Agricultural Trade Liberalization: an International Trade Network Approach

Volume 1 of 2

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: ………………………………………………………..

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ABSTRACT

A number of attempts have been made to facilitate agricultural trade liberalisation over the last decades. In spite of these efforts, trade liberalisation of agricultural and food processed goods has been modest.

It is argued that this lack of trade liberalisation is explained by the existence of governments that are politically biased in the sense that they place anti-trade policies in order to favour powerful sectors in the economy.

While there exists some evidence supporting this argument, it is difficult to assess how these biases influence agricultural trade patterns because existing quantitative modelling approaches do not normally consider simultaneously key aspects that characterise the food industry such as intra-industry trade and the existence of intermediaries in the supply chain with significant market power, among others.

The objective of this thesis is to offer an alternative theoretical model that has the potential to accommodate these key aspects and corresponds to an international trade network model that extends the framework developed by Goyal and Joshi (2006).

The model was solved by means of simulations and the results revealed that policy biased indeed can prevent trade liberalisation of agricultural and food processed goods. However, other factors that apparently have not been reported so far and

that are related to the market power exercised by intermediaries were identified. They correspond to the position of a country in the trade network (i.e. a country occupying a central position in the network is less likely to support trade liberalisation independently of any policy bias), the possibility that global free trade is an unlikely outcome, and the possibility that the world is trapped in an inefficient international trade network.

The results also revealed that the adoption of compensatory lump sum payments across countries (i.e. inter-node transfers) or across sectors within a country (i.e. intra-node transfers) could be used a potential tools to achieve global free trade in agriculture as they can compensate losers from trade by gainers achieving, as a consequence, Pareto improving outcomes.

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DEFINITIONS

Link deletion proof: A network *g* is *link deletion proof* if no country in the world is willing to break an existing agreement.

Strong link deletion proof: A network *g* is *strong link deletion proof* if no country has an incentive to break one or more agreements simultaneously.

Link addition proof: Network *g* is *link addition proof* when the following condition holds: If in this network there is a country willing to sign an agreement with another country, then the latter is not willing to sign this agreement.

Global treaty proof: A network *g* is *global treaty proof* if at least one country $i \in N$ (i.e. the set of countries in the world) is not willing to sign a global agreement.

Pairwise stability: A network *g* is pairwise stable if *g* is link deletion proof as well as link addition proof.

Strongly pairwise stability: A network *g* is strongly pairwise stable if *g* is strong link deletion proof as well as link addition proof.

Global treaty stability: A network *g* is said to be *global treaty stable* if *g* is strong link deletion proof as well as global treaty proof.

Inter-node transfer: An inter-node transfer $T_{ij}(g) \in R$ given from country i to country j in network g is a transfer such as $T_{ij}(g) = -T_{ji}(g)$.

Intra-node transfer: An Intra-node transfer correspond to a lump sum payment given by a particular sector in a country to another sector in the same country.

ABBREVIATIONS

AMS:	Aggregate Measure Support
CAP:	Common Agricultural Policy
DDA:	Doha Development Agenda
Eq:	Equivalent networks
EU:	European Union
FAO:	Food and Agriculture Organization of the United Nations
FTAs	Free Trade Areas
INRA:	National Institute of Agricultural Research
NAFTA:	North American Free Trade Agreement
NRA:	Nominal rate of assistance
OECD:	Organisation for Economic Co-operation and Development
PICTA:	Pacific Island Countries Trade Agreement
PTAs:	Preferential trade agreements
R&D:	Regional trade agreements
RTAs:	Research and development
UNCTAD:	United Nations Conference on Trade and Development
URAA:	Uruguay Round Agreement on Agriculture
USA:	United States of America
WTO:	World Trade Organisation

CHAPTER ONE: Introduction

Lack of agricultural trade liberalisation is a problem that has attracted the interest of researchers over a long period of time. This is reflected in the debate about the unsuccessful attempts to reach a global agreement, as well as to the current trade pattern that is explained mainly by regional agreements across the world. This pattern is characterised by the existence of trade concentrated in clusters of countries that belong to the same regional area, and by the existence of central countries that bridge these clusters by means of trade.

Different explanations have been proposed to explain this phenomenon. However, it is still not well understood at the current state of the knowledge. It is argued in this dissertation that useful insights to explain the current lack of trade liberalisation in agricultural and food processed goods can be achieved by including the main key features that characterise the agricultural sector into a theoretical assessment. These features include the existence of clusters of trade (i.e. regionalism); countries that bridge the clusters (i.e. centrality); evidence of intra-industry trade; existence of intermediaries in the supply chain with significant market power; and governments that are politically biased.

The research gap that was identified in this dissertation is that a suitable theoretical framework able to accommodate all these key features has not been developed so far. In considering this gap, the objective of the current investigation is to propose a modelling approach that has the potential to contribute to filling this gap. It corresponds to an international network framework that extends the contribution by Goyal and Joshi (2006). It is important to clarify that this model can also be applied to other sectors because the features described above are not specific to agriculture. However, the aim of this thesis is to focus the analysis on trade of agricultural and food processed goods.

In order to investigate how the proposed model can explain the lack of agricultural trade liberalisation, this dissertation is organised as follows.

Chapter Two provides a literature review that provides the context of the research in terms of agricultural trade liberalisation. It justifies the research gap and explains why the international trade network model can be used to fill the gap.

Given that the proposed international trade network model is an extension of the model developed by Goyal and Joshi (2006), Chapter Three describes the main features of the original model with the purpose of highlighting the novel extensions to the theoretical framework that have been introduced in this dissertation, namely: the introduction of the farming sector; the use of alternative stability concepts to determine possible stable international trade architecture; and the potential for compensatory lump-sum transfers to lead the world towards global free trade. The main results obtained by Goyal and Joshi are also described in order to used them as a benchmark to assess how the extensions described above cause deviations from the original model. The headline outcome is that the theoretical extensions result in significant deviations from the insights from the Goyal and Joshi model and go some way to providing new insights towards explaining why liberalisation of trade in food and agricultural markets is difficult and which would not be apparent from the Goyal and Joshi model.

Chapter Four formally introduces the proposed international trade model and the main features of this model are discussed. After that, a number of simulations under the concept of *pairwise stability* (i.e. the concept used by Goyal and Joshi) are explored to gain an understanding of the possible networks that the world may reach when countries are involved in bilateral agreements of food processed goods. These simulations consider different assumptions including asymmetry in market size and farmers' productivity.

Chapter Five extends the analysis by introducing two different stability concepts. One of them is referred to as *strongly pairwise stability* and is a refinement of pairwise stability that is more suitable to predict the stability of networks when countries are engaged in bilateral agreements. The other stability concept is a novel contribution of this investigation and was introduced with the purpose of studying agricultural trade liberalisation of food processed goods when countries are involved in global agreements. This concept is named in this dissertation *global treaty stability*.

The main results obtained in Chapters Four and Five are that global free trade is not always stable; multiple equilibria including regionalism can emerge in some determined scenarios; and centrality can emerge and can prevent the signature of a global agreement. In considering these results, Chapter Six explores the use of lump-sum transfers to stabilise global free trade or to break inefficient stable networks in favour of free trade. Two different types of transfers are considered in this chapter: inter-node transfers (i.e. payments across countries); and intra-node payments (compensatory payments across sectors within the same country).

Finally, Chapter Seven discuses and concludes by linking the current international trade network architecture with the results obtained in the previous chapters. The focus is placed on the stability of global free trade; regionalism; centrality; and the adoption of compensatory payments as a potential political tool to favour free trade in food processed goods. Given the observed difficulty in achieving free(r) trade in food and agricultural markets, the framework outlined in this thesis offers a number of new insights that can offer an interpretation for the difficulties in liberalising trade in these markets.

To finish this chapter, note that the network model developed in this thesis uses the number of duty free tariff lines as a measure of trade liberalisation. However, it is a poor proxy of trade liberalisation intensity because it does not necessarily reflect trade flows. Other measures that could be considered are, for example, the bilateral trade intensity ratio (i.e. the ratio of bilateral trade flows between two countries divided by the sum of the total trade flows in these countries) and the openness index (i.e. the ratio between the sum of exports and imports in a country and the country's GDP) (see Calderón et al., 2007; Guerrieri and Caffarelli, 2012). These alternatives will be explored in future research.

CHAPTER TWO: Literature Review

2.1 Introduction

The debate on agricultural trade liberalisation has attracted the attention of researchers and policymakers over a long period of time. The argument that has dominated this debate is that this liberalisation would increase global welfare because this would lead to a more efficient trade system (Anderson, 2016; Anderson et al., 2001). In addition, it has been proposed the idea that trade liberalisation in the agricultural sector would promote global food security by making the international food system more efficient and more responsive to unexpected shocks that might threaten food security (Matthews, 2014). In spite of these arguments, little progress has been made to liberalise the agricultural trade system leading to a substantial body of research that have been developed to provide possible explanations to this fact.

The objective of this chapter is to review the current debate on the issue of agricultural trade patterns and the explanations that have been offered to explain the lack of liberalisation in this sector with the purpose of highlighting the research gap that this thesis aims to fill. For this purpose, the chapter is organised as follows. Since agricultural and food trade form part of the international trade system, a description of the current trends in this system is provided in Section 2.2. After that, Section 2.3 focused on the evolution of agricultural trade liberalisation and the arguments that have been introduced to explain the little progress that have been made to liberalise this sector. Section 2.4 highlights the research gap that is considered in this dissertation and explains why the international network approach has the potential to contributing in filling this gap. Finally, Section 2.5 summarises and concludes.

2.2 Current trends in the international trade system

International trade agreements in general has been carried out by means of three different types of tariff reform agreements (Hartman, 2013): *global or multilateral agreements* (i.e. countries members of the World Trade Organisation (WTO) negotiate the reduction of barriers to trade among them); *preferential trade agreements* (PTAs) (i.e. reciprocal or non-reciprocal preference trade schemes typically between developed and developing nations); and *regional trade agreements* (RTAs) (i.e. reciprocal trade between two or more partners).

It is recognised the fact that global agreements have made little progress and most of the existing agreements correspond to RTAs followed by PTAs, and they have proliferated dramatically from the last decade (Estevadeordal and Suominen, 2004; James, 2006; Freund and Ornelas, 2010; Hartman, 2013; Baier et al., 2014; Maggi, 2014). In fact, about 267 bilateral and regional trade agreements have been reported by the WTO by 2016 (Grossman, 2016). In the case of PTAs, this increase is shown in Figure 2.1.

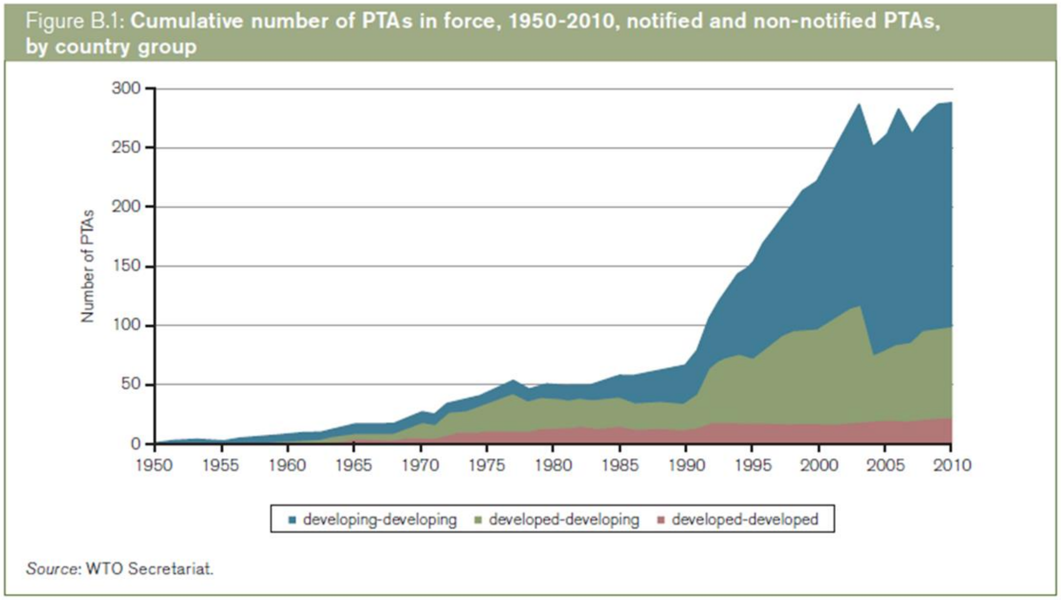


Figure 2.1. Cumulative number of PTAs in force, 1950-2010, notified and non-notified PTAs, by country group

According to this figure, the highest increment in PTAs occurred between developing countries followed by developed and developing countries. The smallest number is found between developed countries.

In relation to RTAs, on the other hand, the database of the WTO shows that these agreements have significantly increased over the last decades[1]. Nowadays, they are concentrated in geographical areas, a fact that is explained by the nature of these agreements. For example, the Asia Pacific Trade Agreement includes Bangladesh, China, India, Republic of Kprea, Lao and Sri Lanka. A useful way to see the main features of the current configuration of RTAs is by means of a network representation as follows.

[1] See https://www.wto.org/english/tratop_e/region_e/regfac_e.htm

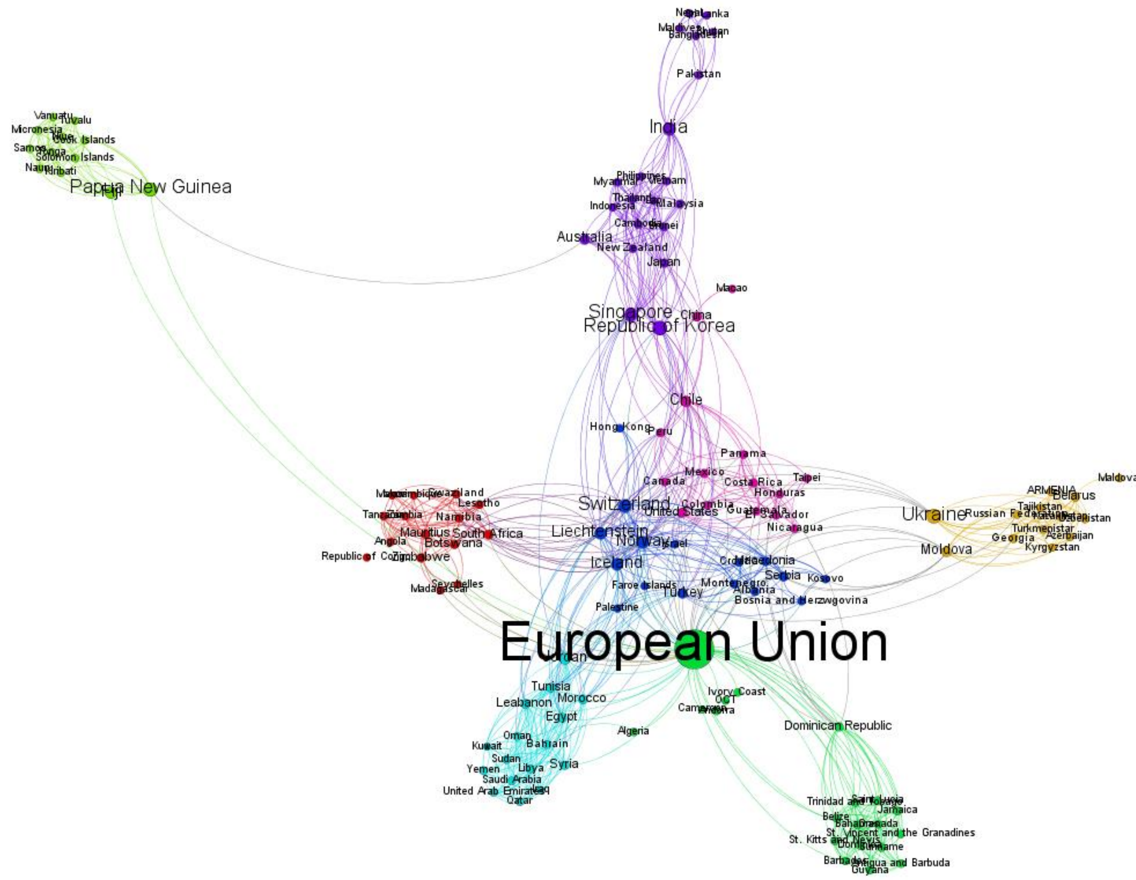


Figure 2.2. RTAs network in 2013 (source: elaborated by the author using Gephi software with statistic of WTO)

In this figure, nodes correspond to countries and links to two-way preferences (i.e. tariff reduction agreements) between countries. For example, NAFTA was introduced by connecting the members of this Free Trade Area with each other.

This figure shows that RTAs are concentrated in clusters containing countries that belong to the same regional area (e.g. Pacific Island Countries Trade Agreement (PICTA) includes countries that are located in Oceania). It also shows bilateral agreements between countries (e.g. agreement between New Zealand and China) or block of countries with single countries (e.g. agreement between EU and Chile) because they also are classified as RTAs in the database of the WTO. In this network configuration, trade across clusters is not as significant as trade within each cluster and they are linked in general by few agreements. Clusters are also connected indirectly by countries or group of countries such as the European Union. In other words, they correspond to central countries in the sense that they bridge several clusters in the network. This configuration has also been noted by Salvatici and Nenci (2017) who explain that countries' efforts to obtain the benefits of trade have led to an intertwined network that is dense, reciprocal and clustered.

In considering the proliferation of regional trade agreements, it is believed that RTAs rather than global agreements would eventually lead to global free trade (Ash and Lejarraga, 2014). However, it is also argued that agreements of this nature may become a stumbling block to multilateral liberalisation (for an early discussion see Lamy, 2002). For example, Baldwin (2006) explains that lobbying groups can prevent further liberalisation when able to influence policymakers. Likewise, Bhagwati et al. (2016) argue that lobbing groups can prevent further liberalisation by pushing non-trade agenda items consisting of intellectual property rights and labour standards. In addition, the same researchers explain that countries can maintain distortions in agriculture by preventing a multilateral agreement. A final example is found in Krishna (2013) who argues that preferential agreements are not monotonic path to multilateral free trade because they can create incentives within member countries against further multilateral liberalisation.

The claim that RTAs can lead to global free trade is partially supported by empirical evidence revealing a positive association between RTAs and trade (see for example Roy, 2014; Baier and Bergstrand 2007). However, the slowdown in the world trade growth may also indicate that a global trade has peaked and what is observed today (see Figure 2.2) is a new normal with weaker levels of trade (Hoekman, 2015). In the context of the new literature on international trade networks, this suggests that the world may be reaching a stable trade network different from global free trade (see for example Goyal and Joshi, 2006). This is also noted by Limao (2016) who explains that the interdependence between RTAs may lead to suboptimal outcomes (i.e. stable networks other than global free trade) from the global perspective.

2.3 The evolution of the agricultural trade system

This section discuses four key aspects that are related to the evolution of the agricultural trade system and that are the basis for the proposed network framework developed in this dissertation (see Section 2.4): attempts to reach a global agreement in agriculture; regional trade agreements; policy biases; and market power in the supply chain of the food processed industry. These aspects are discusses as follows.

2.3.1 Attempts to reach a global agreement in agriculture

The agriculture sector in different industrialised countries has been protected over a long period of time in order to achieve some specific objectives. According to FAO (1988), these objectives were to maintain the parity between farm and non-farm incomes; to guarantee the stability of farm incomes by means of import flow controls; and to ensure food security in order to guarantee a level of food self-sufficiency. The last objective was considered to be particularly important for European countries who saw food security as a fundamental target to avoid the scarcity of food suffered during the Second World War (Gardner, 1996). In order to achieve these objectives, a number of policy support instruments have been adopted to protect agriculture such as price support systems, import barriers, supply controls, export subsidies, and import tariffs (Daugbjerg and Swinbank, 2009; Agro Europe 2006; Frank, 1992; Harris et al., 1983; Marsh and Swanney, 1980).

The policy instruments adopted by industrialised countries to protect agriculture have been criticized because of the costs that they add to the economy (FAO, 1988): costs to consumers; costs to taxpayers; and costs to the economy. Researchers also claim that a further effect of agricultural support policies has been a distortion of the prices of agricultural goods caused by oversupply of food commodities that were disposed in the international market. This led to a decrease in the international prices and an increase in the volatility of these prices (Devadoss, 2006; Hopkins, 1992; Moyer 1992).

During the 1980s, attention turned to the idea of carrying out a global reform with the purpose of eliminating the negative effects of agricultural protection described above (Chung and Veek, 1999). The main argument was that the removal of domestic support policies in developed countries would redirect the production and international trade of agricultural commodities to the most efficient producers. This, in turn, would lead to a more efficient trade system that would increase global welfare (Anderson, 2016; Anderson et al., 2001). This global reform materialised in the form of the Uruguay Round Agreement on Agriculture (URAA) which is the first global agreement in agriculture that has been negotiated by members of the WTO. The idea of the URAA was to follow the liberalisation path of manufacturing, with protection rates continuously declining (Aksoy, 2005). The URAA concluded in December of 1993 and included agreements on three sets of issues referred to as the three pillars: (i) market access; (ii) export competition; and (iii) domestic support.

In the agreement on market access of the URAA, countries agreed to convert all import barriers to their tariff equivalents in a process called tariffication. The conversion of non-tariff measures into tariffs was based on the actual difference between internal and external prices from 1986-1988. Once the tariff equivalents were established, tariffs were supposed to be restricted (Matthews, 2001; Athukorala and Kelegama, 1998). Regarding export competition, the agreement for developed countries consisted of a reduction of export subsidies from the 1986-88 average level by 36% in value and 21% in volume over a period of six years starting in 1995. For developing countries, the agreement involved a reduction of export subsidies by 24% in value and 14% in volume over a period of ten years starting in 1995 (Khor, 2003; Josling, 1998). Finally, the URAA agreement on domestic support is applied specifically to the programmes included in the Amber Box. Programmes under this classification are calculated under the Aggregate Measure Support (AMS) which is determined by calculating a market price support estimate for each commodity receiving such support, plus non-exempt direct payments or any other subsidy not exempted from reduction commitments, less specific agricultural levies or fees paid by producers (OECD, 2000). In the URRA agreement, the AMS was subject to a 20% reduction for developed countries from its 1986-88 base, over six years starting in 1995. For developing countries, the agreement involved a 13% reduction over ten years starting in 1995 (Baffes and de Gorter, 2005; Khor, 2003).

The URAA is considered as an important achievement because it provided for the first time a foundation for establishing a rule-based world trading system that included both developed and developing countries (Athukorala and Kelegama, 1998; Anderson and Morris, 2000; Anderson et al., 2001). However, this agreement has been considered unsuccessful because tariffs in agriculture remained high and also because agricultural trade liberalisation post URAA was modest (Messerlin, 2003; OECD, 2001; Gale, 1995). According to Josling (1998), tariffs on manufactured goods in the second half of the 1990s were of the order of

5-10%. In contrast, agricultural tariffs were on average 40% with tariffs peaks of over 300% revealing that the URRA did little to liberalise trade in agriculture.

The unsuccessful outcome of the URAA has also been noted by Aksoy (2005) who argued that the lack of agricultural trade liberalisation is associated with the actual levels of protection. This author provided some facts supporting this argument. First, the post-Uruguay Round agricultural tariffs remained high and they constituted the major protection policy, accounting for about 70 per cent of the total protection in OECD countries. Second, the magnitude of international trade of agricultural commodities was higher in developed countries which had preferential tariff agreements among them. This can be explained by the absence of tariffs barriers among the partner countries. Third, the expanding groups of agricultural goods, like fruits and vegetables, had low rates of protection in contrast with the declining groups, like grains and coffee, which had high rates of protection in industrial countries. Fourth, the export of protected goods between industrial countries decreased. This is because protection generated greater production, making many industrial countries more self-sufficient.

During the second half of the 1990s, the next step in promoting further integration of the agro-food sector into the multilateral trading system was carried out. This was triggered by three main factors: (i) lack of agricultural trade liberalisation post URAA; (ii) export subsidies and domestic support policies still being used by developed countries after this agreement; and (iii) the mandate in Article 20 of the URAA to hold new negotiations (Young et al., 1999; Coleman and Meilke, 2000;

Josling, 2000; OECD, 2001). These three factors led to new multilateral trade negotiations on agriculture with the purpose of strengthening the disciplines already established under the URAA (Devadoss, 2002). These negotiations were formally included in a round referred to as the Doha Round or the Doha Development Agenda (DDA). The DDA was launched at the World Trade Organisation (WTO)'s Fourth Ministerial Conference in Doha (Qatar) in November 2001, and was planned for conclusion in January 2005 (WTO, 2011b; and Matthews, 2001).

After more than ten years of talks, the Doha Round still did not have a framework (modalities) deal. In fact, the Geneva Ministerial Meeting in December 2009 ended without any substantial progress (Cho, 2010). As a consequence of this Doha's failure, it was suggested by a number of researchers that a global agreement in agriculture might never be attainable (see, for example, Scott and Wilkinson, 2010; Anderson et al., 2013; Bagwell et al., 2016). In this respect, some researchers argue that it would appear difficult to finalise the Doha Round in the near future because the central dossiers of the Doha Round negotiations (i.e. market access for non-agricultural goods and services, domestic agricultural subsidies, and agricultural import tariffs) remain unresolved (Scott, 2017; Koopmann and Stephan, 2014; Wilkinson et al. 2014).

In sum, there are observations from the above discussion. First, agriculture was largely left untouched in early rounds of trade negotiations at least prior to the Uruguay Round. Second, despite being integrated into the WTO framework following the completion of the Uruguay Round, protection in the agricultural sector across many countries is still a dominant feature of world trade. Third, there is a lack of progress in the Doha Round of negotiations in large part reflecting the unwillingness of countries to promote the liberalisation of trade involving food and agricultural products.

2.3.2 Regional trade agreements

The failure of an eventually global agreement in agriculture has not prevented countries from being involved in agricultural trade liberalisation. On the contrary, as explained in Section 2.2, about 267 RTAs have been reported to be in effect by the WTO in 2016 and many of them include both agricultural commodities and food processed goods (see for example Baker et al., 2016; Friel et al., 2016; Parra et al., 2016; Regmi et al. 2005). In fact, most of the existing agreements correspond to free trade areas (FTAs) and current levels of agricultural trade liberalisation are explained mainly by these agreements (Estevadeordal and Suominen, 2004; James, 2006; Freund and Ornelas, 2010; Baier et al., 2014).

It is not clear, however, whether there will be a significant increase in trade of agricultural goods and processed goods in the future. This is because, as explained in Section 2.2, it is argued that it is unlikely that a global free trade will be reached from RTAs. That is, while there is partial evidence suggesting a positive effect of RTAs on trade, there is also evidence suggesting that the world is reaching a normal with weaker levels of trade (Hoekman, 2015). In terms of the network approach, this means that the world may be reaching a stable trade network different from global free trade (Limao, 2016: Goyal and Joshi, 2006). Since agricultural trade liberalisation is mainly explained by the existing RTAs, this suggests a similar trend in the international trade of agricultural and food processed goods.

Unfortunately, it was not possible to find information about RTAs in the agricultural sector in the public domain to support this claim. However, some insights can be obtained from information on trade flows of determined food processed goods available in FAO statistics[2]. In order to show a possible correlation between the paucity of RTAs and trade flows of food processed goods, Chile was considered as an example because this is one of the countries having more FTAs in the world accounting for about 30 agreements in force. This is shown in the following figure.

[2] Available at http://www.fao.org/faostat/en/#data

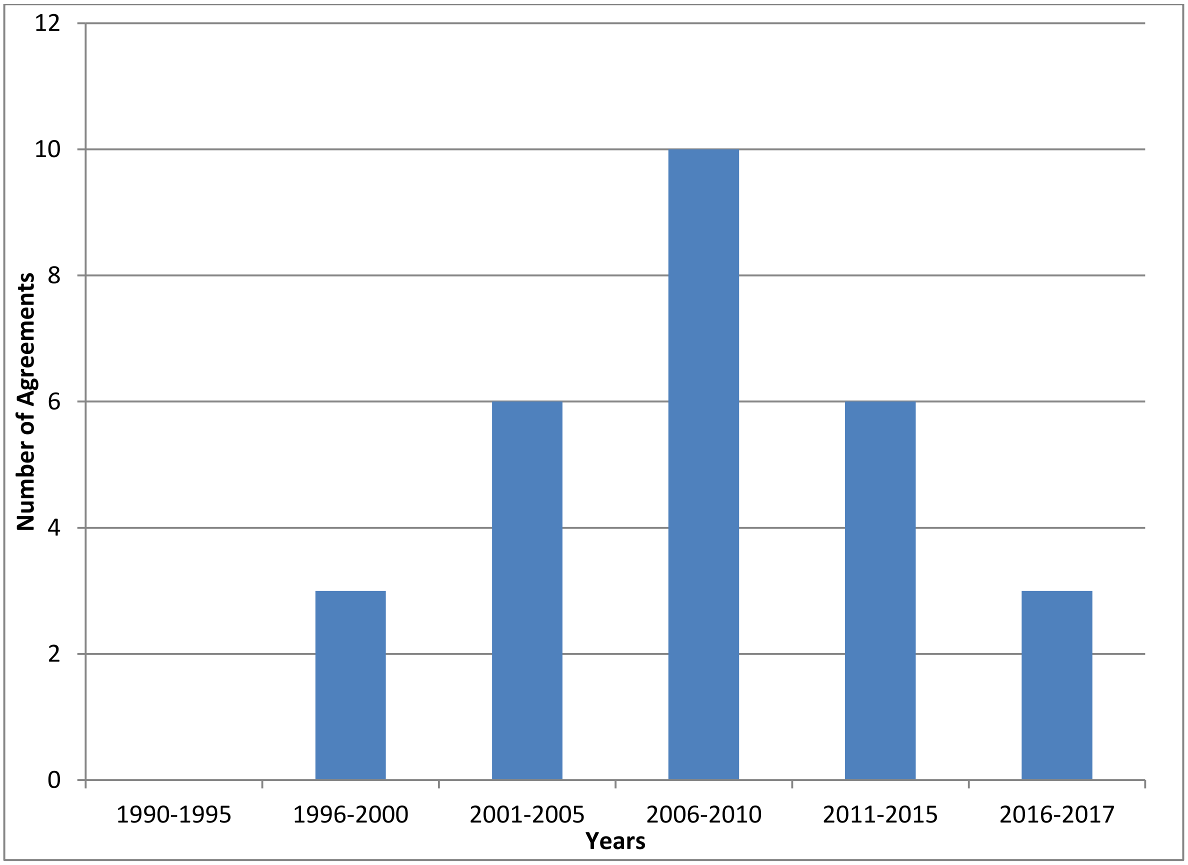


Figure 2.3. Number of RTAs signed in Chile (own's author based on WTO statistics)

According to this figure, Chile started to sign RTAs in the second half of the 1990s. The higher number of agreements per five-year period occurred between 2006 and 2010. After that, there was a decline in the number of new agreements which is consistent with the argument that trade in the world is slowing down (Hoekman, 2015). A similar trend is found in terms of trade flows of some processed foods that are relevant for this country. In order to show this fact, let us consider the cases of beef (i.e. meat, cattle, boneless (beef & veal)) and wine. The following figures show the exports (quantity and value) and imports (quantity and value) of beef from 1987 to 2013.

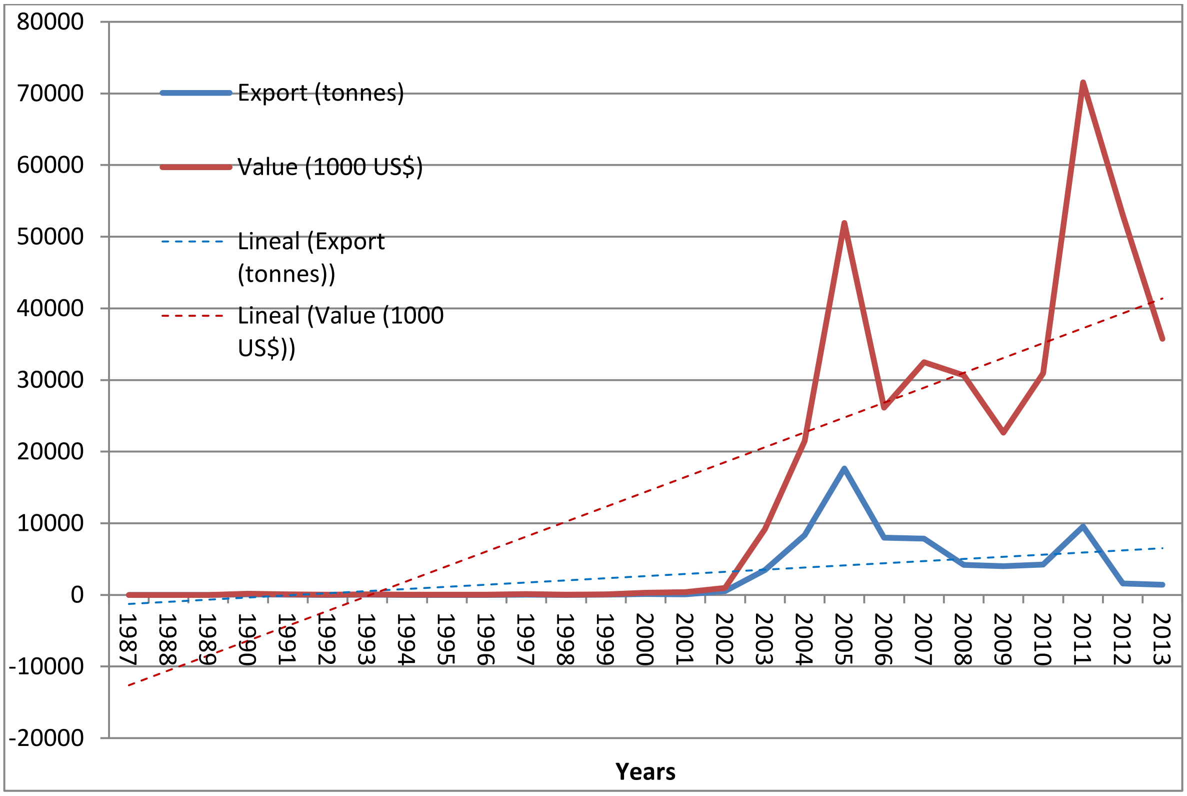


Figure 2.4. Export of beef (meat, cattle, boneless (beef & veal)). Source: Own's author based on FAO statistics

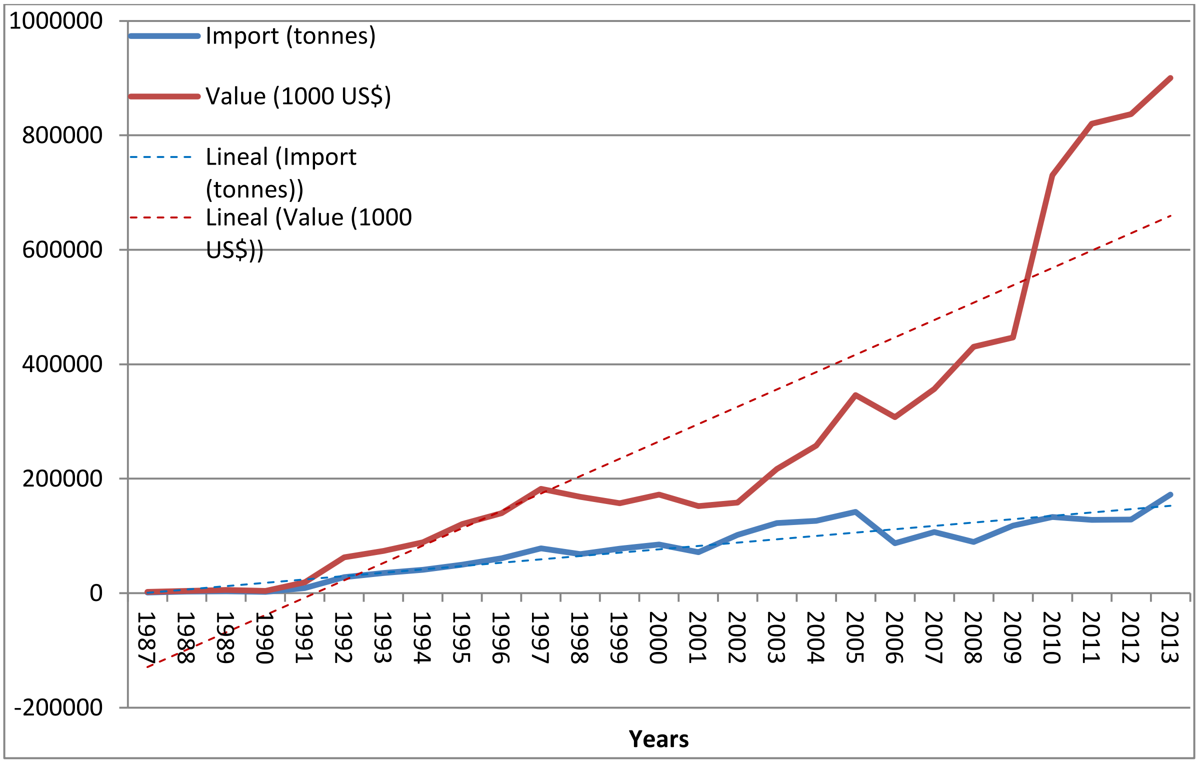


Figure 2.5. Import of beef (meat, cattle, boneless (beef & veal)). Source: Own's author based on FAO statistics

These figures show that there was a significant increment in both exports and imports of beef in terms of quantity and value by the time when RTAs started to be signed in Chile. This is suppoeted by the trend lines used in these figures.

On the other hand, in order to determine whether there is a possible association between the decrease in the number of RTAs in Chile after 2011 and trade flows of beef, the annual growth rate of exports and improts of this good are considered in the following figures.

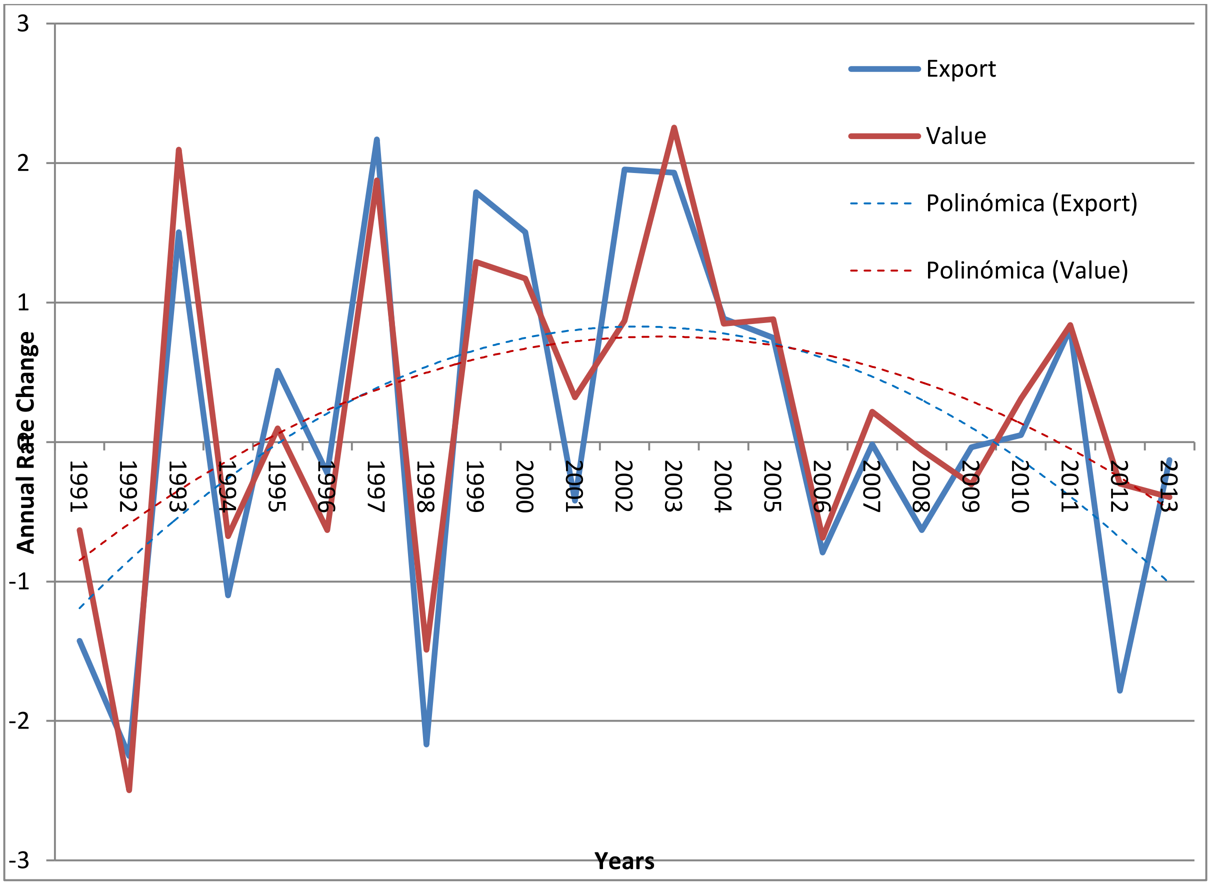


Figure 2.6. Annual growth rate of exported beef (meat, cattle, boneless (beef & veal)). Source: Own's author based on FAO statistics

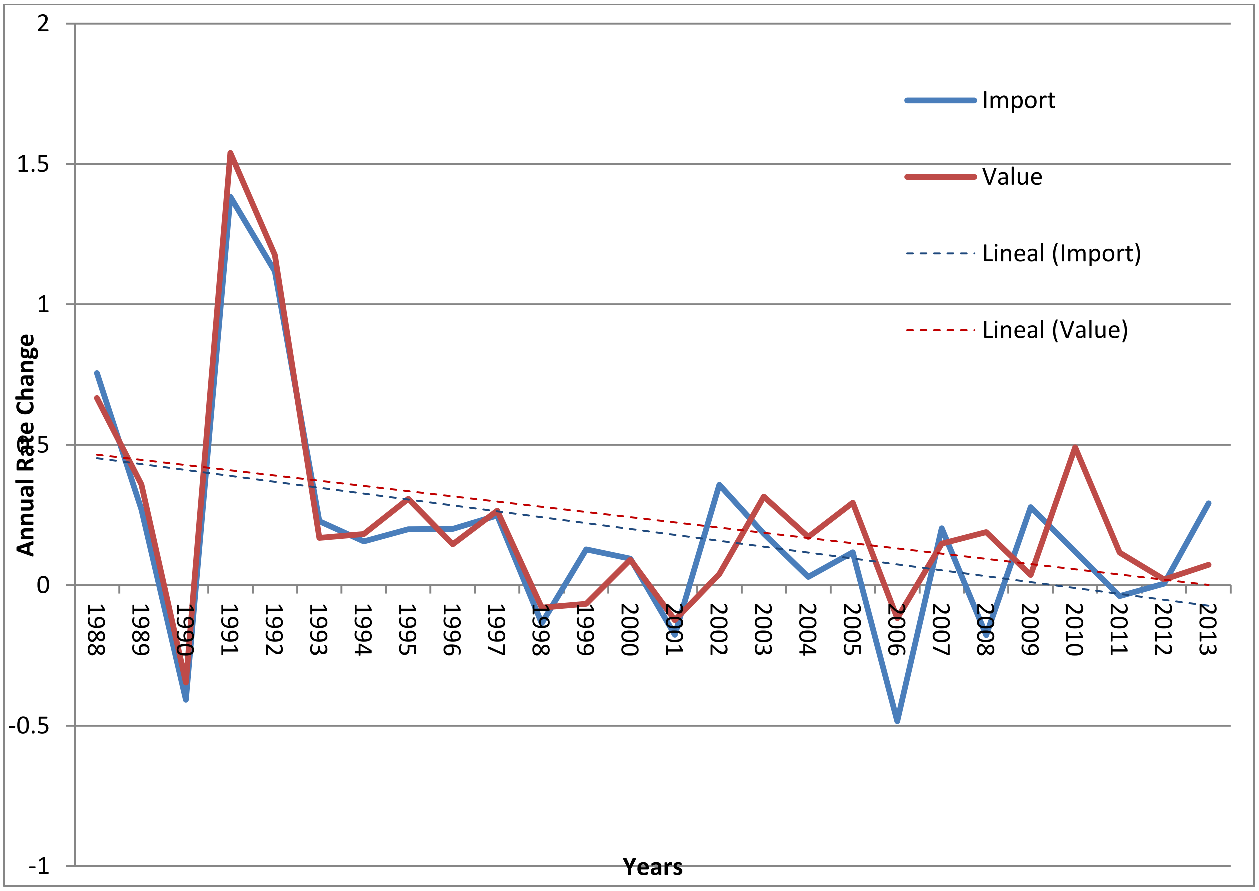


Figure 2.7. Annual growth rate of imported beef (meat, cattle, boneless (beef & veal)). Source: Own's author based on FAO statistics

According to Figure 2.6, the annual rate of expoerted beef increased dramaticly during the periods where more RTAs were signed in Chile. After that, there was a clear decrease in the rates wich is consistent with the decrease in the number of new RTAs. In relation to imports, the association between annual rates and RTAs is not so clear. However, the trend indicates that in general the the annual rates have decreased over the last decades. In considering figures 2.6 and 2.7, it is concluded that the argument claiming that the world is reaching a normal with weaker levels of trade seems to be supported in this example.

Let us now consider the case of wine. Exports and imports (in tonnes and values) of this good and the annular rate changes are presented in the following figures.

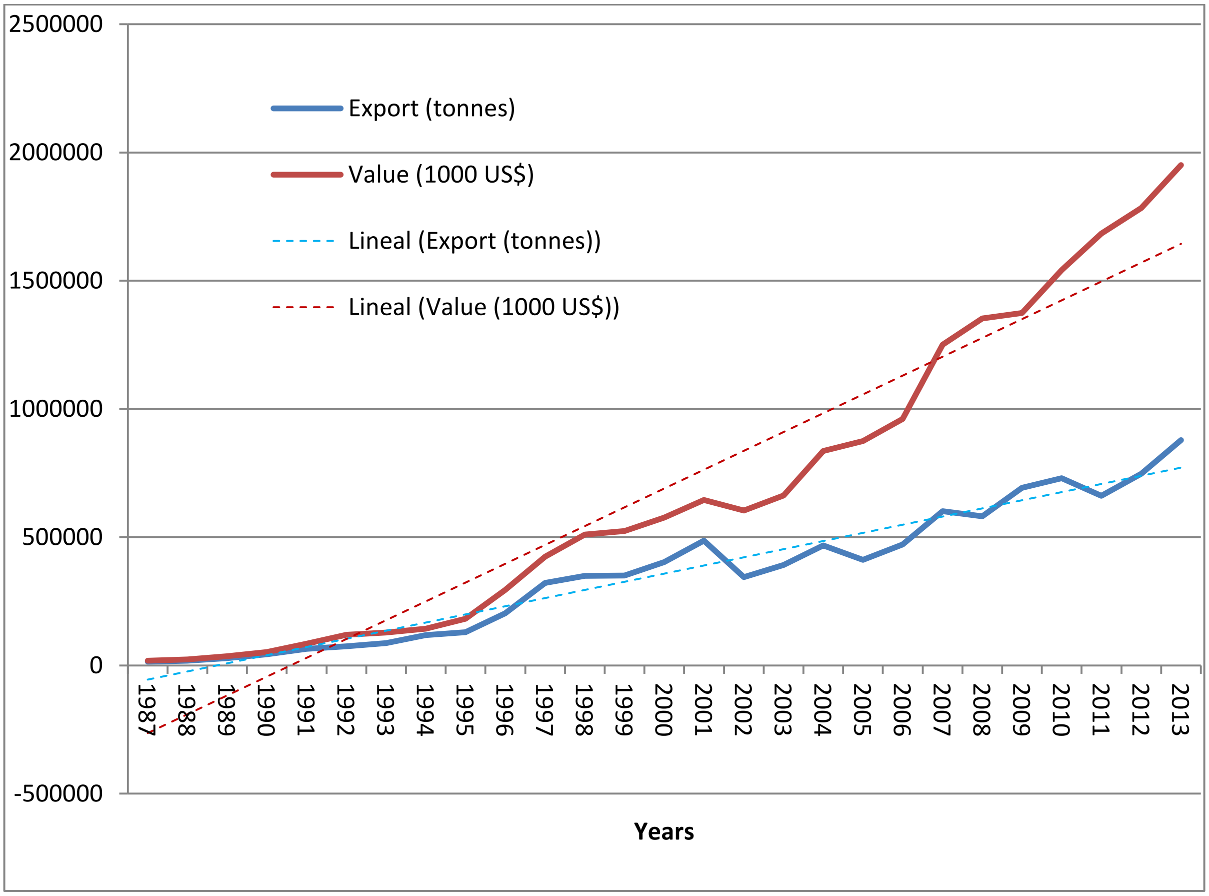


Figure 2.8. Export of wine. Source: Own's author based on FAO statistics

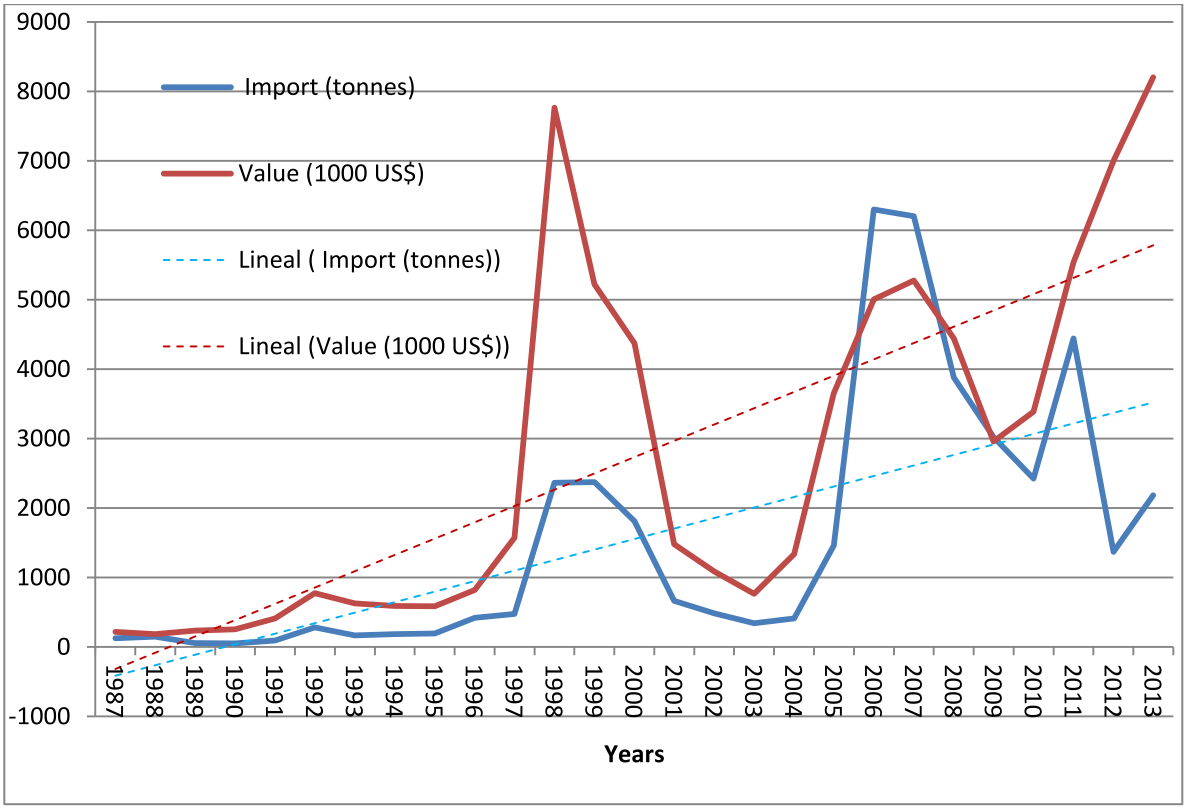


Figure 2.9. Import of wine. Source: Own's author based on FAO statistics

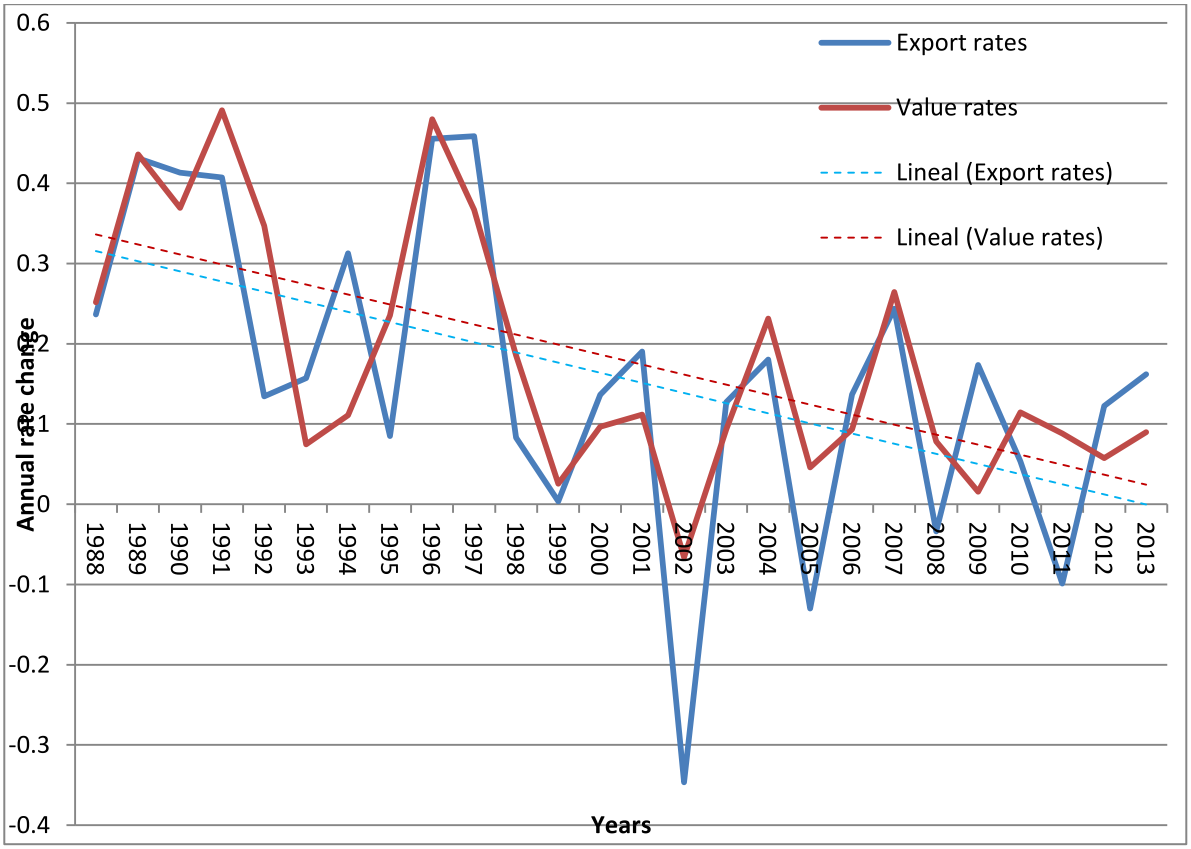


Figure 2.10. Annual growth rate of exported wine. Source: Own's author based on FAO statistics

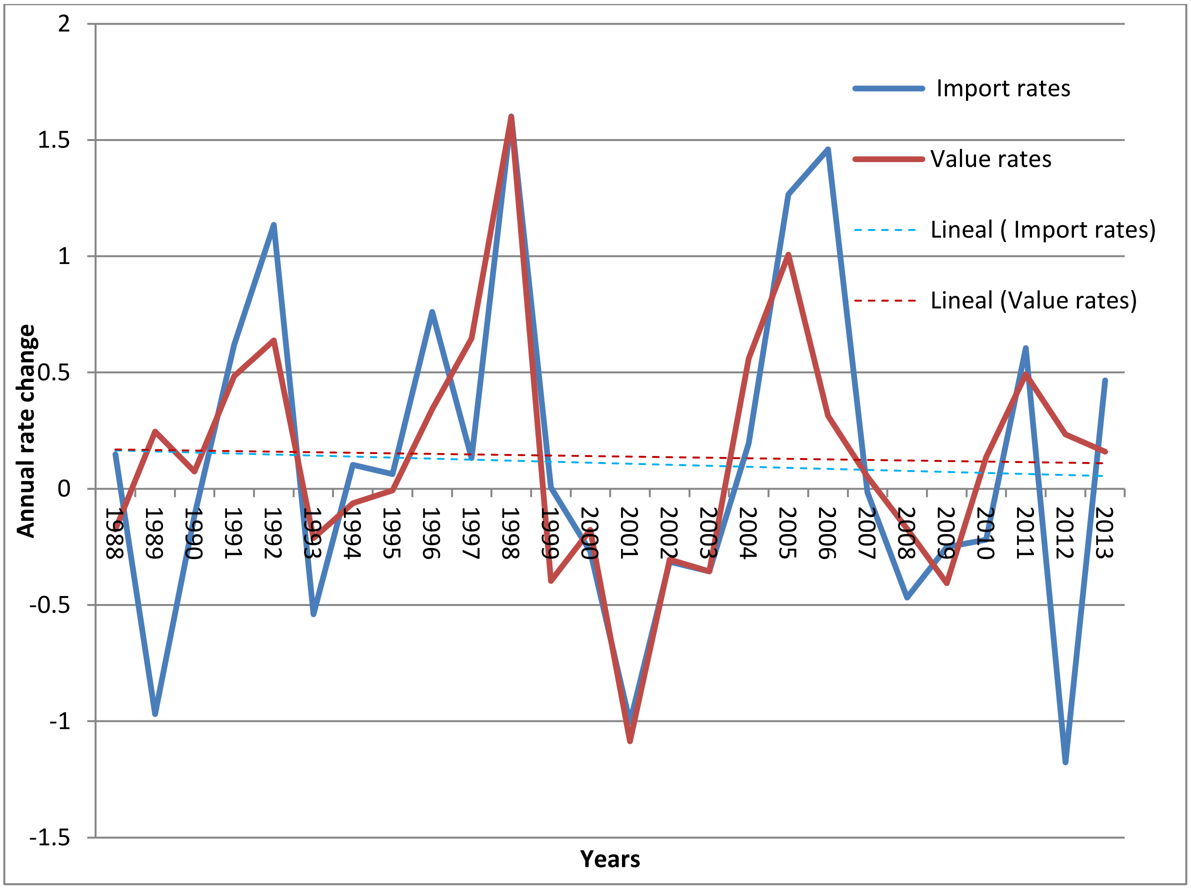


Figure 2.11. Annual growth rate of imported wine. Source: Own's author based on FAO statistics

As in the previous case, these figures show a significant increase in exports and imports of wine by the time when Chile signed a significant number of RTAs, but the annual growth rates of export and import of this good have decreased over the last decades. This evidence, again, is consistent with the suggestion that the world is reaching a new normal with weaker levels of trade and this apparently is also affecting trade of food processed goods.

Another consideration in relation to the association between RTAs and the flow of food processed goods is whether the regionalism shown in figure 2.2 (i.e. RTAs are concentrated in regions and there are countries that bridge these regions by means of bilateral agreements) is also presented in the flows of agricultural and food processed goods. This is indeed a plausible possibility because there exists a correlation between the concentration of RTAs in geographical areas and world trade flows in these areas. In this respect, UNCTAD (2015) points out that a very large part of world trade is clustered around three regions: North America, Europe and East Asia. Trade flows have generally grown for the core regions since 2011, especially those relating to East Asia, but the value of trade flows has contracted in the periphery, especially for Latin America. This is shown in the following figure.

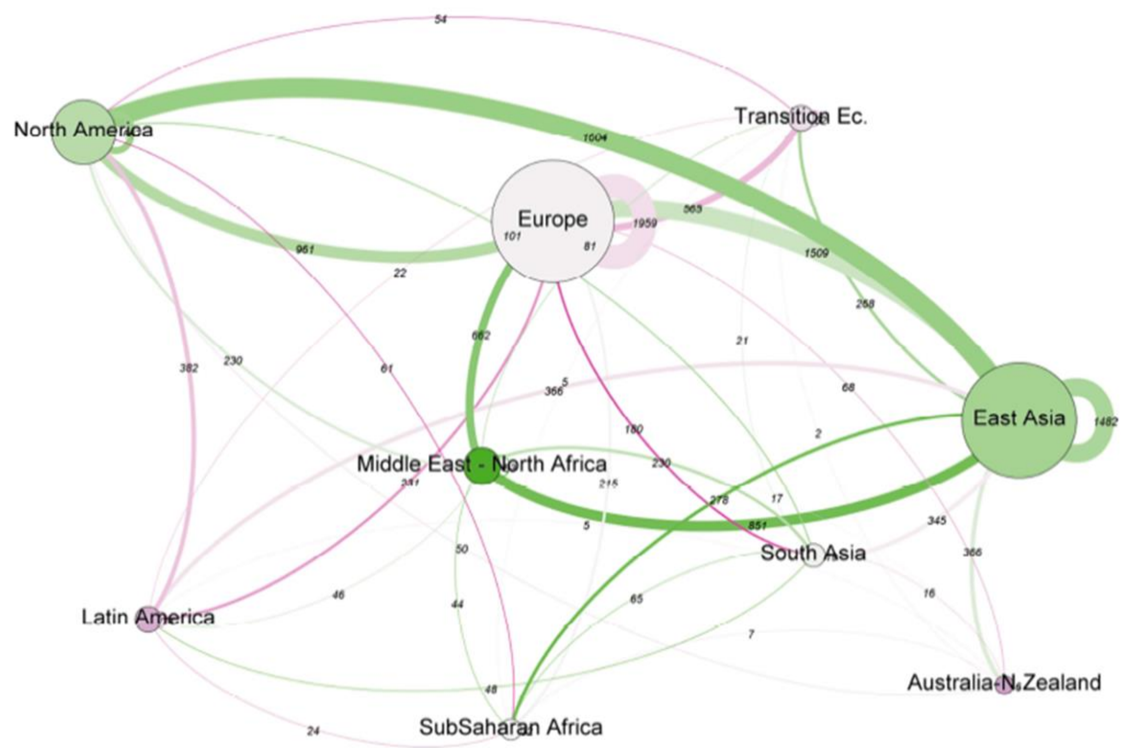


Figure 2.12. World trade flows (source: UNCTAD, 2015)

In order to determine whether a similar pattern exists in trade flows of food processed goods, an online software of the WTO developed in partnership with other organisations was considered to obtain trade flow network graphs of selected goods[3]. Unfortunately it was not possible to consider all the goods and countries that are available in this source because of the limit constraints of this thesis. In considering these constraints, meat and wine were selected to illustrate the flow patterns, and this choice was made because these goods are commonly traded across countries in the world. Likewise, Germany, Spain, France and China where selected to illustrate the fact that flows of trade in these countries have a tendency to be concentrated in regions, even when some of them are major exporters. For example, France is a major exporter of wine (Meloni and Swinnen, 2014) and, as shown below, trade of wine in this country is concentrated in Europe. On the other hand, Chile and USA were selected to show the existence of central countries in terms of trade flows of food processed goods. This choice was made because, according to the statistics of the WTO[4], these countries have a large number of agreements across the world suggesting that centrality is likely in these countries.

In these networks, nodes correspond to countries, links are defined as the role of each country (i.e. the thickness of the link) in terms of either import market share (i.e. buyers) or export market share (i.e. sellers), and the size of the nodes represent the size of market share of each country. This is shown as follows.

[3] Available at https://wits.worldbank.org/GlobalNetwork.aspx?lang=en
[4] See http://rtais.wto.org/UI/PublicMaintainRTAHome.aspx

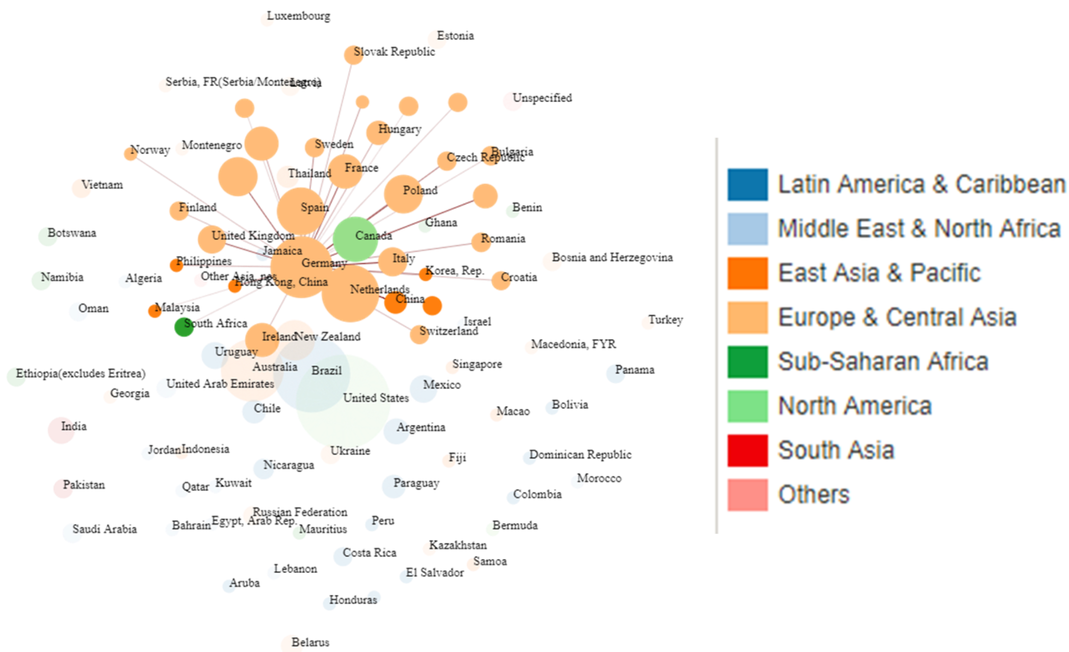


Figure 2.13. Export flow of meat in Germany. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

This figure shows export flows of meat in Germany and is represented in the network as a light orange node. According to this network, Germany exports meat mainly to other European countries (i.e. other light orange nodes) being the United Kingdom, Poland and Italy important partners in terms of export share (i.e. the thickness of the links with these countries). Some of them like the Netherlands and Spain are also larger exporters of meat which is noted by the size of their respective nodes. This is a clear evidence of regionalism in trade. However, exports from Germany to countries located in other continents is also present (e.g.

Canada in light green, South Africa in dark green, and East Asia and Pacific countries in dark orange).

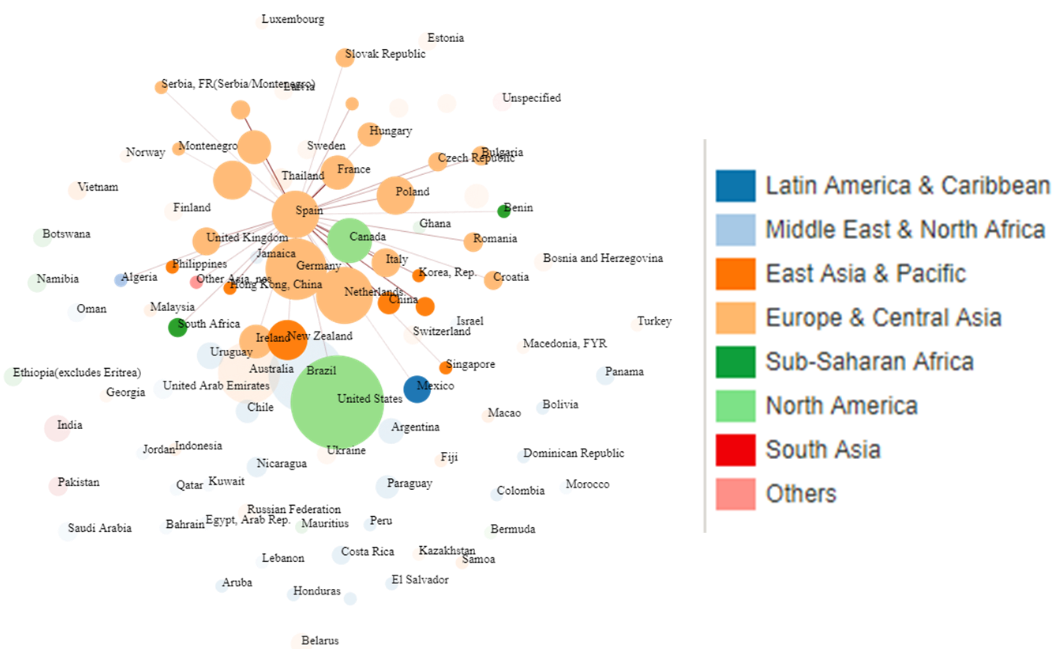


Figure 2.14. Export flow of meat in Spain. Source:

https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

Figure 2.14 shows export flows of meat but from the point of view of Spain. As in the previous figure, there is a clear evidence of regionalism. That is, the main partner countries of Spain are other European countries represented as light orange circles. In considering the thickness of the links, France is an important partner of Spain in terms of export share. However, there are other relevant partner countries located in other continents such as Canada in light green and China in dark orange.

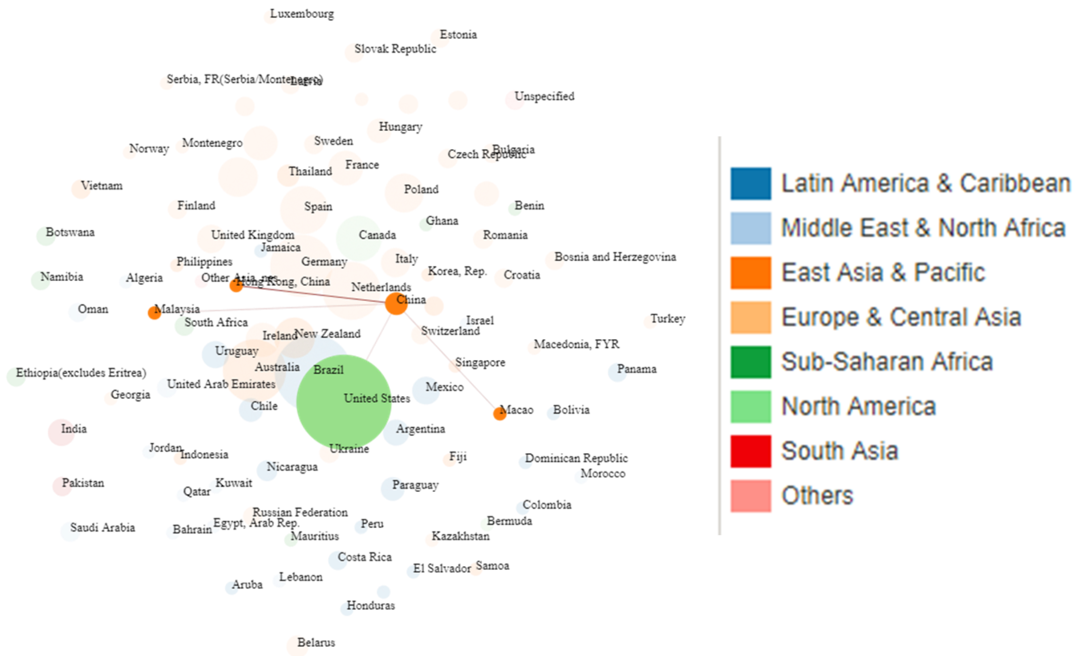


Figure 2.15. Export flow of meat in China. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

The same regionalism pattern is seen in this figure which shows export flows of meat from the point of view of China (note that the small size of the node of this country indicates that this is not a major exporter country in the world). That is, China exports this good mainly to other East Asia and Pacific countries represented in dark orange being Hong Kong an important destination in terms of export share. China also exports meat to the United States, but the export share in this case is not as significant as the export share in other countries in the Region (see the thickness of the links in the figure).

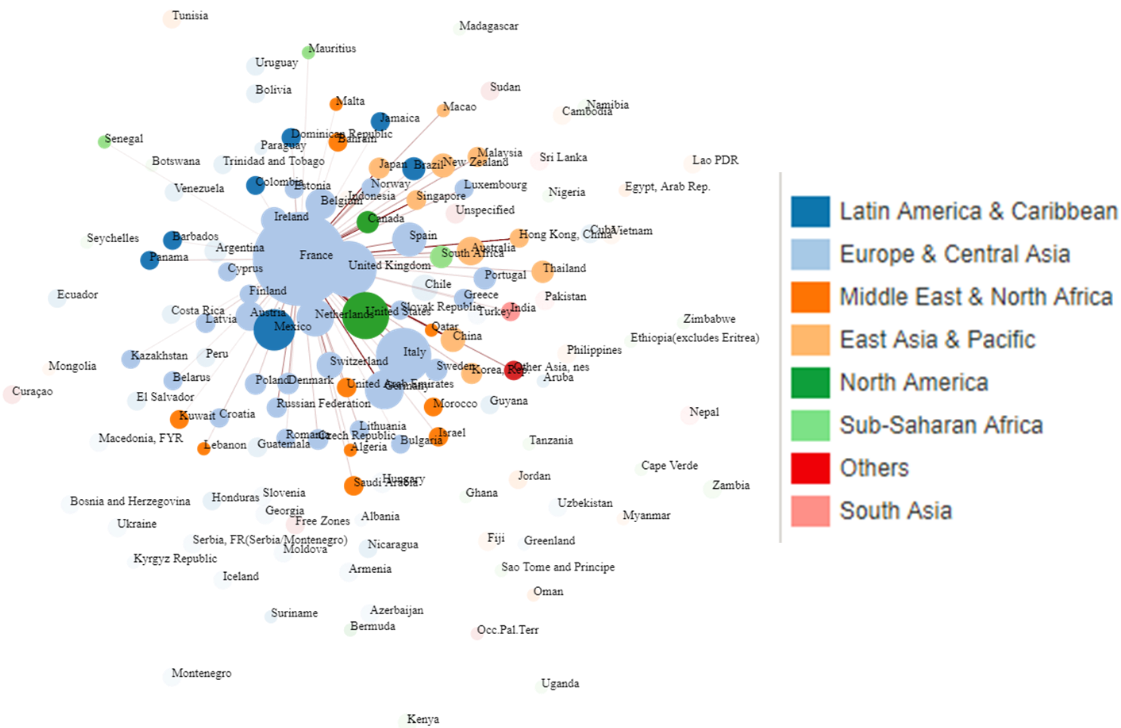


Figure 2.16. Export flow of wine in France. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

According to this figure, regionalism is also present in the market of wine. This network is seen from the point of view of France, a major exporter of this product in terms of global export share (see the size of the node of this country). In spite of this share, the export destinations are mainly European countries (in light blue). However, France also exports wine to countries located in other regions. Important non-European destinations in terms of export share are Canada (in dark green), and several East Asia and Pacific countries (in light orange) such as China, Hong Kong and Singapore.

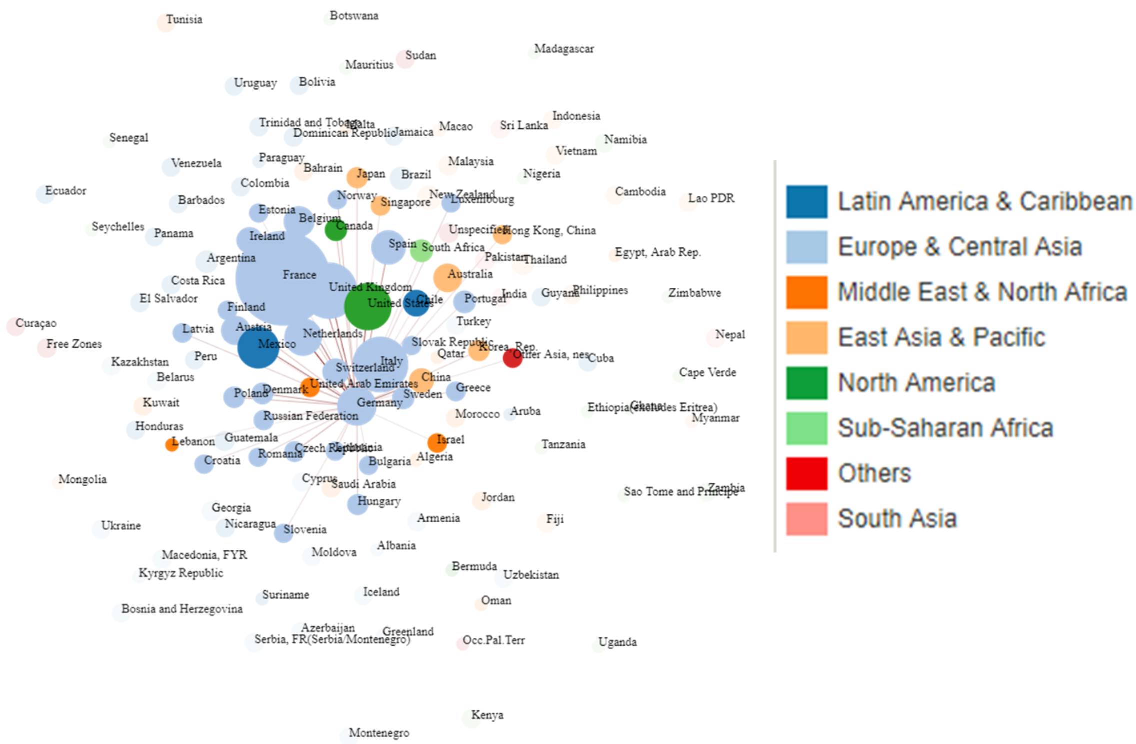


Figure 2.17. Export flow of wine in Germany. Source:

https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

The same regionalism pattern identified in the previous figure is observed in the case of Germany. That is, this country exports wine mainly to other European countries (in light blue) being the UK and the Netherlands relevant destinations in terms of market share. In spite of this regionalism, Germany also exports wine to non-European countries such as Chile and Mexico (in dark blue), Canada and the United States (in dark green), and several East and Middle East countries (in light and dark orange, respectively).

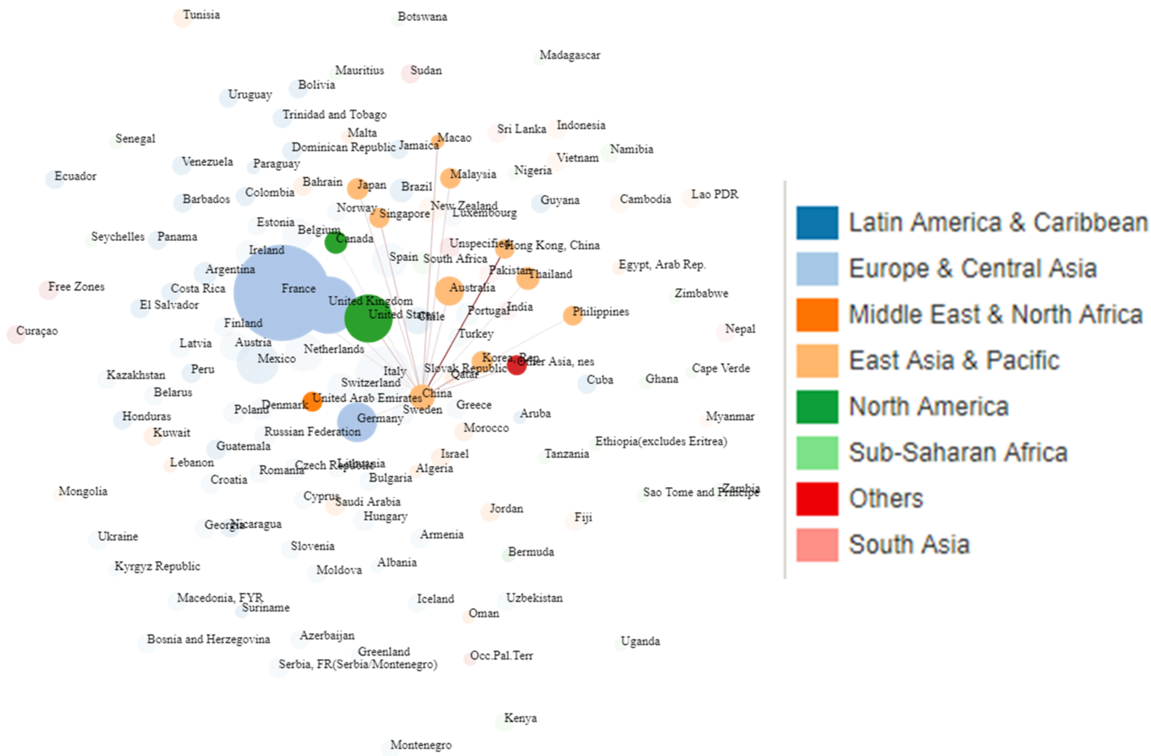


Figure 2.18. Export flow of wine in China. Source:

https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

Figure 2.18 shows that regionalism in the market of wine is also present in the East Asia and Pacific Region. The network in this figure, seen from the point of view of China, shows that this country exports wine mainly to other countries in the region (in light orange) being Hong Kong a major destination in terms of market share (see the thickness of the link between this country and China). However, as in the previous cases, China also exports wine to countries located outside the region such as the United States and Canada in North America (nodes in dark green); and United Kingdom, France and Germany in Europe (nodes in light blue).

In summary, according to Figures 2.13 and 2.14, the destination of export flows of meat from Germany and Spain are mainly European countries. A similar trend is found in other European countries from the same source of information. In contrast, Figure 2.15 shows that China exports meat mainly to East Asian countries (the same trend is found when considering other countries in the region). In relation to wine, similar evidence of regionalism in terms of export trade flows are found in Europa and East Asia (see Figures 2.16, 2.17 and 2.18). For example, France is a major exporter of wine and it supplies countries in several continents. However, there is a concentration of trade in Europe. While this evidence does not imply that the existence of regionalism in RTAs has caused regionalism in trade of food processed goods, it is interesting to note that a possible association exists.

In considering centrality, on the other hand, note in Figure 2.2. that there are countries that link different free trade areas located in different continents by means of bilateral agreements. An example is Chile. The following figures shows that the same trend is found when considering export flows of meat and wine in this country as well as the United States.

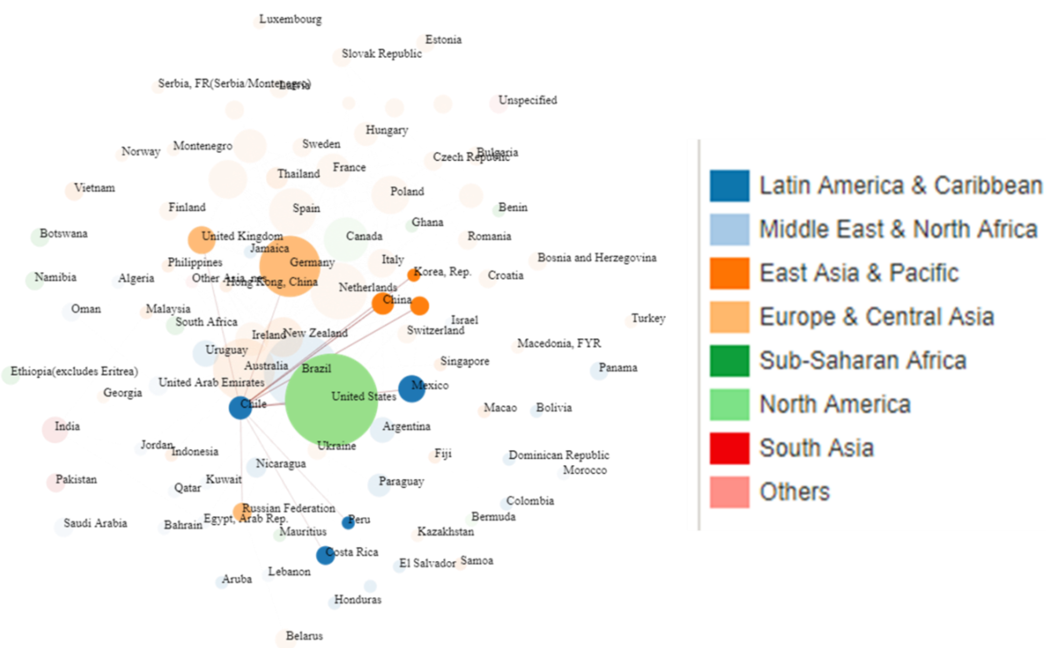


Figure 2.19. Export flow of meat in Chile. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

This figure shows the export flow of meat from the point of view of Chile. According to the network in this figure, Chile is not a major exporter of meat in terms of market share (its node is relatively small in size), and there is no evidence of regionalism. This can be seen from the fact that this country exports this good to a range of destinations that include European countries (in light orange), East Asia and Pacific countries (in dark orange), other Latin American countries (in dark blue), and the United States (in light green). In addition, the most relevant destinations are countries outside the region (see the thickness of the links). This evidence suggests that Chile has a central position in the trade market of meat.

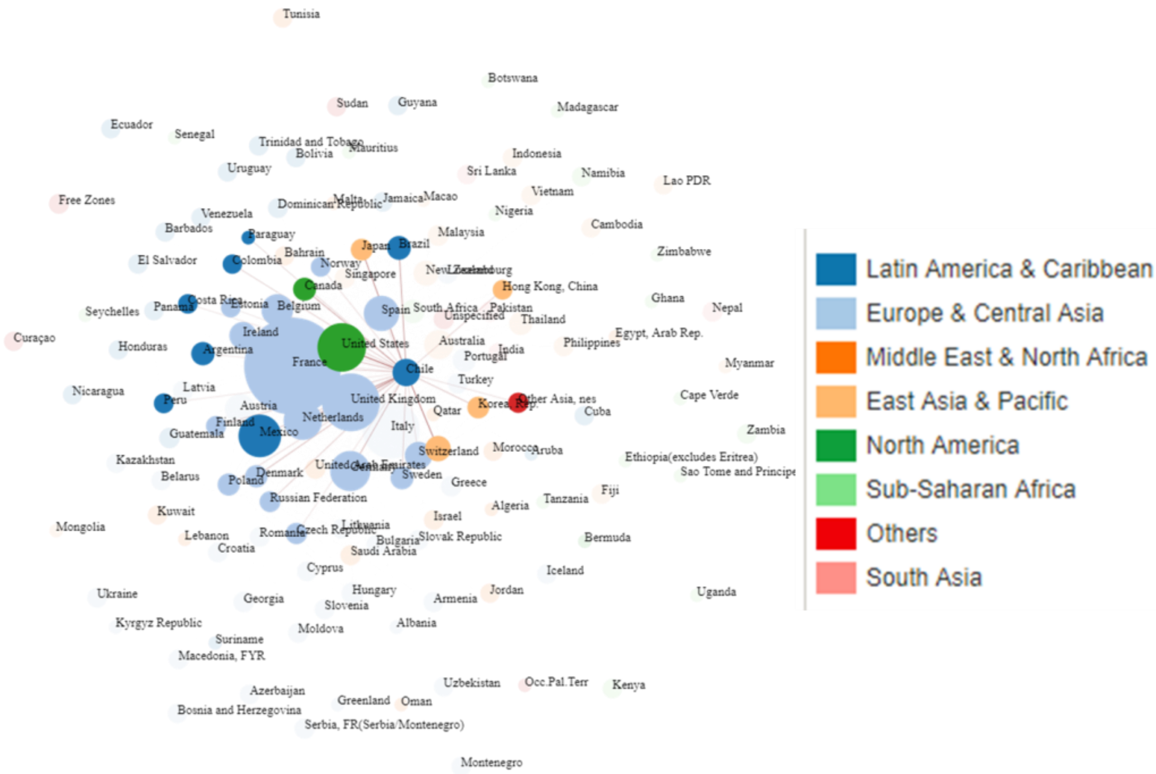


Figure 2.20. Export flow of wine in Chile. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

This figure shows that Chile also occupies a central position in the export network of wine. That is, there is no evidence of regionalism because the destinations of wine from Chile include several countries in Europe (in light blue), Latin America (in dark blue), East Asia and Pacific (in light orange) and North America (in dark green). In considering the thickness of the links, the network shows that the main destinations in terms of export share are countries located in other regions (e.g. United Kingdom, United States and Japan, among others).

