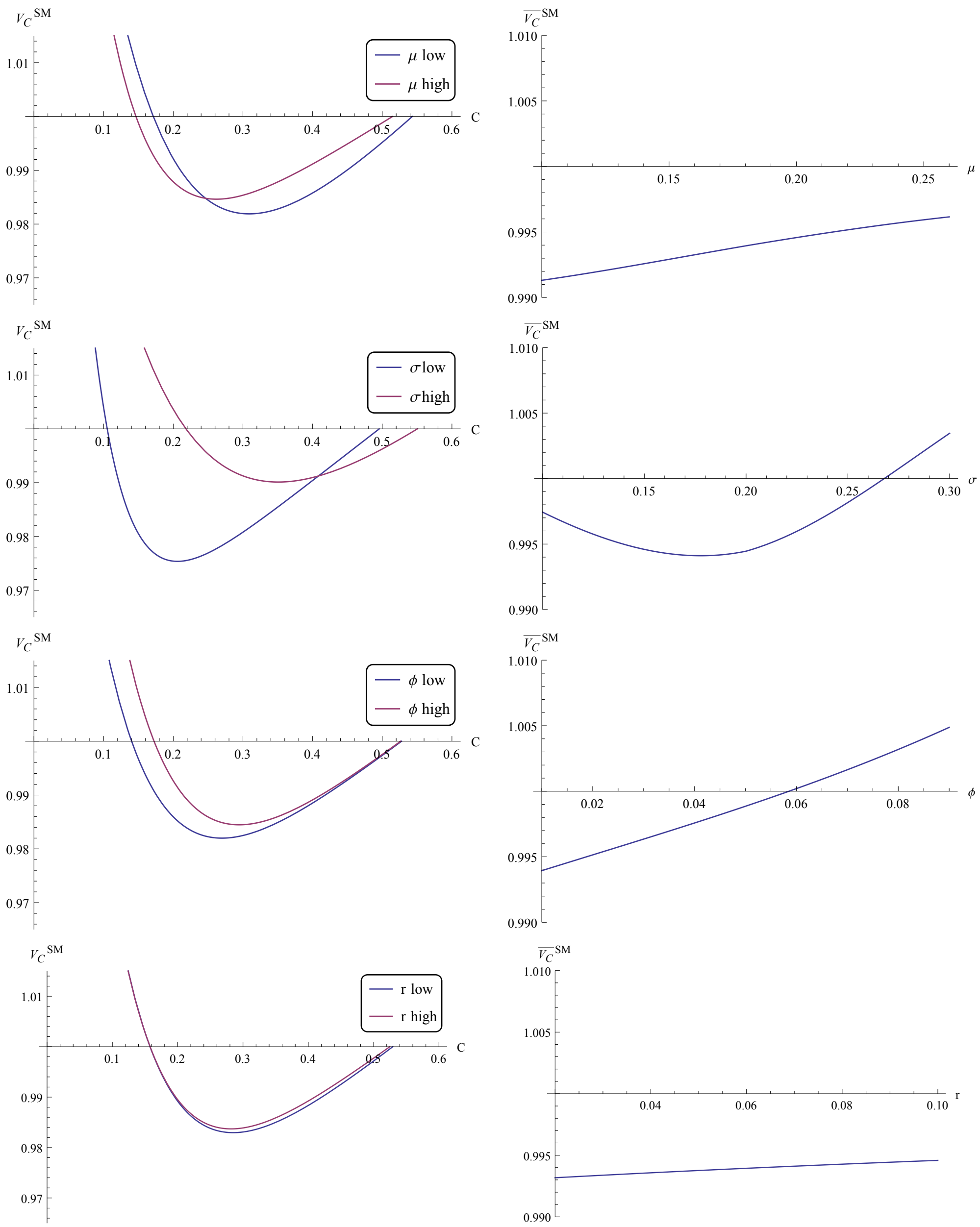

Figure 7: Effect of productivity and economy-wide parameters on the marginal value of cash.

The graphs plot the relation between the remaining parameters of the model and the value of cash. In the left panels, the value of the parameter is varied by 50% from its base case value and the marginal value of cash is plotted against cash levels; the blue line is for the lower value of the parameter, while the purple line is for the higher one. In the right panels, we plot the average marginal value of cash against a range of values of the parameter. The first row shows results for the expected cash flows (μ), the second for the volatility of cash flows (σ), the third for the costs of issuance (ϕ), and the fourth row for the risk-free rate (r).

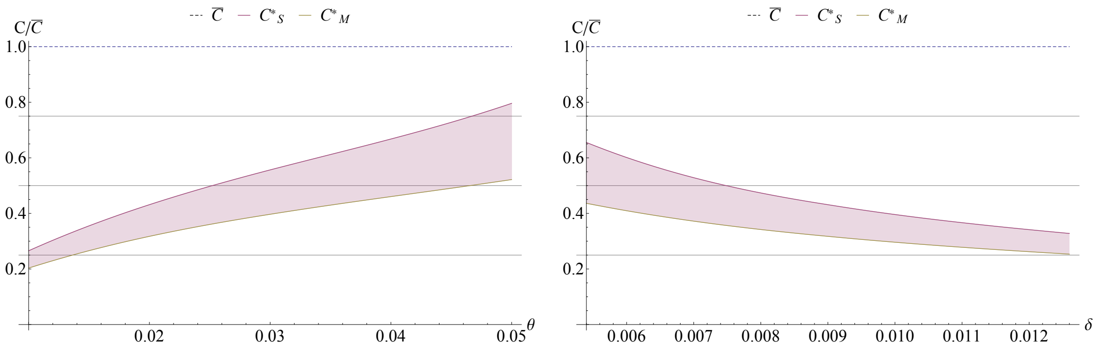

Figure 8: Effect of parameters on risk management incentives.

The graphs plot the thresholds of hedging/gambling preferences for shareholders (C_S^*, purple) and management (C_M^*, golden) with respect to the two variables of main interest, effectiveness of monitoring (θ) and managerial entrenchment (δ). In both panels, thresholds are drawn as a proportion of the payout threshold $\overline{C}$. The shaded area indicates the intersecting range of values for which both parties would benefit from hedging activities.