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Using inertial measurement units to identify medio-lateral ground reaction forces due to walking and swaying

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ABSTRACT

Horizontal ground reaction forces (GRFs) due to human walking and swaying have been investigated (respectively) through direct measurements using a treadmill and a set of force plates. These GRFs have also been measured (or estimated) indirectly using acceleration data provided by inertial measurement units (IMUs).

One motivation for this research has been the lack of published data on these two forms of loading that are generated by movements of the human body in the medio-lateral plane perpendicular to the direction of walking or the direction faced during swaying. The other motivation, following from successful developments in applying IMUs to in-situ vertical GRF measurements, has been to identify best practice for estimating medio-lateral GRFs outside the constraints of a laboratory.

Examination of 852 treadmill measurements shows that medio-lateral GRFs at the first sub-harmonic of pacing rate can exceed 10% of body weight. Using a smaller and more recent set of measurements including motion capture, it has been shown that IMUs can be used to reconstruct these GRFs using a linear combination of body accelerations at each of the lower back and sternum positions. There are a number of potential applications for this capability yet to be explored, in particular relating to footbridge performance.

A separate set of measurements using force plates has shown that harmonic components of medio-lateral dynamic load factors due to on the spot swaying can approach 50% of body weight. Such forces provide a capability to excite horizontal vibration modes of large civil structures with frequencies below 2 Hz that are problematic for mechanical excitation. As with walking, the ability to use IMUs to estimate medio-lateral swaying GRFs outside laboratory constraints has been demonstrated. As for walking a pair of IMUs is needed, but the best linear combination varies strongly between individuals, according to swaying style. In-situ application of indirect measurement has been successfully demonstrated through a very challenging application of system identification of a multi-storey building, including estimation of modal mass.

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1. Introduction: lateral human forces on structures

The effects of lateral human dynamic forces on structures became infamous due to the well-publicised behaviour of two footbridges celebrating the new millennium, Pont de Solferino [1] (now Passerelle Léopold-Sédar-Senghor) and London Millennium Bridge (LMB) [2]. These footbridges exhibited large amplitude lateral vibrations induced by lateral ground reaction forces (GRFs) associated with a passage of crowds of pedestrians. Similar behaviours have since been identified on a number of new [3,4] and also old [5] structures. The issue has been recognised as so critical to footbridge operators in respect of pedestrian comfort and even safety, that provisions against this phenomenon have been made in a number of footbridge design codes [6,7].

Early explanations of the build-up of vibrations relied solely on the notion of synchronisation of pedestrian footsteps to the lateral structural motion [8]. Indeed, Arup – the consulting engineers behind the LMB, describe this mechanism as 'synchronous lateral excitation' (SLE) or 'lateral synchronous vibration' and to this day this seems to be the most often purported cause of excessive lateral response of bridges under the action of walking pedestrians [9]. According to this notion, there exists a form of positive feedback between the pedestrian and structural behaviour whereby the vibrations prompt the pedestrians to change their stride frequency, i.e. half the pacing frequency, such as to coincide with the frequency of structural motion. Moreover, they adjust their phase so as to input energy into the vibrating structure thus increasing the vibration amplitudes, which in turn causes more pedestrians to lock-in their steps [2]. Based on the measurements from the LMB, Arup proposed an empirical model for the critical number of pedestrians at which the structural instability is initiated, where each pedestrian is treated as a source of negative damping to the structure. Although this model seems to agree with some full-scale observations [4], since it is based on the measured bridge response only it lacks insight into pedestrian behaviour hence its general applicability can be considered uncertain. This motivated researchers to seek to explain the mechanics of human lateral stability during walking and the bi-directional exchange of energy between the pedestrian and laterally oscillating structure [10–12]. It has been shown that positive work on the structure can be done by the component of the pedestrian GRF in the medio-lateral plane of the human body (medio-lateral GRF) even if the pedestrian stride frequency does not coincide with the frequency of bridge lateral oscillation. Study of the phenomenon has continued with laboratory experiments using laterally oscillating treadmills [13–15]. The absence of pedestrian synchronisation in the presence of SLE has been observed experimentally in a full-scale investigation [3] and in controlled laboratory conditions [16] so a more comprehensive explanation is required. So far there has been no direct measurement of human lateral forces in situ on a full-scale footbridge experiencing SLE, but while there is clear motivation to do this there are challenges.

The first challenge in doing this is in finding an SLE-prone footbridge not already fixed and whose owner will allow full-scale study. This can, to some extent, be addressed using a laboratory analogue of a swaying bridge [17,18].

The second challenge, that has motivated the research described in this paper, is in measurement of the medio-lateral GRFs synchronous with the bridge response in situ and during SLE. For a laboratory analog, limited direct measurement of medio-lateral GRFs may be possible. However, for in-situ measurements the only possibility seems to be to use wearable sensors such as pressure insoles [19,20] or inertial measurement units (IMUs) [21,22].

The medio-lateral GRFs that have caused problems in footbridges have themselves the potential to be used for the identification of dynamic (modal) properties. The benefit of this approach for vertical GRFs has been proven [23–25] recently by studies on footbridges using inertial measurement units (IMUs) conventionally used in biomechanics research. It is logical to extend the idea to using and measuring medio-lateral GRFs.

1.1. Humans as lateral force generators for experimental investigation of structure dynamic properties

Using humans to force vibrations in structures during dynamic investigations is particularly common in the study of vibration serviceability of floors, footbridges and stadia. Proof testing of the first Wembley Stadium in 1923 used a crowd swaying and stepping [26], and metronome-prompted walking is now standard procedure during vibration serviceability checks for footbridges and floors [27,28].

Until recently only the response to human dynamic loads has been measureable, not the loads themselves. With proven capability to measure in-situ vertical forces due to both walking [23] and jumping [24] it is now possible to study the excitation mechanism in detail as well as to evaluate the dynamic characteristics using standard procedures of experimental modal analysis (EMA) [29].

The idea of EMA using human dynamic forces is attractive for two main reasons. First, it avoids the logistical complexity of mechanical excitation that is heavy, cumbersome and requires an electrical power supply, sometimes driving a hydraulic power pack. Second, the dynamic forces generated can be substantial.

These dynamics forces in the vertical direction (vertical GRFs) are usually defined in terms of their activity rate f_p which for walking and jumping is the footfall rate. The activity rate f_p is also used for bobbing which is like jumping but involves constant double stance. The GRFs are normalised by body weight (W_p), to form the dynamic load factors (DLF) α_n, which can reach almost twice body weight [30]. Typically a GRF is represented as a summation of perfect sinusoids at multiples of f_p and scaled by DLFs:

$$\mathrm{GRF}(t) = W_\mathrm{p}\left[\sum_{n=1}^{N_\mathrm{T}} \alpha_n \sin\left(2\pi n f_\mathrm{p} t + \varphi_n\right)\right] \tag{1}$$

where φ_n is the phase angle of the nth harmonic of f_p and N_T is the total number of harmonics.

These GRFs are greater than those that can be generated by the type of shaker typically used for modal testing [31] for which the maximum sinusoidal force amplitude of 445 N is available only above 2.3 Hz.

The frequency range of GRFs is also self-evidently better matched to natural frequencies of structures sensitive to human dynamic loading. The feasible range of human activity rates f_p is limited to a narrow frequency band up to approximately 4 Hz, depending on the activity, but because the vertical forces are not perfectly sinusoidal, much of the force is contained in higher harmonics.

The capability for forced vibration testing using human 'shakers' has been explored in more detail elsewhere for vertical forces and response [24], so the aim of this paper is to study the nature of human medio-lateral GRFs and their application to study of loading and response mechanisms for lateral vibrations in relevant structures.

Jumping is known to generate medio-lateral GRFs [32], but the greatest interest is for walking, where the largest medio-lateral GRFs are generated at half the footfall or pacing rate, $f_\mathrm{h} = f_\mathrm{p}/2$. Very little information is available about these forces. Published information in Ref. [33] gives a single DLF example as 0.04 for walking, while more recent data summarised in Ref. [11] provide an average DLF = 0.04. Code provision for walking [34] in design for vibration serviceability is for a single medio-lateral DLF value i.e. 0.1; for running, the quoted DLF is 0.2.

There is also very little information available on lateral forces generated by on the spot activities such as stepping or swaying. Deliberate swaying has been used for free vibration testing of tall buildings and masts [35] and there is clear potential for reliable structural characterisation if the forces can be measured.

1.2. Measuring medio-lateral GRFs

In the case of walking, the ideal means of recovering GRFs is to cover the entire walking path with force plates, but this approach is prohibitively expensive and presents problems of data acquisition. Some studies have instrumented short sections of walkway [36,37] but these have not reported medio-lateral GRFs. Instrumented in-soles [38] are an alternative, also complicated by expense and data fusion, but they do offer the possibility of measuring GRFs on a structure in motion.

In the case of jumping or swaying on the spot, individual force plates can be used for measuring GRFs both in the lab and in-situ, although many force plates are too small for reliable use. Even so single force plates have been used for capturing individual footfalls that have been used (with some assumptions) to generate DLFs used in practically all the current the UK for design of floors, stadia, footbridges, etc.

The estimation of the GRFs using measured body kinematics and Newton's Second Law is an appealing alternative, eliminating the need for force sensors at the point of contact:

$$\mathrm{GRF}(t) = \sum_{i=1}^{N} m_i a_i(t) \tag{2}$$

where m_i is the mass of a body part i and a_i is the corresponding acceleration at the centre of mass and the product $m_i a_i$ is the inertia force. By summing inertia forces across N body parts the GRF can be estimated.

Initial investigations with this approach used Coda cameras to track active optical markers in a trial [39] that validated the approach using a force plate as the reference sensor for recovering vertical jumping and bobbing forces. Body part masses for this study were based on cadaver data and data from live humans [40] scaled by mass of the test subject. The same principle has been used in optical motion capture system measurements to estimate vertical GRFs for walking on the spot [41], showing that a marker model with 19 passive markers can reproduce directly measured GRFs reliably.

The key challenge here for applying the principle to field measurements is that for reliable operation, optics-based motion capture is effectively confined to laboratory environments within a limited spatial volume. IMUs are not restricted to laboratory use but their previous use in field measurements [22] has been to recover timing information, not the GRFs themselves.

The discovery [23] that a single carefully placed inertial measurement unit (IMU) can capture the important features of acceleration of a human body centre of mass has led to their application in moving force identification and experimental modal analysis of a footbridge using the vertical component of GRF for walking and jumping [24]. This paper extends the concepts to investigation of the medio-lateral GRF components and their use in forced vibration testing.