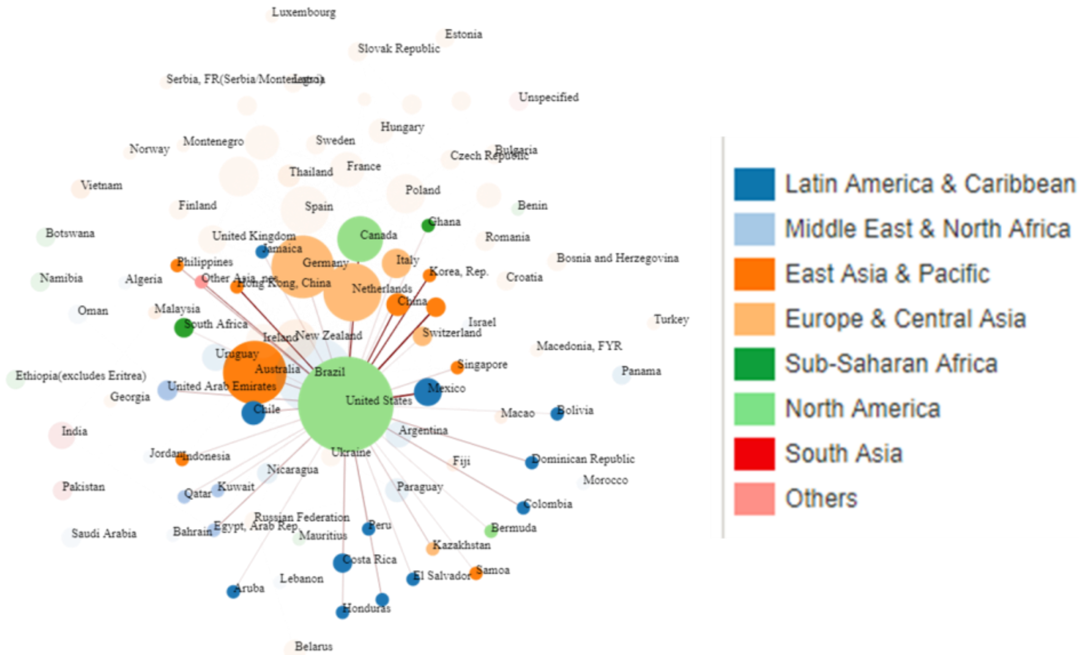


Figure 2.21. Export flow of meat in USA. Source:

https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

According to this figure, the United States also occupies a central position in the market of meat. This country is a major exporter of this product as revealed by the large size of its node, and is connected to several countries located in different continents across the world. In terms of export share of meat, the most relevant destinations correspond to countries in the East Asia and Pacific region and Europe confirming that USA plays a role in linking different regions in the world.

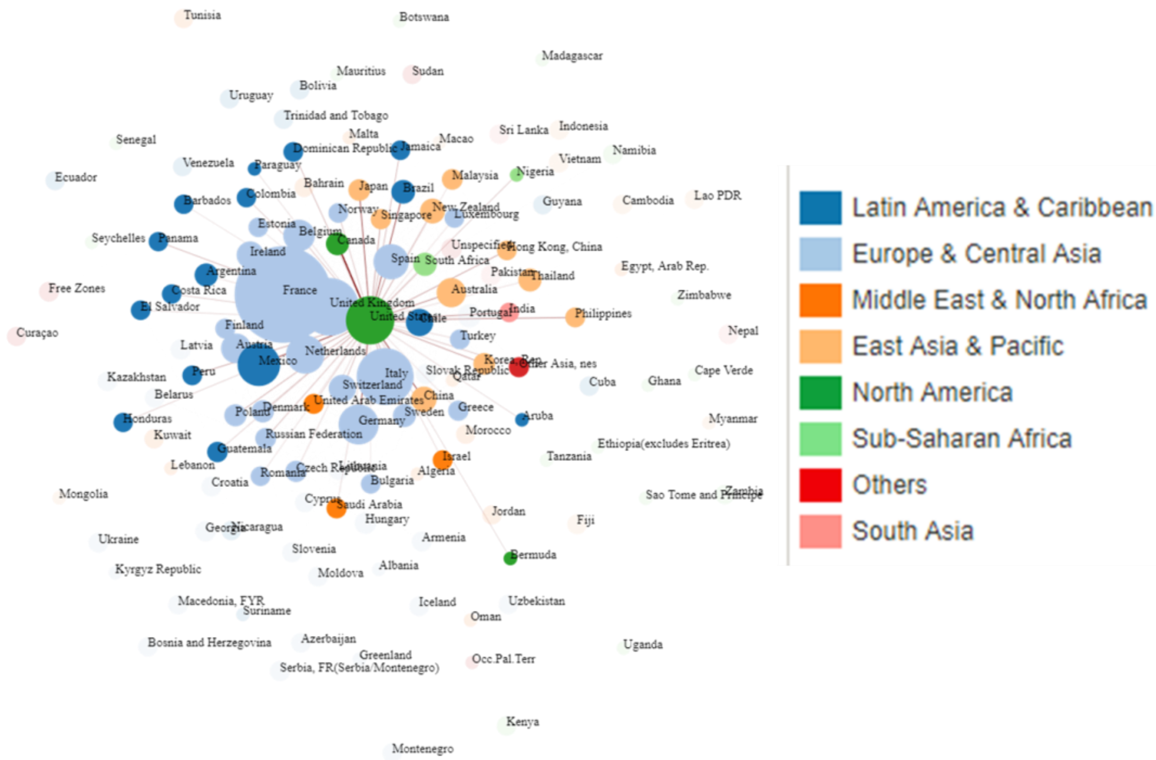


Figure 2.22. Export flow of wine in USA. Source: https://wits.worldbank.org/GlobalNetwork.aspx?lang=en

As in the previous case, this figure shows that USA occupies a central position in the export network of wine in terms of linking different regions across the world. In particular, this country supplies a range of destinations in North America (in dark green), Latin America (in dark blue), Europe (light blue), East Asia and Pacific (light orange), and Middle East (in dark orange).

The main feature that is identified in Figures 2.19, 2.20, 2.21 and 2.22 is that Chile and USA supply meat and wine to countries in different continents. Moreover, most of the countries involved in this trade have an RTA with Chile or USA. For example, the countries that are linked to Chile through export flow of wine in Figure 2.20 have all an international trade agreement with Chile. This evidence suggests that Chile and USA are central countries in the export networks of meat and wine.

In summary, in considering Figures 2.1 and 2.2 and the ones presented in this section, it is suggested a correlation between RTAs and trade flows of food processed goods in three main aspects. Firstly, it appears that trade growth rate flow of these goods is slowing down. Secondly, RTAs and trade flows of food processed goods have in common that they are concentrated in clusters of countries located in determined geographical areas. Thirdly, there are countries that play a central position in the network in the sense that they bridge the existing clusters. These similarities reinforce the claim that current levels of agricultural trade liberalisation reflect the existing regional agreements.

Let us now describe in more detail the liberalisation process of agricultural and food processed goods. In relation to agricultural commodities, this has formally been studied by Fulponi et al. (2011) who found that substantial agricultural trade liberalisation is explained by the RTAs included in their investigation. They found in particular that about 90% of tariffs lines (averaged across individual tariffs concessions and sectors) of agricultural products were duty free by the end of the implementation period. In terms of geographical aggregates, the researchers found that Asia-Pacific agreements (i.e. tariff concessions between Asia-Pacific countries) achieved the highest liberalisation in terms of tariffs lines with 97% being duty free when fully implemented. Latin America agreements (i.e. tariff concessions between Latin American countries) have also achieved a dramatic liberalisation from an initial share of duty free tariff line of 27% to 85% over a period of ten years, and 95% when the implementation is completed. Finally, RTAs concluded between countries from different regions have achieved a more modest trade liberalisation from 68% to 86% of duty free lines at the end of the implementation period. The researchers also aggregated countries into North and South aggregates and found that the average share of duty free tariff lines increased from 28% to 92% in the South-South aggregates, and from 68% to 87% in the North-South aggregates.

However, while much of the literature on agricultural trade liberalisation focuses on bulk or raw commodities (e.g. cereals, rice, sugar and so on), this overlooks the fact that a high proportion of international trade involves sectors downstream from agriculture where trade is in semi-processed or highly processed food products. Regarding food processed goods, trade liberalisation has resulted in large increases in imports and domestic production of highly processed foods (Friel et al., 2016). Moreover, they represent the largest share of agricultural trade, a fact that is reflected as a significant change in dietary habits in several countries across the world (Liapis, 2011, 1012; Thow et al. 2010). In relation to this trade share, trade in food processed goods is dominated by high income OECD countries, followed by emerging economics. However, the share of trade of these goods in low income countries is smaller.

According to Liapis (2011), trade flows of processed foods across countries have at least doubled between 1995 and 2008. In this period, trade among rich countries increased at an average annual rate of 6.1%, and trade among lower income countries grew at an average annual rate of 11.6%. In spite of these rates, trade of processed goods at that time was mainly among high income countries. For example, in 2008 trade flows among rich countries accounted for around US$ 334bn. In contrast, trade flows among low income countries accounted for US$49bn. In relation to trade flows from rich to low income countries accounted for US$60bn, and from low income to rich countries accounted for US$54bn.

Another interesting aspect of international trade of food processed goods is that there is evidence of intra-industry trade of processed goods. An early work by Hartman et al. (1993) adopted the Grubel and Lloyd index (i.e. an index that measures the absolute value of industry i's exports offset by industry i's imports, expressed as a proportion of that industry's total trade) and found high levels of intra-industry trade in meat packing, butter, fluid milk and breakfast cereal, among others. According to the results by these researchers, intra-industry trade is more likely when products are differentiated, when tariffs are similar between countries, when there are economies of scope and when markets are not highly concentrated. On the other hand, Qasmi and Fausti (2001) found using the same index that intra-industry trade of processed food products (e.g. processed cereal, sugar and confectionery, processed fruit and vegetable, and the processed meat) increased between USA, Canada and Mexico since the passage of the NAFTA agreement. However, intra-industry trade did not occur in bulk commodities with little or no processing. Finally, Anderson et al. (2016a) found evidence of intra-industry trade in wine and concluded that the growing demand for wine over the last decades is increasingly being served by new wine exporters, without displacing the historical core of the wine producers.

This evidence and the apparent correlation between the concentration of RTAs in geographical areas and trade flows of food processed goods are both relevant aspects that are considered in this dissertation. They are used in Section 2.4 as antecedents to support the adoption of the proposed international trade network that is developed in the current investigation.

2.3.3 Policy biases

There is an extensive literature in relation to the existence of policy bias in terms of policies that are placed to maximise objectives other than social welfare. In this regard, Rausser et al. (2011) provide a detailed review of the subject and explain that there are conflicts between the public interest and special interests in the design of public policies. In this context, the implementation of a public policy can be the result of manipulation by powerful groups actively engaged in the pursuit of their own self-interest. Evidence of this phenomenon has been found in the agricultural sector in which policy biases arise as a consequence of the intention of governments to put policies in place in order to be re-elected (see for example, de

Gorter and Tsur, 1991; Ames, 1992; de Gorter and Swinnen, 1994; Swinnen, 1994).

Policy bias has also been identified in relation to international trade. For example, Grossman and Helpman (1994) and Grossman (2016) argue that this bias reflects governments' intention to capture voter. In this context Conforti and Salvatici (2004) explain the following: "*In terms of expected total economic benefits, free trade or "strong" trade liberalization would be the dominant strategy for both groups. At the same time, this result highlights the extent to which the analyses that assume a "neutral" government are ineffective for understanding countries' behaviour in the negotiations. Apparently, there are other variables that explain governments' behaviour, such as sensitivity to agricultural lobbies, and the attempts to maintain long standing protection* " (p. 13).

In considering policy biases in international trade, it is argued that the lack of progress in a global agreement in agriculture has been attributed by a number of researchers to the existence of governments that are politically biased in favour of specific groups within a country (see for example Cho, 2010; Regmi et al. 2005; Khor, 2003). According to Anderson et al. (2013), this is reflected in the high level of protection to farmers through policy intervention. For example, tariffs that apply to EU food and agricultural imports are considerably higher compared with other sectors, and non-tariff measures are much more significant (McCorriston, 2018). This evidence is presented in the following figure.

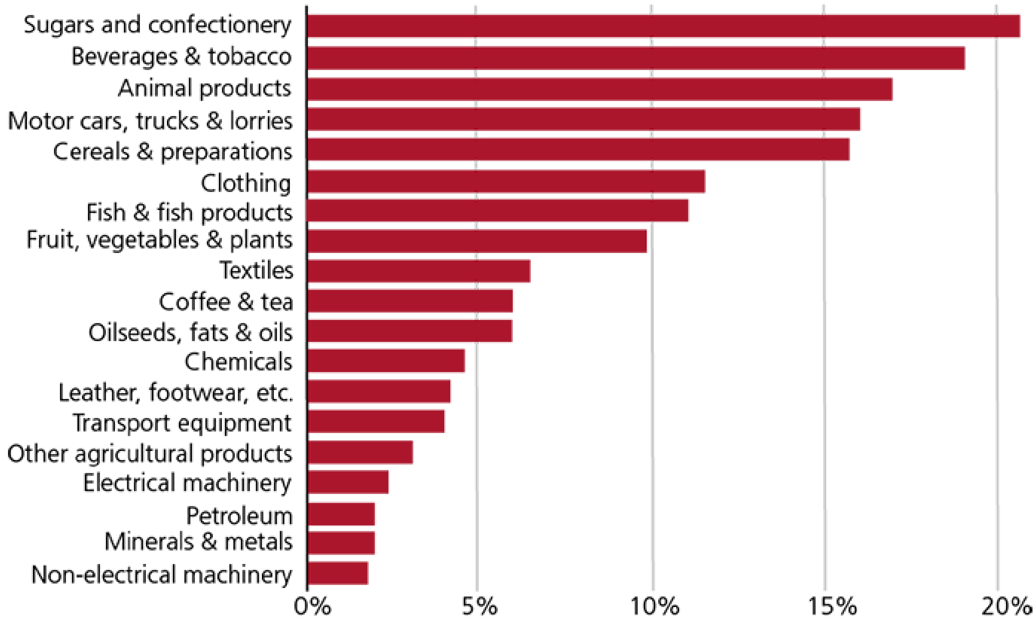


Figure 2.23. EU Average Final Applied Tariffs by Industry (Source: House of Lords, 2017)

Anderson et al (2008) have attempted to present an overview of the extent of protection in the agricultural sector across a wide range of countries by using the nominal rate of assistance (NRA). This is defined as "*the unit value of production at the distorted price less its value at the undistorted free market price expressed as a fraction of the undistorted price*" (p. 681). A positive (negative) value of NRA indicates that governments' policies have increased (decreased) gross return to farmers with respect to the gross return that they would have obtained without policy intervention. The NRA includes all types of assistance to agriculture including import tariffs, export subsidies and domestic support, among others. Figure 2.24 shows the evolution of NRA in some relevant groups of countries considering five-year average from 1980.

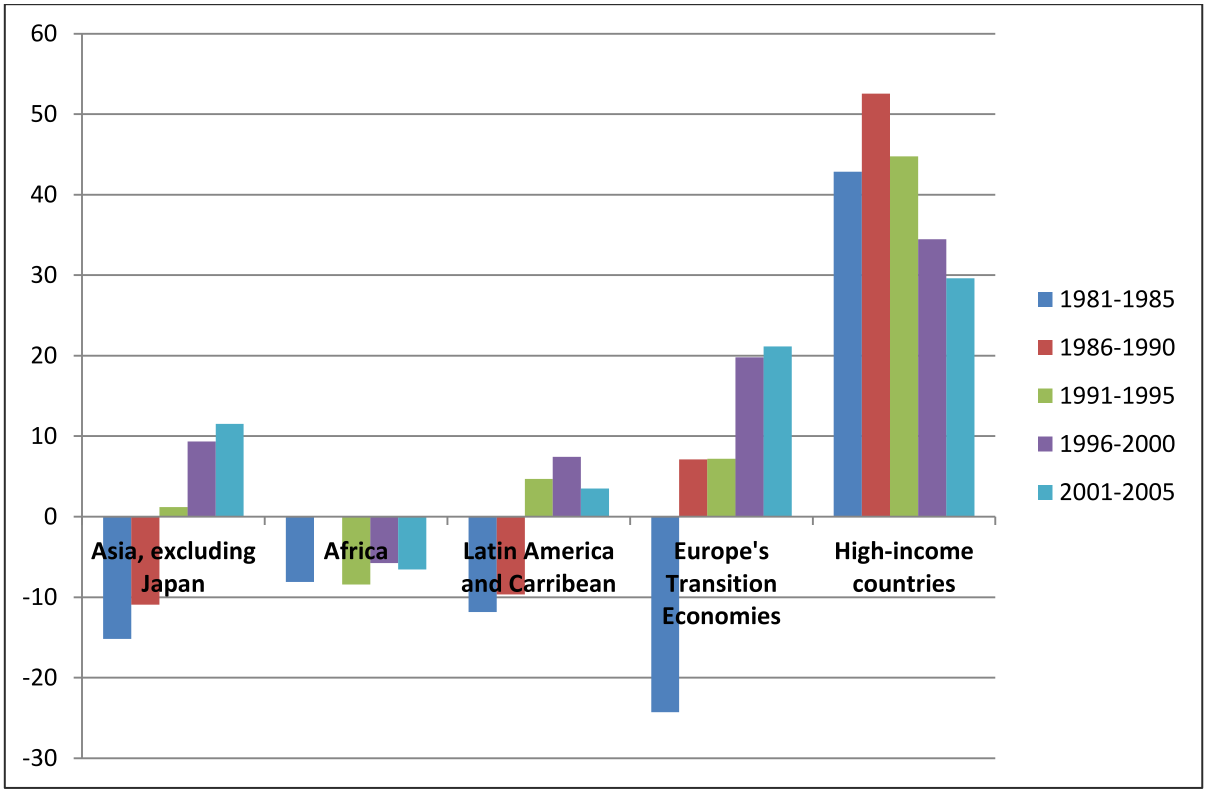


Figure 2.24. Evolution of NRA (Source: Anderson and Valenzuela, 2012)

This figure shows that until 2005, Asian, Latin American and Europe's Transition countries have shifted from taxing (i.e. negative NRA) to assisting agriculture (i.e. positive NRA) in the period of time 1981-2005. African countries, on the other hand, have sustained a taxation policy to agriculture while high-income countries, in contrast, have sustained support to this sector although the level of support has decreased. According to Anderson (2009), this trend suggests that governments have initially taxed agriculture with the purpose of promoting the manufacturing sector. When countries reach certain level of industrialisation, they reverse their agricultural policy in order to protect agriculture. This trend also reveals that the efforts made after the Uruguay Round to reduce the levels of protection to agriculture have been unsuccessful. Moreover, a number of developing countries have increases the levels of support since the 1990s.

In relation to exportable and import-competing agricultural goods, the following figures provide key information based on the NRAs to exportable and import-competing agricultural goods for the same groups of countries.

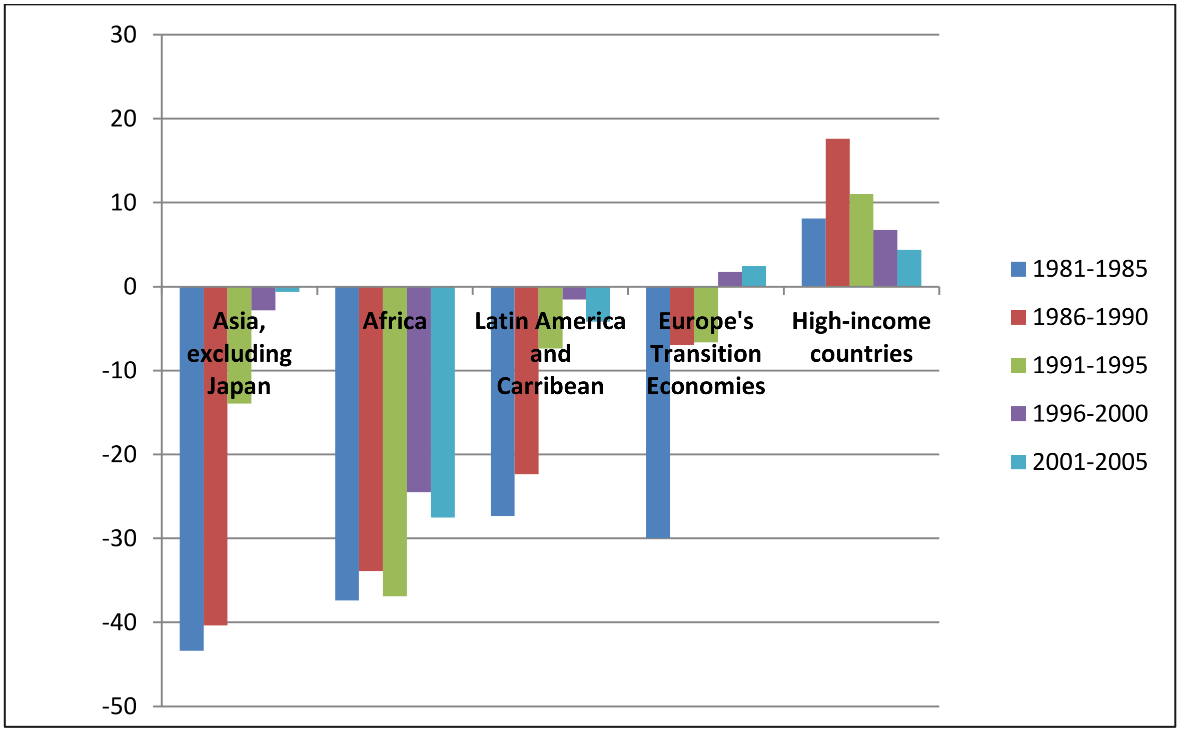


Figure 2.25. NRA to exportable agricultural goods. (Source: Anderson and Valenzuela, 2012)

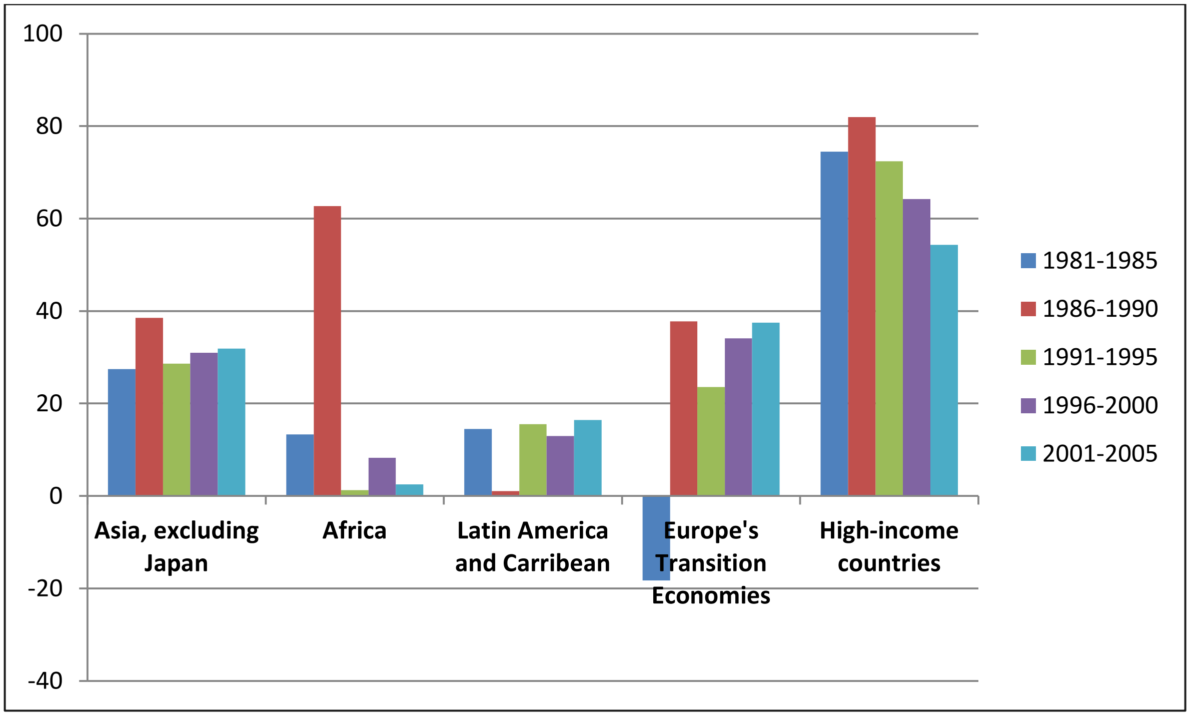
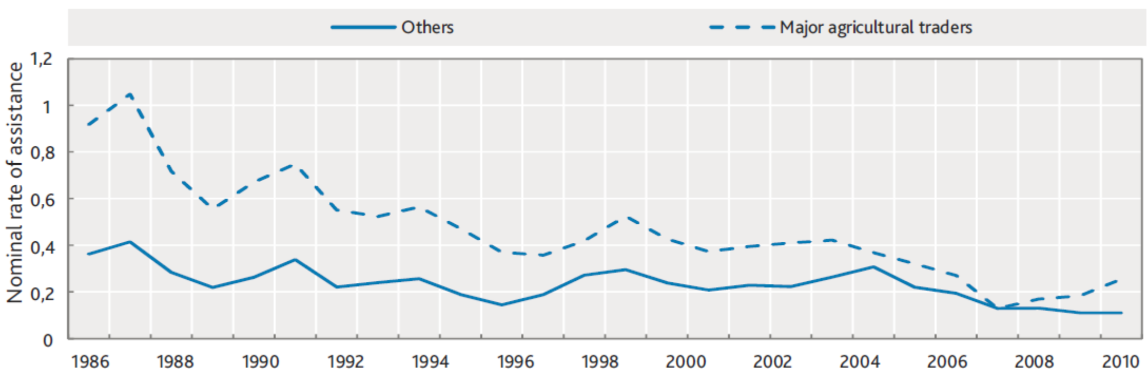


Figure 2.26. NRA to import-competing agricultural goods. (Source: Anderson and Valenzuela, 2012)

According to Figure 2.25, most of the groups of countries have decreased the levels of taxation to the exportable goods. The only exception is the group of high-income countries who have supported their exportable goods, although this support has decreased during the last two decades. Figure 2.26, in contrast, shows that all the groups of countries have supported the import-competing agricultural goods. These figures also show that the levels of support to import-competing agricultural goods in these countries are much larger than the levels of support to exportable goods. This implies that both developing and developed countries have biases in favour of antitrade policies and this supports the argument that policies are placed in order to satisfy political objectives.

It is important to recognise, however, that this conclusion has to be made with caution for three reasons. Firstly, support to agriculture is above average in the top 20 OECD trading nations. This suggests that the impact of distortions on trade could be significant in this group of countries because they accounts for about 70 present of total agro-food exports and imports (Greenville, 2017). Nonetheless, the average NRA from output subsidies in these countries has decreased over the last decades as shown in the following figure.



2.27. Average NRA from output of the major agro-food traders (source: Greenville, 2017)

It appears that this decrease in NRAs has not facilitated a global agreement in agriculture suggesting either that the levels of protection as a consequence of policy biases still remain high, or that there are additional factors that play against an agreement that have not been identified yet.

Secondly, it could be the case that there is protection of the agricultural sector not per se to give farmers higher incomes but to promote food security and this may be a valid policy for the government to pursue and will be reflected in the biased welfare function (see for example Bellemare and Novak, 2017). This means that biased welfare may not necessarily reflect policy biases.

Finally, it may not only be farmers who are protected but that there are high levels of protection involving intermediaries in the food sector. This would indicate that lack of agricultural liberalisation may not only reflect policy biases in favour of the farming sector, but also in favour of other firms such as intermediaries. Actually, as shown in the figure below, protection in terms of non-tariff measures is higher in processed food than in other sectors suggesting some sort of policy bias in higher levels in the food sector supply chain.

Industry	Egger *et al.* (2015)	Industry	Berden *et al.* (2009)
Primary Agriculture Processed Food Beverages and Tobacco	25 48 42	Food, beverages & Tobacco	57
Chemicals, pharmaceuticals	21	Chemicals & Chemical Products	24
Fabricated Metals	38	Basic & Fabricated Metals	12
Motor Vehicles	20	Transport Equipment	22
Electrical Machinery	19	Electrical & Optical Equipment	7
Goods Average	13	Overall Weighted Average (All Sectors)	20

Figure2.28. Comparison of the Tariff (Ad Valorem) Equivalent Effects of Non-Tariff Measures in EU (Source: McCorriston, 2018).

2.3.4 Market power in the food processed industry

Traditional models used to simulate agricultural and food scenarios assume that international markets of agricultural goods operate under perfect competition (Soregaroli and Sckokai, 2011). Examples of these approaches are the AGLINK model (OECD, 1998), the FAPRI model (Devadoss et al, 1993), and the WEMAC model (INRA, 2010). In spite of these commonly used models, the assumption of perfect competition in international markets of agricultural goods have been questioned by a number of researchers. In particular, it is argued that imperfect competition arises from the highly concentration of intermediaries in the food industry. For example, the vertical relationship between suppliers and retailers of fresh and food processed goods in the UK is dominated by nine large retailers, Tesco being the largest of these (Duffy et al., 2003; and White, 2000). In line with this argument, Sexton (2013) explains that food industries with highly concentrated intermediaries have structural oligopolies/oligopsonies, and that even with modest amounts of market power, welfare transfers between groups can be significant.

In the context of international markers of agricultural and food processed goods, on the other hand, McCorriston (2002) explains that the assumption of perfect competition in international markets does not captures the growing market power of food retailers across EU countries. In this regard, this author points out: "*Arguably, it is the high and increasing concentration in food retailing that is the most distinguishing feature of the European food chain. Taken together with the oligopolistic nature of food manufacturing in many European countries, the food* *chain as outlined in Figure 1 is perhaps best described as a successive multi-stage oligopoly. In this case, an oligopolistic sector sells its output to another oligopolistic sector that distributes the final good to consumers*" (p. 354).

The existence of imperfect competition in international markers of agricultural and food processed goods has important implications in terms of international trade policy outcomes. This has formally been explored by Sexton et al. (2007) from a model that considers oligopolies/oligopsonies structures in the food industry. According to these researchers, even relatively modest departures from perfect competition can cause much of the benefits from trade liberalisation to flow to marketing firms instead of producers. They also found that the impact of a trade reforms is affected by the extent of competition in the downstream food sector and the extent of buyer oligopsony power.

2.4 Research gap and the international trade network approach

In order to highlight the research gap that this dissertation aims to contribute to fill, the following key ideas discussed in the previous sections are considered: it is unlikely that a global agreement in agriculture can be signed; trade of agricultural and food processed goods are concentrated in geographical regions and this might reflect regional agreements signed by countries located in the proximity (e.g. Asia Pacific Trade Agreement); the lack of agricultural trade liberalisation seems to be explained by policy biases; there is imperfect competition in the supply chain of food processed goods that are traded internationally that is associated with the existence of potentially powerful intermediaries; and there is evidence of intra-industry trade of food processed goods.

The research gap that was identified is that while these key ideas are well-known, they seldom appear altogether explicitly in quantitative assessments of trade liberalisation. For example, most of the research on agricultural trade assumes inter-industry trade. Likewise, several studies on the industrial organisation of the food sector recognise the importance of limited firms with market power and have made important contributions for the understanding of the impact of imperfect competition on the food industry. However, these contributions seldom features in assessments of trade liberalisation in agricultural markets and where intermediaries with market power coexist with political economy and policy motives by government.

The existence of this research gap has an important implication: it is not possible either to predict outcomes of agricultural trade negotiations or to propose possible policy strategies to facilitate agricultural trade liberalisation from existing modelling approaches because either most of them assume perfect competition in these markets, or they do not consider some of the key ideas described above. That is, they largely ignore the influence of intermediaries in agricultural markets that have the potential to exercise market power as well as the influence of policymakers that are politically biased.

In considering this gap, the following research questions have been established in this thesis:

a) Is the possibility of a global agreement in agriculture influenced by the presence of biased policymakers and intermediaries who exercise market power in the supply chain of food processed goods?

b) Are the existence of FTAs influenced by the presence of biased policy-makers and intermediaries who exercise market power in the supply chain of food processed goods?

c) What policies may be implemented to facilitate agricultural trade liberalisation when there are biased policymakers and intermediaries that exercise market power in the supply chain of food processed goods?

In order to answer these questions, an extended version of the international network model by Goyal and Joshi (2006) was adopted. The main contribution of the thesis is, therefore, the introduction of the key ideas described at the beginning of this section into the debate on agricultural trade liberalisation using an international trade network approach. It would appear that the present dissertation is the first theoretical academic work to consider this extension in the issue of agricultural trade liberalisation[5]. In this respect, Salvatici and Nenci (2017) explain that a network framework to study agricultural trade has not been developed and

[5] There is some research that has included the agricultural sector. However, these works only describe the topological property of the network of selected agricultural goods but not the theoretical foundations that explain these networks (see for example De Benedictis et al., 2014).

they argue that this framework would play a larger role as a tool in agricultural trade analysis. The aim of the research here is to make a contribution to this emerging research area.

The international trade network approach considered in this thesis was independently developed by Goyal and Joshi (2006) and Furusawa and Konishi (2007) as an extension of the social network model proposed by Jackson and Wolinsky (1996). In this extension, countries are represented as nodes and bilateral agreements as links (a formal mathematic description of this model is presented in Chapter Three).

In this setting, what motivates countries to form bilateral agreements or break existing ones depends on the objective function of the government. For example, Furusawa and Konishi (2007) assume that governments care about maximising welfare. In this context, a particular country will have an incentive to sign an agreement with another country if and only if this agreement increases domestic welfare in the former. Likewise, a particular country will break an existing agreement with another country if and only if this allows the government to increase domestic welfare. In order to determine the stability of the network, these authors adopted the pairwise stability concept of Jackson and Wolinsky (1996). According to these researchers, a network is stable if and only if no country has an incentive to break an existing agreement, and if two countries do not have an agreement, then at least one of them is not willing to form one.

The alternative framework developed by Goyal and Joshi (2006) adopted the same equilibrium concept. However, they assumed that governments care about maximising a weighted welfare function in order to determine the influence of policy biases on the stability of the international trade architecture. It was found that the framework by Goyal and Joshi (2006) was the most useful approach to study agricultural trade liberalisation for the following different reasons.

1. The international network model developed by Furusawa and Konishi (2007) is more general and complex in terms of *market structure* because it assumes that firms produce differentiated goods. In contrast, the model of Goyal and Joshi (2006) assume that firms produce a homogeneous good which is traded internationally. Nonetheless, the framework by Goyal and Joshi (2006) is more general in terms of *political incentives* given by their *weighted welfare function*. This makes Goyal and Joshi's model richer and more realistic in terms of political influence and, therefore, more suitable to study agricultural trade liberalisation when allowing for policy biases.

2. The network model by Goyal and Joshi (2006) assumes that a country is composed of two sectors: consumers; and firms that exercise market power. The advantage of this assumption is that these firms can be considered as intermediaries in the extended version of the model developed in this dissertation. In addition, the original version of the model by Goyal and Joshi (2006) is flexible enough to introduce a third sector which corresponds to the farming sector. This can be done by including this sector as an additional

economic group into the weighted welfare function. This extension has already been adopted McCorriston and MacLaren (2012, 2013) but in another research context. The introduction of this sector into this framework has the advantage that the resulting model is based on an explicit description of the supply chain in agriculture. This makes this model a more realistic approach with respect to alternative models proposed by related academic works. The introduction of the farming sector into the model by Goyal and Joshi (2006) is, consequently, one of the main novel contributions of this dissertation.

3. Regarding the weighted welfare function adopted by Goyal and Joshi, it not only can be used to analyse policy incentives within a single country, but also to represent real situations such as heterogeneous policy incentives across countries. In relation to this point, the bias in agricultural policy towards producers in developed countries or towards consumers in developing countries is well-known and this fact can easily be introduced into Goyal and Joshi's framework. Moreover, the proposed extended version of this model can also be used to explore the trade implications of putting policy weights on the food industry (i.e. the intermediaries) in some countries. This extension is important for two reasons. Firstly, the existence of intermediaries exercising market power in the agricultural sector has largely been ignored, and the network model offers an opportunity to fill this gap. Secondly, there is evidence that backs up the assumption that the food industry in some countries is favoured by policy biases. For example, Gawande and Bandyopadhyay (2000) show the following: industries that are well organised

get more protection; the concentration ratio also matters; and when the upstream sector is protected (agriculture in this case), the downstream sector gets more protection (the food industry) which relates to the issue of contingent protection (i.e. protection in one sector is contingent to what happens elsewhere). Another example is the work by Lopez (2008) who, using data for the US food sector, found empirical evidence of political weights in the food manufacturing industry.

4. The majority of the related research has studied the incentives of countries to reduce tariffs on third countries and the welfare trade liberalisation effect. However, this research has taken as given a fixed trading structure meaning that they not evaluate whether this structure is stable (see, for instance, Baldwin, 1999; Bond et al., 2004; Devadoss, 2006). It is for this reason that it is not possible to infer from this research the incentives of countries to sign global or bilateral agreements in agriculture for any trading structure. As a consequence, they cannot be used to determine under which conditions these agreements may lead to global free trade. In contrast, the network model formally analyses the stability of any trading structure making this approach an important extension to existing research in the area of agricultural trade.

5. The fact that the network model assumes oligopolistic international markets makes this framework highly suitable to study the trade implications of having intermediaries exercising market power in the food industry.

6. The proposed model allows for intra-industry trade which is one of the key observations described for the case of food processed goods.

7. While the original network model developed by Goyal and Joshi (2006) was designed to study the formation of bilateral agreements in a network context, this framework can easily be adapted to study global free trade by modifying the stability concept. The introduction of a stability concept of this nature is another novel innovation offered by this thesis making this an additional contribution to the subject (this is formally explained in Chapter Five).

8. Finally, traditional models in economics can broadly be grouped in two groups, namely: (i) models that study the interaction of small groups of individuals (game theory); and (ii) models that study the interaction among large groups (competitive markets and general equilibrium). This is actually the types of models that have been adopted to study issues related to agricultural trade policies (see for example Karp and Perloff, 1994; Deodhar and Sheldon, 1997; Conforti and Salvatici, 2004; Hoekman and Olarreaga, 2004; Han and Lee, 2010). According to Goyal (2015), a number of phenomena appear to arise in between these two extremes and the network model approach has the potential to identify heterogeneous economic behaviour of individual inserted in a network, and how this behaviour is affected by their relative position in the network. The same applies to agricultural trade: the international trade network has the potential to identify phenomena arising in between the two extreme traditional approaches and it

can identify heterogeneous economic behaviour of individual countries inserted in the international network. As shown in this dissertation, this property of the international network model makes it possible to identify, for example, alternative explanations for the failure of a global agreement in agriculture that apparently have not been proposed to far. One of them is the behaviour of countries that occupy a central position in the network (i.e. centrality) as a key factor in explaining the lack of progress in agricultural trade liberalisation.

It important to recognise, however, that the model by Goyal and Joshi (2006) has a major disadvantage. That is, it is very complex in mathematical terms and becomes untreatable under the assumption of endogenous tariffs (i.e. when it is assumed that governments place the tariffs that maximises welfare) a fact that is explicitly recognised by these researchers: "*Given the complexity of the computations involved, we have been unable to completely characterize the nature of stable networks in this setting. We do have some interesting partial results*" (p. 768). The tractability problem not only is present in international trade networks, but in many theoretical applications based on the network approach in general. This is formally stated by Goyal (2015) who explains: "*The tension arises from problems of tractability: models with fully rational agents and general network structures are difficult to analyze, especially in terms of deriving a clear relation between the network structure and individual behaviour. It is also difficult to incorporate heterogeneity in a tractable way within a network model with fully rational agents*" (p. 4).

Given that the international network developed in this dissertation is a much more complex extension of the original model by Goyal and Joshi, the tractability problem has unfortunately been inherited. In order to deal with this problem, some strategies were adopted. They are explained as follows.

Firstly, we have assumed a world composed of four countries. This extension was useful to solve the equations of the model assuming simulated vales of some key parameters for each network that can be formed using four countries. Even using this simplification, it was possible to identify interesting and relevant deviations from the results obtained by Goyal and Joshi (2006). For example, they found that global free trade is always a stable network. In contrast, it was found in this dissertation that global free trade may be unstable when there is a farming sector linked to intermediaries having market power. Note that simulations to extensions of Goyal and Joshi's model have also been adopted by Daisaka and Furusawa (2011).

It is important to highlight the fact that the use of a reduced number of countries to explore the issue of international trade has also been adopted by a number of researchers. For example, Facchini et al. (2013) adopted a three country trade model to investigate the formation of free trade areas and custom unions when governments have political incentives. Chen and Joshi (2010) used a three country trade model to study the effect of having free trade agreements with third countries on a country's incentive to sign an additional one. These researchers cited the work by Goyal and Joshi (2006) to explain that they obtained the same result referred to as *concession erosion* (i.e. profits made by an exporter firm is smaller when the importer country has already a number of agreements). Other examples are found in Saggi and Yildiz (2010, 2011) and Saggi et al. (2013) who studied whether multilateralism and bilateralism may lead to a global free trade using a model that considers three countries. These researchers formally explain that they obtained the same results of Goyal and Joshi (2006) for the case of symmetrical countries: bilateral agreements leads to global free trade. Another example is the research by Seidmann (2009) who also adopted a three countries model to study the formation of bilateral free trade areas, bilateral customs unions and trilateral preferential trading arrangements. This researcher compares his results with those obtained by Goyal and Joshi (2006) to show that when the trade negotiation process is dynamic and when countries are impatient, their dynamic approach can be used to refine the set of pairwise stable networks. Lake (2017) extended the work by Seidmann (2009) and developed a dynamic game network formation model with three countries to explore whether free trade agreements can lead to global free trade when countries are asymmetric. On the other hand, Zu et al. (2011) and Tran and Zikos (2014) adopted a version of Goyal and Joshi' s international trade network model to study the influence of R&D collaboration between firms in different countries on the trade system. In order to carry out this analysis, these researchers assumed a world composed of three countries. These examples illustrate how Goyal and Joshis' model can be used to contrast related framework composed of a reduced number of countries, and this provides support to the approach adopted in this dissertation. Finally, the strategy of considering a reduced number of nodes (i.e. normally three) is a practice that not only has been adopted in the research on international trade, but also in a number of network applications. See for example Kesavayuth and Zikos (2013), Rickman and Zikos (2016) and Lake (2016).

Secondly, in order to deal with the tractability problem, Goyal and Joshi (2006) adopted the following strategy. They developed most of the trade network analysis by assuming exogenous tariffs (i.e. each country establishes a prohibitive tariff avoiding any trade between them. If two countries decide to sign an agreement, then each one offers the other a free market access). This assumption allowed the researchers to simplify the mathematical complexity of the model significantly and to explore the effect of policy biases and asymmetry in market size across countries on the network trade architecture. They also analysed the network model under endogenous tariffs but only for the case of symmetric and politically unbiased countries given the complexity of involved the mathematical computations. Since the international network proposed in this dissertation is an extension of Goyal and Joshi's model, the same approach was adopted. This not only was useful to deal with the tractability problem, but also to identify deviations from the original version of the international trade model. In addition, the current research extends the analysis by exploring other situations that are relevant for the issue of agricultural trade liberalisation and that were not investigated by Goyal and Joshi, namely: asymmetry in policy biases; and biased governments when countries are asymmetric in market size and farmers' productivity. These extensions not only revealed deviations from the original network model, but also provided novel insights that explain some of the international patterns observed in the agricultural sector in the real world.

Thirdly, extreme political biased cases were considered to explore stable international trade networks under the assumption of exogenous tariffs. They correspond to the following: (i) governments are completely biased in favour of consumers (i.e. social welfare is equal to consumer surplus); (ii) governments are completely biased in favour of intermediaries (i.e. social welfare is equal to the total profits made by intermediaries); (iii) governments are completely biased in favour of the farming sector (i.e. social welfare is equal to producer surplus); and (iv) governments are politically unbiased (i.e. social welfare is equal to the unweighted welfare function). These extreme cases were useful to identify general patterns, to make extrapolations and also to focus on non-extreme cases that were found relevant for the debate of agricultural trade liberalisation.

In relation to the general research on international trade networks, some alternative network analyses have been introduced over the last years. However, they have a completely different focus and they are not appropriate to study agricultural trade liberalisation. For example, Pandey and Whalley (2004) studied how individuals' participation in networks (i.e. family members interacting with other family members in the location who value joint consumption, emotional support, etc) can affect the desirability of trade liberalisation under the existence of differential network properties in rural and urban areas. Chaney (2011), on the other hand, developed a network model with the purpose of evaluating the ability of individual exporters to access foreign markets, and how this ability is influenced by the number of connections they have with foreign importers. This network approach differs from the one adopted in this thesis in that it formally analyses how non-tariff barriers may prevent exporters from expanding their trade activities, being the number of links with foreign importers one of these barriers. A final example is the work by Zu et al. (2011) who developed a R&D collaboration network model in the open economy framework. These researchers introduced a double-layer pairwise stability concept to explore the network impact of two types of links across countries: bilateral trade agreements; and research joint venture links between firms in different countries. This research differs from the one developed in this dissertation because the aim of the current investigation is to explore the network trade impact of having intermediaries with market power in the agricultural sector, but not the impact of being involved in R&D collaboration. An exception is the work by Zhang et al. (2014) who developed an extended version of Goyal and Joshi's model to study the evolutionary dynamics of free trade agreement network formation when there are random perturbations that affect the model. These perturbations are defined by the authors as mistakes made by governments when signing bilateral agreements. This approach could be extended by allowing for intermediaries with market power in the food industry and when governments are boundedly rational and make mistakes sometimes. This possibility is considered for future research.

2.5 Summary and Conclusions

Trade liberalisation in the international trade system is explained mainly by the proliferation of regional agreements. This has led to a network characterised by the existence of trade concentrated in clusters of countries that occupy the same geographical area. International trade across clusters exists but is not as significant as trade within the clusters. There are also countries or group of countries that occupy a central position in the network in the sense that they bridge several clusters by means of bilateral trade agreements.

International trade liberalisation of agricultural and food processed goods are concentrated in geographical areas. This might reflect the existence of regional agreements that have been signed by countries located in the proximity (e.g. Asia Pacific Trade Agreement). In this regards, it is argued that lack of progress for additional liberalisation and for a global agreement in agriculture is explained by policy biases of governments who place policies in order to be re-elected.

There are other features associated to the agricultural sector that may explain the current international network configuration of agricultural and food processed goods but that have not fully been explored such as the existence of intermediaries in the supply chain that exercise market power, and intra-industry trade of food processed goods.

In considering all these aspects of the agricultural trade systems, a research gap was identified: while these key aspects are well-known, they seldom appear altogether explicitly in quantitative assessments of trade liberalisation. Given the relevance of the international trade network model by Goyal and Joshi (2006) as a potential tool to contribute to filling this gap, a formal description of this model and is provided in the next chapter.

CHAPTER THREE: The International Trade Network Model

3.1 Introduction

In the previous chapter, a research gap was identified: a modelling approach to study the issue of food and agricultural trade liberalisation that includes intermediaries with potential to exercise market power has not been developed so far. As explained in that chapter, an extension of the international trade network model developed by Goyal and Joshi (2006) is proposed to contribute to filling this gap.

It is argued in this dissertation that this extension offers results that cannot be identified from the contribution by Goyal and Joshi and that can explain some observed patterns in the real world. While these results are fully explored in the next chapters, it is important for illustrative purposes to highlight from the beginning the sources that explain these differences.

In Goyal and Joshi, there are two issues at play when networks evolve: what happens to consumer surplus and what happens to firms' profits. Consumer surplus increases when a country signs additional agreements because this increases the level of competition in the domestic market of this country. This, however, reduces profits from the domestic market for the domestic firm because they receive a lower price for the selling output. But the new agreement offers this firm the opportunity to make additional profits from exporting to other markets. Of course, the wider (more countries) in the network that can trade with each other, the export profit effect will diminish implying that the loss of profits in the domestic market may not be compensated by the export profits. So, depending on the current structure of the network, the gains from an international trade agreement (i.e. gains in consumer surplus and export profits) can either be larger or smaller than the loss of domestic profits and this has important implications in the stability of international trade networks.

On the other hand, when the farming sector is introduced into the model (i.e. the extended version of the model), a new mechanism is added, and this mechanism plays a key role in explaining deviations from the original work by Goyal and Joshis' model. That is, firms (i.e. intermediaries) face a supply function reflecting the monopsonistic power exercised by them. This monopsonistic power implies that the intermediaries face an increasing marginal cost when they increase the level of food processed goods that are traded domestically and in external markets. In other words, they have to pay higher agricultural prices to the farming sector in more integrated networks. Consequently, the export profit effect is weaker not just because of the effect of competing in other export markets, but also because the costs faced by intermediaries rise. This effect is not present in the model by Goyal and Joshi because they assume that firms face a fixed marginal cost that is not affected by the degree of international integration.

The existence of an increasing marginal cost brings new effects on the welfare function when an agreement is signed. Firstly, the balance between the loss of profits in the domestic market and the export profits after the agreement is affected by the highest price that the intermediaries have to pay to the farming sector. This, in turn, can affect the trade-off between the gain in consumer surplus and the loss of profits and, therefore, the network stability. Secondly, the existence of a farming sector adds a new component in the welfare function which is producer surplus. This new component also affects the trade-off between the gains and losses from a bilateral agreement, and this brings important implications to the network stability that are not present in the original model by Goyal and Joshi.

It should also be noted that the effect of the increasing marginal cost on global free trade is also important in this new paradigm: as one additional node is connected, export sales will drive up the costs to intermediaries as they serve all destinations. So, while consumers and farmers may prefer global free trade, to the extent that the government 'cares' about intermediaries, global free trade may not be desirable. This is in contrast to the results by Goyal and Joshi who found that global free trade is always stable independently of any political bias of the government. This example illustrates the relevance of the extended version of the model in explaining current patterns in the real world that cannot be elucidated from the original model by Goyal and Joshi.

As explained above, relaxing the assumption of fixed marginal cost in models of imperfect competition informs about important deviations in terms of decision making of key players in the economy. However, this is also true in wider modelling approaches. For example, it is likely that non-fixed costs exist in other sectors. This means that, in the context of general equilibrium models, an increase in the marginal cost in a sector other than agriculture as a consequence of trade can potentially affect both the outcome in agricultural markets and the outcome of trade negotiations, particularly in cases where governments have biases in favour of specific sectors. This possible interrelation across sectors due to increasing marginal cost is not explored in the international trade network approach considering in this thesis. The reason is because the aim of the current investigation is to extend current approaches to study agricultural trade that normally assume either perfect competition or where the food sector is acknowledged, it plays no formal role in determining the outcomes. For example, the food sector may be introduced where there is a fixed margin between the farm level price and the consumer price. In contrast, the model adopted in this study departs from the conventional literature of agricultural trade and expands beyond the political economy focus in agricultural trade models. Nonetheless, the relevance of the impact of introducing non-fixed marginal costs on interrelated sectors is recognised, and this certainly can be identified from general equilibrium and multi-sectors models. This potential extension is left for future research.

Having described the main key differences between both versions of the model, the objective of the current chapter is to formally introduce the model by Goyal and Joshi and to explain the main results obtained by these researchers. For this purpose, this chapter is organised as follows. Section 3.2, describes the network model developed by Goyal and Joshi (2006) in order to highlight the main characteristics of this framework. Section 3.3 describes the main results obtained by these researchers with the purpose of using them as a benchmark for the extended version of the model. It is also discussed in this section some potential deviations that may arise when the farming sector is introduced into the analysis. Finally, Section 3.4 summarises and concludes the chapter.

3.2 The International Trade Network Model by Goyal and Joshi (2006)

In order to facilitate the description of the original model by Goyal and Joshi, this section was subdivided into two parts. The first one provides an informal (i.e. graphical) description of the international trade network model by Goyal and Joshi (2006) with the purpose of showing the main ideas and principles behind this framework. The second provides a formal (i.e. mathematical) description of the model.

3.2.1 Informal Description of the International Trade Network Model

While the mathematical representation of the International Trade Network by Goyal and Joshi (2006) is complex, the idea behind it is very simple and can easily be understood by using a graphical representation. This is shown in Figure 3.1.

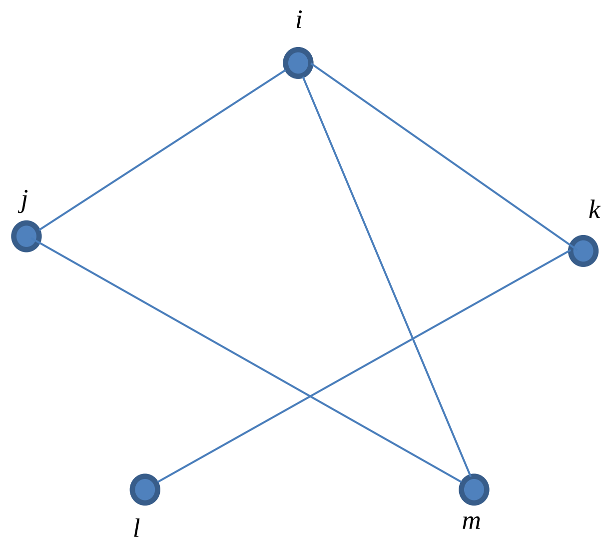


Figure 3.1. The International Trade Network Model

There are five countries in this figure represented as nodes *i*, *j*, *k*, *l* and *m*. Country *i*, in particular, is connected with countries *j*, *k* and *m* by means of links. These links represent bilateral agreements that country *i* has with countries *j*, *k* and *m*. In the figure, country *i* is not connected to country *l* implying that these two countries do not have a free trade agreement with each other. This is formally illustrated by a binary variable $g_{ij} \in \{0,1\}$. If $g_{ij} = 0$, then no agreement exists between countries *i* and *j*. Conversely, if $g_{ij} = 1$, then these countries have an international agreement. Because an agreement between countries *i* and *j* is equivalent to an agreement between countries *j* and *i*, it holds that $g_{ij} = g_{ji}$. Using this terminology, the countries in Figure 3.1 can be characterised in terms of their international agreements as follows: $g_{ij} = g_{ik} = g_{im} = g_{jm} = g_{kl} = 1$; and $g_{il} = g_{jk} = g_{jl} = g_{km} = g_{lm} = 0$. The set of these links is referred to as a network *g*. That is, *g* is a description of the international agreements between the countries in *N*, where *N* is the set of countries in the world. The network in Figure 3.1 is therefore described as $g = \{ g_{ij}$

$=1$; g_{ik} =1; g_{im} =1; g_{jm} =1; g_{kl} = 1; g_{il} = 0; g_{jk} = 0; g_{jl} = 0; g_{km} = 0; g_{lm} = 0}. Now, consider the networks presented in Figures 3.2(a) and 3.2(b):

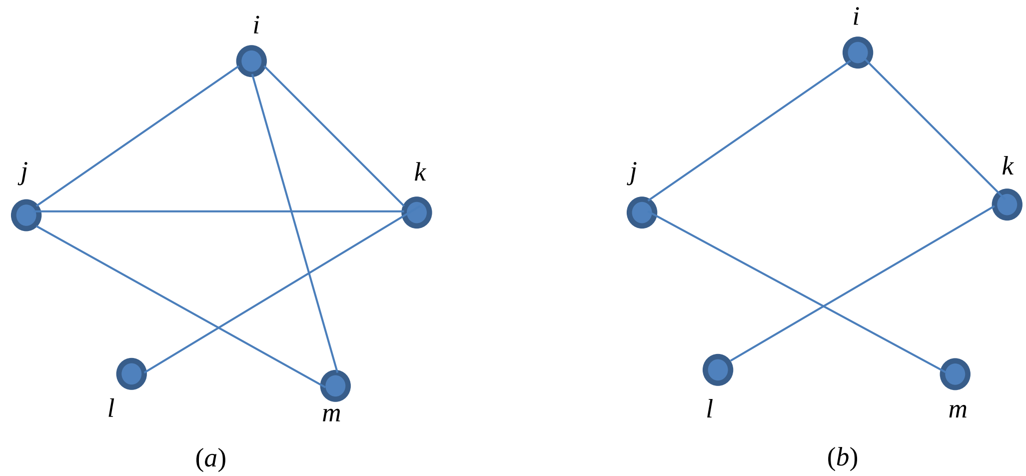


Figure 3.2. Alternative International Networks

The only difference between the network presented in Figure 3.1 (i.e. network *g*) and that presented in Figure 3.2(a) is that in the latter countries *j* and *k* have an international agreement. That is, in Figure 3.1 it holds that g_{jk} = 0 but in Figure 3.2(a) it holds that g_{jk} = 1. In the network terminology, if the network presented in Figure 3.1 is defined as *g*, then the network that results when linking countries *j* and *k* is given by $g + g_{jk}$. Likewise, the only difference between the network presented in Figure 3.1 and that presented in Figure 3.2(b) is that in the latter, countries *i* and *m* have broken their international agreement. That is, in Figure 3.1 it holds that g_{im} = 1 but in Figure 3.2(b) it holds that g_{im} = 0. In the network terminology, if the network presented in Figure 3.1 is defined as *g*, then the network that results when countries *i* and *m* break their agreement is given by *g* –

g_{im}. On the other hand, Figure 3.3 shows two important networks for the current investigation: the complete network and the empty network.

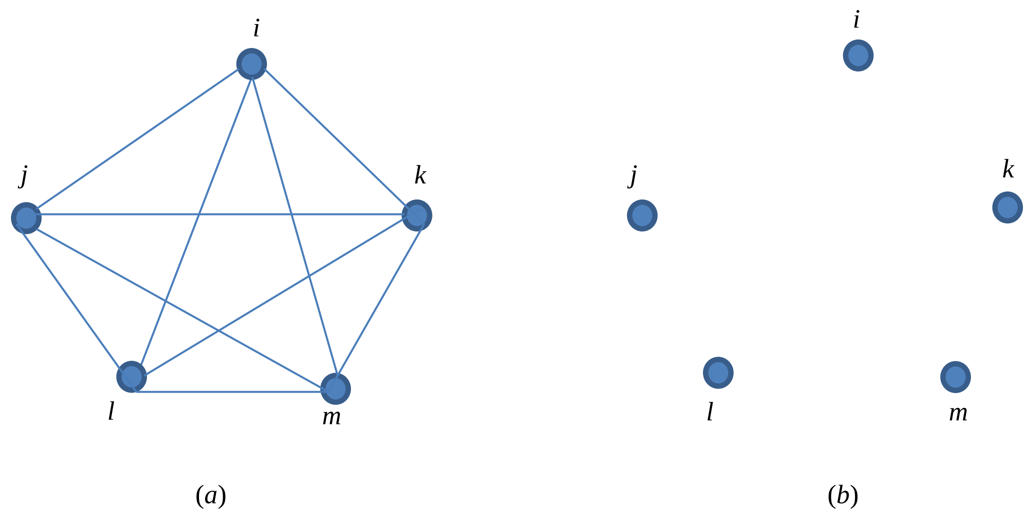


Figure 3.3. The complete and the empty networks

The complete network, denoted by g^c, is presented in Figure 3.3(a). The main characteristic of this network is that all countries have an international agreement with each other. In other words, the complete network corresponds to global free trade. Figure 3.3(b), on the other hand, corresponds to the empty network which is denoted by g^e. The main characteristic of this network is that no country in the world has an international agreement.

In terms of market structure, on the other hand, the original model by Goyal and Joshi (2006) assumes that countries compete in Cournot oligopolistic markets. For example, in Figure 3.1 countries *i*, *j*, *k* and *m* play Cournot in the domestic market of country *i*. Countries *i*, *j*, and *m* play Cournot in the domestic market of country *m*.

Countries j and k play Cournot in the domestic market of country j, and so on. Cournot is relevant in this setting due to the fact that though we start with monopoly in the domestic context, the interaction between firms in different countries will bring about the pro-competitive effects.

3.2.2 Formal Description of the International Trade Network Model

Having informally described the main features of the international trade network model, let us now describe the model using formal mathematical notation. An international agreement between countries i and j is described by a link, given by a binary variable $g_{ij} \in \{0,1\}$ with $g_{ij} = 1$ if an agreement exists between countries i and j and $g_{ij} = 0$ otherwise. A network $g = \{(g_{ij})_{ij \in N}\}$ is a description of the international agreements that exist among a set $N = \{1,\ldots,N\}$ of identical countries, where N is the total number of countries. Networks g^c and g^e are the complete network (*i.e*. $g_{ij} = 1$ for all $i, j \in N$) and the empty network (*i.e*. $g_{ij} = 0$ for all $i, j \in N$) respectively. Let G denote the set of all possible networks, $g + g_{ij}$ denotes the network obtained by replacing $g_{ij} = 0$ in network g by $g_{ij} = 1$, and $g - g_{ij}$ denotes the network obtained by replacing $g_{ij} = 1$ in network g by $g_{ij} = 0$. Let $N_i(g) = \{j \in N: g_{ij} = 1\}$ be the set of countries with whom country i has an international trade agreement in network g. Assume that $i \in N_i(g)$ so that $g_{ii} = 1$. The cardinality of $N_i(g)$ is denoted $\eta_i(g)$. In this model $\eta_i(g)$ is also the number of active firms in country i because of the assumption that each country has only one firm. Note that the domestic firm in country i is included in $\eta_i(g)$ meaning that when a country does not have any agreement, $\eta_i(g) = 1$. Let $L_i(g) = \{(g_{ij})_{ij \in \mathrm{N}} : j \in N_i(g)\}$ be the set of links existing in country i in network g. Note that $g_{ii} \in L_i(g)$. Let $h_i \subset L_i(g) - \{g_{ii}\}$ be a link subset, and let μ_i be the cardinality of h_i. This latter notation is used in the definition of the alternative stability concept adopted in this research. Let $\Gamma(g)$ be a subset of countries in network g. $\Gamma(g)$ is said to be a complete component if: (*i*) $g_{ij} = 1$ for all $i,j \in \Gamma(g)$; and (*ii*) $g_{ik} = 0$ for all $i \in \Gamma(g)$ and all $k \notin \Gamma(g)$. However, $\Gamma(g)$ is said to be an incomplete component if there exists at least two countries $i,j \in \Gamma(g)$ such that $g_{ij} = 0$.

3.2.3 Market Structure adopted by Goyal and Joshi (2006)

In Goyal and Joshi's model, each country in the world has only one firm which produces a homogeneous good that can be traded internationally. When two countries form an agreement, their domestic firms play Cournot in the domestic market of these countries. The selection of Cournot game is appropriate for three reasons. Firstly, the alternative Bertrand oligopoly model leads to the competitive equilibrium under standard assumptions of homogeneous good at constant and identical marginal cost (for a discussion, see Burguet and Sákovics, 2017). The adoption of this model would, therefore, prevent researchers in the area of international trade networks from gaining an understanding of the factors that explain deviations from perfect competition that are observed in the real world. Secondly, intra-industry trade is more likely in Cournot competition. This was formally studied by Brander and Spencer (2015) who found that trade in homogeneous products never takes place under Bertrand competition because Bertrand firms have an incentive to differentiate their products when being exposed to international trade. Since intra-industry trade is observed in the real world, the use of Cournot is more suitable to explain the existence of this type of trade. Finally, given the simplicity of Cournot competition, the use of this approach contributes in decreasing the degree of complexity of the international network model. However, the adoption of the Stackelberg model would be an interesting alternative to study the effects of leader intermediaries in the network when the game is played sequentially (for a recent application of the Stackelberg competition model to study issues related to international trade, see Ferreira and Ferreira, 2018). Given the potential of this oligopoly game to study international trade networks, this extension is left for future research.

In this framework, Goyal and Joshi consider two alternative solutions: (i) the solution under exogenous tariffs (i.e. each country establishes a prohibitive tariff avoiding any trade between them. If two countries decide to sign an agreement, then each one offers the other a free market access); and (ii) the solution under endogenous tariffs (i.e. the tariff that a country applies to non-partner countries is the one that maximises the social welfare function. If two countries decide to sign an agreement, then each one offers the other free market access).

The reason of why Goyal and Joshi adopted two alternative solutions is because the model becomes untractable in mathematical terms when considering endogenous tariffs. This complexity made it impossible to identify the stable networks from the generic equations that resulted when solving the model under endogenous tariffs. In relation to this technical problem, the authors formally state that “Given the complexity of the computations involved, we have been unable to completely characterize the nature of stable networks in this setting” (p. 768). In considering this limitation, the researchers use the endogenous tariffs solution only to explore the stability of global free trade. But in order to identify all potential stable networks, they adopted the exogenous tariffs solution as the analysis becomes much less complex. Thus, conducting the analysis under exogenous tariffs as a first approximation offered by these researchers an easier way to identify relevant results without complicating the analysis in excess. These two solutions considered by these researchers are described as follows.

3.2.3.1 Solution under Exogenous Tariffs

Let $P_i = \alpha_i - Q_i$ be the inverse demand of the unique good in country $i \in N$, where P_i is the price of this good in the domestic market of country i, α_i represents the size of this market, and Q_i is the total quantity of the good demanded in this country. Let $\gamma_i < \alpha_i$ be the marginal cost faced by the domestic firm of country i. It is assumed that all countries are symmetrical (i.e. $\alpha_i = \alpha$ and $\gamma_i = \gamma$ for all $i \in N$). It is also assumed that firms play Cournot in each market where they compete. The equilibrium output of the firm in country i in the domestic market is given by $Q_i^i(g) = (\alpha - \gamma)/(\eta_i(g) + 1)$, and the total output of equilibrium in this market is given by $Q_i(g) = (\alpha - \gamma)\eta_i(g)/(\eta_i(g) + 1)$. Likewise, the equilibrium output of the domestic firm of country i that is sold in country k is given by $Q_k^i(g) = (\alpha - \gamma)/(\eta_k(g) + 1)$.

Consumer surplus in country i (i.e. $CS_i(g)$), the profit that the firm in country i makes in the domestic market (i.e. π_i^i(g)), and the profit that the same firm makes in country k (i.e. π_k^i(g)) are given by $Q_i(g)^2/2$, $(P_i - \gamma)Q_i^i(g)$, and $(P_k - \gamma)Q_k^i(g)$, respectively. By replacing the equilibrium quantities and the inverse demand into these definitions, the following expressions are obtained: (i) $CS_i(g) = (\alpha - \gamma)^2\eta_i(g)^2/2(\eta_i(g) + 1)^2$; (ii) $\pi_i^i(g) = (\alpha - \gamma)^2/(\eta_i(g) + 1)^2$; and (iii) $\pi_k^i(g) = (\alpha - \gamma)^2/(\eta_k(g) + 1)^2$.

Finally, total profit made by the domestic firm of country i in network g is given by $\pi_i(g) = \sum_{k \in N_i(g)} \pi_k^i(g)$. From these expressions and by assuming that $\alpha - \gamma = 1$ without loss of generality, the welfare function becomes:

$$W_i(g) = a_i CS_i(g) + b_i \pi_i(g) = a_i \frac{1}{2} \frac{\eta_i^2(g)}{(\eta_i(g)+1)^2} + b_i \sum_{k \in N_i(g)} \frac{1}{(\eta_k(g)+1)^2} \tag{3.1}$$

Where a_i and b_i represent exogenous weights that the government in country i puts on consumer surplus and total profits, respectively. In this representation if $a_i > b_i$, then the government is biased in favour of consumers. In contrast, if $a_i < b_i$, then the government is biased in favour of the domestic firm. Finally, if $a_i = b_i$, then the government is politically unbiased.

Note that the exogenous tariff creates the benchmark as no trade between nodes in the network as the tariff is prohibitive. There is obviously no need for a formal expression for this tariff as, by definition, between potential partners is zero.

3.2.3.2 Solution under Endogenous Tariffs

Let us now assume endogenous tariffs. Let $T_{ij}(g)$ be the tariff faced by country i in country j and in network g, and let $TR_i(g)$ denotes tariff revenue in country i and in network g. Because both $T_{ij}(g) = T_{ji}(g) = 0$ for all $j \in N_i(g)$ and $Q_i^k(g) = Q_i^l(g)$ for all $k,l \notin N_i(g)$, it holds that $T_{ki}(g) = T_i(g)$ for all $k \notin N_i(g)$. The Cournot equilibrium outputs in the domestic market of country i are: (*i*) $Q_i^j(g) = [1 + (N - \eta_i(g))T_i(g)]/(N + 1)$ for all $j \in N_i(g)$; and (*ii*) $Q_i^k(g) = [1 - (\eta_i(g) + 1)T_i(g)]/(N + 1)$ for all $k \notin N_i(g)$. From these expressions: (i) $CS_i(g) = [N - (N - (\eta_i(g))T_i(g)]^2/2(N + 1)^2$; (ii) $\pi_j^i(g) = [1 + (N - \eta_j(g))T_j(g)]^2/(N + 1)^2$ for all $j \in N_i(g)$; (iii) $\pi_k^i(g) = [1 - (\eta_k(g) + 1)T_k(g)]^2/(N + 1)^2$ for all $k \notin N_i(g)$; (iv) $\pi_i(g) = \sum_{j \in Z_i(g)} \pi_j^i + \sum_{k \notin Z_i(g)} \pi_k^i$; and (v) $TR_i(g) = \{(N - \eta_i(g))T_i(g)[1 - (\eta_i(g) + 1)T_i(g)]\}/(N + 1)$. Using these expressions and assuming that $\alpha - \gamma = 1$ without losing generality, the welfare function becomes:

$$
\begin{aligned}
&W_i(g) = a_i CS_i(g) + b_i \pi_i(g) + c_i TR_i(g) = \\
&a_i \frac{1}{2}\left(\frac{N-(N-\eta_i(g))T_i(g)}{N+1}\right)^2 + c_i (N-\eta_i(g))T_i(g)\left(\frac{1-(\eta_i(g)+1)T_i(g)}{N+1}\right) \quad (3.2) \\
&+ b_i\left[\sum_{j \in N_i(g)}\left(\frac{1+(N-\eta_j(g))T_j(g)}{N+1}\right)^2 + \sum_{k \notin N_i(g)}\left(\frac{1-(\eta_k(g)+1)T_k(g)}{N+1}\right)^2\right]
\end{aligned}
$$

Where a_i, b_i and c_i represent exogenous weights that the government in country i puts on consumer surplus, total profits and tariff revenue, respectively. The optimal tariff that maximises this function corresponds to:

$$T_i^*(g) = \frac{N(c_i - a_i) + c_i + 2b_i}{2c_i(\eta_i(g)+1)(N+1) - (N - \eta_i(g))(a_i + 2b_i)} \tag{3.3}$$

This tariff depends on who the other partners in the network are. They probably vary when asymmetry applies. However, this was not explored by the authors. Nonetheless, interesting insights were obtained by these Goyal and Joshi for the case of symmetric and unbiased governments (i.e. $a_i = b_i = c_i$).

Firstly, in considering this tariff in the welfare function, Goyal and Joshi identified three effects of increasing this tariff on welfare. The first one is that it lowers competition in the domestic market positively affecting the profits made by domestic firm. The second effect is that the lower competition negatively affects consumer surplus. And finally, the third effect is on the aggregate level of tariff revenue. This can be either positive or negative depending on the size of the tariff as can be inferred from the expression (v) above. The first derivative of this expression with respect to the tariff reveals that an increase in tariff will increase tariff revenue only when $T_i(g) < 1/(\eta_i(g) + 1)$. If a tariff is larger, then it will have a large impact on imports negatively affecting tariff revenue.

Secondly, by taking the first derivative of expression 3.3 with respect to the number of agreement in country *i*, the following expression is obtained:

$$\frac{\partial T_i^*(g)}{\partial \eta_i(g)} = -\frac{\left[N(c_i - a_i) + c_i + 2b_i\right]\left[2c_i(N+1) + a_i + 2b_i\right]}{\left[2c_i(\eta_i(g)+1)(N+1) - (N-\eta_i(g))(a_i + 2b_i)\right]^2} \tag{3.4}$$

According to this result, additional agreements signed by country *i* lowers the optimal tariff when $N(c_i - a_i) + c_i + 2b_i$ ≥ 0. Goyal and Joshi concluded this result under the assumption of unbiased governments which satisfies this condition. Note that this condition has to be satisfied in order to be consistent with the tariffs observed in the real world. Otherwise, countries would apply negative tariffs as inferred from Expression 3.3.

Thirdly, it can infer from expression 3.3 that governments biased in favour of consumers have a tendency to place lower optimal tariffs. While this result was not studied by Goyal and Joshi, this can be seen by taking the first derivative of this expression with respect to the weight a_i:

$$\frac{\partial T_i^*(g)}{\partial a_i} = \frac{-N\left[c_i(2\eta_i(g)+1) - 2b_i\right] - N\left[c_i(\eta_i(g)+1) + 2b_i(\eta_i(g)-1)\right] - \eta_i(g)(c_i + 2b_i)}{\left[2c_i(\eta_i(g)+1)(N+1) - (N-\eta_i(g))(a_i + 2b_i)\right]^2} \tag{3.5}$$

The reason is because lowering tariffs increases the level of competition in domestic markets positively affecting consumer surplus which is what it is expected from governments biased in favour of consumers.

Finally, it can be inferred from the first derivative of Expression 3.3 with respect to the weight b_i that governments biased in favour of the domestic firm have a tendency to place higher tariffs. This is shown in the following expression.

$$\frac{\partial T_i^*(g)}{\partial b_i} = \frac{2\left[2c_i(\eta_i(g)+1)(N+1)-(N-\eta_i(g))(a_i+2b_i\right]+2(N-\eta_i(g))}{\left[2c_i(\eta_i(g)+1)(N+1)-(N-\eta_i(g))(a_i+2b_i)\right]^2} \quad (3.6)$$

This result neither was studied by Goyal and Joshi, but the intuition is straightforward. Raising the optimal tariffs reduces the level of competition in the domestic market and this increases the profit made by the domestic firm in this market.

3.2.4 Stability Concept Adopted by Goyal and Joshi (2006)

In order to determine the stability of international trade networks, Goyal and Joshi (2006) adopted a stability concept referred to as pairwise stability. This concept was introduced by Jackson and Wolinsky (1996) and assumes that countries can only break one international trade agreement at a time, and that countries can only form one agreement at time. Under this assumption, a country will break or sign additional international agreements only when the effect of this action on welfare (weighted welfare) is positive. Consequently, a network g is pairwise stable if and only if: (i) $W_i(g) > W_i(g - g_{ik})$ for all $i \in N$; and (ii) if $W_i(g) > W_i(g + g_{ij})$, then $W_j(g) < W_j(g + g_{ij})$. In words, pairwise stability establishes that a network g is stable when no country has an incentive to break an existing agreement (i.e. condition (i)); and if a determined country i has an incentive to sign an agreement with country j, but the latter does not have an incentive to form one with the former.

The main implication of this stability concept is that what determine the existence of bilateral agreements are the gains or losses in social welfare (or weighted welfare function if governments are politically biased). That is, countries in a pairwise stable network would prefer to stay in their current position in the trade network because any change would cause a loss in social welfare. Actually, there could be networks having a country willing to sign an agreement. But this agreement would not be signed because there are not potential partners that would be interested in the agreement because this would mean for them a loss in social welfare.

The current level of welfare in a stable network, however, does not mean that everyone supports this network. For example, consumers would prefer more integrated networks because they offer higher levels of consumer surplus. In contrast, domestic firms would prefer less integrated networks in order to obtain higher profits in less competitive markets. Thus, being in a determined network implies a situation where there exists tension faced by the government that arises from the trade-off between the interests of consumers and domestic firms. However, this trade-off in a pairwise stable network is in balance from the point of view of the government. Of course, this balance can be broken when governments become politically biased in favour of one of these groups of individuals leading to other stable networks. This is the key aspect that the pairwise stability can capture. In considering this property of the pairwise stability, the following section describes the results identified by Goyal and Joshi under different policy biases.

3.3 The results by Goyal and Joshi

The objective of this section is to describe the main results obtained by Goyal and Joshi. These results will be used as a benchmark for the extended version of the model. In order to illustrate the advantage of the extended model to study the issue of agricultural trade liberalisation, it is also discussed possible deviations from the original model when the farming sector is introduced into the analysis.

In relation to the results obtained by Goyal and Joshi (2006) under the assumption of exogenous tariffs, these researchers found that the pairwise stability of networks depends on the weights that policymakers put on the components on the welfare function. In particular, when governments are politically biased in favour of consumers, the pairwise stable network is the complete network and is unique. The reason is because more trade increases competition in the domestic market of the countries in the world. As a consequence, consumers obtain higher levels of consumer surplus in more integrated network. In relation to the extended version of the model, the same result is expected to be found. To understand this prediction, note that free trade increases the quantity of processed food goods that is traded by the intermediaries. This higher quantity has two effects on the supply chain. Firstly, it increases the price paid to producers implying that free trade makes agricultural goods more expensive. Secondly, the higher quantity of processed goods increases the level of competition in the competitor countries implying that intermediaries receive a lower price for these goods in more integrated networks. In considering these effects, it is concluded that having a farming sector into the analysis negatively affects the level of gross margin obtained by intermediaries, but not the gains on consumer surplus as a result of higher competition.

On the other hand, Goyal and Joshi found that when governments are biased in favour of domestic firms, the empty network, the complete network and networks formed of complete components of different size with or without singletons are all pairwise stable. This result is explained by the trade-off faced by domestic firms when an agreement is signed. That is, if a country signs an agreement, the level of competition in the domestic market increases negatively affecting the profit made by the domestic firm in this market. However, this firm makes additional export profits in the new partner country. If the loss in profits in the domestic market is larger than the gain in export profits, then the agreement will not be signed by the country. The same analysis applies when a country is evaluating the possibility of deleting an agreement. If the gain in domestic profit is lower than the loss of export profits when breaking the agreement, then the agreement will not be broken.

In the case of the empty network, the pairwise stability is explained by the fact singletons are unwilling to sign an agreement with each other because it would cause a net loss in profits: the loss of profit in the domestic market offsets the export profits. In the extended version of the model, this net loss in profit would be reinforced by the higher price that intermediaries have to pay to the farming sector after the agreement. It is expected to be found, therefore, that the empty network is also pairwise stable network when there exists a farming sector.

On the other hand, the complete network (i.e. global free trade) is pairwise stable in Goyal and Joshi because breaking an agreement causes a net loss in profits. This is because countries in this network are highly integrated and the gain in profit in the domestic market after breaking an agreement is not significantly large to offsets the loss in the export profits. In relation to the extended version of the model, global free trade might not be stable because of the higher cost faced by the intermediaries in this network. It may be possible that deviating from global free trade would allow the intermediaries to reduce the price paid to the farming sector causing a net gain in profits.

Finally, in relation to the networks composed of complete components of different size with or without singletons, a country in a large complete component has an incentive to sign an agreement with a country that belongs to a small complete component. This is because in this case the loss in domestic profit is lower than the gain in export profit. This is due to the fact that the country in the large component is more integrated implying that the impact of the agreement in increasing competition is not significantly large. In contrast, the country in the small component is less integrated implying that the impact of the agreement on increasing competition would be more severe. As a consequence, this country is unwilling to sign an agreement with a country of the large component. In the extended version of the model, it is expected a similar result for the case of countries in the small component because the intermediaries in these countries face higher costs when signing an agreement with a country of the large component. However, the presence of a farming sector can potentially affect the stability of the complete components themselves. In Goyal and Joshi these components are stable because no country has an incentive to deviate by breaking an existing agreement with another country that belongs to the same component. However, as discussed in the case of global free trade, having an agreement with all countries in the component can be expensive from the point of view of the intermediaries as they have to pay higher prices to the farming sector. This suggests that the pairwise stability of a complete component can be compromised when the farming sector is introduced into the analysis.

A final result identified by Goyal and Joshi is in the case of politically unbiased governments. In this case, global free trade and a network composed of a complete component and a singleton are both pairwise stable. The complete network is stable because no country has an incentive to break an existing agreement. If they did, then the losses in consumer surplus and export profits would not be compensated by the gain in profit in the domestic market as a consequence of the resulting lower competition. In the extended version of the model, it is difficult to predict the stability of this network because there are other effects that are in place. Firstly, the gain in the domestic profit after an agreement is broken is larger given by the lower price that the intermediary has to pay to the farming sector. But there is also a loss in producer surplus for the same reason: farmers get paid a lower price. Consequently, a deviation from global free trade would cause a loss in consumer surplus, producer surplus and export profit that may or may not be compensated by the gain in the domestic profit.

In relation to the other stable network when governments are unbiased, the same analysis applies to the complete components. That is, in Goyal and Joshis' world no country in the complete component is willing to break an existing agreement because this would cause a net loss in welfare. Likewise, in the extended version of the model, it is not clear whether a deviation would cause either a gain or loss in welfare because the additional domestic profit made by the intermediary when paying a lower price to the farming sector may not be enough to compensate the losses in consumer surplus, producer surplus and export profits. In relation to the singleton, on the other hand, this country is unwilling to sign an agreement with any country of the complete component. The reason is because the latter are highly integrated implying that their domestic markets have a high degree of competition. This means that the export profit that the intermediary of the singleton can make after an agreement is signed is not large enough to compensate the loss of domestic profit. This net loss in profits offsets the gain in consumer surplus which is what explains why this country is unwilling to sign an agreement. In the extended version of the model, this incentive may be reversed because the agreement also increases producer surplus and this can change the trade-off balance faced by the government.

On the other hand, the analysis developed by Goyal and Joshi under endogenous tariffs was based on the case of unbiased governments. They found that in this case global free trade is pairwise stable. This stability is explained by the fact that a deviation from global free trade causes a loss in consumer surplus and export profits that are not compensated by the gain in domestic profits and tariff revenue.

As in the case of exogenous tariffs, it is not clear whether the same result holds in the extended version of the model because it is not known the impact of the additional gain in domestic profits as a result of the lower price paid to the farming sector and the loss in producer surplus in the welfare function.

As illustrated in this section, there are possible deviations from the original work by Goyal and Joshi that are attributed to the existence of a farming sector. Potential deviations can also be predicted when countries are asymmetric in terms of market size and farmers productivity. A more detailed explanation of these possible deviations and the rationale behind them is explained in more detail in the next chapter.

3.4 Chapter Summary

The previous chapter identified a research gap which corresponds to the fact that a modelling approach to study the issue of agricultural trade liberalisation that includes intermediaries with potential to exercise market power has not been developed so far. It is argued in this dissertation that key extensions of the international trade network model developed by Goyal and Joshi (2006) can be employed to contribute to filling this gap.

The objectives of this chapter is to introduce the international trade network model by Goyal and Joshi (2006), describe the main results obtained by these researchers, and illustrate potential deviations that are expected to be found when the farming sector is introduced into the analysis. The focus is placed on the alternative mechanisms that explain the differences between both versions of the model.

In the original model by Goyal and Joshi, the mechanism that explains the incentives of a country to sign or break an existing international trade agreement is related to the fact that free trade affects the level of competition in the domestic market of the player countries. An additional agreement increases the level of competition and this causes both a gain in consumer surplus and a gain in export profits. However, this also causes a negative effect on the domestic profits made by the intermediaries. Thus, depending on the current structure of the network, there could be a net loss of profits (i.e. the loss in domestic profit is larger than the gain in export profits) that originates a trade-off between the gain in consumer surplus and the loss of net profits when a trade agreement is signed (and the reverse holds when an existing agreement is broken). In order to deal with this trade-off, governments decide whether to sign or break bilateral agreements in order to favour a net gain in welfare. Nonetheless, this decision is affected when governments are politically biases.

When the farming sector is introduced into the model, an additional mechanism arises. That is, more free trade implies a larger quantity of the processed food good that is traded by the intermediaries. This pushes the price paid to the farming sector up implying that these individuals face higher costs in more integrated networks. Thus, the lower price obtained for the processed food good and the higher cost faced by the intermediaries implies that these individuals suffer a more severe loss in profits in more integrated networks than in Goyal and Joshis' world. This effect of the farming sector on profits can cause important deviations from the results obtained by these researchers. In addition, the farming sector adds a new component in the welfare function with respect to the model by Goyal and Joshi which is producer surplus. This new element can also cause important deviations from the original model because it affects the trade-off faced by governments in the network.

The deviations on the international trade system that arise when introducing the farming sector into the analysis are formally studied in the next section. The analysis considers deviations with respect to the results obtained by Goyal and Joshi under pairwise stability.

CHAPTER FOUR: Stable Trade Networks under Pairwise Stability

4.1 Introduction

One of the key aspects discussed in the literature review is that current empirical evidence has revealed the existence of imperfect competition in domestic and international markets of agricultural and food processed goods. According to this evidence, this imperfection is characterised by powerful intermediaries in the supply chain of food industry who buy agricultural goods to the farming sector and sell proceeded foods in domestic as well as international markets for these goods. These firms exercise market power in two ways, namely: oligopolistic competition in international markets when they compete with other foreign intermediaries; and monopsonistic power when they buy agricultural output from the domestic farming sector.