1.3. Structure of the paper

Initially a large database [42] of 852 walking measurements obtained using an instrumented treadmill and 85 volunteers is used to identify the characteristics of medio-lateral GRFs for walking.

Next a smaller sample of more recent treadmill walking tests is used to evaluate reliability of recovering medio-lateral GRFs using IMUs and the reference (treadmill) GRFs checked against the larger data set. The process for estimating the GRFs is described in detail, highlighting the difficulties in the process which are:

1. correctly resolving the IMUs accelerations to the medio-lateral plane,
2. fusing data streams from IMUs and treadmill using a separate acquisition system and
3. identifying the body location or the weighted combination of body locations required to provide the most reliable GRF estimate.

Medio-lateral walking GRFs are relatively small compared to those that can be generated by deliberate swaying designed to excite lateral vibrations of certain types of structures. Such swaying activity is not natural during human locomotion or (so far) even entertainment and only has the purpose of modal testing. However, the potential of this approach is worth studying as it would greatly simplify study of certain types of structure that are logistically challenging for conventional forced vibration testing and EMA. Hence, using the methodology developed for walking, medio-lateral GRFs are measured for swaying both directly and indirectly with a view to application in field testing.

The paper finishes with such an application, to vibration testing of a multi-storey building.

2. Medio-lateral walking GRFs

Experiments were carried out using an ADAL3D-F instrumented treadmill at the University of Sheffield [42]. This treadmill, which is now out of service, was able to measure the vertical, medio-lateral and anterior-posterior components of the GRFs for each foot for ambulation with belt speeds up to 10 km/h. The treadmill used a pair of belts moving on smooth surfaces supported by piezoelectric triaxial load cells so as to capture the triaxial forces of the left and right feet (see Fig. 1).

2.1. Walking GRF database

From its installation in 2007 until 2014 the treadmill was used to acquire a database of 852 walking force time histories from a diverse set of 85 volunteers: 57 males and 28 females, body mass 75.8 ± 15.2 kg (1σ), height 174.4 ± 8.2 cm, age 29.8 ± 9.1 years [42,43]. Fig. 2 shows two samples from this database, for the same subject but different ends of their walking speed range, as time series and discrete (fast) Fourier transforms (FFTs). The FFTs are (sinusoidal) amplitudes of GRF component normalised by body weight and calculated using a time window T that includes an integer number of left and right footfall pairs to minimise spectral leakage. These GRF samples are from the same subject for two pacing rates. The time

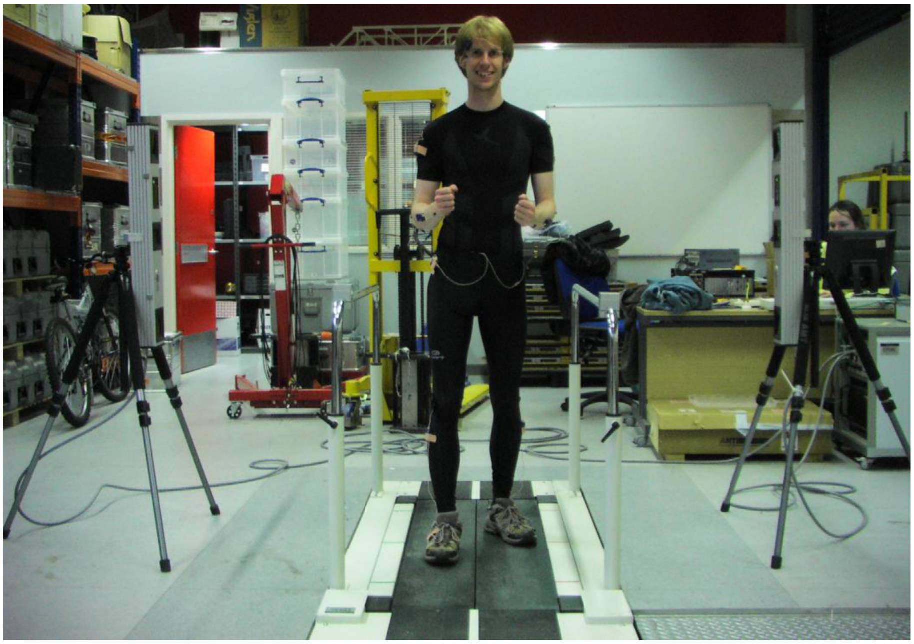


Fig. 1. Student on instrumented treadmill.

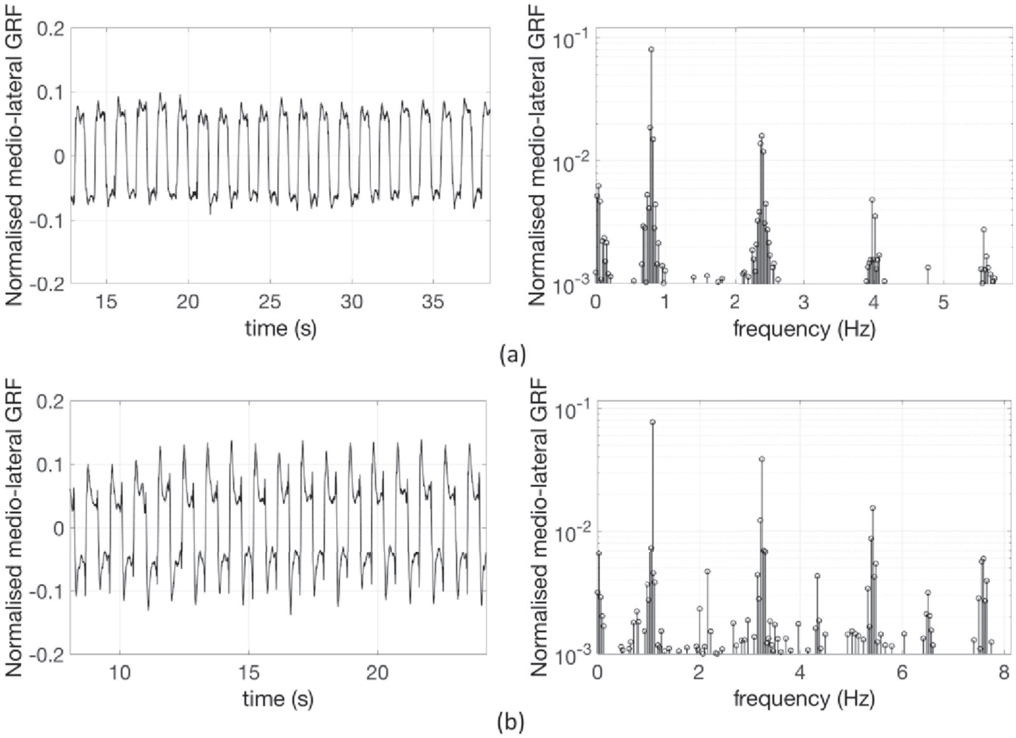


Fig. 2. Samples of medio-lateral GRF directly measured by treadmill as time series and FFT for the same subject at two different pacing rates and speeds: a) 1.66 Hz and 0.66 m/s b) 2.17 Hz and 1.62 m/s.

series of M data points sampled at $f_s = 200$ Hz are shown from $M/4$ to 3M/4 samples for clarity, and FFT line spacing is $f_s/M = 1/T$.

For all subjects, the shape of the GRF time series changes from resembling a square wave with raised corners at low pacing rates to one having enhanced edges at higher pacing rates, with increasing emphasis on higher harmonics.

The natural variability in what is effectively a narrow band random process [44] appears as a spread of spectral lines in Fig. 2. For a perfectly periodic process, there would be a single line representing each harmonic, and the DLF would be simply the value of the FFT line. Because vertical GRFs used to derive DLFs provided in design guidance [45,46] were obtained by replicating a single footfall trace (from a force plate), they are by definition perfectly periodic and will have single lines at multiples of the pacing rate f_p so it is natural to assume the same applies to medio-lateral GRFs.

Since, GRFs obtained from continuous walking using a treadmill are a narrow band random process [44], the DLF is calculated as the square root of the sum of the squares (SRSS) of FFT amplitudes centred on the frequency of the peak value ± 5% for all harmonics i.e. at multiples of f_p. Gait asymmetry results in subharmonics in vertical GRFs at odd multiples of half the pacing rate $f_h = f_p/2$.

For medio-lateral GRFs the fundamental frequency is f_h since the human body centre of mass executes a complete lateral oscillation cycle for every pair of left and right footfalls. What Fig. 2 shows is force at odd multiples of f_h and (remembering the logarithmic scale) negligible force at integer multiples of f_p (even multiples of f_h). This is significant because the constraints of treadmill walking and the certain orientation of the force transducers mean that medio-lateral GRFs are orthogonal to the walking direction. Hence it should be expected that medio-lateral GRFs measured indirectly using IMUs, whose local axes are aligned with the canonical body axes, should also have negligible force at multiples of f_p.

The DLFs obtained from all 852 medio-lateral GRFs are summarised in Fig. 3. The first harmonic component at f_h indicates a DLF well above other published values corresponding to rigid surfaces [33,47] but consistent with the ISO 10137 value [34]. The first harmonic shows limited frequency dependence compared to 3rd, 5th and 7th harmonic components at 3 f_h, 5 f_h and 7 f_h.

Subject to the constant treadmill belt speed, treadmill data naturally include variations in timing, shape, and level even if (like force plate data) real world data freed of laboratory constraints would in some sense be different. They are however the only currently available technology for direct measurement of walking GRFs and they provide the 'gold standard' for checking indirect measurements via Equation (2) using optical markers and IMUs which can then be used as proxies in less restricted environments.

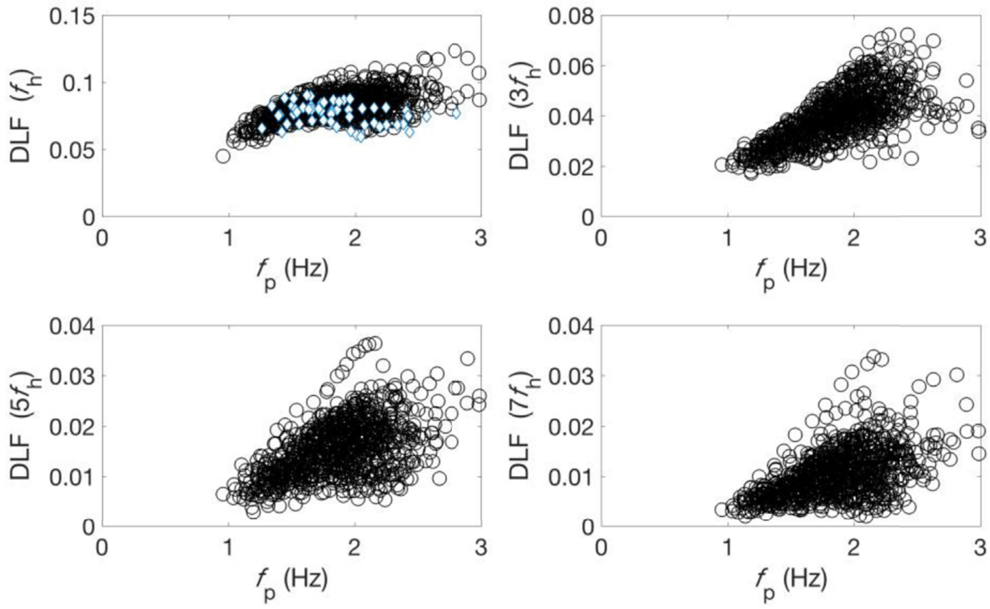


Fig. 3. Lateral harmonics from walking (after [41]). The white diamonds are obtained from the experiments described in section 2.2.

2.2. Walking GRF recovery using optical markers and IMUs

The ADAL treadmill was used in 2014 and 2015 as a reference in two studies on medio-lateral GRF recovery by application of Equation (2). Six student subjects (s1-s6, 33 measurements) participated in 2014 and three subjects participated in 2015 (s7-s9, 24 measurements). Eight male students and one female participated, with weight mean and standard deviation 77 kg and 12.5 kg, respectively. All tests involving human subjects were approved by the College of Engineering, Mathematics and Physical Sciences Research Ethics Committee at the University of Exeter.

In both studies APDM Opal IMUs were used and treadmill analog signals were acquired using the data acquisition function of a Codamotion (Coda) motion capture system. For the 2014 measurements (only), which are reported in Ref. [23], the full-body 3D motion data were recorded using Coda active markers, but these data are not reported here.

The Opal IMUs were set to acquire data at 128 Hz. The Coda system was operated at 100 Hz acquiring treadmill signals for the anterior-posterior, medio-lateral and vertical directions from each belt, a total six channels. Low pass zero lag fourth-order Butterworth digital filter was used to remove noise while preserving the frequency content corresponding to the first four harmonics of f_p and sub-harmonics (multiples of f_h) of the walking GRF signal for the fastest pacing rate.

The marker placement protocol was based on the full-body Plug-in Gait [48] and the marker arrangement is shown in Figs. 4 and 5 for tests conducted in 2014 & 2015, respectively. On both occasions IMUs were attached to seventh cervical (neck) vertebra (C7) using medical tape, and sternum (ST), lower back (LB) and navel (N) using APDM straps. The data from the navel were not used in the analysis as they produced poor quality data compared to the IMUs attached closer to bones.

The procedure described in Ref. [23] for validating the use of IMUs for recovering walking vertical GRFs has been adapted for the case of medio-lateral GRFs. The IMUs were attached without particular concern for orientation since a major role of an IMU is to report the orientation of its local coordinate system (LCS) and axis alignment compared to a world coordinate system (WCS) of (magnetic) north, west and up. The process for identifying the IMU orientation is not specifically described but manufacturer literature states that it uses "… *a state space model with a Kalman Filter for orientation estimation. This approach uses a complex fusion of the accelerometer, gyroscope, and magnetometer.*" The orientation for each sample is reported in the form of a quaternion, which is a four-component (time varying) vector that can be used (offline) to convert accelerations reported by the IMU in LCS to accelerations in WCS. This is the first step in the processing.

Because the IMU and Coda data acquisition operated independently and used differing sample rates, the next step is to resample the treadmill data (at 100 Hz) to the IMU sample rate ($f_s = 128$ Hz) then to align and merge two data streams.

The alignment is a two-step process. Opal IMUs can operate in two modes depending on the distance between a group of synchronised IMUs, using 2.40–2.48 GHz industrial, scientific, and medical band radio for communication. For field use of a set of IMUs in 'synchronised logging' mode, each IMU stores its own data and communicates with others to preserve synchronisation. The IMUs start recording immediately they are removed from the docking station and stop when they are re-docked, so an acquisition session will record a continuous sequence of events. For laboratory use in 'synchronised streaming' mode a wireless access point (WAP) collects data transmitted by each IMU. In this mode, data acquisition can easily be started and stopped in software for individual events and the WAP can be configured to output start and stop voltage trigger signals

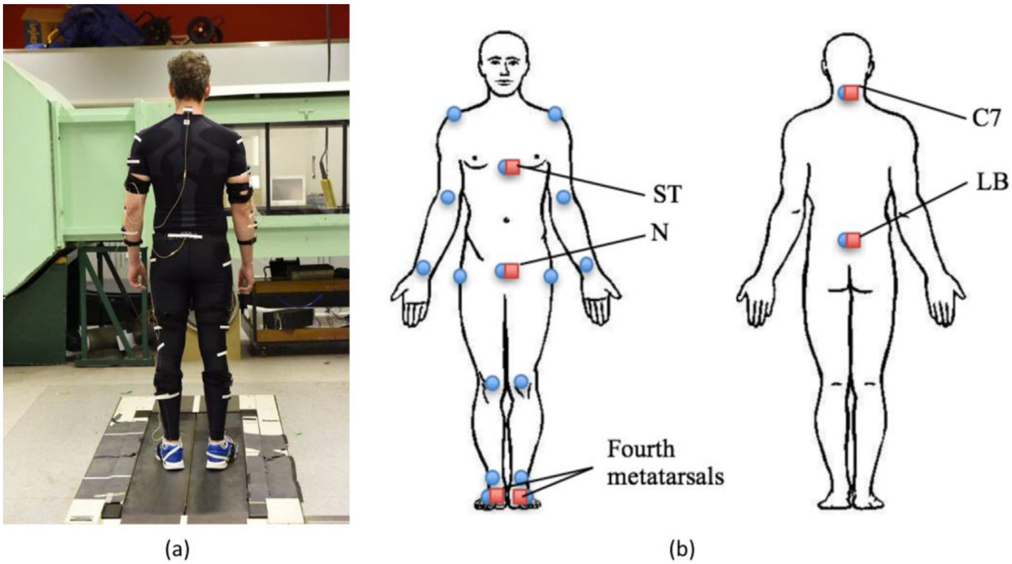


Fig. 4. (a) Subject instrumentation layout for 2014 measurements with s1:s6. (b) location of Coda markers (blue circles) and Opal IMUs (orange squares). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

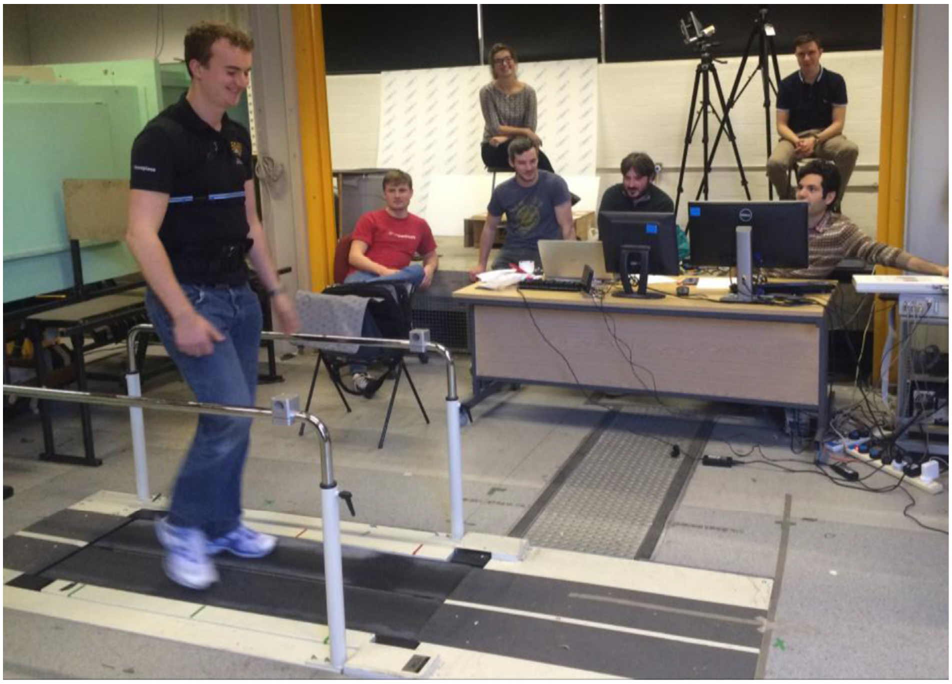


Fig. 5. 2015 measurements with s7:s9.

to enable synchronisation with external devices. In the measurement sequence the Coda system was started before the IMUs so the reference (treadmill or force plate) signal preceding the trigger can be truncated. This provides imperfect alignment so a second alignment step finds a time shift for highest correlation between vertical GRF in the reference and the vertical signal for an IMU at C7. C7 is used because it has been shown [23] to provide the closest match to the (treadmill) reference GRF. The output from this process is GRFs in the treadmill or force plate (reference) coordinate system or RCS and IMU accelerations in WCS.