It was found in the literature review that a framework that includes these intermediaries from a global perspective to analyse agricultural trade policies have not been developed so far. On the contrary, the influence of these firms in the trade system has largely been ignored and most of the analysis in this area still uses theoretical and empirical approaches based on either the assumption of perfect competition or imperfect competition between two single countries.

It is argued in this dissertation that an extended version of the international trade network model developed by Goyal and Joshi (2006) can be adopted to explore trade policies and the potential for trade agreements (including global free trade) when there are intermediaries exercising market power in the food industry from a wider perspective than the current modelling approaches. This is because the original model includes domestic firms that can act as intermediaries in the extended version of the model that is proposed in this dissertation. However, the extended version also includes the farming sector in order to capture the monopsonistic power effect caused by intermediaries on this sector.

The aim of this chapter is to present the extended version of the model by Goyal and Joshi and to use this framework to investigate how the international trade system in the food industry is affected when there are intermediaries with market power and when there is a farming sector. For this purpose, the original results obtained by Goyal and Joshi are used in this context as a benchmark. That is, because the analysis developed by these researchers was carried out using the pairwise stability concept, this chapter uses the same concept to explore deviations from the original model that are attributed to the monopsonistic power exercised by intermediaries on the farming sector. To recall, the pairwise stability concept is defined in Section 3.2.4 as follows. Let D be the set of *link deletion proof networks* (i.e. the set of networks in which no country has an incentive to break an existing agreement); and let A be the set of *link addition proof networks* (i.e. the sets of networks in which signing additional agreements is not feasible). Using these notations, the set of *pairwise stable network*, P, is defined as the intersection between the set of link deletion proof networks and the set of link addition proof networks (i.e. $P = D \cap A$).

As explained in Chapter Three, the extended version of the international trade model is extremely complex in mathematical terms, a problem that has been named the *tractability problem* in other network applications (Goyal, 2015). In order to deal with this problem, simulations based on the assumption of a world composed of four countries were adopted. In spite of this simplification, it was possible to identify a number of deviations from the original work by Goyal and Joshi (2006) that are attributed to the existence of a farming sector in the countries of the world. In these simulations it was found that the existence of a farming sector can either positively or negatively affect free trade depending on the political biases of the governments as well as the existence of asymmetries across countries.

In particular this chapter shows that global free trade is not always stable. This happens when governments are biased in favour of intermediaries. This is explained by the fact that more free trade pushes the agricultural prices up as a consequence of monopsonistic power, and the price of food processed goods down as a consequence of higher competition. This in turn negatively affects the profits made by the intermediaries in more integrated networks.

Another key result is that the farming sector can positively affect free trade when governments are politically unbiased. This is because more trade implies higher agricultural prices and, therefore, higher levels of producer surplus. This gain in producer surplus plus the gain in consumer surplus are both large enough to offsets the associated loss of profits faced by intermediaries. However, when countries are asymmetric in terms of market size, this positive effect vanishes in some networks when a large country is dealing with a small country. This is because the export profit made in the small country in these networks is not large enough to compensate for the loss in both domestic profits and producer surplus. A similar result was found in the case of asymmetric countries in terms of farmers' productivity. In this case a farming sector positively affects the incentives of efficient countries to sign additional bilateral agreements. However, in less efficient ones this is not always the case because an agreement has a significant effect on agricultural prices that causes a more severe impact on the profits made by the intermediaries of these countries.

Finally, other key results found in this chapter are related to the case of governments biased in favour of the farming sector. That is, it was found that when countries are symmetric, this sector contributes to free trade. This is explained by the fact that more trade pushes the price paid to the farming sector up positively affecting producer surplus. However, when countries are asymmetric in market size, this positive effect only holds in countries with similar sizes. However, a large country is unwilling to sign an agreement with a smaller country because the additional export output in the latter is not large enough to compensate for decrease in the output that is sold in the domestic market of the former. This net decrease in total output sold by the intermediary of the large country depresses the agricultural price negatively affecting producer surplus in this country.

The chapter is organised in three main parts. Part 1 deals with theoretical considerations of the work developed in the chapter. It contains Section 4.2 which formally presents the extended version of the model. As in Goyal and Joshi's work, different solutions of the model are considered with the purpose of introducing some relevant considerations such as asymmetries across countries (i.e. differences in market size and farmers' productivity) and different ways by which tariffs are placed (i.e. exogenous vs. endogenous tariffs). Expected possible patterns from the generic equations obtained from these solutions and the intuition behind them are also discussed. Part 2 studies the simulations carried out in this research. Section 4.3 in particular studies the issue of bilateral agreements in agriculture and the pairwise network stability under the assumption of symmetric countries in terms of market size, farmers' productivity and governments' policy biases. This case is used as a benchmark for the analysis of international network stability under different types of asymmetry. Section 4.4 extends the analysis to explore the issue of bilateral agreements under the assumption of asymmetry in market size and farmers' productivity, respectively. Finally, Part 3 contains Section 4.5 which summarises and concludes the chapter.

PART I: Theoretical Considerations of the Proposed Model

In this first part of the chapter, a formal presentation of the extended version of the international model is provided. Key aspects of this model and general patterns that can be identified from this framework are discussed.

4.2. The Extended International Trade Network Model

As explained in Chapter Two, one of the advantages of Goyal and Joshi's model is that it assumes that a country is composed of two economic groups: consumers; and domestic firms that exercise market power. The last group can be considered as intermediaries in the supply chain of agri-food goods which is what is missing in current modelling approaches used to study agricultural trade liberalisation from a global perspective. However, the original version Goyal and Joshi's model is not suitable to study this liberalisation because the farming sector is not included in it.

The first extension of the original model by Goyal and Joshi (2006) that is introduced in this chapter is precisely the introduction of the farming sector. As explained in the previous chapter, this extension adds an important mechanism that contributes in explaining the motivations of private firms and policymakers in relation to the issue of agricultural trade liberalisation. This mechanism contains two main elements. Firstly, the intermediaries in the extended model face a marginal cost that increases in more integrated networks. That is, the additional processed food that is traded in more integrated networks by the intermediaries implies that these firms have to increase the demand for agricultural goods to the farming sector pushing the price paid to farmers up. This means that in the extended version of the model the losses in profits faced by the intermediaries that are attributed to free trade are more severe than in Goyal and Joshi's world. Linking this effect on profits to the motivations of policymakers, there are some networks where policymakers face a trade-off between the increase in consumer surplus and the decrease in net profits (i.e. when the loss of profit in the domestic market is larger than the gain in export profits) when deciding whether to sign a new agreement. This trade-off is affected by the increasing marginal cost faced by the intermediaries and this can potentially affect the decision made by policymakers in relation to signing new agreements or maintaining existing ones. Secondly, the existence of a farming sector adds an additional component to the welfare function which corresponds to producer surplus. This new component with respect to the original model also affects the trade-off faced by governments and, therefore, the pairwise stability of international networks.

In order to introduce the farming sector into the original framework by Goyal and Joshi, a parsimonious version of the food supply chain is adopted where agriculture and food can be seen a chain of vertically linked markets. In this chain, farmers purchase inputs from an upstream input sector. The output produced by farmers is sold to first stage food processors who, in turn, sell processed product to food retailers. Finally, food retailers sell the final product to consumers.

In order to simplify the analysis concerning the identification of international trade networks in agriculture, the food chain considered in this chapter is characterised by two main actors: farmers and intermediaries. Farmers in this case are assumed to be input and output price takers. This assumption was introduced to reflect the fact that farmers are in general highly atomised (Evidence supporting the argument that farmers are price takers can be found, for example, in Brown and Miller (2008)). It is also assumed the existence of one intermediary in each country of the world who purchases the agricultural output produced by domestic farmers and sells a processed food product. In order to keep the model as simple as possible, the transformation process of the agricultural output into a processed food good is assumed to be a fixed proportion (Leontief) technology of agricultural goods. For example, milk supplied by farmers is closely related to the milk that is sold at the retail level. Likewise, cheese is normally produced using a fixed proportion of milk. Specifically, the Leontief relationship between the output of a processed food good (q_i) in an arbitrary country *i* and the output of an agricultural good produced by the farming sector (q_i^f) that is commonly applied in vertical models is given by $q_i(g) = \varphi q_i^f(g)$ and, with no loss in generality, φ = 1(see for example Schmit and Kaiser, 2006).

The advantage of considering a supply chain based on a single homogenous good is that the complexity of the extended version of the international trade network model is significantly reduced. In this model, each intermediary is assumed to have monopsonistic power in the inter-play between farmers and the intermediary and with oligopolistic power at the consumer end through trade, but with monopoly with no trade. A scheme of this food chain description is presented in Figure 4.1.

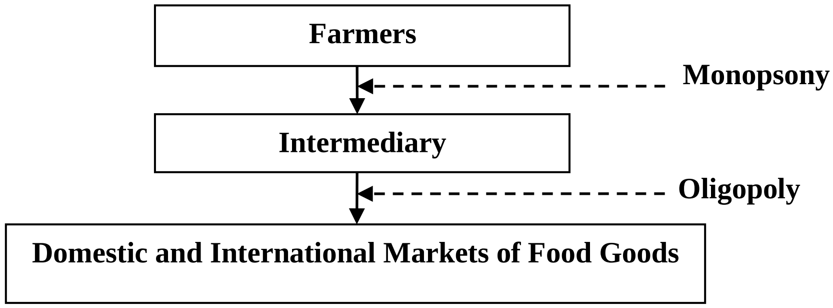


Figure 4.1. Vertically related food chain.

In order to facilitate the explanation of the extended model and its main implications, this section is organised in two sub-sections. Sub-section 4.2.1 describes the solution of the Cournot game under different assumptions: symmetric countries; asymmetry in market size, asymmetry in farmers' productivity; and endogenous tariffs. Sub-section 4.2.2 describes some possible patterns that can be identified from the generic equations obtained in sub-section 4.2.1. The emphasis is placed on the endogeneity that arises when there is a farming sector and how this endogeneity creates a number of externalities that can only exist when there is a farming sector. The second sub-section presents the information that shows key differences between the original model by Goyal and Joshi and the extended version of the model. The discussion here also illustrates why there is a need to rely on simulations given the intractability of obtaining closed form solutions.

4.2.1 Solving the extended international network model

This sub-section presents the main results of the extended model after solving the Cournot game. Intuition behind some of the resulting equations is formally given in sub-section 4.2.2.

4.2.1.1 Solution under the assumptions of exogenous tariffs and symmetric countries

In order to solve the Cournot game of this model, a characterisation of the firms that participate in this framework is provided as follows.

The farming sector

In this model, it is assumed that the farming sector is formed of a single group of farmers who are price takers and produce a homogeneous crop denoted by $q_i^f(g)$ (i.e. this is the total output produced by the farmers in country *i* and in network *g*). It is assumed that this output is the input purchased by the domestic intermediary. Since the latter is the only buyer of this input, this firm faces a non-horizontal inverse supply function of the homogeneous crop (White, 1996):

$$p_i^f(g) = \frac{1}{2}\phi_i q_i^f(g) \tag{4.1}$$

where $p_i^f(g)$ is the price of the homogeneous good that is paid to farmers, ϕ_i is a positive coefficient that captures how sensitive the price paid to farmers to changes in the total output sold by these firms is, and the term ½ is used to simplify terms when maximising a quadratic expression that is originated when substituting the Expression 4.1 into the profit function of the intermediary.

Note that this expression is not directly comparable to the one used by Goyal and Joshi (2006) because it does not included a fixed cost. There are two reasons for not adopting a fixed cost in the extended version of the model. Firstly, the pairwise stability of international networks is not affected when countries face the same fixed cost. Consequently, it can be standardised by assuming that this cost is equal to zero without losing generality. The advantage of using this simplification is that the complexity of the model is reduced because it contains one less parameter. Secondly, as explained above, the additional mechanism that plays a key role in explaining the stability of agricultural trade networks is the increasing marginal cost to free trade. This is why the current dissertation is focussed on this cost rather than on fixed costs

Using the Expression 4.1, producer surplus in network *g* (i.e. $PS_i(g)$) is given by:

$$PS_i(g) = p_i^f q_i^f(g) - \frac{1}{2}\phi_i \int_0^{q_i^f} q_i^f(g)\,dq_i^f(g) \qquad (4.2)$$

To understand the sources of producer surplus that are associated with this expression, let us replace the Equation 4.1 into Equation 4.2 and let us solve the integral. The resulting expression is $PS_i(g) = \frac{1}{2}\phi_i q_i^{f\,2}(g) - \frac{1}{4}\phi_i q_i^{f\,2}(g) = \frac{1}{4}\phi_i q_i^{f\,2}(g)$. A key feature that needs to be considered in this expression is the meaning of the term $q_i^f(g)$. That is, how the total production that is sold by the farming sector to the intermediary affects producer surplus from the point of view of the network approach. To understand this term, assume that the production function of the intermediary is characterised by the following Leontief function: $q_i^f(g) = q_i(g)$, where $q_i(g)$ is the total output of the food processed good that is sold in the domestic market as well as in foreign markets. Now, suppose that the intermediary supplies *N* markets including the domestic market (i.e. this firm exports the food processed food to *N* – 1 countries). This means that the total output traded by the intermediary can be expressed as $q_i(g) = q_1(g) + q_2(g) + ... + q_N(g)$ and, therefore, the producer surplus expression becomes: $PS_i(g) = \frac{1}{4}\phi_i\left(q_1(g) + q_2(g) + ... + q_N(g)\right)^2$

$= \frac{1}{4}\phi_i\left(q_1^2(g) + q_2^2(g) + ... + q_N^2(g) + ...q_i(g)_2 q(g) + ...q_1(g)q_N(g) + ...q_2(g)q_N(g) + ...\right).$

As can be seen in this expression, producer surplus contains square and cross terms, and they correspond to the sources of producer surplus. The square terms reflect the sources of producer surplus that are obtained in specific markets. For example, $q_1^2(g)$ is the source of producer surplus obtained in the domestic market of country 1. The cross terms, on the other hand, represent the sources of producer surplus that arise from the farming price effect of market interaction given the existence of a single farming sector in the country. For example, consider the term $q_1(g)q_N(g)$. In this case the intermediary supplies the food processed good to markets 1 and *N*. However, because this individual buys the agricultural good that is processed and sold to these markets to the same farming sector, the outputs $q_1(g)$ and $q_N(g)$ are introduced into the same single farming supply causing a dramatic increase in the agricultural price. In contrast, if there were different farming sectors, the outputs $q_1(g)$ and $q_N(g)$ would be introduced into different supply function resulting in a less dramatic increase in price. In other world, there would not be cross terms. Note that this approach is appropriate when working in a partial equilibrium model. However, because the farming supply function was introduced exogenously in the network model, and because interactions across different industries are not considered, welfare analysis from the partial equilibrium approach have to be considered with caution.

The intermediary

Since the output produced by the agricultural sector is at the same time the input purchased by the intermediary, the production function of the latter corresponds to a Leontief production function. This function is represented as noted above:

$$q_i(g) = q_i^f(g) \tag{4.3}$$

where $q_i(g)$ is the total output of the processed good that is sold by the intermediary in network g in the domestic and international markets. For convenience, this output is expressed as:

$$q_i(g) = q_1^{(i)}(g) + q_2^{(i)}(g) + ... + q_j^{(i)}(g) + ... + q_{N_i}^{(i)}(g) \tag{4.4}$$

where $q_j^{(i)}(g)$ is the output exported by country i to country j (note that $q_i^{(i)}(g)$ is the output sold by the intermediary in country i in the domestic market). Using expressions 4.1, 4.2 and 4.3, the total cost incurred by the intermediary of country i in network g when exporting to an arbitrary country j is:

$$TC_j^{(i)}(g) = p_i{}^f q_j^{(i)}(g) = \frac{1}{2}\phi_i q_j^{(i)}(g) q_i(g) \tag{4.5}$$

The inverse demand function faced by intermediaries who compete in country j is assumed to be the following linear function:

$$P_j(g) = \alpha_j - Q_j(g) = \alpha_i - \sum_{i \in N_j(g)} q_j^{(i)}(g) \tag{4.6}$$

where $Q_j(g) = \sum_{i \in N_j(g)} q_j^{(i)}(g)$ is total output sold in the domestic market of country j and in network g; and $P_j(g)$ is the retailer price of the processed good in this market. Using expressions 4.5 and 4.6, each intermediary $i \in N_j(g)$ (i.e.

intermediaries competing in country *j*) is assumed to choose $q_j^{(i)}(g)$ that maximises the following profit function:

$$\pi_j^{(i)} = q_j^{(i)}(\alpha_j - \sum_{i \in N_j(g)} q_j^{(i)}(g)) - \frac{1}{2}\phi_i q_j^{(i)}(g) q_i(g) \qquad (4.7)$$

The government

In order to evaluate policy biases in favour of any of the main actors considered in the model (i.e. consumers, intermediaries and producers), the weighted welfare function used by McCorriston and MacLaren (2012, 2013) to represent the objective function of the government was adopted in the current research. This function is given by:

$$W_i(g) = a_i CS_i(g) + b_i \sum_{j \in N_i(g)} \pi_j^{(i)}(g) + c_i PS_i(g) \qquad (4.8)$$

where $CS_i(g) = Q_i^2(g)/2$ is consumer surplus; $\pi_j^{(i)}(g)$ is the profit that the intermediary of country *i* makes in country *j*; $PS_i(g)$ denotes producer surplus; and a_i, b_i and c_i are exogenous weights that the government puts on consumer surplus, total profits made by the domestic intermediary, and producer surplus, respectively.

Solving the game when countries are symmetric

Assume exogenous tariffs and symmetric countries in terms of market size and farmers' productivity (i.e. $\alpha_i = \alpha$ and $\phi_i = \phi$ for all $i \in N$). After solving the Cournot-Nash game played by the intermediaries, the following expression for the output exported by the intermediary in country *i* to country *j* is obtained:

$$q_j^{(i)}(g) = \frac{2\alpha(1+\phi) + \phi \sum_{i \in N_j(g)} q_{i-j}(g) - \phi(1+\phi+\eta_j(g))q_{i-j}(g)}{2(1+\phi+\eta_j(g))(1+\phi)} \tag{4.9}$$

where the term $q_{i-j}(g)$ correspond to the total output exported by country *i* minus the output that this country exports to country *j* (i.e. $q_{i-j}(g) = q_i(g) - q_j^{(i)}(g)$).

Using the Equation 4.9, the total output that is traded by country *i* to the domestic and foreign markets is:

$$q_i(g) = \sum_{j \in N_i(g)} \frac{2\alpha(1+\phi) + \phi \sum_{i \in N_j(g)} q_{i-j}(g) - \phi(1+\phi+\eta_j(g))q_{i-j}(g)}{2(1+\phi+\eta_j(g))(1+\phi)} \tag{4.10}$$

On the other hand, the total output that is traded in the domestic market of country *i* is:

$$Q_i(g) = \frac{2\alpha\eta_i(g) - \phi \sum_{j \in N_i(g)} q_{j-i}(g)}{2(1+\phi+\eta_i(g))} \tag{4.11}$$

This term has a number of implications related to externalities that arise when there is a farming sector. These externalities and the intuition behind this expression are formally discussed in the next sub-section.

Using the Equation 4.11, the following expression for consumer surplus is obtained:

$$CS_i(g) = \frac{1}{2}\left(Q_i(g)\right)^2 \tag{4.12}$$

This expression corresponds to consumer surplus in country *i* and is a monotonic transformation of the total output that is traded in the domestic market of this country. This implies that any externality caused by the existence of a farming sector on this output will affect the component of the welfare function. This is formally discussed in the next sub-section.

The other relevant term that results from the Cournot solution is the profit obtained in country *j* by the intermediary of country *i*:

$$\pi_j^{(i)}(g) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g)\right)^2 \tag{4.13}$$

This profit is a monotonic transformation of the output that is exported by the intermediary in country *i* to country *j*. It is inferred therefore that changes on this output that are caused by the existence of a farming sector will also affect the profit made by this intermediary as discussed in the next subsection.

A related term that is important to consider for the extended version of the model is the total profit made by the intermediary of country *i.* This corresponds to:

$$\pi_i(g) = \sum_{j \in N_i(g)} \frac{(2+\phi)}{2} \left(q_j^{(i)}(g)\right)^2 \tag{4.14}$$

A final result is the following expression for producer surplus:

$$PS_i(g) = \frac{\phi}{4}(q_i(g))^2 = \frac{\phi}{4}\left(\sum_{j \in N_i(g)} q_j^i(g)\right)^2 \tag{4.15}$$

This expression is a monotonic transformation of the total output traded by the intermediary of country *i.* It differs from the total profit function (i.e. Expression 4.14) in that the latter is the sum of monotonic transformations rather than a monotonic transformation of the sum. Some implications of this difference are described in the next sub-section.

Afinal expression that is needed in the extended version of the model is the weighted welfare function. In considering the Expressions 4.12, 4.14 and 4.15, this function is given by:

$$W_i(g) = a_i CS_i(g) + b_i \pi_i(g) + c_i PS_i(g)$$
$$= \frac{a_i}{2}\left(Q_i(g)\right)^2 + b_i \sum_{j \in N_i(g)} \frac{(2+\phi)}{2}\left(q_j^{(i)}(g)\right)^2 + c_i \frac{\phi}{4}\left(\sum_{j \in N_i(g)} q_j^{(i)}(g)\right)^2 \qquad (4.16)$$

The impact of the potential network and the trade-off faced by the government can be seen in this expression: as the network expands, agricultural prices rise in relation to trade *across all nodes*. This benefits producers but impacts on the profits of the intermediary who also face the impact of competition on the output price. The latter impact, however, benefits consumers. The positive net effect of the trade-off and the weights that governments put on the components of the welfare function is what dictates whether a new agreement is signed and whether an existing agreement is maintained.

4.2.1.2 Solution under the Assumptions of Exogenous Tariffs and Asymmetry in Market Size

The solution of the extended international model under the assumption of symmetric countries offers a convenient way to explore the network impact of the farming sector in a world with intermediaries with market power. However, symmetry does not fit with reality. On the contrary, one of the key aspects of the debate on the issue of international trade liberalisation is the relationship between developing and developed countries (see for example Melitz and Ottaviano, 2008; Fulponi et al. 2011). In this regard, it is argued that market size is related to the degree of industrialisation implying that developed countries are more likely to have larger markets (Faber, 2014). Introducing this type of asymmetry into the proposed model not only makes this framework more realistic, but also provides the theoretical basis to identify how the international trade structure is affected when countries have differences in market size and, therefore, differences in the degree of industrialisation.

It is important to clarify the fact that market size does not necessarily reflect country size in terms of area of land. The latter may be relevant on the supply side in determining capability to export. Given the complexity of the international trade model, this possibility is not explored in the current investigation and is left for future research. The use of the term *country size* in this thesis refers to market size and level of industrialisation and is captured by the parameter α in Expression 4.6.

To illustrate how the network can be affected by this type of asymmetry, consider the following key question: are large countries willing to sign international agreements with small countries? While this question seems to have an obvious answer at first, it is not by any means clear whether there is a single answer when the problem is analysed from a network point of view. For example, in some network structures a large country could be unwilling to sign a bilateral agreement with a very small country because the export profit that the large country makes in the very small country is probably too small to compensate the loss in the domestic profit as a consequence of more intense competition. However, in other structures it can be strategically convenient to sign an agreement with a very small country because this can increase the competitive position of the intermediary of the large country when competing with other large countries. The reason is because an agreement with a small country increases the level of competition in the large country and this reduces the price paid to the farming sector. As a consequence, the intermediary faces a lower marginal cost that increases the competitive position of this firm with respect to other large countries. Note that this second possibility is only possible in the extended version of the model because it holds only when there is a farming sector with an upward sloping supply function.

Asymmetry in market size is introduced by means of the same equations presented in the previous subsection. The only difference is that it is assumed now the existence of two subsets of countries. Subset Ω includes all countries having the same market size denoted by α. On the other hand, the subset Ψ includes all the countries having the same market size denoted by $\tilde{\alpha} \neq \alpha$. The cardinality of sets Ω and Ψ is given by N_Ω and N_Ψ, respectively. Since there are only two subsets in the worlds, it holds that $N = N_\Omega + N_\Psi$. Finally, because asymmetry in market size is the only type of asymmetry considered in this case, it is also assumed that $\phi_i = \phi$, $a_i = a$, $b_i = b$ and $c_i = c$ for all $i \in N$.

To accommodate asymmetry, the welfare functions have to be re-written. Let $i \in \Omega$ and $j \in \Psi$. Using Equation 4.16, the welfare functions considered by the governments of countries i and j in network g are, respectively:

$$
\begin{aligned}
W_i(g) &= a_i CS_i(g) + b_i \pi_i(g) + c_i PS_i(g) \\
&= \frac{a_i}{2}\left(Q_i(g)\right)^2 + b_i\left[\sum_{k\in N_i(g)\subset\Omega}\frac{(2+\phi)}{2}\left(q_k^{(i)}(g)\right)^2 + \sum_{l\in N_i(g)\subset\Psi}\frac{(2+\phi)}{2}\left(q_l^{(i)}(g)\right)^2\right] \\
&+ c_i\frac{\phi}{4}\left(\sum_{k\in N_i(g)\subset\Omega} q_k^{(i)}(g) + \sum_{l\in N_i(g)\subset\Psi} q_l^{(i)}(g)\right)^2 = a_i\,\frac{1}{2}\left[\frac{2\alpha\eta_i(g) - \phi\sum_{j\in N_i(g)} q_{j-i}(g)}{2(1+\phi+\eta_i(g))}\right]^2 \\
&+ b_i\frac{(2+\phi)}{2}\left(\sum_{k\in N_i(g)\subset\Omega}\left[\frac{2\alpha(1+\phi) + \phi\sum_{i\in N_j(g)} q_{i-k}(g) - \phi(1+\phi+\eta_j(g))q_{i-k}(g)}{2(1+\phi+\eta_k(g))(1+\phi)}\right]^2 + \sum_{l\in N_i(g)\subset\Psi}\left[\frac{2\tilde{\alpha}(1+\phi) + \phi\sum_{i\in N_j(g)} q_{i-l}(g) - \phi(1+\phi+\eta_l(g))q_{i-l}(g)}{2(1+\phi+\eta_l(g))(1+\phi)}\right]^2\right) \\
&+ c_i\frac{\phi}{4}\left(\sum_{k\in N_i(g)\subset\Omega}\frac{2\alpha(1+\phi) + \phi\sum_{i\in N_j(g)} q_{i-k}(g) - \phi(1+\phi+\eta_j(g))q_{i-k}(g)}{2(1+\phi+\eta_k(g))(1+\phi)} + \sum_{l\in N_i(g)\subset\Psi}\frac{2\tilde{\alpha}(1+\phi) + \phi\sum_{i\in N_j(g)} q_{i-l}(g) - \phi(1+\phi+\eta_l(g))q_{i-l}(g)}{2(1+\phi+\eta_l(g))(1+\phi)}\right)^2
\end{aligned}
\quad (4.17)
$$

$$
\begin{aligned}
W_j(g) &= a_j CS_j(g) + b_j \pi_j(g) + c_j PS_j(g) \\
&= \frac{a_j}{2}\left(Q_j(g)\right)^2 + b_i\left[\sum_{k\in N_j(g)\subset\Omega}\frac{(2+\phi)}{2}\left(q_k^{(j)}(g)\right)^2 + \sum_{l\in N_j(g)\subset\Psi}\frac{(2+\phi)}{2}\left(q_l^{(j)}(g)\right)^2\right] \\
&+ c_i\,\frac{\phi}{4}\left(\sum_{k\in N_j(g)\subset\Omega} q_k^{(j)}(g) + \sum_{l\in N_j(g)\subset\Psi} q_l^{(j)}(g)\right)^2 = a_j\,\frac{1}{2}\left[\frac{2\tilde{\alpha}\eta_j(g) - \phi\sum_{k\in N_j(g)} q_{k-j}(g)}{2(1+\phi+\eta_j(g))}\right]^2 \\
&+ b_j\,\frac{(2+\phi)}{2}\left(\sum_{k\in N_j(g)\subset\Omega}\left[\frac{2\alpha(1+\phi)+\phi\sum_{j\in N_j(g)} q_{j-k}(g) - \phi(1+\phi+\eta_j(g))q_{j-k}(g)}{2(1+\phi+\eta_k(g))(1+\phi)}\right]^2 \right. \\
&\qquad \left. + \sum_{l\in N_j(g)\subset\Psi}\left[\frac{2\tilde{\alpha}(1+\phi)+\phi\sum_{j\in N_j(g)} q_{j-l}(g) - \phi(1+\phi+\eta_l(g))q_{j-l}(g)}{2(1+\phi+\eta_l(g))(1+\phi)}\right]^2\right) \\
&+ c_j\,\frac{\phi}{4}\left(\sum_{k\in N_j(g)\subset\Omega}\frac{2\alpha(1+\phi)+\phi\sum_{j\in N_j(g)} q_{j-k}(g) - \phi(1+\phi+\eta_j(g))q_{j-k}(g)}{2(1+\phi+\eta_k(g))(1+\phi)} \right. \\
&\qquad \left. + \sum_{l\in Nj(g)\subset\Psi}\frac{2\tilde{\alpha}(1+\phi)+\phi\sum_{j\in N_j(g)} q_{j-l}(g) - \phi(1+\phi+\eta_l(g))q_{j-l}(g)}{2(1+\phi+\eta_l(g))(1+\phi)}\right)
\end{aligned}
\tag{4.18}
$$

4.2.1.3 Solution under the Assumptions of Exogenous Tariffs and Asymmetry in Farmers' Productivity

Another type of asymmetry that is relevant for the current investigation corresponds to differences in farmers' productivity. This is important because this asymmetry reflects different degrees of competitiveness across countries that can potentially affect the stability of international trade networks. For example, countries with less efficient farmers imply that the intermediaries in these countries face higher agricultural prices that negatively affect their competitive position. As a consequence, these countries are probably less integrated than more efficient countries in a determined stable network. This can be particularly true when governments are biased in favour of intermediaries. Note that these deviations can only be identified in the proposed framework because the original model by Goyal and Joshi does not consider a variable marginal cost.

Having described some potential effects of asymmetry in farmers' productivity, let us now introduce this asymmetry into the proposed model. In this case, it is assumed that there are two subsets of countries in the world. Countries within a determined subset have the same productivity, and countries that belong to different subsets have different productivity. Let Ω and Ψ be these two subsets. The cardinality of Ω and Ψ is given by N_{Ω} and N_{Ψ}, respectively. Because there are only two subsets in the worlds, it holds that $N = N_{\Omega}$ plus N_{Ψ}. Finally, because asymmetry in farmers' productivity is the only type of asymmetry considered in this case, it is assumed that $\alpha_i = \alpha$, $a_i = a$, $b_i = b$ and $c_i = c$ for all $i \in N$.

Different productivity is captured by parameter δ. In particular it is assumed that $\delta \neq 1$ for all $i \in \Omega$ and $\delta = 1$ for all $k \in \Psi$. The supply functions of the farmers in countries $i \in \Omega$ and $k \in \Psi$ are given respectively by:

$$p_i{}^f(g) = \frac{1}{2}\phi\delta q_i^f(g) \tag{4.19}$$

$$p_k{}^f(g) = \frac{1}{2}\phi q_k^f(g) \tag{4.20}$$

Note that as in the symmetrical case, the fixed cost is assumed to be equal to zero because the aim of this investigation is to explore how international trade structure is influenced when the agricultural price increases as a consequence of monopsonistic power. This is why the intercept in Equations 4.19 and 4.20 was not included. Thus rather than exploring asymmetry across countries by allowing for different fixed costs, this dissertation is focussed on asymmetries in the variable cost that is captured by the parameter δ in the inverse supply functions in Expressions 4.19 and 4.20.

The intuition behind these expressions is as follows. When $\delta > 1$, the impact of an increase in the agricultural output demanded by the intermediary to the farming sector on the agricultural price is stronger than when $\delta = 1$. This is caused by the fact that the farming sector is less efficient when $\delta > 1$. Note that this is equivalent to have asymmetry in terms of monopsonistic power. That is, intermediaries exercise more monoponistic power when $\delta > 1$ since the marginal outlay curve will be correspondingly steeper thus resulting in a relatively greater mark-down in agricultural prices.

Using Expressions 4.19 and 4.20, and because it is assumed that there is only a single group of farmers in each country, producer surplus in countries $i \in \Omega$ and $k \in \Psi$ are given, respectively, by:

$$PS_i(g) = p_i^{\ f}(g)q_i^f(g) - \frac{1}{2}\phi\delta\int_0^{q_i^f} q_i^f(g)dq_i^f(g) \qquad (4.21)$$

$$PS_k(g) = p_k^{\ f}(g)q_k^f(g) - \frac{1}{2}\phi\int_0^{q_k^f} q_k^f(g)dq_k^f(g) \qquad (4.22)$$

The production function of the intermediary is:

$$q_i(g) = q_i^f(g) \qquad (4.23)$$

where, as before, $q_i(g) = q_1^{(i)}(g) + q_2^{(i)} + ... + q_{\eta_i(g)}^{(i)}$. Using expressions 4.19 and 4.23, the total cost faced by the intermediary in country $i \in \Omega$ when exporting the output $q_j^{(i)}(g)$ to country *j* in network *g* is, therefore:

$$TC_j^{(i)}(g) = p_i^{\ f}(g)q_j^{(i)}(g) = \frac{1}{2}\phi\delta q_j^{(i)}(g)q_i(g) \qquad (4.24)$$

Likewise, using expressions 4.20 and 4.23, the total cost faced by the intermediary in country $k \in \Psi$ when exporting to country *j* in network *g* is, therefore:

$$TC_j^{(k)}(g) = p_k^{\ f}(g)q_j^{(k)}(g) = \frac{1}{2}\phi q_j^{(k)}(g)q_k(g) \qquad (4.25)$$

Assume that countries $i \in \Omega$ and $k \in \Psi$ compete Cournot in country j. The inverse demand in the latter country is assumed to be the following linear function:

$$P_j(g) = \alpha - Q_j(g) = \alpha - \sum_{l \in N_j(g)} q_j^{(l)}(g) \tag{4.26}$$

where $Q_j(g) = \sum_{l \in N_j(g)} q_j^{(l)}(g)$ is total output sold in the domestic market of country j and in network g; and $P_j(g)$ is the retailer price of the homogeneous agricultural good in this market. Using expressions 4.24, 425 and 4.26, the profits that the intermediaries in countries $i \in \Omega$ and $k \in \Psi$ make in country j are given, respectively, by:

$$\pi_j^{(i)}(g) = q_j^{(i)}(g)(\alpha - \sum_{l \in N_j(g)} q_j^{(l)}(g)) - \frac{1}{2}\phi\delta q_j^{(i)}(g) q_i(g) \tag{4.27}$$

$$\pi_j^{(k)}(g) = q^{(k)}{}_j(g)(\alpha - \sum_{l \in N_j(g)} q_j^{(l)}(g)) - \frac{1}{2}\phi q_j^{(k)}(g) q_k(g) \tag{4.28}$$

where $\pi_j^{(i)}(g)$ is the profit made by the intermediary of country i in country j and in network g, and $q_j^{(i)}(g)$ is the output sold by this firm in country j and in network g. As in the symmetric case, governments are assumed to maximise a weighted welfare function.

In order to solve the Cournot-Nash game played by the intermediaries in the arbitrary country *j*, it is assumed that $\gamma_j(g)$ countries that compete in this country belong to the subset Ω and the rest $\eta_j(g) - \gamma_j(g)$ countries belong to the subset Ψ. After solving the Cournot-Nash game, the following results are obtained:

$$Q_j(g) = \frac{2\alpha\left[\gamma_j(g)(1+\phi) + (\eta_j(g) - \gamma_j(g))(1+\phi\delta)\right] - \phi\delta(1+\phi)\sum_{i\in\Omega\subset N_j(g)} q_{i-j} - \phi(1+\phi\delta)\sum_{k\in\Psi\subset N_j(g)} q_{k-j}}{2(1+\phi\delta+\gamma_j(g))(1+\phi) + 2(\eta_j(g) - \gamma_j(g))(1+\phi\delta)} \quad (4.29)$$

$$q_j^{(i)}(g) = \frac{2\alpha(1+\phi\delta)(1+\phi) + \phi\delta(1+\phi)\sum_{i\in\Omega\subset N_j(g)} q_{i-j} + \phi(1+\phi\delta)\sum_{k\in\Psi\subset N_j(g)} q_{k-j} - \phi\delta\left[(1+\phi\delta+\gamma_j(g))(1+\phi) + (\eta_j(g) - \gamma_j(g))(1+\phi\delta)\right]q_{i-j}}{2(1+\phi\delta+\gamma_j(g))(1+\phi)(1+\phi\delta) + 2(\eta_j(g) - \gamma_j(g))(1+\phi\delta)^2} \quad (4.30)$$

$$q_j^{(k)}(g) = \frac{2\alpha(1+\phi\delta)(1+\phi) + \phi\delta(1+\phi)\sum_{i\in\Omega\subset N_j(g)} q_{i-j} + \phi(1+\phi\delta)\sum_{k\in\Psi\subset N_j(g)} q_{k-j} - \phi\left[(1+\phi\delta+\gamma_j(g))(1+\phi) + (\eta_j(g) - \gamma_j(g))(1+\phi\delta)\right]q_{k-j}}{2(1+\phi\delta+\gamma_j(g))(1+\phi)^2 + 2(\eta_j(g) - \gamma_j(g))(1+\phi\delta)(1+\phi)} \quad (4.31)$$

$$CS_i(g) = \frac{1}{2}\left(Q_i(g)\right)^2 \quad (4.32)$$

$$CS_k(g) = \frac{1}{2}\left(Q_k(g)\right)^2 \quad (4.33)$$

$$\pi_j^{(i)}(g) = \frac{\left[2(1+\phi\delta) - \phi\right]}{2}\left(q_j^{(i)}(g)\right)^2 \quad (4.34)$$

$$\pi_j^{(k)}(g) = \frac{(2+\phi)}{2}\left(q_j^{(k)}(g)\right)^2 \tag{4.35}$$

$$PS_i(g) = \frac{\phi\delta}{4}\left(\sum_{j \in N_i(g)} q_j^{(i)}(g)\right)^2 \tag{4.36}$$

$$PS_k(g) = \frac{\phi}{4}\left(\sum_{j \in N_k(g)} q_j^{(k)}(g)\right)^2 \tag{4.37}$$

In these formulations, the interpretation of Expressions 4.34 and 4.36 has to be made with caution because the parameter δ appears explicitly in these equations suggesting, at face value, that a country with an inefficient farming sector (i.e. $\delta >$ 1) obtains higher profits and higher producer surplus. However, the effect of $\delta > 1$ also works through $g_j^{(i)}(g)$. Specifically, the parameter δ also affects implicitly the outputs that are traded by the intermediary to different markets (see Expression 4.30). In fact, as shown in the simulation presented in Section 4.5, profits and producer surplus decrease when the parameter δ increases (the only exception is in the empty network as producer surplus is higher in the inefficient country). The effect of the parameter δ on the components of the welfare function is studied in more detail in this simulation.

To finish this sub-section, the expressions that were obtained after solving the Cournot game are used to derive the following expressions for the welfare function in countries $i \in \Omega$ and $j \in \Psi$ are, respectively:

$$W_i(g)=\frac{a}{2}\left(Q_i(g)\right)^2+\frac{b\left[2(1+\phi\delta)-\phi\right]}{2}\sum_{j\in N_i(g)}\left(q_j^{(i)}(g)\right)^2+c\frac{\phi\delta}{4}\left(\sum_{j\in N_i(g)}q_j^{(i)}(g)\right)^2 \tag{4.38}$$

$$W_k(g)=\frac{a}{2}\left(Q_k(g)\right)^2+\frac{b(2+\phi)}{2}\sum_{r\in N_k(g)}\left(q_j^{(k)}(g)\right)^2+c\frac{\phi}{4}\left(\sum_{j\in N_k(g)}q_j^{(k)}(g)\right)^2 \tag{4.39}$$

Note that the parameter δ also appears explicitly in Expression 4.38. However, the same observations made for Expressions 4.34 and 4.36 apply in this case. That is, this parameter also affects the outputs contained in the Expression 4.38, and an increase in this parameter (i.e. a country having a less efficient farming sector) causes a decrease in the level of welfare as revealed by the simulation presented in Section 4.5.

4.2.1.4 Solution under the Assumptions of Endogenous Tariffs

As explained in the previous chapter, Goyal and Joshi extended the analysis by studying the stability of global free trade when tariffs are placed endogenously and when governments are politically unbiased. While this extension provides a more realistic description of the world in terms of the way by which international agreements are negotiated, the complexity of the extended version of the model increases significantly because this increases the level of endogeneity across countries. As a consequence of this complexity, it is not possible to obtain a specific expression for the optimal tariffs. In spite of this disadvantage, some general patterns can be identified from the relevant equations presented below. In general terms, these patterns are consistent with some trends identified by Goyal and Joshi. Firstly, a decrease in tariffs increases the level of competition positively affecting consumer surplus. Secondly, this higher competition negatively affects the profits made by the intermediary in the domestic market. Finally, tariff revenue can either increase or decrease depending on the size of the tariff.

In addition to these patterns, it is possible to identify additional effects of a decrease in tariffs in the extended version of the model. Firstly, a tariff reduction increases competition in the domestic country causing a negative impact on the domestic profit and a positive impact on export profit. Depending on the relative number of links in the competing countries, the loss in domestic profits can be either larger or smaller than the gain in domestic profits. If there is a net loss (gain) in profits, then the intermediary will reduce (increase) the demand for the agricultural good pushing the agricultural price down (up). This decrease in agricultural price means that the intermediary becomes more competitive positively affecting the export profits obtained in other third countries. Secondly, if the intermediary faces a net loss of profits after the tariff is reduced, then the lower demand for the agricultural good will cause a decrease in the price paid to the farming sector negatively affecting producer surplus. These examples illustrate the relevance of extending the analysis for the case of endogenous tariffs. This is shown as follows.

In order to obtain expressions for the model under endogenous tariffs, note that if countries *i* and *j* have an agreement (i.e. g_{ij} = 1), then $T_i^j(g) = T_j^i(g) = 0$, where $T_i^j(g)$ is the tariff applied by country *i* to country *j*. On the other hand, because countries are symmetric, it holds that $q_i^{(k)}(g) = q_i^{(l)}(g)$ for all $k,l \notin N_i(g)$ (i.e. countries that do not have an agreement in country *i* but they export to this country). This implies that $T_i^k(g) = T_i(g)$ for all $k \notin N_i(g)$. Using this simplification, the output exported by country *j* to country *i* (where g_{ij} =1) is the output that results from the following maximisation problem:

$$\max_{q_i^{(j)}} \pi_i^{(j)}(g) = q_i^{(j)}(g)\left(\alpha - \sum_{j\in N_i(g)} q_i^{(j)}(g) - \sum_{k\notin N_i(g)} q_i^{(k)}(g)\right) - \frac{1}{2}\phi q_i^{(j)}(g) q_j(g) \qquad (4.40)$$

The first order condition of this maximisation problem leads to the following expression:

$$q_i^{(j)}(g) = \frac{\alpha - \sum_{j\in N_i(g)} q_i^{(j)}(g) - \sum_{k\notin N_i(g)} q_i^{(k)}(g) - \frac{1}{2}\phi q_{j-i}(g)}{\phi + 1} \qquad (4.41)$$

By taking summation in both sides of this expression and by rearranging terms, the following expression is obtained:

$$\sum_{j\in N_i(g)} q_i^{(j)}(g) = \frac{\eta_i(g)\left(\alpha - \sum_{k\notin N_i(g)} q_i^{(k)}(g)\right) - \frac{\phi}{2}\sum_{j\in N_i(g)} q_{j-i}(g)}{\eta_i(g) + \phi + 1} \qquad (4.42)$$

On the other hand, the output exported by country *k* to country *i* (where g_{ik} =0) is the output that results from the following maximisation problem:

$$\max_{q_i^k} \pi_i^{(k)}(g) = q_i^{(k)}(g)\left(\alpha - T_i(g) - \sum_{j\in N_i(g)} q_i^{(j)}(g) - \sum_{k\notin N_i(g)} q_i^{(k)}(g)\right) - \frac{1}{2}\phi q_i^{(k)}(g) q_k(g) \qquad (4.43)$$

The first order condition of this maximisation problem leads to the following expression:

$$q_i^{(k)}(g) = \frac{\alpha - T_i(g) - \sum_{j\in N_i(g)} q_i^{(j)}(g) - \sum_{k\notin N_i(g)} q_i^{(k)}(g) - \frac{1}{2}\phi q_{k-i}(g)}{\phi + 1} \qquad (4.44)$$

By taking summation in both sides of this expression and by rearranging terms, the following expression is obtained:

$$\sum_{k\notin N_i(g)} q_i^{(k)}(g) = \frac{\left(N - \eta_i(g)\right)\left(\alpha - T_i(g) - \sum_{k\in N_i(g)} q_i^{(j)}(g)\right) - \frac{1}{2}\phi \sum_{k\notin N_i(g)} q_{k-i}(g)}{N - \eta_i(g) + \phi + 1} \qquad (4.45)$$

By substituting Equation 4.45 into 4.42, an expression for $\sum_{k\in N_i(g)} q_i^{(j)}(g)$ is obtained:

$$\sum_{j\in N_i(g)} q_i^{(j)}(g) = \frac{\begin{array}{l} 2\alpha\eta_i(g)(\phi+1) + 2\eta_i(g)(N-\eta_i(g))T_i(g) \\ +\phi\eta_i(g)\sum_{k\notin N_i(g)} q_{k-i} - \phi(N-\eta_i(g)+\phi+1)\sum_{j\in N_i(g)} q_{j-i} \end{array}}{2(\phi+1)(N+\phi+1)} \tag{4.46}$$

Likewise, by substituting Equation 4.42 into 4.45, an expression for $\sum_{k\notin N_i(g)} q_i^{(k)}(g)$ is given by:

$$\sum_{k\notin N_i(g)} q_i^{(k)}(g) = \frac{\begin{array}{l} 2\alpha(N-\eta_i(g))(\phi+1) - 2(N-\eta_i(g))(\eta_i(g)+\phi+1)T_i(g) \\ +\phi(N-\eta_i(g))\sum_{j\in N_i(g)} q_{j-i} - \phi(\eta_i(g)+\phi+1)\sum_{k\notin N_i(g)} q_{k-i} \end{array}}{2(\phi+1)(N+\phi+1)} \tag{4.47}$$

The total output in equilibrium that is demanded by Country i is obtained by adding expressions 4.46 and 4.47. This output is given by:

$$Q_i(g) = \frac{2\alpha N - 2(N-\eta_i(g))T_i(g) - \phi\sum_{k\notin N_i(g)} q_{k-i} - \phi\sum_{j\in N_i(g)} q_{j-i}}{2(N+\phi+1)} \tag{4.48}$$

The outputs in equilibrium exported by countries $j \in N_i(g)$ and $k \notin N_i(g)$ exported to country *i* are obtained by substituting expression 4.48 into expressions 4.41 and 4.44, respectively. These outputs are:

$$q_i^{(j)}(g) = \frac{2\alpha(\phi+1) + 2(N-\eta_i(g))T_i(g) + \phi \sum_{k \notin N_i(g)} q_{k-i}(g) + \phi \sum_{j \in N_i(g)} q_{j-i}(g) - \phi(N+\phi+1)q_{j-i}}{2(\phi+1)(N+\phi+1)}$$

$$= \frac{2\alpha(\phi+1) + 2(N-\eta_i(g))T_i(g) + \phi \sum_{k \notin N_i(g)} q_{k-i}(g) + \phi \sum_{l \in N_i(g)-\{j\}} q_{l-i}(g) - \phi(N+\phi)q_{j-i}}{2(\phi+1)(N+\phi+1)}$$

(4.49)

$$q_i^{(k)}(g) = \frac{2\alpha(\phi+1) - 2(\eta_i(g)+\phi+1)T_i(g) + \phi \sum_{k \notin N_i(g)} q_{k-i}(g) + \phi \sum_{j \in N_i(g)} q_{j-i}(g) - \phi(N+\phi+1)q_{k-i}}{2(\phi+1)(N+\phi+1)}$$

$$= \frac{2\alpha(\phi+1) - 2(\eta_i(g)+\phi+1)T_i(g) + \phi \sum_{u \notin N_i(g)-\{k\}} q_{u-i}(g) + \phi \sum_{j \in N_i(g)} q_{j-i}(g) - \phi(N+\phi)q_{k-i}}{2(\phi+1)(N+\phi+1)}$$

(4.50)

By using Equation 4.48 and the generic definition for consumer surplus, $CS_i(g) = Q_i^2(g)/2$, the following expression for consumer surplus is obtained:

$$CS_i(g) = \frac{1}{8}\left(\frac{2\alpha N - 2(N-\eta_i(g))T_i(g) - \phi \sum_{k \notin N_i(g)} q_{k-i} - \phi \sum_{j \in N_i(g)} q_{j-i}}{N+\phi+1}\right)^2 = \frac{a_i}{2}(Q_i(g))^2 \quad (4.51)$$

Profit made by country $j \in N_i(g)$ in country i is obtained by introducing expressions 4.48 and 4.49 into Equation 4.40. This profit is given by:

$$\pi_i^{(j)}(g) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g)\right)^2 \quad (4.52)$$

Likewise, the profit made by country $k \notin N_i(g)$ in country i is obtained by introducing Expressions 4.48 and 4.50 into Equation 4.43. This profit is given by:

$$\pi_i^{(k)}(g) = \frac{(\phi+2)}{2}\left(q_i^{(k)}(g)\right)^2 \tag{4.53}$$

In relation to producer surplus, it is defined:

$$PS_i(g) = \frac{\phi}{4}\left(\sum_{j \in N_i(g)} q_i^{(j)}(g) + \sum_{k \notin N_i(g)} q_i^{(k)}(g)\right)^2 \tag{4.54}$$

Tariff revenue is defined as:

$$TR_i(g) = \sum_{k \notin N_i(g)} T_i(g) q_i^{(k)}(g) \tag{4.55}$$

Using Expressions 4.51, 4.52, 4.53, 4.54 and 4.55, welfare in country i is defined as:

$$\begin{aligned} W_i(g) &= a_i CS_i(g) + b_i \pi_i(g) + c_i PS_i(g) + d_i TR_i(g) \\ &= \frac{a_i}{2}\left(Q_i(g)\right)^2 + b_i\left\{\sum_{j \in N_i(g)} \frac{(\phi+2)}{2} q_j^{(i)}(g) + \sum_{k \notin N_i(g)} \frac{(\phi+2)}{8} q_k^{(i)}(g)\right\} \\ &+ c_i \frac{\phi}{4}\left(\sum_{j \in N_i(g)} q_j^{(i)}(g) + \sum_{k \notin N_i(g)} q_k^{(i)}(g)\right)^2 + d_i \sum_{k \notin N_i(g)} T_i(g) q_i^{(k)}(g) \end{aligned} \tag{4.56}$$

where a_i, b_i, c_i and d_i are the weights that the government puts on consumer surplus, profits, producer surplus and tariff revenue, respectively.

This weighted welfare function is the objective function of the government. In order to obtain the optimal endogenous tariff, this expression has to be maximised. As explained above, unfortunately it is not possible to obtain a generic expression for this tariff given the high degree of endogeneity and complexity of the model. However, optimal tariffs were obtained using a simulation (see Section 4.3.2). In this simulation, a system of sixteen equations was employed. Each equation represents the output traded by an intermediary to a determined market. Solving by substitution, it was possible to express the sixteen equations in function of the tariffs applied by each country. These equations were introduced into the welfare function presented in expression 4.56. After maximising this expression, optimal endogenous tariffs were obtained for each network considered in the simulation.

4.2.2 Some general possible patterns

As noted, one of the main disadvantages of the proposed international network model is its complexity. This complexity arises from the fact that the addition of a farming sector creates high degree of endogeneity across countries that are explained by the influence of this sector on the marginal cost faced by the intermediaries. In order to deal with this endogeneity, expressions for the optimal outputs traded by these firms that are needed in the weighted welfare function can only be obtained by solving a matrix system. Unfortunately, given the large number of parameters in the model, solving a generic NxN matrix is virtually impossible. It is for this reason that simulations were adopted to obtain numerical solutions for the involved equations. In order to simplify the analysis, the simulations consider a world composed of four countries. The reason for this choice is because this is the minimum number of countries that can be used to explain observed features in the real world such as regionalism, centrality (countries that are highly connected with other less connected countries), among other considerations. Even with this small number of countries, solving the model becomes a challenging task because it involves in several cases solving a matrix of 16x16 (i.e. each intermediary can supply up to four countries).

In spite of this simplification, it is still possible to identify some possible patterns from the equations presented in the previous section. The objective of this subsection is to show these patterns and the intuition behind them and the equations involved. However, before doing this exercise, it is important to explain first the endogeneity of the extended model because this is a key aspect that needs to be considered in the description of possible general patterns of the model, and also because this is the key difference between the model by Goyal and Joshi and the extended version of the model.

4.2.2.1 The endogeneity of the model

As mentioned above, the extended version of the model has a high degree of endogeneity that is caused by the existence of a farming sector with an upward slopping supply function. To illustrate this endogeneity, consider the networks presented in the following figure.

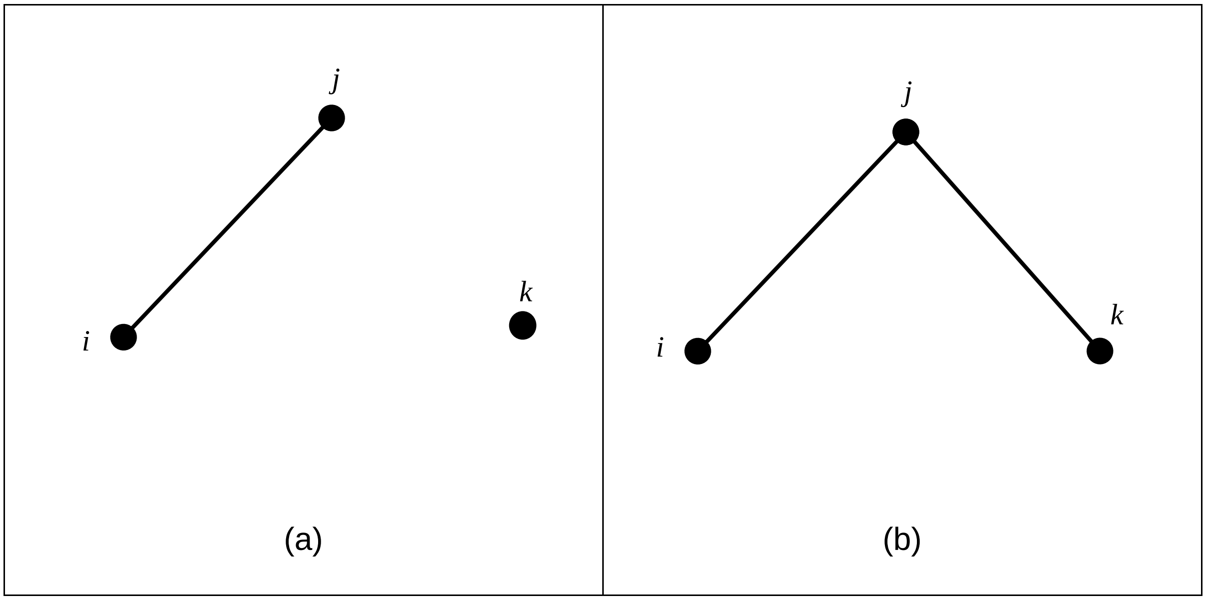


Figure 4.2. Networks composed of three countries

Figure 4.2(a) shows a network formed of countries *i*, *j* and *k*. In this network, referred to as network *g*, countries *i* and *j* have an agreement, and country *k* is a singleton. Figure 4.2(b), shows the network that results when countries *j* and *k* sign an agreement. This network is referred to as network $g + g_{jk}$. The idea of this exercise is to show that as a consequence of endogeneity, the output sold by the intermediary of country *i* in the domestic market of this country (i.e. $g_i^{(i)}(g)$) is affected when countries *j* and *k* sign the agreement. To show this, consider the generic Equation 4.9. According to this equation, the output sold by the intermediary of country *i* in country *j* depends on the size of the market in the latter (i.e. α), the number of agreements in country *j* (i.e. $\eta_i(g)$, the level of monopsonistic power exercised by the intermediaries (i.e. ϕ), and the outputs that are exported by the partner countries of county *j* to other markets (i.e. $\sum_{i \in N_j(g)} q_{i-j}(g)$). These outputs reflect the endogeneity of the model as is illustrated as follows.

Let us use this equation to get an expression for the output sold by the intermediary of country *i* in the domestic market of this country in the networks presented in Figures 4.2(a) and 4.2(b):

$$q_i^{(i)}(g) = \frac{2\alpha(\phi+1) + \phi q_j^{(j)}(g) - \phi(\phi+2)q_j^{(i)}(g)}{2(\phi+3)(\phi+1)} \tag{4.57}$$

$$q_i^{(i)}(g+g_{jk}) = \frac{2\alpha(\phi+1) + \phi q_j^{(j)}(g+g_{jk}) + \phi q_k^{(j)}(g+g_{jk}) - \phi(\phi+2)q_j^{(i)}(g+g_{ik})}{2(\phi+3)(\phi+1)} \tag{4.58}$$

By subtracting these equations, the following expression is obtained:

$$q_i^{(i)}(g+g_{jk}) = q_i^{(i)}(g) + \frac{\begin{array}{c}\phi\left[q_j^{(j)}(g+g_{jk}) - q_j^{(j)}(g)\right] + \phi q_k^{(j)}(g+g_{jk}) \\ -\phi(\phi+2)\left[q_j^{(i)}(g+g_{ik}) - q_j^{(i)}(g)\right]\end{array}}{2(\phi+3)(\phi+1)} \tag{4.59}$$

An interesting feature of this expression is that when ϕ = 0 (i.e. when there is no monopsonistic power), this expression converges to $q_i^{(i)}(g+g_{jk}) = q_i^{(i)}(g)$. This means that a new agreement between countries *j* and *k* does not affect the output sold by the intermediary of country *i* in the domestic market of this country when there is not a farming sector. This is actually what the original model by Goyal and

Joshi predicts. However, when there is a farming sector, a number of externalities that affects this output arise.

The first externality is captured by the term $\phi\left[q_j^{(j)}(g+g_{jk})-q_j^{(j)}(g)\right]$. In this term, it holds that $q_j^{(j)}(g+g_{jk})<q_j^{(j)}(g)$ because the output that is sold by the intermediary of country *j* in the domestic market of this country decreases as a result of the higher level of competition after the agreement between countries *j* and *k* is signed. Thus, because the term $\phi\left[q_j^{(j)}(g+g_{jk})-q_j^{(j)}(g)\right]$ is negative, it is seen from the point of view of the intermediary of country *i* as a negative externality on the output $q_i^{(i)}(g+g_{jk})$. The reason is because a decrease in $q_j^{(j)}(g)$ implies that the intermediary of country *j* pays a lower price to the farming sector after the agreement. This lower marginal cost allows this firm to increase the output that is exported to the existing partner country *i* increasing the level of competition in the latter which is what explains the negative impact on $q_i^{(i)}(g+g_{jk})$.

The second externality on $q_i^{(i)}(g+g_{jk})$ is captured by the term $\phi q_k^{(j)}(g+g_{jk})$ in Equation 4.59. This is the additional output that is exported by the intermediary of country *j* to country *k* after the agreement between these countries and corresponds to a positive externality on $q_i^{(i)}(g+g_{jk})$ from the point of view of the intermediary of country *i*. The reason is because the additional output that is exported by the intermediary of country *j* pushes the price paid to the farming sector up. In response to this higher cost, this intermediary reduces to some extent the output that is exported to the existing partner country *i.* This makes the domestic market of the latter less competitive positively affecting $q_i^{(i)}(g+g_{jk})$. In considering the first and second externalities, it is concluded that the net effect of the actions of the intermediary of country *j* on the output sold by the intermediary of country *i* in the domestic market depends on whether the decrease in $q_j^{(j)}(g)$ is larger or smaller than the additional export output $q_k^{(j)}(g+g_{jk})$.

The final externality is captured by the term $-\phi(\phi+2)\left[q_j^{(i)}(g+g_{ik})-q_j^{(i)}(g)\right]$ in Equation 4.59. In this case it holds that $q_j^{(i)}(g+g_{ik})<q_j^{(i)}(g)$ because the output that is exported by the intermediary of country *i* to country *j* is negatively affected by the agreement signed by the latter with country *k* as a consequence of higher competition. This implies that the term $-\phi(\phi+2)\left[q_j^{(i)}(g+g_{ik})-q_j^{(i)}(g)\right]$ is positive meaning that this corresponds to a positive externality on $q_i^{(i)}(g+g_{jk})$ from the point of view of the intermediary in country *i*. This is explained by the fact that a decrease in the output sold by this firm to country *j* reduces the price paid to the farming sector in country *i*. As a result of this lower cost, the intermediary of the latter country increases the output that is sold in the domestic market of this country.

This simple example demonstrates how the endogeneity of the model creates externalities on the output that is traded by the intermediaries of the world. Obviously the number of externalities increases in larger networks which is why the model becomes untractable in mathematical terms when there are *N* countries.

In spite of this limitation, a generalisation can be proposed from the example given in Figure 4.2. That is, it is expected that the positive externalities on $q_i^{(i)}(g+g_{jk})$ dominate the negative externalities. While this cannot be proved without solving the model, this is inferred from the fact that the higher competition in country *j* resulting from the new agreement causes a general decrease in the output that is sold by an existing partner in this market. This lower output causes a decrease in the price paid to the farming sector by the intermediary of the partner country positively affecting the output that is traded by this firm to other third markets. This is confirmed in the simulations developed in the sections below.

The effect of the endogenous externalities on the outputs that are traded by partner countries to third markets is the basis for the identification of possible patterns from the equations representing the components of the welfare function. This is because these components depend on these outputs as can be seen in the equations obtained in the previous section (see for example Equation 4.16). In considering these externalities, some possible patterns are described as follows.

4.2.2.2 Consumer surplus

The first possible pattern that is considered in this part is associated with the effect of free trade on consumer surplus. To explore this effect, note first that according to equation 4.12, consumer surplus in an arbitrary country *i* is a monotonic transformation of the total output that is sold in the domestic market of this country (i.e. $Q_i(g)$). In considering the example presented in Figure 4.2, this implies that the externalities caused by the endogeneity of the model also affect this component of the welfare function. To see this, consider the first derivative of Equation 4.12 with respect to the term $\eta_i(g)$:

$$\frac{\partial CS_i(g)}{\partial \eta_i(g)} = \frac{\partial CS_i(g)}{\partial Q_i(g)} \frac{\partial Q_i(g)}{\partial \eta_i(g)} = Q_i(g) \frac{\partial Q_i(g)}{\partial \eta_i(g)} \qquad (4.60)$$

Consider now the total output that is traded in the domestic market of country *i* which is represented by Equation 4.11. The first derivative of this expression with respect to the term $\eta_i(g)$ is:

$$\frac{\partial Q_i(g)}{\partial \eta_i(g)} = \frac{2\alpha(1+\phi) + \phi \sum_{j \in N_i(g)} q_{j-i}(g) - \phi(1+\phi+\eta_i(g)) \frac{\partial \sum_{j \in N_i(g)} q_{j-i}(g)}{\partial \eta_i(g)}}{2(1+\phi+\eta_i(g))^2} \qquad (4.61)$$

Note that in Goyal and Joshi's world the parameter $\phi = 0$ implying that this derivative converges to:

$$\frac{\partial Q_i(g)}{\partial \eta_i(g)} = \frac{\alpha}{(1+\eta_i(g))^2} > 0 \qquad (4.62)$$

This means that in their model it always holds that $\frac{\partial CS_i(g)}{\partial \eta_i(g)} > 0$. That is, additional agreements always increase consumer surplus because these agreements increase the level of competition in country *i*. In this case the individual output that is traded by existing partners in this country decrease but the addition of new players increases the aggregate level of output.

In contrast, in the extended version of the model this is not so clear because the externalities on the outputs that are traded by the partner countries in third markets (i.e. the endogeneity characteristic of the model) affects the derivative shown in Expression 4.61. These externalities are captured by the terms $\phi \sum_{j \in N_i(g)} q_{j-i}(g)$ and $-\phi(1+\phi+\eta_i(g))\dfrac{\partial \sum_{j \in N_i(g)} q_{j-i}(g)}{\partial \eta_i(g)}$ in Equation 4.61. The first term is a positive externality on the total output and is explained by the fact that higher competition decreases the marginal cost faced by the intermediaries of the partner countries positively affecting the individual output that is sold by these countries in the domestic market of country *i*. The impact of this decrease in marginal cost depends on the existing levels of output already traded in third markets (i.e. $\sum_{j \in N_i(g)} q_{j-i}(g)$).

The second externality is expected to be negative as argued in the previous sub-section. This is because the effect of bilateral agreements on the output sold by the partner countries to third countries is positive. That is, the resulting higher competition in country *i* lowers the price paid to the farming sector by the partner countries positively affecting the output exported by these countries to third markets. But this increase mitigates the effect of the first externality on the agricultural price because the increase in the output exported to third markets increases the cost faced by the intermediaries. While it is not possible to determine which of these externalities dominate, it is conjectured that the positive influence of bilateral agreements on the total output that is traded in country *i* (see Equation 4.62) dominates a net negative externality because the first and second externalities have a tendency to cancel each other. If this is the case, then it would hold that $\frac{\partial CS_i(g)}{\partial \eta_i(g)} > 0$. An important implication of this possibility is that if this inequality holds, then the only pairwise stable network when governments are biased in favour of consumers is global free trade. The reason is because when $\frac{\partial CS_i(g)}{\partial \eta_i(g)} > 0$, this country would always be willing to sign an additional agreement, and would never be willing to break an existing one. Since this would hold for an arbitrary country *i*, the same would be valid for the rest of the countries. This prediction is explored in the simulations offered in the next sections.

A second possible pattern is related to the effect of market size on consumer surplus. In order to show this pattern, consider again the Equation 4.12. The first derivative of this expression with respect to the parameter *α* is:

$$\frac{\partial CS_i(g)}{\partial \alpha} = \frac{\partial CS_i(g)}{\partial Q_i(g)} \frac{\partial Q_i(g)}{\partial \alpha} = Q_i(g) \frac{\partial Q_i(g)}{\partial \alpha} \tag{4.63}$$

Now consider the Equation 4.11. The first derivative of this expression with respect to the parameter *α* is:

$$\frac{\partial Q_i(g)}{\partial \alpha} = \frac{2\eta_i(g) - \phi \dfrac{\partial \sum_{j \in N_i(g)} q_{j-i}(g)}{\partial \alpha}}{2(1+\phi+\eta_i(g))} \tag{4.64}$$

Note that in Goyal and Joshi, this derivative converges to $\frac{\partial Q_i(g)}{\partial \alpha} = \frac{2\eta_i(g)}{2(1+\eta_i(g))} > 0$ meaning that in the original network model it always holds that $\frac{\partial CS_i(g)}{\partial \alpha} > 0$. That is, an increase in the size of the domestic market of country *i* increases the total output that is traded in this country pushing the price paid by consumers down and, therefore, increasing the level of consumer surplus.

The same pattern is expected to be found in the extended version of the model. However, this effect is reinforced by the externality presented in the Expression 4.64. This is captured by the term $-\phi \frac{\partial \sum_{j \in N_i(g)} q_{j-i}(g)}{\partial \alpha}$ in that expression. This term is positive because an increase in the market size of country *i* positively affects the output that is traded in this market by the partner countries. As a result, the intermediaries of these countries have to pay higher prices to the farming sector negatively affecting the output that these firms trade in other third markets. This implies that the derivative in Expression 4.64 is positive and, therefore, $\frac{\partial CS_i(g)}{\partial \alpha} > 0$.

This result has important implications for the case of asymmetry in market size developed in Subsection 4.2.1.2. In considering the welfare Expressions 4.17 and

4.18, it is concluded not only that large countries obtain higher levels of consumer surplus, but also that the detrimental effect of the externality on the output traded by partner countries to other markets is more severe in large countries. By contrast, in a very small country this externality is not present. To see this, consider the following limit applied to Equation 4.11:

$$\lim_{\alpha \to 0} Q_i(g) = \frac{-\phi \lim_{\alpha \to 0} \sum_{j \in N_i(g)} q_{j-i}(g)}{2(1+\phi+\eta_i(g))} = 0 \tag{4.65}$$

In this expression, $\lim_{\alpha \to 0} \sum_{j \in N_i(g)} q_{j-i}(g)$ is zero because the optimal outputs that are traded by partner countries after solving the model depend on the coefficient *α*. Now, because $Q_i(g)$ = 0 in the limit shown in expression 4.65, consumer surplus is also zero as this is a monotonic transformation of this output. This is an obvious result because the level of consumer surplus in a country with an irrelevant domestic market tends to zero. But this means that a very small country is harmless in terms of the externality on third countries identified in the Expression 4.64. As shown in the simulation for the case of asymmetric countries in market size, this difference between large and small countries explains interesting deviations from the symmetrical case in terms of the possible stable international networks when governments are biased in favour of consumers.

Another possible pattern is associated with the effect of an increase in the general level of monopsonistic power in all countries of the world on consumer surplus.

This is explained as follows. The first derivative of Expression 4.12 with respect to the parameter ϕ is:

$$\frac{\partial CS_i(g)}{\partial \phi} = \frac{\partial CS_i(g)}{\partial Q_i(g)}\frac{\partial Q_i(g)}{\partial \phi} = Q_i(g)\frac{\partial Q_i(g)}{\partial \phi} \tag{4.66}$$

Now consider the Equation 4.11. The first derivative of this expression with respect to ϕ is:

$$\frac{\partial Q_i(g)}{\partial \phi} = \frac{-(1+\phi+\eta_i(g))\phi\frac{\partial \sum_{j\in N_i(g)} q_{j-i}(g)}{\partial \phi} - (1+\eta_i(g))\sum_{j\in N_i(g)} q_{j-i}(g) - 2\alpha\eta_i(g)}{2(1+\phi+\eta_i(g))^2} \tag{4.67}$$

According to this equation, there are three factors that affect this derivative. The first one corresponds to the term $-(1+\phi+\eta_i(g))\phi\frac{\partial \sum_{j\in N_i(g)} q_{j-i}(g)}{\partial \phi}$. It is difficult to predict the sign of this term because a generalised increase in the level of monopsonistic power increases the cost faced by all the intermediaries of the world and this negatively reduces the output that is traded in different markets. However, the decrease in the output that is traded in the domestic market of country *i* makes this country less competitive and this mitigates the decrease in output the output exported to other countries. Therefore it is not clear how this externality affects the output that is sold in country *i* when monopsonistic power increases. On the other hand, the term $-(1+\eta_i(g))\sum_{j\in N_i(g)} q_{j-i}(g)$ indicates that an increase in the output that is traded by partner countries to third countries reduces the impact of an increase of monopsonistic power on the total output sold in country *i.* This is explained by the fact that more output traded to third markets increases the marginal cost faced by famers negatively affecting the output that is sold by the partner countries in country *i*. Thus, if in addition this cost is increased exogenously (i.e. if monopsonistic power increases), then the negative impact on the output sold in country *i* is more severe. Finally, the term $-2\alpha\eta_i(g)$ indicates that the negative effect of increasing the level of monopsonistic power on the total output sold in country *i* is amplified when the size of the market in this country is larger. This happens because a larger market size is associated with more output sold in the country and this implies that intermediaries pay a higher agricultural price. Therefore when the intermediaries are already paying a higher agricultural price, the negative effect of an increase in monopsonistic power on the total output sold in country *i* is more severe. In considering these effects, it is not possible to predict with certainty the net effect of a global increase in the total output sold in country *i*. But it is likely that this effect is negative because intermediaries in general face a higher marginal cost. This would mean that it is likely that the Expression 4.67 is negative suggesting that an increase in monopsonistic power may lower consumer surplus (i.e. $\frac{\partial CS_i(g)}{\partial \phi} < 0$). This prediction is studied in the simulations developed in this chapter.

In relation to the case of asymmetric countries in terms of farmers' productivity, it is more difficult to analyse the effect of key variables on consumer surplus from the generic equations used in this case. This is due to the complexity of these expressions and the significant number of externalities involved (see Equations 4.29 and 4.32). Likewise the case of endogenous tariffs is extremely complex because the endogeneity of the model increases through the impact of marginal cost on tariffs and vice versa.

In summary, some possible patterns on consumer surplus were possible to identify from some of the equations presented in the previous section. However, it was not possible to deal with the case of asymmetry in farmers' productivity and endogenous tariffs. Neither was also possible to gain a full understanding of the possible stable networks that may arise in the extended version of the model. However, these considerations are fully explored in the simulations presented below.

4.2.2.3 Profits

The Equation 4.13 represents the profit obtained by the intermediary of country *i* in country *j*. This is a monotonic transformation of the output that is exported by this intermediary to this country (i.e. Equation 4.9). As in the case of consumer surplus, this equation has implicitly some externalities that capture the endogeneity of the model. In order to show these externalities, consider the effect of an increase in the number of agreement in country *j* on the profit made by the intermediary of country *i* on the former country:

$$\frac{\partial \pi_j^{(i)}(g)}{\partial \eta_j(g)} = \frac{\partial \pi_j^{(i)}(g)}{\partial q_j^{(i)}(g)} \frac{\partial q_j^{(i)}(g)}{\partial \eta_j(g)} = (2+\phi) q_j^{(i)}(g) \frac{\partial q_j^{(i)}(g)}{\partial \eta_j(g)} \tag{4.68}$$

Consider equation 4.9. The first derivative of this expression with respect to the term $\eta_j(g)$ is:

$$\frac{\partial q_i^{(j)}(g)}{\partial \eta_j(g)} = \frac{(1+\phi+\eta_j(g))(1+\phi)\left[\phi \frac{\partial \sum_{v \in N_j(g)-\{i\}} q_{v-j}(g)}{\partial \eta_j(g)} - \phi(1+\phi+\eta_j(g)) \frac{\partial q_{i-j}(g)}{\partial \eta_j(g)} - \phi q_{i-j}(g)\right] - (1+\phi)\left[2\alpha(1+\phi) + \phi \sum_{v \in N_j(g)-\{i\}} q_{v-j}(g) - \phi(\phi+\eta_j(g)) q_{i-j}(g)\right]}{2(1+\phi+\eta_j(g))^2 (1+\phi)^2} \tag{4.69}$$

Note in this equation that when there is no monopsonistic power (i.e. when $\phi = 0$), this expression converges to:

$$\frac{\partial q_i^{(j)}(g)}{\partial \eta_j(g)} = -\frac{\alpha}{(1+\eta_j(g))} < 0 \tag{4.70}$$

This means that in Goyal and Joshi's world, additional agreements signed by country *j* decreases the output exported in this country by the intermediary of country *i.* This is not surprising because this model only considers the effect of higher competition on this output that result from more trade. By introducing the

Expression 4.70 into 4.68 it is concluded therefore that in Goyal and Joshi always holds that $\frac{\partial \pi_j^{(i)}(g)}{\partial \eta_j(g)}$ < 0. That is, higher competition in country *j* negatively affects the profit made by the intermediary of country *i.*

In the extended model, however, this is more complex because it also considers the externalities caused by the farming sector. Unfortunately the expression 4.69 is too complex to fully understand these externalities. However, some intuition can be provided as follows. When the number of agreements increases in country *j*, the domestic market of this country becomes more competitive negatively affecting the output that is traded in this market by the competitor partner countries. This decreases the price paid to the farming sector by the intermediaries of these countries. This causes a number of effects in the network. Firstly, the resulting lower agricultural price paid by these firms mitigates the negative effect of competition on their exported output. Secondly, this lower cost also affects the level of output that is traded by the partner countries to third markets. As a result, of this increase in output the agricultural price is also impacted to some extent. This generates other externalities on the output that is traded by these countries. As can be seen from this analysis, it is difficult to assess the net effect of these externalities. However, it is expected that the increase in the number of agreements by country *j* has a net negative impact of the profit made by the intermediary of country *i*. This is because it is likely that opposite externalities have a tendency to cancel each other. It is impossible to confirm this prediction without solving the model. However, this is explored in the simulations presented below.

The relevance of the externalities caused by the farming sector not only is relevant to identify the effect of free trade on the profits made by the intermediaries, but also to understand the trade-off faced by these firms when a new agreement is signed or when an existing agreement is broken. To illustrate the role of the externalities on this trade off, consider the following example. Suppose that country *i* is only connected to country *j*. In this case the total profit made by the intermediary of this country is $\pi_i(g) = \pi_i^{(i)}(g) + \pi_j^{(i)}(g)$ where $\pi_i^{(i)}(g) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g)\right)^2$ is the profit made in the domestic market and $\pi_j^{(i)}(g) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g)\right)^2$ is the export profit made in country *j*. Now suppose that country *i* signs an agreement with country *k*. In this case the total profit made by the intermediary of country *i* is $\pi_i(g+g_{ik})$ = $\pi_i^{(i)}(g+g_{ik})$ + $\pi_j^{(i)}(g+g_{ik})$ + $\pi_k^{(i)}(g+g_{ik})$, where $g+g_{ik}$ is the network that results after the agreement between countries *i* and *k* is signed. As explained in above for Expression 4.9, it is expected that the output that is sold by an intermediary in a determined market decreases as the number of players increases in this market. If this holds, then $\pi_i^{(i)}(g)$ > $\pi_i^{(i)}(g+g_{ik})$. In other words, the intermediary of this country faces a loss in the domestic market after the agreement given by the higher degree of competition under the assumption that this inequality is not reversed by the externalities. On the other hand, the new agreement offers the intermediary of country *i* access to the domestic market of country *k*. This implies that this firm makes the new export profit $\pi_k^{(i)}(g+g_{ik})$. Thus, if the loss of profits in the domestic market is larger than the additional export profit (i.e. if $\pi_i^{(i)}(g)$ – $\pi_i^{(i)}(g+g_{ik})$ > $\pi_k^{(i)}(g+g_{ik})$), then the intermediary will not support the agreement.

This trade-off is also present in the original model by Goyal and Joshi. However, in the extended model this trade-off can directly and indirectly be affected when there is a farming sector. The direct effect is on the price that the intermediary of country *i* has to pay to farmers. That is, when this country signs the agreement, the domestic market becomes more competitive reducing in this way the price paid to the farming sector. This lower marginal cost can have a positive effect on the output that is exported to the existing partner country *j* meaning that the export profits made by the intermediary of country *i* in country *j* increases (i.e. $\pi_j^{(i)}(g + g_{ik})$ > $\pi_j^{(i)}(g)$). However, the new output that is exported to the new partner country *k* has the opposite effect meaning that the profit made in country *j* decreases. The net effect on this profit will depend on the relative number of links that countries *i* and *k* have before the agreement.

This differs from the model by Goyal and Joshi because countries are not interdependent through the influence of the farming sector on the marginal cost faced by the intermediaries. That is, in Goyal and Joshi it always holds that $\pi_j^{(i)}(g + g_{ik})$ = $\pi_j^{(i)}(g)$. On the other hand, the indirect effect arises when country *j* has already an agreement with country *k*. Thus, if the latter signs an agreement with country *i*, then the marginal cost faced by the intermediary in country *k* will be affected. Now, because this country is already connected to country *j*, this will affect the degree of competition in the latter. This in turn will affect the export profit made by the intermediary of country *i* in country *j*. This example illustrates the high degree of endogeneity that is present when there is a farming sector, and this is the key difference between the original model by Goyal and Joshi and the proposed extension.

The same complexity is present when changing other parameters such as those representing market size and the level of monopsonistic power. And the model becomes even more complex when countries are asymmetric in terms of farmers' productivity and when tariffs are placed endogenously. As a consequence of this complexity, it is not possible to fully understand the possible stable networks when governments are biased in favour of firms. This is why the simulations were necessary to identify the nature of the extended network and to make reasonable predictions concerning possible stable structures of the international trade system of processed food goods.

4.2.2.4 Producer surplus

As explained in Section 4.2.1.1, the expression for producer surplus (see Equation 4.15) is a monotonic transformation of the total output traded by the intermediary of country *i* and differs from the total profit function (i.e. Expression 4.14) in that the latter is the sum of monotonic transformations rather than a monotonic transformation of the sum. This difference has important implications for the stability of international networks because the producer surplus function has cross terms that increase significantly the complexity of the model. To see this fact, consider this simple version of Expression 4.15: $PS_i(g) = \frac{\phi}{4}\left(q_i^{(i)}(g) + q_j^{(i)}(g)\right)^2 =$

$\frac{\phi}{4}q_i^{(i)2}(g) + \frac{\phi}{2}q_i^{(i)}(g)q_j^{(i)}(g) + \frac{\phi}{4}q_j^{(i)2}(g)$. In this case the cross term $\frac{\phi}{2}q_i^{(i)}(g)q_j^{(i)}(g)$ contributes to producer surplus because it reflects the interaction that there exists between the domestic markets of countries *i* and *j* and the influence of this interaction of the price paid to the farming sector. That is, because the intermediary demands the agricultural good to the same farming sector, exporting an additional output to country *k* will increase the price paid to this sector and this will also affect the output that is traded in the domestic market. Because the cross term is positive, it is inferred therefore that this interaction has a positive net effect on the price paid to the farming sector.

While it is not possible to derive a clear pattern from the generic equations representing producer surplus under different assumptions, it can be proposed the idea that the farming sector may be favoured by global free trade as this increases the general level of agricultural prices throughout the interaction of the outputs in the involved cross terms. This prediction is explored in the simulations developed below.

4.2.2.5 A final comment

The analysis developed in this sub-section reveals that while it is possible to propose some possible patterns from the generic equations of the extended version of the model, it is not possible to fully understand the impact of key variables on the international trade system. This is a consequence of the endogeneity of the model that is caused by the presence of a farming sector that not only connects the intermediaries through the impact on marginal cost, but also originates a number of externalities that affect the output traded by these firms. As a result of this complexity, it is not possible to assess the trade impact on welfare and, therefore, the possible behaviour of policymakers.

This endogeneity is the key difference between the model by Goyal and Joshi and the extended version of this model. In the original model there are no externalities and this is why Goyal and Joshi were able to derivate general conclusions for the case of exogenous tariffs. However, they were unable to identify the possible stable networks under endogenous tariffs because the original model becomes intractable in mathematical terms. Obviously this also holds in the extended version because it is even more complex. This is why this dissertation is focussed mainly on the solutions under exogenous tariffs. However, endogenous tariffs are also considered in some simulations that comprise symmetry across countries.

In spite of the complexity of the model, the use of simulations not only confirmed some of the patterns suggested in this subsection, but also allowed the identification of stable networks and the analysis of relevant implications for the issue of agricultural trade liberalisation. The study of these simulations is the topic of the following sections.

PART II: SIMULATIONS DEVELOPED IN THE RESEARCH

As explained in the previous part, it is not possible to obtain general results from the proposed model as a consequence of its high level of endogeneity. It is for this reason that the model was solved using simulations. The objective of the second part of the current chapter is to explain and discuss the main results obtained from the simulations carried out in this dissertation. The calculations carried out in these simulations are presented in appendices A, B, C and D, and the numerical results are shown in the tables in Appendix E. Only the tables obtained from the first simulation (see Section 4.3.1.1) are also presented in this chapter with the purpose of illustrating how they have to be read.

The material covered in this part is focussed on the cases of political unbiased governments, governments biased in favour of intermediaries, and governments biased in favour of the farming sector. The reason for this choice is because it is unlikely to find countries with governments biased in favour of consumers as they are more atomised implying that it is more difficult for them to influence policymakers. Nonetheless, a detailed analysis of governments biased in favour of consumers is provided in Appendix F.

4.3 Simulations for bilateralism under symmetric countries

As explained in the literature review, the original work by Goyal and Joshi (2006) adopted the traditional pairwise stability concept to determine the stability of international trade networks when countries are involved in bilateral trade agreements. Using this concept these researchers found the following results. For the case of symmetric countries and exogenous tariffs, they found that when governments are biased in favour of consumers, the only pairwise stable network is global free trade; when governments are unbiased, the pairwise stable networks are global free trade and a network formed of a complete component and a singleton; and when governments are biased in favour of firms (i.e. intermediaries in the extended version of the model), the stable networks corresponds to global free trade, the empty network, and networks formed of one or more complete components of different size with or without singletons. For the case of symmetric countries and endogenous tariffs, Goyal and Joshi found that global free trade is pairwise stable. However, the stability of other networks was not explored by these researchers. Finally, they partially explored the issue of bilateral agreements under exogenous tariffs, asymmetry in market size, and different firms' cost structure. In this analysis, they found that smaller countries have greater incentives to form trade agreements than larger countries, and that low-cost countries have greater incentives to form trade agreements than high-cost countries. However, the stability of international networks under different policy biases was not studied by Goyal and Joshi.

In order to determine deviations from these results, a number of simulations were carried out in this section. Note that in these simulations the values for the slopes and intercepts of the various functions that have been used and that the ordering of these results are unlikely to change with alternative values. The figure summarises the set of possible networks that can be formed with countries *i*, *j*, *k* and *l*. These networks are shown in the following figure.

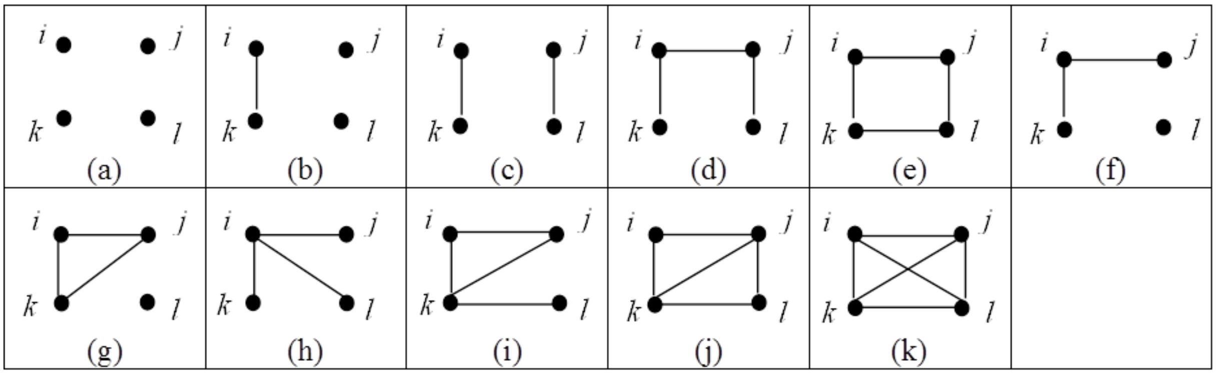


Figure 4.3. Possible network architectures formed with countries *i*, *j*, *k* and *l*.