A second conversion is then required to rotate (yaw) IMU signals in WCS to the RCS. The rotation angle could be obtained from the angle between RCS and magnetic north based on local magnetic declination data and maps and drawings of the

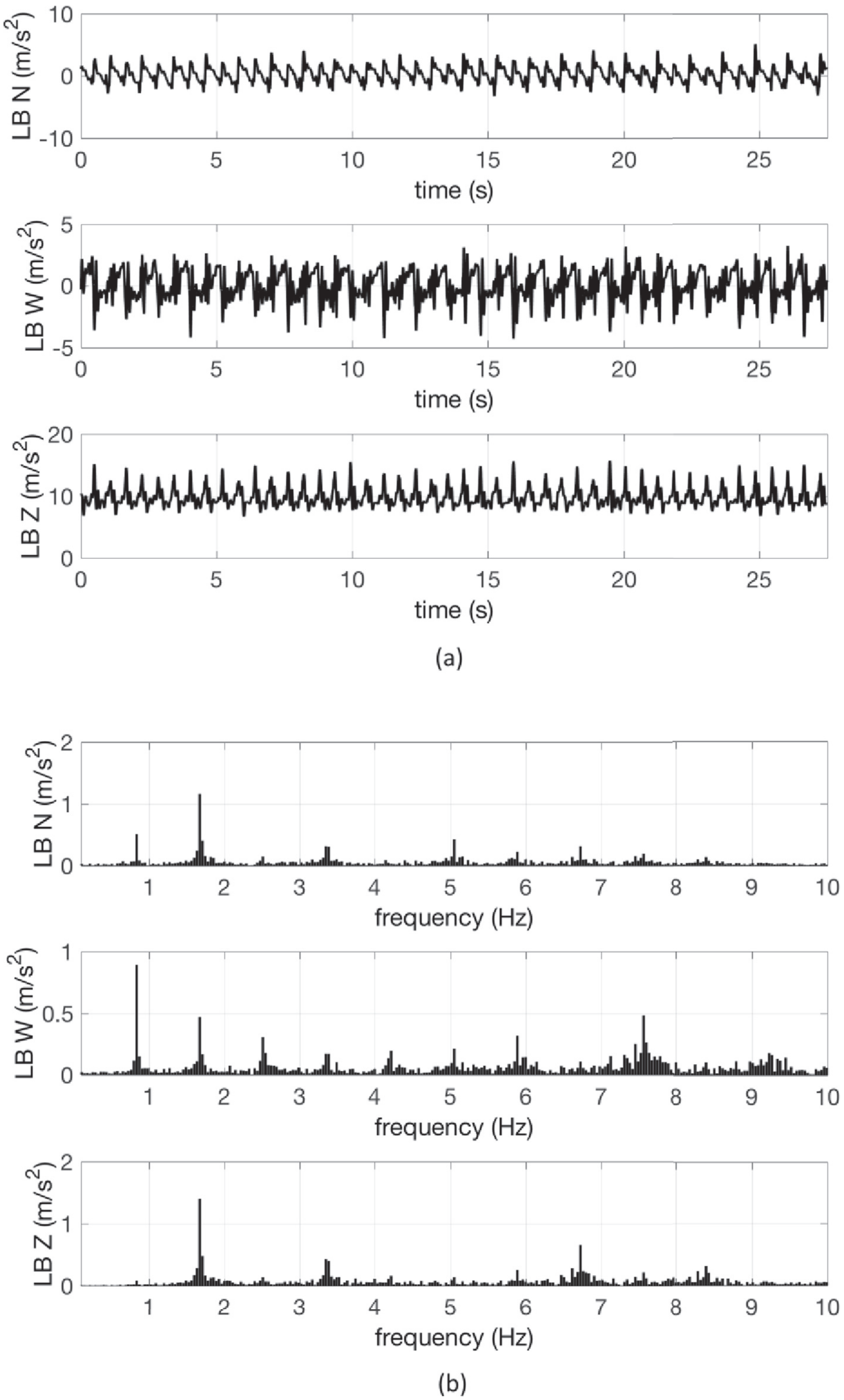


Fig. 6. Raw acceleration data from IMU on lower back (LB), in WCS, for subject s6 at natural pacing rate. N-north, W = west, Z = up, (a) as time series and (b) as FFTs.

reference system location. For the treadmill at University of Sheffield, the anterior-posterior axis (walking direction) is 14° west of north. This means that in WCS the north component of GRF should already be dominated by anterior-posterior GRF which occurs at f_p while the west component should be dominated by medio-lateral GRF at f_h.

Fig. 6 shows, as time series (M samples) and as FFT lines (with spacing f_s/M), one example of raw IMU accelerations in WCS, before alignment with treadmill data, for lower back (LB) during pacing at 2 Hz. The character of vertical and anterior-posterior GRFs is clear and as expected in the N (north) and Z (up) components respectively, but it is less clear that the west (W) component for the IMU signal contains a medio-lateral GRF signal with the nature shown in Fig. 2. This leads to the question of whether the IMU magnetometers provide for a reliable orientation consistent with the true orientation, and whether there might be an alternative approach.

One such approach to obtain an improved alignment of data dominated by medio-lateral forces expected at f_h and odd harmonics is to rotate (yaw) the WCS acceleration data about the vertical axis so as to maximise the force component at f_h. The resulting 'optimal rotations' were found to differ between IMUs for the same subject so this approach was abandoned. Before

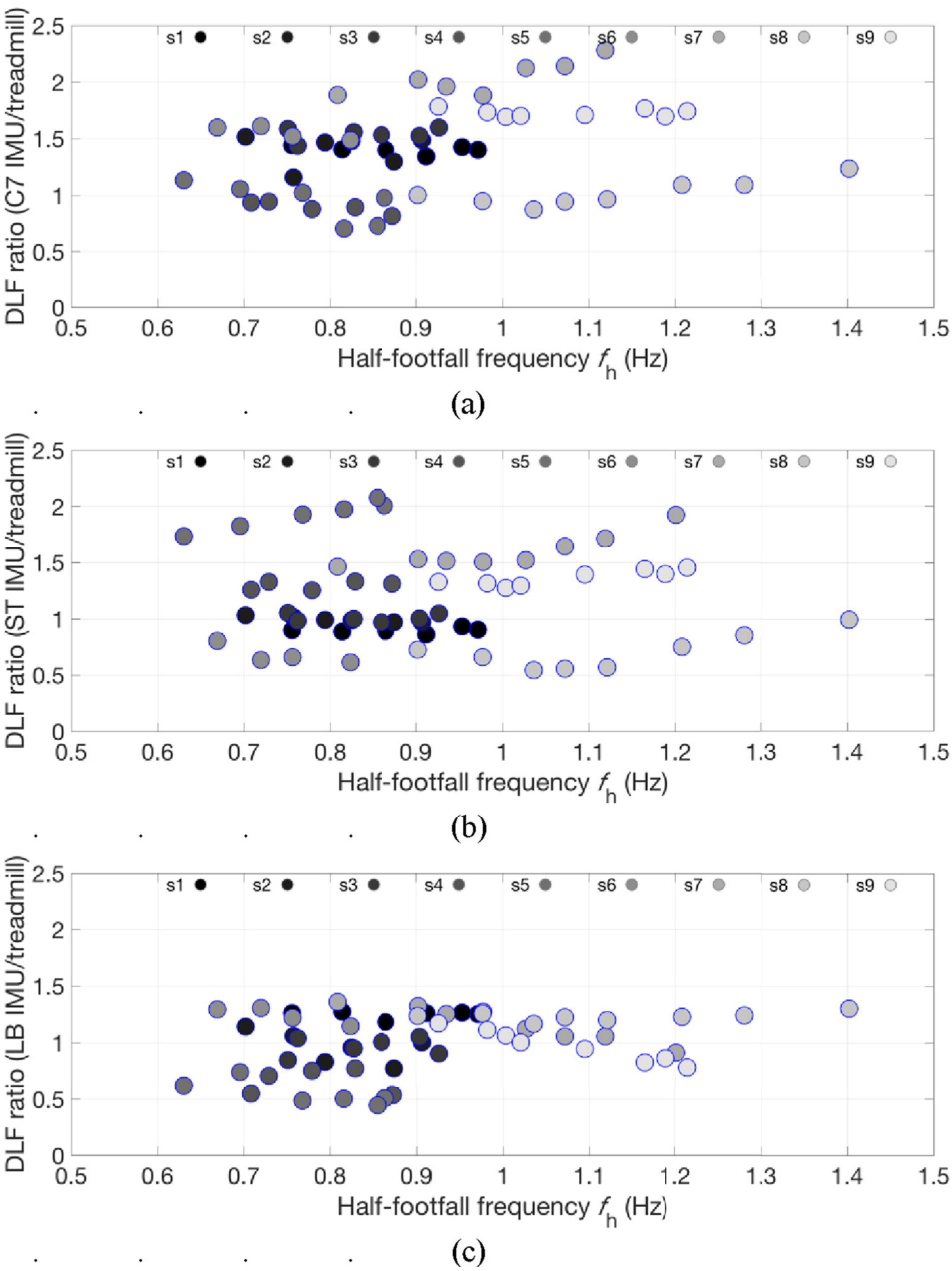


Fig. 7. Comparison of DLFs from individual IMUs with treadmill DLFs for a) C7, b) sternum (ST) and c) lower back (LB).

reverting to the default 14° rotation, the reliability of the IMU magnetometers for absolute and relative accuracy was checked in laboratory and outdoor environments close to a range of different buildings around the campus of the University of Exeter and confirmed to be within 5° in most cases and (with one exception) not exceeding 20°.

No IMU measurement had been made for an IMU aligned exactly to the treadmill axis (to check its reliability in that specific case) so the default method used the 14° rotation before the time alignment step.

The final step in the process compares the medio-lateral GRFs obtained by IMU to the reference GRFs. As previously mentioned, because a continuously recorded GRF is a narrow-band process, DLFs are obtained using SRSS of FFT line amplitudes and comparison is expressed as a ratio of GRFs obtained using IMUs to the directly measured reference values. Fig. 7 shows the comparison result for each IMU (C7, ST, LB) for the entire data set of 2014 and 2015 measurements, shading-coded according to test subject. There is large scatter, reduced somewhat for LB measurements.

The scatter for ratios for a single IMU could be due to trunk sway, which varies among subjects with different gait styles, so a better approach would be a linear combination of IMU signals instead of the single IMU approach that works well for vertical GRFs [23]. The ideal approach would be an optimisation of a set of weighting factors for the four markers, summing to unity, but a semi-qualitative approach was adopted for just two markers due to the strong requirement for minimal instrumentation. Considering that pedestrians rotate their trunks about the anterior-posterior axis to some degree during the gait cycle, a mix of upper trunk (C7, ST) and lower trunk (N, LB) should better reflect the inertia forces. Hence all possible pairs of C7 with N or LB, and of ST with N or LB were examined individually in time and frequency domains and collectively by convergence of DLFs around unity. Navel (N) data were less useful and while C7 was the best marker for recovering vertical GRFs [23], ST worked better for recovering medio-lateral walking GRFs, in combination with lower back (LB) data.

A weighting of 30% lower back (LB) and 70% sternum (ST) appears to work best in terms of minimum standard deviation around unity, and is a moderate compromise between best weighting for LB of 40% for 2014 data and 20% for 2015 data. Fig. 8 shows an example comparison for the record corresponding to Fig. 6, showing the time series of M samples from 3M/8 to 5M/8 samples in Fig. 8(a) and the FFT (with resolution f_s/M) in the vicinity of f_h in Fig. 8(b). The comparison for DLFs over all nine subjects is shown in Fig. 9.

All records were used to generate the result, although it is clear from cross-talk between f_h and f_p frequency components, and from other quality indicators such as fit of vertical GRFs and intra-subject pacing rate variability that some data points could be excluded. These indicators could be used to judge reliability of field-acquired IMU data for reconstructing medio-lateral GRFs.

Better results for a specific individual could be possible if a reference (treadmill) is always available to check, but the complexity and cost of such a device defeats the simplicity of the process for general application. Hence applying a small adjustment and adding a tolerance for data obtained using two markers would be a good approach. Given the stronger higher harmonic components in IMU data compared to the reference it would not be advisable to use the method directly for components at $3f_h$, $5f_h$ etc. However, this might not be true for medio-lateral GRFs for a different type of motion such as

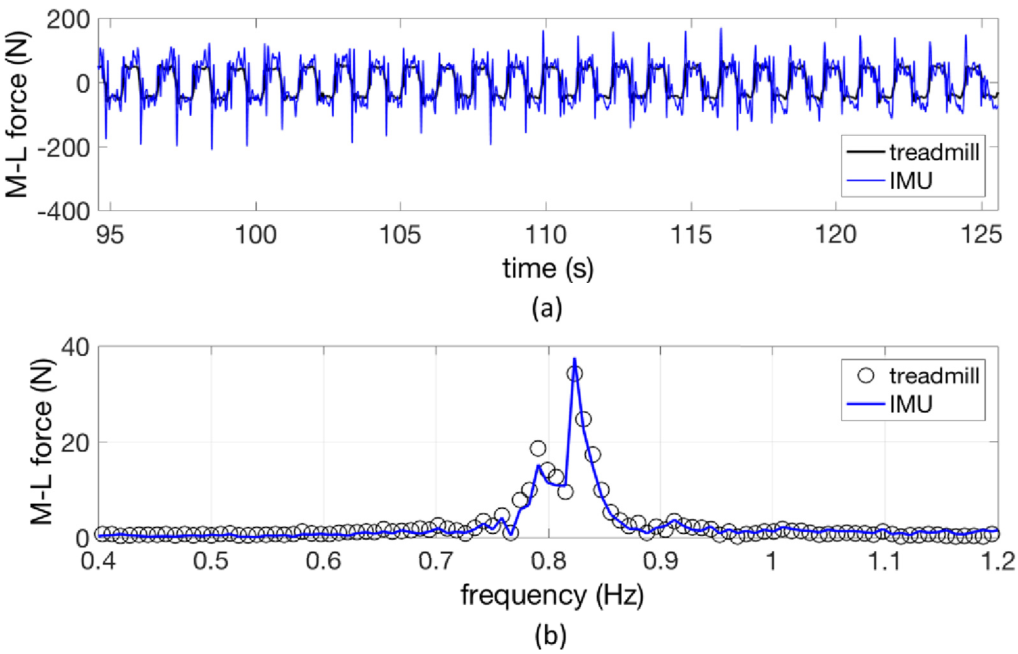


Fig. 8. Comparison of 30% lower back (LB) and 70% sternum (ST) signal with treadmill force for the record of Fig. 6 as: (a) time series and (b) FFT.

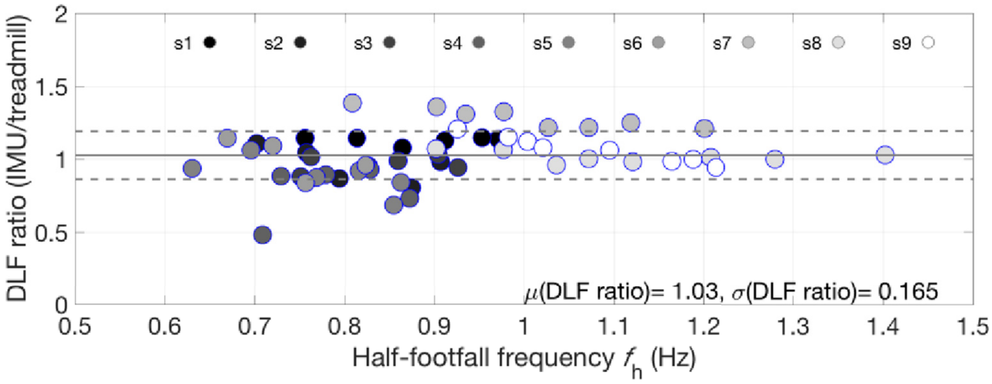


Fig. 9. Combining IMUs: ratio of DLF using IMU to DLF using treadmill for all subjects using comparison data of which Fig. 8 is an example.

swaying on the spot, where contribution at higher harmonics can be very relevant and where IMUs could more reliably represent forces at higher harmonics.

In summary, the best approach to indirect recovery of medio-lateral walking GRFs appears to be to transform the IMU data to WCS, then to apply a second rotation about the vertical axis to align with the walking direction and finally to multiply body mass by a linear combination of lower back and sternum accelerations. The second rotation can be based on a static measurement (if available) from an IMU aligned with the walking direction, alternatively using the known alignment of walking direction. In most cases differences in IMU and actual alignment should lead to small cosine errors.

3. Swaying GRFs

Research to date on GRFs generated by swaying has focused on the effects of coordinated spectator swaying on grandstands. These studies have therefore usually considered the context of the vibrations by simulating the stadium environment and swaying motion, for example [49] where swaying was used to induce motion of a platform behaving as a horizontal single degree of freedom oscillator. That study measured horizontal swaying forces with DLFs approximately 0.2, but dropping to 0.07 when swaying and platform natural frequencies coincided, and the authors found it impossible to sway at frequencies above 2 Hz. More recently [50] sway forces due to swaying while standing or sitting were studied along with forces generated as a side effect of jumping, again aimed at application to assembly structures such as grandstands. Sway forces were measured at five frequencies up to 1.5 Hz, with DLFs peaking at 0.15. Apparently in all these studies swaying was always with double stance simulating conditions on assembly structures.

On the other hand, the maximum levels of sway forces that could be generated to shake buildings and towers for free vibration testing [35] have not yet been measured. The technique has been demonstrated for the seven storey University of Exeter Physics Building (Fig. 10 (a)). This is a substantial reinforced concrete structure with 22 m × 16 m floor plan for storeys

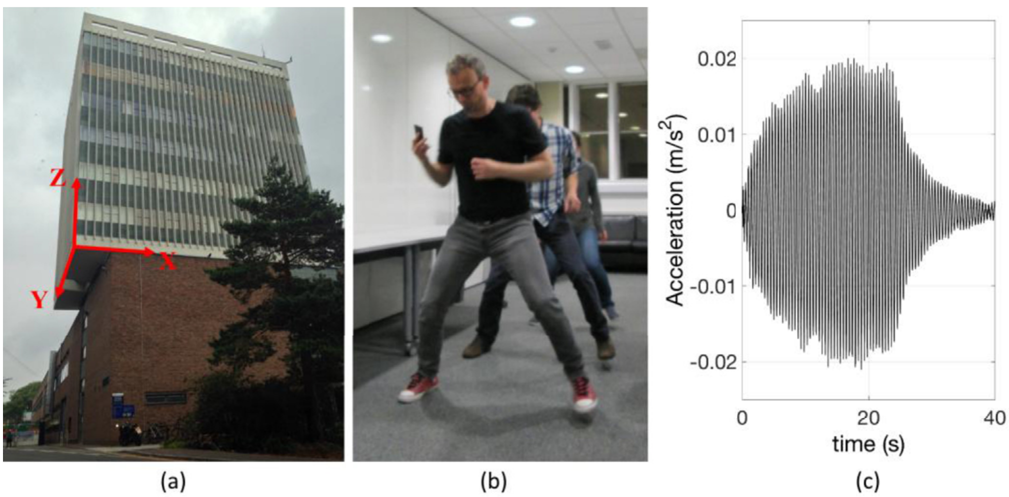


Fig. 10. Vibration testing of University of Exeter Physics building a) Physics Building, b) human shakers and c) build-up of response and decay on ceasing.

2–7. Ambient vibration data established that the fundamental natural frequency in the y-direction as denoted on the left plot in Fig. 10(a) is approximately 2.1 Hz. Hence three researchers (*not* instrumented with IMUs) attempted to excite this mode by stepping at $f_p = 4.2$ Hz, timed by a metronome to produce a cyclic lateral force at $f_h = 2.1$ Hz, as shown in Fig. 10(b). The induced response, shown in Fig. 10(c), was half the peak allowable acceleration level for vibration serviceability in a one-year return period according to [34].

The exercise shows that human forcing is an attractive proposition for certain types of structure where forced vibration may be unfeasible. It would be particularly attractive if the forcing function could be reliably measured, so that classical experimental modal analysis techniques could be applied to the input/output data without the considerable logistical expense of mechanical shaking. To test the hypothesis a laboratory study was used to compare medio-lateral GRFs obtained by IMUs with reference GRFs.

3.1. Swaying GRF measurement using force plate array

A series of experiments on 15th September 2016 at Shuguang Hospital, Shanghai, was designed to measure the medio-lateral GRFs during swaying both directly (as a reference) using an array of four force plates and indirectly using Opal IMUs. The arrangement of four AMTI force pates (Fig. 11) ensured that the complete GRF was recorded with very little restriction on stance so that feet could be planted quite far apart for maximum effect.

Test subjects were s10 (weight 868 N), s11 (875 N) and s12 (645 N), swaying with metronome-prompted stepping rates from a sedate 40 footfalls per minute to a very energetic 240 footfalls per minute ($f_p = 0.667$ Hz to $f_p = 4$ Hz), corresponding to sway frequencies of $f_h = 0.333$ Hz to $f_h = 2$ Hz. For these datasets, the test subjects alternately lifted left and right feet. In addition, s10 swayed without lifting feet at frequencies from $f_h = 0.417$ Hz to $f_h = 1.167$ Hz, above which it was physically impossible to maintain a double stance, keeping both feet on the ground at all times. Selection of participants and the test protocol satisfied the requirements of the Tongji University Medical Ethics Committee.

Fig. 12 shows three sample time series of M samples at $f_s = 128$ Hz and their corresponding FFTs with resolution f_s/M. Review of the time series shows that as f_h increases, the shape of the medio-lateral GRFs (when lifting feet) begins with sharp accentuated corners as with Fig. 12(a), leading to relatively strong odd harmonics to high order. The corners flatten to make the footfall sequence more closely resemble a square wave (Fig. 12(b)), increasing the third harmonic at the expense of higher harmonics, while at the highest frequencies the GRFs adopt a triangular shape with relatively weak third harmonic (Fig. 12(c)).