Note in this figure that some networks are omitted. For example, country *l* in network *g* in this figure is a singleton. Similar network architectures could have been introduced in order to represent the cases when the other countries are singleton. However, information about these networks can be inferred from network *g* as a result of the assumption of symmetrical countries.

4.3.1 Bilateralism under exogenous tariffs and symmetric countries

In order to capture the effect of different levels of monopsonistic power on the international trade stability when countries are symmetric and when tariffs are established exogenously, three simulations were carried out using the networks presented in Figure 4.3. Each of these simulations corresponds to different levels of monopsonistic power associated with specific values of the parameter ϕ_i in equation 4.1: $\phi_i = 0$; $\phi_i = 0.5$; and $\phi_i = 1.5$ for all $i \in N$. In these simulations, non-monopsonistic power implies ϕ_i = 0 which corresponds, by definition, to the original model by Goyal and Joshi (2006). The other values for ϕ_i were selected in accordance to related empirical research that have found values for this parameter smaller than one (see, for example, McCorriston and MacLaren, 2013). In this context values of ϕ_i equal to 0.5 and 1.5 are considered as representing large and very large degrees of monoponistic power, respectively.

On the other hand, it was also assumed that market size (i.e. the parameter α_i in the inverse demand function in Equation 4.6) is equal to one in all countries of the world without losing generality given the assumption of symmetrical countries. The mathematical computations carried out in these simulations are presented in Appendix A. These simulations are reported below.

4.3.1.1 Simulation 1: ϕ_i = 0 and α_i = 1 for all i $\in$ N.

As explained above, this simulation converges to the original model by Goyal and Joshi (2006) because in this case there is no monopsonistic power (i.e. ϕ_i = 0). Consequently, the results obtained in this part will be used as a benchmark to evaluate deviations from the original international trade network model when there is a farming sector.

The results of the simulation in terms of consumer surplus, profits made by the intermediary, and welfare are presented in the following tables (note that producer surplus was omitted in this simulation because it is zero when ϕ_l = 0 as can be inferred from Equation 4.16).

Table 4.1. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1250	0.1250	0.1250	0.1250
b	0.2222	0.1250	0.2222	0.1250
c	0.2222	0.2222	0.2222	0.2222
d	0.2813	0.2813	0.2222	0.2222
e	0.2813	0.2813	0.2813	0.2813
f	0.2813	0.2222	0.2222	0.1250
g	0.2813	0.2813	0.2813	0.1250
h	0.3200	0.2222	0.2222	0.2222
i	0.2813	0.2813	0.3200	0.2222
j	0.2813	0.3200	0.3200	0.2813
k	0.3200	0.3200	0.3200	0.3200

Table 4.2. Profits made by the intermediary

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2500	0.2500	0.2500	0.2500
b	0.2222	0.2500	0.2222	0.2500
c	0.2222	0.2222	0.2222	0.2222
d	0.2361	0.2361	0.1736	0.1736
e	0.1875	0.1875	0.1875	0.1875
f	0.2847	0.1736	0.1736	0.2500
g	0.1875	0.1875	0.1875	0.2500
h	0.3733	0.1511	0.1511	0.1511
i	0.1650	0.1650	0.2761	0.1511
j	0.1425	0.2050	0.2050	0.1425
k	0.1600	0.1600	0.1600	0.1600

Table 4.3. Welfare.

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3750	0.3750	0.3750	0.3750
b	0.4444	0.3750	0.4444	0.3750
c	0.4444	0.4444	0.4444	0.4444
d	0.5174	0.5174	0.3958	0.3958
e	0.4688	0.4688	0.4688	0.4688
f	0.5660	0.3958	0.3958	0.3750
g	0.4688	0.4688	0.4688	0.3750
h	0.6933	0.3733	0.3733	0.3733
i	0.4463	0.4463	0.5961	0.3733
j	0.4238	0.5250	0.5250	0.4238
k	0.4800	0.4800	0.4800	0.4800

The case of politically unbiased governments

Let us start the analysis by considering the case of unbiased governments (i.e. governments care about maximising an unweighted welfare function). According to the information presented in Table 4.3, there are networks that are preferred for some countries but they are not pairwise stable and, therefore, cannot be reached permanently. For example, network *j* is the preferred network for country *j* because the highest level of welfare in this country (i.e. 0.5250) is reached in this network. The reason is because country *j* has already agreements with all countries of the world (see Figure 4.3) implying that the domestic market of this country has a high degree of competition as a consequence of these agreements. As a result, consumers in this country obtain high levels of consumer surplus.

At the same time, the domestic firm of country *j* makes high profits in countries *i* and *l* because the degree of competition in these countries is lower as they are not fully integrated. However, this network is not pairwise stable because these countries have an incentive to sign a bilateral agreement (i.e. passing from network *j* to *k* in Figure 4.3) as this increases the level of welfare in each country from 0.4238 to 0.4800. Because this agreement only depends on the decisions made by the governments of countries *i* and *l*, country *j* cannot remains in its preferred network.

In considering these unstable preferred networks and using the information presented in Table 4.3, it is inferred that network *k* is link deletion proof because no country in this network has an incentive to break an existing link. This can be seen from the fact that when countries *i* and *l* break their agreement, welfare in each country decreases from 0.4800 to 0.4238 (i.e. when passing from network *k* to network *j*). Following this reasoning, it is inferred that the set of link deletion proof networks is *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *j*, *k, Eq*} where *Eq* denotes the networks that are equivalent to the networks included in this set[6].

On the other hand, Network *a* is not link addition proof because countries *i* and *k* have an incentive to sign an agreement (i.e. passing from network *a* to network *b*) because this agreement increases welfare in both countries from 0.3750 to 0.4444. Following this reasoning, it is concluded that the set of link addition proof networks is *A* = {*g*, *k*, *Eq*}.

[6] For example, the network in which countries *i* and *j* have a link and countries *k* and *l* are singletons is equivalent to network *b* in Figure 4.3 given the assumption of symmetrical countries. The former was not included in this figure because its stability is inferred from the analysis of network *b*. However, it has to be included in the set *D* because this set includes all networks in which countries are unwilling to break an existing link.

Now, remember that pairwise stability requires that: (1) no country has an incentive to break an existing agreement (i.e. the link deletion proof condition); and (2) a new agreement is not feasible because at least one country is unwilling to sign this agreement (i.e. the link addition proof condition). This implies that the set of pairwise stable networks is found in the intersection of the sets of link deletion and link addition proof networks. In the case of unbiased governments this is given by: $P = D \cap A = \{g, k, Eq\}$. This result is not surprising because the international network model proposed in this dissertation converges to the original model developed by Goyal and Joshi (2006) when $\phi_i = 0$, and the result has already been reported by these researchers in Proposition 1 of their article (see Goyal and Joshi 2006, 755).

The case of politically biased governments

Let us now determine the stable networks for the case of biased governments in favour of consumers. In considering Table 4.1 it is concluded that $D = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $A = \{k\}$. This means that the set of pairwise stable networks in this case is $P = D \cap A = \{k\}$. That is, global free trade. Likewise, in considering Table 4.2 it is inferred that $D = \{a, e, g, k, Eq\}$ and $A = \{a, b, g, i, k, Eq\}$. This implies that the set of pairwise stable networks when governments are biased in favour of the intermediary is given by $P = D \cap A = \{a, g, k, Eq\}$. As in the unbiased case, these results are not surprising because they have already been reported by Goyal and Joshi (2006, 760-761) in Propositions 4 and 5 of their work.

The intuition behind these results is discussed as follows.

Discussion

In relation to the intermediaries, these firms always face a trade off when an agreement is either signed or broken. That is, when a country signs an agreement, they lose profits in the domestic market as a consequence of the higher level of competition and, at the same time, they make additional profits in the new partner country after the agreement (the opposite happens when an agreement is broken). Consequently, the intermediaries are better off only when the loss of domestic profit is smaller than the gain in profits in new partner countries. But this depends on the existing number of agreements in both partner countries. In other words, this depends on both the current architecture of the network and the relative position of countries in the network.

This is why there are multiple equilibriums when governments are biased in favour of the intermediaries. Depending on the current architecture of the network, some intermediaries may be better off or worse off after an agreement. For example, the empty network is stable because there is a net loss of profits when two singletons sign an agreement. Likewise, network *g* is stable because breaking a link causes a net loss of profits in countries *i*, *j* and *k* (i.e. the gain in profits in the domestic market in these countries is smaller than the loss of profits in foreign countries after a link is broken). These countries would be willing to sign an agreement with country *l* because the additional profit that they can make in that country is larger than the loss of domestic profits caused for the higher degree of competition. However, the opposite happens in country *l*. This country is in autarky in network *g* implying the intermediary in this country makes high profits in the domestic market because this firm is a monopolist. Consequently, signing an agreement with either *i*, *j* or *k* would cause a significant loss of profit in the domestic market that is not compensated by the additional profit in the new foreign country. Finally, global free trade is stable because the gain in profits in the domestic market is not compensated by the loss of profit in the ex-partner foreign country after an agreement is broken.

In the case of unbiased governments, the set of pairwise stable networks is smaller than the set that is observed when governments are biased in favour of domestic firms. The reason is because consumers positively influence the formation of bilateral agreements as this increases consumer surplus (see Appendix F). For example, the empty network is only stable when governments are biased in favour of intermediaries because the loss of domestic profit is not compensated by the gain in profits in the foreign country when an agreement is signed. However, when consumer surplus is included in the welfare function, this net loss of profit is compensated by the gain in consumer surplus caused by the higher degree of competition after the agreement. This affects the link addition proof condition of the empty network that was valid for the case of biased governments in favour of the intermediaries. In the unbiased case, this condition does not hold because the countries in the empty networks have an incentive to form a bilateral agreement.

In summary, the main conclusions obtained from this simulation are: (i) when governments are biased in favour of consumers, only global free trade is pairwise stable; (ii) the number of pairwise stable networks increases as more weight is given to the intermediaries in the welfare function; (iii) global free trade is always stable independently of the weights given to the components of the welfare function[7]; and (iv) the empty network is pairwise stable when governments are biased in favour of intermediaries.

4.3.1.2 Simulation 2: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation includes the farming sector into the network approach which is captured by the monopsonistic power exercised by the intermediaries. That is, in this case it is assumed that the price paid by the intermediaries to the farming sector increases as the quantity supplied by the rural sector increases (see Equation 4.1). However, this increase is moderate because it is assumed in this simulation that the degree of market power is not very high (i.e. $\phi_i = 0.5$). The relevant information obtained from the simulations are summarised in Tables E.4, E.5, E.6 and E.7 in Appendix E.

[7] This is a consequence of the stability of global free trade in the extreme cases when governments are biased in favour of consumers and intermediaries. In the first case, it holds that $CS_i(g^c) \geq CS_i(g^c - g_{ik})$ for all $i,k \in N$, and in the second case it holds that $\pi_i(g^c) \geq \pi_i(g^c - g_{ik})$ for all $i,k \in N$. Let multiply the first inequality by a non-negative arbitrary constant a, and the second by an non-negative arbitrary constant b. By adding the resulting inequalities, it is obtained $aCS_i(g^c) + b\pi_i(g^c) \geq aCS_i(g^c - g_{ik}) + b\pi_i(g^c - g_{ik})$ for all $i,k \in N$. But this means that global free trade is pairwise stable for any non-negative weight (a or b) put on the components of the welfare function.

<u>The case of politically unbiased governments</u>

The sets of link deletion proof and link addition proof networks that are needed to identify the pairwise stable networks for the case of unbiased countries can be inferred from the information given in Table E.7. In considering this information, it was found that these sets correspond to *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *i*, *j*, *k*, *Eq*} and *A* = {*k*}. Consequently, the set of pairwise stable networks in this case is given by *P* = *D* ∩ *A* = {*k*}. This is an interesting result because this means that when governments are political unbiased, the inclusion of the farming sector has a positive effect on free trade. This is inferred from the fact that the stable networks in Goyal and Joshi's model are networks *g* (and equivalents) and *k*. But the stability of the former is broken when there is a farming sector and the only stable network in this case is *k*. That is, global free trade.

<u>The case of politically biased governments</u>

In this simulation there are three types of biases: bias in favour of consumers (see Appendix F); firms; and the farming sector. The information that is needed to identify the pairwise stable networks under these biases is presented in Tables E.4, E.5 and E.6 in Appendix E.

When governments are biased in favour of the intermediaries, the sets of link deletion and link addition proof networks are given by *D* = {*a*} and *A* = { *a*, *b*, *c*, *d*, *e*, *f*, *g*, *i*, *j*, *k*, *Eq*}, respectively, a result that was inferred from the information presented in Table E.5. This implies that the set of pairwise stable networks in this case is $P = D \cap A = \{a\}$. This is different from the results by Goyal and Joshi. As shown in subsection 4.3.1.1, the pairwise stable networks in their model are *a*, *g* (and equivalents) and *k*. This difference has two main implications. Firstly, because network *a* is the empty network, it is inferred that the addition of the farming sector plays against free trade when governments are biased in favour of intermediaries. Secondly, the claim made by Goyal and Joshi that global free trade (i.e. network *k*) is always pairwise stable independent of the governments' political biases does not hold when the farming sector is included because in this case the stability of network *k* is broken.

On the other hand, in considering Table E.6, it was inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the farming sector are given by $D = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $A = \{k\}$. Consequently, the set of pairwise stable networks in this case is given by $P = D \cap A = \{k\}$. This result suggests that the inclusion of the farming sector into the network approach has a positive effect on the formation of bilateral agreements when governments are biased in favour of farmers as the only stable network in this case is global free trade.

<u>Discussion</u>

The results obtained in this simulation reveal that the international trade structure is strongly influenced by the presence of a farming sector in each country. The effect of this sector on the stability of international trade networks is explained by the monopsonistic power exercised the intermediaries who, in contrast to the approach by Goyal and Joshi, have to pay a price for the agricultural good that increases in proportion to the output sold by the farmers (see Equation 4.1). The effect of this non-fixed marginal cost faced by the intermediaries on trade stability is explained as follows.

Let us consider first the case of governments biased in favour of intermediaries. In comparing the results obtained in this simulation with the previous one, it is concluded that governments are more willing to break existing agreements when there is a farming sector and less willing to sign new ones (i.e. the number of networks in the set of link deletion proof decreases and the number of networks in the set of link addition proof networks increases when introducing the farming sector). The reason is because more trade increases the total output that is sold by the intermediaries in the domestic and foreign markets. Since the price paid to farmers increases in proportion to this output (i.e. increasing marginal cost), the total cost faced by these firms in more integrated networks is significantly higher and negatively affects the profit obtained by them in these networks. As a consequence, if governments are biased in favour of intermediaries, they will break links in order to help them to obtain higher profits in less integrated networks until network *a* is reached. That is, until the empty network is reached which is the network where the intermediaries make the highest level of profits as they pay lowest price to farmers. In contrast, when there is no monopsonistic power (i.e. in Goyal and Joshi's world), the intermediaries face a fixed-marginal cost implying that these firms sell more output in more integrated networks with the same marginal cost making, therefore, higher profits. As a result, biased governments in favour of intermediaries are less inclined to break links which explains why the number of stable networks in the previous simulation is larger.

On the other hand, when governments are biased in favour of the farming sector, only free trade is the pairwise stable network. This is also explained by the fact that the intermediaries face a non-fixed marginal cost that increases as the output purchased by these firms increases. This non-fixed marginal cost implies that farmers are paid higher prices when they sell higher output volumes positively affecting producer surplus. Thus, because free trade increases the output that is purchased by the intermediaries, the farmer sector will support additional agreements. The main implication of this finding is that the farming sector exercises a positive effect on free trade when governments are symmetric and biased in favour of this sector.

In considering these extreme cases, it is clear that the pairwise stability of global free trade depends on whether the governments put more weight on the intermediaries in the welfare function. For example, the results revealed that when governments are unbiased, only global free trade is pairwise stable. The reason is because the positive influence that consumers and the farming sector exercise on free trade offsets the negative influence exercised by the intermediary (see Appendix F for the case of governments biased in favour of consumers). It is inferred, therefore, that deviations from global free trade can only be observed in a world where governments put more weight on intermediaries under the assumption that the latter have monopsonistic power, a fact that does not hold in Goyal and Joshi's world. This confirms that the existence of a farming sector can strongly affect the stability of the international trade system and global free trade in particular.

4.3.1.3 Simulation 3: ϕ_i = 1.5 and α_i = 1 for all $i \in N$.

This is the last simulation of this section and was introduced to explore the effects of very high levels of monopsonistic power on the international trade system. The information that is needed for this analysis is presented in Tables E.8, E.9, E.10 and E.11 is Appendix E.

In considering these tables, it was found that the relevant sets of networks are the same as the ones identified for the case of moderate degree of monopsonistic power (i.e. when ϕ_i = 0.5). That is, when governments are unbiased, the sets of link deletion proof, link addition proof and pairwise stable networks corresponds to D = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *i*, *j*, *k*, *Eq*}, A = {*k*} and $P = D \cap A$ = {*k*}, respectively. When governments are biased in favour of consumers, these sets correspond to D = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *Eq*}, A = {*k*} and $P = D \cap A$ = {*k*} (see Appendix F). When governments are biased in favour of the intermediaries, these sets are given by D = {*a*}, A = { *a*, *b*, *c*, *d*, *e*, *f*, *g*, *i*, *j*, *k*, *Eq*}, and $P = D \cap A$ = {*a*}. Finally, when governments are biased in favour of the farming sector, these sets correspond to D = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *Eq*}, A = {*k*} and $P = D \cap A$ = {*k*}.

This result reinforces the conclusion that monopsonistic power can strongly affect the international trade system when there is a farming sector. The reason, as in the previous simulation, is due to the fact that intermediaries face higher marginal cost in more integrated networks because the higher output volume sold in these markets pushes the price paid to the farming sector up negatively affecting the profit made by the former.

4.3.2 Bilateralism under endogenous tariffs and symmetric countries

One of the most challenging aspects of the proposed international trade network model is the analysis of trade stability when tariffs are established endogenously. This is because endogenous tariffs into the model generate high degree of endogeneity across countries making it very difficult to solve the model even when having only four countries in the world. This endogeneity arises from the fact that tariffs affect the output in equilibrium in each market of the world and, therefore, the cost incurred by intermediaries when buying the output to the farming sector given the monopsonistic power exercised by the former. This cost, in turn, affects the optimal tariff in each country, and this, in turn, the output in equilibrium in each market, and so on. Under exogenous tariffs, the endogeneity problem of tariffs is not present and this is why it easier to analyse considerations concerning asymmetry and policy biases. For example, policy biases under exogenous tariffs can be explored by multiplying weights on the simulated values of consumer surplus, profits and producer surplus in the weighted welfare function. However, this cannot be done when tariffs are endogenous because the simulated values of these variables depend on specific tariffs that are different for each bias assumed in the weighted welfare function.

In recognising this problem, the current investigation is focused mainly on the simplest case of homogeneous and unbiased countries under endogenous tariffs given the complexity of the analysis (this is the same strategy adopted by Goyal and Joshi (2006) to study the issue of international trade networks under endogenous tariffs). However, partial extensions are explored to analyse the effect of policy biases on the pairwise stability of global free trade. The mathematical calculations carried out for the simulations under the assumption of endogenous tariffs are presented in Appendix B.

4.3.2.1 Simulation 4: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

Similar to the case of exogenous tariffs, this simulation converges to the original model by Goyal and Joshi (2006) because there is no monopsonistic power (i.e. $\phi_i = 0$). In this case, these researchers only studied the pairwise stability of global free trade but they did not investigate the stability of other networks given the complexity of the analysis. This is formally stated by Goyal and Joshi (2006) who point out: "*Given the complexity of the computations involved, we have been unable to completely characterize the nature of stable networks in this setting. We do have some interesting partial results*" (p.768).

The simulation developed with four countries was very useful to extend the original work by these researchers because it was possible to assess the pairwise stability of networks other than free trade when governments are unbiased. The results obtained from this exercise were used to assess the influence of monopsonistic power on the trade system when tariffs are placed endogenously. The information needed for this analysis is summarised in Tables E.12, E.13, E.14 and E.15 in Appendix E.

According to the information presented in Table E.15, the set of link deletion proof and link addition proof networks when governments are unbiased correspond to $D = \{a, b, c, e, g, j, k, Eq\}$ and $A = \{g, k\}$, respectively. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A = \{g, k, Eq\}$. This result is the same as the one obtained for the case of unbiased countries, exogenous tariffs, and $\phi = 0$ suggesting that this finding remains robust through the assumptions of endogenous and exogenous tariffs.

In order to understand this result, an analysis of the components of the welfare function based on the information presented in Tables E.12, E.13 and E.14 is carried out as follows. Before showing this analysis, however, it is important to highlight the fact that this exercise does not have to be interpreted as the incentives of different groups of firms when governments are biased in favour of them. The reason is because the information presented in these tables was obtained from tariffs that maximise an un-weighted welfare function. In relation to this point, the relevant information that is needed to analyse the incentives of interest groups when governments are biased has to be obtained from optimal tariffs that maximise a weighted welfare function. This extension is explored in Section 4.3.2.4.

According to the information presented in Table E.12, consumer surplus always increases when new agreements are signed, and always decreases when existing ones are broken. This is explained, as in the case of exogenous tariffs, by the fact that more trade increases competition in domestic markets (i.e. reduces the market power exercised by intermediaries) positively affecting consumer surplus. On the other hand, the information in Table E.13 reveals that in networks *a*, *e*, *g*, *j*, *k* and equivalents, the total profit made by at least one intermediary in the world decreases when its country breaks an existing agreement. Likewise, the profit made by at least one intermediary in the world in networks *a*, *b*, *g*, *k* and equivalents decreases when its country signs an additional agreement. This implies that, as in the case of exogenous tariffs, these firms would not support signing a new or breaking and existing agreement in networks *a*, *g*, *k* or equivalents. Again, as in the case of exogenous tariffs, this is a reflection of the trade-off faced by the intermediaries: new agreements increases the profit made by these firms in foreign countries but reduces the profit made in the domestic market as a consequence of higher competition, and the opposite happens when an agreement is broken. Thus, these firms would support signing or breaking an agreement depending on whether this implies a net gain in profits. Finally, it is inferred from the information presented in Table E.14 that tariff revenue decreases when governments sign an agreement and its maximum value in a country is reached when this country does not have any international agreement. The reason is because according to the information presented in Appendix B, it is in this condition where countries place the highest tariff to imported processed food.

In comparing these figures with the pairwise stable networks described above, it is concluded that the pairwise stability of networks *g* (and equivalents) and *k* are explained by the profits made by the intermediaries, and the pairwise stability of the latter network is reinforced by the high level of consumer surplus that is obtained in global free trade. This positive influence of consumer surplus on trade is large enough to offset the negative influence of tariff revenue.

4.3.2.2 Simulation 5: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

Let us now explore the pairwise stability of international trade networks when there is a farming sector in each country of the world. For this purpose, consider the information presented in Tables E.16, E.17, E.18, E.19 and E.20 in Appendix E.

In considering the information presented in Table E.20, it is inferred that the set of link deletion and link deletion proof networks correspond to $D = \{a, b, c, d, e, f, g, j, k, Eq\}$ and $A = \{g, k, Eq\}$, respectively. This implies that the set of parwise stable networks in this case is given by $P = D \cap A = \{g, k, Eq\}$. This result is interesting because, in contrast to the case of exogenous tariffs, it reveals that the farming sector does not affect the pairwise stability of network *g* (and equivalents) when $\phi =$ 0.5. To see this, note that the set of pairwise stable networks identified in the current simulation is exactly the same as the one when $\phi = 0$ (see Section 4.3.2.1). This differs from the case of exogenous tariffs because, as discussed in 4.3.1, the stability of network *g* is broken when there is a farming sector. It is explained in that section that more trade implies higher levels of exported output and this pushes the price paid to farmers up reducing the profit made by the intermediaries. Consequently, the level of profits made by the latter increases when the country breaks and existing agreement, and this gain in profit is larger than the loss of consumer surplus which is why the pairwise stability of network *g* is broken. In the case of endogenous tariffs, in contrast, this stability is not broken because firms do not lose profits when an agreement is broken.

The reason is due to the fact that breaking an agreement adds an additional cost to intermediaries which corresponds to the higher tariffs paid to non-partner countries as can be inferred in Equations 4.50 and 4.53. Thus, while the deviation from network *g* helps these firms to reduce total costs by paying lower prices to the farming sector, they have to incur in a higher cost which corresponds to higher tariff levels in non-partner countries. This trade off in cost is clearly identified in these equations which show the profit made by the intermediary in country *k* in the domestic market of a non-partner country *i*. This profit implicitly includes a total cost function given by $TC(g) = T_i(g)q_i^{(k)}(g) + \frac{1}{2}\phi q_i^{(k)}(g)q_k(g)$. Using Expression 4.1, this total cost function becomes $TC(g) = T_i(g)q_i^{(k)}(g) + p_k^f(g)q_k(g)$.

As can be seen in this expression, the cost function has two components: one related to the tariff faced in country *i* (i.e. $T_i(g)q_i^{(k)}(g)$), and the other one related to the price paid to the farming sector (i.e. $p_k^f(g)q_k(g)$). Thus, when a country in network *g* (and equivalents) breaks an existing agreement, the first component increases, but the second one decreases. The increase in the first component is large enough to prevent the pairwise stability of this network to be broken. In contrast, when tariffs are placed exogenously, only the second component is present in the cost function implying that when a country breaks an agreement, total cost decreases unambiguously positively affecting the profit made by the intermediary and, consequently, breaking the deletion proof property of the pairwise stability in this network.

The information presented in Table E.18 reveals that producer surplus always decreases when an existing agreement is broken. This implies that the farming sector will not support the decision of deleting a trade link with another country when governments are unbiased. This is consistent with the finding obtained for the case of exogenous tariffs. However, there is a difference in relation to decisions concerning the signature of new agreements. It was shown in Section 4.3.1 that when tariffs are placed exogenously, the farming sector will always support new agreements. However, when tariffs are endogenous, this not always holds. In particular, Table E.18 shows that producer surplus in country *l* decreases when this country signs a new agreement. This implies that the farming sector in this country will not support such an agreement. The reason for this result is explained by the fact that the agreement causes a significant decrease in the output sold by the intermediary of country *l* in the domestic market which is not compensated by the increase in output sold in foreign countries after the agreement. This is inferred from the information presented in Appendix B: the increase in output exported from country *l* after the signature of the agreement between country *l* and *k* (i.e. after passing from network *g* to network *i*) is equal to 0.0561, and a decrease in the output sold in the domestic market by the intermediary of country *l* is equal to 0.0626. This implies that there is a net decrease in the output purchased to the farming sector of this country equal to 0.0626 – 0.0561 = 0.0065. This net decrease in output pushes the price paid to farmers down negatively affecting producer surplus.

This result only happens when tariffs are endogenous because before the agreement country *l* is already exporting output to foreign countries having high levels of competition. Thus the increase in output that is exported after the agreement is not as large as the decrease in the output sold in the domestic market of this country. In contrast, when tariffs are exogenous, country *l* does not export any level of output to foreign countries before the agreement. Thus, when the agreement is signed, the increase in exported output is large enough to offsets the decrease in output sold in the domestic market. The main implication of this finding is that the parwise stability of network *g* and equivalents when tariffs are endogenous and when governments are unbiased is reinforced when there is a farming sector. In other works, the existence of inefficient stable pairwise stable networks is reinforced when there is a farming sector.

In relation to the other components of the welfare function, the information presented in Tables E.16, E.17 and E.19 reveals the same patterns than the ones identified in the previous simulation, namely: (i) consumer surplus always increases when new agreements are signed, and never decreases when existing ones are broken; (ii) the intermediaries would not support signing a new or breaking and existing agreement in networks *a*, *g*, *k* or equivalents; and (iii) tariff revenue decreases when governments sign an agreement and its maximum value in a country is reached when this country does not have any international agreement.

In considering the results obtained in this simulation, it is concluded that the effect of the farming sector on trade liberalisation is not strong enough to break the inefficient pairwise stable networks identified in the case when there is no monopsonistic power, when governments are unbiased, and when tariffs are determined endogenously.

4.3.2.3 Simulation 6: $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation was introduced to determine the effect of the farming sector on the trade system when intermediaries exercise very high levels of monopsonistic power. The information employed in this simulation is presented in Tables E.21, E.22, E.23, E.24 and E.25 in Appendix E.

The information presented in Table E.25 reveals that the sets of link deletion and link addition proof networks are D = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *j*, *k, Eq*} and A = {*g*, *k, Eq*}. This implies that the set of pairwise stable networks in this case is $P = D \cap A$ = {*g*, *k, Eq*}. This is the same result as the ones obtained for the cases when $\phi = 0$ and $\phi = 0.5$. It is concluded, therefore, that the influence of the farming sector is not strong enough to affect the pairwise stability of networks when governments are unbiased and when tariffs are placed endogenously for levels of monopsonistic power equal or smaller than 1.5.

In spite of this result, it can be inferred from Tables E.22 and E.23 that high levels of monosonistic power influence the incentives of intermediaries and the farming sector in some networks. For example, it was shown in the simulations for the cases of $\phi = 0$ and $\phi = 0.5$ that in networks *a* and *b*, not all intermediaries support the signature of new agreements. However, this does not hold when $\phi = 1.5$ because in this case all the intermediaries support an additional agreement suggesting that the farming sector exercise a positive influence towards free trade on intermediaries for very high levels of monopsonistic power. This result is explained by a combination of three interrelated effects in which some of them operate in opposite direction. They correspond to the market power effect (i.e. intermediaries lose profits in the domestic market in more integrated networks as a consequence of higher competition but at the same time make additional profit in foreign markets); the monopsonistic effect (i.e. the increase in the quantity of output exported to other countries in more integrated networks pushes the price paid to farmers up negatively affecting the profit made by the intermediaries); and the tariff effect (more trade reduces the level of tariffs applied by non-partner countries positively affecting the profit made in these countries).

In order to illustrate how the relevance of these three effects changes under different levels of monoponistic power, let us consider for illustrative purposes networks *a* and *b.* As explained above, when ϕ = 0 and ϕ = 0.5, not all intermediaries support the signature of new agreements in network *a*. However, this does not hold when ϕ = 1.5 because in this case all the intermediaries support an additional agreement. To understand this result, consider Figure 4.4. This figure is based on the information presented in Appendix B and shows the changes in output, profits and tariffs when countries *i* and *k* signs an agreement (i.e. when passing from network *a* to *b*) for different values of the parameter ϕ. The symbols ↑ and ↓ denote an increase and a decrease in a determined variable, respectively, and a bar over a variable means that this variable does not change after the agreement.

	Country *i* **ϕ = 0**	**Country *j*** **ϕ = 0**	**Country *k*** **ϕ = 0**	**Country *l*** **ϕ = 0**
Output **Profit** **Tariff**	$\downarrow q_i^{(i)}$ $\overline{q_j^{(i)}}$ $\uparrow q_k^{(i)}$ $\overline{q_l^{(i)}}$ $\downarrow \pi_i$ $\downarrow T_i$	$\uparrow q_i^{(j)}$ $\overline{q_j^{(j)}}$ $\uparrow q_k^{(j)}$ $\overline{q_l^{(j)}}$ $\uparrow \pi_j$ $\overline{T_j}$	$\uparrow q_i^{(k)}$ $\overline{q_j^{(k)}}$ $\downarrow q_k^{(k)}$ $\overline{q_l^{(k)}}$ $\downarrow \pi_k$ $\downarrow T_k$	$\uparrow q_i^{(l)}$ $\overline{q_j^{(l)}}$ $\uparrow q_k^{(l)}$ $\overline{q_l^{(l)}}$ $\uparrow \pi_l$ $\overline{T_l}$
	Country *i* **ϕ = 0.5**	**Country *j*** **ϕ = 0.5**	**Country *k*** **ϕ = 0.5**	**Country *l*** **ϕ = 0.5**
Output **Profit** **Tariff**	$\downarrow q_i^{(i)}$ $\uparrow q_j^{(i)}$ $\uparrow q_k^{(i)}$ $\uparrow q_l^{(i)}$ $\downarrow \pi_i$ $\downarrow T_i$	$\uparrow q_i^{(j)}$ $\downarrow q_j^{(j)}$ $\uparrow q_k^{(j)}$ $\uparrow q_l^{(j)}$ $\uparrow \pi_j$ $\downarrow T_j$	$\uparrow q_i^{(k)}$ $\uparrow q_j^{(k)}$ $\downarrow q_k^{(k)}$ $\uparrow q_l^{(k)}$ $\downarrow \pi_k$ $\downarrow T_k$	$\uparrow q_i^{(l)}$ $\uparrow q_j^{(l)}$ $\uparrow q_k^{(l)}$ $\downarrow q_l^{(l)}$ $\uparrow \pi_l$ $\downarrow T_l$
	Country *i* **ϕ = 1.5**	**Country *j*** **ϕ = 1.5**	**Country *k*** **ϕ = 1.5**	**Country *l*** **ϕ = 1.5**
Output **Profit** **Tariff**	$\downarrow q_i^{(i)}$ $\downarrow q_j^{(i)}$ $\uparrow q_k^{(i)}$ $\downarrow q_l^{(i)}$ $\uparrow \pi_i$ $\downarrow T_i$	$\uparrow q_i^{(j)}$ $\uparrow q_j^{(j)}$ $\uparrow q_k^{(j)}$ $\downarrow q_l^{(j)}$ $\uparrow \pi_j$ $\uparrow T_j$	$\uparrow q_i^{(k)}$ $\downarrow q_j^{(k)}$ $\downarrow q_k^{(k)}$ $\downarrow q_l^{(k)}$ $\uparrow \pi_k$ $\downarrow T_k$	$\uparrow q_i^{(l)}$ $\downarrow q_j^{(l)}$ $\uparrow q_k^{(l)}$ $\uparrow q_l^{(l)}$ $\uparrow \pi_k$ $\uparrow T_l$

Figure 4.4. Effects of an agreement signed by countries *i* and *k* when the initial network is *a.*

According to this figure, when there is no monopsonistic power (i.e. ϕ = 0), the agreement between countries *i* and *k* causes a decrease in the output sold in the domestic market of these countries, and an increase in the output exported to the new partner countries. However, the output exported to non-partner counties (i.e. countries *j* and *l*) remains the same. The net change in output is, however, negative and this explains why the agreement causes a net loss of profits in each of the new partner counties. On the other hand, this agreement generates a positive externality in countries *j* and *l*. That is, the tariffs applied by countries *i* and *k* decrease after the agreement positively affecting the quantity of the output exported by countries *j* and *l* to the former. As a result, the intermediaries in countries *j* and *l* make higher profits after the agreement, even when the profits that these intermediaries make in the domestic market do not change (this is because the tariffs in these countries are not affected by the new agreement). In terms of the incentives of the intermediaries of countries *i* and *k*, they will not support the agreement because this would cause a net loss of profits for them.

Let us now consider the case when $\phi = 0.5$. According to Figure 4.4, in this case all the variables included in the analysis are affected by the new agreement proving the high degree of endogeneity of the model when there is monopsonistic power in a world with endogenous tariffs. In order to understand the implications of this endogeneity, the effects of the agreement on countries *i* and *k* is analysed first.

Figure 4.4 shows that when these two countries sign an agreement, the output that is sold by the intermediaries of these countries in the domestic market decreases as a consequence of the higher competition. This negatively affects the profit obtained by these firms from these markets. The new agreement causes an increase in the output exported by the intermediaries of countries *i* and *k* to the new partner countries. In addition, the output exported to non-partner countries (i.e. *j* and *l*) also increases as a consequence of the tariff reduction in these countries after the signature of the new agreement (note that this decrease in tariffs in countries *j* and *l* is an externality effect cause by the agreement between countries *I* and *k*). This additional output implies that the intermediaries of countries *i* and *k* make additional profits in foreign countries (both the new partners and the existing non-partner countries) after the agreement because, as shown in Equations 4.52 and 4.53, the profit that an intermediary makes in a determined market is a monotonic transformation of the output sold by these firms in that market.

According to the information presented in Appendix B, the decrease in the quantity of output sold by these firms in the domestic market is compensated by the increase in the quantity of exported output. This implies that the intermediaries pay higher prices to the farming sector after the agreement because this price is proportional to the total output sold by these firms. This additional cost and the loss of market power in the domestic market is reflected as a net loss of profit faced by the intermediaries in countries *i* and *k*.

In relation to the non-partner countries *j* and *l*, on the other hand, the agreement is beneficial for them because the resulting tariff reduction in countries *i* and *k* allow the former to export more output to the latter countries. This higher output, however, pushes the price paid to the farming sector in countries *j* and *l* up negatively affecting the profit made by the intermediaries in these countries. In order to decrease the pressure on this price, these countries reduce the optimal tariff in order to increase the level of competition in their domestic markets. This higher level of competition negatively affects the profit made by the intermediaries in countries *j* and *l* in the domestic market, but increases the profit that they make in the rest of the countries as a consequence of the decrease in the pressure of the price paid to the farming sector. As a consequence, these intermediaries obtain higher net profits after the agreement between countries *i* and *k*. Finally, because the intermediaries of the latter countries lose net profits after the agreement, they are unwilling to support this agreement implying that the incentives of these firms are the same as the ones in the case when $\phi = 0$.

The final case presented in Figure 4.4 corresponds to the one in which intermediaries exercise very high levels of monopsonistic power (i.e. $\phi = 1.5$). This case in interesting because it is here where it is possible to see how the existence of a farming sector may reverse the incentives of intermediaries to support of reject a new agreement. As shown in this figure, the same direct effects between countries *i* and *k* are verified when these two countries sign an agreement: the output sold in the domestic market decreases as a consequence of the higher level of competition after the agreement; and the output exported to the new partner countries increases as a consequence of the tariff reduction resulting from the agreement.

What is different from the previous cases is that when monopsonistic power is very high, the agreement causes a net gain in the profit made by the intermediaries of the new partner countries *i* and *j* implying that under this degree of monopsonistic power, these firms support the new agreement. This result is explained by the externality caused by the agreement on countries *j* and *l*. These countries increase the optimal tariff in response to the agreement. As a result, the output that is sold by the intermediaries of these countries in the domestic market increases as a consequence of the lower degree of competition, and the output exported to countries *i* and *k* also increases as a consequence of the lower tariffs in these countries. This higher output pushes the price paid to the farming sector up negatively affecting the profit obtained by the intermediaries in country *j* and *l*. However, this higher price is cushioned by the lower output that is exported to the non-new partner countries (because the higher tariff in these countries). On the other hand, the decrease in the output exported by the intermediaries of countries *i* and *k* to the non-partner countries pushes the price paid to the farming sector by these firms down positively affecting the profits that they make in the new partner country. This gain in profit is large enough to offset the loss of profit in the domestic market which is why the intermediaries in countries *i* and *k* make higher profits after the agreement.

In relation to the farming sector, the information presented in table E.23 revealed another case of a change of incentives when monopsonistic power is very high. As shown in the previous case (i.e. when $\phi = 0.5$), producer surplus always decreases when an existing agreement is broken. However, when tariffs are endogenous, the farming sector does not always support the signature of a new agreement. In particular, it was shown that producer surplus in country *l* and network *g* decreases when this country signs a new agreement because the total output purchased to the farming sector in this country decreases pushing the price paid to farmers down. This, in turn, implies that the farming sector of this country will not support such an agreement.

This incentive is reversed when there is very high level of monopsonistic power because, according to Table E.20, producer surplus always decreases when an agreement is broken and always increases when a new agreement is signed in any country and in any network. The reason of why the incentive of the farming sector in country *l* and network *g* is reversed is because the decrease in output in the domestic is not as large as the gain in output exported to foreign countries after the agreement. This implying that more output is purchased to the farming sector after the agreement and this pushes the price paid to this sector up positively affecting producer surplus.

Finally, the information presented in Tables E.21 and E.22 revealed that consumer surplus always increases (decreases) and tariffs revenue always decreases (increases) when an agreement is signed (broken). This is the same result obtained in the previous simulations suggesting that the response of these variables to different trade patterns remains robust through different degrees of monopsonistic power.

In conclusion, it is proved in this section that, as in the cease of exogenous tariffs, the existence of a farming sector has the potential to influence the incentives of different groups of firms in favour free trade when governments are politically unbiased. However, in contrast to the case of exogenous tariffs, when tariffs are determined endogenously this positive influence is not strong enough to affect the pairwise stability of the networks identified in the case when there is no monopsonistic power.

4.3.2.4 The case of biased governments

A key result identified in the case of exogenous tariffs that revealed a clear deviation from the original model by Goyal and Joshi (2006) is that global free trade is not always stable when governments are biased in favour of intermediaries and when there is a farming sector. The reason is because more trade implies more output sold in foreign countries and this, in turn, implies higher prices paid to the farming sector negatively affecting the profit made by the intermediaries. As a result, the pairwise stability of global free trade is broken because biased government will deviate with the purpose of helping the intermediaries to achieve higher profits in less integrated networks.

The aim of this subsection is to show that the stability of global free trade can also be broken when governments are biased in favour of the intermediaries in a world where tariffs are placed endogenously. However, it was not possible to generalise the results for larger networks given the mathematical tractability problem. In spite of this, partial results were obtained to show that the finding concerning the destabilising effect of the farming sector on the pairwise stability of free trade is robust. In order to show this fact, the following simulations based on the information presented in Appendix B are considered.

Table 4.4. Tariffs and profits

	$\phi = 0$			**$\phi = 0.5$**			**$\phi = 1.5$**		
Simulations for Policy bias	**Tariff $T_i(j)$**	**Profit $\pi_i(j)$**	**Profit $\pi_i(k)$**	**Tariff $T_i(j)$**	**Profit $\pi_i(j)$**	**Profit $\pi_i(k)$**	**Tariff $T_i(j)$**	**Profit $\pi_i(j)$**	**Profit $\pi_i(k)$**
(7) $a_i = c_i = d_i = 0.40$; and $b_i = 1$	0.1765	0.1388	0.1600	0.1360	0.1209	0.1280	0.1153	0.0887	0.0914
(8) $a_i = c_i = d_i = 0.30$; and $b_i = 1$	0.2371	0.1413	0.1600	0.1764	0.1224	0.1280	0.1393	0.0893	0.0914
(9) $a_i = c_i = d_i = 0.20$; and $b_i = 1$	0.3793	0.1561	0.1600	0.2837	0.1344	0.1280	0.2031	0.0930	0.0914
(10) $a_i = c_i = 0$; $d_i = 0.50$; and $b_i = 1$	0.2500	0.1425	0.1600	0.2180	0.1257	0.1280	0.1927	0.0922	0.0914

Table 4.5. Welfare

	$\phi = 0$		$\phi = 0.5$		$\phi = 1.5$	
Simulations for Policy bias	**Weighted Welfare $W_i(j)$**	**Weighted Welfare $W_i(k)$**	**Weighted Welfare $W_i(j)$**	**Weighted Welfare $W_i(k)$**	**Weighted Welfare $W_i(j)$**	**Weighted Welfare $W_i(k)$**
(7) $a_i = c_i = d_i = 0.40$ and $b_i = 1$	0.2599	0.2880	0.2190	0.2304	0.1594	0.1646
(8) $a_i = c_i = d_i = 0.30$ and $b_i = 1$	0.2270	0.2560	0.1938	0.2048	0.1414	0.1463
(9) $a_i = c_i = d_i = 0.20$ and $b_i = 1$	0.2192	0.2240	0.1769	0.1792	0.1260	0.1280
(10) $a_i = c_i = 0$; $d_i = 0.50$; and $b_i = 1$	0.1425	0.1600	0.1289	0.1280	0.0954	0.0914

The first columns in Tables 4.4 and 4.5 show different biases that were used to develop four simulations referred to as (7), (8), (9) and (10). For example, the first one assumes that governments place a weight equal to 1 to the intermediaries and a weight equal to 0.4 to the rest of the components of the welfare function. The other columns in Table 4.4 show the tariff in country *i* when this country deviates from global free trade by passing from network *k* to network *j* (i.e.$T_i(j)$), and the profit that the intermediary of this country makes in networks *j* and *k* (i.e. $\pi_i(j)$ and $\pi_i(k)$) under different levels of monopsonistic power (i.e. $\phi = 0$; $\phi = 0.5$; $\phi = 1.5$) for each of the four simulations. Likewise, the columns in Table 4.5 show the values of the weighted welfare function of country *i* in networks *j* and *k* (i.e. $W_i(j)$ and $W_i(k)$) under different levels of monopsonistic power for each of the four simulations.

The introduction of these simulations has two objectives. The first one is to show that intermediaries may influence governments' incentives to deviate from global free trade. That is, these firms may influence the selection of tariffs that maximise a weighted welfare function that allow them to obtain higher profits in less integrated networks. The second objective is to show that this influence is stronger when the level of monopsonistic power is higher. This is shown as follows.

According to Simulations (7), (8) and (9) in Table 4.4, when the government in country *i* places less weight on the components of the welfare function other than the intermediary, the optimal tariff increases positively affecting the profit made by the latter. When there is no monopsonistic power, this increase in profit is not strong enough to reach the level of profit in global free trade. For example, when the weight placed on the components of the welfare function other than the intermediary is equal to 0.2 (i.e. Simulation (9)), the intermediary obtains a profit equal to 0.1561 which is smaller than the profit obtained in global free trade and equal to 0.1600. In contrast, when there is monopsonistic power, the increase in profit can actually offset the profit in global free trade with lower tariffs. For example, when $\phi = 1.5$, the intermediary in country *i* makes a profit equal to 0.0930 which is larger than the profit that this firm makes in global free trade which corresponds to 0.0914. Moreover, this higher profit in network *j* is obtained with a tariff equal to 0.2031 which is smaller than the tariffs that maximises a similar weighted welfare function but with lower levels of monopsonistic power.

The higher profit that the intermediary can make in network *j* in Simulation (9) when $\phi = 0.5$ and $\phi = 1.5$ is, however, not feasible because, as shown in Table 4.5, the value of welfare in this simulation is still larger in global free trade implying that the biased government in country *i* is not willing to deviate from the complete network *k*. This is explained by two facts. Firstly, the level of consumer surplus and producer surplus is higher in global free trade. Secondly, the weight that the government puts on these components of the welfare function is still large enough for welfare to be larger in global free trade.

Simulation (10) shows a case of a government that is strongly biased in favour of the intermediary and chooses an optimal tariff that causes a deviation from global free trade. In particular, Table 4.4 shows that the profit made by the intermediary in country *i* is larger in network *j* only when the monopsonitic power is given by ϕ = 1.5. Nonetheless, under this policy bias, the stability of global free trade is broken even when ϕ = 0.5, but not when ϕ = 0. This is shown in Table 4.5: welfare is larger in network *j* in simulation (4) when the intermediary exercises monoponistic power implying that under this market imperfection the pairwise stability of global free trade is broken.

What is interesting about this result is that the pairwise stability of this network can be broken even when the intermediary obtains higher profits in global free trade. As shown in the case of ϕ = 0.5, this happens when the gain in tariff revenue after the deviation from global free trade is larger than the loss of profits made by the intermediary implying that the government has in this case an incentive to break an existing agreement. However, when ϕ = 1.5, the deviation causes both a gain in the profit made by the intermediary and a gain in tariff revenue. The reason that explains why the intermediary makes higher profits after the deviation is because under very high levels of monopsonistic power, the price paid to the farming sector is too high when the world is in global free trade, and this price is reduced when the country deviates from global free trade.

In summary, the main implication of the results presented in this part is that global free trade is not always stable when governments are biased in favour of intermediaries and this result also holds when tariffs are determined endogenously.

4.4 Simulations on bilateralism under asymmetric countries

In the previous section, the pairwise stability of international networks under the assumption of symmetric countries was studied. This analysis provided interesting insights about international trade patterns in agriculture and the stability of free trade. The objective of this section is to extend this analysis in order to explore the issue of agricultural trade liberalisation in a more realistic world characterised by asymmetric countries. Ttwo types of asymmetry are considered in this study: (1) asymmetry in market size: and (2) asymmetry in farmer's productivity.

In order to determine the pairwise stable networks under these asymmetries, different simulations were carried out in this section. As in the previous section, they consider the set of possible networks that can be formed with countries *i*, *j*, *k* and *l*. These networks are shown in the following figure.

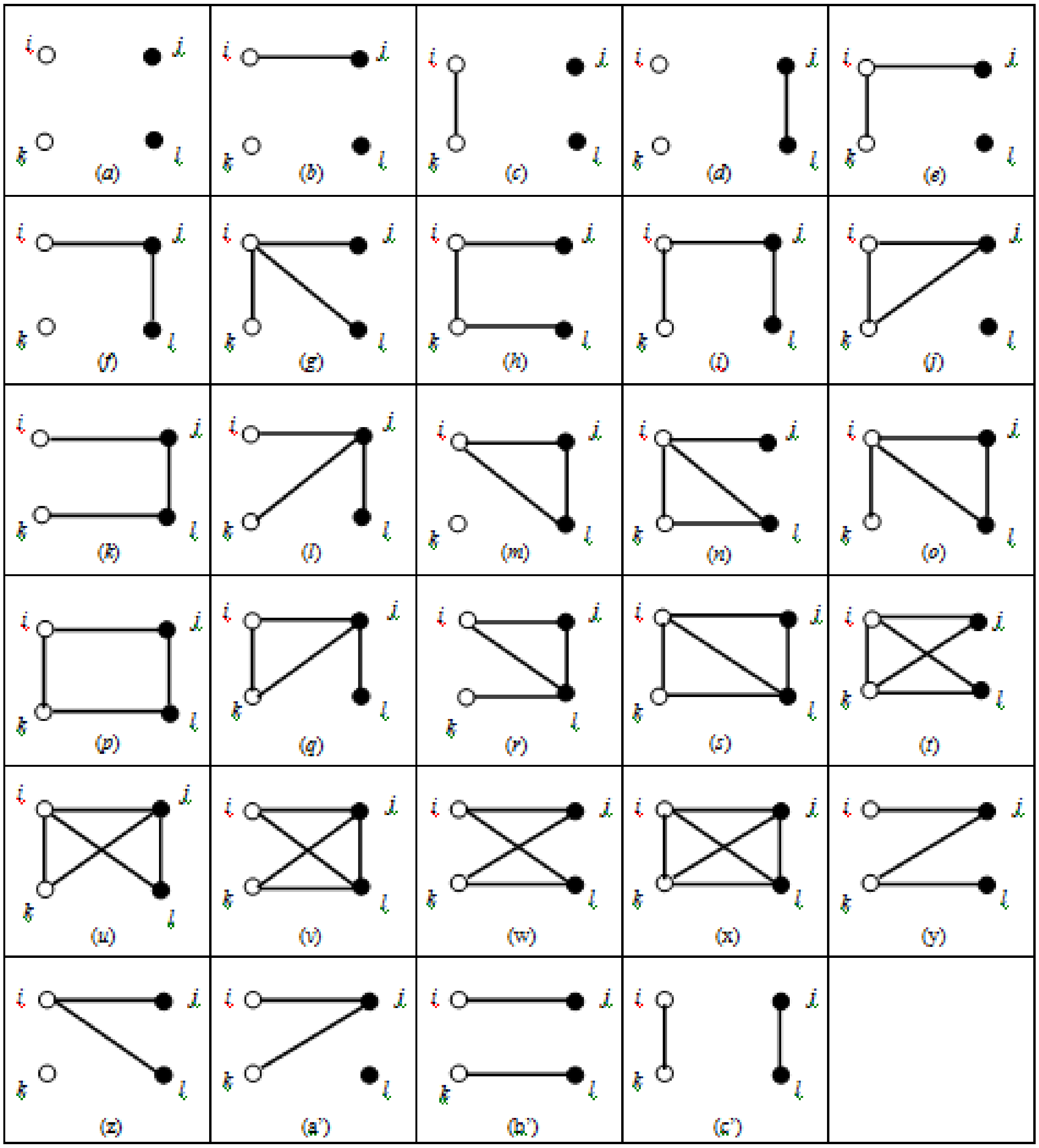


Figure 4.5. Possible network architectures formed with countries *i*, *j*, *k* and *l*.

This figure assumes two sets of countries. The first set contains countries *i* and *k* and are represented with a white circle each. The second set is formed of countries *j* and *l* and are represented with a black circle each. Countries *i* and *k* are assumed to be symmetric, and countries *j* and *l* are also assumed to be symmetric. However, countries that belong to different sets are assumed to be asymmetric.

In this figure the number of networks that have to be analysed is larger than in the case of symmetrical countries. This is because similar network architectures are not necessarily equivalent. For example networks *g* and *l* have the same architecture corresponding to the star network (i.e. a network with a central country connected to the rest of the countries, but the latter are only connected to the central country). While these networks have the same architecture, they are different because their central countries are asymmetric implying that numerical simulations in these networks have to be different.

Note that as in the previous section, some networks were omitted. For example, country *l* in network *j* in this figure is a singleton. A similar network can be considered when country *j* is the singleton and the rest have agreements with each other. However, information about this network can be inferred from network *j* because countries *j* and *l* are symmetric in this simulation.

4.4.1 Bilateralism under exogenous tariffs and asymmetry in market size

Asymmetry in market size is introduced by assuming that countries *i* and *k* have the same market size denoted by α, and countries *j* and *l* have the same market size denoted by $\tilde{\alpha} \neq \alpha$ (see Section 4.2.1.2). Using this assumption, six simulations were developed (i.e. simulations 11, 12, 13, 14, 15, and 16). The three first simulations considers the extreme case when $\tilde{\alpha}$ = 0. That is, they consider the case when countries *j* and *l* are extremely small in the sense that they don't have a domestic market. This extreme case is used as a benchmark limiting behaviour or boundary to gain an understanding of the incentive of large countries to trade with very small countries under different levels of monopsonistic power. This assumption is relaxed in the next three simulations with the purpose of studying the incentive of large countries to trade with middle size countries under different levels of monoxonistic power. In these simulations it is assumed $\tilde{\alpha} = 0.5$. That is, countries *j* and *l* are small but still have a significant domestic market.. The mathematical computations carried out in the simulations are show in Appendix C. The results of the simulations are presented as follows.

4.4.1.1 Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$ for all $i \in N$.

In this simulation it is assumed that there is no monopsonistic power (i.e. $\phi_i = 0$). This implies that this simulation converges to the original model by Goyal and Joshi (2006) under asymmetry in market size. The results obtained in this part will, therefore, be used as a benchmark to evaluate deviations from the original international trade network model when there is a farming sector.

The results of the simulation in terms of consumer surplus (see Appendix F), profits made by the intermediary, and welfare are presented in Tables E.26, E.27 and E.28.

<u>The case of politically unbiased governments</u>

Let us consider first the information presented in Table E.28. From this table it is inferred that the set of link deletion proof networks is *D* = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *m*, *n*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} where *Eq*, as before, denotes the networks that are equivalent to the networks included in this set[8]. On the other hand, the set of link addition proof networks is *A* = {*m*, *t*, *x*, *z*, *Eq*}. In considering these sets, it is concluded therefore that the set of pairwise stable networks in the case of unbiased governments is given by: $P = D \cap A$ = {*m*, *t*, *x*, *z*, *Eq*}. This result was not explored by Goyal and Joshi (2006) because these researchers only focussed on the stability of global free trade when countries are asymmetric in market size. However, the results revealed that global free trade is not the only parwise stable network in their model.

<u>The case of politically biased governments</u>

Using the information presented in Table E.27 it is inferred that *D* = {*a*, *d*} and *A* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *x*, *y*, *z*, *c*', *Eq*}. This implies that the set of pairwise stable networks when governments are biased in favour of intermediaries is given by $P = D \cap A$ = {*a*, *d*}.

[8] For example, in network *e* country *i* is connected to countries *j* and *k*. A similar network is the one where country *k* is connected to countries *i* and *l*. This network has been omitted because information about this network can be inferred from network *e* as a consequence of symmetry between counties *i* and *k*, and countries *j* and *l*.

Discussion

Let us first consider the case of unbiased governments. The stability of network *m* in this case is explained by the fact that the singleton (i.e. country *k*) is unwilling to sign an agreement with any of the other countries. This happens because this country is large and an agreement with a small country only causes a loss of profit in the domestic market as a consequence of higher competition. This loss of profit is not compensated by additional profit made in a small country because the latter does not have a relevant domestic market (i.e. it is very small). The gain in consumer surplus that the larger country obtains after the agreement with a small country as a consequence of higher competition does not compensate the loss of profit either, and this is why such an agreement causes a net loss of welfare in the large country. Country *k* is not willing to sign an agreement with the other large country *i* either because the latter has already high level of competition in the domestic market (i.e. country *i* is already connected to the small countries *j* and *l*). As a consequence, the gain in consumer surplus and profit in country *i* if they sign an agreement are not large enough to compensate the loss of profit in the domestic market of country *k*. The same facts discussed for network *m* explains the stability of network *z*.

Another interesting observation in relation to network *m* is that this network is also stable when countries are symmetric (see Section 4.3.1.1). However, network *j* is not stable even when having the same architecture. The reason is because network *j* is not link addition proof implying that there are at least two countries willing to sign an agreement. For example, countries *k* and *l* have an incentive to form the agreement. For country *k* the gain in consumer surplus offsets the loss of profits after the agreement. For country *l*, on the other hand, the agreement allows this country to obtain positive profits in country *k* without losing domestic profits and consumer supplies (because this country is very small and does not have domestic market). In conclusion, networks composed of a complete component and a singleton are all pairwise stable in the symmetric countries case. However, in the asymmetric case this only holds when the singleton is a large country.

The stability of network *t*, on the other hand, is explained by the fact that the small countries *j* and *l* are indifferent about signing an agreement. This is because the agreement will not change consumer surplus or profits in these countries as a consequence of not having a relevant domestic market.

Finally the stability of global free trade (i.e. network *x*) is explained by the fact that the small countries are indifferent about breaking an agreement, and large countries face a net decrease in welfare when deviating from global free trade (i.e. the gain in profit in the domestic market as a consequence of the lower level of competition after breaking an agreement is not large enough to compensate the loss in consumer surplus and the profit made in the ex-partner country).

Let us now consider the case of countries biased in favour of intermediaries. In this case there are two pairwise stable networks: *a* and *d*. The stability of network *a* is explained by the fact that no large country in autarky is willing to sign an agreement. If they did, then the gain in consumer surplus plus the additional profits made in the new partner country (this profit is cero if the new country is a small one) are not large enough to compensate the loss of profits made in the domestic market (i.e. the monopoly profit). On the other hand, small countries are indifferent about signing an agreement with one another because they don't have domestic market implying that they would obtain cero profits. These countries would be willing to sign an agreement with large countries in order to benefit from getting access to large markets. However, as explained above, countries would not sing this agreement.

On the other hand, network *d* is stable because the small countries do not have an incentive to break the existing agreement as a consequence of not having a domestic market. They are willing to sign an agreement with large countries in order to make large profits in these countries. But the latter are not willing to sign an agreement with small countries because this agreement does not allow them to get profits in the small countries. On the contrary, the agreement would increase competition in the domestic market of the large countries negatively affecting the profit made by the intermediaries of these countries. Finally, large countries would be unwilling to sing an agreements with one another for the same reason given above for network *a*.

Note that the results obtained in this simulation are the ones obtained from the original framework by Goyal and Joshi (2006). According to these researchers, global free trade is always pairwise stable and this claim is used by Goyal and

Joshi to suggest the use of bilateral agreements to reach global free trade. However, as seen in this simulation, this is not always the case. When governments are biased in favour of intermediaries, networks *a* and *d* are pairwise stable but not global free trade. Consequently, the claim made by Goyal and Joshi has to be considered with caution.

4.4.1.2 Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ *and* $\phi = 0.5$ *for all* $i \in N$.

This simulation introduces the farming sector into the analysis. This is done by assuming moderate level of monopsonistic power (i.e. $\phi = 0.5$ for all $i \in N$). The relevant information that is needed for this simulation is presented Tables E.29, E.30, E.31 and E.32 in Appendix E.

<u>*The case of politically unbiased governments*</u>

In considering Table E.32 it is inferred that the sets of link deletion proof and link addition proof networks are *D* = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *m*, *n*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} and *A* = {*m*, *t*, *x*, *z*, *Eq*}, respectively. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A$ = {*m*, *t*, *x*, *z*, *Eq*}.

The case of politically biased governments

The information that is needed to identify the pairwise stable networks under biased governments in favour of consumers, firms and the farming sector is presented in Tables E.29, E.30 and E.31, respectively.

In considering Table E.30 it is inferred that the sets of link deletion and link addition proof networks when governments are biased in favour of the intermediaries are *D* = {*a*, *d*} and *A* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *x*, *y*, *z*, *a*', *c*', *Eq* }, This implies that the set of pairwise stable networks in this case is $P = D \cap A$ = {*a*, *d*}, respectively. This is the same result than the one obtained in the previous simulation. On the other hand, in considering Table E.31, it is inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the farming sector are given by *D* = {*a*, *c*, *d*, *c*', *Eq*} and *A* = {*c*, *e*, *g*, *h*, *i*, *j*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *x*, *c*', *Eq*}. Consequently, the set of pairwise stable networks in this case is given by $P = D \cap A$ = {*c*, *c*'}.

Discussion

The results revealed that the pairwise stability of international networks is not affected when there is a farming sector, when monopsonistic power is moderate and when governments are unbiased or biased in favour of intermediaries. This implies that the influence of the farming sector is not large enough to affects the network stability under these conditions.

When governments are biased in favour of the farming sector, the pairwise stable networks (i.e. *c* and *c*') reflect some forms of regionalism. For example network *c*' contains two blocks of countries. One of them is composed of large countries (i.e. countries *i* and *k*) and the other block is composed of small countries (i.e. countries *j* and *l*). As explained in the literature review, this type of regionalism exists in the real world. That is, large countries, referred to as *countries of the north*, have higher degree of international trade of agricultural products between them. The same happens with small countries which are referred to as *countries of the south*. However, international trade between countries of the north and the south is significantly lower. According to the results obtained in the current simulation, this regionalism is explained by the monopsonistic and oligopolistic power exercised by the intermediaries, and by asymmetry in market size. This is explained as follows by taking network *c*' as an example (note that this explanation also applies to network *c*).

If the large countries broke their agreement, then the total output sold by the intermediaries in the domestic market would increase because this market would become less competitive. However, the intermediaries would stop exporting to the ex-partner country. The increase in output sold in the domestic market is not large enough to offsets the decrease in the output exported to the ex-partner country implying that breaking the agreement would cause a net decrease in the total output sold by the intermediaries. This decrease in output implies that the farming sector would receive a lower price for their production negatively affecting producer surplus. This explains why the large countries are unwilling to break their existing agreement when they are biased in favour of the farming sector.

On the other hand, the large countries are unwilling to sign an agreement with small countries because this would increase the level of competition in the domestic markets of the former reducing the total output sold by the intermediaries. These firms would be unable to compensate this decrease by exporting new output to the small countries because the domestic market of these counties is very small. Thus, an agreement with a small country would cause a net decrease in the output sold by the intermediaries of the large countries. This, in turn, would decrease the price paid to the farming sector in these countries negatively affecting producer surplus. In relation to the small countries, they are indifferent between having or breaking their agreement because their domestic markets are very small and any change would not cause changes in producer surplus (note that this is what explains the stability of *c* as well). However, they would be willing to sign an agreement with a large country because this would cause a significant gain in produce. This is explained by the fact that the intermediaries of small countries would be able to access large markets. This would cause an increase in the output sold by these firms pushing the price paid to the farming sector up, and therefore, increasing producer surplus in small countries.

In summary, it is predicted that bilateral agreements in a world formed of biased governments in favour of the farming sector leads to regionalism when countries are asymmetric in market size. In contrast, when countries are symmetric (see

Section 4.3.1.2), the only pairwise stable network is global free trade. It is concluded therefore that the existence of a farming sector that is supported by biased governments favours free trade when countries are symmetric, and prevents global free trade when the world of composed of large and very small countries.

4.4.1.3 Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ *and* $\phi = 1.5$ *for all* $i \in N$.

The objective of this simulation is to determine whether the results obtained in the previous one are affected when intermediaries exercise larger levels of monopsonistic power (i.e. when $\phi = 1.5$) The information used in this analysis is presented in Tables E.33, E.34, E.35 and E.36 in Appendix E.

The case of politically unbiased governments

In considering Table E.36 it is concluded that the sets of link deletion proof and link addition proof networks when governments are politically unbiased are *D* = {*a*, *c*, *d*, *e*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} and *A* = {*t*, *x*}, respectively. Therefore the set of pairwise sable networks in this case is $P = D \cap A$ = {*t*, *x*}. The number of networks in this set is smaller than the number of pairwise stable networks identified in the previous simulation (i.e. when $\phi = 0.5$). This implies that as monopsonistic power increases, the number of pairwise stable networks decreases in the case of unbiased governments and asymmetry in market size.

<u>The case of politically biased governments</u>

The information presented in Table 4.34 revealed that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the intermediaries are D = {*a*, *d*} and A = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*}, respectively. Therefore the set of pairwise stable networks in this case is $P = D \cap A$ = {*a*, *d*}. The same set of pairwise stable networks was found in the previous simulation. However, the set of link addition proof networks is different. In the case of high level of monopsonitic power (i.e. ϕ = 1.5), this set is larger implying that the number of networks in which countries are unwilling to sigh new agreements increases as the level of monopsonistic power increases.

On the other hand, in considering Table E.35, it is concluded that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the farming sector are D = {*a*, *c*, *d*, *c*', *Eq*} and A = {*c*, *e*, *g*, *h*, *i*, *j*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *x*, *c*', *Eq*}, respectively. Therefore the set of pairwise stable networks in this case is $P = D \cap A$ = {*c*, *c*'}. This is the same result than the one obtained in the previous section. It is inferred therefore that the incentives of countries to break or sign bilateral agreements is not affected under moderate or large levels of monopsonitic power.

Discussion

This discussion is focussed on the cases that revealed deviations from the previous simulation.

The first deviation was found in the case of politically unbiased governments. In this case the pairwise stability of networks *m* and *z* is broken because these networks are not link addition proof any longer when the level of monopsonistic power is high. In both networks this is explained by the incentives of the singleton large country *k*. When monopsonistic power is moderated (i.e. $\phi = 0.5$) this country is not willing to sign any agreement with any other country, but this incentive is reversed when monopsonistic power is high. This is explained as follows.

If country *k* signed an agreement with the other large country *l*, then the domestic market of the former would become more competitive positively affecting consumer surplus. However, this higher competition would negatively affect the profits made by the intermediary of country *k* in the domestic market. The additional profit that this firm would make in the new partner country is not large enough to compensate the loss of profit in the domestic market implying that the agreement would cause a net loss of profits. This loss is reinforced by the fact that the total output traded in the domestic market and the new partner country increases after the agreement as can be seen form the information presented in Appendix C. This higher quantity of output means that the intermediary has to pay a higher price to the farming sector after the agreement given the existing monopsonistic power. However, this higher price and the higher quantity of traded output positively affect producer surplus. Thus, when monopsonistic power is moderate, the gain in consumer surplus and producer surplus is not large enough to offset the loss of profits faced by the intermediary and this explains why in this case country *k* is unwilling to sign the agreement. In contrast, when monopsonistic power is high, the gain in producer surplus is more significant implying that this gain plus the gain in consumer surplus offsets the net loss of profits. As a consequence, an agreement increases welfare which is what explains why country *k* is willing to sign an agreement with the other large country when monopsonistic power is high. It is concluded, therefore, that in the asymmetric case in terms of market size and unbiased countries, the existence of a farming sector in a world with high level of monopsonistic power positively affects free trade in networks having large singleton countries because this sector increase the incentives of these countries to sign bilateral agreements.

The second deviation identified in this simulation corresponds to the case of governments biased in favour of intermediaries. While the same pairwise stable networks were found under different levels of monopsonistic power, the number of networks in the set of link addition proof networks changed. In particular, it was found that when the level of monopsonistic power increased from $\phi = 0.5$ to $\phi = 1.5$, networks *k*, *v*, *w* and *b*' become link addition proof. This is because when $\phi = 0.5$, the larger countries *i* and *k* in these networks are willing to sign an agreement. However, this incentive is reversed when $\phi = 1.5$.

To understand this change, note that an agreement between these countries increases the level of competition in their domestic market negatively affecting the profits made by the intermediaries in these markets. But at the same time these firms get access to the new partner country. Thus, the large countries will be willing to sign the agreement when the gain in profits in the new partner country is larger than the loss of profits in the domestic market. But this depends on the impact of the agreement on the price paid to the farming sector. In both cases (i.e. when from ϕ = 0.5 and when ϕ = 1.5) the agreement increases the total output sold by the intermediaries of the large countries. This pushes the price paid to the farming sector. When monopsonistic power is moderate, this additional marginal cost is not strong enough. As a consequence, the profit made in the new partner countries is larger than the loss of profits in the domestic market. This is why networks *k*, *v*, *w* and *b*' are not link addition proof in this case: the large countries have an incentive to sign the agreement. In contrast, when the level of monopsonistic power is large, the increase in marginal costs after the agreement is large enough to reverse this difference in profits. That is, when ϕ = 1.5, the loss of profits in the domestic market offsets the gain of profits in the new partner given the high marginal cost faced by the intermediaries after the agreement. As a result, the large countries in networks *k*, *v*, *w* and *b*' do not have an incentive to sign an agreement. This explains why these networks are link addition proof for the case of high level of monpsonistic power.

It is concluded therefore that the farming sector negatively affects the formation of bilateral agreements between large countries when intermediaries exercise high levels of monopsonistic power, when governments are biased in favour of these firms, and when countries are asymmetric in market size.

4.4.1.4 Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0 for all i $\in$ N.

The next three simulation were introduced with the purpose of identifying the pairwise stable networks when there are large and medium size countries (i.e. α = 1 and $\tilde{\alpha}$ = 0.5, respectively) under different degrees of monopsonistic power. The current simulation in particular considers the case when there is no monopsonistic power. That is, Goyal and Joshi's world when there are large and medium size countries. The information used in this simulation is presented in Tables E.37, E.38 and E.39 in Appendix E.

The case of politically unbiased governments

In considering Table E.39 it is concluded that when governments are politically unbiased the sets of link deletion proof and link addition proof are *D* = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *m*, *n*, *p*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} and *A* = {*m*, *x*, *Eq*}, respectively. This implies that the set of pairwise stable networks is *P* = *D* $\cap$ *A* = {*m*, *x*, *Eq*}.

The case of politically biased governments

Using the information presented in Table E.38 it is inferred that the link deletion proof and link addition proof networks when governments are biased in favour of the intermediaries are *D* = {*a*} and *A* = {*a*, *b*, *c*, *d*, *f*, *i*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *u*, *x*, *y*, *c*', *Eq*}, respectivelly. This implies that the set of pairwise stable networks in this case is $P = D \cap A = \{a\}$.

Discussion

The result obtained for the case of unbiased governments is different from the one obtained in the simulation assuming the existence of both large and very small countries without monopsonistic power (see Simulation 11 in Section 4.4.1.1). In that case, the stable networks are {*m*, *t*, *x*, *z Eq*}. This means that in Goyal and Joshi's world, the number of stable pairwise networks becomes smaller when small countries are replaced by medium size countries and the networks that become unstable are networks *t* and *x*.

To understand this fact, remember that it was found in Simulation 11 that the small countries *j* and *l* in networks *t* and *x* are indifferent about having an agreement with each other because they don't have domestic markets. In contrast, when countries *j* and *l* are medium size, they have relevant domestic markets that origin a gain in consumer surplus and a net gain in profits when these countries sign an agreement. The gain in consumer surplus is explained by the higher level of competition in the domestic markets of these countries after the agreement. Likewise, the net gain in profits is explained by the fact that the additional profit that the intermediaries make in the new partner country offsets the loss of profits in the domestic market caused by the higher level of competition.

On the other hand, when governments are biased in favour of the intermediaries, the only stable network is the empty network. This also differs from the results obtained in Simulation 11. In that case network *d* is also pairwise stable and this is explained by the fact that the very small countries *j* and *l* are indifferent about having an agreement with each other as a result of not having domestic markets. In contrast, when these countries are medium size, they have an incentive to break their existing agreement because the gain in profits as a result of the decrease in market power offsets the loss of profit that the intermediaries made in the ex-partner countries.

In summary it is concluded that in Goyal and Joshi's world the governments of very small and medium size countries who trade with large countries have different incentives towards bilateral agreements and this is explained by the existence of domestic markets in these countries. In the case of unbiased governments of medium size countries, these governments have an incentive to sign an agreement in some key networks because this causes a net gain in social welfare. Likewise, governments of medium size countries that are biased in favour of consumers have an incentive to sign an agreement because this causes a gain in consumer surplus (see Appendix F). In contrast governments of medium size countries that are biased in favour of the intermediaries have an incentive to break an existing agreement because this causes a net gain in profits.

4.4.1.5 Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

The objective of this simulation is to introduce the farming sector into the model when networks are formed of large and medium size countries. This is reflected by the assumption that intermediaries exercise a moderate level of monopsonistic power. (i.e. ϕ_i = 0.5). The relevant information that is needed to determine the pairwise stable networks in this case is presented in Tables E.40, E.41, E.42 and E.43 in Appendix E.

The case of politically unbiased governments

The sets of link deletion proof and link addition proof networks were inferred from the information given in Table E.43 and correspond to *D* = {*a*, *c*, *d*, *e*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} and *A* = {*x*}. Consequently, the set of pairwise stable networks in this case is given by $P = D \cap A$ = {*x*}.

The case of politically biased governments

In this simulation there are three types of biases: bias in favour of consumers (see Appendix F); firms; and the farming sector. The information that is needed to identify the pairwise stable networks under these biases is presented in Tables E.40, E.41 and E.42.

The information presented in Table E.41 revealed that when governments are biased in favour of the intermediaries, the sets of link deletion and link addition proof networks *D* = {*a*} and *A* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *x*, *y*, *z*, *a*', *c*', *Eq*}, respectively. This implies that the set of pairwise stable networks in this case is $P = D \cap A = \{a\}$. On the other hand, in considering Table E.42, it is inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the farming sector are given by *D* = {*a*, *b*, *c*, *d*, *e*, *g*, *h*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *b*', *c*', *Eq*} and *A* = {*x*, *c*'}. Consequently, the set of pairwise stable networks in this case is given by $P = D \cap A = \{x, c'\}$.

Discussion

As in the case of symmetric counties, the result for the case of unbiased governments shows that the pairwise stability of networks other than global free trade is broken when there is a farming sector implying that this sector positively affects international trade. This is inferred from the fact that the stable networks in Goyal and Joshi's model (see the previous simulation) are networks *m* (and equivalents) and *x* (i.e. global free trade). This is explained by the incentive of country *k* in network *m*. When there is moderate monopsonistic power, producer surplus in this country increases after the agreement because the total output traded by the intermediary in this country increases pushing the price paid to farmers up. This positive effect on producer surplus and consumer surplus are together strong enough to offsets the net loss of profits made by the intermediary and this explains why the government of country *k* is willing to sign an agreement and why network *m* is not pairwise stable in this case.

This result also differs from the case of large and very small countries with moderate monopsonistic power (see Section 4.4.1.2). In that case there are several pairwise stable networks. But the stability of the networks other than global free trade is broken when replacing very small countries with medium size countries. For example, the pairwise stability of networks *t* and *z* is broken in the current simulation. This is explained by the fact that when countries *j* and *l* are medium size, they have a relevant domestic market that offers them a net gain in welfare after the agreement is signed. This is because the gain in consumer surplus plus the gain in producer surplus are together large enough to offsets the net loss of profits faced by the intermediary in these countries.

In relation to the case of governments biased in favour of intermediaries, the results revealed that only the empty network is pairwise stable when there are large and medium size countries with moderate monopsonistic power. This result differs from the case of large and very small countries with moderate levels of monopsonistic power (see Section 4.4.1.2). In that simulation network *d* is also pairwise stable and this is explained by the fact that these countries are indifferent about signing an agreement with each other because they have irrelevant domestic markets. That is, an agreement between the very small countries *j* and *l* will not allow the intermediaries to export the food processed good to the new partner countries because their domestic markets are very small. In contrast, when countries *j* and *l* are medium size, their domestic markets are large enough to allow the intermediaries to compensate the loss of profit in their domestic markets with the additional profits that they make in the new partner countries.

Finally, when governments are biased in favour of the farming sector, global free trade and network *c*' are the pairwise stable network in current simulation. This result is interesting because it shows that regionalism arises when there is asymmetry in market size. To see this, remember that in the symmetrical case, only global free trade is pairwise stable when governments are biased in favour of the farming sector and when the level of monopsonistic power is moderate (see Section 4.3.1.2). However, when there are large and medium size countries, the large countries in network *c*' are unwilling to sign an agreement with the medium size countries. The reason is explained by the fact that the latter have smaller domestic markets. Thus the additional output that can be sold in the medium size countries is not large enough to compensate the decrease of output sold in the domestic market of the large countries. This net loss of output implies that the farming sector obtain a lower price for their production which explains why the agreement decreases the level of producer surplus in large countries and why network *c'* is pairwise stable in the current simulation.

The results obtained in the current simulation also differ from the case of large countries and very small countries with moderate monopsonistic power (see Section 4.4.1.2). In that case global free trade is not pairwise stable but network *c* is. In relation to global fee trade, when there are medium size countries rather than very small ones, the intermediaries of the large countries can export a significant quantity of output to the medium size country that compensates the decrease of output in the domestic market as a consequence of the higher competition. Thus the gain in total output traded by the intermediaries in these countries pushes the price paid to farmers up positively affecting producer surplus in global free trade which is what explains the stability of this network in the current simulation.

In relation to network *c*, on the other hand, this network is pairwise stable in the case of large and very small countries because the latter are indifferent about signing an agreement as a consequence of their very small domestic markets. These countries are willing to sign an agreement with large countries because it can help them to export the food processed output positively affecting producer surplus. However, large countries are unwilling because an agreement with very small countries causes a net decrease in producer surplus. In contrast, in the case of large and medium size countries, the latter have an incentive to sign an agreement with each other because they have relevant domestic markets that can be filled with exports that give the farmers higher levels of producer surplus. This is why network *c* is not pairwise stable in the current simulation.

4.4.1.6 Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$ for all $i \in N$.

This last simulation in the section of asymmetry in market size introduced to study the effects of very high levels of monopsonistic power on the international trade system. The information that is needed for this analysis is presented in Tables E.44, E.45, E.46 and E.47 in Appendix E.

The case of politically unbiased governments

In considering Table E.47 it is concluded that the sets of link deletion proof, link addition proof and pairwise stable networks are *D* = {*a*, *c*, *d*, *e*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *z*, *c*', *Eq*} and *A* = {*t*, *x*}, respectively. This implies that the set of pairwise stable networks in this case is $P = D \cap A$ = {*t*, *x*}.

The case of politically biased governments

It was found from Table E.45 that when governments are biased in favour of the intermediaries, the sets of link deletion proof, link addition proof and pairwise stable networks are *D* = {*a*}, *A* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *c*', *Eq*}, and $P = D \cap A$ = {*a*}, respectively.

On the other hand, in considering Table E.46 it was found that when governments are biased in favour of the farming sector, the sets of link deletion proof, link addition proof and pairwise stable networks are *D* = {*a*, *b*, *c*, *d*, *g*, *h*, *j*, *m*, *n*, *o*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *b*', *c*', *Eq*}, *A* = {*x*, *c*'}, and $P = D \cap A$ = {*x*, *c*'}, respectively.

<u>Discussion</u>

A first deviation with respect to the previous simulation was identified in the case of unbiased governments. In this case it was found that when monopsonistic power is high, network *t* becomes pairwise stable. The reason is because in this network the medium size countries *j* and *l* are indifferent about signing an agreement with each other. In contrast, when the level of monopsonistic power is moderate, these counties are willing to sign the agreement. This difference is explained by the higher cost that intermediaries have to face when monopsonistic power is high. That is, an agreement between countries *j* and *l* causes a gain in consumer surplus in these countries as a result of the higher competition in their domestic markets. However, this higher competition negatively affects the profits made by the intermediaries of these countries in the domestic market. This decrease is compensated to some extent by the additional profits that they make in the new partner country. Nonetheless, because countries *j* and *l* are medium size, the gain in profits from exports is not large enough to fully compensate the loss of profits in the domestic market implying that the agreement causes a net loss of profits.

This is also explained by the higher price that the intermediaries have to pay to the farming sector because the agreement causes a net increase in the total output that is traded. The farming sector, on the other hand, is better off because the higher price and the higher level of output that is traded by the intermediary implies that they obtain higher levels of producer surplus after the agreement. Thus, when monopsonistic power is high, the net loss of profits faced by the intermediaries is more severe because this means that they have to face a much larger marginal cost. This loss is just equal to the gain in consumer surplus and producer surplus, and this is why the governments of the medium size countries are indifferent about signing the agreement. It is concluded therefore that high levels of monopsonistic power have a negative effect on the trade system when governments are unbiased and when the world is formed of large and medium size countries because this creates multiple pairwise equilibriums.

Regarding the case of governments biased in favour of intermediaries, it was found that networks *v* and *w* become link addition proof in the current simulation. This happens because the large countries *i* and *k* in these networks are unwilling to sign an agreement with each other when monopsonistic power is high. In this case the agreement increases the level of competition in the domestic market of the large countries causing a decrease in the output sold by the intermediaries of these countries in the domestic market. However, they increase the export output to the new partner large country and this increase is larger than the decrease in the domestic output. This net increase pushes the price paid to the farming sector up negatively affecting the profits made by the intermediaries after the agreement. This suggests therefore that countries' unwillingness to sign new bilateral agreements is reinforced under this level of monopsonistic power.

Finally, in relation to the case of governments biased in favour of the farming sector, two deviations were identified with respect to the previous simulation. The first one corresponds to the fact that networks *e*, *p* and *q* are not link deletion proof when monopsonistic power is high. This is because large countries in these networks have an incentive to break their existing agreements with a medium size country. To understand this result, note that the large countries in these networks are already connected to other large countries meaning that their domestic markets are relatively competitive. This implies that the gain in output in the domestic market of a large country after an agreement with a medium size country is broken is not significantly large given the existing level of competition. This gain in output in the domestic market can be either larger or smaller than the decrease in the export output after the agreement is broken depending on the effect of this action on the cost faced by the intermediary of the large country. That is, when monopsonistic power is high, breaking this agreement lowers the cost faced by the intermediary in the large country and this reinforces the increase in output sold in the domestic market as a consequence of lower competition. This gain in output is large enough to compensate the decrease in the output that was exported in the ex-partner medium size country. Now, because producer surplus is a monotonic transformation of the total output that is traded by the intermediary, breaking the agreement with the medium size country positively affects producer surplus in the large country. However, when monopsonistic power is moderate, breaking the agreement does not contribute significantly in the reduction of the cost faced by the intermediary of the large country. As a result, the increase in the output in the domestic market is not large enough to compensate the decrease in the output that was exported before the agreement was broken. It is concluded, therefore, that high levels of monopsonistic power have a negative effect on trade because it increases the incentives of large countries to break existing agreements with medium size countries.

The second deviation with respect to the previous simulation corresponds to the fact that network *y* becomes link deletion proof when monopsonistic power if high. This is the opposite of the deviation described above for networks *e*, *p* and *q* and this is explained by the fact that the large country *i* is not willing to break the existing agreement with the medium size country *j*. To understand this result, note that this country is only connected to country *j*. As a consequence, the level of competition in the domestic market of country *i* is low. In contrast, the medium size country *j* is already connected to all the large countries in the network implying that the level of competition in the domestic market of this country is high. Thus, when the agreement between countries *i* and *j* is broken, the gain in output in the large country *i* is larger than the decrease in the export output in the medium size country *j*. Thus, from the point of view of the large country, when monopsonistic power is moderate, the gain in output in the domestic market offsets the loss of export output. This means that breaking the agreement causes a net increase in output sold by the intermediary of the large country positively affecting producer surplus. This is why network *y* is not deletion proof in the previous simulation. However, when monopsonistic power is high, the large increase in the output sold in the domestic market of the large country significantly increases the price paid to the farming sector in this country. This higher cost mitigates the increase of this output to the extent that it is not large enough to compensate the decrease in the export output. This implies that when monopsonistic power is high, there is a net decrease in the output sold by the intermediary of the large country when the agreement is broken and, therefore, a decrease in producer surplus. This is why network *y* is link deletion proof in the current simulation. This result suggests, consequently, that high levels of monopsonistic power may prevent large countries from breaking existing links with medium size countries when they have low degree of international integration.

In summary, it was found in this simulation that while high monopsonistic power does not always affect the pairwise stability of the networks identified in the previous simulation, it affects the incentives of countries in non-stable networks. In particular it was found that high degree of monopsonistic power plays against free trade in the cases of unbiased and biased governments.

4.4.2 Bilateralism under exogenous tariffs and asymmetry in farmers' productivity

A key result obtained in the previous simulations is that monopsonistic power has an important effect on the architecture and stability of international networks of food processed goods. The reason is because this power makes free trade more expensive to intermediaries as they have to pay higher prices to farmers as a consequence of the higher total quantity of the good that is traded domestically and internationally. This finding was developed assuming that all the intermediaries in the world exercise the same monopsonistic power. Given the relevance of this result, the current simulation extends the analysis with the purpose of exploring the pairwise stability of international networks when intermediaries exercise different monopsonistic power. That is, when there are intermediaries in some countries who buy the output from more efficient farmers, and there are intermediaries in other countries who buy the output from less efficient farmers.

As explained in Section 4.2.1.3, asymmetry in farmers' productivity is captured by the parameter δ in Equation 4.19. Using this parameter, two sets of countries are defined. The set Ω = {*i*, *j*} contains the countries in the networks having the same productivity coefficient $\delta \neq 1$. On the other hand, the set Ψ = {*j*, *l*} contains the countries having the same productivity coefficient $\delta = 1$. Using this definition, all networks were partitioned into two groups of counties: the efficient countries (i.e. countries *j*, *l*); and the inefficient countries (i.e. *i*, *k*). The networks considered in this simulation are presented in Figure 4.5 and the calculations are presented in Appendix D.

4.4.2.1 Simulation 17: $\delta = 3$ for Ω = {i, k}; $\delta = 1$ for Ψ = {j, l}; $\phi = 0.5$; and $\alpha = 1$.

The information that was used in this simulation is presented Tables E.48, E.49, E.50 and E.51 in Appendix E.

The case of politically unbiased governments

In considering Table E.51 it is inferred that the sets of link deletion proof and link addition proof networks are *D* = {*a*, *b*, *c*, *d*, *h*, *i*, *j*, *m*, *n*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*',*b*', *c*', *Eq*} and *A* = {*m*, *x*}, respectively. Consequently, the set of pairwise stable networks in this case is given by *P* = *D* ∩ *A* = {*m*, *x*}.

<u>The case of politically biased governments</u>

It was inferred from Table E.49 that when governments are biased in favour of the intermediaries, the sets of link deletion and link addition proof networks are given by *D* = {*a*} and *A* = {*a*, *b*, *c*, *d*, *e*, *f*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*}, respectively. Consequently the set of pairwise stable networks in this case is *P* = *D* ∩ *A* = {*a*}. On the other hand, in considering Table E.50, it was inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of the farming sector are given by *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and *A* = {*x*}. Consequently, the set of pairwise stable networks in this case is given by *P* = *D* ∩ *A* = {*x*}.

<u>Discussion</u>

The results of the current simulation when governments are biased in favour of either consumers, intermediaries or the farming sector are the same as the ones obtained for the cases of symmetric countries with different levels of monosonistic power (see Sections 4.3.1.1, 4.3.1.2, 4.3.1.3 and Appendix F). This means that the same conclusions discussed in these simulations applies to the case of asymmetry in farmers' productivity when governments are politically biased.

On the other hand, the result obtained for the case of politically unbiased governments is different from the one obtained in the case of symmetrical governments without monopsonistic power (i.e. Goyal and Joshi's world with symmetrical countries presented in Section 4.3.1.1).

In that simulation it was found that the pairwise stable networks are global free trade and a network composed of a complete component and a singleton. The current simulation revealed that global free trade is also pairwise stable when there is asymmetry in farmers' productivity. However, the other stable network is a network formed of a complete component and a singleton having a less efficient farming sector. This means that Goyal and Joshi's is only consistent with the current simulation when the singleton has an intermediary that exercise higher levels of monopsonistic power. This is explained as follows. In Goyal and Joshi's world the network with the complete component and the singleton is stable because the latter country is unwilling to sign an agreement. That is, the gain in consumer surplus as a consequence of the higher competition after an agreement plus the additional profits made in the new partner country are not large enough to compensate the loss of profits in the domestic market of the singleton.

In the current simulation this is reversed when the singleton is efficient because in addition to the gain in consumer surplus and the additional profit made in the new partner country there is also a gain in producer surplus that reflects the higher level of output that is traded after the agreement. This three positive sources of welfare

are large enough to offsets the loss of profits in the domestic market. In contrast, when the singleton is inefficient, the additional gain in welfare from producer surplus is not large enough to contribute in compensate the loss of profit in the domestic market. This reflects the fact that the intermediary in this country faces a higher marginal cost than intermediaries in more efficient countries. It is concluded therefore that the existence of a farming sector contributes to free trade in countries that have a more efficient farming sector.

This result also differs from the cases of symmetrical countries with different monopsonistic power (see Sections 4.3.1.1, 4.3.1.2 and 4.3.1.3). In these cases only global free trade is pairwise stable. In considering this result it was concluded in these simulations that the farming sector have a positive effect on global free trade because producer surplus has a significant contribution in the welfare function. But as explained above, the result obtained in the current simulation revealed that this conclusion only holds for countries that have an efficient farming sector. In considering these differences, it is predicted that in the real world higher levels of international trade of food processed goods can be found in more efficient countries with politically unbiased governments. That is, in countries where intermediaries exercise lower levels of monopsonistic power.

PART III: Summary, Discussion and Conclusions

4.5 Summary and conclusions

This chapter introduces the proposed international trade network and explains the fact that the introduction of the farming sector into the original framework by Goyal and Joshi (2006) creates high level of endogeneity. This happens because this sector translates as non-fixed marginal cost from the point of view of intermediaries. That is, increasing the demand for agricultural goods by these firms pushes the price paid to the farming sector up. There are two main implications of this effect. Firstly, more free trade increases agricultural prices but reduces the price of food processed good given by higher competition negatively affecting the gross margin made by the intermediaries in more integrated networks. Secondly, the high level of endogeneity caused by the farming sector makes the theoretical model intractable in mathematical terms. This means that it is difficult to obtain generalisations from the model as it cannot be solved for a generic number of countries.

In considering the endogeneity problem, the proposed model was solved by means of simulations that assume the existence of a world composed of four countries under different assumptions (i.e. exogenous and endogenous tariffs, asymmetry in market size, and asymmetry in farmers' productivity). The aim of these simulations is to identify deviations from the original work by Goyal and Joshi that are attributed to the existence of a farming sector. That is, the aim is to assess how the international trade architecture is affected when intermediaries face a non-fixed marginal cost. In this context, three types of deviations were explored: cases when global free trade becomes unstable; cases when other networks become unstable; and cases when multiple stable networks emerge.

In relation to the fist type of deviations, it was found that global free trade becomes unstable mainly when governments are biased in favour of intermediaries. This happens, as noted above, because more trade increases agricultural prices but reduces the price of food processed goods negatively affecting the profits made by these firms. This result was found in all the simulations developed in this chapter. In considering this result, it is concluded therefore that the farming sector exercise a negative effect on free trade when governments are biased in favour of intermediaries.

In order to understand the economic mechanism behind this finding, let us consider the impact of trade liberalisation upstream and downstream in the supply chain in, say, country *i*. In relation to the upstream segment, when this country signs new agreements, the domestic market becomes more competitive implying that the intermediary obtains a lower price in this market after the new agreements are in force. At the same, this firm gets access to the domestic market of the new partner countries but the price obtained in these markets decreases as trade liberalisation progress because countries become more competitive. As a result, the total output that is exported by the intermediary in country *i* increases but the price that this firms obtains in foreign markets decreases resulting in a net loss of profits as a result of trade liberalisation. Consumers in country *i* are better off as a result of the increase in competition in this country. In relation to the downstream segment, on the other hand, the increase in the total output that is traded by the intermediary as international trade progresses increases the demand for the agricultural good pushing the price paid to the farming sector up positively affecting the return obtained by this sector. In summary, the impact of trade liberalisation upstream and downstream is reflected as an increase in total output that is traded by the intermediary, a decrease in net profits, and increase in consumer surplus, and an increase in producer surplus.

In order to illustrate the upstream and downstream effects of trade liberalisation described above, consider as an example the Simulation 2 (i.e. symmetrical countries; moderate level of monopsonistic power given by $\phi = 1$ in Equation 4.1; and market size given by $\alpha = 1$ in Expression 4.6 in all countries). Let us assume that all countries are in autarky and suppose that countries *i* and *k* decide to sign a bilateral agreement which each other (i.e. passing from network *a* to network *b* in Figure 4.3). According to the information presented in Appendix A, the intermediary of country *i* sells a total output of 0.4000 in the domestic market in autarky and, according to equation 4.6, the price that this firm obtains in this market is 0.6000. At that price the intermediary makes a total profit of 0.2000. Now, when country *i* signs the agreement with country *k*, the output that is sold in the domestic market decreases from 0.4000 to 0.2667 as a consequence of the higher competition, but at the same time, this firm exports an output of 0.2667 to the new partner country. This means that the total output that is traded by the intermediary of country *i* increases from 0.4000 to 0.5334 (i.e. 2x0.2667) after the agreement. According to the Equation 4.6, the price that the intermediary receives in each market after the agreement is equal to 0.4666. As a result of this price, this firm makes a profit of 0.0889 in each market. This means that after the agreement, the total profit made by the intermediary of country *i* decreases from 0.2000 to 0.1778 (i.e. 2x0.0889). Let us consider now the downstream segment in this example. As explained above, the total output traded by the intermediary of country *i* increases from 0.4000 to 0.5334. Because of the Leontief production function of this firm, this implies that the demand for the agricultural good increases by the same amount. Using the Equation 4.1, this means that the price paid to the farming sector increases from 0.1000 to 0.1333 after the agreement resulting in an increase of producer surplus from 0.0200 to 0.0356.

Regarding the cases when other networks become unstable, it was found that the farming sector positively affects free trade when governments are politically unbiased because the presence of this sector breaks pairwise stable networks other than global free trade. This is explained by the fact that more trade increases the demand for agricultural goods pushing the price paid to the farming sector up. This gain in producer surplus plus the gain in consumer surplus due to higher competition are both large enough to offsets the net loss of profits made by the intermediaries. Nonetheless, two exceptions were found.

Firstly, it was found in the simulation that assumes a world composed of large and very small countries that the farming sector positively affects the incentives of large countries to sign more agreements. However, this does not happen in very small countries because they markets are too small to obtain significant gains from trade. Consequently, this result suggests that more trade would be expected in large countries in a world composed of large and very small countries with politically unbiased governments.

Secondly, it was found that when countries are asymmetric in farmers' productivity, pairwise stable networks that contain a singleton becomes unstable under the presence of a farming sector when the singleton is an efficient country. The reason is because these countries obtain a net gain in welfare when signing a bilateral agreement that is reinforced by the gain in producer surplus resulting from higher agricultural prices. In contrast, this increase in price is too high in inefficient singletons in terms of the negative effect on the profits made by the intermediary of these countries. This causes a net loss in welfare when an agreement is signed. It is concluded therefore that in this case the farming sector has a positive effect on trade in countries having a more efficient farming sector.

Finally, in relation to deviations that consider the emerging of new stable networks with respect to Goyal and Joshi's world, the following results were found. Firstly, when governments are biased in favour of the farming sector and when countries are symmetric, only global free trade is stable. The reason is because, as explained above, more free trade implies that farmers obtain higher agricultural prices and, therefore, higher levels of producer surplus. In this context, the farming sector positively affects free trade. However, when countries are asymmetric in market size, regionalism of the south-north emerges. That is, large countries are willing to sign bilateral agreements with each other but are unwilling to sign agreements with smaller countries.

The reason is because signing an agreement with a smaller country increases the level of competition in the domestic market of the large country. This causes a decrease in the output that is sold by the intermediary of this country in the domestic market that is not compensated by the additional export output that is exported to the new smaller country partner. This net decrease in output pushes the price paid to the farming sector in the large county down negatively affecting producer surplus in this country. It is concluded therefore that in this case the farming sector only favour free trade in blocks of countries of similar size leading to regionalism.

It was also found that several networks emerged in the case of unbiased governments when the world is composed of large countries and very small countries. In this case there are several stable networks where large countries are unwilling to sign agreements with very small countries because the gain in profits in these countries is irrelevant. As a result, there is not significant gain in export profits to compensate the loss in consumer surplus and producer surplus in the large countries given by the resulting higher competition after an agreement is signed.

On the other hand, note this chapter is focussed on the cases of unbiased governments or governments biased in favour of either intermediaries or the farming sector. The reason is because it is less likely to find in the real world governments biased in favour of consumers. Nonetheless, this analysis was developed in Appendix F and some interesting theoretical results were obtained.

It was found in all the simulations that global free trade is pairwise stable when governments are biased in favour of consumers. The reason is because more trade increases competition and this positively affects consumer surplus.

However, global free trade is not the only stable network when there are asymmetries across countries. For example, in the simulation that assumes a world composed of large and very small countries, networks in which all countries have agreements with each other except the very small countries is also pairwise stable. This is because the latter countries are indifferent about signing an agreement with each other because this does not offer significant gains in consumer surplus as a consequence of their small domestic markets.

Likewise, when countries are asymmetric in farmers' productivity, less efficient countries are unwilling to sign an agreement with large countries because the additional export output in the large country pushes the agricultural price in the less efficient country up. In order to mitigate this increase, the intermediary of the latter country decreases the output sold in the domestic market depressing in this way the level of competition and, therefore, negatively affecting consumer surplus. This suggests therefore that regionalism of the south-north type can also emerge when governments are biased in favour of consumers and when there is asymmetry in farmers' productivity.

A summary of the main results obtained in this chapter is presented in the Table 4.6. On the other hand, Table 4.7 shows the cases where global free trade is not pairwise stable and when regionalism arises.

Table 4.6. Summary of the results found in the simulations

Simulation	Unbiased	Biased in favour of consumers	Biased in favour of intermediaries	Biased in favour of farming sector
1: Symmetric without farming sector	Two pairwise stable networks. One of the is global free trade	Global free trade is the only pairwise stable network	Multiple equilibria including global free trade	NA
2: Symmetric with moderate monopsonistic power	Farming sector positively affects trade: it breaks the inefficient stable network	Idem	Farming sector negatively affects trade: Countries are unwilling to keep their agreements	Farming sector positively affects trade: Only global free trade is stable.
3: Symmetric with high monopsonistic power	Idem	Idem	Idem	Idem
4: Symmetric without farming sector and endogenous tariffs	The same as in simulation 1	NA	NA	NA
5: Symmetric with moderate monopsonistic power and endogenous tariffs	Farming sector does not affect the pairwise stability of the networks in simulation 4	NA	NA	NA
6: Symmetric with high monopsonistic power and endogenous tariffs	Idem	NA	GFT can be broken	NA
11: Large and very small countries without farming sector	Multiple equilibria including global free trade	Two pairwise stable networks. Network *t* in Figure 4.5 and global free trade	Empty network and Regionalism emerges: single block formed of very small countries	NA
12: Large and very small countries with moderate monopsonistic power	Idem. Farming sector does not affect pairwise stability	Idem	Idem	Regionalism emerges: Blocks composed of same size countries

13: Large and very small countries with high monopsonistic power	Farming sector favours free in large singleton countries. The number of stable networks decreases	Idem	Idem. Farming sector prevents bilateralism between large countries in non-stable networks	Idem
14: Large and medium size countries without farming sector	Number of stable networks decreases with respect to simulation 11. Medium size countries willing to sign agreements with each other	Global free trade is the only pairwise stable network	Regionalism is lost: medium size countries unwilling to keep their agreements. Only empty network is stable	NA
15: Large and medium size countries with moderate monopsonistic power	Farming sector favours free trade. Only global free trade is stable	Global free trade is the only pairwise stable network	Idem	Regionalism emerges. Global free trade is stable
16: Large and medium size countries with high monopsonistic power	Farming sector negatively affects free trade. More pairwise stable networks emerge	Two pairwise stable network: global free trade and network *c* in Figure 4.5	Idem Unwillingness to sign bilateral agreements is reinforced	Large countries with several connections are more willing to break agreements with medium size countries. The opposite happens with large countries having few connections
17: Asymmetry in farmers' productivity	Farming sector favour trade only in more efficient countries	Global free trade is the only pairwise stable network	The same as in the symmetric case with different degree of monopsonistic power	The same as in the symmetric case with different degree of monopsonistic power

Table 4.7. Simulations where regionalism can emerge. Cells in red are the cases where global free trade is not paiwise treaty stable

Simulation	**Unbiased**	**Biased in favour of consumers**	**Biased in favour of intermediaries**	**Biased in favour of farming sector**
1: Symmetric without farming sector				NA
2: Symmetric with moderate monopsonistic power				
3: Symmetric with high monopsonistic power				
4: Symmetric without farming sector and endogenous tariffs		NA	NA	
5: Symmetric with moderate monopsonistic power and endogenous tariffs		NA	NA	NA
6: Symmetric with high monopsonistic power and endogenous tariffs		NA		NA
11: Large and very small countries without farming sector			Regionalism	
12: Large and very small countries with moderate monopsonistic power			Regionalism	Regionalism
13: Large and very small countries with high monopsonistic power			Regionalism	Regionalism
14: Large and medium size countries without farming sector				NA
15: Large and medium size countries with moderate monopsonistic power				Regionalism
16: Large and medium size countries with high monopsonistic power		Regionalism		Regionalism
17: Asymmetry in farmers' productivity				

CHAPTER FIVE: Stable Trade Networks under Alternative Stability Concepts

5.1 Introduction

In the previous chapter, the stability of international trade networks was studied using the pairwise stability concept. The main results obtained in that chapter are that when there is a farming sector global free trade is not always stable, and free trade may be prevented depending on the political biases of governments, the position of countries in the networks and the existence of asymmetries across countries.

Pairwise stability is a useful concept to predict as a first approximation the possible stable international trade architecture when countries sign bilateral agreements. This is why pairwise stability has been used as a benchmark in the current investigation. However, this concept has two main disadvantages. Firstly, it does not consider cases when countries break two or more agreements simultaneously. This implies that the set of pairwise stable networks might be overestimated by the traditional pairwise stability concept and this may affect predictions on international trade patterns in the real world. Secondly, the pairwise stability concept can only identify stable networks when countries are involved in bilateral agreements. However, it cannot inform about the possible stable networks when countries are involved in global agreements which is actually one of the approaches promoted by the World Trade Organisation in what is referred as the Doha Round.

In considering these disadvantages, the objective of this chapter is to extend the analysis by introducing two alternative stability concepts and to use them in the study of agricultural trade liberalisation. One of them is the *strongly pairwise stability* concept that allows countries to break two or more agreements simultaneously. The other concept is a novel stability concept developed in this dissertation referred to as *global treaty stability* and has the potential to identify stable networks when countries are involved in global agreements.

The chapter is organised as follows. Section 5.2 formally introduced the proposed alternative stability concepts. It also explains how these concepts provide some insights to the issue of agricultural trade liberalisation. Section 5.3 studies bilateralism under strongly pairwise stability. Section 5.4 studies global agreements under global treaty stability. As in the previous chapter, the focus is placed on the cases of politically unbiased governments and governments biased in favour of either intermediaries or the farming sector. This is because it is less likely to find in the real world cases of governments biased in favour of consumers. Nonetheless, a detailed analysis of this type of biases is provided in Appendix F. Finally, section 5.5 concludes the chapter.

5.2 Introducing Alternative Stability Concepts

Researchers in the area of International Trade Networks have adopted the pairwise stability concept developed by Jackson and Wolinsky (1996) to identify the set of stable international networks (see for instance Goyal and Joshi, 2006;

Furusawa and Konishi, 2007; Zu et al. 2011). Pairwise stability establishes that an international trade network is stable when no country has an incentive to break an existing international agreement between them, and, if two countries are not involved in an international agreement, then at least one of them does not have an incentive to form one. Formally, let $S_i(g)$ be the objective function of the government of country *i* in network *g*. A network *g* is pairwise stable if and only if: (*i*) $S_i(g) > S_i(g - g_{ik})$ for all $i \in N$ (i.e. no country is willing to break an existing agreement); and (*ii*) if $S_i(g + g_{ij}) > S_i(g)$, then $S_j(g + g_{ij}) < S_j(g)$ (i.e. if country *i* has an incentive to sign an agreement with country *j*, then the latter does not have an incentive to sign an agreement with country *i*). Note that bilateral deals often have exclusions that apply to agricultural trade and here there may be a deal covering other sectors but not agriculture. This aspect of bilateral agreements is not considered by the original model by Goyal and Joshi (2006) because their model assumes trade of a single commodity. The same strategy is considered in the current chapter as this allows focusing on key aspects of food processed goods trade without complicating the model in excess. Nonetheless, a more complex investigation of bilateral agreements is left for future research.

The reason of why pairwise stability has been adopted to identify stable networks is because the traditional Nash equilibrium concept in a network framework produces unrealistic equilibria. This is formally explained by Bloch and Jackson (2006): "*It is easy to see that the concept of Nash stability is too weak as a concept for modelling network formation when links are bilateral, as it allows for too many* *equilibrium networks. For instance, the empty network is always a Nash network, regardless of the payoff structure* (p. 309)".

While the pairwise stability has the ability to identify different stable international networks when countries are involved in bilateral agreements, there is potential for refinements that can be used to reduce the set of stable international networks. To see this, note that pairwise stability assumes that countries cannot break or sign more than one agreement simultaneously. This constitutes a strong assumption for the analysis of international trade liberalisation.

Firstly, it is reasonable to assume that signing and putting in force several bilateral agreements simultaneously is unrealistic given the large amount of resources that each bilateral negotiation demands. Nonetheless, breaking two or more agreements simultaneously is not unrealistic as this depends only on decisions made by single countries (i.e. governments cannot be forced to maintain several agreements if they don't want to). This suggests that an appropriate stability concept to study bilateral trade agreements is the one that allows countries to: (i) break two or more agreements simultaneously; and (ii) sign one agreement at time.

Secondly, a global agreement involves a commitment made by all the countries in the world. This is equivalent to sign all possible bilateral agreements by all the countries of the world simultaneously, a fact that is not captured by pairwise stability. In addition, a global agreement can only be sustained if no country has an incentive to deviate from the agreement by breaking one or more agreements simultaneously. This suggests, therefore, that an appropriate stability concept to study the issue of global agreements in agriculture is the one that allows countries to: (i) sign all the possible bilateral agreements simultaneously (i.e. to sign a global agreement); and (ii) break one or more agreements simultaneously. These considerations were introduced into the original model of Goyal and Joshi (2006) as stability concept extensions. They are described as follows.

5.2.1. A Stability Concept to Study Bilateral Agreements

Given the disadvantages of using the pairwise stability to study the issue of bilateral agreements from an international trade network point of view, it was considered that a more suitable equilibrium concept would be that of *strongly pairwise stability*. This concept was formally studied by Gilles et al. (2006), Gilles and Sarangi (2010), and Gilles et al. (2012). It was first proposed as an extension by Jackson and Wolinsky (1996) and referred to as pairwise Nash equilibrium by Bloch and Jackson (2006). Strongly pairwise stability has the property that countries are allowed to break multiple links at the same time. Moreover, the set of strongly pairwise stable networks is equal to the intersection of the sets of Nash stable networks and pairwise stable networks (Bloch and Jackson, 2006). That is, $\Omega = P \cap NE$, where Ω, P, and NE are the sets of strongly pairwise stable networks, pairwise stable networks, and Nash equilibrium networks, respectively. The main implication of this property is that the set of strongly pairwise stable networks is a subset of the set of pairwise stable networks. This is a useful property that is considered in this thesis to determine the stability of international trade networks under strongly pairwise stability.

In order to define strongly pairwise stability in the terms of the International Trade Network model, let us consider some concepts adapted from Gilles et al. (2006), Gilles and Sarangi (2010), and Gilles et al. (2012): (*i*) The marginal benefit of country *i* when breaking an international agreement with country *j* is: $D_i(g,g_{ij}) = S_i(g) - S_i(g - g_{ij}) \in R$; and (*ii*) the marginal benefit in country *i* when deleting (simultaneously) $h_i \in L_i(g)$ international agreements is $D_i(g,h_i) = S_i(g) - S_i(g - h_i) \in R$.

Using these concepts, Gilles et al. (2006), Gilles and Sarangi (2010), and Gilles et al. (2012) define:

(a) A network $g \in G$ is *link deletion proof* if for every player $i \in N$ and every neighbour $j \in N_i(g)$ it holds that $D_i(g,g_{ij}) \geq 0$. Let $D \subset G$ be the set of link deletion proof networks.

(b) A network $g \in G$ is *strong link deletion proof* if for every player $i \in N$ and every $h_i \in L_i(g)$ it holds that $D_i(g,h_i) \geq 0$. Let $D_S \subset G$ be the set of strong link deletion proof networks.

(c) A network $g \in G$ is *link addition proof* if $S_i(g + g_{ij}) > S_i(g)$ implies that $S_j(g + g_{ij}) < S_j(g)$ for all $i,j \in N$. Let $A \subset G$ be the set of link addition proof networks.

The researchers used these definitions to establish the following equilibrium concepts:

(1) A network $g \in G$ is pairwise stable if g is link deletion proof as well as link addition proof. Let $P = D \cap A \subset G$ be the set of pairwise stable networks. This is the stability concept used originally by Goyal and Joshi (2006).

(2) A network $g \in G$ is strongly pairwise stable if g is strong link deletion proof as well as link addition proof. Let $\Omega = D_S \cap A \subset G$ be the set of strongly pairwise stable networks. This is the stability concept adopted in this chapter.

Pairwise stability and strongly pairwise stability have in common that both of them are link addition proof. That is, both stability concepts allow countries to form only one agreement at time. However, they differ in that the former is link deletion proof and the latter is strong deletion proof meaning that strongly pairwise stability allows countries to break several agreements simultaneously.

5.2.2. A Stability Concept to Study Global Trade Agreements

No concept that is suitable to study global trade agreements in agriculture using a network approach was found in the literature. It is for this reason that the concept described in this section is an additional novel contribution of the current chapter. This concept was named in this thesis *Global Treaty Stability*[9].

The global treaty stability proposed in this dissertation is an extension of strongly pairwise stability that replaces the link addition proof condition by an alternative condition that has been named *global treaty proof.* This is explained as follows. Let the marginal benefit of country i when forming a global agreement be $\Gamma_i(g^c) = W_i(g^c) - W_i(g)$. A network $g \in G$ is *global treaty proof* if for at least one country $i \in N$ it holds that $\Gamma_i(g^c) \leq 0$. In words, a network $g \in G$ is *global treaty proof* if at least one country $i \in N$ does not have an incentive to form a global agreement. Let Γ be the set of global treaty proof networks and GT be the set of global treaty stable networks. Using this definition, a network g is said to be *global treaty stable* if g is strong link deletion proof as well as global treaty proof (i.e. $g \in GT = \Gamma \cap D_S$). That is, network g is global treaty stable if: (i) no country has an incentive to break one or more international agreements; and (ii) at least one country is not willing to form a global trade agreement.

It is important to highlight the fact that in contrast to strongly pairwise stability, the set of global treaty stable networks is not a subset of pairwise stable network. This is because the global treaty proof condition of this stability concept is not a subset of the link addition proof condition that characterises the pairwise stability concept. This means that it may be possible to identify global treaty stable networks that are not pairwise stable. That is, it may be possible to find different results from those

[9] This contribution was published in the Bulleting of Economic Research. See May (2016). See Appendix G.

obtained by Goyal and Joshi (2006). This possibility is explored in the simulations considered in the current chapter.

5.2.3 New insights of the proposed stability concepts

The proposed stability concepts have the potential to inform about new results in relation to the original theoretical work by Goyal and Joshi as well as the extension introduced in the previous chapter to study the issue of agricultural trade liberalisation. This is explained as follows.

Regarding the strongly pairwise stability concept, remember that it differs from pairwise stability in that the latter cannot capture cases when countries break two or more agreements simultaneously. This is because pairwise stability assumes that countries can only break a single agreement at time. To illustrate how the results from Goyal and Joshi may be affected when relaxing this assumption, consider the following analysis.

Suppose that an arbitrary network g is pairwise stable. When a country in this network breaks a single agreement in Goyal and Joshis' world, its domestic market becomes less competitive. Now, because network g is pairwise stable, the gain in the profit made by the intermediary of this country in the domestic market after the agreement is broken is not large enough to offsets the loss in consumer surplus, and the profit made in the ex-partner country. However, this balance on the welfare function may be reversed when countries are allowed to break two or more agreements simultaneously. That is, it may be the case that if a country in network *g* breaks several agreements simultaneously, the gain in profits in the domestic market is much larger because the level of competition is reduced significantly. The resulting increase in the domestic profit can potentially be large enough to offset the loss in consumer surplus and the profit made in the ex-partner countries. If this was the case, then network *g* would not be strongly pairwise stable even when being pairwise stable.

In the extended version of the model this possibility is more likely when governments have a tendency to be biased in favour of intermediaries. This is because more trade increases agricultural prices implying that intermediaries face higher costs and get paid a lower price for the finished food good given the higher competition. Consequently, breaking several links simultaneously can significantly increase the profits made by these individuals in the domestic market to the extent of offsetting any loss in welfare from consumer surplus, export profits and producer surplus. In contrast, it is more likely that breaking several links simultaneously will not affect the stability of networks when governments are biased in favour of the farming sector because, as explained above, farmers get paid higher agricultural prices in more liberalised networks and breaking one or more agreements will cause a decrease in producer surplus.

This, of course, can be reinforced in some cases when countries are asymmetric in Goyal and Joshis' world. For example, a large country may have an incentive to break simultaneously its existing agreements with small countries in order reduce the level of competition in its domestic market in order to favour their intermediaries. The increase in domestic output can be large enough to offset the loss of export profits obtained from the small countries. However, this is not so clear when there is a farming sector because breaking several links with small countries can also increase the cost faced by the intermediaries. These examples illustrate some possible implications of replacing the pairwise stability with the strongly pairwise stability. This is explored in detail in Section 5.3.

Regarding the global treaty stability concept, on the other hand, it has the ability to identify what countries in the world are willing or unwilling to sign a global agreement. In this context, centrality becomes a key feature of the analysis under this stability. Centrality is referred to a country that is highly connected to other countries that have a small number of connections. The former is said to have a central position in the network. To illustrate why centrality is important in the analysis of global free trade agreements, consider as an example the star network. That is, a network where the central country is connected to all countries of the world (i.e. the non-central countries) and the non-central countries are only connected to the central one.

It is not difficult to infer in this example that the central country is unwilling to sign a global agreement in Goyal and Joshis' world. This is because this country has already an agreement with all countries implying that it has reached the highest level of competition in the domestic market and, therefore, the highest level of consumer surplus. In addition, the central country makes high levels of export profits because non-central countries have low levels of competition as they are only connected to the former country. Thus, a global agreement does not offer the central country gains in consumer surplus. However, the agreement increases competition in non-central countries negatively affecting the export profits made by the central country.

In the extended version of the model, the unwillingness to sign an agreement by a central country may be weakened or reinforced when there is a farming sector depending on several factors such as policy biases and the level of monopsonistic power, among others. For example, a central government that has a tendency to be biased in favour of the farming sector may be less willing to sign an agreement. This is because a global agreement increases the level of competition in non-central countries negatively affecting the output that is exported by the central country. Now, because producer surplus depends on the total output that is traded by the intermediary, this means that a global agreement decreases producer surplus reinforcing the unwillingness of the central country to sign the global agreement. However, the situation is not so clear when governments have a tendency to be biased in favour of intermediaries because a global agreement also lowers the agricultural price and, therefore, the cost faced by the intermediaries.

This example illustrates the role of centrality when studying the issue of global agreements. This centrality and other relevant considerations are formally studied in Section 5.4.

Having discussed some possible implications of adopting alternative stability concepts, a detailed analysis of how these concepts affect the international network stability is presented in the following sections.

5.3 Bilateralism under strongly pairwise stability

In order to identify the strongly pairwise stable networks in the simulations presented in the previous chapter, the following sets defined in Section 5.2.1 will be used: (i) strong link deletion proof networks (i.e. D_S); link addition proof networks (i.e. A); and strongly pairwise stable networks (i.e. $\Omega = D_S \cap A$).

Because the set of strongly pairwise stable networks is a subset of pairwise stable networks (Gilles et al., 2006; Gilles and Sarangi, 2010; Gilles et al., 2012), this section only reports the cases when these sets are different. That is, only the simulations that revealed deviations from the analysis developed in the previous chapter.

5.3.1 Simulation 1: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

In the previous chapter it was found in the case of exogenous tariffs, symmetric countries with governments biased in favour of intermediaries, and not monopsonistic power (i.e. the original model Goyal and Joshi with exogenous tariffs) that the set of pairwise stable networks is $P = \{a, g, k, Eq\}$ (see Figure 4.3). However, this differs from the set of strongly pairwise stable networks. To see this, consider Table 4.2. From this table it is inferred that the sets of strong link deletion proof and link addition proof are D_S = {*a*} and *A* = {*a*, *b*, *g*, *i*, *k*, *Eq*}, respectively. This implies that the set of strongly pairwise networks in this case is $\Omega = D_S \cap A$ = {*a*}, that is, the empty network. This result revealed that biased governments in favour of intermediaries without monoponistic power can favour these individuals by breaking multiple links simultaneously. This is because the resulting higher competition in the domestic market is strong enough to cause a net gain in total profit. That is, the gain in the profit made in the domestic market is large enough to offsets the loss of profits made in the ex-partner countries.

It can also be inferred from the results obtained in the previous chapter that the incentives of governments to break one or more agreements simultaneously are reinforced when there is a farming sector. This is because this sector makes free trade more expensive to intermediaries as these individuals have to face higher marginal costs in more integrated networks. This can be seen for example when considering Simulations 2 and 3 in the previous chapter (see Sections 4.3.1.2 and 4.3.1.3). In these simulations the sets of pairwise stable networks and strongly pairwise stable networks are the same and correspond to the empty network. This means that no network other than the empty network can be sustained because free trade is expensive from the point of view of the intermediaries. This is due to the fact that more output is traded by the intermediaries in more integrated networks as can be seen in the calculations developed for networks *a*, *g* and *k* in Appendix A. This higher total output pushed the price to the farmers up which is why the intermediaries face higher costs when there is more free trade in these simulations.

5.3.2 Simulations 4 and 5: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$; and $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

A similar deviation was found in the cases of endogenous tariffs, symmetric countries with governments biased in favour of intermediaries, no monopsonistic power (i.e. the original model Goyal and Joshi with exogenous tariffs), and moderate levels of monopsonistic power (see Sections 4.3.2.1 and 4.3.2.2). It was found in the previous chapter that the set of pairwise stable networks in these simulations is $P = \{a, g, k, Eq\}$ (see Figure 4.3). However, this differs from the set of strongly pairwise stable networks which, according to Tables E.13 and E.17 in Appendix E, correspond to $\Omega = D_S \cap A = \{a\}$, that is, the empty network. In relation to Simulation 4, this difference is explained by the fact that breaking all the existing agreements by a determined country increase market power in the domestic market helping the intermediary to make higher profits in this market that offsets the loss in export profits. Nonetheless, this gain is mitigated to some extent by the optimal endogenous tariffs. That is, in contrast to the deviation explained in the previous subsection, the existence of optimal tariffs implies that a country that breaks all its agreements still imports a reduced level of food processed good causing a certain degree of competition in the domestic market. In spite of this mitigating factor, the gain in domestic profit after breaking the agreements offsets the loss of export profits.

Regarding simulation 5, on the other hand, the same factors discussed above explain why the only strongly pairwise stable network is the empty network. However, as explained in the previous subsection, the existence of a farming sector reinforces the gain in profits in the domestic market because less trade lowers the cost faced by the intermediaries.

5.3.3 Simulations 11 and 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0 for all i $\in$ N; and α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5 for all i $\in$ N.

Other deviations were found in the case of unbiased governments in networks formed of large and very small countries without monopsonistic power (i.e. Goyal and Joshis' world) and the case of unbiased governments in networks formed of large and very small countries with moderate level of monopsonistic power. In both cased it was found in Sections 4.4.1.1 and 4.4.1.2 of Chapter Four that the set of pairwise stable networks is *P* = {*m*, *t*, *x*, *z, Eq*} (see Figure 4.5). In contrast, as can be inferred from Tables E.28 and E.32, the set of strongly pairwise stable networks is Ω = {*t*, *x*}.

In both cases it can be seen that networks *m* and *z* are pairwise stable but not strongly pairwise stable. The reason is related to the incentives of the unbiased government of the large country *i* in these networks. Welfare in this country decreases when the agreement with one of the very small countries is broken. In contrast, welfare increases when the large country *i* breaks simultaneously the existing agreements with the very small.

To understand this finding, note that breaking an agreement increases the level of market power in the domestic market of the large country *i*. In the first case, this higher level of market power causes an increase in the profit made by the intermediary in this market but at the same time a decrease in consumer welfare. In the second, in addition to this gain in profits and loss in consumer surplus, there is also an increase in producer surplus because the higher level of market power in the large country translates into a higher level of output sold in this market that pushes the price paid to the farming sector up. In both cases the intermediary does not face a loss of profits made in the ex-partner country because the domestic market of this country is very small (i.e. the level of profits obtained in this market is irrelevant from the point of view of the intermediary of the large country). Consequently, the trade-off faced by the unbiased government in the first case consists of the gain in profits in the domestic market vs. the loss of consumer surplus after an agreement is broken, and the trade off in the second case consists of the gains in profits in the domestic market and producer surplus vs. the loss in consumer surplus.

According to the results, when the government breaks a single agreement, the loss of consumer surplus is larger than the gains implying a net loss of welfare. This explains why networks *m* and *z* are both pairwise stable in both simulations. In contrast, when the government breaks all the existing agreements with the very small countries simultaneously, the gains offset the loss in consumer surplus implying a net gain in welfare. This is why in this case networks *m* and *z* are not strongly pairwise stable in both cases.

It is concluded therefore that when there is no monopsonistic power or when the level of monopsonistic power is moderate, unbiased government of large countries that are connected only with small countries have an incentive to break all their agreements with these countries. Note however that as long as large countries have some agreements with other large countries, their incentives to break their agreements with the very small countries are reversed. This is why networks *t* and *x* are also strongly pairwise stable in the simulations considered in this subsection. This happens because the level of competition in the domestic markets of the large countries is relatively high when they have an agreement with each other. This implies that the gain in market power when deleting simultaneously the agreements with the very small countries is not large enough to cause a net gain in welfare.

Finally, it can be inferred from Simulation 13 (see Section 4.4.1.3 in Chapter Four) that when the level of monopsonistic power is high in networks composed of large and very small countries, the sets of pairwise stable and strongly pairwise stable networks are the same and corresponds to $P = D_S = \{t, x\}$. This means that under high levels of monopsonistic power, the existence of a farming sector plays against free trade in networks where large countries are only connected to very small countries (i.e. networks *m* and *z* in Figure 4.5). The reason is because the gain in producer surplus when breaking one or more agreements simultaneously is significantly high under these levels of monoponistic power. This gain plus the gain in profits in the domestic market are both large enough to offset the loss in consumer surplus caused by the lower level of competition after one or more agreements are broken.

5.4 Global agreements under global treaty stability

The analysis of a global agreement is based on the following sets of networks defined in Section 5.2.2: (i) strong link deletion proof networks (i.e. D_S); global treaty proof networks (i.e. Γ); and global treaty stable network, (i.e. $GT = D_S \cap \Gamma$).

Note as pointed out in Section 5.2.2 that the set of global treaty networks is not a subset of pairwise stable networks. This is because the link addition proof condition of pairwise stability does not include cases where countries are willing to sign several links simultaneously which is what characterises a global international agreement. As a consequence, the stable global treaty networks that are studied in this section do not have to be the same as the ones identified in the previous chapter. This is shown as follows.

5.4.1 Global agreements under exogenous tariffs and symmetric countries

This section studies the network stability when countries are involve in global agreements under the assumption that tariffs are placed exogenously and that countries are symmetric in terms of market size and farmers' productivity. The analysis considers the same three simulations that were carried out using the networks presented in Figure 4.3 (see Section 4.3). Each of these simulations corresponds to different levels of monopsonistic power associated with specific values of the parameter ϕ_i in equation 4.1: $\phi_i = 0$; $\phi_i = 0.5$; and $\phi_i = 1.5$ for all $i \in N$. These simulations are explained as follows.

5.4.1.1 Simulation 1: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

As explained in 4.3.1.1, this simulation converges to the original model by Goyal and Joshi (2006) because in this case there is no monopsonistic power (i.e. $\phi_i = 0$). The results obtained in this part will be used as a benchmark to evaluate deviations when the farming sector is introduced into the analysis.

The case of politically unbiased governments

In considering the information presented in Table 4.3 it was found that the sets of strong link deletion proof networks and global treaty proof networks are $D_S = \{a, b, c, d, e, f, g, j, k, Eq\}$ and $\Gamma = \{d, f, h, i, j, k, Eq\}$, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{d, f, j, k, Eq\}$.

The case of politically biased governments

Using Table 4.2 it is concluded that when governments are biased in favour of intermediaries, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *Eq*}, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*a*}.

Discussion

Let us first consider the case of politically unbiased governments. According to the results, there are four global treaty stable networks: *d*, *f*, *j* and *k*. The stability of networks *d*, *f* and *j* is explained by the fact that they contain at least one country that is not willing to sign a global agreement. The stability of network *k*, on the other hand, is explained by the fact that no country in this network is willing to break one or more agreements simultaneously. These networks are shown in Figure 5.1. The countries that are not willing to sign a global agreement are depicted as nodes with an eccentric circle.

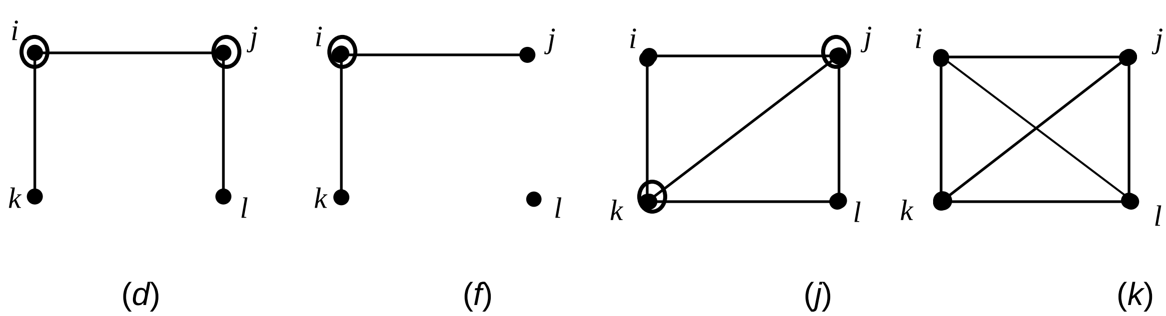


Figure 5.1. Global treaty stable networks when governments are unbiased

According to this figure, the countries that are unwilling to sign a global agreement in networks *d*, *f* and *j* are those who have a central position in the network. That is, countries having on average a larger number of agreements. To illustrate why these countries are unwilling to sign a global agreement, consider network *f*. In this network the domestic market of country *i* is more competitive than the domestic market of the other countries. This is because country *i* has bilateral agreements with countries *j* and *k* implying that these three countries compete in the domestic market of the former. This high level of competition offers the consumers in country *i* high levels of consumer surplus. On the other hand, the intermediary in this country obtains a low level of profits in the domestic market as a consequence of competition. However, this low profit is compensated by the profits that this individual makes in countries *j* and *k* as domestic market of these countries are less competitive because there are less players competing in these markets. Thus the ability of country *i* to obtain large levels of consumer surplus and profits is due to its privileged position in the network.

Now, if this country signed a global agreement, then there would be a gain in consumer surplus because its domestic market would become even more competitive. However, this would also cause a loss of profit in the domestic market of countries *i*, *j* and *k* because all these markets would become more competitive after the agreement. According to the results, this loss in profits offsets the gain in profits that the intermediary would make in the new partner-countries implying that the global agreement would cause a net loss of profits that cannot be compensated by the gain in consumer surplus. This is why country *i* is unwilling to sign such an agreement. In contrast, countries who are located more far away from the centre of the network (i.e. networks that have less agreements than country *i*) are willing to sign the global agreement because this would cause a net gain in welfare. That is, the gain in consumer surplus as a consequence of higher competition offsets the net loss of profits.

The same explanation applies to networks *d* and *j*. That is, the privileged position of countries *i* and *j* in network *d*, and the privileged position of countries *j* and *k* in network *j* is what explains why these countries are unwilling to sign a global agreement in these networks.

In relation to network *k*, on the other hand, its global treaty stability is explained by the fact that this network is strong link deletion proof. That is, no country in this network is willing to break one or more agreements simultaneously. If they did, then gain in profit in their domestic market as a consequence of lower competition would not be large enough to compensate for the loss in consumer surplus and the loss of profits that the intermediaries made in the ex-partner countries.

Let us now consider the case of governments biased in favour of intermediaries. According to the results, the only global treaty stable network in this case is network *a*, that is, the empty network. This is because a global agreement increases the level of competition in the domestic markets of the countries in the network. This causes a loss of profits in these markets that is not compensated by the additional profits that the intermediaries make in the new partner-countries.

In considering the results presented in this simulation, it is concluded that in Goyal and Joshis' world global free trade is a possible outcome when countries are involved in a global agreement and when governments are politically unbiased or biased in favour of consumers (see Appendix F). However, this possible outcome is not unique. There are other global treaty stable networks that contain countries who are unwilling or indifferent about signing a global agreement in agriculture. This is a consequence of their central position in the network. Finally, biased governments in favour of intermediaries are unwilling to sign a global agreement in agriculture because the associated high level of competition causes a net loss in profits.

5.4.1.2 Simulation 2: $\phi_i = 0.5$ *and* $\alpha_i = 1$ *for all* $i \in N$*.*

This simulation introduces the farming sector in order to assess how moderate levels of monopsonistic power (i.e. $\phi_i = 0.5$) affects the global treaty stable networks identified in the previous case.

The case of politically unbiased governments

The information presented in Table E.7 in Appendix E revealed that the sets of strong link deletion proof networks and global treaty proof networks are $D_S = \{a, b, c, d, e, f, g, i, j, k, Eq\}$ and $\Gamma = \{d, f, h, i, j, k, Eq\}$, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{d, f, i, j, k, Eq\}$.

The case of politically biased governments

When governments are biased in favour of intermediaries (see Table E.5 in Appendix E), the sets of strong link deletion proof networks and global treaty proof networks are $D_S = \{a\}$ and $\Gamma = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{a\}$. On the other hand, using Table E.6 it is inferred that when governments are biased in favour of the farming sector, the sets of strong link deletion proof networks and global treaty proof networks are $D_S = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $\Gamma = \{f, h, i, j, k, Eq\}$, respectively. Therefore the set of global treaty stable networks in this case is $GT = D_S \cap \Gamma = \{f, h, i, j, k, Eq\}$.

Discussion

According to the results, the global treaty stability of the networks identified in the previous simulation in the case of biased governments in favour of intermediaries is not affected when there is a farming sector. This means that Goyal and Joshis' model generates results that are robust under this bias and when countries are involved in global agreements.

A deviation was found in the case of unbiased governments. That is, when the agricultural sector is introduced into the model, the network *i* in Figure 4.3 becomes global treaty stable. This is explained by the incentives of the government in country *l* in this network. In the previous simulation, this country is unwilling to keep the agreement with country *k* because breaking this agreement causes a net gain in welfare. In contrast, when the farming sector is introduced, producer surplus in country *l* decreases when this country breaks the agreement with country *k*. This happens because the total output that is traded by the intermediary of the former decreases after the agreement is broken. This lower level of output implies that the farming receives a lower price negatively affecting producer surplus. This loss in producer surplus and the loss of consumer surplus are both large enough to offset the net gain in profits and this explains why the unbiased government of country *l* is unwilling to break the agreement when there is a farming sector. It is concluded therefore that the presence of this sector has a positive effect on international trade because it positively affects the incentives of governments of low integrated countries to maintain their existing agreements.

In spite of the positive effect of the farming sector, this effect is not large enough to break the global treaty stability of network *i* in favour of a global agreement. This is because the central country of this network, country *k*, obtains a higher level of welfare in this network than in global free trade, even when there is a farming sector. This is explained by the fact that this country is already connected to all countries of the world. As a consequence, a global agreement will not offer this country access to new markets. Nonetheless, a global agreement increases the level of competition in these markets negatively affecting the profits made by the intermediary of the central country. This country also faces a loss in producer surplus that is caused by the decrease in the output that is traded by this country when all the markets in the world become more competitive after the agreement. This lower output translates into a lower price that is paid to the farming sector in the central country that explains the loss in producer surplus.

Finally, the global agreement causes a loss in consumer surplus in the central country as a consequence of the decrease in the output that is sold in the domestic market of this country after the agreement. This happens because the total output that is traded by non-central countries increases when these countries get access to new markets after the agreement. This higher output pushes the price paid to the farming sector in these countries up. In response to this higher marginal cost, the intermediaries of the non-central countries adjust by reducing the output that is sold in the domestic market of the central country negatively affecting consumer surplus. Thus, because the central country faces a loss in welfare after the agreement, it is concluded that the presence of a farming sector reinforces the incentives of central countries to prevent a global agreement in agriculture.

In summary, the results for the case of unbiased governments revealed that the existence of a farming sector has a positive effect on international trade in non-central countries and a negative effect on central countries. This again proves the advantage of studying agricultural trade liberalisation using an international network approach as it allow to identify heterogeneous behaviour in the network.

Let us now analyse the case of biased governments in favour of farmers. The stable networks are presented in Figure 5.2.

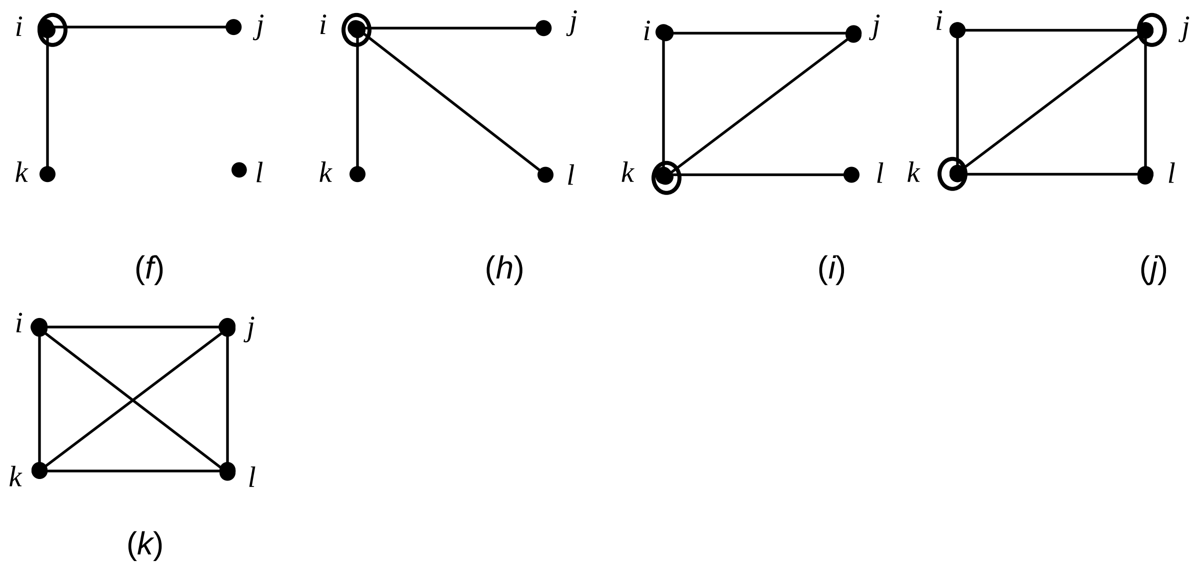


Figure 5.2. Global treaty networks when governments are biased in favour of the farming sector

In this figure the countries that are unwilling to sign a global agreement are identified as nodes with the highlighted circle. In networks *f*, *h*, *i* and *j* these countries occupy a central position in the network. The reason of why these countries are unwilling to sign a global agreement is because this position allows them to obtain higher levels of producer surplus than in global free trade. This is a consequence of the decrease in the quantity of output that is traded by the central countries after a global agreement is signed. That is, central countries have already access to all markets in the world implying that a global agreement does not offer them access to new markets. However, this agreement increases the level of competition in non-central countries and this, in turn, causes a decrease in the output that is exported to these countries. As a result of this lower output, the farming sector in the central countries receives a lower price negatively affecting producer surplus.

Network *f* is an exception because the central country is not fully connected. However, the same explanations apply here because the additional output that is exported by the central country to the new partner country *l* is not large enough to offsets the decrease in the output traded in other markets as a consequence of the higher competition after the global agreement. It is concluded therefore that the existence of a farming sector negatively affects the outcome of a global agreement negotiation because this causes a negative effect on producer surplus in countries that occupy a central position in the network.

In relation to network *k* (i.e. global free trade), on the other hand, the global stability of this network is explained by the fact that no country is willing to break one or more agreements simultaneously. If they did, then the increase in the output sold in the domestic market would not be enough to compensate the decrease in the output that was exported in the ex-partner countries. This net decrease would reduce the price paid to the farming sector negatively affecting producer surplus.

5.4.1.3 Simulation 3: $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation was introduced with the purpose of investigating whether the results obtained under moderate levels of monopsonistic power remains robust when this power is high.

The same results were obtained with only one exception. This corresponds to the stability of network *f* in the case of governments biased in favour of the farming sector. When monopsonistic power is moderate, this network is global treaty stable. However, when monopsonistic power is high, this network becomes unstable. The reason is because when monopsonistic power is high, the additional output that is exported to the new partner country *l* offsets the decrease in output caused by the higher competition positively affecting producer surplus. This happens because the intermediary of the singleton country reduces the output sold in the domestic market in a greater magnitude in order to compensate for the higher marginal cost that the intermediary faces when exporting to the new partner countries. This implies that the level of competition in this market is lower when monopsonistic power is higher. This lower competition is what explains why the output that is exported by the central country to country *l* is more significant and large enough to compensate for the decrease in output sold in the other markets. It is concluded therefore that when monopsonistic power is high, the existence of a farming sector have a positive effect on trade on central countries that are not fully connected in the network.

5.4.2 Global agreements under endogenous tariffs and symmetric countries

This section extends the analysis with the purpose of determining whether the global treaty stable networks identified in the previous simulations are affected when tariffs are placed endogenously. As in the previous chapter, the analysis only considers the case of symmetrical countries with unbiased governments given the complexity of the mathematical computations involved. However, some partial analyses of the components of the welfare function are provided.

5.4.2.1 Simulation 4: $\phi_i = 0$ *and* $\alpha_i = 1$ *for all* $i \in N$.

This simulation assumes that there is not monopsonistic power in the model (i.e. $\phi_i = 0$) implying that it converges to the original model by Goyal and Joshi (2006). The results will be used to explore how the introduction of the farming sector affects the international trade structure of processed goods.

According to the information presented in Table E.15 in Appendix E, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *b*, *c*, *e*, *g*, *k, Eq*} and Γ = {*d*, *f*, *h*, *i*, *j*, *k, Eq*}, respectively. This implies that the set of global treaty stable networks when governments are unbiased is $GT = D_S \cap \Gamma$ = {*k*}.

This result is different from the one obtained under exogenous tariffs and non-monopsonistic power (see Section 5.4.1.1). In that simulation, networks *d*, *f*, *j* and *k* are all global treaty stable. However, when tariffs are placed endogenously, networks *d*, *f* and *j* become unstable. The reason that explains this change is because there is at least one country in these networks that is willing to break one or more agreements simultaneously.

Using the information presented in Table E.15, it was found that these countries correspond to non-central countries (see Figure 4.3). In particular, it was found that country *k* has an incentive to break the agreement with country *i* and country *l* has an incentive to break the agreement with country *j* in network *d*; countries *j* and *k* are willing to break their existing agreements with country *i* in network *f*; and countries *i* and *l* have an incentive to break all their agreements simultaneously in network *j*.

To understand why the non-central countries have an incentive to break one or more agreement, consider as an example network *d*. If country *k* in this network deleted its existing link with the central country *i*, then the following changes in the components of the welfare function would be observed. Firstly, this would cause a decrease in consumer surplus because the domestic market of country *k* would become less competitive. Secondly, for the same reason, the profit made by the intermediary of country *k* in this market would increase. Thirdly, this intermediary would face a loss in profits in the ex-partner country after the agreement was broken. Finally, there is an additional gain when an agreement is broken and corresponds to the additional tariff revenue obtained by the government. This additional tariff revenue plus the gain of profits in the domestic market are both large enough to offset the losses in consumer surplus and the profit made in the ex-partner country. This net gain in welfare after an agreement is broken explains why the non-central countries are not willing to maintain their existing agreements.

On the other hand, the instability of networks *d*, *f* and *j* and the global treaty stability of global free trade (i.e. network *k*) suggest that when governments are unbiased and when tariffs are placed endogenously, countries in different networks will either break or sign some agreements until they reach a network in which all countries of the world will be willing to sign a global agreement. In Figure 4.3, these networks correspond to *a*, *b*, *c*, *e* and *g*. This optimistic view, however, only holds under the assumption of politically unbiased governments. To see this, remember that it is shown in Chapter Four in the context of pairwise stability that intermediaries can potentially make additional profits by influencing biased policymakers to place tariffs that destabilises global free trade. This possible deviation by biased governments is discussed in more detail in the next simulations.

5.4.2.2 Simulation 5 and 6: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in$ N; and $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in$ N.

In considering Tables E.20 and E.25 in Appendix E, it was found the sets of strong link deletion proof networks and global treaty proof networks in the simulations that assume moderate (i.e. $\phi_i = 0.5$) and high levels of moopsonistic power (i.e. $\phi_i = 1.5$) are the same. They correspond to $D_S = \{a, b, c, d, e, f, g, j, k, Eq\}$ and $\Gamma = \{d, f, h, i, j, k, Eq\}$, respectively. This implies that the set of global treaty stable networks in both simulations is $GT = D_S \cap \Gamma = \{d, f, j, k, Eq\}$.

This result has three main implications. Firstly, when there is a farming sector, several networks become global treaty stable. To understand this result, remember that we saw in the previous simulation that in Goyal and Joshis' world under endogenous tariffs only global free trade is global treaty stable. The reason is explained as follows.

When there is a farming sector, networks *d*, *f* and *j* become global treaty stable because an additional loss arises when the agreement with a central country is broken: producer surplus in the non-central country decreases because the farming sector obtains a lower price after the agreement is broken. Now, because the loss in consumer surplus plus the loss of profits made in the ex-partner country plus the loss in producer surplus are together larger than the gain in profits in the domestic market plus the gain in tariffs revenue, the non-central countries are unwilling to break the agreement with the central country which is what explains why *d*, *f* and *j* become global treaty stable. It is concluded therefore that the farming sector has a positive effect on trade because it prevents non-central countries from breaking their existing agreement with central countries.

The second implication is that the analysis developed under exogenous tariffs offers a reasonable approximation to analysis under endogenous tariffs because both of them concluded the same: the farming sector has a positive effect on trade because it prevents non-central countries from breaking existing agreement with central countries. Given the complexity of the study of networks under endogenous tariffs, adopting exogenous tariffs as a first approximation seems to be a good strategy.

Finally, the global treaty stability of global free trade suggests that this network could potentially be reached. However, as stated in the previous simulation, this only holds in the case of politically unbiased governments. As shown in Chapter Four in the context of pairwise stability, intermediaries can influence biased policymakers to place tariffs that can destabilise global free trade and this deviation is more likely when monopsonistic power if high. This also applies to the case of global treaty stability because if global free trade is not link deletion proof (which is what happens when the intermediary influences the biased government), then it is not strong link deletion proof either. And if it is not strong link deletion proof, it cannot be global treaty stable. It is concluded therefore that a global agreement in agriculture is unlikely when there is a farming sector and when governments are biased in favour of intermediaries.

5.4.3 Global agreements under exogenous tariffs and asymmetry in market size

A key characteristic in the real world is that countries are heterogeneous in terms of market size. According to the results obtained in the previous chapter, this type of asymmetry can affect the international network stability in the context of pairwise stability. The objective of this section is to extend the analysis to determine whether asymmetry in market size can also affect the global treaty stability when countries are involved in global agreements. As in Chapter Four, the strategy adopted in this section consists of analysing first the interaction between large countries and very small countries. The attention is then placed to the case of large and medium size countries.

5.4.3.1 Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$ for all $i \in N$.

This simulation considers a world composed of large and very small countries without monopsonistic power. That is, it corresponds to Goyal and Joshis' world under this type of asymmetry.

The case of politically unbiased governments

In considering the information presented in Table E.28 in Appendix E, it is inferred that the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *n, p*, *q*, *s*, *t*, *u*, *x*, *c', Eq*} and Γ = {*c*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *n, o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *b'*, *c', Eq*}, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*c*, *e*, *h*, *i*, *j*, *n, p*, *q*, *s*, *t*, *u*, *x*, *c', Eq*} (see Figure 4.5).

The case of politically biased governments

According to the information presented in Table E.27 in Appendix E, the sets of strong link deletion proof networks and global treaty proof networks when governments are biased in favour of intermediaries are D_S = {*a*, *d*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n, o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c', Eq*}, respectively. Therefore the set of global treaty stable networks in this case is $GT = D_S \cap \Gamma$ = {*a*, *d*}.

Discussion

Let us consider first the results obtained when governments are unbiased. The global treaty stable networks in this case are presented in Figure 5.3. In this figure large countries are represented as white nodes, very small countries as black nodes, and countries that are unwilling to sign a global agreement are represented as nodes with highlighted circles.

In comparing this figure with Figure 5.1 above, it is possible to identify several differences from the results obtained in the symmetrical countries case in Goyal and Joshis' world (see Section 5.4.1.1). Firstly, while central countries in several global treaty stable networks are still the ones who are unwilling to sign a global agreement, in most of the cases these central countries are large ones. For example, consider network *e* in Figure 5.3. In this network country *i* is unwilling to sign a global agreement because it has a privileged position in this architecture. That is, this large central country makes high export profits in the other large country *k* because the domestic market of the latter is less competitive (i.e. it contains only two competitor countries: *i* and *k*). This high profit offsets the loss in profits that is made in the domestic market of the central country as a consequence of the higher competition (i.e. it contains three competitor countries: *i*, *j* and *k*). Consequently, having the agreement with the large country *k* allows the large central country *i* to obtain a net gain in profits.

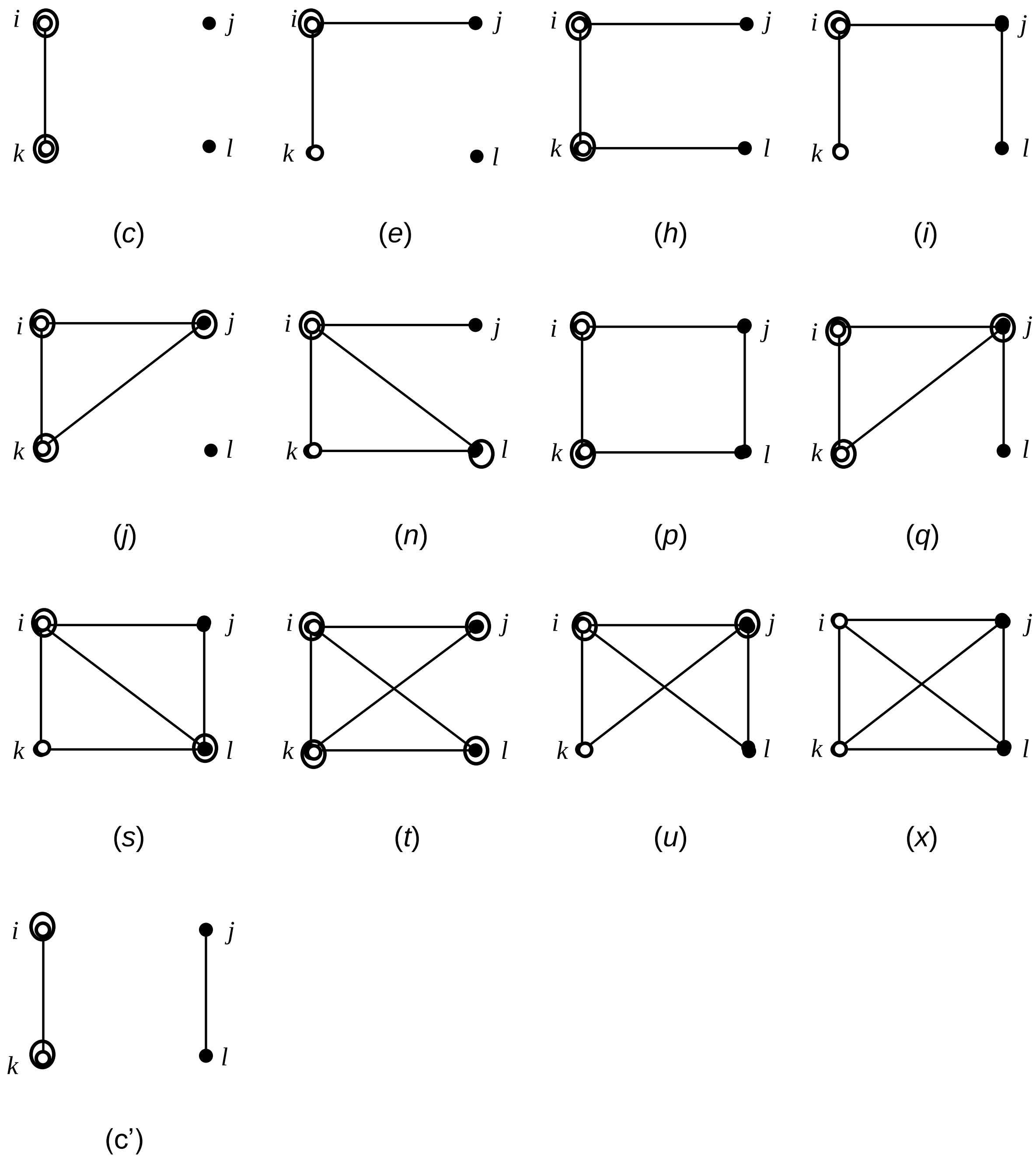


Figure 5.3. Global treaty networks when governments are unbiased

On the other hand, the agreements with countries *k* and *j* makes the domestic market of country *i* more competitive positively affecting consumer surplus. Finally, the large central country *i* does not make profits in the very small country *j* because the domestic market of the latter is very small. However, the central country is willing to keep the agreement with the small country because the latter increases the level of competition in the latter positively affecting consumer surplus. Thus, if country *i* signed a global agreement, then the loss of profits in the domestic and foreign markets as a consequence the higher competition would be large enough to offsets the gain in consumer surplus.

Now, consider network *f* in figure 4.5. This network has the same architecture as network *e*. However they differ in that the central country in network *f* is the very small country *j*. This country is also unwilling to sign a global agreement. This global agreement would affect neither the profit made in the domestic market in country *l* nor consumer surplus because the domestic markets of these countries are very small. However, the additional profit that the very small country *j* would make in the new partner country *k* is not large enough to compensate the loss of profit in the large country *i* as a consequence of the higher level of competition after the agreement.

This network, however, is not global treaty stable because the non-central large country is willing to break the agreement with the very small central country *j*. Breaking this agreement would make the domestic market of the large country less competitive positively affecting the profit made by the intermediary in this market.

This would also decrease consumer surplus. But the gain in profits is larger than the loss in consumer surplus which explains why the large country *i* is unwilling to keep the link with the central country *j*. It is concluded therefore that while central countries do not have an incentive to sign a global agreement, networks with large central countries are in general global treaty stable.

Secondly, there exist some exceptions to the conclusion given in the previous paragraph. That is, there are some global treaty stable networks that contain a very small central country. In Figure 5.3, these networks correspond to networks *q*, *s* and *u*. The stability of these networks is explained by the political motivations of large countries.

To illustrate this fact, consider as an example network *q*. In this network, the very small central country *j* is connected to the non-central very small country *l* and the non-central large countries *i* and *k.* The very small countries are indifferent about keeping or breaking their agreement because this will not affect their level of welfare as a result of the very small size of their domestic markets. However, the very small central country is willing to maintain the agreements with the large countries because this offers the former to obtain high profits in these countries. The central country is not willing to sign a global agreement because this would increase the level of competition in the large non-central countries negatively affecting the level of profits made in these countries. On the other hand, the large non-central countries are willing to keep the agreement with the very small central country because this helps the former to increase competition in their domestic markets and, therefore, to increase the level of consumer surplus. It is concluded therefore that the global treaty stability of networks that contains a very small central country depends on the incentives of large countries to maintain their agreements with the central country.

Third, it was found in the symmetric countries case that only central countries in global treaty stable networks are unwilling to sign a global agreement in agriculture. A key difference with respect to this case is that when countries are asymmetric in market size, there are global treaty stable networks with non-central countries that are also unwilling to sign an agreement. Examples of these networks are networks *c*, *j*, *n*, *q*, *s*, *t*, *u*, and *c*'. In these networks the unwillingness of non-central large countries is explained by the fact that they do not obtain additional significant profits when trading with very small countries. On the contrary, opening access to these countries makes the domestic markets of the large countries more competitive negatively affecting the profits made by the latter. This loss in profits is large enough to offsets the gain in consumer surplus caused by the higher competition.

On the other hand, non-central very small countries unwilling to sign a global agreement are always connected to large countries. This is the reason why they are against the agreement: a global agreement makes the domestic markets of large countries more competitive negatively affecting the profit made by non-central very small countries. This finding implies that occupying a central position in a network is a sufficient but not a necessary condition for a country to be against a global agreement.

Fourthly, a last difference with respect to the symmetrical case is that regionalism of the south-north type can arise when there is asymmetry in market size in the context of global treaty stability. This is reflected in networks *c* and *c*' in Figure 5.3. In these networks, the very small countries are indifferent about maintaining or breaking their existing agreement because this does not affect their levels of welfare as a consequence of the small size of their domestic markets. However, they are willing to sign a global agreement because this would offer them open access to the large countries.

The large countries, on the other hand, are willing to maintain their agreement with each other because it allows them to increase the level of competition in their domestic markets positively affecting consumer surplus and also to obtain additional profits in the partner country. However, these countries are against a global agreement because this would increase competition in their domestic markets causing a net loss in welfare after given access to the small countries.

Let us now consider the case of biased governments in favour of intermediaries. The results revealed that in this case the global stable networks are networks *a* and *d*. The stability of these networks is explained by the incentives of the large countries. These countries are unwilling to have any agreement with very small countries because they do not obtain significant profits in the latter. On the contrary, very small countries increase the level of competition in very large countries negatively affecting the profits made by the intermediaries of these countries. In addition, larger countries are unwilling to keep an agreement with each other because the additional profits that the make in the new partner country is not large enough to compensate for the loss of profits in the domestic market as a result of the higher competition. This is why the large countries in the global stable networks *a* and *d* are singletons.

In summary, it is concluded that when governments are unbiased and when there are large and very small countries in Goyal and Joshis' world, several networks become global treaty stable implying that global free trade is unlikely under this asymmetry. In these networks, central countries (normally large countries) are always against a global agreement because this causes a net loss in welfare. Non-central countries can also be against a global agreement depending on the structure of the network. Finally, if a network contains a block composed of large countries (i.e. regionalism of the south-north type), then these countries would not sign a global agreement because offering access to small countries would negatively affect their levels of welfare.

On the other hand, when governments are biased in favour of intermediaries, only networks where large countries are singletons can be global treaty stable. This is because they are unwilling to increase the level of competition in their domestic market from a global agreement as this negatively affect the profits made by the intermediaries in these countries.

5.4.3.2 Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$ for all i $\in$ N.

Having identified the stable networks when there is not a farming sector in a world composed of large and very small countries, this simulation investigates how this stability is affected when this sector is introducing into the analysis. This is done by assuming that intermediaries exercise moderate levels of monopsonistic power (i.e. $\phi = 0.5$ in all i $\in$ *N*)

The case of politically unbiased governments

In considering the information presented in Table E.32 in Appendix E it is concluded that the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *n*, *p*, *q*, *s*, *t*, *u*, *x*, *c'*} and Γ = {*b*, *c*, *e*, *f*, *g*, *i*, *j*, *k*, *l*, *n, o*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *b'*, *c', Eq*}, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*c*, *e*, *i*, *j*, *n*, *q*, *s*, *t*, *u*, *x*, *c', Eq*}.

The case of politically biased governments

It is inferred from Table E.30 in Appendix E that when governments are biased in favour of intermediaries, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *d*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n, o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c', Eq*}, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*a*, *d*}. On the other hand, using the information presented in Table E.33, it is concluded that when governments are biased in favour of the farming sector, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *c*, *d*, *c'*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *c'*, *Eq*}, respectively. Therefore the set of global treaty stable networks in this case is $GT = D_S \cap \Gamma$ = {*a*, *c*, *d*, *c'*}.

<u>*Discussion*</u>

The results for the case of unbiased governments revealed deviations from the results obtained in the previous simulation. In particular, the results revealed that when the farming sector is introduced into the analysis, networks *h* and *p* become unstable. This is explained by the fact that the large countries in these networks are unwilling to sign a global agreement when there is not a farming sector, but this incentive is reversed when intermediaries exercise a moderate level of monopsonistic power. This happens because a global agreement also decreases the level of producer surplus in the large countries. While this loss implies an additional negative effect of a global agreement from the point of view of the large countries, this also implies that the intermediaries of large countries pay a lower price to the farming sector in global free trade. This mitigates the decrease in profits which is what explains why in this case the gain in consumer surplus offsets the net decrease in profits and producer surplus.

Let us now consider the case of politically biased governments in favour of the farming sector, networks *a*, *c*, *d* and *c*' are global treaty stable. This is different from the case of symmetrical countries with moderate monopsonistic power (see Section 5.4.1.2 and Figure 5.2) in that centrality becomes irrelevant because large central countries are better off by breaking their agreements with the very small countries. This is because breaking these links decreases the level of competition in large countries positively affecting the profits made by intermediaries. This increase is accompanied by an increase in the output sold by these individuals in this market pushing the price paid to the farming sector up. This, in turn, increases producer surplus in the large countries. It is for this reason that under this type of asymmetry, it is regionalism rather than centrality what explains the difference with respect to the symmetrical case. This also explains why in the asymmetric case global free trade is not stable. That is, biased large countries in favour of the farming sector have an incentive to deviate from global free trade by breaking their links with very small countries in order to increase producer surplus.

In summary, the current simulation revealed that some of the results obtained in Goyal and Joshis' world under asymmetry in market size remain robust when the farming sector is introduced into the analysis and when monopsonistic power is moderate. However, there are some deviations that prove the fact that the presence of a farming sector can affect the international pattern of food processed goods.

Firstly, it was found that larger unbiased countries that are connected to each other may have an incentive to form a global agreement that includes very small countries. This is because the latter increases competition in the large countries reducing pressure on the price paid to farming sector mitigating the loss of profits that the intermediaries face in global free trade.

Secondly, the results revealed that when the world is composed of large and very small countries, it is regionalism rather than centrality what explains the unwillingness of large countries to sign a global agreement in agriculture. This also explains why in the asymmetric case global free trade is not stable. In this case biased governments in favour of the farming sector can increase producer surplus by breaking their links with very small countries.

5.4.3.3 Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ *and* $\phi = 1.5$ *for all* $i \in N$.

This last simulation in the case of a world composed of large and very small countries is introduced to determine how the international trade structure of processed agricultural goods is affected when the level of monopsonistic power is high (i.e. when $\phi = 1.5$ for all $i \in N$).

It was found that increasing the level of monopsonistic power does not cause deviations when governments are biased in favour of either consumers (see Appendix F), intermediaries or the farming sector. However, when governments are unbiased, the number of networks that are included in the sets of strong link deletion proof and global treaty stable networks increases. In the case of the set of strong link deletion proof networks, the new networks are *g*, *m*, *o* and *z* and, in the case of global treaty networks, the new networks are *g* and *o*.

In relation to networks are *g*, *m*, *o* and *z*, they are strong link deletion proof networks when monopsonistic power is high because the incentives of some large countries in these networks (i.e. country *i* in networks *m* and *z*, and country *k* in networks *g* and *o*) are reversed. In relation to networks *m* and *z*, the large country *i* is only connected to the very small countries *j* and *l* in these networks. Thus, when the former country breaks these agreements, market power increases in the domestic market positively affecting the output that is sold by the intermediary in this market. This does not cause a decrease in export output because the domestic market of the very small countries is irrelevant from the point of view of the very large country *i.* As a consequence, there is a net increase in output in the latter country that pushes the agricultural price up. While this implies a gain in producer surplus, it also implies that the gain in domestic profit is mitigated by the higher cost faced by the intermediary. Thus, when monopsonistic power is high, this mitigating effect is strong enough to make the gains in domestic profit and producer surplus not large enough to compensate the loss in consumer surplus. This is why in these networks the large country *i* is willing to maintain its agreements with the very small countries. That is, this is why networks *m* and *z* are strong link deletion proof when monopsonistic power is high.

In relation to networks *g* and *o*, on the other hand, the incentive of the large country *k* is reversed when there is high level of monopsonistic power. To understand this fact, note that this country is only connected to the other large country *i*. Thus when country *k* breaks the link with country *i*, there is an increase in domestic output and a decrease in export profit from the point of view of the large country *K*. According to the results obtained in Appendix C, the decrease in export output dominates implying that breaking the agreement causes a net decrease in output in country *k*. As a result of this decrease, the farming sector in this country gets paid a lower price that reinforces the gain in domestic profits by the intermediary but reduces producer surplus. Consequently, when monopsonistic power is high, the loss in producer surplus is significantly high to the extent that the gain in domestic profit is not large enough to compensate for the losses in producer surplus, consumer surplus and export profits. This is why under this level of monopsonistic power two networks *g* and *o* are strong link deletion proof.

Regarding the set of global treaty networks, the reason of why networks *g* and *o* belong to this set when monopsonistic power is high is a consequence of the fact that they become strong link deletion proof as explained above.

In considering the results obtained in the current simulations, the following implications are highlighted. Firstly, a high level of monopsonistic power has a positive effect on free trade because it prevents some countries from breaking their existing agreements with other countries. That is, the number of networks that belong to the set of strong link deletion proof network increases. However, it also plays against a global agreement because it increases the number of stable global treaty stable networks. Secondly, it is interesting to notice that centrality is not so relevant when the world is composed of large and very small countries and when the level of monopsonistic power is moderate (see the previous simulation). This is because large countries are in general unwilling to sign agreements with very small countries. However, when monopsonistic power is high, centrality becomes relevant again as proved by the stability of networks *g* and *o*. In these networks, a central large country is willing to keep its agreements with very small countries because this increases the level of competition in the domestic market of the former. This not only increase consumer surplus in the central country, but also reduces pressure on the price paid to the farming sector mitigating the high cost paid by the intermediary of this country which offers them a better competitive position in the network.

5.4.3.4 Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ *and* $\phi = 0$ *for all* $i \in N$.

The next three simulations are introduced to study global agreements when the world is composed of large and medium size countries (i.e. $\alpha = 1$ and $\tilde{\alpha} = 0.5$, respectively) under different degrees of monopsonistic power. The current simulation in particular considers the case when there is not monopsonistic power (i.e. Goyal and Joshi's world).

<u>*The case of politically unbiased governments*</u>

In considering the information presented in Table E.39 in Appendix E, it was found that the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *c*, *d*, *e*, *h*, *i*, *j*, *m*, *n*, *p*, *s*, *t*, *u*, *w*, *x*, *z*, *c'*, *Eq*} and Γ = { *b*, *c*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *c'*, *Eq*}, respectively. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*c*, *e*, *h*, *i*, *j*, *n*, *p*, *s*, *t*, *u*, *w*, *x*, *c'*, *Eq*}.

The case of politically biased governments

When governments are biased in favour of intermediaries (see Table E.38 in Appendix E), the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*}, respectively. This implies that the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*a*}.

Discussion

The results found in this simulation for the case of unbiased governments are in general the same as the ones identified for the case of large countries and very small countries (see Section 5.4.3.1). This suggests that these results remain robust when the world is composed of large countries and either medium size or very small countries. However, three deviations were identified.

The first one is related to network *t*. In Section 5.4.3.1, it was found that this network is global treaty stable because all countries in this network are indifferent about signing a global agreement. However, when countries *j* and *l* are medium size, their domestic markets are relevant. If a link is formed between these countries, then they increase welfare because they obtain additional export profits and a gain in consumer surplus due to higher competition that offsets the loss of domestic profits. In contrast, the large countries are against this agreement because this makes the domestic markets of the medium size countries more competitive negatively affecting their export profits. It is interesting to notice then that in both cases network *t* is global treaty stable. However, what changes are the incentives of the countries that belong to this network.

The second deviation corresponds to network *q*. It was found in section 5.4.3.1 that this network is global treaty stable. However, it is not stable when the world is composed of large and medium size governments. This is explained by the incentive of country *l* which is only connected to network *j*. Thus, when these countries are medium size, country *l* is better off by breaking the link with country *j* because the gain in domestic profit as a consequence of the resulting lower competition offsets the losses of consumer surplus and export profits. This explains why in the current simulation network *q* is not global treaty stable.

The last deviation identified for the case of unbiased governments is related to network *w*. That is, when the world is composed of large and very small countries, this network is not global treaty stable. In contrast, when the world is composed of large and medium size countries, this network becomes stable. This is explained by the incentives of the large country *k* which is only connected to countries *j* and *l* in this network. When the latter countries are very small, country *k* is willing to break simultaneously the agreements with these countries *j* and *l* because in autarky the gain in domestic profits offsets the loss in consumer surplus. In contrast, when countries *j* and *l* are medium size, breaking these agreements also causes a loss of export profits. This loss and the loss of consumer surplus cannot be compensated by the gain in domestic profits and this is why network *w* becomes global treaty stable in the current simulation.

Let us now consider the case of biased governments in favour of intermediaries, it was found in Section 5.4.3.1 that when the world is composed of large and very small countries, networks *a* and *d* in Figure 4.5 are both global treaty stable. However in the current simulation only network *a* is stable. This difference is explained by the incentives of countries *j* and *l*. That is, when these countries are medium size, they have large enough domestic markets to obtain a net gain in profits (i.e. gain in domestic profits offsets the loss in export profits) by breaking their agreement. This is why this network is not global treaty stable in a world composed of large and medium size countries.

In summary, it is concluded from the results obtained in this simulation that, while several results remains robust, there are some deviations that are explained by the more active role of medium size countries. In contrast to very small countries, medium size countries are not indifferent about certain trade decisions and this is what causes these deviations.

Having discussed the case of large and medium size countries in Goyal and Joshis' world, the attention is placed now on how the international trade structure is affected when there is a farming sector. This is explored in the next simulations.

5.4.3.5 Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

In this simulation, it is assumed a world composed of large and medium size countries with intermediaries that exercise moderate levels of monopsonistic power (i.e. $\phi_i = 0.5$).

The case of politically unbiased governments

In considering the information presented in Table E.43 in Appendix E, it is concluded that the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *c*, *d*, *e*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *x*, *z*, *c'*, *Eq*}, Γ = {*b*, *c*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *b'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*c*, *e*, *g*, *h*, *i*, *j*, *n*, *o*, *q*, *s*, *t*, *u*, *x*, *c'*, *Eq*}, respectively.

The case of politically biased governments

Using the information presented in Table E.41 in Appendix E, it is inferred that when governments are biased in favour of intermediaries, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*}, Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*a*}, respectively. On the other hand, using the information presented in Table E.42 it is concluded that when governments are biased in favour of the farming sector, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *b*, *c*, *d*, *e*, *g*, *h*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *w*, *x*, *z*, *b'*, *c'*, *Eq*}, Γ = {*c*, *e*, *g*, *h*, *i*, *j*, *l*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*c*, *e*, *g*, *h*, *j*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *w*, *x*, *c'*, *Eq*}, respectively.

<u>Discussion</u>

Let us start the discussion with the case of unbiased governments. According to the results, the sets of strong link deletion proof, global treaty proof, and global treaty networks are all affected when the farming sector is introduced into the international network model.

In relation to the first set, networks *g*, *o* and *q* become strong link deletion proof and network *w* does not belong to this set any longer when monopsonistic power is moderate (i.e. ϕ = 0.5). Regarding networks *g* and *o*, this change is explained by the incentive of the large country *k*, and for network *q* this is explained by the incentive of the medium size country *l*. That is, when there is not a farming sector, these countries are better off in autarky because the gain in domestic profit as a consequence of lower competition offsets the loss of consumer surplus and the loss of export profits. However, when there is a farming sector, this gain in domestic profits is mitigated to some extent by the higher cost faced by the intermediary of these countries. In addition, when country *k* breaks its agreements in networks *g* or *o* and when country *l* breaks its agreement in network *q*, there is a net decrease in the total output that is traded by the intermediary of these countries and this translates as a lower level of producer surplus. As a result of these changes, the gain in domestic profits is not large enough to compensate the losses in consumer surplus, export profits and producer surplus.

On the other hand, network *w* is not strong link deletion proof when there is moderate monopsonistic power because country *k* has an incentive to break its existing agreements with the medium size countries *j* and *l*. The reason is because this country is only linked to medium size countries implying that it obtains relatively low levels of export profits. Thus, when the large country *k* breaks its entire links, there is a net increase in the output that is traded by the intermediary pushing the price paid to the farming sector up. While this mitigates to some extent the gain in domestic output, it also increases the level of producer surplus. As a result, being in autarky is an optimal choice for country *k* because the gains in domestic profits and producer surplus offset the losses in consumer surplus and export profits. To finish this analysis, it is interesting to notice that the behaviour of a country depends on its position in the network and the current architecture of the network. For example, as discussed above, country *k* is better off in autarky when the network is *w*, and is better off when keeping its agreements when the networks are *g* and *o*. This illustrates the advantage of working with the network approach. That is, it is possible to identify individual behaviour that depends on the nature of the current network.

In relation to the set of global treaty proof networks for the case of unbiased governments, the results revealed that network *p* does not belong to this set when there is a moderate level of monopsonistic power. However, network *b'* becomes global treaty proof. Regarding network *p* this change is explained by the incentives of the large countries *i* and *k*. These countries are already connected in this network which implies that a global agreement only allows them to form a link with an additional medium size country. This means that the resulting additional export profit plus the gain in consumer surplus is not large enough to offset the loss in export profits in existing partner countries and the loss of domestic profits. This is why that when there is no farming sector, these countries are unwilling to sign a global agreement.

On the other hand, when monopsonistic power is moderate, the same effects in profits and consumer surplus are caused by a global agreement. However, this agreement also causes a decrease in the total output that is traded by the intermediaries of the large countries as a consequence of the resulting higher competition which implies that these firms pay lower agricultural prices after the agreement. This negatively affects producer surplus, but also mitigates the net loss in profits because the intermediaries face lower costs. These effects of agricultural prices make the gain in consumer surplus and the gain in export profits in the new partner country large enough to offset the losses in domestic profits, export profits in existing partner-countries and producer surplus. Thus, because in the current simulation the large countries are better off in global free trade, they have an incentive to support a global agreement and this is why network *p* is not global treaty proof in this case.

In relation to network *b'*, on the other hand, it becomes global treaty proof because the incentives of the medium size countries *j* and *l* are reversed. To understand this change, note that these countries are only connected to a large country. Thus, when there is a moderate level of monopsonistic power, the export expansion after the global agreement causes a net increase in the total output that is traded by the intermediaries of the medium size countries *j* and *l*. This pushes the price paid to the farming sector up positively affecting producer surplus but negatively affecting the profits made by these intermediaries as they face a higher cost. As a result, the gains in consumer surplus, export profits in new partner countries and producer surplus after the global agreement are not large enough to compensate the losses in domestic profits and the export profits in existing partner countries. This is why in the current simulation the medium size countries are unwilling to sign a global agreement (i.e. why they are global treaty proof).

Finally, the results revealed that set of global treaty stable networks in the case of unbiased governments in the current simulation includes three new networks (networks *g*, *o* and *q*), but it does not includes networks *p* and *w*. This is a consequence of the changes described above for the sets of strong link deletion proof and global treaty proof networks.