GRFs peak for a stepping rate around 180 footfalls per minute ($f_h = 1.5$ Hz) and it is difficult (but not impossible) to sway faster than 240 footfalls per minute, corresponding to horizontal structural modes above 2 Hz.

Medio-lateral DLFs for the set of 53 records are summarised in Fig. 13 for first and third harmonics, reflecting these observations. The third harmonic peaks around $3f_h = 2.5$ Hz and declines slowly, showing the potential to excite modes above 2 Hz.

Fig. 11. Swaying on force plate array.

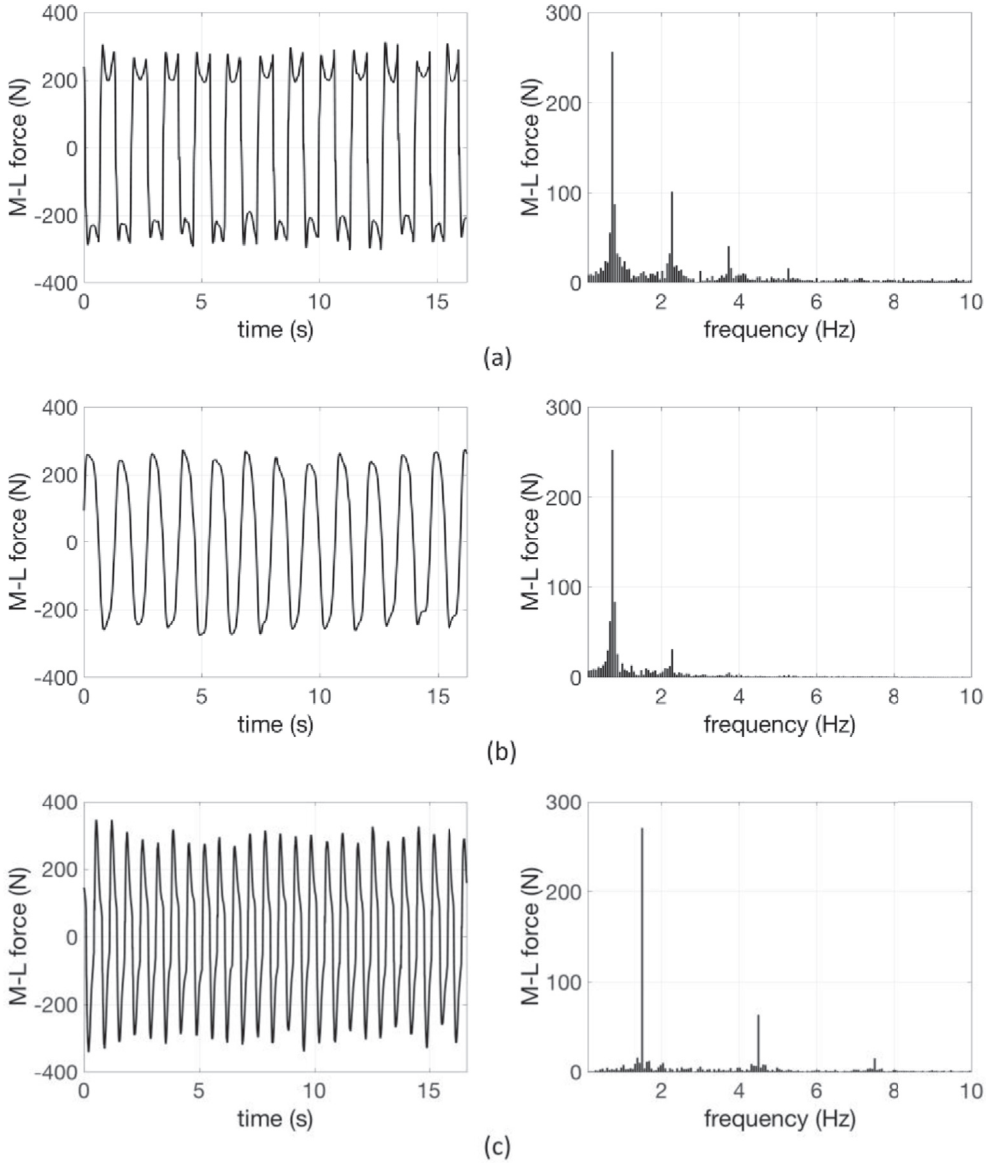


Fig. 12. Sample swaying medio-lateral forces as time series and as FFTs. (a) s10 swaying at 90 footfalls per minute ($f_h = 0.75$ Hz) (b) s10 swaying at same rate but without lifting feet (c) s12 swaying at 180 footfalls per minute ($f_h = 1.5$ Hz).

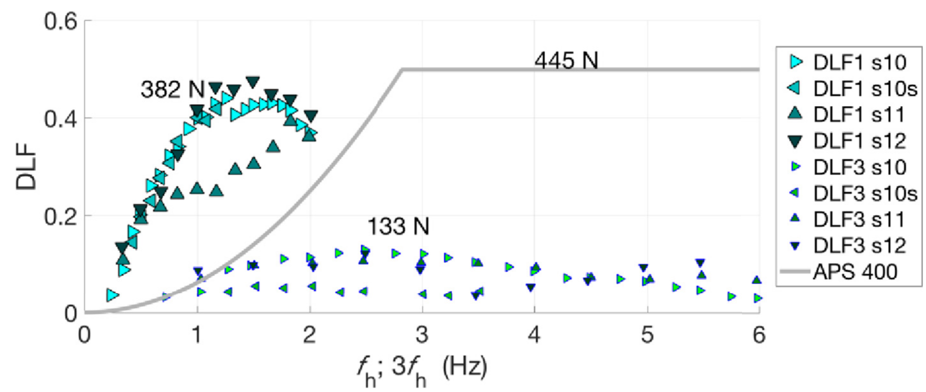


Fig. 13. DLFs for swaying showing peak force amplitude and (for reference) the APS 400 characteristic.

For comparison with capability of mechanical shakers, the theoretical maximum force output for an APS 400 shaker (APS Dynamics Ltd) is shown by the gray line in Fig. 13. This force characteristic is based on the shaker stroke (158 mm), mass of armature plus the standard set of reaction masses and peak motor force (445 N), and in real world conditions is not easily achieved. The theoretical DLF is obtained based on shaker total mass of 91 kg so it can be compared (almost) directly with s10 and s11 DLFs. The 'human shaker' is clearly more effective below 2 Hz.

The APS 400 is commonly used for forced vibration testing of a range of structures, although its mass of 73 kg for the motor body and armature and 18 kg for the masses makes it logistically challenging to manage on construction sites. Fig. 14(a) shows an example of this type of shaker in use for modal testing of Changi Mezzanine Bridge, Singapore [51] where the force output at the first lateral mode natural frequency was below 100 N. The rising force characteristic of the APS 400 (Fig. 13) can be shifted to a lower frequency band (reaching maximum value at 0.5 Hz) to achieve better performance at low frequencies by setting the shaker to drive a slip table loaded with 600 kg of extra mass (Fig. 14(b)). Even more powerful systems can be arranged using hydraulic actuators driving a slip table, but such arrangements are even more challenging logistically. The trolley arrangement (Fig. 14(b)) while remaining a 'gold standard' has been used once only, since the process of shipping and assembling almost 1000 kg of equipment per shaker dominates all other considerations. The opportunity to use the test team themselves as shakers for low frequency structures is very attractive, trading off some reduced accuracy and controllability with far simpler and much more efficient site application.

3.2. Swaying GRF recovery using IMUs and comparison with force plate data

For the Shuguang Hospital study, Opal IMUs were attached to test subjects at C7 (taped), navel (N, using a tensioned belt), sternum (ST, taped), lower back (LB, taped), forehead (H, taped) and right foot (RF, using a tensioned strap). A similar process to that described in 2.2 was adopted to compare IMU and reference GRF data.

First the IMU data were converted to WCS, then the independently collected force plate data were resampled to the IMU sample rate. Next, force plate data were time-aligned with the IMU data. The acquisition system provided no means for acquiring an IMU trigger signal so a single alignment step was used based on stamping three times on the force plate and aligning C7 and force plate vertical signals. Without pre-alignment by trigger signal, the starting point for searching for the optimal alignment included a manual step of approximate visual alignment using a graphical cursor.

As for the treadmill walking data, optimal alignment was checked by maximising content at f_h, but results were inconsistent, so the actual alignment of the force plate axis to the known direction of magnetic north was used.

GRFs obtained using IMUs were compared to the reference (force plate) DLFs using the same process used with the treadmill data. A variety of combinations of ST, LB and C7 were tried (in pairs) for the four data sets: s10, s11 and s12 swaying with no requirement for double stance and for s10 with both feet always on the force plate.

Fig. 15 shows the comparison (to force plate data) for combination of 10% of ST and 90% of LB for s10 in the two types of swaying in the same way that Fig. 8 compares walking M-L forces using a treadmill. Where a comparison can be made for the lower range of f_h, lifting feet induces a larger component of third harmonic.

Among the four data sets there was no overall best result, so the best combination is shown in Fig. 16 for each case, i.e. for each test subject due to their personal styles of swaying. C7 data are not used in any of the combinations since they provide worse comparisons than using ST. The average ratios are remarkably close to unity with small standard deviations whose calculation weights the individual values by the correlation functions in time domain, and which are reflected by the shading (dark is high correlation coefficient).

(a) (b)

Fig. 14. APS 400 electrodynamic shaker (a) in use for footbridge testing and (b) with high-mass slip tables to extend low frequency capability.