Let us now consider the case of politically biased governments in favour of intermediaries. The results revealed no differences with respect to the previous simulation suggesting that the introduction of a farming sector does not affect the international trade structure when governments have this bias.

Finally, in relation to the case of governments biased in favour of the farming sector, deviations were identified with respect to the simulation assuming symmetrical countries with moderate monopsonistic power (see Section 5.4.1.2). Firstly, in the symmetrical countries case, it was found that all networks are strong link deletion proof. That is, no country has an incentive to break one or more agreements simultaneously. However, when the world is composed of large and medium size countries and when monopsonistic power is moderate, networks that contains medium size countries with high degree of centrality (e.g. networks *f*, *i*, *h*, *l*, *r*, *v*, *y*, and *a'*) are not strong link deletion proof.

The reason is because non-central large countries have an incentive to break their agreement with the medium size country because the domestic market of the latter is not large enough to make significant profits and also because the degree of competition in this market is high given by the centrality of the country. Thus, if a large country breaks an existing agreement, the increase in output traded by the intermediary of this country in the domestic market will be larger than the decrease in export output. This net increase in output pushes the agricultural price up positively affecting producer surplus in the large country. This is why, in the asymmetrical case networks with medium size countries occupying a central position in the networks are not strong link deletion proof. However, this does not happen when central countries are large countries (e.g. network *g*). It is inferred therefore that in the real world it is more likely to find large countries occupying a central position in the network.

Secondly, it was found in the symmetrical case that when monopsonistic power is moderate and when governments are biased in favour of the farming sector, networks containing countries that occupy a central position are global treaty stable because they are unwilling to sign a global agreement (see the discussion provided in Section 5.4.1.2). However, when the world is composed of large and medium size countries, only networks with central large countries and networks composed of blocks of countries of the north-south type can be global treaty stable. As explained in the previous paragraph, this is because networks containing central medium size countries cannot be global treaty stable as large countries are unwilling to keep their agreements with them. It is concluded therefore that when the world is composed of large and medium size countries and when monopsonistic power is moderate, it is expected to be found global treaty stable networks having large central countries as well as networks characterised by regionalism of the north-south type.

5.4.3.6 Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$ for all $i \in N$.

To finish with the issue of asymmetry in market size, this last simulation in this section studies the global treaty stability of the international trade system when the world is composed of large countries and medium size countries and when monopsonistic power is high (i.e. $\phi = 1.5$). The results are shown as follows.

The case of politically unbiased governments

In considering the information presented in Table E.47 in Appendix E it is concluded that the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *c*, *d*, *e*, *g*, *h*, *i*, *j*, *m*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *x*, *z*, *c'*, *Eq*}, Γ = {*b*, *c*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *b'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*c*, *e*, *g*, *h*, *i*, *j*, *n*, *o*, *q*, *s*, *t*, *u*, *x*, *c'*, *Eq*}, respectively.

The case of politically biased governments

Using the information presented in Table E.45 in Appendix E it is inferred that when governments are biased in favour of intermediaries, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*}, Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*a*}, respectively. On the other hand, using the information presented in Table E.46 it is concluded that when governments are biased in favour of the farming sector, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *b*, *c*, *d*, *h*, *j*, *m*, *n*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *b'*, *c'*, *Eq*}, Γ = {*c*, *e*, *g*, *h*, *i*, *j*, *l*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *a'*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*c*, *h*, *j*, *n*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *c'*, *Eq*}, respectively.

Discussion

According to the results obtained in the current and previous simulations, increasing monopsonistic power does not affect the international trade system when governments are unbiased or when governments are biased in favour of intermediaries. However, deviations were found in the cases of governments biased in favour of consumers and in favour of the farming sector. This is explained as follows (see Appendix F for the case of governments biased in favour of consumers).

In the case of governments biased in favour of intermediaries, the increase in monopsonistic power affected the sets of strong link deletion proof and global treaty stable networks. In the first set, networks *v* and *y* become strong link deletion proof. However, networks *e*, *g*, *o*, *p* and *q* are not included in this set. The reason is explained by the incentives of the large countries contained in these networks. That is, when monopsonistic power is high, trading the processed food is costly for the intermediary of the large country. In this case an agreement with a medium size country that is highly integrated does not help the intermediary to export a relevant quantity of the processed good. But it increases competition in the domestic market of the large country releasing in this way pressure on the agricultural price. This lower cost allows the intermediary of this country to increase the output that is sold in the domestic market, and this additional output is what increases producer surplus when the agreement is maintained. In other words, breaking the agreement is not beneficial for the large country because it negatively affects producer surplus when monopsonistic power is high, and this is why network *v* is strong link deletion proof in this case.

In relation to networks *e*, *g*, *o*, *p* and *q*, on the other hand, they are not strong link deletion proof for the opposite reason. That is, in this case when monopsonistic power is high, large countries are unwilling to keep their agreements with medium size countries that have few links. The reason relies on the fact that the domestic markets of these medium size countries are less competitive as a consequence of the small number of agreements. This lower competition implies that the output that is exported by the intermediary of a large country is relevant and contributes in increasing the price paid to the farming sector. This higher cost negatively affects the output that is sold by the intermediary of the large country in the domestic market. As a result, the agreement causes a net decrease in the output that is traded by the intermediary of the large country negatively affecting producer surplus. This is why this country is better off when breaking the agreement.

In relation to the set of global treaty stable networks, the results revealed that high levels of monopsonistic power influence the number and composition of the networks that are included in this set. This is explained by the changes described above for the case of governments biased in favour of the farming sector.

In summary, the results obtained in this simulation revealed that increasing monopsonistic power can affect the international trade structure when governments are biased in favour of either consumers or the farming sector. In the case of consumers, high levels of monopsonistic power negatively affect free trade because it becomes costlier from the point of view of medium size countries that can increase consumer surplus in less integrated networks. In the case of governments biased in favour of the farming sector, high levels of monopsonistic power affects the incentives of large countries in different ways depending on whether they are connected to medium size countries with high or low degree of trade integration. If a large country is connected to medium size countries that are highly integrated, then this country will keep the agreements in order to lower the cost faced by intermediaries. In contrast, if the large country is connected to medium size countries with low degree of integration, then the agreement will be broken because this will allow the large country to lower cost, increase domestic output and producer surplus. Finally, the scope of a global agreement is reduced when there is a high degree of monopsonistic power and when governments are biased in favour of consumers or the farming sector.

5.4.4 Global agreements under exogenous tariffs and asymmetry in farmers' productivity

The simulations developed in this chapter have revealed that the existence of a farming sector strongly influences the international trade architecture and the incentives of countries to sign a global agreement. This is because monopsonistic power exercised by the intermediaries directly affects the cost faced by them and producer surplus, and indirectly consumer surplus throughout the externalities caused by this type of imperfection.

Given the relevance of monopsonistic power in the extended version of the international trade model offered in this dissertation, the objective of this section is to extend the analysis to determine whether asymmetry in farmers' productivity (i.e. different levels of monopsonistic power across countries) affects countries' incentives to sign a global agreement. For this purpose, the same simulation developed in Section 4.4.2 is considered in this analysis. This is presented as follows.

5.4.4.1 Simulation 17: $\delta = 3$ *for* $\Omega = \{i, k\}$; $\delta = 1$ *for* $\Psi = \{j, l\}$; $\phi = 0.5$; *and* $\alpha = 1$.

The information that was used in this simulation is presented in Tables E.48, E.49, E.50 and E.51 (see Appendix E). From these tables, the following relevant sets of networks were obtained.

The case of politically unbiased governments

In considering Table E.51 it is inferred that the sets of strong link deletion proof and global treaty proof networks are D_S = {*a*, *b*, *c*, *d*, *h*, *i*, *j*, *m*, *n*, *p*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*} and Γ = {*c*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *c'*, *Eq*}, respectively. Consequently, the set of global treaty stable networks in this case is given by $GT = D_S \cap \Gamma$ = {*c*, *h*, *i*, *j*, *n*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *c'*, *Eq*}.

The case of politically biased governments

It is inferred from Table E.49 in Appendix E that when governments are biased in favour of the intermediaries, the sets of strong link deletion and global treaty proof networks are D_S = {*a*} and Γ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*}, respectively. Consequently the set of global treaty stable networks in this case is $GT = D_S \cap \Gamma$ = {*a*}. On the other hand, it was inferred from Table E.50 that the sets of strong link deletion proof and global treaty proof networks when governments are biased in favour of the farming sector are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and Γ = {*c*, *e*, *g*, *h*, *i*, *j*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *y*, *z*, *a*', *c*', *Eq*}, respectively. Consequently, the set of global treaty stable networks in this case is $GT = D_S \cap \Gamma$ = {*c*, *e*, *g*, *h*, *i*, *j*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *y*, *z*, *a*', *c*', *Eq*}.

Discussion

Let us first consider the case of politically unbiased governments. In this case, deviations were found in the sets of strong link deletion proof, global treaty proof and global treaty stable networks with respect to the simulations that consider monopsonistic power with symmetrical countries (see Sections 5.4.1.2 and 5.4.1.3). In relation to the first set, the configuration captured by network *f* in Figure 4.3 when countries are symmetric is not strong link deletion proof in the current simulation (see networks *e* and *f* in Figure 4.5). This happens because the non-central inefficient country in networks *e* and *f* in Figure 4.5 has an incentive to break the existing agreement with the central country. This happens because when the agreement is broken, there is a net decrease in the total output that is traded by the intermediary of country the inefficient country pushing the agricultural price down. As a result of this lower cost, the gain in domestic profit due to lower competition is reinforced to the extent that this gain offsets the losses in producer surplus, consumer surplus and export profits.

On the other hand, when governments are symmetric, the configuration represented by network *i* in Figure 4.3 is strong link deletion proof. When countries are asymmetric, this only happens when the less connected country is an efficient one (i.e. networks *n* and *q* in Figure 4.5). However, when the less connected country is inefficient, this country has an incentive to break its existing agreement for the same reason explained above.

In relation to the set of global treaty proof networks under the assumption of unbiased governments, it was found in the symmetrical case that the configuration represented by network *g* in Figure 4.3 is not global treaty proof meaning that all countries in this configuration are willing to sign a global agreement. However, in the asymmetric case this configuration is global treaty proof when the singleton is an efficient country (i.e. network *j* in Figure 4.5) but not when the singleton is an inefficient one (i.e. network *m* in Figure 4.5). This is explained by the incentive of the efficient country *j* in these networks. In network *l* this country is only connected to the inefficient countries *i* and *k*. Thus because this country has a better competitive position than the inefficient countries, it makes high profits in these countries. A global agreement is therefore not a good option for country *j* because the other efficient country *l* will compete in the same markets reducing the profits that it makes in the inefficient countries causing a net loss in welfare.

In contrast, in network *m* the efficient country *j* is connected to the efficient country *l* and the inefficient country *i*. Thus, because country *j* is already competing with the efficient country *l*, a global agreement does not cause a significant loss of export profits in the existing partner countries. On the contrary, it allows the efficient country *j* to access the inefficient country *k* that implies a significant increase of export profits in the later as well as a gain in consumer surplus given by higher competition. This is why network *j* is global treaty proof, but network *m* is not.

On the other hand, it was found that network *w* in the asymmetric case (see Figure 4.5) is global treaty proof. However, the equivalent network when countries are symmetric (i.e. network *e* in Figure 4.3) is not global treaty proof. This is explained by the incentives of the inefficient countries *i* and *k*. These countries are unwilling to sign a global agreement because it causes a significant increase in the output that is traded by the intermediaries of these countries. Now, because monopsonistic power in these countries is high with respect to the efficient countries, this increase in output causes a significant increase in the agricultural price in countries *i* and *k* negatively affecting the profits made by the intermediaries of these countries. This loss in profits is too large to be compensated by the gains in producer surplus and consumer surplus.

Let us now consider the case of politically biased governments. According to the results obtained in the current simulation, there are no differences between the symmetric (see Sections 5.4.1.2 and 5.4.1.3) and the asymmetric cases when governments are biased in favour of either consumers (see Appendix F) or firms. However, deviations were found when governments are biased in favour of the farming sector. Firstly, in the symmetrical case, the configuration represented by network *f* in figure 4.3 is global treaty stable when there is moderate level of monopsonistic power (see Section 5.4.1.2), but it is not global stable when monopsonistic power is high (see Section 5.4.1.2).

In contrast, when countries are asymmetric in farmers' productivity, this configuration is stable in most of the cases (see networks *e*, *z* and *a'* in Figure 4.5), except network *f* in Figure 4.5. The reason is because in this network all countries have an incentive to sign a global agreement because this causes an expansion of export output that increases producer surplus. However, in the other related configurations the country that occupies a central position in the network faces a decrease in producer surplus when a global agreement is signed. What these configurations have in common is that the efficient countries are not connected, and this is the key feature that explains why a central country is unwilling to sign the global agreement. That is, when an agreement is signed, there is a significant increase in competition in the efficient countries that depress the output that is traded by the intermediaries of these countries in the domestic market.

This also negatively affects the export output of the central country in the existing partner efficient country reducing in this way pressure on the agricultural price in the former country. This lower cost causes an increase in the output that is traded by the intermediary of the central country in the domestic market. However, this gain and the additional export output in the new partner countries after the agreement are not large enough to compensate the loss of export output in the existing partner countries. This implies that the agreement causes a net decrease in output in the central country and this, in turn, decreases producer surplus in this country which is why it is unwilling to sign the global agreement. In contrast, this does not happen in network *f* in Figure 4.5 because the efficient countries are already connected and, as a result, a global agreement does cause the competitive externality on the central country that was described above.

Other network configurations that become global treaty stable in the asymmetric case are the ones represented by networks *h*, *l* and *y* in Figure 4.5 (which are equivalent to network *d* in Figure 4.3 in the symmetrical case), and the one represented by network *j* in Figure 4.5 (which is equivalent to network *g* in Figure 4.3 in the symmetrical case). In all these networks, there are inefficient countries that are unwilling to sign a global agreement because this causes a significant increase in competition when efficient countries are fully incorporated. This competition effect causes a net decrease in the output that is traded by the intermediary of the inefficient country negatively affecting producer surplus. A final network structure that becomes global treaty stable in the case of asymmetric countries is network *c'*. This case is interesting because this is a form of regionalism of the south-north type and is explained by the unwillingness of inefficient countries to sign a global agreement for the same reason described above: this agreement strongly increases competition by fully incorporating the efficient countries which negatively affect producer surplus in the inefficient countries. Thus, asymmetry in farmers' productivity can also lead to regionalism when governments are biased in favour of the farming sector.

In summary, it is concluded from the results obtained in the current simulation that asymmetry in farmers productivity plays in general against free trade when governments are unbiased because inefficient countries have less incentives to keep their agreements. The only exception is the star network with an efficient central country. Only this configuration becomes strong link deletion proof implying that under this type of asymmetry, efficient countries that occupy a central position in the network prevents countries from breaking their existing agreements.

It is also inferred that when governments are unbiased and when countries are asymmetric in farmers' productivity, there are more network structures having countries unwilling to sign a global agreement. This explains why the number of networks in the set of global treaty stable networks increases when countries are asymmetric in farmers' productivity.

Finally, asymmetry in farmers' productivity also influences the international trade structure when governments are biased in favour of the farming sector. In this case, new network configurations become global treaty stable and this is explained mainly by the unwillingness of inefficient countries to sign a global agreement. That is, a global agreement causes a significant impact on competition when fully integrating more efficient countries. This higher competition negatively affects the output that is traded by the intermediaries of the inefficient countries and this, in turn, decreases producer surplus in these countries. One example of these networks is a networks composed of two blocks, one formed of efficient countries and the other of inefficient countries. This network is interesting because it shows that asymmetry in farmers' productivity can also lead to regionalism of the south-north type.

5.5 Summary and conclusions

The objective of this chapter is to extend the analysis of international trade networks by adopting two alternative stability concepts to the traditional pairwise stability. The first one, referred to as strongly pairwise stability, allows countries to break one or more agreements simultaneously and it was adopted to determine the stability of international networks when countries are involved in bilateral agreements. The second stability concept, referred to as global treaty stability, was introduced to identify stable networks when countries sign global agreements. The results obtained in the current chapter and Appendix F for the case of governments biased in favour of consumers are described as follows.

In relation to the strongly pairwise stability, the results revealed few deviations with respect to the traditional pairwise stability proving that the latter can be considered as a reasonable stability concept to study bilateral agreements. However, key deviations corresponds to the following cases: (1) biased governments in favour of intermediaries with symmetrical governments; and (2) unbiased governments in networks formed of large and very small countries.

In the first case, it was found that governments have an incentive to break all their agreements simultaneously in order to increase the profits made by the intermediaries, and this deviation is reinforced when there is a farming sector because breaking these links reduces the cost faced by these firms.

In the second case, it was found that the number of stable networks decreases because large unbiased governments that are not connected to other large countries have an incentive to break their existing agreements with very small countries and this happens when there is no monopsonistic power or when this type of imperfection is moderate. Note however that this incentive is reversed when large countries are connected to other large countries. The reason is because having agreements with very small countries increases the level of competition in the domestic markets of large countries releasing pressure on the price paid to the farming sector. This, in turn, allows the intermediaries of large countries to improve their competitive position as they face lower costs.

In relation to global treaty stability, several new results were identified. Firstly, when countries are symmetrical and when tariffs are placed exogenously, centrality (i.e. a country that have a significant number of agreements with countries that have few agreements) plays a key role in explaining the unwillingness of some countries to sign a global agreement, and this is in general reinforced when there is a farming sector. This is explained by the fact that a country benefits when occupying a central position in the network. On the one hand, its domestic market is highly competitive given the large number of agreements which implies that consumer surplus is high in the central country. On the other hand, a central country exports a significant amount of output to third countries because they have less competitive domestic markets as a consequence of their few agreements. This allows a central country to obtain high levels of profits and producer surplus.

This country is unwilling to support a global agreement because it increases competition in third countries negatively affecting producer surplus and profits, and also because this competition increases the level of trade in third countries. As a result, the intermediaries of these countries pay higher agricultural prices that cause a decrease in the output that is exported to the central country negatively affecting consumer surplus.

Secondly, when countries are symmetric, when tariffs are placed endogenous, and when there is no monopsonistic power (i.e. in Goyal and Joshis' world), the only global treaty stable network is global free trade. However, when there is a farming sector, centrality becomes an important factor in explaining the unwillingness of countries to sign a global agreement. This difference relies on the fact that non-central countries have an incentive to break their agreements with the central country when there is not a farming sector. However, when this sector is present, this incentive is reversed given the contribution of the agreement in increasing producer surplus in the non-central countries.

Thirdly, when the world is composed of large and very small countries under the assumption of exogenous tariffs, unbiased governments and non-monopsonistic power, centrality becomes relevant mainly in large countries because they are unwilling to sign a global agreement that includes very small countries. This is because the latter increases the level of competition in large countries negatively affecting profits made in these countries. However, there are some structures where very small countries are also unwilling to sign a global agreement and this happens when these countries are already connected to large countries. Finally, regionalism of the south-north type is also a possible outcome and this happens because welfare in large countries in a trading block is reduced when very small countries are included as they increase the level of competition without offering significant export profits.

On the other hand, when there is a farming sector, the number of stable networks that include countries that are unwilling to sign a global agreement increases and this is more evident when monopsonistic power is high. This is due to the effect of the agreement in increasing the cost faced by the intermediaries in the player countries. However, when monopsonistic power is moderate, the farming sector can impact in favour of a global agreement in some networks structures where large countries are already connected with each other and when they have an agreement with a very small country. In these structures, a global agreement does not cause a significant expansion of exports in the large countries. But it increases the level of competition in their domestic markets that lowers the prices paid to the farming sector. This lower cost mitigates the loss of profits caused by higher competition to the extent that the gain in consumer surplus offsets the losses in profits and producer surplus.

Fourthly, when the world is composed of large and very small countries under the assumption of exogenous tariffs and politically biased governments, the following results were found. When governments are biased in favour of consumers, all networks are global treaty stable independently of the existence of a farming sector. This is because very small countries are indifferent about signing a global agreement because this does not offer them significant gains in consumer surplus.

Alternatively, when governments are biased in favour of the intermediaries, both the empty network and a network in which only the very small countries are connected are global treaty stable. This is explained by the fact that the very small countries are indifferent about keeping or breaking their agreement with each other because this does not offer them significant gains in profits. Finally, when governments are biased in favour of the farming sector, regionalism becomes a likely outcome for two reasons.

On the one hand, large countries are unwilling to include very small countries in a global agreement because of the decrease in producer surplus that results from the agreement. On the other hand, very small countries are indifferent about breaking their agreements with each other because there is no significant change in producer surplus.

Fifthly, when the world is composed of large and medium size countries under the assumption of exogenous tariffs, unbiased governments and non-monopsonistic power, the same general patterns than the ones identified in the case of a world composed of large and very small countries were identified. The only difference is that in contrast to the very small countries, medium size countries are not indifferent about certain decisions because their domestic markets have a relevant size that allow these countries to obtains certain gains in profits and consumer surplus. This is more evident when there is monopsonistic power because in this case the composition of the networks in the set of global treaty stable networks changes. New existence of networks in this set is explained by the incentives of medium size countries to keep their agreements as this allow them to obtain net gains in welfare. However, other networks become unstable because the unwillingness of central countries to sign a global agreement is reversed as they can make relevant export profits in medium size countries. In any case, centrality and regionalism are still key factors that explain in general the unwillingness of countries to sign a global agreement.

Sixthly, when the world is composed of large and medium size countries under the assumption of exogenous tariffs and politically biased governments, the following results were found. When governments are biased in favour of consumers, centrality becomes the main factor in explaining countries' unwillingness to sign a global agreement when there is no monopsonistic power or when this power is moderate. In this case, global treaty stable networks are unwilling to sign a global agreement because the resulting increase in trade in non-central countries increase agricultural prices in these countries reducing the export to the central country and negatively affecting consumer surplus. However, when monopsonistic power is high, several networks become global treaty stable suggesting that the high cost faced by intermediaries in global free trade has a detrimental effect on welfare on large countries and countries that occupy a central position.

Under this level of monopsonistic power, regionalism is also a possible outcome. However, in this case the medium size countries are the ones that are against a global agreement because this agreement causes a significant export expansion to the large countries pushing the price paid to the farming sector down. This lowers the total output that is traded by the intermediary in the domestic market negatively affecting consumer surplus. On the other hand, when governments are biased in favour of intermediaries, the same results were obtained for all levels of monopsonistic power: the only global treaty stable network in this case is the empty networks. This happens in general because higher competition decreases the output made by the intermediary and this is reinforced by the higher costs that they face when there is more free trade. Finally, when governments are biased in favour of the farming sector, centrality (mainly in large countries) and regionalism become relevant in explaining countries' unwillingness to sign a global agreement. However, when monopsonitic power is high, the number of networks in the set of global treaty stable networks decreases because large countries are unwilling to keep their agreements with medium size countries.

Finally, when there is asymmetry in farmers' productivity, it was found that this type of asymmetry plays in general against free trade when governments are unbiased because inefficient countries have fewer incentives to keep their agreements. The only exception is the star network with an efficient central country. It was also found that the number of stable networks containing countries that are unwilling to sign a global agreement increases with respect to the symmetrical case. This explains why the number of networks in the set of global treaty stable networks increases when countries are asymmetric in farmers' productivity.

Asymmetry in farmers' productivity also influences the international trade structure when governments are biased in favour of the farming sector. In this case new network configurations become global treaty stable and this is explained mainly by the unwillingness of inefficient central countries to sign a global agreement. On the other hand, regionalism is also a possible outcome and this is explained by the unwillingness of inefficient countries to sign a global agreement. This is because this agreement incorporates the efficient countries that have a significant impact on competition that negatively affect producer surplus in inefficient countries.

A summary of the main results found in the simulations for the case of global treaty stable networks is presented in Table 5.1. On the other hand, Table 5.2 shows the cases where centrality and regionalism become relevant. It also shows in what cases global free trade is not global treaty stable.

Having described some of the most relevant stable networks under strongly pairwise stability and global treaty stability, the attention is placed now on how to break inefficient stable networks by means of lump sum transfers. This is the topic of the next section.

Table 5.1. Summary of the results found in the simulations

Simulation	Unbiased	Biased in favour of consumers	Biased in favour of intermediaries	Biased in favour of farming sector
1: Symmetric without farming sector	Global treaty stable networks related to centrality	Global treaty stable networks related to centrality: indifferent central countries	Only the empty network (autarky)	NA
2: Symmetric with moderate monopsonistic power	Farming sector positively affects trade on non-central countries but negatively on central	Idem. However central countries are unwilling rather than indifferent	Idem	Negative effect of the farming sector on trade in central countries
3: Symmetric with high monopsonistic power	Idem	Idem	Idem	Reverse incentives of central countries in some networks
4: Symmetric without farming sector and endogenous tariffs	Some networks become unstable as central countries are willing to break agreements	NA	NA	NA
5: Symmetric with moderate monopsonistic power and endogenous tariffs	More global treaty networks become stable. Farming sector prevents breaking agreements of non-central countries.	NA	NA	NA
6: Symmetric with high monopsonistic power and endogenous tariffs	Idem	NA	NA	NA
11: Large and very small countries without farming sector	Mainly large countries unwilling to sign a global agreement. Regionalism emerges.	All networks are global treaty stable. Very small countries indifferent about this agreement	Empty network and Regionalism emerges: blocks of large and very small countries	NA
12: Large and very small countries with moderate monopsonistic power	Idem. However there are networks where large countries are willing to sign with very small. Centrality less important.	Idem	Idem	In contrast to the symmetrical case, centrality is not relevant. Regionalism emerges.
13: Large and very small countries with high monopsonistic power	Farming sector prevents links to be broken. Number of global treaty stable networks increases. Centrality becomes relevant for large countries.	Idem	Idem	Idem
14: Large and medium size countries without farming sector	Mainly large countries unwilling to sign a global agreement. Regionalism emerges.	Centrality becomes relevant	Regionalism is lost: small countries unwilling to keep their agreements	NA

15: Large and medium size countries with moderate monopsonistic power	Change in the composition of the set of global treaty stable networks	Incentives to sign a global agreement increase: more competition in non-central countries increases consumer surplus in central. However, there are more incentives to break links unilaterally	Idem	Blocks become stable (regionalism) and networks with central countries
16: Large and medium size countries with high monopsonistic power	Idem	Farming sector negatively affects free trade: consumer surplus increases in less integrate networks	Idem	Large countries have incentives to break agreements with medium size countries that have low degree of integration
17: Asymmetry in farmers' productivity	Inefficient countries less incentives to keep their agreements, except the star network with efficient networks in the centre.	The same as in the symmetric case	The same as in the symmetric case	Regionalism emerges: less efficient countries unwilling to sign global agreements that include efficient countries

Table 5.2. Simulations where centrality and regionalism can emerge. Cells in red are the cases where global free trade is not global treaty stable

Simulation	Unbiased	Biased in favour of consumers	Biased in favour of intermediaries	Biased in favour of farming sector
1: Symmetric without farming sector	Centrality	Centrality		NA
2: Symmetric with moderate monopsonistic power	Centrality	Centrality		Centrality
3: Symmetric with high monopsonistic power	Centrality	Centrality		Centrality
4: Symmetric without farming sector and endogenous tariffs		NA	NA	
5: Symmetric with moderate monopsonistic power and endogenous tariffs	Centrality	NA	NA	NA
6: Symmetric with high monopsonistic power and endogenous tariffs	Centrality	NA	NA	NA
11: Large and very small countries without farming sector	Centrality Regionalism	Centrality Regionalism	Regionalism	
12: Large and very small countries with moderate monopsonistic power	Centrality Regionalism	Centrality Regionalism	Regionalism	Regionalism
13: Large and very small countries with high monopsonistic power	Centrality Regionalism	Centrality Regionalism	Regionalism	Regionalism
14: Large and medium size countries without farming sector	Centrality Regionalism	Centrality		
15: Large and medium size countries with moderate monopsonistic power	Centrality Regionalism	Centrality		Centrality Regionalism
16: Large and medium size countries with high monopsonistic power	Centrality Regionalism	Centrality Regionalism		Centrality Regionalism
17: Asymmetry in farmers' productivity	Centrality Regionalism	Centrality		Centrality Regionalism

CHAPTER SIX: Compensatory Payments

6.1 Introduction

A key result that was obtained by Goyal and Joshi (2006) in the original network model is that global free trade is the efficient network because it is in this network where global welfare (i.e. the sum of welfare in each country of the world) is maximised. Since in this framework, global free trade is always pairwise stable, these researchers used this result to support the claim that bilateralism is a reasonable strategy to reach this network (ie. bilateralism is a path towards global free trade). In line with this research, Furusawa and Konishi (2005) developed a closely related model and found that while global free trade is the efficient network, it is not pairwise stable when countries have different levels of industrialization. However, this network can be stabilised by means of international compensatory payments across countries. The reason is because gains from trade are Pareto improving in this model implying that as long as the gainers could compensate the losers and still be better off, there are (potential) gains from trade. Consequently, if payments across countries are adopted to compensate losers, then governments would be willing to sign bilateral agreements until global free trade is reached.

The use of this type of payments has been considered before in the literature of networks in general. The aim is to use transfers between nodes (i.e. inter-node transfers) with the purpose of reaching the efficient network defined as the one that maximises the aggregate level of countries’ payoffs. In this research, the tension between efficiency and stability can eventually be addressed by using side-payments. That is, the efficient network can be stabilised when nodes have the facility to make these payments. However, this strategy only works in some cases that depend on the network model under consideration and the assumptions that are included in these models (see Jackson and Wolinsky, 1996; Bloch and Jackson, 2007).

In relation to the current research, the use of compensatory payments (i.e. inter-node transfers) is a reasonable strategy to facilitate agricultural trade liberalisation for several reasons.

Firstly, it is also the case that bilateral agreements do have transfers. For example, the CAP is actually a type of wealth re-distribution among member states and, in the agreement allowing a country (e.g. Norway) access to the European Single Market, this country has to pay a contribution to the EU budget which is then distributed across EU member states (Shucksmith et al. 2005). Likewise, the *aid for trade* introduced by the World Trade Organisation is a type of transfer from developed to less developed countries to help the latter to engage in international trade (Silva and Nelson, 2012; Huhne et al. 2014). These examples illustrate the fact that the introduction of side-payments to favour agricultural trade liberalisation may be feasible.

Secondly, the results obtained in Chapters Four and Five revealed that global free trade is not always stable when there is a farming sector. This suggests therefore that the use of compensatory payments could potentially be adopted to stabilise global free trade in order to take advantage of gains from trade of food processed goods. While a similar approach was adopted by Furusawa and Konishi (2005), the current research differs in the type of problem that is considered. That is, in Furusawa and Konishis' framework, the instability of global free trade is explained only by differences in the level of industrialisation between countries. In contrast, in the current research, as revealed in the previous chapters, the instability of global free trade is explained by the existence of a farming sector that affects the marginal cost faced by intermediaries, existence of asymmetry in market size across countries, existence of asymmetry in farmers' productivity across countries, and governments' political biases. These sources of instability are the focus of the current chapter.

Thirdly, the stability of global free trade does not mean that this network can be reached when there are multiple equilibriums because countries can eventually be trapped in an inefficient equilibrium. The existence of multiple equilibria not only was identified by Goyal and Joshi, but also in the extended model in the simulations developed in the previous chapters. In this context, the adoption of compensatory payments may eventually be used to break inefficient equilibriums in favour of free trade.

Fourthly, while in Goyal and Joshi (2006) and Furusawa and Konishi (2005) global free trade is the efficient network, it was found in the current investigation that there are cases where other networks are efficient (e.g. when there is asymmetry in market size and when there is a farming sector). The use of compensatory payments can also be considered as an alternative to reach these networks.

Fifthly, the adoption of side-payments to stabilise the efficient network have only been considered in the context of pairwise stability. This chapter extends this approach in order to explore also how free trade can be favoured in the context of strongly pairwise stability and global treaty stability. In relation to the latter stability concept, this extension implies that compensatory payments not only have the potential to favour bilateral agreements, but also global agreements which is one of the aims of the WTO in relation to agricultural goods.

Finally, this chapter also explores the use of *intra-node payments* (i.e. compensatory payments within a country e.g. from consumers to farmers in the same country) as an alternative tool to either break inefficient networks or reach global free trade. The main advantage of this alternative is that it can be used to complement inter-node payments particularly when their applicability is limited. For example, it is reasonable to adopt an inter-node payment from a developed country to a less developed one in order to favour trade. However, payments across rich countries or from poor countries to developed countries may be politically unfeasible. In this respect, intra-node payments can potentially be used in these cases as they do not involve transfers across countries.

This chapter is organised as follows. Section 6.2 discusses the issue of efficiency and stability in international networks and explains what networks in the simulations developed in the previous chapters are efficient. Section 6.3 formally introduces and defines the inter-node and intra-node payments. Section 6.4 studies the impact of these payments on the international trade system when countries are involved in bilateral and global agreements. The analysis is focussed only on the simulations that consider the farming sector because the aim of introducing these payments in the context of the current investigation is to provide insights of how to facilitate agricultural trade liberalisation. Finally, Section 6.5 summarises and concludes.

6.2 Efficiency vs. stability

In the network literature, a network is said to be efficient when the aggregate level of nodes' payoff is maximised in this network (Bloch and Jackson, 2007). Formally, let *N* = {1, 2, …, N} be the set of nodes, *G* = {a, b, …, G} the set of networks that can be formed with the nodes in *N*, and $S_i(g)$ the payoff obtained by node *i* in network g. Using these notations, network g is said to be efficient when $\sum_i^N S_i(g) > \sum_i^N S_i(h)$ for all network $h \neq g$ in *N*.

This definition in the context of international trade implies that the efficient network is the one that maximises global welfare. In the simulations developed in the previous chapters for the case of symmetrical countries, the efficient networks are presented in the following table.

Table 6.1. Global welfare and efficient networks for symmetrical countries

	Simulation					
Network	1	2	3	4	5	6
a	1.5000	1.2000	0.8568	1.7356	1.4124	1.0288
b	1.6388	1.3112	0.9364	1.8054	1.4592	1.0480
c	1.7776	1.4224	1.0160	1.8756	1.5040	1.0748
d	1.8264	1.4572	1.0400	1.8910	1.5146	1.0818
e	1.8752	1.5000	1.0712	1.9068	1.5264	1.0900
f	1.7326	1.3811	0.9855	1.8482	1.4867	1.0647
g	1.7814	1.4250	1.0176	1.8640	1.4993	1.0735
h	1.8132	1.3480	0.9005	1.8867	1.5083	1.0771
i	1.8620	1.4817	1.0570	1.9022	1.5219	1.0869
j	1.8976	1.5148	1.0810	1.9134	1.5306	1.0932
k	1.9200	1.5360	1.0980	1.9200	1.5360	1.0972

In this table, the first column shows the networks that were considered in the six simulations assuming symmetric countries (see Figure 4.3). The other columns show global welfare in each network used in simulations 1, 2, 3, 4, 5 and 6. These numbers were obtained by adding each line in Tables E.3, E.7, E.11, E.15, E.20, and E.25 in Appendix E. Finally, the numbers in red correspond to the highest global welfare in each simulation.

According to this table, network *k* (i.e. global free trade) is the efficient network in all simulations. This is not surprising for simulations 1 and 4 because they correspond to the original model of Goyal and Joshi under exogenous and endogenous tariffs, respectively. In the rest of the simulations, the introduction of the farming sector does not affect the efficiency of global free trade in the sense that it is still the most efficient network. This is explained by the fact that countries obtain in global free trade, high levels of consumer surplus as a consequence of high competition and high levels of producer surplus as a consequence of the large quantity of output that is exported. These gains are large enough to offsets the lower level of profits made by the intermediaries.

In Goyal and Joshi global free trade is pairwise stable independently of any political bias of governments. This means that in this paradigm compensatory payments are not needed to stabilise global free trade. However, when governments are biased in favour of intermediaries, when there is a farming sector, and when tariffs are placed exogenously, only the empty network is pairwise stable (see Simulations 2 and 3 in Chapter Four). Likewise, in the context of strongly pairwise stability and global treaty stability, global free trade is not stable in Simulations 1, 2, 3, 4 and 5 when governments are biased in favour of intermediaries. Thus, because global free trade offers a higher level of global welfare, it is reasonable to use in these cases compensatory payments to direct the world towards this network. This may be done by means of inter-node payments as well as intra-node payments because consumers and the farming sector would be willing to compensate the intermediaries for the loss in profits when passing from inefficient networks to global free trade as long as the gainers and losers are all better off after the payment. In the particular case of intra-node payments, this would be an option to deal with cases involving politically biased governments in favour of trade losers. That is, because these governments care about the payoffs obtained by trade losers rather than aggregate welfare, a redistribution of wealth from trade gainers to losers within a country can lead to a Pareto improving change when reaching a more integrated network.

On the other hand, there are several cases where global free trade is stable (i.e. pairwise, strongly pairwise or global treaty stable) but not unique. For example, when governments are unbiased, network *g* is also pairwise stable in simulations 1, 4, 5 and 6, and when governments are biased in favour of the farming sector, networks *h*, *i*, *j* and *k* are all global treaty stable in Simulations 5 and 6. The problem in cases of multiple stable networks is that the world could eventually reach an inefficient stable network and remains trapped in this network. Under this circumstance, a compensatory may be used to break the stability of the inefficient network in order to reach global free trade.

Let us now consider the efficient networks in the simulations developed under the assumption of asymmetric countries. This is shown in the following table.

Table 6.2. Global welfare and efficient networks for asymmetrical countries

	Simulation						
Network	11	12	13	14	15	16	17
a	0.7500	0.6000	0.4284	0.9376	0.7500	0.5356	1.1102
b	0.8194	0.6876	0.5227	1.0244	0.8280	0.6024	1.2123
c	0.8888	0.7114	0.5080	1.0764	0.8612	0.6152	1.1804
d	0.7500	0.6000	0.4284	0.9724	0.7776	0.5554	1.2214
e	0.9132	0.7668	0.5847	1.1182	0.9034	0.6602	1.2067
f	0.8194	0.6876	0.5228	1.0477	0.8573	0.6139	1.2889
g	0.9244	0.7985	0.6339	1.1468	0.9278	0.6874	1.2889
h	0.9376	0.8178	0.6548	1.1600	0.9486	0.7030	1.3212
i	0.9132	0.7668	0.5847	1.1415	0.9227	0.6748	1.3554
j	0.9376	0.7978	0.6144	1.1485	0.9326	0.6841	1.2961
k	0.8888	0.7752	0.6172	1.1230	0.9402	0.7028	1.3638
l	0.8888	0.7456	0.6809	1.1200	0.8846	0.6427	1.3410
m	0.8438	0.7259	0.5735	1.0780	0.8761	0.6432	1.3246
n	0.9488	0.8304	0.6667	1.1771	0.9597	0.7136	1.3496
o	0.9244	0.7985	0.6339	1.1586	0.9417	0.6952	1.4578
p	0.9376	0.8178	0.6548	1.1718	0.9564	0.7090	1.3792
q	0.9376	0.7978	0.6144	1.1688	0.9460	0.6964	1.3632
r	0.9132	0.7919	0.6297	1.1503	0.9321	0.6867	1.3480
s	0.9488	0.8304	0.6667	1.1859	0.9670	0.7175	1.3919
t	0.9600	0.8466	0.6846	1.1942	0.9850	0.7336	1.3754
u	0.9488	0.8360	0.6731	1.1859	0.9670	0.7175	1.3919
v	0.9376	0.8178	0.6548	1.1776	0.9582	0.7074	1.4084
w	0.9386	0.8178	0.6548	1.1718	0.9564	0.7090	1.3792
x	0.9600	0.8466	0.6846	1.2000	0.9820	0.7308	1.4106
y	0.9132	0.7919	0.6297	1.1415	0.9283	0.6853	1.3425
z	0.8438	0.7259	0.5735	1.0662	0.8670	0.6388	1.2658
a’	0.8888	0.7453	0.5673	1.0997	0.8844	0.6356	1.2820
b’	0.8888	0.7752	0.6172	1.0112	0.9060	0.6692	1.3144
c’	0.8888	0.7114	0.5080	1.1112	0.8888	0.6350	1.2916

In these simulations, global free trade corresponds to network *x* (see Figure 4.5). As shown in Table 6.2, this network is the unique efficient network only in simulation 14. That is, in a world composed of large countries and medium size countries without a farming sector (i.e. Goyal and Joshis’ world under this type of asymmetry). However, when the world is composed of large and very small countries (i.e. Simulations 11, 12 and 13), network *t* and global free trade are both efficient networks and this holds independently of the existence of a farming sector.

The reason of why network *t* is also efficient is related to the fact that in this network only the very small countries *j* and *l* are not connected (see Figure 4.5). Thus, if they signed an agreement, then the impact on global welfare would be irrelevant given their very small domestic markets. On the other hand, when the world is composed of large and medium size countries with a farming sector (i.e. Simulations 15 and 16), only network *t* is the efficient network. This is explained by the fact that the farming sector increases the cost faced by the intermediaries in more integrated networks. Thus, if the medium size countries *j* and *l* broke their agreement, they would stop exporting the processed food good to each other negatively impacting the price paid to the farming sector in these countries. In response to this lower cost, the intermediaries in countries *j* and *l* would increase the level of exports to the existing partner large countries increasing the level of competition in these countries. This would cause a significant increase in consumer surplus in the large countries. At the same time, the domestic markets of countries *j* and *l* would become less competitive positively affecting the output that is exported by the large countries to the former. This would cause and increase in profits and producer surplus in the large countries. These positive gains in large countries when the agreement between the medium size countries is broken are large enough to offset the losses faced by the latter and this is why network *t* is the efficient under this type of asymmetry.

Finally, according to the information presented in Table 6.2, network *o* is the unique efficient network when there is asymmetry in farmers' productivity (i.e. Simulation 17). The reason of why this network is the efficient network is explained by the large contribution of the inefficient country *i* to global welfare. This country occupies a central position in the network implying that the domestic market in this country has a high level of competition. As a consequence, consumer surplus in this country is large. The high level of competition in the central country also significantly decreases the price paid to the farming sector. This lower cost allows the intermediary of country *i* to be more competitive and to obtain higher export profits. These gains obtained by the inefficient country for occupying a central position are large enough to offset the losses in other countries causing a positive net contribution to global welfare. This is why network *o* is the efficient network when countries are asymmetric in farmers' productivity.

The existence of efficient networks other than global free trade suggests that reaching less integrated networks may not necessarily be a bad outcome when there are asymmetries across countries. However, putting the effort on these networks is against the spirit of the WTO. In spite of this, the current chapter investigates how compensatory payments can be used to reach both global free trade and also the other efficient networks in the simulations presented in Table 6.2. It is also explored how inefficient stable networks can be broken. How inter and intra-node payments can be used for these purposes is the topic of the next section.

6.3 Inter-node and intra-node payments

Inter-node and intra-node payments are lump sum transfers that are given by gainers from free trade (i.e. either bilateral or global free trade) to losers in order to achieve a Pareto improvement outcome reflected as a more efficient stable network (note however that it may be the case that a third country that is not involved in transfer activities may be worse off after more free trade is created as a consequence of transfers because this may cause an externality in this country. In other words, transfers may only be Pareto improving for countries that are involved in transfer activities depending on the nature of the current network. This is explained in more detail in the next sections).

Inter-node payments in particular correspond to lump sum transfers that are given by gainer countries to loser countries. That is, they are transfers across countries. In contrast, intra-node payments correspond to lump sum transfers given by gainer groups within a country to loser groups within the same country. That is, they are transfers across groups of people that belong to the same country. How these payments can be used to stabilise the efficient network or break inefficient ones is explained next.

6.3.1 Inter-node payments

The description and formal definition of an inter-node payment given in this subsection was obtained from Furusawa and Konishi (2005, 153). Following these researchers, a transfer from country i to country j is defined as the amount $T_{ij}(g) \in R$ given from i to j in network g such that $T_{ij}(g) = -T_{ji}(g)$. A transfer system of the network g is $T(g) = (T_{ij}(g))_{(i,j)\in g}$ such that $T_{ij}(g) = -T_{ji}(g)$ for any link $g_{ij} \in g$. A network with transfers $(g, T(g))$ is a pair of a network and an associated transfer system. Country i's payoff under $(g, T(g))$ is given by $S_i(g, T(g)) = S_i(g) + \sum_{j\in N} T_{ji}(g)$.

As explained above, an inter-node payment can be used for two purposes: to stabilise global free trade; and to break inefficient stable networks in favour of free trade. This is formally explained as follows.

6.3.1.1 Inter-node transfers to stabilise global free trade

As in Furusawa and Konishi (2005), to stabilise global free trade requires a redefinition of pairwise stability that includes the inter-node transfers across countries. This approach is extended to include also the strongly pairwise stability and the global treaty stability concepts. For this purpose, the definitions for the sets of link addition proof, link deletion proof, and strong link deletion proof proposed by Gilles et al. (2006), Gilles and Sarangi (2010), and Gilles et al. (2012) described in Section 3.4.1 are considered in this sub-section to define the concepts of pairwise and strongly pairwise stability with transfers. The definition of the set of strong link deletion proof proposed by these researchers and the definition of the set of global treaty proof proposed in this dissertation (see Section 5.2.2) are also employed to re-define the global treaty stability concept.

Formally, let $D_i(g, T(g), g_{ij}) = S_i(g, T(g)) - S_i(g - g_{ij}) \in R$ be the marginal benefit of country i when breaking an international agreement with country j and when cancelling the inter-node transfers between these countries. On the other hand, let $D_i(g, T(g), h_i) = S_i(g, T(g)) - S_i(g - h_i) \in$ R be the marginal benefit in country i when deleting (simultaneously) $h_i \in L_i(g)$ international agreements and when cancelling all the inter-node transfers between the ex-partner countries. Finally, let $\Gamma_i(g^c, T(g^c)) = S_i(g^c, T(g^c)) - S_i(g)$ be the marginal benefit of country i when forming a global agreement with inter-node transfers. Using these concepts, the following definitions are considered:

(a) A network $g \in G$ is *link deletion proof* with inter-node transfers if for every player $i \in N$ and every neighbour $j \in N_i(g)$ it holds that $D_i(g, T(g), g_{ij}) \geq 0$. That is, no country has an incentive to cut a link and thereby eliminate the transfer from (or to) that partner. Let $D_T \subset G$ be the set of link deletion proof networks with inter-node transfers.

(b) A network $g \in G$ is *strong link deletion proof* with inter-node transfers if for every player $i \in N$ and every $h_i \in L_i(g)$ it holds that $D_i(g, T(g), h_i) \geq 0$. That is, no country has an incentive to cut one or more links simultaneously and thereby eliminate the transfer(s) from (or to) the ex-partner(s). Let $D_{ST} \subset G$ be the set of strong link deletion proof networks with inter-node transfers.

(c) A network $g \in G$ is *link addition proof* with inter-node transfers if $S_i(g + g_{ij}) > S_i(g, T(g))$ implies that $S_j(g + g_{ij}) < S_j(g, T(g))$ for all $i,j \in N$. That is, for any pair of unlinked countries, at least one of them has no incentive to form the link with any feasible inter-node transfer. Let $A_T \subset G$ be the set of link addition proof networks with inter-node transfers.

(d) A network $g \in G$ is *global treaty proof* with inter-node transfers if for at least one country $i \in N$ it holds that $\Gamma_i(g^c, T(g^c)) \leq 0$. In words, a network $g \in G$ is *global treaty proof* with inter-node transfers if at least one country $i \in N$ does not have an incentive to form a global agreement with any feasible inter-node transfer. Let Γ_T be the set of global treaty proof networks with inter-node transfers.

Used these definitions, following equilibrium concepts are defined:

(1) A network $g \in G$ is pairwise stable with inter-node transfers if g is link deletion proof with inter-node transfers as well as link addition proof with inter-node transfers. Let $P_T = D_T \cap A_T \subset G$ be the set of pairwise stable networks with inter-node transfers.

(2) A network $g \in G$ is strongly pairwise stable with inter-node transfers if g is strong link deletion proof with inter-node transfers as well as link addition proof with inter-node transfers. Let $\Omega_T = D_{ST} \cap A_T \subset G$ be the set of strongly pairwise stable networks with inter-node transfers.

(3) A network g is said to be global treaty stable with inter-node transfers if g is strong link deletion proof with inter-node transfers as well as global treaty proof

with inter-node transfers. Let $GT_T = D_{ST} \cap \Gamma_T \subset G$ be the set of global treaty stable networks with inter-node transfers.

6.3.1.2 Inter-node transfers to break inefficient stable networks in favour of free trade

An inefficient network g is either pairwise or strongly pairwise stable when two conditions are satisfied: no country in this network is willing to break an existing agreement (or several agreements simultaneously for the case of strongly pairwise stability); and if a country is willing to sign an agreement with another country, then the latter is not willing to sign this agreement. In considering these conditions, it is inferred that the stability of this network can be broken in favour of free trade when the second condition (i.e. proof link addition) is altered by an inter-node transfer. Because the pairwise stability and strongly pairwise stability share the same condition, this strategy can be adopted to break both types of stability.

Formally, assume that the inefficient network g is either pairwise or strongly pairwise stable. In addition, assume that it holds for countries i,j that $S_i(g + g_{ij}) > S_i(g)$ and $S_j(g + g_{ij}) < S_j(g)$ (i.e. country i has an incentive to sign an agreement with country j but the latter is not willing to sign the agreement). An inter-node transfer system $T_{ij}(g + g_{ij})$ given in network $g + g_{ij}$ is said to be able to break the stability of the inefficient network g in favour of free trade when $S_i(g + g_{ij}, T(g + g_{ij})) > S_i(g)$ and $S_j(g + g_{ij}, T(g + g_{ij})) > S_j(g)$. That is, this transfer system can break network g when country j is compensated by the loss after free trade and when country i is still willing to sign the agreement after the agreement. Note that this condition implicitly states that countries *i* and *j* are both better off after the agreement. But it does not consider the welfare effect of the agreement on third countries. It is for this reason that this transfer is not considered as a Pareto improving tool. It is considered a tool that can potentially lead the world to the efficient network.

Now, suppose that the inefficient network *g* is global treaty stable. As before, this stability requires two conditions: no country has an incentive to break one or more agreements simultaneously; and at least one country is not willing to sign a global agreement. This stability can be broken in favour of global free trade when the second condition is altered by an inter-node transfer. Formally, assume that $S_i(g^c) < S_i(g)$ in country *i*. An inter-node transfer system $T(g^c)$ given in network g^c can break the global treaty stability of *g* when $S_i(g^c, T(g^c)) > S_i(g)$. That is, when country *i* is compensated by the loss from trade by means of the transfer system.

Having described the concepts that are needed to explore the role of inter-node payments as free trade facilitators, the attention is placed now on the intra-node transfers. This is discussed in the next subsection.

6.3.2 Intra-node payments

Intra-node payments correspond to lump sum transfers that are given by a particular sector in a country to another sector in the same country (e.g. from consumers to the farming sector in a determined country). They correspond to a welfare redistribution strategy that compensates a loser sector from free trade by a gainer sector in a way that all the sectors in the country are better off. That is, it is a Pareto improving strategy from the point of view of the country (note however that a third country may negatively be affected by this transfer through the resulting free trade implying that it may not be Pareto improving from the point of view of the world). The advantage of this type of transfer is that it only depends on the decision made by a single country because it involves domestic welfare redistribution. As a consequence, it is easier to implement because it does not requires the consent of several countries. However, as it is demonstrated in Corollary 6.2 and Proposition 6.3 below, the disadvantage of this transfer is that it only works in cases of politically biased governments.

Let us now offer a formal definition for intra-node transfers. Let's assume the existence of two sets of sectors in country following generic welfare function in country *j*: $S_j(g) = S_j^{(u)}(g) + S_j^{(v)}(g)$ where $S_j^{(u)}(g)$ and $S_j^{(v)}(g)$ are the payoffs obtained by sectors *u* and *v* in country *j* and network *g*, respectively. An intra-node transfer is defined as the amount $Tr_{uv}(g) \in R$ given from sector *u* to sector *v* in country *j* and network *g* such that $Tr_{uv}(g) = -Tr_{vu}(g)$. That is, this is a compensatory payment that benefits sector *v*. This definition can also be extended to more sectors as follows. Let $U = \{u_1, \ldots, u_n\}$ be the sectors in country *j* that pay intra-node transfers in network *g*, and let $V = \{v_1, \ldots, v_m\}$ be the sectors that receive these transfers. An intra-node transfer system is defined in this case as *Tr*(*g*) such as $\sum_{i \in U} Tr_{u_i v}(g) = -\sum_{k \in V} Tr_{u v_k}(g)$.

As in the case of inter-node transfers, an intra-node payment can also be used to stabilise global free trade and to break inefficient stable networks in favour of free trade. This is discussed as follows.

6.3.2.1 Intra-node transfers to stabilise global free trade

The same approach considered to stabilise global free trade in the case of inter-node transfers is adopted in the case of intra-node transfers. That is, the stability concepts are redefined in order to account for these transfers. This is done as follows.

Let $D_i(g, Tr(g), g_{ij}, Tr(g - g_{ij})) = S_i(g, Tr(g)) - S_i(g - g_{ij}, Tr(g - g_{ij})) \in R$ be the marginal benefit of country i when breaking an international agreement with country j and when $Tr(g) \neq Tr(g - g_{ij})$. On the other hand, let $D_i(g, Tr(g), h_i, Tr(g - h_i)) = S_i(g, Tr(g)) - S_i(g - h_i, Tr(g - h_i)) \in R$ be the marginal benefit in country i when deleting (simultaneously) $h_i \in L_i(g)$ international agreements and when $Tr(g) \neq Tr(g - h_i)$. Finally, let $\Gamma_i(g^c, Tr(g^c)) = S_i(g^c, Tr(g^c)) – S_i(g)$ be the marginal benefit of country i when forming a global agreement with intra-node transfers. Using these concepts, the following definitions are considered:

(a) A network $g \in G$ is *link deletion proof* with intra-node transfers if for every player $i \in N$ and every neighbour $j \in N_i(g)$ it holds that $D_i(g, Tr(g), g_{ij}, Tr(g - g_{ij})) \geq 0$. That is, no country has an incentive to cut a link and thereby modify the transfers from (or to) different sectors within the country. Let $D_{Tr} \subset G$ be the set of link deletion proof networks with intra-node transfers.

(b) A network $g \in G$ is *strong link deletion proof* with intra-node transfers if for every player $i \in N$ and every $h_i \in L_i(g)$ it holds that $D_i(g, Tr(g), h_i, Tr(g - h_i)) \geq 0$. That is, no country has an incentive to cut one or more links simultaneously and thereby modify the transfers from (or to) different sectors within the country. Let $D_{STr} \subset G$ be the set of strong link deletion proof networks with intra-node transfers.

(c) A network $g \in G$ is *link addition proof* with intra-node transfers if $S_i(g + g_{ij}, Tr(g + g_{ij})) > S_i(g)$ implies that $S_j(g + g_{ij}) < S_j(g)$ for all $i,j \in N$. That is, if country i is willing to sign an agreement with country j after adopting a intra-node transfer system, then country j is unwilling to sign this agreement. Let $A_{Tr} \subset G$ be the set of link addition proof networks with intra-node transfers.

(d) A network $g \in G$ is *global treaty proof* with intra-node transfers if for at least one country $i \in N$ it holds that $\Gamma_i(g^c, Tr(g^c)) \leq 0$. In words, a network $g \in G$ is *global treaty proof* with intra-node transfers if at least one country $i \in N$ does not have an incentive to form a global agreement with any feasible intra-node transfer. Let Γ_{Tr} be the set of global treaty proof networks.

Used these definitions, following equilibrium concepts are defined:

(1) A network $g \in G$ is pairwise stable with intra-node transfers if g is link deletion proof with intra-node transfers as well as link addition proof with intra-node transfers. Let $P_{Tr} = D_{Tr} \cap A_{Tr} \subset G$ be the set of pairwise stable networks with intra-node transfers.

(2) A network $g \in G$ is strongly pairwise stable with intra-node transfers if g is strong link deletion proof with intra-node transfers as well as link addition proof with intra-node transfers. Let $\Omega_{Tr} = D_{STr} \cap A_{Tr} \subset G$ be the set of strongly pairwise stable networks with intra-node transfers.

(3) A network g is said to be global treaty stable with intra-node transfers if g is strong link deletion proof with intra-node transfers as well as global treaty proof with intra-node transfers. Let $GT_{Tr} = D_{STr} \cap \Gamma_{Tr} \subset G$ be the set of global treaty stable networks with intra-node transfers.

6.3.2.2 Intra-node transfers to break inefficient stable networks in favour of free trade

As in the case of inter-node transfers, an inefficient pairwise or strongly pairwise stable network can be broken by intra-node payments in favour of free trade by altering the link addition proof condition of countries that are unwilling to sign additional agreements. Before explaining how this can be done, it is important to describe first a desirable property that an intra-node payment has to complete. This property corresponds to the idea that an intra-node transfer system has to be

Pareto improving from the point of view of the country that adopts this payment. This is explained in detail as follows. Let us consider the following generic welfare function in country j that is composed of $U = \{u_1,\ldots, u_n\}$ sectors that pay intra-node transfers $\sum_{i\in U} Tr_{u_i v}(g) = -\sum_{k\in V} Tr_{uv_k}(g)$ and $V = \{v_1,\ldots, v_m\}$ sectors that receive these transfers: $S_j(g) = \sum_{i\in U} S_j^{(u_i)}(g) + \sum_{k\in V} S_j^{(v_k)}(g)$. Suppose now that $\sum_{i\in U} S_j^{(u_i)}(g+g_{ij})$ > $\sum_{i\in U} S_j^{(u_i)}(g)$ for all $u_i \in U$ and $\sum_{k\in V} S_j^{(v_k)}(g+g_{ij})$ < $\sum_{k\in V} S_j^{(v_k)}(g)$. That is, suppose that all the sectors in U are better off after an agreement between countries i and j is signed and all sectors V are worse off. We say that an intra-node transfer system given in network $g + g_{ij}$ is Pareto improving from the point of view of country j when the following conditions are satisfied:

(i) $\sum_{i\in U} S_j^{(u_i)}(g+g_{ij}) - \sum_{i\in U} Tr_{u_i v}(g) = \sum_{i\in U} S_j^{(u_i)}(g+g_{ij}, Tr(g+g_{ij})) > \sum_{i\in U} S_j^{(u_i)}(g)$

(ii) $\sum_{k\in V} S_j^{(v_k)}(g+g_{ij}) + \sum_{k\in V} Tr_{uv_k}(g) = \sum_{k\in V} S_j^{(v_k)}(g+g_{ij}, Tr(g+g_{ij})) > \sum_{k\in V} S_j^{(v_k)}(g)$

Condition (i) states the all the sectors that pay transfers in network $g + g_{ij}$ are better off in this network than in network g, and condition (ii) states that all the the sectors that receives the transfers in network $g + g_{ij}$ are better off in this network than in network g.

The two conditions outlined above is a desirable property of any transfer from the point of view of the society. However, it has an important disadvantage: intra-node transfers can only work in cases of biased governments. To see this, let us consider the following propositions.

Proposition 6.1. Intra-node transfers used in network $g + g_{ij}$ can only be Pareto improving from the point of view of the country that uses these transfers when unweighted welfare in this country in network $g + g_{ij}$ is larger than unweighted welfare in network g.

Proof. Suppose that the two conditions in (i) and (ii) are satisfied. By adding these conditions, it is obtained the following expression: $\sum_{i \in U} S_j^{(u_i)}(g+g_{ij})$ + $\sum_{k \in V} S_j^{(v_k)}(g+g_{ij})$ > $\sum_{i \in U} S_j^{(u_i)}(g)$ + $\sum_{k \in V} S_j^{(v_k)}(g)$. But for definition, this implies that $S_j(g+g_{ij})$ > $S_j(g)$, and the proof is complete. □

According to this proposition, a Pareto improving intra-node transfer can only exist when welfare in the country that uses this transfer increases after this country signs an agreement. This result implies the use of this tool is restricted to cases where there is a gain in welfare, but governments are concerned about the payoffs obtained from trade losers rather than welfare. This is shown in the following corollary and proposition.

Corollary 6.2. Intra-node transfers cannot be used to break inefficient pairwise or strongly pairwise stable networks when the government that adopt this tool is politically unbiased.

Proof. This corollary is a consequence of Proposition 6.1. To see this, assume that the inefficient network *g* is either pairwise or strongly pairwise stable and assume that governments are politically unbiased. Given this stability, it holds for at least one country $j \in N$ that $S_j(g + g_{ij}) < S_j(g)$ (i.e. the link addition proof condition). That is, there is at least one country *j* in the network that is unwilling to sign an additional agreement. But because welfare in country *j* in network $g + g_{ij}$ is smaller than in network *g*, it is concluded that this country cannot use a Pareto improving intra-node transfer to reverse the inequality above as revealed in Proposition 6.1. □

The idea behind this result is when $S_j(g + g_{ij}) < S_j(g)$, country *j* does not have enough resources to compensate the losers when passing from network *g* to network $g + g_{ij}$. As a consequence, this inequality cannot be reversed and, therefore, the inefficient network cannot be broken in favour of free trade when governments are unbiased. However, this strategy can work when governments are biased in favour of the loser sectors. This is shown in the following proposition.

Proposition 6.3. Consider the following generic welfare function in country *j*: $S_j(g) = \sum_{i \in U} S_j^{(u_i)}(g) + \sum_{k \in V} S_j^{(v_k)}(g)$. Suppose that the government of this country is biased in favour of the sectors in $V = \{v_1, \ldots, v_m\}$ and that $\sum_{k \in V} S_j^{(v_k)}(g + g_{ij}) < \sum_{k \in V} S_j^{(v_k)}(g)$ (i.e. this government is unwilling to sign an agreement with country *i* because this causes a decrease in welfare in these sectors). This inequality can be reversed by an intra-node transfer as long as $S_j(g+g_{ij}) > S_j(g)$ (i.e. as long as total welfare in this country in network $g + g_{ij}$ is larger than in network g).

Proof. By using the generic welfare function, the inequality $S_j(g+g_{ij}) > S_j(g)$ can be expressed as $\sum_{i\in U} S_j^{(u_i)}(g+g_{ij}) + \sum_{k\in V} S_j^{(v_k)}(g+g_{ij}) > \sum_{i\in U} S_j^{(u_i)}(g) + \sum_{k\in V} S_j^{(v_k)}(g)$. By rearranging terms, this can be expressed as $\sum_{i\in U} S_j^{(u_i)}(g+g_{ij}) - \sum_{i\in U} S_j^{(u_i)}(g) > \sum_{k\in V} S_j^{(v_k)}(g) - \sum_{k\in V} S_j^{(v_k)}(g+g_{ij})$. Now, let ε be the amount of transfer that makes indifferent the sectors in $V = \{v_1,\ldots, v_m\}$ from networks g and $g + g_{ij}$. That is, $\varepsilon = \sum_{k\in V} S_j^{(v_k)}(g) - \sum_{k\in V} S_j^{(v_k)}(g+g_{ij})$. By replacing this term in the previous inequality, it is obtained $\sum_{i\in U} S_j^{(u_i)}(g+g_{ij}) - \sum_{i\in U} S_j^{(u_i)}(g) > \varepsilon$. This expression indicates that there are sufficient resources in country j from the sectors in $U = \{u_1,\ldots, u_n\}$ to compensate the sectors in $V = \{v_1,\ldots, v_m\}$ when passing from network g to network $g + g_{ij}$ and when $S_j(g+g_{ij}) > S_j(g)$.□

This result indicates that an intra-node transfer can be used to break inefficient pairwise and strongly pairwise stable networks when there are sufficient resources to compensate the loser sectors from free trade. That is, when $S_j(g+g_{ij}) > S_j(g)$. In this case the inequality $\sum_{k\in V} S_j^{(v_k)}(g+g_{ij}) < \sum_{k\in V} S_j^{(v_k)}(g)$ of the link addition proof condition can be reversed.

Formally, assume that g is an inefficient pairwise or strongly pairwise stable network in which country i is willing to sign an agreement with country j, but the latter is not willing to sign the agreement. In addition, suppose that the government of country j is biased in favour of the sectors in $V = \{v_1, \ldots, v_m\}$. Network g can be broken in favour of free trade when the following condition holds: if $\sum_{k \in V} S_j^{(v_k)}(g + g_{ij})$ < $\sum_{k \in V} S_j^{(v_k)}(g)$, then $\sum_{k \in V} S_j^{(v_k)}(g + g_{ij}, Tr(g + g_{ij}))$ > $\sum_{k \in V} S_j^{(v_k)}(g)$.

In relation to global treaty stability, on the other hand, the same considerations hold. That is, an intra-node transfer can be used to break an inefficient global treaty stable network only when the government of the country that uses this payment is politically biased and when welfare in this country in global free trade is larger than in the inefficient network. This can easily be proven by replacing in the previous analysis network $g + g_{ij}$ by g^c. Thus, for the case of an inefficient global treaty stable network g, this network can be broken in favour of global free trade under the assumption of government biased in favour of the sectors in $V = \{v_1, \ldots, v_m\}$ the following condition holds: if $\sum_{k \in V} S_j^{(v_k)}(g^c)$ < $\sum_{k \in V} S_j^{(v_k)}(g)$ in some countries $j \in N$, then $\sum_{k \in V} S_j^{(v_k)}(g^c, Tr(g^c))$ > $\sum_{k \in V} S_j^{(v_k)}(g)$.

6.3.3 Final remarks

It is important to clarify that inter-node and intra-node transfers should not be seen as competitor tools. They can assist on free trade in different contexts. For example, in a world where governments are politically unbiased, only inter-node transfers can be used. In contrast, there are cases with biased governments in which intra-node transfers offer better results. Moreover, there are cases when both types of transfers are needed to reach global free trade. These possibilities are discussed in the next sections taking as a reference the simulations carried out in the previous chapters.

6.4 Trade effects of transfers on the international trade stability

It is argued in this dissertation that the adoption of inter-node and intra-node side payments can potentially facilitate agricultural trade liberalisation because they can be used to compensate losers from trade from the gainers. In order to show this possibility, this section explores how these payments can both stabilise the efficient network or break inefficient stable ones in favour of free trade. For this purpose, the simulations that include the farming sector and that were analysed in the previous chapters are considered in this section. This is explored as follows.

6.4.1 Simulations under the assumption of exogenous tariffs and symmetric countries

There are two simulations under the assumptions of exogenous tariffs and symmetric countries that include the farming sector. One of them (i.e. Simulation 2 in Chapters Four and Five) considers the case of moderate levels of monopsonistic power (i.e. ϕ_i = 0.5), and the other simulation (i.e. Simulation 2 in Chapters Four and Five) considers the case of high levels of monopsonistic power (i.e. ϕ_i = 1.5).

6.4.1.1Simulation 2: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

The case of politically unbiased governments

According to the results obtained in Sections 4.3.1.2 and 5.3, the pairwise and strongly pairwise stable network in this simulation when governments are politically unbiased is network *k* in Figure 4.3. Since this corresponds to global free trade and is also the efficient network, a transfer in this case is not needed.

On the other hand, the results obtained in Section 5.4.1.2 revealed that the global treaty stable networks are *d*, *f*, *i*, *j*, *k* in Figure 4.3. Because global free trade is global treaty stable, a transfer is only needed to break the inefficient networks *d*, *f*, *i* and *j* in order to facilitate the signature of a global agreement. Now, as shown in Corollary 6.2 and Proposition 6.3, when governments are unbiased, only inter-node transfers can be used to break the stability of these networks. To show that these transfers can indeed be used to facilitate a global agreement in the current simulation, consider the following table.

Table 6.3. Inter-node transfer to break inefficient networks under unbiased governments

	Country				
Difference of welfare between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
$k-d$	-0.0177	-0.0177	0.0571	0.0571	0.0788
$k-f$	-0.0479	0.0594	0.0594	0.084	0.1549
$k-i$	0.0231	0.0231	-0.065	0.0731	0.0543
$k-j$	0.0372	-0.0266	-0.0266	0.0372	0.0212

The first column in this table shows the difference in welfare between global free trade (i.e. network *k*) and a determined stable inefficient network. For example, *k* – *d* is the difference in welfare between global free trade and the inefficient network *d*. The second, third, fourth and fifth columns correspond to this difference in numerical terms in each country (these figures were obtained from Table E.7 in Appendix E). For example, welfare in country *i* when passing from network *d* to network *k* decreases by 0.0177. Finally, the last column is the sum of the numbers in each row and represents the net gain in global welfare when passing from the inefficient network to global free trade.

Table 6.3 reveals that there are always enough resources in the world to finance inter-node transfers in order to break inefficient networks. For example, suppose that the world is trapped in network *d*. If countries signed a global agreement, then welfare in countries *i* and *j* would decrease by 0.0177 and welfare in countries *k* and *l* would increase by 0.0571. Now, because the total gain in welfare is larger than the total loss, there is a net gain in welfare (i.e. net gain in global welfare) of 0.0788. This suggests, consequently, that the gainer countries *k* and *l* would be willing to pay inter-node transfers to the loser countries *i* and *j* in order to sign a global agreement. In considering the global treaty stability concept with transfers (see Section 6.3.1.1), this agreement would be stable. Moreover, these transfers do not need to be paid for ever to sustain global free trade because, as demonstrated in Section 5.4.1.2, this network is also stable without transfers. This is explained by the fact that once in global free trade, no country has an incentive to deviate unilaterally by breaking on or more links simultaneously. In other works, the intra-node payment can be used to break an inefficient network in order to induce countries to sign a global agreement. But when the agreement is signed, the payments can be cancelled. The same holds for the other inefficient networks considered in Table 6.3. This result illustrates, therefore, that a tool of this nature can be used to facilitate a global agreement by compensating loser countries from trade.

The case of politically biased governments

Let us now consider the case of governments biased in favour of consumers. According to the results obtained in in Sections 4.3.1.2 and 5.3, global free trade is the only pairwise and strongly pairwise stable network in this simulation implying that transfers are not needed in this case. On the other hand, the results obtained in Section 5.4.1.2 revealed that networks *h*, *i*, *j* and *k* are global treaty stable. Using the same analysis as the one conducted for the case of unbiased governments, it is concluded that there are enough resources to finance inter-node transfers to break the inefficient networks *h*, *i* and *j.* This is inferred from the information presented in Table 6.4.

Table 6.4. Inter-node transfers to break inefficient networks when governments are biased in favour of consumers

	Country				
Difference of consumer surplus between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global consumer surplus gain
$k-h$	-0.0127	0.0816	0.0816	0.0816	0.2321
$k-i$	0.0279	0.0279	-0.0059	0.068	0.1179
$k-j$	0.0289	-0.0027	-0.0027	0.0289	0.0524

This table shows the change in consumer surplus when passing from the inefficient network to global free trade. For example, if the world is trapped in network *h*, passing from this network to global free trade causes a loss in consumer surplus in country *i* by 0.0127 and a gain in countries *j*, *k* and *l* by 0.0816. Because there is a net global consumer surplus when passing to global free trade, the gainer countries *j*, *k* and *l* have an incentive to pay inter-node transfers to the loser countries, and the same holds for the other inefficient networks. As before, this strategy can be adopted to induce the signature of a global agreement and this payment can be cancelled when global free trade is reached. In intra-node payment, however, cannot work in this case. To see this, consider the following table.

Table 6.5. Intra-node transfers to break inefficient networks when governments are biased in favour of consumers

	Country			
Difference between welfare (W) and consumer surplus (CS) in the same loser country in a determined network	*i*	*j*	*k*	*l*
$W-CS$ in network *h*	0.2881			
$W-CS$ in network *i*			0.1840	
$W-CS$ in network *j*		0.2031	0.2031	
$W-CS$ in network *k*	0.1792	0.1792	0.1792	

The first column of this table represents the available welfare resources other than consumer surplus that there are in a loser country in a determined network, and the other columns show the numerical value of this difference (these figures were obtained by subtracting from welfare in Table E.7 the corresponding values of consumer surplus in Table E.4 in Appendix E). This difference corresponds therefore to the available resources that belong to the intermediary and the farming sector in the loser country. For example, country *i* is a loser country when passing from network *h* to global free trade as shown in Table 6.5. In the inefficient network *h*, the available resources in the hands of the intermediary and the farming sector is the amount of 0.2881.

Now, if countries signed an agreement (i.e. passing from network *h* to network *k*), the available resources that belong to the intermediary and the farming sector would decrease from 0.2881 to 0.1792 as can be seen in the last row of Table 6.5. This means that the global agreement would cause a net decrease in the resources available in these sectors implying that at least one of these sectors would be worse off after the agreement and, consequently, would be unwilling to pay the intra-node transfer to compensate consumers contradicting the desirable property of Pareto improving from the point of view of the loser country. The same situation holds in loser countries in other inefficient networks. This is why intra-node payments that are Pareto improving cannot be used in this case.

In relation to the case of governments biased in favour of intermediaries, the results obtained in in Sections 4.3.1.2 and 5.3 revealed that the empty network is the only pairwise and strongly pairwise stable network in this simulation implying that transfers are need in this case to favour free trade. A suitable strategy to achieve global free trade in the current simulation when countries are involved in bilateral agreements is the use of intra-node transfers. The idea is to use these transfers sequentially from the empty network (i.e. network *a* in Figure 4.3) in order to follow a path of networks that leads to global free trade. There are different feasible paths and the one that has been selected for illustrative purposes is the following: $a \to b \to c \to d \to e \to j \to k$, where the arrows represents the path from an initial network to the next one (e.g. $a \to b$ means the path starting from network *a* to network *b*). The information that is needed to determine the feasibility of this path is presented in Tables 6.6 and 6.7.

Table 6.6. Welfare minus profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1000	0.1000	0.1000	0.1000
b	0.1778 – 0.0223 = 0.1555	0.1000	0.1778 – 0.0223 = 0.1555	0.1000
c	0.1778 – 0.0223 = 0.1555	0.1778 – 0.0223 = 0.1555	0.1778 – 0.0223 = 0.1555	0.1778 – 0.0223 = 0.1555
d	0.2315 – 0.0300 = 0.2015	0.2315 – 0.0300 = 0.2015	0.1705 - 0.0437 = 0.1268	0.1705 – 0.0437 = 0.1268
e	0.2250 – 0.0502 = 0.1748	0.2250 – 0.0502 = 0.1748	0.2250 – 0.0502 = 0.1748	0.2250 – 0.0502 = 0.1748
j	0.2150 – 0.0684 = 0.1466	0.2650 – 0.0547 = 0.2103	0.2650 – 0.0547 = 0.2103	0.2150 – 0.0684 = 0.1466
k	0.2560 – 0.0723 = 0.1837	0.2560 = 0.0723 = 0.1837	0.2560 – 0.0723 = 0.1837	0.2560 – 0.0723 = 0.1837

Table 6.7. Profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.2000	0.2000	0.2000
b	0.1778 + 0.0223 = 0.2001	0.2000	0.1778 + 0.0223 = 0.2001	0.2000
c	0.1778 + 0.0223 = 0.2001	0.1778 + 0.0223 = 0.2001	0.1778 + 0.0223 = 0.2001	0.1778 + 0.0223 = 0.2001
d	0.1702 + 0.0300 = 0.2002	0.1702 + 0.0300 = 0.2002	0.1564 + 0.0437 = 0.2001	0.1564 + 0.0437 = 0.2001
e	0.1500 + 0.0502 = 0.2002	0.1500 + 0.0502 = 0.2002	0.1500 + 0.0502 = 0.2002	0.1500 + 0.0502 = 0.2002
j	0.1318 + 0.0684 = 0.2002	0.1456 + 0.0547 = 0.2003	0.1456 + 0.0547 = 0.2003	0.1318 + 0.0684 = 0.2002
k	0.1280 + 0.0723 = 0.2003	0.1280 + 0.0723 = 0.2003	0.1280 + 0.0723 = 0.2003	0.1280 + 0.0723 = 0.2003

Table 6.6 shows the available resources that belong to consumer surplus and the farming sector (i.e. welfare in Table E.7 minus profits in Table E.6 in a determined country. See Appendix E) minus the intra-node transfer that is paid by these

sectors to the intermediary. Table 6.7, on the other hand, is the profit made by the intermediary plus the intra-node transfer paid by consumers and the farming sector. For example, in network *a* in Table 6.6, the resources that belong to consumers and the farming sector in country *i* have a value of 0.1778. From this amount, an intra-node payment of 0.0223 is subtracted and the resulting resources in the hands of consumers and the farming sector have a value of 0.1555. In Table 6.7, this transfer is paid to the intermediary in country *i*. As a result, the resources in hands of this firm increases from 0.1778 to 0.2001.

Having described the information contained in Tables 6.6 and 6.7, let us know explain how the intra-node transfers in these tables can help the world to reach global free trade in the suggested path of networks. According to Figure 4.3, passing from network *a* to network *b* requires countries *i* and *k* to sign a bilateral agreement. However, this causes a decrease in the profit made by the intermediaries of these countries from 0.2000 to 0.1778 (see Table E.5 in Appendix E). Thus, since governments are biased in favour of these firms, they are not willing to sign the agreement. However, if the intermediaries of countries *i* and *k* were compensated by an intra-node transfer of 0.0223 paid by consumers and the farming sector (see the row for network *b* in Table 6.6), then they would obtain the amount of 0.2001 which is larger than the profits in network *a*. (see the row for network *b* in Table 6.7). As a result, countries *i* and *k* would be willing to sign the agreement. If this agreement is signed, the intermediaries of these countries would be better off as can be seen in Table 6.7. Consumers and the farming sector in these countries would also be better off because they would obtain the amount of

0.1555 which is larger than the amount of 0.1000 obtained in network *a* as can be seen in Table 6.6. In other words, the transfer is a Pareto improving payment from the point of view of countries *i* and *k* as required.

Now, suppose that the agreement is signed. In order to reach network *c* from network *b* requires countries *j* and *l* to sign an agreement. As revealed in Tables 6.6 and 6.7, the same results hold. That is, intra-node transfers paid consumers and the farming sector in countries *j* and *l* would make all the sectors in these countries better of implying that this agreement would be signed. By following the same reasoning for the suggested path of networks, it is inferred that global free trade can be reached with intra-node transfers. Moreover, global free trade would be both pairwise and strongly pairwise stable as it is concluded from the definition of pairwise and strongly pairwise stability with intra-node transfers defined in Section 6.3.2.1.

In the relation to global treaty stability for the case of governments biased in favour of intermediaries, the results obtained in Section 5.4.1.2 revealed that network *a* (i.e. the empty network) is the only global treaty stable network. It is difficult to justify the use inter-node transfers to reach global free trade from the empty network in this case because a global agreement causes a loss of profits in all countries of the world. This can be seen in Table E.5. In this table, all countries obtain a profit of 0.1280 in global free trade (i.e. network *k*) which is smaller than the profit of 0.2000 that is made in the inefficient network *a*. In other words, all countries of the world should be compensated to be willing to sign the agreement.

Nonetheless, an intra-node transfer can potentially solve this conflict. To see this, note that according to Table E.5 the intermediaries in each country of the world (i.e. countries *i*, *j*, *k* and *l*) should be compensated by at least the amount of 0.072 to be willing to support a global agreement (i.e. 0.2000 – 0.1280). Now, note by subtracting profits from welfare (i.e. subtracting the figures in Table E.5 from Table E.7) that consumers and the farming sector in each country can increase the value of their resources from 0.1000 to 0.2560. That is, a global agreement would offer them a gain of 0.156. But this amount is larger than the compensation required by the intermediaries. This means that in global free trade there are enough resources from consumers and the farming sector to compensate the intermediaries. This suggests, therefore, that intra-node transfers can be used to achieve a global agreement when governments are biased in favour of intermediaries. Moreover, this finding illustrates the fact that these payments can constitute a possible alternative tool when inter-node transfers are not feasible.

Finally, let us consider the case of governments biased in favour of the farming sector. In this case it was found in Sections 4.3.1.2 and 5.3 that global free trade is the only pairwise and strongly pairwise stable network implying that a transfer is not needed to favour free trade. On the other hand, the results obtained in Section 5.4.1.2 revealed that networks *h*, *i*, *j* and *k* are global treaty stable. In this case, only inter-node transfers can be used to break the global treaty stability of networks *h*, *i*, and *j* in favour of a global agreement. To see that this is the case, consider the following table (the figures in this table were obtained from Table E.6 in Appendix F).

Table 6.8. Inter-node transfers to break inefficient networks when governments are biased in favour of the farming sector

	Country				
Difference of producer surplus between global free trade and the inefficient stable network	i	j	k	l	Net global producer surplus gain
$k-f$	-0.0049	0.0206	0.0206	0.0312	0.0675
$k-h$	-0.0276	0.0257	0.0257	0.0257	0.0495
$k-i$	0.0091	0.0091	-0.015	0.0233	0.0265
$k-j$	0.0121	-0.0063	-0.0063	0.0121	0.0116

According to this table, loser countries in terms of producer surplus can be compensated by gainers by transferring payments from the farming sector of the latter to the former. This is because a global agreement causes a net gain in producer surplus (see the last column in the table).

Now, consider Table 6.9 to show that intra-node transfers cannot work in this case (the figures in this table were obtained by subtracting producer surplus in Table E.6 from welfare in Table E.7 in Appendix E).

Table 6.9. Welfare minus producer surplus in countries that are unwilling to sign a global agreement

	Country				
Difference between welfare and producer surplus in inefficient stable network and global free trade	i	j	k	l	Net global producer surplus gain
f	0.3758				0.0675
h	0.4268				0.0495
i			0.3828		0.0265
j		0.3531	0.3531		0.0116
k	0.3328	0.3328	0.3328		

This table shows the available resources in the hands of consumers and intermediaries in countries that are unwilling to sign a global agreement (these countries can be identified in Table 6.8 and corresponds to the ones that face a net decrease in producer surplus when passing from the inefficient network to global free trade). According to Table 6.9, the resources that belong to consumers and intermediaries decrease when passing from an inefficient global treaty stable network to network *k*. This implies that there are not enough resources in global free trade to compensate the farming sector from the other sectors because this would not cause a Pareto improvement from the point of view of the loser countries. This illustrates again the claim that intra-node and inter-node payments are not substitutes and they can work in different contexts.

Final comments

In considering the analysis developed in this simulation it is concluded therefore that inefficient pairwise, strongly pairwise and global treaty stable networks can be broken in favour of free trade by means of transfers in the current simulation. The reason is because there are sufficient resources either across countries or within a country to compensate losers from trade by using these transfers. However, the type of transfer (i.e. intra-node or inter-node) that can be used for this purpose depends on the context and also on the type of policy bias.

6.4.1.2 Simulation 3: $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in N$.

The same results obtained in the previous case are in verified when $\phi_i = 1.5$ as can be inferred from the information presented in Tables E.8, E.9, E.10 and E.11 in Appendix E. This implies that inter-node and intra-node transfers can also be used to break inefficient networks in favour of free trade when countries are symmetric, tariffs are placed exogenously, and when monopsonistic power is high.

6.4.2 Simulations under the assumption of endogenous tariffs and symmetric countries

Simulations that assume endogenous tariffs were carried out for the case of symmetrical countries with unbiased governments and farming sectors (i.e. Simulations 5 and 6). As shown in Table 6.1, in these simulations global free trade (i.e. network *k*) is the efficient network and therefore the target of compensatory payments is to help the world to reach this network.

Unfortunately, it was not possible to fully study cases of biased governments because the model becomes untractable in mathematical terms. However, some partial results were obtained in Section 4.3.2.4. They are considered in the current section in the context of compensatory payments.

The case of politically unbiased governments

There are two simulations that include the farming sector in a world with endogenous tariffs and unbiased governments. They are studied as follows.

6.4.2.1 Simulation 5: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation assumes moderate levels of monopsonistic power, symmetrical countries and unbiased governments. It was found in Sections 4.3.2.2 and 5.3.2 that the pairwise and strongly pairwise stable networks in this case correspond to networks *g* and *k* in Figure 4.3. It was also found in Section 5.4.2.2 that the global treaty stable networks in this simulation are *d*, *f*, *j* and *k* in the same figure. This result revealed therefore the existence of inefficient stable networks: the pairwise and strongly pairwise stable network *g*; and the global treaty stable networks *d*, *f* and *j*. Because governments are unbiased in this case, only inter-node transfers can be used to break these inefficient stable networks as proven in Corollary 6.2. However, this instrument is not needed to stabilise global free trade because this network is already pairwise, strongly pairwise and global treaty stable.

Let us first consider the inefficient pairwise and strongly pairwise stable network *g*. A path of networks that can be followed from bilateral agreements that lead to global free trade is $g \rightarrow i \rightarrow j \rightarrow k$. The information that is needed to determine whether this path can be facilitated is presented in Table 6.10 (this information was obtained from Table E.20 in Appendix E).

Table 6.10. Welfare with inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
g	0.3756	0.3756	0.3756	0.3725
i	0.3764	0.3764	0.4027 – 0.0062 = 0.3965	0.3664 + 0.0062 = 0.3726
j	0.3741	0.3912	0.3912	0.3741
k	0.3840	0.3840	0.3840	0.3840

According to Figure 4.3, passing from network *g* to network *i* requires countries *k* and *l* to sign a bilateral agreement. According to Table E.20, country *k* is willing to sign this agreement because this would increase welfare in this country from 0.3756 to 0.4027. However, country *i* does not support this agreement because this would cause a decrease in welfare in this country from 0.3725 to 0.3664. Now, if country *k* paid to country *i* an inter-node payment to compensate this loss, then this agreement would be signed. This is shown in Table 6.10. In this table, country *k* pays a transfer of 0.0062 to country *i*. As a result, both countries are better off after the agreement because welfare in both countries increases. As can be seen in this table, this is the only inter-node transfer that is needed in this path because once network *i* reached, countries *j* and *l* have an incentive to form an agreement which is required to pass from network *i* to network *j*. Likewise, once network *j* is reached, countries *i* and *l* have an incentive to sign a bilateral agreement which is what is required to reach network *k*, that is, global free trade.

Let us now explore whether the inefficient global treaty stable networks d, f and j can be broken by means of inter-node transfers. This is shown in Table 6.11 (the information in this table was obtained from Table E.20 in Appendix E).

Table 6.11. Inter-node transfer to break inefficient networks under unbiased governments

	Country				
Difference of welfare between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
$k-d$	-0.0042	-0.0042	0.0149	0.0149	0.0214
$k-f$	-0.009	0.0213	0.0213	0.0157	0.0493
$k-j$	0.0099	-0.0072	-0.0072	0.0099	0.0054

This table shows the welfare gains/losses when passing from the inefficient network to global free trade (i.e. network *k*). For example, welfare in countries *i* and *j* decreases by 0.0042 when passing from network *d* to network *k*, and welfare in countries *k* and *l* increases by 0.0149. The last column in Table 6.11 shows that the aggregate gain in welfare when passing from network *d* to *k* is positive implying that the gainer countries *k* and *l* have enough resources in global free trade to compensate the loser countries *i* and *j* is a global agreement is signed. The same holds for the other inefficient networks as can be seen in this table.

In considering the results obtained in this simulation, it is concluded therefore that inter-node transfers have the potential to assist countries to reach global free trade in a world where tariffs are placed endogenously and when countries have a farming sector.

6.4.2.2 Simulation 6: $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation also assumes endogenous tariffs and politically biased governments. However, it assumes high level of monopsonistic power (i.e. ϕ_i =

1.5). According to the results obtained in Sections 4.3.2.3 and 5.3, the pairwise and strongly pairwise stable networks in this case are *g* and *k*. On the other hand, it was found in Section 5.4.2.2 that the global treaty stable networks are *d*, *f*, *j* and *k*.

The same stable networks were found in the previous case (see Section 6.4.2.1) and the same results concerning the use of inter-node transfers hold in the current simulation. It is concluded therefore that inter-node transfers can also facilitate free trade when tariffs are placed endogenously in a world with high levels of monopsonistic power.

The case of politically biased governments

As explained above, it was not possible to fully study the case of endogenous tariffs when governments are biased. However, partial simulations based on the information presented in Appendix B were developed 4.3.2.4 (see Simulations 7, 8, 9 and 10 in Tables 4.4 and 4.5). The aim of these simulations was to show that it is possible to identify cases where biased governments have an incentive to deviate from global free trade. An example is found in Simulation 10. In this case, when governments are biased in favour of intermediaries and take into account tariff revenue (i.e. the government puts cero weight to consumer surplus and producer surplus, a weight equal to one to the profits made by the intermediary, and a weight equal 0.5 to tariff revenue) and when monopsonistic power is high (i.e. ϕ_i = 1.5), global free trade becomes unstable because a government obtains a higher level of weighted welfare in network *j* by breaking an existing agreement.

Based on the same information obtained from Appendix B, it is possible to show that an intra-node network can be used to stabilise global free trade in Simulation 10. To see this, note that consumer surplus and producer surplus in a determine country in global free trade sum together the amount of 0.1829. If this country breaks an existing agreement, this amount decreases to 0.1602 implying a net loss of 0.0227 for these sectors. On the other hand, according to Table 4.5 the weighted welfare in this country is 0.0914, and 0.0954 when deviating by breaking an existing link. This means that the deviation causes a gain in weighted welfare of 0.0040. In considering these figures, it is concluded that consumers and the farming sector have enough resources compensate for the 0.0040 in case the government decides to stay in global free trade. This illustrates that in the context of endogenous tariffs and biased governments, intra-node transfers have the potential to facilitate free trade.

6.4.3 Simulations under the assumptions of exogenous tariffs and asymmetry in market size

Four simulations that include the farming sector were developed under the assumption of asymmetric countries in terms of market size. Two of them (Simulations 12 and 13) consider the case of large and very small countries under different levels of monopsonistic power. The other two simulations (Simulations 15 and 16) consider the case of large and medium size countries, also under different levels of monopsonistic power. The way in which compensatory transfers can be used in these simulations is explained as follows.

6.4.3.1 Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$ for all $i \in N$.

This simulation assumes the existence of large countries and very small country with low degree of monosonistic power. According to Table 6.2, in this simulation there are two efficient networks: network *t* in which all countries have an agreement with each other except the very small countries *j* and *l*; and network *x* that corresponds to global free trade (see Figure 4.5). In the following analysis it is studied how to reach these efficient networks by means of compensatory payments.

The case of politically unbiased governments

It was found in Sections 4.4.1.2 and 5.3 that when governments are politically unbiased in the current simulation, networks *m*, *t*, *x*, *z* are pairwise stable networks, and networks *t* and *x* (i.e. the efficient networks) are also strongly pairwise stable. This finding suggests that compensatory payments may be used to break the inefficient pairwise stable networks *m* and *z*. Because governments are unbiased, only inter-node payments can be used in this case as proved in Corollary 6.2 and proposition 6.3.

Note before proposing a suitable path to reach one of the efficient networks that there are sub-paths that are not feasible because there is no change in welfare in any of the countries. For example, a logical path to break the two inefficient pairwise stable networks is to pass from *z* to *m* which requires the very small countries *j* and *l* to sign an agreement. However, according to Table E.32 in Appendix E all countries obtain the same level of welfare in both networks implying that there is no incentive for compensation. This happens in any sub-path that requires an agreement between the very small countries *j* and *l*. The reason is because the domestic markets of these countries are very small to make any significant change in the network system in terms of welfare when signing an agreement.

It is inferred from this limitation that only the efficient network *t* can be reached from the inefficient network *z*. In contrast, the efficient network *x* (i.e. global free trade) can be reached from the inefficient network *m* because in this network the very small countries *j* and *l* have already an agreement with each other. Having clarified why two different paths are needed for the inefficient networks, the following paths to reach the efficient networks for networks *z* and *m* are proposed, respectively: $z \rightarrow g \rightarrow n \rightarrow t$; and $m \rightarrow o \rightarrow s \rightarrow x$ (note that there are other possibilities as well). The information that is needed to determine whether these paths are feasible is presented in the following tables (the figures in these table were obtained from the information presented in Table E.32).

Table 6.12. Feasible path to reach the efficient network *t* from network *z* by means of inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
z	0.2901	0.0679	0.3000	0.0679
g	0.3928 - 0.0028 = 0.3900	0.0542	0.2973 + 0.0028 = 0.3001	0.0542
n	0.3527	0.0557	0.3131	0.1089
t	0.3326	0.0907	0.3326	0.0907

Table 6.13. Feasible path to reach the efficient network *x* from network *m* by means of inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
m	0.2901	0.0679	0.3000	0.0679
o	0.3928 - 0.0028 = 0.3900	0.0542	0.2973 + 0.0028 = 0.3001	0.0542
s	0.3527	0.0557	0.3131	0.1089
x	0.3326	0.0907	0.3326	0.0907

Table 6.12 shows how the inefficient network *z* can be broken in order to reach the efficient network *t* through the path $z \rightarrow g \rightarrow n \rightarrow t$. In this case, passing from network *z* to network *g* requires the large countries *i* and *k* to sign a bilateral agreement (see Figure 4.5). According to table E.32, welfare in country *i* increases from 0.2901 to 0.3928 after the agreement is signed, and welfare in country *k* decreases from 0.3000 to 0.2973. This suggests that network *z* can be broken in favour of the agreement by means of an inter-node payment paid by country *i* to country *k* to compensate for the welfare loss. In Table 6.12, it is considered as an example an inter-node transfer of 0.0028. If this transfer is paid, then both countries obtain a higher level of welfare after the agreement (i.e. country *i* obtains 0.3900 and country *k* obtains 0.3001). This means that both countries have the incentive to sign the agreement with the transfer. Now, suppose that the agreement is signed. Passing from network *g* to network *n* requires countries *k* to *l* to sign an agreement. However, according to Table 6.12, welfare in these two countries increases after the agreement even without any transfer implying that this agreement will be signed. The same holds for the sub-path between network *n* and

t. In this case passing from network *n* to network *t* requires an agreement between countries *k* and *j* and this is feasible because this agreement increase welfare in these countries. In conclusion, it is possible to reach the efficient network *t* from the inefficient network *z* when the latter is broken by means of an inter-node transfers. What it is not possible, however, is to reach the other efficient network *x* (i.e. global free trade) because an agreement between the very small countries does not favour any country in terms of welfare gains. That is, there are no available beneficiaries to pay compensatory payments.

Let us now consider the second path from the inefficient network *m* to global free trade. According to the information presented in Table 6.13, the same conclusions obtained for the first path applies in this case. That is, an inter-node payment paid by country *i* to compensate the welfare loss in country *k* can be used to break the inefficient network *m.* This, once network *o* is reached from network *m*, there are always a pair of countries willing to sign an agreement until global free trade is reached. The only difference with respect to the previous case is that the efficient network that is reached is global free trade. The reason is because the very small countries *j* and *l* have already an agreement with each other implying that the lack of gainers to promote this agreement is not an issue in this case.

On the other hand, it was found in Section 5.4.3.2 that the global treaty stable networks in this case are networks *c*, *e*, *i*, *j*, *n*, *q*, *s*, *t*, *u*, *x*, and *c'*. The following table shows whether inter-node transfers can be used to break the inefficient global treaty stable networks *c*, *e*, *i*, *j*, *n*, *q*, *s*, *u*, and *c'*.

Table 6.14. Inter-node transfer to break inefficient networks under unbiased governments

	Country				
Difference of welfare between the efficient network (*t* or *x*) and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
Efficient network – c	-0.0231	0.0907	-0.0231	0.0907	0.1352
Efficient network – e	-0.0379	0.0100	0.0170	0.0907	0.0798
Efficient network – i	-0.0379	0.0100	0.0170	0.0907	0.0798
Efficient network – j	0.0002	-0.0423	0.0002	0.0907	0.0488
Efficient network – n	-0.0201	0.0350	0.0195	-0.0182	0.0162
Efficient network – q	0.0002	-0.0423	0.0002	0.0907	0.0488
Efficient network – s	-0.0201	0.0350	0.0195	-0.0182	0.0162
Efficient network – u	-0.0236	-0.0183	0.0175	0.0350	0.0106
Efficient network – c'	-0.0231	0.0907	-0.0231	0.0907	0.1352

According to this table, the gainers when passing from any inefficient network to an efficient one have enough resources to compensate the losers as can be inferred from the last column of this table. As a result, inter-node transfers can be used to facilitate the signature of a global agreement.

The case of politically biased governments

According to the results obtained in Sections 4.4.1.2 and 5.3, the pairwise and strongly pairwise stable networks when governments are biased in favour of consumers are networks *t* and *x*. Since these networks are efficient in the current simulation, compensatory transfers are not needed in this case.

On the other hand, the results obtained in Section 5.4.3.2 revealed that all the networks are global treaty stable when governments are biased in favour of consumers. As explained in that section, one of the reasons is because the very small countries are indifferent about signing a global agreement because this does not significantly affect the level of consumer surplus as a consequence of having very small domestic markets. The other reason is because large countries obtain high levels of consumer surplus in some inefficient networks when they occupy a privileged position in the network.

A possible strategy to break inefficient global treaty stable networks in this case is the adoption of inter-node transfers by gainer countries in order to either compensate loser countries from free trade or motivate very small countries to sign a global agreement. It is inferred from the information presented in Table E.29 in Appendix E that this strategy would work for all the inefficient networks. For example, consider network *j*. Passing from this network to global free trade increases consumer surplus in the large countries *i* and *k* by 0.0425. However, consumer surplus in the very small countries *j* and *l* remains the same. This suggests therefore that the larger countries can pay a transfer to the very small countries to motivate them to sign a global agreement.

The other possibility is to use intra-node payments. However this alternative can work in some networks only. For example, consider network *p*. This network is global treaty stable because the very small countries *j* and *l* are indifferent about signing a global agreement as the agreement would not increase the level of consumer surplus in these countries. However, the intermediaries and the farming sector would be better off as can be inferred from Tables E.29 and E.32. By subtracting consumer surplus from welfare, it is inferred that these sectors would increase their available resources by 0.0133 if a global agreement is signed. This suggests therefore that a joint intra-node transfer paid by the intermediary and the farming sector to consumers can be used to break the inefficient network *p* in favour of a global agreement. Now, consider network *j*. This network is also global treaty stable because the very small countries are indifferent about signing a global agreement. However in this case an intra-node transfer cannot be used to break this network. To see why, note from the information presented in Tables E.29 and E.32 that a global agreement would increase the resources of the intermediary and the farming sector in country *l* by 0.0907. However, it would decrease the resources of the intermediary and the farming sector in country *j* by 0.0423. This implies that these sectors would not be willing to pay a transfer to consumers and, as a consequence, the global agreement would not be signed.

In relation to the case of governments biased in favour of intermediaries, on the other hand, it was found in Sections 4.4.1.2 and 5.3 that the pairwise and strongly pairwise stable networks are *a* and *d*. Unfortunately inter-node transfers cannot always be used to break these networks and lead to the efficient network. The reason is because in many paths there are only loser countries or indifferent countries implying that there are not available gainers to compensate losers. For example, according to Table E.30 in Appendix E, passing from network *a* to *c* does not create profit gainers. Intra-node transfers have the ability to break the inefficient networks *a* and *d* and to lead the world towards the efficient network. As in the case of governments biased in favour of consumers, two different paths are needed for these inefficient networks and each of these paths can only lead to one of the efficient networks (i.e. network *t* or *x*). The following paths are proposed for these networks corresponds to: $a \rightarrow c \rightarrow e \rightarrow h \rightarrow n \rightarrow t$; and $d \rightarrow c \rightarrow i \rightarrow p \rightarrow s \rightarrow x$. The following Tables shows how intra-node payments used in the first path can be used to reach the efficient network *t* (the information in these tables was obtained from Tables E.30 and E.32).

Table 6.15. Welfare minus profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1000	0.0000	0.1000	0.0000
c	0.1779 - 0.0223 = 0.1556	0.0000	0.1779 - 0.0223 = 0.1556	0.0000
e	0.2298 - 0.0595 = 0.1703	0.0073	0.1749	0.0000
h	0.2281	0.0070	0.2281 - 0.0374 = 0.1907	0.0070
n	0.2617 - 0.0125 = 0.2492	0.0051	0.2221	0.0179
t	0.2570	0.0151	0.2507 - 0.0155 = 0.2415	0.0151

Table 6.16. Profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.0000	0.2000	0.0000
c	0.1778 + 0.0223 = 0.2001	0.0000	0.1778 + 0.0223 = 0.2001	0.0000
e	0.1407 + 0.0595 = 0.2002	0.0734	0.1407	0.0000
h	0.1034	0.0704	0.1034 + 0.0374 = 0.1408	0.0704
n	0.091 + 0.0125 = 0.1035	0.0506	0.0910	0.0910
t	0.0756	0.0756	0.0756 + 0.0155 = 0.0911	0.0756

Table 6.15 shows the resources that belong to consumers and the farming sector (i.e. welfare minus profits) and the intra-node transfers paid to the intermediaries in the selected path of networks, and Table 6.16 shows the profit made by the intermediaries of countries *i*, *j*, *k* and *l* in these networks and the transfers that have been paid to them. To see how these transfer lead to the efficient network *t*, consider for example the pass from network *a* to network *c* which requires a bilateral agreement signed by countries *i* and *k* (see Figure 4.5). According to Table E.30, these countries are unwilling to sign the agreement because it causes

a decrease in profits from 0.2000 to 0.1778. However, the joint resources of consumers and the farming sector (i.e. consumer surplus plus producer surplus) increases from 0.1000 to 0.1779 as can be inferred from Tables E.30 and E.32. This means that these sectors have an incentive to compensate the intermediaries of countries *i* and *k* if the agreement between these countries is signed. This is shown in Table 6.15: consumers and the farming sector pay an intra-node transfer of 0.0223 in network *c*. Table 6.16, on the other hand, shows the resulting payoff that farmers obtain when they are given the transfers. According to this table, the resulting payoff in countries *i* and *k* is 0.2001 which is larger than the profits in network *a*. This implies according to the link addition proof condition of the pairwise and strongly pairwise stability concepts with transfer that the agreement will be signed and, therefore, the stability of network *a* will be broken. Following the same line of reasoning, it can be seen in Tables 6.15 and 6.16 that intra-node transfers lead to the efficient network *t*.

Now consider the other path for the inefficient network *d*. According to Tables 6.17 and 6.18, the same figures as in the previous case were obtained from Tables E.30 and E.32 implying that intra-nodes in this path can also break the inefficient network *d* and lead to global free trade.

Table 6.17. Welfare minus profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
d	0.1000	0.0000	0.1000	0.0000
c'	0.1779 - 0.0223 = 0.1556	0.0000	0.1779 - 0.0223 = 0.1556	0.0000
i	0.2298 - 0.0595 = 0.1703	0.0073	0.1749	0.0000
p	0.2281	0.0070	0.2281 - 0.0374 = 0.1907	0.0070
s	0.2617 - 0.0125 = 0.2492	0.0051	0.2221	0.0179
x	0.2570	0.0151	0.257 - 0.0155 = 0.2415	0.0151

Table 6.18. Profits made by the intermediary with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
d	0.2000	0.0000	0.2000	0.0000
c'	0.1778 + 0.0223 = 0.2001	0.0000	0.1778 + 0.0223 = 0.2001	0.0000
i	0.1407 + 0.0595 = 0.2002	0.0734	0.1407	0.0000
p	0.1034	0.0704	0.1034 + 0.0374 = 0.1408	0.0704
s	0.091 + 0.0125 = 0.1035	0.0506	0.0910	0.0910
x	0.0756	0.0756	0.0756 + 0.0155 = 0.0911	0.0756

In considering this result, it is interesting to note that intra-node payments do not require very small countries to compensate large countries for trade losses. However, this can be required in some networks when using inter-node transfers. That is, while inter-node networks cannot lead to the efficient network as explained above, they can eventually break specific networks. For example, passing from network *a* to *b* is feasible with an inter-node transfer paid by the very small country *j* to the large country *i*. This is an ethical issue (e.g. it is difficult to support a compensatory transfer paid by a poor African country to the United States) that is beyond the scope of this dissertation. Nonetheless, the ability to break inefficient networks without requiring transfers across countries can be considered as an advantage of intra-node transfers within this context.

Let us now analyse the case of global treaty stable networks when governments are biased in favour of intermediaries. According to Section 5.4.3.2, the global treaty stable networks in this case are networks *a* and *d* (see Figure 4.5) and they are stable because the large countries *i* and *k* are unwilling to sign a global agreement as this agreement decrease the profits made by the intermediaries of these countries (this is shown in Table E.30). According to the information presented in Tables E.30 and E.32, these networks cannot be broken in favour of a global agreement. To see this, note that according to Table E.30, when passing from network *a* or *d* to network *x* (i.e. global free trade in Figure 4.5), profits in the large countries *i* and *k* decrease by 0.1244 and profits in the very small countries *j* and *l* increase by 0.0756. This means that gain in profit in the latter countries are not large enough to compensate the large countries.

On the other hand, intra-node networks can broke these networks in favour of a global agreement. This can be inferred from the same tables. That is, consumer surplus plus producer surplus in the large countries *i* and *k* increase by 0.1570 which is enough to compensate the decrease in profits of 0.1244 when passing from network *a* or *d* to global free trade. Again, this result reveals that intra-node transfers are more effective than inter-node transfers when governments are biased in favour of intermediaries.

Finally, consider the case of governments biased in favour of the farming sector. According to the results obtained in Sections 4.4.1.2 and 5.3, the pairwise and strongly pairwise stable networks in this case are *c* and *c'* (i.e. regionalism). In this case, an inter-node transfer cannot be used to break these networks. To see this, consider the inefficient network *c*. This network can be broken following two possible sub-paths (see Figure 4.5): $c \rightarrow c'$; and $c \rightarrow e$. According to Table E.31, there is no change in producer surplus in any country when passing from this network to network *c'*. This implies that this path does not generate gainers able to compensate losers. On the other hand, passing from network *c* to network *e* causes a loss in producer surplus in each large country by 0.0082 and a gain in producer surplus in the very small country *j* by 0.0073. This implies that the farming sector in the very small country does not have enough resources to compensate the farming sectors in the large countries for trade losses. The same holds for the other inefficient network *c'*. In this case, the possible sub-path is from this network to network *i.* However the conclusion is obtained from the information presented in Table E.31: producer surplus in the very small country is not large enough to compensate the farming sectors in the large countries.

The alternative intra-node transfers tool, however, can assist in this case. As before, two different alternative paths are needed for each inefficient stable network to reach any of the efficient networks. That is, because the very small countries *j* and *l* are not linked (see Figure 4.5) in network *c*, any suitable path can only lead to the efficient network *t* as forming a link between these countries does not affect the level of producer surplus as a consequence of the very small domestic markets in these countries. Likewise, because these countries have already an agreement with each other in network *c'*, any suitable path from this network can only lead to network *x* (i.e. global free trade). The following path are proposed to show that intra-node transfers can break inefficient networks and lead to an efficient one: $c \rightarrow e \rightarrow h \rightarrow n \rightarrow t$; and $c' \rightarrow i \rightarrow p \rightarrow s \rightarrow x$.

In relation to the first path, consider the information presented in Tables 6.19 and 6.20.

Table 6.19. Welfare minus producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c	0.3201	0.0000	0.3201	0.0000
e	0.3431 - 0.0083 = 0.3348	0.0734	0.2882	0.0000
h	0.3108	0.0703	0.3108 - 0.0068 = 0.3040	0.0703
n	0.3348 - 0.0029 = 0.3319	0.0506	0.2952	0.0910
t	0.3175	0.0756	0.3175 - 0.0029 = 0.3146	0.0756

Table 6.20. Producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c	0.0356	0.0000	0.0356	0.0000
e	0.0274 + 0.0083 = 0.0357	0.0073	0.0274	0.0000
h	0.0207	0.0071	0.0207 + 0.0068 = 0.0275	0.0071
n	0.0179 + 0.0029 = 0.0208	0.0051	0.0179	0.0179
t	0.0151	0.0151	0.0151 + 0.0029 = 0.0180	0.0151

Table 6.19 shows the resources of consumers and the intermediaries and the intra-node payments given to the farming sector. Table 6.20, on the other hand, shows producer surplus and the transfers received from consumers and the intermediaries. By analysing this table, it is concluded that in all these networks there are at least two countries willing to sign an agreement implying that the efficient network *t* can be reached from the inefficient network *c* by means of intra-node payments. The same conclusion is obtained for the second proposed path. That is, global free trade can be reached from network *c'* by means of these payments as inferred from Tables 6.21 and 6.22.

Table 6.21. Welfare minus producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.3201	0.0000	0.3201	0.0000
i	0.3431 - 0.0083 = 0.3348	0.0734	0.2882	0.0000
p	0.3108	0.0704	0.3108 - 0.0068 = 0.3040	0.0704
s	0.3348 - 0.0029 = 0.3319	0.0506	0.2952	0.0910
x	0.3175	0.0756	0.3175 - 0.0029 = 0.3146	0.0756

Table 6.22. Producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.0356	0.0000	0.0356	0.0000
i	0.0274 + 0.0083 = 0.0357	0.0073	0.0274	0.0000
p	0.0207	0.0070	0.0207 + 0.0068 = 0.0725	0.0070
s	0.0179 + 0.0029 = 0.0208	0.0051	0.0179	0.0179
x	0.0151	0.0151	0.0151 + 0.0029 = 0.0180	0.0151

Regarding global treaty stability for the case of governments biased in favour of the farming sectors, it was found in Section 5.4.3.2 that the global treaty stable networks in this case are *a*, *c*, *d*, *c'* (see Figure 4.5). In this case, inter-node transfers can only be used to break the inefficient networks *a* and *d* in favour of a global agreement. This is inferred from the information presented in Table 6.23.

Table 6.23. Inter-node transfer to break inefficient networks for the case of biased governments in favour of the farming sector

	Country				
Difference of producer surplus between the efficient network (*t* or *x*) and the inefficient stable network	*i*	*j*	*k*	*l*	Net global producer surplus gain
Efficient network – a	-0.0049	0.0151	-0.0049	0.0151	0.0204
Efficient network – c	-0.0205	0.0151	-0.0205	0.0151	-0.0108
Efficient network – d	-0.0049	0.0151	-0.0049	0.0151	0.0204
Efficient network – c'	-0.0205	0.0151	-0.0205	0.0151	-0.0108

As can be seen in this table, only in networks *a* and *d* it holds that a change from these networks to an efficient one generates a net positive gain in producer surplus. That is, only in these networks the very small countries *j* and *l* have sufficient resources in terms of producer surplus to compensate the farming sectors in the large countries. However, this does not hold in the other inefficient networks *c* and *c'*.

This finding has two main implications. Firstly, inter-node transfers cannot be used in all cases; and when it can be used, very small countries have to compensate large countries which is difficult to support.

The same results hold for the alternative intra-node transfers. That is, this tool can only break the inefficient networks *a* and *d*. To see this, note that when passing from these networks to global free trade producer surplus decreases in the large countries by 0.0049. However, as it inferred from Tables E.31 and E.32, consumer surplus plus profits (i.e. welfare minus producer surplus) in each large country increase by 0.0375 which is large enough to compensate the loss of 0.0049 faced by the farming sector. In relation to the other inefficient networks *c* and *c'*, on the other hand, the loss in producer surplus in each large country is 0.0205. But in this case, the sum of consumer surplus and producer surplus also decreases by 0.0026 implying that there are insufficient resources available in the large countries to compensate the farming sector suggesting that regionalism is difficult to break. In spite of this disappointing result, there is still a possibility. That is, consumer surplus plus profits in the very small countries increase by 0.0756 when passing from either network *c* or network *c'* to global free trade. This suggests that if these sectors have enough resources to compensate the farming sectors in the very large countries in order to break regionalism. However, as explained above, it is difficult to support payments given by very small countries to large countries. Nonetheless, this possibility exists in theoretical terms.

6.4.3.2 Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$ for all $i \in N$.

This simulation assumes the existence of large countries with very small countries with high level of monopsonistic power (i.e. $\phi_i = 1.5$) and the efficient networks correspond to networks *t* and *x* (see Table 6.2).

The results obtained in Sections 4.4.1.3, 5.3, and 5.4.3.3 revealed that the pairwise, strongly pairwise, and global treaty stable networks are the same as those identified in the previous simulation. The only exception is the case of unbiased governments. In the current simulation the pairwise and strongly pairwise stable networks when governments are unbiased are network *t* and *x*. Since these networks are efficient, no transfer is needed in this case. On the other hand, the global treaty stable networks in this case are networks *c*, *e*, *g*, *i*, *j*, *n*, *o*, *q*, *s*, *t*, *u*, *x*, *c'*. This set differs from the previous case in that networks *g* and *o* are also global treaty stable when monopsonistic power is high. Apart from these differences, all the results obtained in the previous simulation also holds in the current simulation. This suggests therefore that these results remain robust through different levels of monopsonistic power.

6.4.3.3 Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

This simulation assumes a world composed of large and medium size countries with intermediaries that exercise moderate levels of monopsonistic power (i.e. ϕ_i = 0.5). In contrast to the two previous simulations, only network *t* is the efficient network. In spite of this, this section explores how to reach this network and also global free trade by means of transfers as reaching the latter is one of the aims of the WTO and also because it is the second best network in terms of the level of global welfare that can be obtained in any network.

The case of politically unbiased governments

The results obtained in Sections 4.4.1.5 and 5.3 revealed that network *x* (i.e. global free trade) is the only pairwise and strongly pairwise stable network when governments are unbiased. As explained above, this a desirable network and also the second best in terms of efficiency. However, the efficient network *t* can still be reached and stabilised from global free trade by paying the medium size countries *j* and *l* inter-node compensatory payments in order to break their existing network (i.e. passing from global free trade to network *t* requires these countries to break their agreement). This can be inferred from Table E.43 in Appendix E. That is, when passing from network *x* to *t*, welfare in each large country increases by 0.0048 and welfare in the medium size countries decreases by 0.0033. This proves that a transfer can be used in this case to break the agreement. However, it is difficult to support this particular transfer because breaking agreements is against the spirit of the WTO.

On the other hand, it was found in Section 5.4.3.5 that the global treaty stable networks when governments are unbiased are *c*, *e*, *g*, *h*, *i*, *j*, *n*, *o*, *q*, *s*, *t*, *u*, *x* and *c'*. While global free trade is a second best option in this simulation, it can be reached by means of a global agreement with inter-node transfers. This is inferred from the following table.

Table 6.24. Inter-node transfer to break inefficient networks under unbiased governments

	Country				
Difference of welfare between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
$X-c$	-0.0187	0.0791	-0.0187	0.0791	0.1208
$X-e$	-0.0386	0.0207	0.0174	0.0791	0.0786
$X-g$	-0.0644	0.0419	0.0348	0.0419	0.0542
$X-h$	-0.0046	0.0213	-0.0046	0.0213	0.0334
$X-i$	-0.0327	0.0068	0.0165	0.0687	0.0593
$X-j$	-0.0009	-0.0279	-0.0009	0.0791	0.0494
$X-n$	-0.0258	0.0401	0.0173	-0.0093	0.0223
$X-o$	-0.0542	0.0304	0.0337	0.0304	0.0403
$X-q$	0.0020	-0.0395	0.0020	0.0715	0.0360
$X-s$	-0.0190	0.0323	0.0194	-0.0177	0.0150
$X-u$	-0.0190	-0.0177	0.0194	0.0323	0.0150
$X-c'$	-0.0187	0.0653	-0.0187	0.0653	0.0932

The last column of this table shows that there are always enough resources when passing from an inefficient network to global free trade to compensate loser countries from a global agreement.

The case of politically biased governments

As shown in Sections 4.4.1.5 and 5.3, the pairwise and strongly pairwise stable networks when governments are biased in favour of consumers is global free trade. As explained above, this is a second best and desirable network. On the other hand, the global treaty stable networks in this case are *g*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v* and x (see Section 5.4.3.5). They cannot in general be broken by intra-node transfers. For example, consider network *g*. This network is global treaty stable because country *i* is unwilling to sign a global agreement. It is concluded from Tables E.40 and E.43 that passing from network *g* to network *x* decreases the level of profits plus producer surplus in this country by 0.571 implying that the intermediary and the farming sector do not have enough resources to compensate consumers when signing a global agreement. In contrast, inter-node payments are effective in this case. This is inferred from the fact that passing from an inefficient network to global free trade always generates a net gain in global welfare as shown in Table 6.25.

Table 6.25. Inter-node transfer to break inefficient networks under biased governments in favour of consumers

	Country				
Difference of welfare between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
$X-g$	-0.0073	0.0137	0.0809	0.0137	0.1010
$X-n$	-0.0037	0.0128	0.0330	0.0064	0.0485
$X-o$	-0.0049	0.0040	0.0795	0.0040	0.0826
$X-q$	0.0344	-0.0020	0.0344	0.0117	0.0785
$X-r$	0.0290	0.0047	0.0771	-0.0025	0.1083
$X-s$	-0.0021	0.0053	0.0330	-0.0008	0.0354
$X-u$	-0.0021	-0.0008	0.0330	0.0053	0.0354
$X-v$	0.0316	-0.0013	0.0316	-0.0013	0.0606

Let us now consider the case of governments biased in favour of intermediaries. According to Sections 4.4.1.5 and 5.3, network *a* (see Figure 4.5) is the only pairwise and strongly pairwise in this case. According to the information presented in Table E.41, this network cannot be broken with an inter-node transfer financed by gainer intermediaries. For example, profits in country *j* increases by 0.0647 when passing from network *a* to network *b*. But this gain in profits is not large enough to compensate the loss in profits of 0.0853 faced by the intermediary of country *i*. Likewise, passing from network *a* to either network *c* or *d* only generates losers in terms of profits. In contrast, a suitable alternative to break network *a* in favour of either the efficient network *t* or global free trade is the adoption of intra-node transfers. To illustrate that this is possible, the following paths are proposed, one to reach the efficient network *t* and the other to reach global free trade: $a \rightarrow c \rightarrow e \rightarrow h \rightarrow n \rightarrow t$; and $a \rightarrow c \rightarrow e \rightarrow h \rightarrow p \rightarrow s \rightarrow x$. The following tables were developed with the information presented in Tables E.41 and E.43 and show how these paths can be led by intra-node transfers.

Table 6.26. Welfare minus profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1000	0.0250	0.1000	0.0250
c	0.1778 - 0.0223 = 0.1555	0.0250	0.1778 - 0.0223 = 0.1555	0.0250
e	0.2324 - 0.0571 = 0.1753	0.0449	0.1724	0.0250
h	0.2274	0.0455	0.2274 - 0.0331 = 0.1943	0.0455
n	0.2638 - 0.0153 = 0.2485	0.0430	0.2192	0.0630
t	0.2567	0.0629	0.2567 - 0.0155 = 0.2412	0.0629

Table 6.27. Profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.0500	0.2000	0.0500
c	0.1778 + 0.0223 = 0.2001	0.0500	0.1778 + 0.0223 =0.2001	0.0500
e	0.1431 + 0.0571 = 0.2002	0.0885	0.1471	0.0500
h	0.1141	0.0873	0.1141 + 0.0331 = 0.1472	0.0873
n	0.0989 + 0.0153 = 0.1142	0.0710	0.1004	0.1004
t	0.0850	0.0879	0.085 + 0.0155 = 0.1005	0.0879

Table 6.28. Welfare minus profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1000	0.0250	0.1000	0.0250
c	0.1778 - 0.0223 = 0.1555	0.0250	0.1778 - 0.0223 = 0.1555	0.0250
e	0.2324 - 0.0571 = 0.1753	0.0449	0.1724	0.0250
h	0.2274	0.0455	0.2274 - 0.0331 = 0.1943	0.0455
p	0.2245	0.0592 - 0.0043 = 0.0549	0.2245	0.0592 - 0.0043 = 0.0549
s	0.2591 - 0.0147 = 0.2444	0.0548	0.2178	0.0750
x	0.2535	0.0707	0.2535 - 0.0164 = 0.2371	0.0707

Table 6.29. Profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.0500	0.2000	0.0500
c	0.1778 + 0.0223 = 0.2001	0.0500	0.1778 + 0.0223 = 0.2001	0.0500
e	0.1431 + 0.0571 = 0.2002	0.0885	0.1471	0.0500
h	0.1141	0.0873	0.1141 + 0.0331 = 0.1472	0.0873
p	0.1114	0.0831 + 0.0043 = 0.0874	0.1114	0.0831 + 0.0043 = 0.0874
s	0.0968 + 0.0147 = 0.1115	0.0670	0.0997	0.0968
x	0.0834	0.0834	0.0834 + 0.0164 = 0.0998	0.0834

Tables 6.26 and 6.27 show that in any sub-path of the path that lead to the efficient network there are available resources in the loser country from consumers and the farming sector to compensate the intermediaries. Likewise, Tables 6.28 and 6.29 shows the same but for the path that leads to global free trade.

In relation to global treaty stability for the case of biased governments in favour of intermediaries, it was found in Section 5.4.3.5 that the global treaty stable network is network *a* (see Figure 4.5). In this case, inter-node transfers financed by gainer intermediaries cannot be used to break network *a* in favour of a global agreement. This can be inferred from the information presented in Table E.41. That is, profits in each of the large countries *i* and *k* decreases by 0.1166, and profits in each medium size county increase by 0.0334. This means that the intermediaries of the latter have not enough resources to compensate the intermediaries of the latter if a global agreement is signed. Nonetheless, intra-node transfers can be used for this purpose. This is inferred from the information presented in Tables E.31 and E.33. According to this information, consumer surplus plus producer surplus in each large countries *i* and *k* increase by 0.1535 which is large enough to compensate the loss in profits of 0.1166 faced by the intermediaries of these countries.

Finally, consider the case of governments biased in favour of the farming sector. According to the results obtained in Sections 4.4.1.5 and 5.3, the pairwise and strongly pairwise stable networks in this case are networks *c'* and *x*. Because the latter is global free trade and is the second best in this simulation in terms of efficiency, the focus is placed on how to break network *c'*. This network is a form of regionalism of the south-north type (see Figure 4.5) and can be broken towards global free trade by means of inter-node transfers. To see this, consider the following path of networks: $c' \rightarrow i \rightarrow p \rightarrow s \rightarrow x$. In considering the information presented in Table E.42, the following results concerning this path was obtained.

Table 6.30. Producer surplus with inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.0356	0.0089	0.0356	0.0089
i	0.0350 + 0.0007 = 0.0357	0.0221 - 0.0007 = 0.0214	0.0288	0.0088
p	0.0297	0.0212	0.0297	0.0212
s	0.0323	0.0182	0.0261	0.0323
x	0.0288	0.0288	0.0288	0.0288

According to this table, an inter-node payment is only needed to break the stability of network *c'*. That is, it is inferred from Figure 4.5 that passing from network *c'* to network *i* requires an agreement between the large country *i* and the medium size country *j*. If this agreement is signed, producer surplus in the former country decreases from 0.0356 to 0.0350, and producer surplus in the latter increases from 0.0089 to 0.0221. This means that the gain in producer surplus in the medium size country is large enough to compensate the large country by paying, for example, a transfer of 0.0007. Thus once network *a* is broken, there are always at least two countries willing to sign bilateral agreement in any network in the path. For example, passing from network *p* to network *s* requires an agreement between countries *i* and *l* (see Figure 4.5). As shown in Table 6.30, these countries have an incentive to sign this agreement because producer surplus increases in both countries.

Intra-node transfers can also be used to break network *a* and to lead to global free trade. To see this, consider the same path and the following results obtained from Tables E.42 and E.43 in Appendix E.

Table 6.31. Welfare minus producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.3200	0.0799	0.3200	0.0799
i	0.3346 - 0.0007 = 0.3339	0.1252	0.2916	0.0766
p	0.3062	0.1211	0.3062	0.1211
s	0.3236	0.1036	0.2914	0.1395
x	0.3081	0.1253	0.3081	0.1253

Table 6.32. Producer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.0356	0.0089	0.0356	0.0089
i	0.0350 + 0.0007 = 0.0357	0.0221	0.0288	0.0088
p	0.0297	0.0212	0.0297	0.0212
s	0.0323	0.0182	0.0261	0.0323
x	0.0288	0.0288	0.0288	0.0288

According to Tables 6.31 to 6.32, a joint intra-node transfer of 0.0007 paid by consumers and the intermediary of the large country *i* to the farming sector in this country can break network *a*. The same as in the case of the inter-node transfer discussed in Table 6.30, once network *a* is broken, there are always at least two countries willing to sign an agreement in any network in the path.

In relation to the results obtained for the case of governments biased in favour of the farming sector, there are two aspects that are interesting to discuss. Firstly, while both types of transfers can lead to global free trade, intra-node transfers may be preferred form an ethical point of view. The reason is because it is difficult to support the use of payments given by medium size countries to compensate large countries. This ethical issue is beyond the scope of this dissertation. Nonetheless, this argument may be used in future normative research to support the use of intra-node transfers in this case.

Secondly, reaching the efficient network *t* in this simulation requires the medium size countries *j* and *l* to break their existing agreement in any feasible path. This is against the aim of the WTO and as such, this corresponds to an ethical issue that is beyond the scope of this dissertation. However, the possibility to lead to this network exists in theory. For example, breaking this agreement in global free trade (i.e. passing from network *x* to network *t*) decreases producer surplus in each medium size country by 0.0036, but increases total welfare in each large country by 0.0048. This implies that a joint inter-node transfer paid by consumers, the intermediary and the farming sector of the large country has the potential to lead to the efficient network.

On the other hand, it was found in Section 5.4.3.5 that the global treaty stable networks when governments are biased in favour of the farming sector are *c*, *e*, *g*, *h*, *j*, *n*, *o*, *p*, *q*, *s*, *t*, *u*, *w*, *x* and *c'*. In this case, inter-node transfers can be used to break inefficient global treaty networks in order to sign a global agreement. This is inferred from the following table (the information presented in this table was obtained from Table E.42).

Table 6.33. Inter-node transfer to break inefficient in favour of a global agreement

	Country				
Difference of producer surplus between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net producer surplus gain
$X-c$	-0.0068	0.0238	-0.0068	0.0238	0.0340
$X-e$	-0.0092	0.0121	0.0000	0.0238	0.0267
$X-g$	-0.0134	0.0150	0.0038	0.0150	0.0204
$X-h$	-0.0030	0.0124	-0.0030	0.0124	0.0188
$X-j$	-0.0025	-0.0025	-0.0025	0.0238	0.0163
$X-n$	-0.0066	0.0149	0.0013	0.0013	0.0109
$X-o$	-0.0087	0.0102	0.0038	0.0102	0.0155
$X-p$	-0.0009	0.0076	-0.0009	0.0076	0.0134
$X-q$	-0.0006	-0.0082	-0.0006	0.0202	0.0108
$X-s$	-0.0035	0.0106	0.0027	-0.0035	0.0063
$X-u$	-0.0035	-0.0035	0.0027	0.0106	0.0063
$X-w$	0.0076	-0.0009	0.0076	-0.0009	0.0134
$X-c'$	-0.0068	0.0199	-0.0068	0.0199	0.0262

The last column in this table shows that passing from an inefficient global treaty stable network to global free trade generates a net world gain in producer surplus implying that there are enough resources from the farming sector in the gainer countries to compensate the loser countries.

6.4.3.4 Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$ for all i $\in$ N.

This is the last simulation that considers asymmetry in market size and assumes a world composed of large and medium size countries with high levels of monoponistic power.

The results obtained in Sections 4.4.1.6, 5.3 and 5.4.3.6 are in general the same as the ones obtained in the previous simulation. As a consequence, the same general conclusions for the use of compensatory payments hold in the current simulation as can be inferred from Tables E.44, E.45, E.46 and E.47 (See Appendix E). However there are some differences that are explored as follows.

Firstly, the pairwise and strongly pairwise stable networks when governments are politically unbiased are networks *t* and *x* (see Figure 4.5). In contrast, in the previous simulation only the latter is stable. This suggests that the efficient network *t* and the second best network *x* (i.e. global free trade) are both possible outcomes when governments are unbiased. As a result, no compensatory payments are needed in this case.

Secondly, the pairwise and strongly pairwise stable networks when governments are biased in favour of consumers is network *x* and network *c'*. The latter is only presented in this simulation under this bias suggesting that regionalism is a possible outcome when governments are biased in favour of consumers. According to the information presented in Table E.44, this network can be broken and lead to either global free trade or the efficient network *t* by either inter-node or intra-node transfers. To see this, consider the following path that lead to global free trade: $c' \rightarrow i \rightarrow p \rightarrow s \rightarrow x$. The ability of inter-node and intra-node transfers to facilitate this path is inferred from the following tables.

Table 6.34. Consumer surplus with inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.0726	0.0181	0.0726	0.0181
i	0.1114 - 0.0020 = 0.1094	0.0162 + 0.0020 = 0.0182	0.0762	0.014
p	0.1131	0.014	0.1131 - 0.0001 = 0.1130	0.0140 + 0.0001 = 0.0141
s	0.1367	0.0127	0.1092	0.0153
x	0.1335	0.0143	0.1335	0.0143

Table 6.35. Welfare minus consumer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.1814	0.0454	0.1814	0.0454
i	0.1526	0.0991 - 0.0020 = 0.0971	0.1549	0.0504
p	0.1285	0.0989	0.1285	0.0989 - 0.0001 = 0.0988
s	0.1188	0.0862	0.1198	0.1188
x	0.1088	0.1088	0.1088	0.1088

Table 6.36. Consumer surplus with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
c'	0.0726	0.0181	0.0726	0.0181
i	0.1114	0.0162 + 0.0020 = 0.0181	0.0762	0.0140
p	0.1131	0.014	0.1131	0.0140 + 0.0001 = 0.0141
s	0.1367	0.0127	0.1092	0.0153
x	0.1335	0.0143	0.1335	0.0143

According to Figure 4.5, breaking network *c'* requires an agreement between the large country *i* and the medium size country *j* (i.e. passing from network *c'* to network *i*). According to Table 6.34, this agreement decreases consumer surplus in country *j* from 0.0181 to 0.0162 and increases consumer surplus in country *i* from 0.0726 to 0.1114. Thus, an inter-node transfer of 0.0020 by the latter to the former can break this network. Following this reasoning, it can be inferred from this table that this path is feasible with this type of transfers.

Tables 6.35 and 6.36, on the other hand, show a similar result when using a join intra-node transfer paid to consumers by the intermediary and the farming sector. For example, a transfer of 0.0020 paid to consumers is large enough to break network *c'*.

6.4.4 Simulations under the assumptions of exogenous tariffs and asymmetry in farmers' productivity

This section considers the simulation that was developed to identify possible international trade structures when countries are asymmetric in terms of farmers' productivity. In this case it is assumed that countries *i* and *k* have a less productive farming sector, and countries *j* and *l* have more efficient farming sectors.

According to Table 6.2, network *o* in Figure 4.5 is the efficient network and network *x* (i.e. global free trade) is a second best network in terms of efficiency. The efficiency of network *o* is explained by the fact that the high cost faced by the intermediary in a less productive country is reduced when this country occupies a central position in the network because this increases the level of competition in the market. This lower cost has positive effects on global welfare. The problem is that network *o* has a relatively low degree of integration implying that reaching this network is not in the spirit of the WTO. In spite of this problem, this section explores how to reach the efficient as well as global free trade. The use of compensatory payments to reach these networks is explained as follows.

6.4.4.1 Simulation 17: $\delta = 3$ for $\Omega = \{i, k\}$; $\delta = 1$ for $\Psi = \{j, l\}$; $\phi = 0.5$; and $\alpha = 1$.

<u>*The case of politically unbiased governments*</u>

The results obtained in Sections 4.4.2.1 and 5.3 revealed that the pairwise and strongly pairwise stable networks in this simulation are networks *m* and *x*. Because network *x* is the desirable network and second best in terms of efficiency, the focus is placed on how to break network *m*.

As explained in Corollary 6.2 and proposition 6.3, only inter-node transfers can be used when governments are unbiased. This tool in the current simulation has the potential to break network *m* and lead the world to both network *o* and global free trade. To see this, consider the following proposed path: $m \rightarrow o \rightarrow s \rightarrow x$. The way by which this path is facilitated by an inter-node transfer is inferred from the following table.

Table 6.37. Welfare with inter-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
m	0.2793	0.3951	0.2551	0.3951
o	0.3927 - 0.0149 = 0.3778	0.4124	0.2403 + 0.0149 = 0.2552	0.4124
s	0.3030	0.3810	0.2569	0.4510
x	0.2854	0.4199	0.2854	0.4199

According to Figure 4.5, passing from network *m* to network *o* requires the inefficient countries *i* and *k* to sign a bilateral agreement. As shown in Table 6.37, this can be achieved by means of a transfer of 0.0149 paid by the gainer country *i* to the loser country *k*. This allows the world to reach the efficient network *o*. From this network, there are always at least two countries willing to sign an agreement in the path until global free trade is reached. This means that the transfer can assist the world to reach the first and second best networks in this simulation. Note, however, that to remain in the efficient network *o* would require an additional transfer to prevent countries *k* and *l* to form an agreement (see Figure 4.5). This is against of the spirit of the WTO. However, this possibility exists in theoretical terms. To see this, note from Table 6.37 that passing from network *o* to network *s* increase welfare in countries *k* and *l* by 0.017 and 0.0386, respectively. In contrast, welfare in countries *i* and *j* decreases by 0.0748 and 0.0314, respectively. This means that the latter countries have enough resources to prevent the agreement by paying a joint inter-node payment of at least 0.0403. On the other hand, countries *j* and *k* have an incentive to sign an agreement with each other in network *o* (i.e. passing from network *o* to network *u* in Figure 4.5). This agreement also has to be prevented in order to stabilise the efficient network *o*. Using a similar approach, it is inferred from Table E.51 that this agreement can indeed be prevented.

On the other hand, it was found in Section 5.4.4.1 that the global treaty stable networks when governments are unbiased in the current simulation are *c*, *h*, *i*, *j*, *n*, *q*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, and *c'*. The following table shows that inter-node transfers can be used to break these networks in favour of global free trade.

Table 6.38. Inter-node transfer to break inefficient in favour of a global agreement

	Country				
Difference of welfare between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global welfare gain
$X-c$	-0.0048	0.1199	-0.0048	0.1199	0.2302
$X-h$	-0.0083	0.0530	-0.0083	0.0530	0.0894
$X-i$	-0.0325	-0.0075	0.0210	0.0742	0.0552
$X-j$	0.0022	-0.0098	0.0022	0.1199	0.1145
$X-n$	-0.0493	0.0725	0.0151	0.0227	0.0610
$X-q$	0.0153	-0.0675	0.0153	0.0843	0.0474
$X-s$	-0.0176	0.0389	0.0285	-0.0311	0.0187
$X-t$	-0.0246	0.0422	-0.0246	0.0422	0.0352
$X-u$	-0.0176	-0.0311	0.0285	0.0389	0.0187
$X-v$	0.0175	-0.0164	0.0175	-0.0164	0.0022
$X-w$	-0.0105	0.0262	-0.0105	0.0262	0.0314
$X-y$	0.0282	-0.0097	-0.0228	0.0724	0.0681
$X-z$	-0.0387	0.0766	0.0303	0.0766	0.1448
$X-a'$	0.0298	-0.0509	0.0298	0.1199	0.1286
$X-c'$	-0.0048	0.0643	-0.0048	0.0643	0.1190

The last column of this table shows that there is always a global gain in welfare when passing from an inefficient global treaty stable network to global free trade. This means that there are always enough resources in the world to compensate loser countries when signing a global agreement from any inefficient global treaty stable network.

<u>The case of politically biased governments</u>

When governments are biased in favour of consumers, only global free trade is pairwise and strongly pairwise stable (see Sections 4.4.2.1 and 5.3). Because this is a desirable and second best network, the use of transfers is not considered in this case. On the other hand, the global treaty stable networks under this political bias correspond to *g*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v* and *x* (see Section 5.4.4.1). In this case, intra-node transfers cannot be used to break these networks in favour of a global agreement. This is inferred from the information presented in Table E.48. For example, this agreement decreases consumer surplus in country *i* when passing from network *g* to global free trade by 0.0110. This cannot be compensated by a joint transfer paid by the intermediary and the farming sector of this country because the agreement also decreases their joint resources by 0.0685. In contrast, inter-node transfers are effective in this case as can be seen in the following table.

Table 6.39. Inter-node transfer to break inefficient in favour of a global agreement

	Country				
Difference of consumer surplus between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global consumer surplus gain
$X-g$	-0.0110	0.0592	0.0886	0.0592	0.1960
$X-l$	0.0596	-0.0133	0.0596	0.0322	0.1381
$X-n$	-0.0077	0.0569	0.0356	0.0356	0.1204
$X-o$	-0.0577	0.0146	0.0857	0.0146	0.0572
$X-q$	0.0365	-0.0062	0.0365	0.0308	0.0976
$X-r$	0.0111	0.0111	0.0713	-0.0087	0.0848
$X-s$	-0.0028	0.0146	0.0356	-0.0028	0.0446
$X-t$	-0.0020	0.0358	-0.0020	0.0358	0.0676
$X-u$	-0.0028	-0.0028	0.0356	0.0146	0.0446
$X-v$	0.0144	-0.0037	0.0144	-0.0037	0.0214

This table shows in the last column that the gain in global consumer surplus is large enough to compensate loser countries from a global agreement.

Let us now consider the case of governments biased in favour of the intermediaries. In this case, only network *a* is both pairwise and strongly pairwise stable (see Sections 4.4.2.1 and 5.3). In this case inter-node transfers paid by gainer intermediaries cannot be used to break this network. For example, it is inferred from Table E.49 that passing from this network to network *b* decreases the profit made by the intermediary of country *i* by 0.0630 and increases the profit made by the intermediary of country *j* by 0.0312. This gain is not large enough to compensate the intermediary of the former country. From the same table, it is inferred that passing from network *a* to either network *c* or network *d* does not generate gainers in terms of profits implying that there are no intermediaries willing to pay compensatory inter-node transfers. In contrast, intra-node transfers have the potential to break this network and to lead the world to either the efficient network *o* or global free trade. To see this, consider the following proposed path: $a \rightarrow c \rightarrow e \rightarrow g \rightarrow o \rightarrow s \rightarrow x$. The following tables show that this path can be facilitated by means of intra-node transfers.

Table 6.40. Welfare minus profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0714	0.1000	0.0714	0.1000
c	0.1269 - 0.0205 = 0.1064	0.1000	0.1269 - 0.0205 = 0.1064	0.1000
e	0.1960 - 0.0497 = 0.1463	0.1466	0.0983 - 0.0710 = 0.0273	0.1000
g	0.2473 - 0.0542 = 0.1931	0.1395	0.1101	0.1395 - 0.0035 = 0.1360
o	0.2818	0.2076	0.1131	0.2076
s	0.2183	0.2003	0.1683 - 0.0387 = 0.1296	0.2449
x	0.2106	0.2348	0.2106 - 0.0526 = 0.1580	0.2348

Table 6.41. Profits with intra-node transfers

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1837	0.2000	0.1837	0.2000
c	0.1633 + 0.0205 = 0.1838	0.2000	0.1633+ 0.0205 = 0.1838	0.2000
e	0.1342 + 0.0497 = 0.1839	0.2187	0.1129 + 0.0710 = 0.1839	0.2000
g	0.1298 + 0.0542 = 0.1840	0.1966	0.1295	0.1966 + 0.0035 = 0.2001
o	0.1109	0.2048	0.1272	0.2048
s	0.0847	0.1807	0.0886 + 0.0387 = 0.1273	0.2061
x	0.0748	0.1851	0.0748 + 0.0526 = 0.1274	0.1851

Table 6.40 shows the networks where join transfers have to be paid by consumers and the farming sector, and Table 6.41 shows the networks where these payments are received by the intermediaries. The payoffs in the latter table show that in any

network in the path there are at least two countries willing to sign a bilateral agreement. For example, passing from network *o* to network *s* requires countries *k* and *l* to sign an agreement (see Figure 4.5). This is feasible because the payoffs with transfers increase in both countries when the agreement is signed. This table also shows that the efficient network *o* and global free trade can both be reached by adopting intra-node transfers.

On the other hand, it was found in Section 5.4.4.1 that network *a* is also the only global treaty stable network when governments are biased in favour of the intermediaries. In this case, inter-node transfers cannot be used to facilitate a global agreement because passing from network *a* to global free trade decreases the profits made by the intermediaries of all countries in the network as is inferred from Table E.49. This means that all countries need to be compensated implying that there are not incentives to pay transfers across countries. In contrast, intra-node transfers can assist the world to sign a global agreement. To see this, note that according to the information presented in Table E.49, the profits made by the intermediary of each inefficient country *i* and *k* decreases by 0.1089 when passing from network *a* to global free trade. In contrast, consumer surplus plus producer surplus increase by 0.1392. This implies that consumers and the farming sector in these countries have enough resources to compensate the intermediaries. The same holds for the efficient countries *j* and *l* (i.e. profits in these countries decrease by 0.0149 and consumer surplus plus producer surplus increase by 0.1348). This finding suggests therefore that a suitable tool to facilitate a global agreement when governments are biased in favour of the intermediaries corresponds to intra-node transfers.

Finally, let us consider the case of governments biased in favour of the farming sector. According to Sections 4.4.2.1 and 5.3, in this case only global free trade is both pairwise and strongly pairwise stable. Again, because this is a desirable network and is also the second best option in terms of efficiency, no transfer is needed to deal with the issue of pairwise and strongly pairwise stability. On the other hand, according to the results obtained in Section 5.4.4.1, the global treaty stable network when governments are biased in favour of the farming sector are *c*, *e*, *g*, *h*, *i*, *j*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *y*, *z*, *a'* and *c'*. The use of Intra-node transfers is not the most suitable tool to break these networks in favour of a global agreement because they only work in some of them. For example, passing from network *c* to global free trade decreases producer surplus in countries *i* and *k* by 0.0046, but it increases profits plus consumer surplus by 0.0194 as can be inferred from tables E.50 and E.51 in Appendix E. This means that, in this particular case, consumers and the intermediaries have together enough resources to pay a transfer to the farming sector. Conversely, passing from network *e* to global free trade decreases producer surplus in country *i* by 0.0140 and consumer surplus plus profits by 0.0176. This implies that in this case consumers and the intermediary in this country do not have enough resources to compensate the farming sector.

A suitable strategy that can effectively be used in this case corresponds to the adoption of inter-node transfers. This is inferred from the following table (this table was obtained from the information contained in Table E.50).

Table 6.42. Inter-node transfer to break inefficient in favour of a global agreement

	Country				
Difference of producer surplus between global free trade and the inefficient stable network	*i*	*j*	*k*	*l*	Net global producer surplus gain
$X-c$	-0.0046	0.0540	-0.0046	0.0540	0.0988
$X-e$	-0.0140	0.0305	0.0125	0.0540	0.0830
$X-g$	-0.0258	0.0361	0.0118	0.0361	0.0582
$X-h$	-0.0044	0.0306	-0.0044	0.0306	0.0524
$X-i$	-0.0069	0.0110	0.0028	0.0390	0.0459
$X-j$	-0.0029	0.0037	-0.0029	0.0540	0.0519
$X-l$	0.0171	-0.0251	0.0171	0.0407	0.0498
$X-n$	-0.0145	0.0341	0.0024	0.0123	0.0343
$X-o$	-0.0135	0.0126	0.0119	0.0126	0.0236
$X-q$	0.0022	-0.0166	0.0022	0.0400	0.0278
$X-r$	0.0088	0.0193	0.0168	-0.0081	0.0368
$X-s$	-0.0049	0.0199	0.0067	-0.0073	0.0144
$X-t$	-0.0076	0.0163	-0.0076	0.0163	0.0174
$X-u$	-0.0049	-0.0073	0.0067	0.0199	0.0144
$X-v$	0.0095	-0.0034	0.0095	-0.0034	0.0122
$X-y$	0.0120	0.0057	-0.0010	0.0346	0.0513
$X-z$	-0.0036	0.0353	0.0192	0.0353	0.0862
$X-a'$	0.0120	-0.0037	0.0120	0.0540	0.0743
$X-c'$	-0.0046	0.0384	-0.0046	0.0384	0.0676

This table shows that there is always a gain in global producer surplus when passing from an inefficient network to global free trade (see the last column in this table). This confirms the claim that inter-node transfers can assists the world to reach a global agreement in a world composed of asymmetric countries in terms of farmers' productivity and when governments are biased in favour of the farming sector.

6.5 Summary and conclusions

It was found in the previous chapters that different networks can be either pairwise, strongly pairwise or global treaty stable depending on the existence of asymmetries across countries and governments' policy biases. This chapter extends the analysis with the purpose of exploring how to break inefficient stable networks and to reach efficient ones by means of compensatory payments that are given by gainers from free trade to losers.

The chapter starts defining the concept of efficiency as those networks that offer the highest possible global welfare among all possible international trade configurations. It also explains the fact that, in many cases, efficient networks are not stable and this is one of the arguments that have been used to propose the use of compensatory side-payments to stabilize these networks.

In the current investigation, global free trade is the efficient network in all the simulations that assume symmetrical countries. However, when asymmetry is introduced into the model, other networks becomes efficient. In particular, two networks become efficient when the world is composed of large and very small countries: global free trade; and a network in which all countries have an agreement with each other except the very small countries. The latter network is efficient because an agreement between the very small countries does not significantly affect global welfare as a consequence of the very small domestic markets in these countries. When the world is composed of large and medium size countries, only the network in which all countries have an agreement with each other except the medium size countries is efficient. The reason is because when these countries sign the agreement (i.e. global free trade), the increase in the output that is exported to each other significantly increases the price paid to the farming sector in these countries. In order to offsets this higher cost, the intermediaries of these countries reduce the level of export to the existing large country partners causing a net decrease in global welfare. This is why global free trade is not efficient in this case. However, it is the second most efficient network and, as such, it is considered in these simulations as a second best.

Finally, when there is asymmetry in terms of farmers' productivity, a network with centre a less productive country that is connected to another less productive country and to productive countries having an agreement with each other is the efficient network. This is explained by the fact that the central position of the less efficient country increases the level of competition in this country cushioning the high price that is paid to the farming sector. This allows the intermediary of this country to be more competitive and this positively affects the level of global welfare. In spite of the fact that global free trade is not the efficient network in this case, it is the second best one.

The chapter continues by defining and describing two different types of compensatory transfers that have the potential to lead to free trade. One of them is referred to as inter-node transfers and has normally been proposed in the literature to deal with the problem of efficiency and stability. That is, this is a tool that can be used to stabilise the efficient network when it is unstable by allowing payments across nodes (or countries in the presented study). In a related article by Furusawa and Konishi (2005), this transfer was explored as a potential tool to stabilise global free trade when countries have different levels of industrialisation. However, the adoption of this type of payment in the current investigation differs from the work by these researchers in that the instability of the efficient network is explained by the existence of a farming sector that affects the marginal cost faced by intermediaries, the existence of asymmetry in market size across countries, the existence of asymmetry in farmers' productivity across countries, and governments' political biases. It also differs in terms of the use because inter-node payments in the current dissertation not only are used as a tool that is able to stabilise the efficient network, but also to break inefficient stable ones in favour of free trade.

The other type of compensatory transfers referred to in this investigation as intra-node transfers is a novel contribution of this dissertation. The reason is because the transfers considered in the literature of networks used to reach the efficient network correspond to payments across nodes, but not transfers within a node itself. In the current investigation they correspond to payments given by gainers from free trade within a country to losers that belong to the same country (e.g. from consumers to the farming sector). It is proved in this chapter that intra-node payments to be Pareto improving can only be used in cases when governments are politically biased.

The ability of inter-node and intra-node transfers to stabilise the efficient network or to break existing ones is also explored in this study by using them in the simulations developed in the previous chapters. It was found that in some simulations, these transfers are not needed because the stable network is the efficient one (or the desirable global free trade when it is the second best) implying that the world would reach this network without payments. However, in cases where this network is not stable or when the world is trapped in a stable inefficient stable network, the following general trends were found. Firstly, inter-node transfers are more suitable to favour either bilateral or global agreements when governments are unbiased. Secondly, when governments are biased in favour of consumers, inter-node transfers are normally more effective than intra-node transfers, although the latter can work in some simulations that involve asymmetry in market size. Thirdly, intra-node transfers are more effective to lead to global free trade when governments are biased in favour of intermediaries. Finally, when governments are biased in favour of the farming sectors, inter-node or intra-node transfers are more effective depending on the simulation.

It is important to highlight the fact that inter-node and intra-node payments are abstract concepts. However, there exist some transfers that are consistent with these payments such as the aid for trade. While existing transfers have normally been used in different contexts (e.g. intra-node or inter-node transfers can be adopted with the purpose of breaking inefficient networks in favour of free trade rather than facilitating trade in developing countries), their current adoption suggests that the proposed payments can indeed be considered as suitable compensatory strategies to reach more integrated networks. This possibility is discussed in more detail in the next chapter.

In relation to the work by Furusawa and Konishi (2005), the results obtained in this chapter offer new insights that were not explored by these researchers. Firstly, it was also found that asymmetry affects the stability of global free trade. However, the type of asymmetry explored in the current investigation is different suggesting that transfers can be adopted to stabilise global free trade under a wider range of asymmetry across countries. Secondly, the current investigation extends the work by Furusawa and Konishi by exploring how to break inefficient networks. In contrast, these researchers only explored how to stabilise global free trade. This difference has important implications in terms of political strategies that can potentially be adopted to deal with real situations such us facilitating trade between existing blocks of countries, among others. Thirdly, the results obtained in this chapter shows that transfers not only have the potential to favour bilateral agreements as in the case by Furusawa and Konishi, but also global agreements. This offers new alternatives for current global negotiations that have been unsuccessful. Finally, these researchers only consider the use of inter-node transfers. However, as shown in the current investigation, intra-node transfers can become important complementary tools particularly in cases where inter-node payments are less effective.

Based on these results, a summary of the types of transfers that are more effective to facilitate free trade in different simulations and policy biases is presented in the following table.

Table 6.43. Types of transfers that are more effective to facilitate free trade in different simulations and policy biases

Simulations considered in the chapter	Unbiased governments	Biased in favour of consumers	Biased in favour of intermediaries	Biased in favour of the farming sector
2. Symmetric countries and exogenous tariffs. Low monopsonistic power	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Not needed. b) Global agreements: Inter-node
3. Symmetric countries and exogenous tariffs. High monopsonistic power	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Not needed. b) Global agreements: Inter-node
5. Symmetric countries and endogenous tariffs. Low monopsonistic power	a) Bilateral agreements: Inter-node b) Global agreements: Inter-node	Not solved because of endogeneity compexity	Not solved because of endogeneity compexity	Not solved because of endogeneity compexity
6. Symmetric countries and endogenous tariffs. High monopsonistic power	a) Bilateral agreements: Inter-node b) Global agreements: Inter-node	Not solved because of endogeneity compexity	Not solved because of endogeneity compexity	Not solved because of endogeneity compexity
12. Large and very small countries. Endogenous tariffs. Low monopsonistic power	a) Bilateral agreements: Inter-node b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node and intra-node in some networks	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Intra-node b) Global agreements: Inter-node and intra-node but none of them fully effective
13. Large and very small countries. Endogenous tariffs. High monopsonistic power	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node and intra-node in some networks	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Intra-node b) Global agreements: Inter-node and intra-node but none of them fully effective
15. Large and medium size countries. Endogenous tariffs. Low monopsonistic power	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Inter-node and intra-node b) Global agreements: Inter-node
16. Large and medium size countries. Endogenous tariffs. High monopsonistic power	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Inter-node and intra-node b) Global agreements: Inter-node	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Inter-node and intra-node b) Global agreements: Inter-node
17. Asymmetry in farmers' productivity. Endogenous tariffs	a) Bilateral agreements: Inter-node b) Global agreements: Inter-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node	a) Bilateral agreements: Intra-node b) Global agreements: Intra-node	a) Bilateral agreements: Not needed b) Global agreements: Inter-node and in some networks intra-node

CHAPTER SEVEN: Discussion and Conclusions

7.1 Introduction

The aim of this dissertation is to offer an international trade network model that is able to address key aspects related to international trade of food processed goods: global agreements in agriculture have been unsuccessful; trade of agricultural and food processed goods are concentrated in geographical regions; the lack of agricultural trade liberalisation seems to be explained by policy biases; there is imperfect competition in the supply chain of food processed goods that are traded internationally that is associated with the existence of intermediaries with potential market power; and there is evidence of intra-industry trade of food processed goods. The reason for developing the international trade network is because these well-known key ideas are not considered explicitly in quantitative assessments of trade liberalisation making this the research gap that this thesis aims to contributing to fill.

Based on the simulations developed in the previous sections, a number of new insights were found from the international trade network proving that this framework has the potential to inform about trade outcomes that cannot fully be identified from alternative models that deal with the issue of international trade of agricultural and food processed goods. The objective of this chapter is to conclude the current study by highlighting some key results that can be used to provide alternative explanations to current trends observed in the real world. It also outlines limitations of the international trade network approach and potential avenues for future investigation.

The chapter is organised as follows. Section 7.2 explains the mechanism by which oligopoly/oligopsony affects the outcome of trade policies from the international network approach point of view and contrasts with related research. Section 7.3 discusses how this mechanism can explain the observed lack of agricultural trade liberalisation in the real world. In particular, the discussion is focussed on three key aspects explored in the previous chapters: the stability of global free trade; regionalism; and centrality. Section 7.4 extends the previous section by discussing how the inter-node and intra-node transfers might be adopted to facilitate trade in the real world. Finally, Section 7.5 shows some limitations of the international network approach adopted in this thesis and potential avenues for future research.

7.2 Effect of market power on trade policy

As explained in the previous chapters, the mechanism by which market power affects the outcome of trade policies is related to oligopoly and oligopsony power exercised by the intermediaries in each country. In relation to oligopoly, more trade increases the level of competition in domestic markets thereby reducing profits made by intermediaries in these markets and increasing consumer surplus. The former effect will lead to rejection of additional trade agreements unless the loss of domestic profits is compensated by the additional profits made in the new partner countries. This, in turn, depends on the current network structure. For example, if a country initially had a low degree of trade integration (i.e. oligopoly is strong in the domestic market of this country) with respect to a potential partner (i.e. oligopolistic power is less strong), then the intermediary of the former would reject a trade agreement because the gain in export profits would not be large enough to offset the loss of domestic profits. On the other hand, consumers have in general an incentive to support additional agreements because of the positive effect on consumer surplus. As demonstrated by Goyal and Joshi (2006), in this paradigm the outcome of a trade policy depends on whether policymakers are politically biased in favour of either consumers or firms. In the first case, only global free trade is the outcome. However, in the second case, the world can reach different potential stable network structures but where global free trade is only one potential outcome.

In relation to oligopsony, on the other hand, this type of market power increases the marginal cost faced by the intermediaries as the world becomes more integrated. The reason is because more export of food processed goods increases the demand for agricultural goods pushing the price paid to the farming sector up. As a consequence, the presence of the farming sector reinforces the intermediaries' unwillingness to support more trade. The effect of oligopsony on the outcome of a trade policy depends, therefore, on whether policymakers are biased in favour of consumers, the farming sector, or the intermediaries. If governments are biased in favour of consumers, the likely outcome is global free trade because more trade increases consumer surplus as a consequence of more competition, and increases producer surplus as farmers receive higher agricultural prices.

However, when governments are biased in favour of intermediaries, less integrated networks are the possible outcome of trade policies.

There are some deviations to these general possible trade policy outcomes when there is asymmetry across countries. For example, the farming sector in a large country may not support an agreement with a small country because this can cause a net decrease in the output sold by the intermediary of the former pushing the price paid to the farming sector down. In spite of these deviations, the underlying mechanism is the same: the farming sector affects the marginal cost faced by intermediaries and this has important implications in terms of welfare redistribution across sectors.

As explained in the literature review, this mechanism has largely be ignored in quantitative assessment of agricultural trade because in standard models, it is normally assumed that international markets of agricultural and food processed goods operate under perfect competition (and indeed that the links between agriculture and downstream intermediaries are ignored). However, as demonstrated in this thesis, welfare redistribution arising from this mechanism provides new insights in terms of agricultural trade policy suggesting that the issue of agricultural trade liberalisation should be analysed from alternative angles that include imperfect competition.

Efforts to accommodate market power have indeed been made in previous research. For example, Sexton *et al*. (2007) developed a model to explore the issue of oligopoly/oligosony in the context of agricultural trade when developed country food markets are vertically-linked to developing countries. In their model, a primary agricultural product is produced in a developing country and is processed and sold in a developed economy. However, it is important to highlight the fact that the issue addressed by these researchers is different from the one considering in this thesis. Firstly, they analyse export of a primary agricultural good from a developing to a developed country via an intermediary and not directly traded on world markets.. In contrast, the network model developed in the previous chapters considers the case of intra-industry trade of agricultural and food processed goods across countries in an industry with intermediaries with market power. Secondly, Sexton *et al*. (2007) focus the analysis on two countries rather than the set of countries that form part of the network. Finally, they analyse the potential impact of reducing the tariff of a primary good produced in a developing country when the food sector in developed countries are highly concentrated rather than the case where all agri-food sectors in the network may be characterised by imperfect competition.

In contrast, the current investigation analyses the effects of market power on the incentives of policymakers to sign trade agreements and the resulting possible stable networks. In spite of these differences, what it is common in both research strands is the fact that introducing intermediaries with market power into the analysis has important welfare redistribution implications in terms of agricultural trade policy that cannot be identified under the assumption of perfect competition.

Some of these implications in the context of the international trade network framework are discussed as follows.

7.3 Assessing the lack of agricultural trade liberalisation in the real world

The main ideas related to agricultural trade liberalisation discussed in the literature review are that global agreements in agriculture have been unsuccessful, agricultural trade is concentrated in geographic areas, and this apparently reflects the fact that preferential agreements have been signed mainly by countries located in the proximity. The common explanation for this lack of agricultural trade liberalisation is that this reflects the existence of biased policymakers that place policies in order to favour some sectors in the economy (see for example Anderson et al., 2013; Cho, 2010; Regmi et al. 2005; Khor, 2003).

The objective of this section is to explore this argument and to summarise the new insights that may explain the lack of trade liberalisation in the agri-food sector. The new insights are not by any means substitutes of the traditional arguments. On the contrary, they should be considered as complementary factors that may contribute in explaining this lack of trade. According to the international trade network approach, these factors not only include policy biases, but also the stability of global free trade, the central position of some countries in the network, and the possibility of being trapped in an stable network other than global free trade. These possibilities are discussed as follows.

7.3.1 The stability of global free trade

One possibility that can explain lack of trade liberalisation of agricultural and food processed goods is that global free trade cannot be reached because it is not stable (i.e. neither pairwise nor globally treaty stable). This means that no matter what efforts are made to sign a global agreement or to encourage the formation of bilateral and RTAs, if global free trade is not stable, then the world will eventually reach a stable network different from free trade.

According to the simulations developed in the previous chapters, this possibility can only arise when there are governments in the network that are politically biased. This reinforces the view of other researchers who argue that policy bias is a key factor in explaining the lack of agricultural trade liberalisation. However, what is new in the current research is that it was possible to infer from the network approach the type of biases and the circumstances that prevent global free trade. That is, as shown in the summaries in Tables 4.7 for the case of pairwise stability and 5.2, for the case of global treaty stability, in most of the simulations global free trade is not stable when governments are biased in favour of intermediaries. The only exception is the original work by Goyal and Joshi (2006) for the case of symmetrical countries suggesting that this is only a particular case when considering a broader picture of the problem. The main force in place is the effect of the farming sector on the marginal cost faced by the intermediaries. As explained in the previous section, more trade increases the price paid to intermediaries negatively affecting the profits made by these firms. As a result, biased policymakers have an incentive to break existing agreements in order to protect the interests of intermediaries.

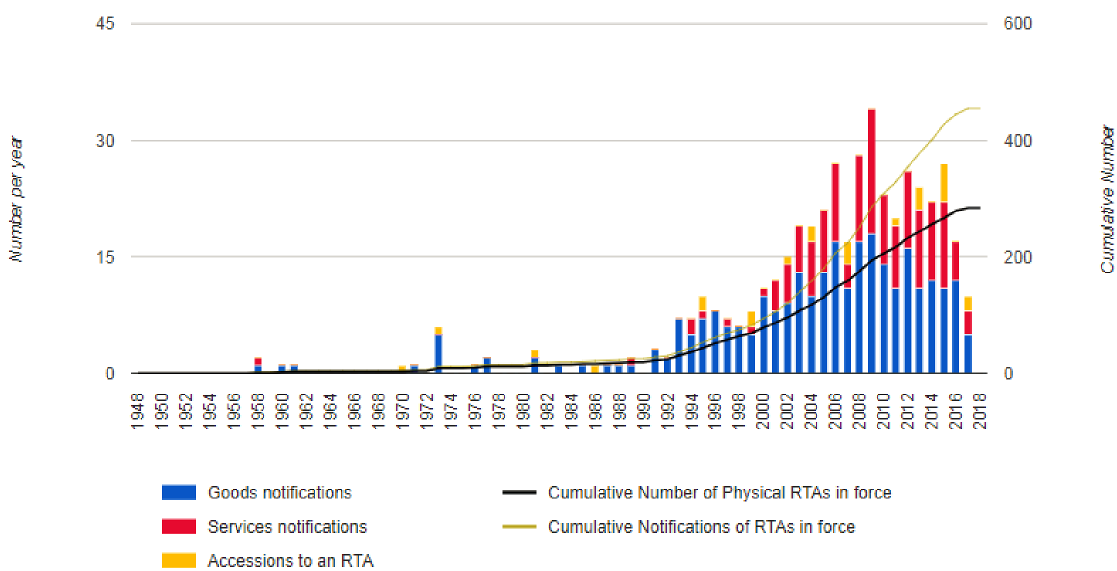
There are other cases where global free trade is not stable and correspond to the ones when governments of large countries are biased in favour of the farming sector. In these cases, large countries have an incentive to break existing agreements with very small countries because the latter have irrelevant domestic markets. By breaking one of these agreements, the increase in domestic output as a consequence of lower competition offsets the decrease in output that was exported to the very small country. This higher net output traded by the intermediary of the large country pushes the price to the farming sector up positively affecting producer surplus.

In summary, if lack of trade reflects the instability of global free trade, then there are either biased governments in favour intermediaries or there are large countries with governments biased in favour of the farming sector.

7.3.2 Regionalism

Another possible explanation for the lack of agricultural trade liberalisation is that the world is either trapped or is reaching a stable network different from global free trade. This is consistent with evidence of the growing trend towards preferential agreements in the global economy. This is explained as follows.

It has been noted a slowdown in the world trade growth that may indicate that global trade has peaked and what is observed today is a new normal with weaker levels of trade (Hoekman, 2015). This is consistent with the current trend of RTAs: as shown in the following figure, there is a slowdown in the number of RTAs in force per year.



Note: Notifications of RTAs: goods, services & accessions to an RTA are counted separately. Physical RTAs: goods, services & accessions to an RTA are counted together. The cumulative lines show the number of notifications/physical RTAs currently in force.

Figure 7.1. RTAs currently in force (by year of entry into force), 1948-2917. Source: WTO Secretariat. Available at http://rtais.wto.org/UI/charts.aspx#

Based on this evidence, suppose that the infrastructure of trade of agricultural and food processed goods is indeed reaching a stable network different from global free trade. What is it the nature of this network? Before answering this question, it is important to clarify that the network model is silence in relation to geographical areas. However, using the facts that the trade flow of agricultural and food processed goods is concentrated in geographical areas, the type of regionalism that characterise this sector consists of blocks of countries having agreements with each other. According to the results obtained in the previous chapters (see Tables 4.7 and 5.2), these agreements correspond to free trade areas as each country in the network model can choose its own trade policy. In these agreements, regionalism can emerge under different circumstances (see network *c'* in Figure 4.5). The ones that are considered in this section are only related to cases that do not include governments biased in favour of consumers because, as inferred from the discussion developed in the literature review (see Section 2.3.3), this type of bias is unlikely to be found in the real world. These cases are described as follows.

Under pairwise and strongly pairwise stability, regionalism emerges when the world is composed of large and very small countries or medium size countries, when there exists either moderate or high levels of monopsonistic power, and when governments are biased in favour of the farming sector (i.e. Simulations 12 and 13). In this case, large countries are only willing to sign agreements with each other. This is because signing an agreement with a small country or with a medium size country increases the level of competition in the domestic market of a large country causing a net decrease in the total output traded by the intermediary of this country (i.e. the decrease in the output that is traded by this firm in the domestic market of the large country as a consequence of the higher competition after an agreement is not compensated by the additional output that is exported to a smaller size country). This decrease in output pushes the price paid to the farming sector of the large country down negatively affecting producer surplus.

Regionalism that consists of blocks of countries having agreements with each other can also emerge under global treaty stability. In particular, when the world is composed of large countries and very small or medium size countries and under different levels of monopsonistic power (i.e. Simulations 11, 12, 13, 14, 15 and 16), this regionalism can emerge when governments are unbiased or biased in favour of the farming sector. In the first case, unbiased governments of large countries are unwilling to sign a global agreement because this implies having trade connections with smaller size countries. These connections increase the level of competition in the domestic market of the large countries positively affecting consumer surplus. But this gain is not large enough to compensate the net loss of profits faced by the intermediaries (i.e. the decrease in domestic profit is not compensate by the additional profits made in the smaller size countries) and the net loss of producer surplus faced by the farming sector (i.e. the decrease in output in the domestic market is not compensated by the additional output that is exported to the smaller size implying a net decrease in output that pushes the agricultural price down). On the other hand, when governments are biased in favour of the farming sector, regionalism emerges because large countries are unwilling to sign a global agreement. As explained above, this happens because the farming sector of large countries face a decrease in produce surplus when forming links with very small countries.

Regionalism under global treaty stability can also emerge when there is asymmetry in terms of farmers' productivity and when governments are either politically unbiased or biased in favour of the farming sector (i.e. Simulations 17). In the first case, inefficient countries in a block are unwilling to sign a global agreement because this causes a significant increase in competition when efficient countries are fully integrated in the network. That is, while this competition increases consumer surplus in all countries, it also causes a net decrease in the profits made by the intermediaries of the inefficient countries and a net decrease in the output that is traded by these firms negatively affecting producer surplus. The gain in consumer surplus is not large enough to offset the losses in profits and producer surplus which is why inefficient countries are unwilling to sign a global agreement. In the second case, inefficient countries in a block having biased governments in favour of the farming sector are not willing to sign a global agreement for the same reason: this agreement reduces producer surplus in these countries.

In summary, the possibility that lack of trade liberalisation in the agri-food sector reflects that the world is reaching a stable network composed of blocks is only possible when there are asymmetries across countries. In the context of bilateralism, this can only happen when governments are biased in favour of the farming sector and when countries are asymmetric in terms of market size. In the context of global agreements, on the other hand, this type of regionalism can only arise when there is asymmetry in either market size or farmers' productivity and when governments are politically unbiased or biased in favour of the farming sector.

7.3.3 Centrality

Another possible explanation for the lack of trade liberalisation in the agri-food sector is centrality, that is, the existence of countries that occupy a central position in the network (i.e. countries having a significant number of links with countries with few links). These countries have an incentive to block trade initiatives because centrality offers them higher levels of welfare compared with global free trade. This is because having a significant number of agreements increases the level of competition in the domestic market of a central country positively affecting consumer surplus. At the same time, having agreements with less integrated countries allows the intermediary of a central country to make high export profits that are large enough to compensate the low profits in the domestic market resulting from competition. Farmers also benefit because the output that is exported to peripheral countries offsets the low output that is traded domestically as a consequence of competition. This higher net output implies that farmers receive higher prices and, therefore, higher producer surplus when the country is in a central position. This is why, in general, a central country is unwilling to support additional trade independently of any policy bias (there are some exceptions when countries are asymmetric).

The main implication of the privileged position of a central country in a network is that this country has an incentive to prevent both the formation of bilateral agreements by non-central countries and the signature of global free trade. This is because these agreements cause negative externalities on consumer surplus, profits and producer surplus in a central country. In relation to consumer surplus, when non-central countries become more integrated after a bilateral or global agreement is signed, the level of trade in these countries increases implying that the marginal cost faced by the intermediaries increases as a consequence of monopsonistic power. In order to mitigate to some extent this higher cost, the intermediaries of the non-central countries reduce the level of export to the central country reducing the level of competition in the latter and, therefore, negatively affecting consumer surplus. In relation to profits, on the other hand, more trade in non-central countries increases the level of competition in these countries negatively affecting the profits made by the intermediary of the central country. Finally, producer surplus is also affected because more competition in non-central countries reduces the amount of output that is traded by the intermediary of the central country negatively affecting the price paid to the farming sector.

Regarding the formation of bilateral agreements by non-central countries, it is unlikely that these agreements are prevented by a central country because this is a decision that depends on third countries. However, this country has the power to block the formation of global agreements because it is directly involved in this collective decision. Moreover, it was found in all the simulations developed in this thesis that there are always networks with central countries unwilling to sign a global agreement (i.e. there are always global treaty proof networks) implying that the incentives to block a global agreement is present in both stable and unstable networks. This means that even if the world is not reaching a stable network, the signature of a global agreement is unlikely.

To finish this section, note that the traditional argument that lack of free trade in the agri-food industry is explained by policy biases does not necessarily hold when there are central countries. As shown in Table 5.2, in all the simulations developed in this thesis, it was found for the unbiased governments case that centrality is a possible stable outcome. The main implication of this result is that the elimination of policy biases is not a sufficient condition to facilitate agricultural trade liberalisation.

7.3.4 Final comments

According to the discussion summarised above, the main factors identified by the international network framework that can explain the current lack of trade in the agri-food sector are policy biases, instability of global free trade (i.e. global free trade is neither pairwise stable nor global treaty stable implying that it cannot be reached), being in a stable network different from global free trade (e.g. regionalism), and the existence of countries occupying a central position in the network. In relation to these factors, it is important to highlight the fact that they are not exclusive of each other. On the contrary, they can coexist and this fact may be easier to identify in more complex network analysis. For example, it could be possible that in some regions of the network, centrality is more relevant, and in others policy biases. It is also possible that in a sequence of unstable networks some factors become less relevant as the world converges to a determined stable network. This means that the traditional argument of policy biases to explain the lack of trade is only one possibility and has to be considered within the current network context.

Finally, based on the results obtained in this thesis it is argued that trade liberalisation agreements can be considered either as temporary or permanent shocks depending on the current international network, the existence of policy biases, and the type of asymmetry across countries. For example, in unstable networks with governments biased in favour of intermediaries, trade agreements can be temporary because there are countries in these networks that are willing to break at least one existing agreement. In contrasts, trade agreements can be permanent in networks that lead to more integrated structures such as global free trade.

7.4 Inter-node and intra-node transfers

The analysis developed in Chapter Six revealed that trade can be promoted by means of inter-node and intra-node transfers (see Table 6.43). This knowledge is considered in this section to explain how these payments may be used as a potential tool to facilitate free trade in the current network configuration of the agri-food sector.

In relation to the possibility that global free trade is neither pairwise stable nor global treaty stable, it is explained in Section 7.3.1 that this always occurs when governments are biased in favour of intermediaries. According to Table 6.43, the suitable tool to facilitate both bilateral and global agreements in this case is intra-node transfers. This is because more integrated networks offer higher levels of welfare. As a result, the gains in consumer surplus and producer surplus in a country after an agreement is signed are both large enough to compensate the losses in profits faced by the intermediaries. In contrast, in this case inter-node payments cannot be adopted because if all countries were biased in favour of these firms, all of them would require transfers across countries to be compensated.

Global free trade is also not a stable outcome when countries are asymmetric in terms of market size and when governments are biased in favour of farmers. In particular, when countries are involved in bilateral agreements, intra-node transfers can be used by large countries in some stages of a path from an inefficient network to global free trade in order to compensate the farming sector for losses in producer surplus. However, when countries are involved in global agreements, the use of intra-node can only work when the world is trapped in some determined stable inefficient networks. If this is not the case, then transfers obtained from welfare gains in smaller countries would be required to compensate the farming sector of large countries.

Let us consider now the case when the world is trapped in an inefficient network as the factor that prevents trade liberalisation. As explained in Section 7.3.2, the network structure that reflects the current trade pattern in the agri-food sector is the one composed of blocks with countries having agreements with each other (i.e.

network *c’* in Figure 4.5). In this case, when the pairwise stability of this network is caused by political biases in favour of the farming sector, it can be broken by means of intra-node transfers adopted by large countries in some stages of a path that leads to global free trade. However, when the global treaty stability of this network arises when governments are unbiased or biased in favour of the farming sector, only inter-node payments that involve transfers from small countries to large ones can be used. This reflects the fact that the small countries are willing to pay for access to the larger market though their ability to do so is limited by the potential gains they can acquire as they are only a small country.