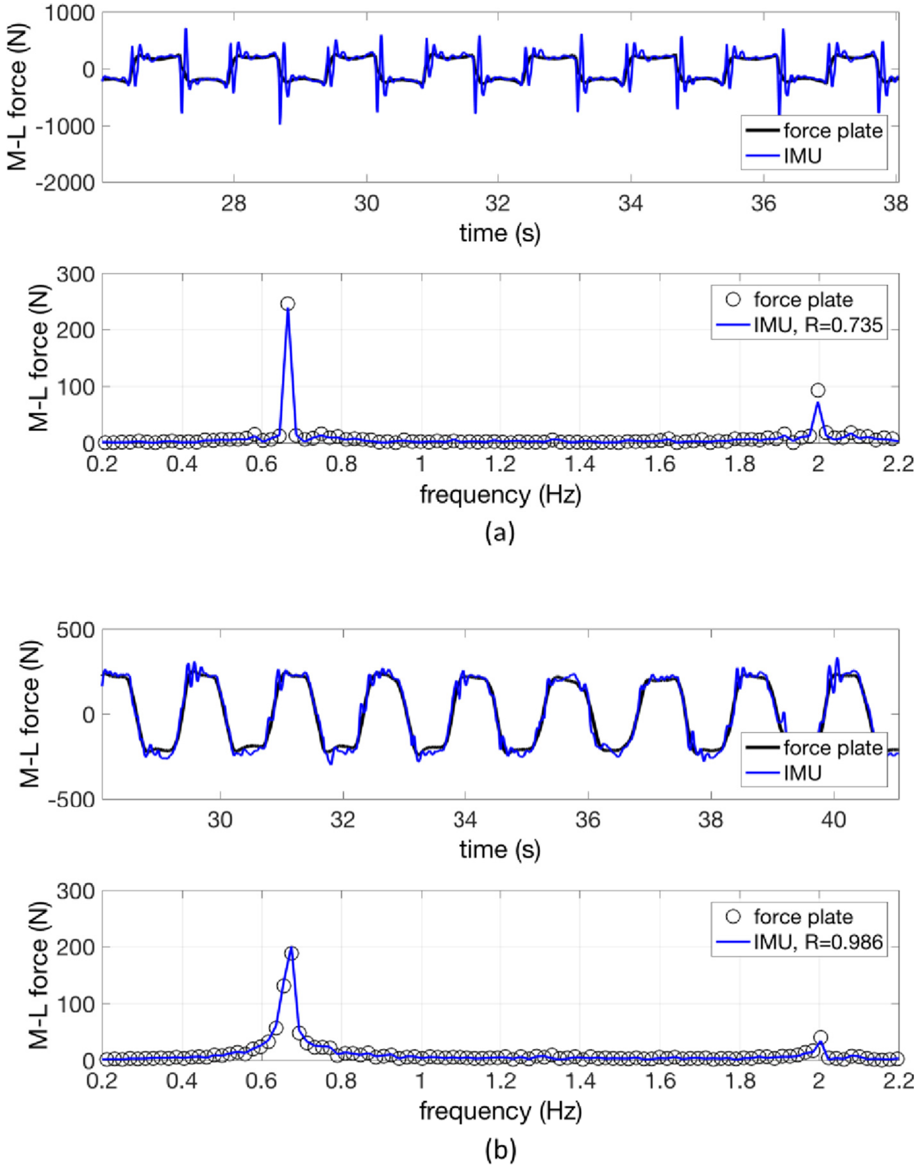


Fig. 15. Comparison of 90% lower back (LB) and 10% sternum (ST) signal with treadmill force for s10 at 80 footfalls per minute ($f_h = 0.667$ Hz) as time series and as FFT (a) with and (b) without lifting feet.

4. Application of human shaker

Two potential applications of IMUs for investigating the effect of lateral GRFs on structural response in situ are to study human-structure interaction (HSI) on footbridges and for forced vibration testing of a range of structures for which horizontal modes are important (including footbridges). Study of HSI merits a separate investigation and also requires access to a swaying footbridge, but there are many opportunities to evaluate the capability for modal testing. A simple example is illustrated for the Tumu Building, Fig. 17, which houses the College of Civil Engineering at Tongji University.

The building is an irregular shape, and there has been no modal test of the building, hence it presents an ideal opportunity. A simple experiment was set up using a pair of low-noise (model: Lance) accelerometers and a set of Opals IMUs. The low noise accelerometers were used based on experience with the University of Exeter Physics Building (Fig. 10(a)). In that study IMUs were used only to measure structural response, and it was found that the 128 $\mu g/\sqrt{Hz}$ noise floor resulted in a very noisy signal. The low-noise (~1 $\mu g/\sqrt{Hz}$) accelerometers used in parallel produced a much clearer signal so the dual-sensor approach was used at Tumu, with a pair of (Lance) piezoelectric accelerometers measuring the transverse response, in the weak (narrow, Y_T) direction.

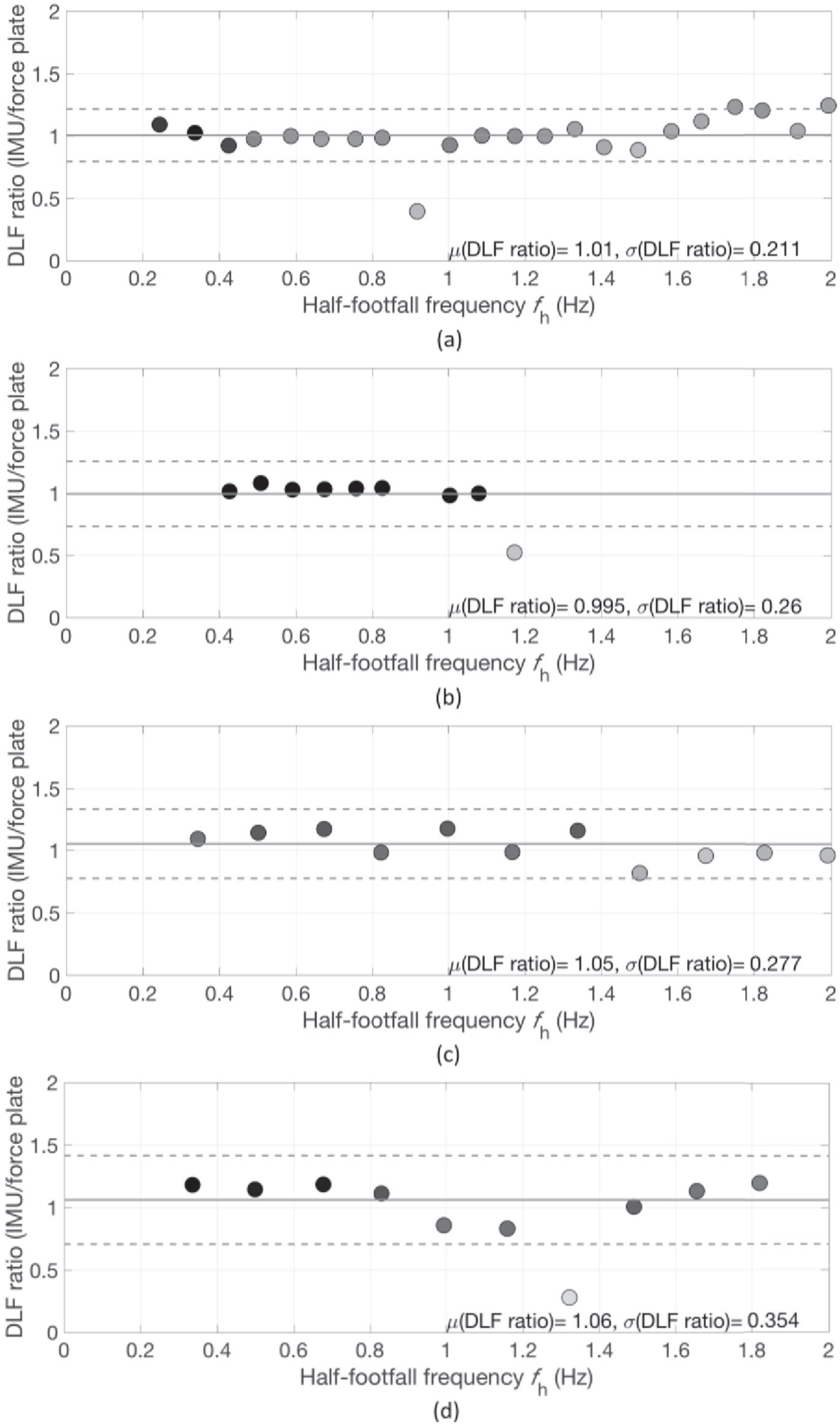
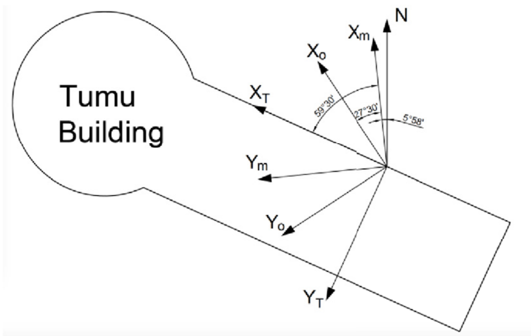


Fig. 16. Combining IMUs: ratio of (DLF using IMU) to (DLF using force plates) for three subjects and four data sets using comparison data of which Fig. 8 is an example. (a) s10 with 10% ST and 90% LB (b) s10s with 45% ST and 55% LB (c) s11 with 100% ST (d) s12 with 55% ST and 45% LB.

Among the various swaying exercises, the most useful was with a single test subject (s10), with IMUs attached to C7, LB and ST. One IMU was fixed to and aligned with the Lance accelerometer.

a)



b)

Fig. 17. Tumu Building, Tongji University a) front view, b) plan at measurement point. X_T, Y_T are Tumu Building axes with shaking and Lance accelerometer alignment in Y_T direction. X_m, Y_m are magnetic north and west and X_o, Y_o are orientation of Opal on Lance accelerometer according to quaternion data.

Building natural frequency was estimated as 1.623 Hz using low level ambient acceleration signals acquired using the Lance accelerometers, so stepping at metronome-prompted 194 footfalls per minute was used in an attempt to generate maximum response. It was found that for the response levels generated, the building responded most with a stepping rate of 190 footfalls per minute. Acceleration data for the building during one of the four short swaying sequences are shown in Fig. 18. The data have been band-pass filtered (1–2 Hz) and reveal clear build up and fast decay of building response. Despite the filtering, the IMU (Opal) attached to the Lance accelerometer remains very noisy.

The Lance accelerations are already aligned transverse to the building axis (Y_T direction in Fig. 17) and IMU signals were rotated to align with the same axis. The orientation of the IMU on the Lance accelerometer with respect to magnetic north and west (X_m, Y_m) was found to be 27.5° (X_o, Y_o) using the quaternion data from the IMU. While this differed from the true building orientation (59.5°), the quaternion-based IMU orientation was used to align body-mounted IMU accelerations to the building coordinate system because the same quaternion data had been used to convert the accelerations in IMU LCS to accelerations in WCS. In other words any error in the quaternion estimate is assumed to be common to all IMUs and is therefore cancelled. The Lance and IMU signals were time-aligned using a similar procedure to the foot-tapping for the force plate alignment, instead shaking the pair of (Lance and Opal) sensors gently to induce a common and recognisable signal.

4.1. System identification using IMU data

Data of Fig. 18 were used to estimate modal properties, specifically natural frequency, damping ratio *and* modal mass. Natural frequency and damping estimation alone can rely on simple fitting of an exponentially decaying sine wave to free decay data, a variant of the logarithmic decrement procedure. However, estimation of modal mass is a much harder proposition, in fact it is extremely rare to find full-scale studies of tall buildings where modal mass estimation is reported, and rare in general for all civil structures.

System identification (modal parameter estimation) was performed on the four swaying sequences of the sample data shown in Fig. 18. Force (input) used 10% of sternum and 90% of lower back (0.1ST+0.9LB) scaled by body mass (89 kg) and response (output) used the Lance signal.

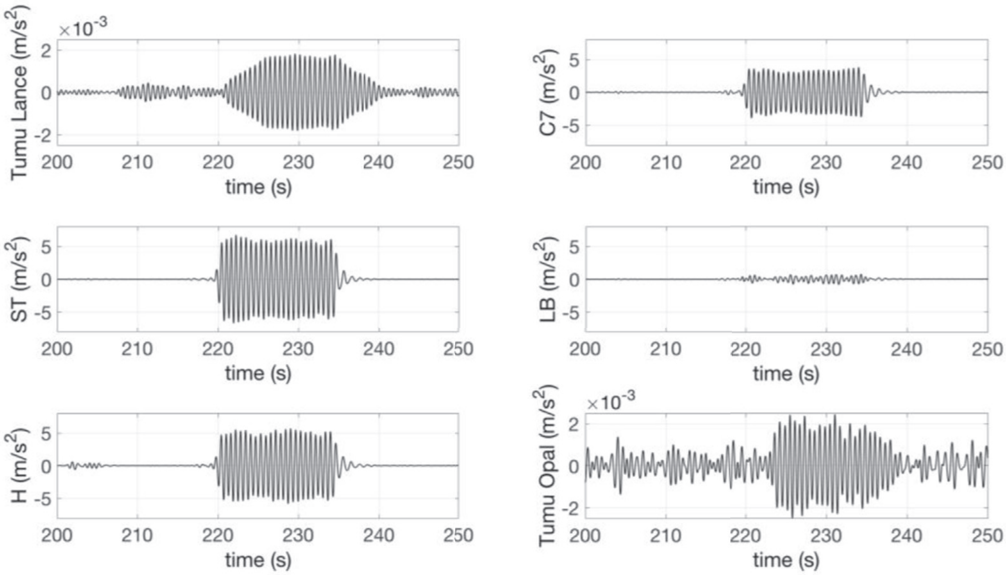


Fig. 18. Lance and IMU accelerometers data signals in sensors or WCS with s10 swaying close to 190 footfalls per minute.

The transfer function estimation tool of the MATLAB system identification toolbox [52] was used to estimate models with two poles and one zero. Fig. 19 shows fitting of the third swaying sequence; the fitted model is:

$$\frac{a_1 s + a_2}{s^2 + a_3 s + a_4} \tag{3}$$

where $s = \mathrm{i}\omega$ and which represents a receptance function [28]

$$\frac{1/\mathrm{k_n}}{(1 - \omega/\omega_\mathrm{n})^2 + 2\mathrm{i}\zeta_\mathrm{n}(\omega/\omega_\mathrm{n})}. \tag{4}$$

This is ratio of displacement rather than acceleration-to force, but the fit did not work when the numerator of Equation (3) was adjusted to be quadratic in s to represent the inertance function [29] (ratio of acceleration to force). The simple fix was to correct a_2 (which is at least 100 time larger than a_1) using the (almost constant) squared circular frequency. The fit of the model response to the (Lance) acceleration response in Fig. 19 corresponds to natural frequency ($f_n = \omega_\mathrm{n}/2\pi$) 1.617 Hz, damping ($\zeta_\mathrm{n}$) 2.6% and modal mass ($m_\mathrm{n} = \mathrm{k_n}\omega_\mathrm{n}^2$) 2447 tonnes.

From the four jump sequences the estimates are summarised as:

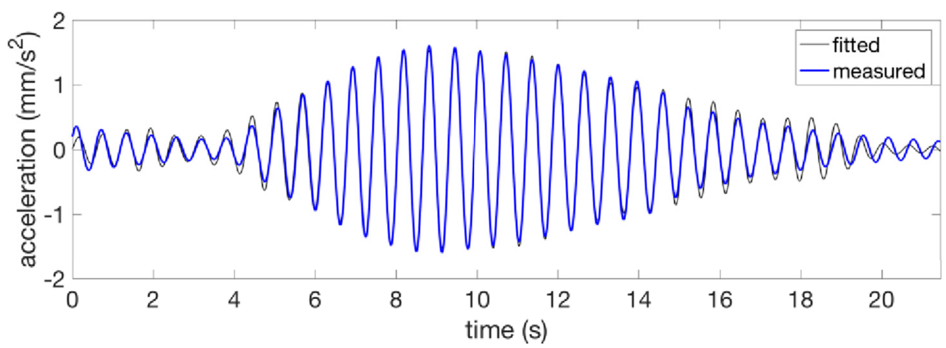


Fig. 19. Curve fit of model (Equation (3)) using IMU-estimated forces to acceleration measuring by Lance accelerometer.

$\mu(m_n) = 2970$ tonnes $\sigma(m_n) = 1140$ tonnes
$\mu(f_n) = 1.6905$ Hz $\sigma(f_n) = 0.008$ Hz
$\mu(\zeta_n) = 2.8\%$ $\sigma(\zeta_n) = 0.1\%$

The modal mass variance reflects the challenge in its identification which is problematic even with conventional input-output data. [53] reports an error margin of 15% compared to modal mass estimates using a finite element model (FEM) but such an approach can be highly inaccurate where mode shapes are not straightforward. An alternative approach is to assume a simple mode shape and use construction drawings to estimate mass distribution. For Tumu Building, assuming linear mode shape suggests a figure of 4000 tonnes, on the edge of the experimental estimates. Due to the complicated shape of the building it is more likely that the experimental estimate is the more reliable, even with the high variance.

5. Discussion

In principle, the most reliable estimation of medio-lateral GRFs through application of Newton's Second Law would require tracking of a large number of points on the human body, which is not feasible in open space using optical systems. So far, the commercially available IMUs with the required capability are relatively expensive (in the range of a few thousand dollars each) so they could not yet be used in large numbers. There is also a desire to reduce the scale of data acquisition since one aim of in-situ recovery of GRFs is to investigate coordination, loading and HSI for multiple footbridge pedestrians [25]. It was initially hoped that a single IMU would be a reasonable proxy but it was found to be necessary to use two. For walking, the general rule seems to be a 30% weighting of lower back and 70% of sternum accelerations, while for swaying it seems to be necessary to use different combinations for different test subjects, due to very different swaying styles. In both cases it seems remarkable that even with some apparently poor-quality data, the two-IMU proxy works well.

The biggest difficulty in this research has been the reliability and believability of IMU orientation data. The obvious tactic of rotating data to resolve acceleration (hence force) components at f_h and f_p into separate directions fails because of large inter and intra-subject variability. The alternative is to trust either actual orientation or orientation realised with the IMU. To check the latter requires a separate IMU measurement in a known direction (i.e. treadmill or force plate axis), but the importance of this tactic has only been revealed after the investigation. The tactic should, however be used in field application of the method.

Applications to measuring medio-lateral walking GRFs in-situ remain to be explored, but the human shaker has been evaluated and found to work in limited applications. In fact, the limitations are similar to those applying to mechanical shaking. Specifically, the vibration mode should be distinct (in frequency) from other modes and it should be below 2 Hz. The human shaker experiment on the University of Exeter Physics Building (Fig. 10) was repeated with instrumented test subjects with the aim of system identification as for the Tumu Building. The attempt was unsuccessful primarily because the building fundamental mode frequency in the apparent stiffer direction is quite close, complicating system identification. Further, it appeared that IMU orientation using quaternion data experienced unusually strong local variation so the tactic of measuring in a known direction (i.e. the natural axes of the building for measuring its acceleration response) failed. The achievement reflected in Fig. 10 could be repeated with some practice and greater care but that partially defeats the purpose of the simple approach. The method is likely to be most effective for taller buildings and towers with fundamental mode frequencies not close but below 1 Hz, and for footbridges.

Finally, the method is not limited to a single human shaker. Following the principle identified in Ref. [53] for jumping, the shaking is limited to only a few cycles, which is enough to build up response but not enough that slight differences in building frequency have any significant effect. Hence a few well-coordinated volunteers could multiply the effect almost linearly. An extra measurement on the Tumu Building used two people to test this, successfully.

6. Conclusions

Using a large dataset of treadmill-measured ground reaction forces (GRFs), medio-lateral dynamic load factors (DLFs) exceeding 0.1 have been identified for first sub-harmonic of walking pacing rate. These DLFs are larger than those previously published based on single footfall (force plate) measurements.

Inertial measurement units (IMUs) have been used successfully as a proxy for direct measurements of medio-lateral walking GRFs, using a linear combination of body accelerations measured at the lower back (LB) and sternum (ST) positions, scaled by body mass. Based on 57 measurements form nine test subjects, the mean DLF so measured was 3% higher than the direct measurement with coefficient of variation 16.5%.

The main difficulty with the approach is the correct alignment of the IMU data with the walking data, although the resulting cosine error based on a set of alignment checks around University of Exeter campus shows that usually this will have little impact on the outcome. Orientation estimated using a fixed IMU can mitigate the error.

So far, the ability to measure medio-lateral walking GRFs in-situ has not been explored, but applications could include investigations into coordination of pedestrian groups and energy flow in resonance excitation and synchronous lateral excitation of footbridges.

For swaying, direct measurements of medio-lateral GRFs for three test subjects using a set of force plates show that DLFs can reach almost 0.5, with peak forces approaching 0.4 kN.

Unlike walking, there appears to be no ideal single linear combination of IMU body locations, but in each case a combination of ST and LB accelerations can be used as a reliable proxy. The reason for not being able to find a single best linear combination is not yet identified but is likely to be due to very different swaying styles that can be expressed far more than for walking.

It is possible to use the IMU-estimated GRFs for system identification of sway modes of civil structures, including estimation of modal mass. Modal mass is rarely reported in examples of system identification as it is notoriously difficult to measure accurately, so the reported application to a tall building is particularly useful.

To mitigate the effects of IMU accelerometer noise on low level vibration measurements (e.g. in tall buildings) it is suggested to use simultaneous measurements with a low noise accelerometer, manually providing a clearly identifiable common signal to both sensors for synchronisation. Alignment information from the same structure-mounted IMU can also be used to orient the body-mounted IMU data.

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