Finally, in considering centrality as the factor that prevents trade liberalisation, it is concluded when comparing Tables 5.2 and 6.43 that at least in stable networks, lump sum transfers can be used to compensate countries for trade losses. In the case of central countries, it is more likely that inter-node are needed to compensate these countries because passing from an inefficient network to more integrated ones normally causes a decrease in consumer surplus, profits and producer surplus.

In summary, it is concluded that it might be possible to facilitate free trade in the agi-food sector in the current network by means of intra-node and inter-node payments. However, the specific type of payment that would be needed would depend on the type of policy bias and whether the aim is to facilitate either bilateral of global agreements. In some cases, breaking this network might require transfers from smaller countries to large countries.

7.5 A note on trade-of-terms

The results obtained in this dissertation can also be analysed from the point of view of the terms-of-trade (i.e. the ratio between a country's export prices to its import prices). However, before explaining this, an important aspect of the model needs to be clarified.

The literature on this topic is based on the idea that the terms-of-trade can be manipulated by means of tariffs. In particular, in a competitive world, large countries have an incentive to place inefficient unilateral policies in order to shift costs onto other countries and to take advantage of the resulting terms-of-trade effect. In contrast, small countries cannot modify the terms-of-trade implying that they have an incentive to place optimal tariffs (Bagwell and Staiger, 2010). In the competitive paradigm, a bilateral agreement arises when the mutual tariff reduction that is discriminatory against third countries will improve their terms of trade against these countries (Bagwell and Staiger, 2005). On the other hand, when markets are not competitive, other externalities arise from tariffs policies such as the firm-delocation externality (i.e. an increase in tariffs changes the balance of competition favouring domestic firms) and the profit-shifting externality (i.e. and increase in tariffs extracts profits from third countries) (Maggi, 2014). According to Bagwell and Staiger (2012), even when these externalities are present, the rationale for a trade agreement is to remedy the inefficient terms-of-trade caused by restrictions in trade volume.

Unfortunately it is not possible to study governments' incentives to manipulate the terms-of-trade by means of tariffs from the international trade network model developed in this thesis because, as in Goyal and Joshi (2006), most of the analysis was developed under the assumption of exogenous tariffs. This is explicitly recognised by these authors who explain that the assumption that countries commit to zero tariffs in free trade agreements made to rule out terms-of-trade deviations (see Goyal and Joshi, 2006, Page 754). In spite of this limitation, it is still possible to identify interesting aspects of the terms-of-trade from the model under the assumption of exogenous tariffs. That is, some countries can benefit from the terms-of-trade depending on the current network structure and the relative position that these countries occupy in the network. This is explained as follows.

It is inferred from the results obtained in this dissertation that countries have an incentive to stay in a central position of the network and to block further trade liberalisation in order to take advantage of the terms-of-trade. The reason relies on the fact that central countries can access foreign markets that are less integrated and, therefore, obtain high export prices. At the same time, because the domestic markets of central countries are more competitive, the price that consumers pay for imported good is lower. However, this advantage is no so evident when there are non-central small countries because the lower domestic price in a central country may not be coupled with higher export prices in the former even if they are less competitive. That is, even if a non-central small country has a low degree of competition in the domestic market, the price in this market can be lower than the price in the central country when the domestic market of the former is very small. This suggests, therefore, that a terms-of-trade advantage is present in countries that occupy a central position in the network and that are connected mainly to large and medium size countries. This terms-of-trade advantage is another possible reason that explains the current lack of agricultural trade liberalisation.

7.6 Limitations and potential avenues for future research

There are a number of limitations of the proposed international network model that is important to highlight some of which will form the basis of a future research agenda.

First, this thesis focuses the analysis of trade agreements on tariffs. However, current negotiations are much more complex and include a number of international rules governing domestic policies used to mitigate adverse trade effects (this complex negotiation approach is referred to as *deep integration*. For a discussion, see Young, 2017). These rules include product standards such as sanitary and phytosanitary standards that are relevant for agriculture and the food industry. An analysis of deep integration from an international trade network point of view is left for future research.

Second, given the complexity and the problem of mathematical tractability, it was not possible to obtain results for large networks. As a consequence, it was not possible to assess, for example, whether centrality within blocks of countries rather than the full network can eventually explain the lack of trade liberalisation. This would provide alternative explanations for the unwillingness to sign agreements by countries located in different blocks across the world. The issue of tractability in large networks is also a particular challenge especially in the case of endogenous tariffs.

Third, this thesis considered concentration as the source of imperfect competition. However, as noted by Sexton (2013), imperfect competition can also arise when goods are differentiated (e.g. product heterogeneity, different product quality and brands). Unfortunately it was not possible to consider this source of imperfection given the complexity of the model.

Fourth, it was assumed the existence of a single stage intermediary. However, there are cases of successive oligopoly/oligopsony that characterise the supply chain of the food industry that may influence trade outcomes as policy biases (see Sexton *et al*. 2007).

Fifth, given the likelihood of networks that are inconsistent with global free trade, it would be interesting to consider the potential impacts of shocks through alternative forms of networks and, indeed, whether concerns about price volatility also relate to the formation of specific trade networks.

Sixth, the analysis on bilateral agreements was based on both pairwise and strongly pairwise stability. A possible extension would be the adoption of Pairwise Farsighted Stability proposed by Zhang et al. (2013). The aim of this stability is to compare myopia with farsightedness in an otherwise fixed framework. That is, if there is a farsightedly improving path from the current network to another one, then each country's decision is motivated by the final attainment of the latter.

Seventh, there is a growing attention of global value chains (GVCs) in the economics literature. This concept is defined as the value added of all activities that are directly and indirectly needed to produce a final product, and is identified in the country-industry where the last stage of production takes place (Timmer et al., 2014). Because different stages of the production process can be located across different countries, the expansion of GVCs has strongly increased economic interdependence. This effect has recently attracted the attention of researchers working in the area of agricultural trade because of the potential impact on the food industry (see for example Salvatici and Nenci, 2017). The network model developed in this thesis does not account for interdependence caused by the GVCs because this model assumes that all the stages in the production of the food processed good takes place in the same country. An interesting extension of the model would be, therefore, to allow for interdependency by assuming that intermediaries purchase the agricultural good from farming sectors located in other countries, and to explore how the stable networks identified in the current investigation are affected by this extension.

Eighth, this thesis shows that inter-node and intra-node transfers can be used to reach global free trade through determined network paths. A possible interesting extension in relation to this finding would be to compare different paths in terms of efficiency and costs.

Ninth, a comparison of the costs implied by inter-node vs. intra-node transfers required to achieve the same network could be undertaken in future research.

Finally, the current thesis revealed theoretical results and novel insights to the issue of agriculture and food processed goods that, apparently, have not been reported so far. Potential extensions would be, therefore, the development of empirical works to determine whether these results hold in the real world. All these potential extensions are left for future research.

Agricultural Trade Liberalization: an International Trade Network Approach

Volume 2 of 2

Submitted by Daniel Esteban May Montana to the University of Exeter
as a thesis for the degree of
Doctor of Philosophy in Economics
In January 2018

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: …………………………………………………………..

APPENDIX A

Simulations for the case of symmetrical countries under exogenous tariffs

Network a

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_a)$ = $q_j^{(j)}(g_a)$ = $q_k^{(k)}(g_a)$ = $q_l^{(l)}(g_a)$ = $\frac{\alpha}{\phi+2}$; $CS_i(g_a) = CS_j(g_a) = CS_k(g_a) = CS_l(g_a)$ = $\frac{1}{2}\left(q_i^{(i)}(g_a)\right)^2$; $\pi_i(g_a)$ = $\pi_j(g_a)$ = $\pi_k(g_a)$ = $\pi_l(g_a)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_a)\right)^2$; and $PS_i(g_a) = PS_j(g_a) = PS_k(g_a) = PS_l(g_a)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_a)\right)^2$.

Simulation 1: α = 1 and ϕ = 0

$q_i^{(i)}(g_a)$ = $q_j^{(j)}(g_a)$ = $q_k^{(k)}(g_a)$ = $q_l^{(l)}(g_a)$ = $\frac{1}{2} = 0.5000$; $CS_i(g_a) = CS_j(g_a) = CS_k(g_a) = CS_l(g_a)$ = $\frac{1}{2}(0.5000)^2 = 0.1250$; $\pi_i(g_a)$ = $\pi_j(g_a)$ = $\pi_k(g_a)$ = $\pi_l(g_a)$ = $(0.5000)^2 = 0.2500$; and $PS_i(g_a) = PS_j(g_a) = PS_k(g_a) = PS_l(g_a) = 0$.

Simulation 2: $\alpha = 1$ and $\phi = 0.5$

$q_i^{(i)}(g_a) = q_j^{(j)}(g_a) = q_k^{(k)}(g_a) = q_l^{(l)}(g_a) = \frac{1}{2.5} = 0.4000$; $CS_i(g_a) = CS_j(g_a) = CS_k(g_a) = CS_l(g_a) = \frac{1}{2}(0.4000)^2 = 0.0800$; $\pi_i(g_a) = \pi_j(g_a) = \pi_k(g_a) = \pi_l(g_a) = \frac{2.5}{2}(0.4000)^2 =$ 0.2000; and $PS_i(g_a) = PS_j(g_a) = PS_k(g_a) = PS_l(g_a) = \frac{0.5}{4}(0.4000)^2 = 0.0200$.

Simulation 3: $\alpha = 1$ and $\phi = 1.5$

$q_i^{(i)}(g_a) = q_j^{(j)}(g_a) = q_k^{(k)}(g_a) = q_l^{(l)}(g_a) = \frac{1}{3.5} = 0.2857$; $CS_i(g_a) = CS_j(g_a) = CS_k(g_a) = CS_l(g_a) = \frac{1}{2}(0.2857)^2 = 0.0408$; $\pi_i(g_a) = \pi_j(g_a) = \pi_k(g_a) = \pi_l(g_a) = \frac{3.5}{2}(0.2857)^2 =$ 0.1428; and $PS_i(g_a) = PS_j(g_a) = PS_k(g_a) = PS_l(g_a) = \frac{1.5}{4}(0.2857)^2 = 0.0306$.

Network b

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_b) = q_k^{(i)}(g_b) = q_k^{(k)}(g_b) = q_i^{(k)}(g_b) = \frac{2\alpha}{3\phi+6}$; $q_j^{(j)}(g_b) = q_l^{(l)}(g_b) = \frac{\alpha}{\phi+2}$; $CS_i(g_b) = CS_k(g_b) = \frac{1}{2}(q_i^{(i)}(g_b) + q_i^{(k)}(g_b))^2$; $CS_j(g_b) = CS_l(g_b) = \frac{1}{2}\left(q_j^{(j)}(g_b)\right)^2$; $\pi_i^{(i)}(g_b) = \pi_k^{(i)}(g_b) = \pi_k^{(k)}(g_b) = \pi_i^{(k)}(g_b) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_g)\right)^2$; $\pi_j^{(j)}(g_b) = \pi_l^{(l)}(g_b) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_b)\right)^2$; $PS_i(g_b) = PS_k(g_b) = \frac{\phi}{4}(q_i^{(i)}(g_b) + q_k^{(i)}(g_b))^2$; and $PS_j(g_b) = PS_l(g_b) = \frac{\phi}{4}\left(q_j^{(j)}(g_b)\right)^2$.

Simulation 1: α = 1 and ϕ = 0

$q_i^{(i)}(g_b)$ = $q_k^{(i)}(g_b)$ = $q_k^{(k)}(g_b)$ = $q_i^{(k)}(g_b)$ = $\frac{2}{6}$ = 0.3333; $q_j^{(j)}(g_b)$ = $q_l^{(l)}(g_b)$ = $\frac{1}{2}$ = 0.5000; $CS_i(g_b)$ = $CS_k(g_b)$ = $\frac{1}{2}(0.3333 + 0.3333)^2$ = 0.2222; $CS_j(g_b)$ = $CS_l(g_b)$ = $\frac{1}{2}(0.5000)^2 = 0.1250$; $\pi_i^{(i)}(g_b)$ = $\pi_k^{(i)}(g_b)$ = $\pi_k^{(k)}(g_b)$ = $\pi_i^{(k)}(g_b)$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(j)}(g_b)$ = $\pi_l^{(l)}(g_b)$ = $(0.5000)^2$ = 0.2500; and $PS_i(g_b)$ = $PS_j(g_b)$ = $PS_k(g_b)$ = $PS_l(g_b)$ = 0.

Simulation 2: α = 1 and ϕ = 0.5

$q_i^{(i)}(g_b)$ = $q_k^{(i)}(g_b)$ = $q_k^{(k)}(g_b)$ = $q_i^{(k)}(g_b)$ = $\frac{2}{1.5+6} = 0.2667$; $q_j^{(j)}(g_b)$ = $q_l^{(l)}(g_b)$ = $\frac{1}{0.5+2}$ = 0.4000; $CS_i(g_b)$ = $CS_k(g_b)$ = $\frac{1}{2}(0.2667 = 0.2667)^2$ = 0.1422; $CS_j(g_b)$ = $CS_l(g_b)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $\pi_i^{(i)}(g_b)$ = $\pi_k^{(i)}(g_b)$ = $\pi_k^{(k)}(g_b)$ = $\pi_i^{(k)}(g_b)$ = $\frac{2.5}{2}(0.2667)^2$ = 0.0889; $\pi_j^{(j)}(g_b)$ = $\pi_l^{(l)}(g_b)$ = $\frac{2.5}{2}(0.4000)^2$ = 0.2000; $PS_i(g_b)$ = $PS_k(g_b)$ = $\frac{0.5}{4}(0.2667 + 0.2667)^2$ = 0.0356; and $PS_j(g_b)$ = $PS_l(g_b)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200.

Simulation 3: $\alpha = 1$ and $\phi = 1.5$

$q_i^{(i)}(g_b) = q_k^{(i)}(g_b) = q_k^{(k)}(g_b) = q_i^{(k)}(g_b) = \frac{2}{4.5+6} = 0.1905$; $q_j^{(j)}(g_b) = q_l^{(l)}(g_b) = \frac{1}{1.5+2} = 0.2857$; $CS_i(g_b) = CS_k(g_b) = \frac{1}{2}(0.1905+0.1905)^2 = 0.0726$; $CS_j(g_b) = CS_l(g_b) = \frac{1}{2}(0.2857)^2 = 0.0408$; $\pi_i^{(i)}(g_b) = \pi_k^{(i)}(g_b) = \pi_k^{(k)}(g_b) = \pi_i^{(k)}(g_b) = \frac{3.5}{2}(0.1905)^2 = 0.0635$; $\pi_j^{(j)}(g_b) = \pi_l^{(l)}(g_b) = \frac{3.5}{2}(0.2857)^2 = 0.1428$; $PS_i(g_b) = PS_k(g_b) = \frac{1.5}{4}(0.1905+0.1905)^2 = 0.0544$; and $PS_j(g_b) = PS_l(g_b) = \frac{1.5}{4}(0.2857)^2 = 0.0306$.

Network c

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_k^{(k)}(g_c) = q_i^{(k)}(g_c) = q_j^{(j)}(g_c) = q_l^{(j)}(g_c) = q_l^{(l)}(g_c) = q_j^{(l)}(g_c) = \frac{2\alpha}{3\phi+6}$; $CS_i(g_c) = CS_j(g_c) = CS_k(g_c) = CS_l(g_c) = \frac{1}{2}\left(q_i^{(i)}(g_c)+q_i^{(k)}(g_c)\right)^2$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_k^{(k)}(g_c) = \pi_i^{(k)}(g_c) = \pi_j^{(j)}(g_c) = \pi_l^{(j)}(g_c) = \pi_l^{(l)}(g_c) = \pi_j^{(l)}(g_c) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_c)\right)^2$; and $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = \frac{\phi}{4}\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)\right)^2$.

Simulation 1: α = 1 and ϕ = 0

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_k^{(k)}(g_c) = q_i^{(k)}(g_c) = q_j^{(j)}(g_c) = q_l^{(j)}(g_c) = q_l^{(l)}(g_c) = q_j^{(l)}(g_c) = \frac{2}{6} =$ 0.3333; $CS_i(g_c) = CS_j(g_c) = CS_k(g_c) = CS_l(g_c) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_k^{(k)}(g_c) = \pi_i^{(k)}(g_c) = \pi_j^{(j)}(g_c) = \pi_l^{(j)}(g_c) = \pi_l^{(l)}(g_c) = \pi_j^{(l)}(g_c) = (0.3333)^2 =$ 0.1111; and $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = 0$.

Simulation 2: α = 1 and ϕ = 0.5

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_k^{(k)}(g_c) = q_i^{(k)}(g_c) = q_j^{(j)}(g_c) = q_l^{(j)}(g_c) = q_l^{(l)}(g_c) = q_j^{(l)}(g_c) = \frac{2}{1.5+6} = 0.2667$; $CS_i(g_c) = CS_j(g_c) = CS_k(g_c) = CS_l(g_c) = \frac{1}{2}(0.2667 + 0.2667)^2 = 0.1422$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_k^{(k)}(g_c) = \pi_i^{(k)}(g_c) = \pi_j^{(j)}(g_c) = \pi_l^{(j)}(g_c) = \pi_l^{(l)}(g_c) = \pi_j^{(l)}(g_c) = \frac{2.5}{2}(0.2667)^2 = 0.0889$; and $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = \frac{0.5}{4}(0.2667 + 0.2667)^2 = 0.0356$.

Simulation 3: α = 1 and ϕ = 1.5

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_k^{(k)}(g_c) = q_i^{(k)}(g_c) = q_j^{(j)}(g_c) = q_l^{(j)}(g_c) = q_l^{(l)}(g_c) = q_j^{(l)}(g_c) = \frac{2}{4.5+6} = 0.1905$; $CS_i(g_c) = CS_j(g_c) = CS_k(g_c) = CS_l(g_c) = \frac{1}{2}(0.1905 + 0.1905)^2 = 0.0726$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_k^{(k)}(g_c) = \pi_i^{(k)}(g_c) = \pi_j^{(j)}(g_c) = \pi_l^{(j)}(g_c) = \pi_l^{(l)}(g_c) = \pi_j^{(l)}(g_c) =$

$\frac{3.5}{2}(0.1905)^2$ = 0.0635; and $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = \frac{1.5}{4}(0.1905+0.1905)^2$ = 0.0544.

Network d

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_d) = q_j^{(j)}(g_d) = q_j^{(i)}(g_d) = q_i^{(j)}(g_d) = \frac{2\alpha(\phi+1)+\phi q_k^{(k)}(g_d)-\phi(\phi+2)q_k^{(i)}(g_d)}{3\phi^2+12\phi+8}$; $q_k^{(i)}(g_d) = q_l^{(j)}(g_d) = \frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_d)-2\phi(\phi+2)q_i^{(i)}(g_d)}{2(\phi+3)(\phi+1)}$; $q_i^{(k)}(g_d) = q_j^{(l)}(g_d) = \frac{2\alpha(\phi+1)+2\phi q_k^{(i)}(g_d)+2\phi q_i^{(i)}(g_d)-\phi(\phi+3)q_k^{(k)}(g_d)}{2(\phi+4)(\phi+1)}$; $q_k^{(k)}(g_d) = q_l^{(l)}(g_d) = \frac{2\alpha(\phi+1)+2\phi q_i^{(i)}(g_d)-\phi(\phi+2)q_i^{(k)}(g_d)}{2(\phi+3)(\phi+1)}$; $CS_i(g_d) = CS_j(g_d) = \frac{1}{2}(q_i^{(i)}(g_d) + q_i^{(j)}(g_d) + q_i^{(k)}(g_d))^2$; $CS_k(g_d) = CS_l(g_d) = \frac{1}{2}\left(q_k^{(i)}(g_d)+q_k^{(k)}(g_d)\right)^2$; $\pi_i^{(i)}(g_d) = \pi_j^{(j)}(g_d) = \pi_j^{(i)}(g_d) = \pi_i^{(j)}(g_d) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_d)\right)^2$; $\pi_k^{(i)}(g_d) = \pi_l^{(j)}(g_d) = \frac{(2+\phi)}{2}\left(q_k^{(i)}(g_d)\right)^2$; $\pi_i^{(k)}(g_d) = \pi_j^{(l)}(g_d) = \frac{(2+\phi)}{2}\left(q_i^{(k)}(g_d)\right)^2$; $\pi_k^{(k)}(g_d) = \pi_l^{(l)}(g_d) = \frac{(2+\phi)}{2}\left(q_k^{(k)}(g_d)\right)^2$; $PS_i(g_d) = PS_j(g_d) = \frac{\phi}{4}\left(q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_k^{(i)}(g_d)\right)^2$; and $PS_k(g_d) = PS_l(g_d) = \frac{\phi}{4}\left(q_i^{(k)}(g_d)+q_k^{(k)}(g_d)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 0 & -\frac{\phi}{3\phi^2+12\phi+8} \\ \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_d) \\ q_k^{(i)}(g_d) \\ q_i^{(k)}(g_d) \\ q_k^{(k)}(g_d) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+4} \\ \frac{\alpha}{\phi+3} \end{pmatrix}$$

<u>*Simulation 1: α = 1 and ϕ = 0*</u>

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_d) \\ q_k^{(i)}(g_d) \\ q_i^{(k)}(g_d) \\ q_k^{(k)}(g_d) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.3333 \\ 0.2500 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_d) = q_j^{(j)}(g_d) = q_j^{(i)}(g_d) = q_i^{(j)}(g_d) = q_i^{(k)}(g_d) = q_j^{(l)}(g_d) = 0.2500$; $q_k^{(i)}(g_d) = q_l^{(j)}(g_d) = q_k^{(k)}(g_d) = q_l^{(l)}(g_d) = 0.3333$; $CS_i(g_d) = CS_j(g_d) = \frac{1}{2}(0.2500 + 0.25000 + 0.2500)^2 = 0.2813$; $CS_k(g_d) = CS_l(g_d) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $\pi_i^{(i)}(g_d) = \pi_j^{(j)}(g_d) = \pi_j^{(i)}(g_d) = \pi_i^{(j)}(g_d) = \pi_i^{(k)}(g_d) = \pi_j^{(l)}(g_d) = (0.2500)^2 = 0.0625$; $\pi_k^{(i)}(g_d) = \pi_l^{(j)}(g_d) = \pi_k^{(k)}(g_d) = \pi_l^{(l)}(g_d) = (0.3333)^2 = 0.1111$; and $PS_i(g_d) = PS_j(g_d) = PS_k(g_a) = PS_l(g_a) = 0$.

Simulation 2: α = 1 and ϕ = 0.5

The output matrix in this case corresponds to:

$$\begin{pmatrix} 1 & 0.0847 & 0 & -0.0339 \\ 0.2381 & 1 & -0.0476 & 0 \\ -0.0741 & -0.0741 & 1 & 0.1296 \\ -0.0952 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_d) \\ q_k^{(i)}(g_d) \\ q_i^{(k)}(g_d) \\ q_k^{(k)}(g_d) \end{pmatrix} = \begin{pmatrix} 0.2034 \\ 0.2857 \\ 0.2222 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_d) = q_j^{(j)}(g_d) = q_j^{(i)}(g_d) = q_i^{(j)}(g_d) = 0.1916$; $q_k^{(i)}(g_d) = q_l^{(j)}(g_d) = 0.2505$; $q_i^{(k)}(g_d) = q_j^{(l)}(g_d) = 0.2189$; $q_k^{(k)}(g_d) = q_l^{(l)}(g_d) = 0.2779$; $CS_i(g_d) = CS_j(g_d) = \frac{1}{2}(0.1916 + 0.1916 + 0.2189)^2 = 0.1813$; $CS_k(g_d) = CS_l(g_d) = \frac{1}{2}(0.2505 + 0.2779)^2 = 0.1396$; $\pi_i^{(i)}(g_d) = \pi_j^{(j)}(g_d) = \pi_j^{(i)}(g_d) = \pi_i^{(j)}(g_d) = \frac{2.5}{2}(0.1916)^2 = 0.0459$; $\pi_k^{(i)}(g_d) = \pi_l^{(j)}(g_d) = \frac{2.5}{2}(0.2505)^2 = 0.0784$; $\pi_i^{(k)}(g_d) = \pi_j^{(l)}(g_d) = \frac{2.5}{2}(0.2189)^2 = 0.0599$; $\pi_k^{(k)}(g_d) = \pi_l^{(l)}(g_d) = \frac{2.5}{2}(0.2779)^2 = 0.0965$; $PS_i(g_d) = PS_j(g_d) = \frac{0.5}{4}(0.1916 + 0.1916 + 0.2505)^2 = 0.0502$; and $PS_k(g_a) = PS_l(g_a) = \frac{0.5}{4}(0.2189 + 0.2779)^2 = 0.0309$.

Simulation 3: α = 1 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1603 & 0 & -0.0458 \\ 0.4667 & 1 & -0.0667 & 0 \\ -0.1091 & -0.1091 & 1 & 0.2455 \\ -0.1333 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_d) \\ q_k^{(i)}(g_d) \\ q_i^{(k)}(g_d) \\ q_k^{(k)}(g_d) \end{pmatrix} = \begin{pmatrix} 0.1527 \\ 0.2222 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_d)$ = $q_j^{(j)}(g_d)$ = $q_j^{(i)}(g_d)$ = $q_i^{(j)}(g_d)$ = 0.1346; $q_k^{(i)}(g_d)$ = $q_l^{(j)}(g_d)$ = 0.1704; $q_i^{(k)}(g_d)$ = $q_j^{(l)}(g_d)$ = 0.1656; $q_k^{(k)}(g_d)$ = $q_l^{(l)}(g_d)$ = 0.2015; $CS_i(g_d)$ = $CS_j(g_d)$ = $\frac{1}{2}$(0.1346 + 0.1346 + 0.1656)^2 = 0.0945; $CS_k(g_d)$ = $CS_l(g_d)$ = $\frac{1}{2}(0.1704+0.2015)^2$ = 0.0692; $\pi_i^{(i)}(g_d)$ = $\pi_j^{(j)}(g_d)$ = $\pi_j^{(i)}(g_d)$ = $\pi_i^{(j)}(g_d)$ $=\frac{3.5}{2}(0.1346)^2$ = 0.0317; $\pi_k^{(i)}(g_d)$ = $\pi_l^{(j)}(g_d)$ $=\frac{3.5}{2}(0.1704)^2$ = 0.0508; $\pi_i^{(k)}(g_d)$ = $\pi_j^{(l)}(g_d)$ $=\frac{3.5}{2}(0.1656)^2$ = 0.0480; $\pi_k^{(k)}(g_d)$ = $\pi_l^{(l)}(g_d)$ $=\frac{3.5}{2}(0.2015)^2$ = 0.0711; $PS_i(g_d)$ = $PS_j(g_d)$ = $\frac{1.5}{4}(0.1346+0.1346+0.1704)^2$ = 0.0725; and $PS_k(g_a)$ = $PS_l(g_a)$ = $\frac{1.5}{4}(0.1656+0.2015)^2$ = 0.0505.

Network e

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_e)$ = $q_j^{(i)}(g_e)$ = $q_k^{(i)}(g_e)$ = $q_i^{(j)}(g_e)$ = $q_j^{(j)}(g_e)$ = $q_l^{(j)}(g_e)$ = $q_i^{(k)}(g_e)$ = $q_k^{(k)}(g_e)$ = $q_l^{(k)}(g_e)$ = $q_j^{(l)}(g_e)$ = $q_k^{(l)}(g_e)$ = $q_l^{(l)}(g_e)$ = $\frac{\alpha}{2\phi+4}$; $CS_i(g_e)$ = $CS_j(g_e)$ = $CS_k(g_e)$ = $CS_l(g_e)$ = $\frac{1}{2}\left(q_i^{(i)}(g_e)+q_i^{(j)}(g_e)+q_i^{(k)}(g_e)\right)^2$; $\pi_i^{(i)}(g_e)$ = $\pi_j^{(i)}(g_e)$ = $\pi_k^{(i)}(g_e)$ = $\pi_i^{(j)}(g_e)$ = $\pi_j^{(j)}(g_e)$ = $\pi_l^{(j)}(g_e)$ = $\pi_i^{(k)}(g_e)$ = $\pi_k^{(k)}(g_e)$ = $\pi_l^{(k)}(g_e)$ = $\pi_j^{(l)}(g_e)$ = $\pi_k^{(l)}(g_e)$ = $\pi_l^{(l)}(g_e)$ =

$\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_e)\right)^2$; and $PS_i(g_e)$ = $PS_j(g_e)$ = $PS_k(g_e)$ = $PS_l(g_e)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)\right)^2$.

<u>*Simulation 1: α = 1 and ϕ = 0*</u>

$q_i^{(i)}(g_e)$ = $q_j^{(i)}(g_e)$ = $q_k^{(i)}(g_e)$ = $q_i^{(j)}(g_e)$ = $q_j^{(j)}(g_e)$ = $q_l^{(j)}(g_e)$ = $q_i^{(k)}(g_e)$ = $q_k^{(k)}(g_e)$ = $q_l^{(k)}(g_e)$ = $q_j^{(l)}(g_e)$ = $q_k^{(l)}(g_e)$ = $q_l^{(l)}(g_e)$ = $\frac{1}{4}$ = 0.2500; $CS_i(g_e)$ = $CS_j(g_e)$ = $CS_k(g_e)$ = $CS_l(g_e)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $\pi_i^{(i)}(g_e)$ = $\pi_j^{(i)}(g_e)$ = $\pi_k^{(i)}(g_e)$ = $\pi_i^{(j)}(g_e)$ = $\pi_j^{(j)}(g_e)$ = $\pi_l^{(j)}(g_e)$ = $\pi_i^{(k)}(g_e)$ = $\pi_k^{(k)}(g_e)$ = $\pi_l^{(k)}(g_e)$ = $\pi_j^{(l)}(g_e)$ = $\pi_k^{(l)}(g_e)$ = $\pi_l^{(l)}(g_e)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_e)$ = $PS_j(g_e)$ = $PS_k(g_e)$ = $PS_l(g_e)$ = 0.

<u>*Simulation 2: α = 1 and ϕ = 0.5*</u>

$q_i^{(i)}(g_e)$ = $q_j^{(i)}(g_e)$ = $q_k^{(i)}(g_e)$ = $q_i^{(j)}(g_e)$ = $q_j^{(j)}(g_e)$ = $q_l^{(j)}(g_e)$ = $q_i^{(k)}(g_e)$ = $q_k^{(k)}(g_e)$ = $q_l^{(k)}(g_e)$ = $q_j^{(l)}(g_e)$ = $q_k^{(l)}(g_e)$ = $q_l^{(l)}(g_e)$ = $\frac{1}{1+4}$ = 0.2000; $CS_i(g_e)$ = $CS_j(g_e)$ = $CS_k(g_e)$ = $CS_l(g_e)$ = $\frac{1}{2}(0.2000+0.2000+0.2000)^2$ = 0.1800; $\pi_i^{(i)}(g_e)$ = $\pi_j^{(i)}(g_e)$ = $\pi_k^{(i)}(g_e)$ = $\pi_i^{(j)}(g_e)$ = $\pi_j^{(j)}(g_e)$ = $\pi_l^{(j)}(g_e)$ = $\pi_i^{(k)}(g_e)$ = $\pi_k^{(k)}(g_e)$ = $\pi_l^{(k)}(g_e)$ = $\pi_j^{(l)}(g_e)$ = $\pi_k^{(l)}(g_e)$ = $\pi_l^{(l)}(g_e)$ = $\frac{2.5}{2}(0.2000)^2$ = 0.0500; and $PS_i(g_e)$ = $PS_j(g_e)$ = $PS_k(g_e)$ = $PS_l(g_e)$ = $\frac{0.5}{4}$(0.2000 + 0.2000 + 0.2000)^2 = 0.0450.

Simulation 3: α = 1 and ϕ = 1.5

$q_i^{(i)}(g_e) = q_j^{(i)}(g_e) = q_k^{(i)}(g_e) = q_i^{(j)}(g_e) = q_j^{(j)}(g_e) = q_l^{(j)}(g_e) = q_i^{(k)}(g_e) = q_k^{(k)}(g_e) = q_l^{(k)}(g_e) = q_j^{(l)}(g_e) = q_k^{(l)}(g_e) = q_l^{(l)}(g_e) = \frac{1}{3+4}$ = 0.1429; $CS_i(g_e) = CS_j(g_e) = CS_k(g_e) = CS_l(g_e) = \frac{1}{2}(0.1429+0.1429+0.1429)^2$ = 0.0918; $\pi_i^{(i)}(g_e) = \pi_j^{(i)}(g_e) = \pi_k^{(i)}(g_e) = \pi_i^{(j)}(g_e) = \pi_j^{(j)}(g_e) = \pi_l^{(j)}(g_e) = \pi_i^{(k)}(g_e) = \pi_k^{(k)}(g_e) = \pi_l^{(k)}(g_e) = \pi_j^{(l)}(g_e) = \pi_k^{(l)}(g_e) = \pi_l^{(l)}(g_e) = \frac{3.5}{2}(0.1429)^2$ = 0.0357; and $PS_i(g_e) = PS_j(g_e) = PS_k(g_e) = PS_l(g_e) = \frac{1.5}{4}(0.1429 + 0.1429 + 0.1429)^2$ = 0.0689.

Network f

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_f) = \frac{\alpha(\phi+1)+\phi q_j^{(j)}(g_f)-\phi(\phi+3)q_j^{(i)}(g_f)}{(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_f) = q_k^{(i)}(g_f) = \frac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_f)-\phi(\phi+2)q_i^{(i)}(g_f)}{3\phi^2+10\phi+6}$; $q_j^{(j)}(g_f) = q_k^{(k)}(g_f) = \frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_f)+\phi q_j^{(i)}(g_f)-\phi(\phi+2)q_i^{(j)}(g_f)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_f) = q_i^{(k)}(g_f) = \frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_f)-\phi(\phi+2)q_j^{(j)}(g_f)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_f) = \frac{\alpha}{\phi+2}$; $CS_i(g_f) = \frac{1}{2}\left(q_i^{(i)}(g_f)+q_i^{(j)}(g_f)+q_i^{(k)}(g_f)\right)^2$; $CS_j(g_f) = CS_k(g_f) = \frac{1}{2}(q_j^{(i)}(g_f) + q_j^{(j)}(g_f))^2$; $CS_l(g_f) = \frac{1}{2}\left(q_l^{(l)}(g_f)\right)^2$; $\pi_i^{(i)}(g_f) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_f)\right)^2$; $\pi_j^{(i)}(g_f) = \pi_k^{(i)}(g_f) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g_f)\right)^2$;

$\pi_j^{(j)}(g_f) = \pi_k^{(k)}(g_f) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_f)\right)^2$; $\pi_i^{(j)}(g_f) = \pi_i^{(k)}(g_f) = \frac{(2+\phi)}{2}\left(q_i^{(j)}(g_f)\right)^2$;

$\pi_l^{(l)}(g_f) = \frac{(2+\phi)}{2}\left(q_l^{(l)}(g_f)\right)^2$; $PS_i(g_f) = \frac{\phi}{4}(q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_k^{(i)}(g_f))^2$; $PS_j(g_f) = PS_k(g_f)$

$= \frac{\phi}{4}\left(q_i^{(j)}(g_f) + q_j^{(j)}(g_f)\right)^2$; and $PS_l(g_f) = \frac{\phi}{4}\left(q_l^{(l)}(g_f)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & 0 \\ \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & 1 & 0 & -\frac{\phi}{3\phi^2+10\phi+6} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} \\ 0 & -\frac{\phi}{(\phi+4)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+4)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_j^{(j)}(g_f) \\ q_i^{(j)}(g_f) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{2\alpha(\phi+1)}{3\phi^2+10\phi+6} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+4} \end{pmatrix}$$

Simulation 1: $\alpha = 1$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_j^{(j)}(g_f) \\ q_i^{(j)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.3333 \\ 0.3333 \\ 0.2500 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f) = q_i^{(j)}(g_f) = q_i^{(k)}(g_f) = 0.2500$; $q_j^{(i)}(g_f) = q_k^{(i)}(g_f) = q_j^{(j)}(g_f) = q_k^{(k)}(g_f) = 0.3333$; $q_l^{(l)}(g_f) = 0.5000$; $CS_i(g_f) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$;

$CS_j(g_f)$ = $CS_k(g_f)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_l(g_f)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $\pi_i^{(i)}(g_f)$ = $\pi_i^{(j)}(g_f)$ = $\pi_i^{(k)}(g_f)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_f)$ = $\pi_k^{(i)}(g_f)$ = $\pi_j^{(j)}(g_f)$ = $\pi_k^{(k)}(g_f)$ = $(0.3333)^2$ = 0.1111; $\pi_l^{(l)}(g_f)$ = $(0.5000)^2$ = 0.2500; and $PS_i(g_f)$ = $PS_j(g_f)$ = $PS_k(g_f)$ = $PS_l(g_f)$ = 0.

Simulation 2: α = 1 and ϕ = 0.5

The output matrix in this case corresponds to:

$$\begin{pmatrix} 1 & 0.2593 & -0.0741 & 0 \\ 0.1064 & 1 & 0 & -0.0426 \\ -0.0476 & -0.0476 & 1 & 0.1190 \\ 0 & -0.0741 & 0.0926 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_j^{(j)}(g_f) \\ q_i^{(j)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.2553 \\ 0.2857 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f)$ = 0.1794; $q_j^{(i)}(g_f)$ = $q_k^{(i)}(g_f)$ = 0.2454; $q_j^{(j)}(g_f)$ = $q_k^{(k)}(g_f)$ = 0.2804; $q_i^{(j)}(g_f)$ = $q_i^{(k)}(g_f)$ = 0.2144; $q_l^{(l)}(g_f)$ = 0.4000; $CS_i(g_f)$ = $\frac{1}{2}(0.1794+0.2144+0.2144)^2$ = 0.1850; $CS_j(g_f)$ = $CS_k(g_f)$ = $\frac{1}{2}(0.2454+0.2804)^2$ = 0.1382; $CS_l(g_f)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $\pi_i^{(i)}(g_f) = \frac{2.5}{2}(0.1794)^2$ = 0.0402; $\pi_j^{(i)}(g_f)$ = $\pi_k^{(i)}(g_f)$ = $\frac{2.5}{2}(0.2454)^2$ = 0.0753; $\pi_j^{(j)}(g_f)$ = $\pi_k^{(k)}(g_f)$ = $\frac{2.5}{2}(0.2804)^2$ = 0.0983; $\pi_i^{(j)}(g_f)$ = $\pi_i^{(k)}(g_f)$ = $\frac{2.5}{2}(0.2144)^2$ = 0.0575; $\pi_l^{(l)}(g_f)$ = $\frac{2.5}{2}(0.4000)^2$ = 0.2000; $PS_i(g_f)$ = $\frac{0.5}{4}(0.1794+0.2454+0.2454)^2$ = 0.0561; $PS_j(g_f)$ = $PS_k(g_f)$ = $\frac{0.5}{4}(0.2144+0.2804)^2$ = 0.0306; and $PS_l(g_f)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200.

Simulation 3: α = 1 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.4909 & -0.1091 & 0 \\ 0.1892 & 1 & 0 & -0.0541 \\ -0.0667 & -0.0667 & 1 & 0.2333 \\ 0 & -0.1091 & 0.1909 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_j^{(j)}(g_f) \\ q_i^{(j)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1802 \\ 0.2222 \\ 0.1818 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f)$ = 0.1227; $q_j^{(i)}(g_f)$ = $q_k^{(i)}(g_f)$ = 0.1657; $q_j^{(j)}(g_f)$ = $q_k^{(k)}(g_f)$ = 0.2039; $q_i^{(j)}(g_f)$ = $q_i^{(k)}(g_f)$ = 0.1610; $q_l^{(l)}(g_f)$ = 0.2857; $CS_i(g_f)$ = $\frac{1}{2}(0.1227+0.1610+0.1610)^2$ = 0.0989; $CS_j(g_f)$ = $CS_k(g_f)$ = $\frac{1}{2}(0.1657+0.2039)^2$ = 0.0683; $CS_l(g_f)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $\pi_i^{(i)}(g_f)$ = $\frac{3.5}{2}(0.1227)^2$ = 0.0263; $\pi_j^{(i)}(g_f)$ = $\pi_k^{(i)}(g_f)$ = $\frac{3.5}{2}(0.1657)^2$ = 0.0480; $\pi_j^{(j)}(g_f)$ = $\pi_k^{(k)}(g_f)$ = $\frac{3.5}{2}(0.2039)^2$ = 0.0728; $\pi_i^{(j)}(g_f)$ = $\pi_i^{(k)}(g_f)$ = $\frac{3.5}{2}(0.1610)^2$ = 0.0454; $\pi_l^{(l)}(g_f)$ = $\frac{3.5}{2}(0.2857)^2$ = 0.1428; $PS_i(g_f)$ = $\frac{1.5}{4}(0.1227+0.1657+0.1657)^2$ = 0.0773; $PS_j(g_f)$ = $PS_k(g_f)$ = $\frac{1.5}{4}(0.1610+0.2039)^2$ = 0.0499; and $PS_l(g_f)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306.

Network g

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_g)$ = $q_j^{(i)}(g_g)$ = $q_k^{(i)}(g_g)$ = $q_i^{(j)}(g_g)$ = $q_j^{(j)}(g_g)$ = $q_k^{(j)}(g_g)$ = $q_i^{(k)}(g_g)$ = $q_j^{(k)}(g_g)$ = $q_k^{(k)}(g_g)$ = $\frac{\alpha}{2\phi+4}$; $q_l^{(l)}(g_g)$ = $\frac{\alpha}{\phi+2}$; $CS_i(g_g)$ = $CS_j(g_g)$ = $CS_k(g_g)$ = $\frac{1}{2}\left(q_i^{(i)}(g_g)+q_i^{(j)}(g_g)+q_i^{(k)}(g_g)\right)^2$; $CS_l(g_g)$ = $\frac{1}{2}\left(q_l^{(l)}(g_g)\right)^2$; $\pi_i^{(i)}(g_g)$ = $\pi_j^{(i)}(g_g)$ = $\pi_k^{(i)}(g_g)$ = $\pi_i^{(j)}(g_g)$ = $\pi_j^{(j)}(g_g)$ = $\pi_k^{(j)}(g_g)$ = $\pi_i^{(k)}(g_g)$ = $\pi_j^{(k)}(g_g)$ = $\pi_k^{(k)}(g_g)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_g)\right)^2$; $\pi_l^{(l)}(g_g)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_g)\right)^2$; $PS_i(g_g)$ = $PS_j(g_g)$ = $PS_k(g_g)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)\right)^2$; and $PS_l(g_g)$ = $\frac{\phi}{4}\left(q_l^{(l)}(g_g)\right)^2$.

Simulation 1: α = 1 and ϕ = 0

$q_i^{(i)}(g_g)$ = $q_j^{(i)}(g_g)$ = $q_k^{(i)}(g_g)$ = $q_i^{(j)}(g_g)$ = $q_j^{(j)}(g_g)$ = $q_k^{(j)}(g_g)$ = $q_i^{(k)}(g_g)$ = $q_j^{(k)}(g_g)$ = $q_k^{(k)}(g_g)$ = $\frac{1}{4}$ = 0.2500; $q_l^{(l)}(g_g)$ = $\frac{1}{2}$ = 0.5000; $CS_i(g_g)$ = $CS_j(g_g)$ = $CS_k(g_g)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_l(g_g)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $\pi_i^{(i)}(g_g)$ = $\pi_j^{(i)}(g_g)$ = $\pi_k^{(i)}(g_g)$ = $\pi_i^{(j)}(g_g)$ = $\pi_j^{(j)}(g_g)$ = $\pi_k^{(j)}(g_g)$ = $\pi_i^{(k)}(g_g)$ = $\pi_j^{(k)}(g_g)$ = $\pi_k^{(k)}(g_g)$ = $(0.2500)^2$ = 0.0625; $\pi_l^{(l)}(g_g)$ = $(0.5000)^2$ = 0.2500; and $PS_i(g_g)$ = $PS_j(g_g)$ = $PS_k(g_g)$ = $PS_l(g_g)$ = 0.

Simulation 2: α = 1 and ϕ = 0.5

$q_i^{(i)}(g_g) = q_j^{(i)}(g_g) = q_k^{(i)}(g_g) = q_i^{(j)}(g_g) = q_j^{(j)}(g_g) = q_k^{(j)}(g_g) = q_i^{(k)}(g_g) = q_j^{(k)}(g_g) = q_k^{(k)}(g_g) = \frac{1}{1+4} = 0.2000$; $q_l^{(l)}(g_g) = \frac{1}{0.5+2} = 0.4000$; $CS_i(g_g) = CS_j(g_g) = CS_k(g_g) = \frac{1}{2}(0.2000+0.2000+0.2000)^2 = 0.1800$; $CS_l(g_g) = \frac{1}{2}(0.4000)^2 = 0.0800$; $\pi_i^{(i)}(g_g) = \pi_j^{(i)}(g_g) = \pi_k^{(i)}(g_g) = \pi_i^{(j)}(g_g) = \pi_j^{(j)}(g_g) = \pi_k^{(j)}(g_g) = \pi_i^{(k)}(g_g) = \pi_j^{(k)}(g_g) = \pi_k^{(k)}(g_g) = \frac{2.5}{2}(0.2000)^2 = 0.0500$; $\pi_l^{(l)}(g_g) = \frac{2.5}{2}(0.4000)^2 = 0.2000$; $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) = \frac{0.5}{4}(0.2000+0.2000+0.2000)^2 = 0.0450$; and $PS_l(g_g) = \frac{0.5}{4}(0.4000)^2 = 0.0200$.

Simulation 3: α = 1 and ϕ = 1.5

$q_i^{(i)}(g_g) = q_j^{(i)}(g_g) = q_k^{(i)}(g_g) = q_i^{(j)}(g_g) = q_j^{(j)}(g_g) = q_k^{(j)}(g_g) = q_i^{(k)}(g_g) = q_j^{(k)}(g_g) = q_k^{(k)}(g_g) = \frac{1}{3+4} = 0.1429$; $q_l^{(l)}(g_g) = \frac{1}{1.5+2} = 0.2857$; $CS_i(g_g) = CS_j(g_g) = CS_k(g_g) = \frac{1}{2}(0.1429+0.1429+0.1429)^2 = 0.0918$; $CS_l(g_g) = \frac{1}{2}(0.2857)^2 = 0.0408$; $\pi_i^{(i)}(g_g) = \pi_j^{(i)}(g_g) = \pi_k^{(i)}(g_g) = \pi_i^{(j)}(g_g) = \pi_j^{(j)}(g_g) = \pi_k^{(j)}(g_g) = \pi_i^{(k)}(g_g) = \pi_j^{(k)}(g_g) = \pi_k^{(k)}(g_g) = \frac{3.5}{2}(0.1429)^2 = 0.0357$; $\pi_l^{(l)}(g_g) = \frac{3.5}{2}(0.2857)^2 = 0.1428$; $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) = \frac{1.5}{4}(0.1429+0.1429+0.1429)^2 = 0.0689$; and $PS_l(g_g) = \frac{1.5}{4}(0.2857)^2 = 0.0306$.

Network h

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_h)$ = $\frac{2\alpha(\phi+1)+3\phi q_j^{(j)}(g_h)-3\phi(\phi+4)q_j^{(i)}(g_h)}{2(\phi+5)(\phi+1)}$; $q_j^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_l^{(i)}(g_h)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_h)-\phi(\phi+2)q_i^{(i)}(g_h)}{4\phi^2+12\phi+6}$; $q_i^{(j)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_i^{(l)}(g_h)$ = $\frac{2\alpha(\phi+1)+3\phi q_j^{(i)}(g_h)-\phi(\phi+2)q_j^{(j)}(g_h)}{2(\phi+5)(\phi+1)}$; $q_j^{(j)}(g_h)$ = $q_k^{(k)}(g_h)$ = $q_l^{(l)}(g_h)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_h)+2\phi q_j^{(i)}(g_h)-\phi(\phi+2)q_i^{(j)}(g_h)}{2(\phi+3)(\phi+1)}$; $CS_i(g_h)$ = $\frac{1}{2}\left(q_i^{(i)}(g_h)+q_i^{(j)}(g_h)+q_i^{(k)}(g_h)+q_i^{(l)}(g_h)\right)^2$; $CS_j(g_h)$ = $CS_k(g_h)$ = $CS_l(g_h)$ = $\frac{1}{2}\left(q_j^{(i)}(g_h)+q_j^{(j)}(g_h)\right)^2$; $\pi_i^{(i)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_h)\right)^2$; $\pi_j^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_l^{(i)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_k)\right)^2$; $\pi_i^{(j)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_i^{(l)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_h)\right)^2$; $\pi_j^{(j)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_h)\right)^2$; $PS_i(g_h)$ = $\frac{\phi}{4}(q_i^{(i)}(g_h)$ + $q_j^{(i)}(g_h)$ + $q_k^{(i)}(g_h)$ + $q_l^{(i)}(g_h))^2$; and

$PS_j(g_h)$ = $PS_k(g_h)$ = $PS_l(g_h)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{3\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{3\phi}{2(\phi+5)(\phi+1)} \\ \frac{\phi(\phi+2)}{4\phi^2+12\phi+6} & 1 & -\frac{\phi}{4\phi^2+12\phi+6} & 0 \\ 0 & -\frac{3\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+5)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+5} \\ \frac{2\alpha(\phi+1)}{4\phi^2+12\phi+6} \\ \frac{\alpha}{\phi+5} \\ \frac{\alpha}{\phi+3} \end{pmatrix}$$

Simulation 1: $\alpha = 1$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.3333 \\ 0.2000 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h) = q_i^{(j)}(g_h) = q_i^{(k)}(g_h) = q_i^{(l)}(g_h) = 0.2000$; $q_j^{(i)}(g_h) = q_k^{(i)}(g_h) = q_l^{(i)}(g_h) = q_j^{(j)}(g_h) = q_k^{(k)}(g_h) = q_l^{(l)}(g_h) = 0.3333$; $CS_i(g_h) = \frac{1}{2}(0.20000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.3200$; $CS_j(g_h) = CS_k(g_h) = CS_l(g_h) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $\pi_i^{(i)}(g_h) = \pi_i^{(j)}(g_h) = \pi_i^{(k)}(g_h) = \pi_i^{(l)}(g_h) = (0.2000)^2 = 0.0400$; $\pi_j^{(i)}(g_h) = \pi_k^{(i)}(g_h) = \pi_l^{(i)}(g_h) = \pi_j^{(j)}(g_h) = \pi_k^{(k)}(g_h) = \pi_l^{(l)}(g_h) = (0.3333)^2 = 0.1111$; and $PS_i(g_h) = PS_j(g_h) = PS_k(g_h) = PS_l(g_h) = 0$.

Simulation 2: α = 1 and ϕ = 0.5

The output matrix in this case corresponds to:

$$\begin{pmatrix} 1 & 0.4091 & 0 & -0.0909 \\ 0.0962 & 1 & -0.0385 & 0 \\ 0 & -0.0909 & 1 & 0.0758 \\ -0.0476 & -0.0952 & 0.2381 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2308 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h)$ = 0.1135; $q_j^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_l^{(i)}(g_h)$ = 0.2269; $q_i^{(j)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_i^{(l)}(g_h)$ = 0.1820; $q_j^{(j)}(g_h)$ = $q_k^{(k)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0.2694; $CS_i(g_h)$ = $\frac{1}{2}$(0.1135 + 0.1820 + 0.1820 + 0.1820)^2 = 0.2175; $CS_j(g_h)$ = $CS_k(g_h)$ = $CS_l(g_h)$ = $\frac{1}{2}(0.2269+0.2694)^2$ = 0.1232; $\pi_i^{(i)}(g_h)$ = $\frac{2.5}{2}(0.1135)^2$ = 0.0161; $\pi_j^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_l^{(i)}(g_h)$ = $\frac{2.5}{2}(0.2269)^2$ = 0.0644; $\pi_i^{(j)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_i^{(l)}(g_h)$ = $\frac{2.5}{2}(0.1820)^2$ = 0.0414; $\pi_j^{(j)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $\frac{2.5}{2}(0.2694)^2$ = 0.0907; $PS_i(g_h)$ = $\frac{0.5}{4}(0.1135+0.2269+0.2269+0.2269)^2$ = 0.0788; and $PS_j(g_h)$ = $PS_k(g_h)$ = $PS_l(g_h)$ = $\frac{0.5}{4}(0.1820+0.2694)^2$ = 0.0255.

Simulation 3: $\alpha = 1$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.7615 & 0 & -0.1385 \\ 0.1591 & 1 & -0.0455 & 0 \\ 0 & -0.1385 & 1 & 0.1615 \\ -0.0667 & -0.1333 & 0.4667 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.1515 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h) = 0.0661$; $q_j^{(i)}(g_h) = q_k^{(i)}(g_h) = q_l^{(i)}(g_h) = 0.1476$; $q_i^{(j)}(g_h) = q_i^{(k)}(g_h) = q_i^{(l)}(g_h) = 0.1454$; $q_j^{(j)}(g_h) = q_k^{(k)}(g_h) = q_l^{(l)}(g_h) = 0.1784$; $CS_i(g_h) = \frac{1}{2}(0.0661 + 0.1454 + 0.1454 + 0.1454)^2 = 0.1262$; $CS_j(g_h) = CS_k(g_h) = CS_l(g_h) = \frac{1}{2}(0.1476 + 0.1784)^2 = 0.0531$; $\pi_i^{(i)}(g_h) = \frac{3.5}{2}(0.0661)^2 = 0.0076$; $\pi_j^{(i)}(g_h) = \pi_k^{(i)}(g_h) = \pi_l^{(i)}(g_h) = \frac{3.5}{2}(0.1476)^2 = 0.0381$; $\pi_i^{(j)}(g_h) = \pi_i^{(k)}(g_h) = \pi_i^{(l)}(g_h) = \frac{3.5}{2}(0.1454)^2 = 0.0370$; $\pi_j^{(j)}(g_h) = \pi_k^{(k)}(g_h) = \pi_l^{(l)}(g_h) = \frac{3.5}{2}(0.1784)^2 = 0.0557$; $PS_i(g_h) = \frac{1.5}{4}(0.0661 + 0.1476 + 0.1476 + 0.1476)^2 = 0.0971$; and $PS_j(g_h) = PS_k(g_h) = PS_l(g_h) = \frac{1.5}{4}(0.1454 + 0.1784)^2 = 0.0393$.

Network i

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_i)+\phi q_k^{(k)}(g_i)+\phi q_l^{(k)}(g_i)-\phi(\phi+3)q_j^{(i)}(g_i)-\phi(\phi+2)q_k^{(i)}(g_i)}{2\phi^2+9\phi+8}$; $q_j^{(i)}(g_i)$ = $q_i^{(j)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_i)+\phi q_k^{(k)}(g_i)+\phi q_l^{(k)}(g_i)-\phi(\phi+3)q_i^{(i)}(g_i)-\phi(\phi+2)q_k^{(i)}(g_i)}{2\phi^2+9\phi+8}$;

$q_k^{(i)}(g_i)$ = $q_k^{(j)}(g_i)$ = $\frac{2\alpha(\phi+1)+2\phi q_i^{(k)}(g_i)+\phi q_l^{(k)}(g_i)+\phi q_l^{(l)}(g_i)-\phi(\phi+3)q_i^{(i)}(g_i)-\phi(\phi+3)q_j^{(i)}(g_i)}{2(\phi+5)(\phi+1)}$;

$q_i^{(k)}(g_i)$ = $q_j^{(k)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_i)+2\phi q_k^{(i)}(g_i)+\phi q_i^{(i)}(g_i)-\phi(\phi+3)q_k^{(k)}(g_i)-\phi(\phi+3)q_l^{(k)}(g_i)}{3\phi^2+13\phi+8}$;

$q_k^{(k)}(g_i)$ = $\frac{2\alpha(\phi+1)+2\phi q_i^{(i)}(g_i)+2\phi q_j^{(i)}(g_i)+\phi q_l^{(l)}(g_i)-2\phi(\phi+4)q_i^{(k)}(g_i)-\phi(\phi+4)q_l^{(k)}(g_i)}{2(\phi+5)(\phi+1)}$;

$q_l^{(k)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_k^{(l)}(g_i)-2\phi(\phi+2)q_i^{(k)}(g_i)-\phi(\phi+2)q_k^{(k)}(g_i)}{2(\phi+3)(\phi+1)}$; $q_k^{(l)}(g_i)$ = $\frac{2\alpha(\phi+1)+2\phi q_i^{(i)}(g_i)+2\phi q_j^{(i)}(g_i)+2\phi q_i^{(k)}(g_i)+\phi q_l^{(k)}(g_i)-\phi(\phi+4)q_l^{(l)}(g_i)}{2(\phi+5)(\phi+1)}$; $q_l^{(l)}(g_i)$ = $\frac{2\alpha(\phi+1)+2\phi q_i^{(k)}(g_i)+\phi q_k^{(k)}(g_i)-\phi(\phi+2)q_k^{(l)}(g_i)}{2(\phi+3)(\phi+1)}$; *CSi*(*gi*) = *CSj*(*gi*) = $\frac{1}{2}\left(q_i^{(i)}(g_i)+q_i^{(j)}(g_i)+q_i^{(k)}(g_i)\right)^2$; *CSk*(*gi*) = $\frac{1}{2}\left(q_k^{(i)}(g_i)+q_k^{(j)}(g_i)+q_k^{(k)}(g_i)+q_k^{(l)}(g_i)\right)^2$; *CSl*(*gi*) = $\frac{1}{2}\left(q_l^{(k)}(g_i)+q_l^{(l)}(g_i)\right)^2$; $\pi_i^{(i)}(g_i)$ = $\pi_j^{(j)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_i)\right)^2$; $\pi_j^{(i)}(g_i)$ = $\pi_i^{(j)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_i)\right)^2$; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(j)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_k^{(i)}(g_i)\right)^2$; $\pi_i^{(k)}(g_i)$ = $\pi_j^{(k)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_i^{(k)}(g_i)\right)^2$; $\pi_k^{(k)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_i)\right)^2$; $\pi_l^{(k)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_l^{(k)}(g_i)\right)^2$; $\pi_k^{(l)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_k^{(l)}(g_i)\right)^2$; $\pi_l^{(l)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_i)\right)^2$; *PSi*(*gi*) = *PSj*(*gi*) =

$\dfrac{\phi}{4}\left(q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_k^{(i)}(g_i)\right)^2$; $PS_k(g_i) = \dfrac{\phi}{4}\left(q_i^{(k)}(g_i)+q_j^{(k)}(g_i)+q_k^{(k)}(g_i)+q_l^{(k)}(g_i)\right)^2$; and

$PS_l(g_i) = \dfrac{\phi}{4}\left(q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix}
1 & \frac{\phi(\phi+3)}{2\phi^2+9\phi+8} & \frac{\phi(\phi+2)}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & 0 & 0 \\
\frac{\phi(\phi+3)}{2\phi^2+9\phi+8} & 1 & \frac{\phi(\phi+2)}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & 0 & 0 \\
\frac{\phi(\phi+3)}{2(\phi+5)(\phi+1)} & \frac{\phi(\phi+3)}{2(\phi+5)(\phi+1)} & 1 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\
-\frac{\phi}{3\phi^2+13\phi+8} & -\frac{\phi}{3\phi^2+13\phi+8} & -\frac{2\phi}{3\phi^2+13\phi+8} & 1 & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & 0 & 0 \\
-\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & \frac{\phi(\phi+4)}{(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\
0 & 0 & 0 & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\
-\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} \\
0 & 0 & 0 & -\frac{\phi}{(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) \\ q_k^{(l)}(g_i) \\ q_l^{(l)}(g_i)
\end{pmatrix}
=
\begin{pmatrix}
\frac{2\alpha(\phi+1)}{2\phi^2+9\phi+8} \\ \frac{2\alpha(\phi+1)}{2\phi^2+9\phi+8} \\ \frac{\alpha}{\phi+5} \\ \frac{2\alpha(\phi+1)}{3\phi^2+13\phi+8} \\ \frac{\alpha}{\phi+5} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+5} \\ \frac{\alpha}{\phi+3}
\end{pmatrix}$$

<u>*Simulation 1: $\alpha = 1$ and $\phi = 0$*</u>

The output matrix in this case is given by:

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) \\ q_k^{(l)}(g_i) \\ q_l^{(l)}(g_i)
\end{pmatrix}
=
\begin{pmatrix}
0.2500 \\ 0.2500 \\ 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.3333 \\ 0.2000 \\ 0.3333
\end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_j^{(i)}(g_i)$ = $q_i^{(j)}(g_i)$ = $q_i^{(k)}(g_i)$ = $q_j^{(k)}(g_i)$ = 0.2500; $q_k^{(i)}(g_i)$ = $q_k^{(j)}(g_i)$ = $q_k^{(k)}(g_i)$ = $q_k^{(l)}(g_i)$ = 0.2000; $q_l^{(k)}(g_i)$ = $q_l^{(l)}(g_i)$ = 0.3333; $CS_i(g_i)$ = $CS_j(g_i)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_k(g_i)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_l(g_i)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $\pi_i^{(i)}(g_i)$ = $\pi_j^{(j)}(g_i)$ = $\pi_j^{(i)}(g_i)$ = $\pi_i^{(j)}(g_i)$ = $\pi_i^{(k)}(g_i)$ = $\pi_j^{(k)}(g_i)$ = $(0.2500)^2$ = 0.0625; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(j)}(g_i)$ = $\pi_k^{(k)}(g_i)$ = $\pi_k^{(l)}(g_i)$ = $(0.2000)^2$ = 0.0400; $\pi_l^{(k)}(g_i)$ = $\pi_l^{(l)}(g_i)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_i)$ = $PS_j(g_i)$ = $PS_k(g_i)$ = $PS_l(g_i)$ = 0.

Simulation 2: α = 1 and ϕ = 0.5

The output matrix in this case corresponds to:

$$\begin{pmatrix} 1 & 0.1346 & 0.0962 & -0.0385 & -0.0385 & -0.0385 & 0 & 0 \\ 0.1346 & 1 & 0.0962 & -0.0385 & -0.0385 & -0.0385 & 0 & 0 \\ 0.1061 & 0.1061 & 1 & -0.0606 & 0 & -0.0303 & 0 & -0.0303 \\ -0.0328 & -0.0328 & -0.0656 & 1 & 0.1148 & 0.1148 & 0 & 0 \\ -0.0606 & -0.0606 & 0 & 0.2727 & 1 & 0.1364 & 0 & -0.0303 \\ 0 & 0 & 0 & 0.2381 & 0.1190 & 1 & -0.0476 & 0 \\ -0.0606 & -0.0606 & 0 & -0.0606 & 0 & -0.0303 & 1 & 0.1364 \\ 0 & 0 & 0 & -0.0952 & -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) \\ q_k^{(l)}(g_i) \\ q_l^{(l)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.2308 \\ 0.2308 \\ 0.1818 \\ 0.1967 \\ 0.1818 \\ 0.2857 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_j^{(i)}(g_i)$ = $q_i^{(j)}(g_i)$ = 0.2081; $q_k^{(i)}(g_i)$ = $q_k^{(j)}(g_i)$ = 0.1643; $q_i^{(k)}(g_i)$ = $q_j^{(k)}(g_i)$ =0.1786; $q_k^{(k)}(g_i)$ = 0.1348; $q_l^{(k)}(g_i)$ = 0.2360; $q_k^{(l)}(g_i)$ = 0.1858; $q_l^{(l)}(g_i)$ = 0.2870; $CS_i(g_i)$ = $CS_j(g_i)$ = $\frac{1}{2}(0.2081+0.2081+0.1786)^2$ = 0.1769; $CS_k(g_i)$ =

$\frac{1}{2}(0.1643+0.1643+0.1348+0.1858)^2$ = 0.2107; $CS_l(g_i)$ = $\frac{1}{2}(0.2360+0.2870)^2$ = 0.1368; $\pi_i^{(i)}(g_i)$ = $\pi_j^{(j)}(g_i)$ = $\pi_j^{(i)}(g_i)$ = $\pi_i^{(j)}(g_i)$ = $\frac{2.5}{2}(0.2081)^2$ = 0.0541; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(j)}(g_i)$ = $\frac{2.5}{2}(0.1643)^2$ = 0.0337; $\pi_i^{(k)}(g_i)$ = $\pi_j^{(k)}(g_i)$ = $\frac{2.5}{2}(0.1786)^2$ = 0.0399; $\pi_k^{(k)}(g_i)$ = $\frac{2.5}{2}(0.1348)^2$ = 0.0227; $\pi_l^{(k)}(g_i)$ = $\frac{2.5}{2}(0.2360)^2$ = 0.0696; $\pi_k^{(l)}(g_i)$ = $\frac{2.5}{2}(0.1858)^2$ = 0.0432; $\pi_l^{(l)}(g_i)$ = $\frac{2.5}{2}(0.2870)^2$ = 0.1030; $PS_i(g_i)$ = $PS_j(g_i)$ = $\frac{0.5}{4}(0.2081 + 0.2081 + 0.1643)^2$ = 0.0421; $PS_k(g_i)$ = $\frac{0.5}{4}(0.1786+0.1786+0.1348+0.2360)^2$ = 0.0662; and $PS_l(g_i)$ = $\frac{0.5}{4}(0.1858+0.2870)^2$ = 0.0279.

Simulation 3: $\alpha = 1$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2596 & 0.2019 & -0.0577 & -0.0577 & -0.0577 & 0 & 0 \\ 0.2596 & 1 & 0.2019 & -0.0577 & -0.0577 & -0.0577 & 0 & 0 \\ 0.2077 & 0.2077 & 1 & -0.0923 & 0 & -0.0462 & 0 & -0.0462 \\ -0.0438 & -0.0438 & -0.0876 & 1 & 0.1971 & 0.1971 & 0 & 0 \\ -0.0923 & -0.0923 & 0 & 0.5077 & 1 & 0.2538 & 0 & -0.0462 \\ 0 & 0 & 0 & 0.4667 & 0.2333 & 1 & -0.0667 & 0 \\ -0.0923 & -0.0923 & 0 & -0.0923 & 0 & -0.0462 & 1 & 0.2538 \\ 0 & 0 & 0 & -0.1333 & -0.0667 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) \\ q_k^{(l)}(g_i) \\ q_l^{(l)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.1923 \\ 0.1923 \\ 0.1538 \\ 0.1460 \\ 0.1538 \\ 0.2222 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_j^{(i)}(g_i)$ = $q_i^{(j)}(g_i)$ = 0.1503; $q_k^{(i)}(g_i)$ = $q_k^{(j)}(g_i)$ = 0.1194; $q_i^{(k)}(g_i)$ = $q_j^{(k)}(g_i)$ = 0.1214; $q_k^{(k)}(g_i)$ = 0.0905; $q_l^{(k)}(g_i)$ = 0.1542; $q_k^{(l)}(g_i)$ = 0.1465; $q_l^{(l)}(g_i)$ = 0.2102; $CS_i(g_i)$ = $CS_j(g_i)$ = $\frac{1}{2}(0.1503+0.1503+0.1214)^2$ = 0.0890; $CS_k(g_i)$ = $\frac{1}{2}(0.1194+0.1194+0.0905+0.1465)^2$ = 0.1132; $CS_l(g_i)$ = $\frac{1}{2}(0.1542+0.2102)^2$ = 0.0664; $\pi_i^{(i)}(g_i)$ = $\pi_j^{(j)}(g_i)$ = $\pi_j^{(i)}(g_i)$ = $\pi_i^{(j)}(g_i)$ = $\frac{3.5}{2}(0.1503)^2$ = 0.0395; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(j)}(g_i)$ = $\frac{3.5}{2}(0.1194)^2$ = 0.0249; $\pi_i^{(k)}(g_i)$ = $\pi_j^{(k)}(g_i)$ = $\frac{3.5}{2}(0.1214)^2$ = 0.0258; $\pi_k^{(k)}(g_i)$ = $\frac{3.5}{2}(0.0905)^2$ = 0.0143; $\pi_l^{(k)}(g_i)$ = $\frac{3.5}{2}(0.1542)^2$ = 0.0416; $\pi_k^{(l)}(g_i)$ = $\frac{3.5}{2}(0.1465)^2$ = 0.0376; $\pi_l^{(l)}(g_i)$ = $\frac{3.5}{2}(0.2102)^2$ = 0.0773; $PS_i(g_i)$ = $PS_j(g_i)$ = $\frac{1.5}{4}$(0.1503 + 0.1503 + 0.1194)^2 = 0.0662; $PS_k(g_i)$ = $\frac{1.5}{4}(0.1214+0.1214+0.0905+0.1542)^2$ = 0.0891; and $PS_l(g_i)$ = $\frac{1.5}{4}(0.1465+0.2102)^2$ = 0.0477.

Network j

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_j)$ = $q_l^{(l)}(g_j)$ = $\frac{\alpha(\phi+1)+2\phi q_j^{(j)}(g_j)+\phi q_i^{(j)}(g_j)-\phi(\phi+3)q_j^{(i)}(g_j)}{(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_j^{(l)}(g_j)$ = $q_k^{(l)}(g_j)$ = $\frac{2\alpha(\phi+1)+4\phi q_i^{(j)}(g_j)+2\phi q_j^{(j)}(g_j)-\phi(\phi+3)q_i^{(i)}(g_j)}{3\phi^2+15\phi+10}$; $q_j^{(j)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_k^{(j)}(g_j)$ = $q_j^{(k)}(g_j)$ = $\frac{2\alpha(\phi+1)+2\phi q_i^{(i)}(g_j)+2\phi q_j^{(i)}(g_j)-2\phi(\phi+3)q_i^{(j)}(g_j)}{3\phi^2+15\phi+10}$; $q_i^{(j)}(g_j)$ = $q_l^{(j)}(g_j)$ =

$q_i^{(k)}(g_j) = q_l^{(k)}(g_j) = \dfrac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_j)-2\phi(\phi+2)q_j^{(j)}(g_j)}{3\phi^2+12\phi+8}$; $CS_i(g_j) = CS_l(g_j) = \frac{1}{2}\left(q_i^{(i)}(g_j) + q_i^{(j)}(g_j) + q_i^{(k)}(g_j)\right)^2$; $CS_j(g_j) = CS_k(g_j) = \frac{1}{2}\left(q_j^{(i)}(g_j)+q_j^{(j)}(g_j)+q_j^{(k)}(g_j)+q_j^{(l)}(g_j)\right)^2$; $\pi_i^{(i)}(g_j) = \pi_l^{(l)}(g_j) = \dfrac{(2+\phi)}{2}\left(q_i^{(i)}(g_j)\right)^2$; $\pi_j^{(i)}(g_j) = \pi_k^{(i)}(g_j) = \pi_j^{(l)}(g_j) = \pi_k^{(l)}(g_j) = \dfrac{(2+\phi)}{2}\left(q_j^{(i)}(g_j)\right)^2$; $\pi_j^{(j)}(g_j) = \pi_k^{(k)}(g_j) = \pi_k^{(j)}(g_j) = \pi_j^{(k)}(g_j) = \dfrac{(2+\phi)}{2}\left(q_j^{(j)}(g_j)\right)^2$; $\pi_i^{(j)}(g_j) = \pi_l^{(j)}(g_j) = \pi_i^{(k)}(g_j) = \pi_l^{(k)}(g_j) = \dfrac{(2+\phi)}{2}\left(q_i^{(j)}(g_j)\right)^2$; $PS_i(g_j) = PS_l(g_j) = \dfrac{\phi}{4}\left(q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)\right)^2$; and $PS_j(g_j) = PS_k(g_j) = \dfrac{\phi}{4}\left(q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)+q_l^{(j)}(g_j)\right)^2$.

The output quantities involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \dfrac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & -\dfrac{2\phi}{(\phi+4)(\phi+1)} & -\dfrac{\phi}{(\phi+4)(\phi+1)} \\ \dfrac{\phi(\phi+3)}{3\phi^2+15\phi+10} & 1 & -\dfrac{2\phi}{3\phi^2+15\phi+10} & -\dfrac{4\phi}{3\phi^2+15\phi+10} \\ -\dfrac{2\phi}{3\phi^2+15\phi+10} & -\dfrac{2\phi}{3\phi^2+15\phi+10} & 1 & \dfrac{2\phi(\phi+3)}{3\phi^2+15\phi+10} \\ 0 & -\dfrac{2\phi}{3\phi^2+12\phi+8} & \dfrac{2\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_j^{(j)}(g_j) \\ q_i^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha}{\phi+4} \\ \dfrac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \dfrac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \dfrac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \end{pmatrix}$$

Simulation 1: α = 1 and ϕ = 0

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_j^{(j)}(g_j) \\ q_i^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.2000 \\ 0.2000 \\ 0.2500 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j) = q_l^{(l)}(g_j) = q_i^{(j)}(g_j) = q_l^{(j)}(g_j) = q_i^{(k)}(g_j) = q_l^{(k)}(g_j) = 0.2500$; $q_j^{(i)}(g_j) = q_k^{(i)}(g_j) = q_j^{(l)}(g_j) = q_k^{(l)}(g_j) = q_j^{(j)}(g_j) = q_k^{(k)}(g_j) = q_k^{(j)}(g_j) = q_j^{(k)}(g_j) = 0.2000$; $CS_i(g_j) = CS_l(g_j) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_j) = CS_k(g_j) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.3200$; $\pi_i^{(i)}(g_j) = \pi_l^{(l)}(g_j) = \pi_i^{(j)}(g_j) = \pi_l^{(j)}(g_j) = \pi_i^{(k)}(g_j) = \pi_l^{(k)}(g_j) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_j) = \pi_k^{(i)}(g_j) = \pi_j^{(l)}(g_j) = \pi_k^{(l)}(g_j) = \pi_j^{(j)}(g_j) = \pi_k^{(k)}(g_j) = \pi_k^{(j)}(g_j) = \pi_j^{(k)}(g_j) = (0.2000)^2 = 0.0400$; and $PS_i(g_j) = PS_j(g_j) = PS_k(g_j) = PS_l(g_j) = 0$.

Simulation 2: α = 1 and ϕ = 0.5

The output matrix in this case corresponds to:

$$\begin{pmatrix} 1 & 0.2593 & -0.1481 & -0.0741 \\ 0.0959 & 1 & -0.0548 & -0.1096 \\ -0.0548 & -0.0548 & 1 & 0.1918 \\ 0 & -0.0678 & 0.1695 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_j^{(j)}(g_j) \\ q_i^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.1644 \\ 0.1644 \\ 0.2034 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j)$ = $q_l^{(l)}(g_j)$ = 0.2135; $q_j^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_j^{(l)}(g_j)$ = $q_k^{(l)}(g_j)$ = 0.1729; $q_j^{(j)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_k^{(j)}(g_j)$ = $q_j^{(k)}(g_j)$ = 0.1492; $q_i^{(j)}(g_j)$ = $q_l^{(j)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_l^{(k)}(g_j)$ = 0.1898; $CS_i(g_j)$ = $CS_l(g_j)$ = $\frac{1}{2}(0.2135 + 0.1898 + 0.1898)^2$ = 0.1759; $CS_j(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}(0.1729 + 0.1492 + 0.1492 + 0.1729)^2$ = 0.2075; $\pi_i^{(i)}(g_j)$ = $\pi_l^{(l)}(g_j)$ = $\frac{2.5}{2}(0.2135)^2$ = 0.0570; $\pi_j^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_j^{(l)}(g_j)$ = $\pi_k^{(l)}(g_j)$ = $\frac{2.5}{2}(0.1729)^2$ = 0.0374; $\pi_j^{(j)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\frac{2.5}{2}(0.1492)^2$ = 0.0278; $\pi_i^{(j)}(g_j)$ = $\pi_l^{(j)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_l^{(k)}(g_j)$ = $\frac{2.5}{2}(0.1898)^2$ = 0.0450; $PS_i(g_j)$ = $PS_l(g_j)$ = $\frac{0.5}{4}(0.2135 + 0.1729 + 0.1729)^2$ = 0.0391; and $PS_j(g_j)$ = $PS_k(g_j)$ = $\frac{0.5}{4}(0.1898 + 0.1492 + 0.1492 + 0.1898)^2$ = 0.0575.

Simulation 3: α = 1 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.4909 & -0.2182 & -0.1091 \\ 0.1720 & 1 & -0.0764 & -0.1529 \\ -0.0764 & -0.0764 & 1 & 0.3439 \\ 0 & -0.0916 & 0.3206 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_j^{(j)}(g_j) \\ q_i^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1274 \\ 0.1274 \\ 0.1527 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j)$ = $q_l^{(l)}(g_j)$ = 0.1557; $q_j^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_j^{(l)}(g_j)$ = $q_k^{(l)}(g_j)$ = 0.1286; $q_j^{(j)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_k^{(j)}(g_j)$ = $q_j^{(k)}(g_j)$ = 0.1040; $q_i^{(j)}(g_j)$ = $q_l^{(j)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_l^{(k)}(g_j)$ = 0.1311; $CS_i(g_j)$ = $CS_l(g_j)$ = $\frac{1}{2}(0.1557 + 0.1311 + 0.1311)^2$ = 0.0873; $CS_j(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}(0.1286 + 0.1040 + 0.1040 + 0.1286)^2$ = 0.1082; $\pi_i^{(i)}(g_j)$ = $\pi_l^{(l)}(g_j)$ = $\frac{3.5}{2}(0.1557)^2$ = 0.0424; $\pi_j^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_j^{(l)}(g_j)$ = $\pi_k^{(l)}(g_j)$ = $\frac{3.5}{2}(0.1286)^2$ = 0.0289; $\pi_j^{(j)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\frac{3.5}{2}(0.1040)^2$ = 0.0189; $\pi_i^{(j)}(g_j)$ = $\pi_l^{(j)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_l^{(k)}(g_j)$ = $\frac{3.5}{2}(0.1311)^2$ = 0.0301; $PS_i(g_j)$ = $PS_l(g_j)$ = $\frac{1.5}{4}(0.1557 + 0.1286 + 0.1286)^2$ = 0.0639; and $PS_j(g_j)$ = $PS_k(g_j)$ = $\frac{1.5}{4}(0.1311 + 0.1040 + 0.1040 + 0.1311)^2$ = 0.0829.

Network k

In considering the equations presented in Section 3.3.1.1 it holds that $q_i^{(i)}(g_k)$ = $q_j^{(i)}(g_k)$ = $q_k^{(i)}(g_k)$ = $q_l^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_k^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_i^{(k)}(g_k)$ = $q_j^{(k)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_i^{(l)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_k^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = $\frac{2\alpha}{5\phi + 10}$; $CS_i(g_k)$ = $CS_j(g_k)$ = $CS_k(g_k)$ = $CS_l(g_k)$ = $\frac{1}{2}\left(q_i^{(i)}(g_k) + q_i^{(j)}(g_k) + q_i^{(k)}(g_k) + q_i^{(l)}(g_k)\right)^2$; $\pi_i^{(i)}(g_k)$ = $\pi_j^{(i)}(g_k)$ = $\pi_k^{(i)}(g_k)$ = $\pi_l^{(i)}(g_k)$ = $\pi_i^{(j)}(g_k)$ = $\pi_j^{(j)}(g_k)$ = $\pi_k^{(j)}(g_k)$ = $\pi_l^{(j)}(g_k)$ = $\pi_i^{(k)}(g_k)$ = $\pi_j^{(k)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = $\pi_l^{(k)}(g_k)$ = $\pi_i^{(l)}(g_k)$ = $\pi_j^{(l)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = $\pi_l^{(l)}(g_k)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_k)\right)^2$; and $PS_i(g_k)$ = $PS_j(g_k)$ = $PS_k(g_k)$ = $PS_l(g_k)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_k) + q_j^{(i)}(g_k) + q_k^{(i)}(g_k) + q_l^{(i)}(g_k)\right)^2$.

Simulation 1: α = 1 and ϕ = 0

$q_i^{(i)}(g_k) = q_j^{(i)}(g_k) = q_k^{(i)}(g_k) = q_l^{(i)}(g_k) = q_i^{(j)}(g_k) = q_j^{(j)}(g_k) = q_k^{(j)}(g_k) = q_l^{(j)}(g_k) = q_i^{(k)}(g_k) = q_j^{(k)}(g_k) = q_k^{(k)}(g_k) = q_l^{(k)}(g_k) = q_i^{(l)}(g_k) = q_j^{(l)}(g_k) = q_k^{(l)}(g_k) = q_l^{(l)}(g_k) =$ 0.2000; $CS_i(g_k) = CS_j(g_k) = CS_k(g_k) = CS_l(g_k) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 =$ 0.3200; $\pi_i^{(i)}(g_k) = \pi_j^{(i)}(g_k) = \pi_k^{(i)}(g_k) = \pi_l^{(i)}(g_k) = \pi_i^{(j)}(g_k) = \pi_j^{(j)}(g_k) = \pi_k^{(j)}(g_k) = \pi_l^{(j)}(g_k) = \pi_i^{(k)}(g_k) = \pi_j^{(k)}(g_k) = \pi_k^{(k)}(g_k) = \pi_l^{(k)}(g_k) = \pi_i^{(l)}(g_k) = \pi_j^{(l)}(g_k) = \pi_k^{(l)}(g_k) = \pi_l^{(l)}(g_k) = (0.2000)^2 = 0.0400$; and $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k) = 0$.

Simulation 2: α = 1 and ϕ = 0.5

$q_i^{(i)}(g_k) = q_j^{(i)}(g_k) = q_k^{(i)}(g_k) = q_l^{(i)}(g_k) = q_i^{(j)}(g_k) = q_j^{(j)}(g_k) = q_k^{(j)}(g_k) = q_l^{(j)}(g_k) = q_i^{(k)}(g_k) = q_j^{(k)}(g_k) = q_k^{(k)}(g_k) = q_l^{(k)}(g_k) = q_i^{(l)}(g_k) = q_j^{(l)}(g_k) = q_k^{(l)}(g_k) = q_l^{(l)}(g_k) =$ 0.1600; $CS_i(g_k) = CS_j(g_k) = CS_k(g_k) = CS_l(g_k) = \frac{1}{2}(0.1600 + 0.1600 + 0.1600 + 0.1600)^2 =$ 0.2048; $\pi_i^{(i)}(g_k) = \pi_j^{(i)}(g_k) = \pi_k^{(i)}(g_k) = \pi_l^{(i)}(g_k) = \pi_i^{(j)}(g_k) = \pi_j^{(j)}(g_k) = \pi_k^{(j)}(g_k) = \pi_l^{(j)}(g_k) = \pi_i^{(k)}(g_k) = \pi_j^{(k)}(g_k) = \pi_k^{(k)}(g_k) = \pi_l^{(k)}(g_k) = \pi_i^{(l)}(g_k) = \pi_j^{(l)}(g_k) = \pi_k^{(l)}(g_k) = \pi_l^{(l)}(g_k) = \frac{2.5}{2}(0.1600)^2 = 0.0320$; and $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k) = \frac{0.5}{4}(0.1600 + 0.1600 + 0.1600 + 0.1600)^2 = 0.0512$.

Simulation 3: α = 1 and ϕ = 1.5

$q_i^{(i)}(g_k)$ = $q_j^{(i)}(g_k)$ = $q_k^{(i)}(g_k)$ = $q_l^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_k^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_i^{(k)}(g_k)$ = $q_j^{(k)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_i^{(l)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_k^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = 0.1143; $CS_i(g_k)$ = $CS_j(g_k)$ = $CS_k(g_k)$ = $CS_l(g_k)$ = $\frac{1}{2}(0.1143+0.1143+0.1143+0.1143)^2$ = 0.1045; $\pi_i^{(i)}(g_k)$ = $\pi_j^{(i)}(g_k)$ = $\pi_k^{(i)}(g_k)$ = $\pi_l^{(i)}(g_k)$ = $\pi_i^{(j)}(g_k)$ = $\pi_j^{(j)}(g_k)$ = $\pi_k^{(j)}(g_k)$ = $\pi_l^{(j)}(g_k)$ = $\pi_i^{(k)}(g_k)$ = $\pi_j^{(k)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = $\pi_l^{(k)}(g_k)$ = $\pi_i^{(l)}(g_k)$ = $\pi_j^{(l)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = $\pi_l^{(l)}(g_k)$ = $\frac{3.5}{2}(0.1143)^2$ = 0.0229; and $PS_i(g_k)$ = $PS_j(g_k)$ = $PS_k(g_k)$ = $PS_l(g_k)$ = $\frac{1.5}{4}(0.1143+0.1143+0.1143+0.1143)^2$ = 0.0784.

APPENDIX B

Simulations for the case of symmetrical countries under endogenous tariffs

Network a

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_a) = \frac{2\alpha(\phi+1) + 6T_i(g) + \phi\begin{pmatrix} q_j^{(j)}(g_a) + q_k^{(j)}(g_a) + q_l^{(j)}(g_a) + q_j^{(k)}(g_a) + q_k^{(k)}(g_a) \\ q_l^{(k)}(g_a) + q_j^{(l)}(g_a) + q_k^{(l)}(g_a) + q_l^{(l)}(g_a) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_a) + q_k^{(i)}(g_a) + q_l^{(i)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_a) = \frac{2\alpha(\phi+1) - 2(2+\phi)T_j(g) + \phi\begin{pmatrix} q_i^{(j)}(g_a) + q_k^{(j)}(g_a) + q_l^{(j)}(g_a) + q_i^{(k)}(g_a) + q_k^{(k)}(g_a) \\ q_l^{(k)}(g_a) + q_i^{(l)}(g_a) + q_k^{(l)}(g_a) + q_l^{(l)}(g_a) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_a) + q_k^{(i)}(g_a) + q_l^{(i)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_a) = \frac{2\alpha(\phi+1) - 2(2+\phi)T_k(g) + \phi\begin{pmatrix} q_i^{(j)}(g_a) + q_j^{(j)}(g_a) + q_l^{(j)}(g_a) + q_i^{(k)}(g_a) + q_j^{(k)}(g_a) \\ q_l^{(k)}(g_a) + q_i^{(l)}(g_a) + q_j^{(l)}(g_a) + q_l^{(l)}(g_a) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_a) + q_j^{(i)}(g_a) + q_l^{(i)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_a) = \frac{2\alpha(\phi+1) - 2(2+\phi)T_l(g) + \phi\begin{pmatrix} q_i^{(j)}(g_a) + q_j^{(j)}(g_a) + q_k^{(j)}(g_a) + q_i^{(k)}(g_a) + q_j^{(k)}(g_a) \\ q_k^{(k)}(g_a) + q_i^{(l)}(g_a) + q_j^{(l)}(g_a) + q_k^{(l)}(g_a) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_a) + q_j^{(i)}(g_a) + q_k^{(i)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_i(g)+\phi\begin{pmatrix}q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_j^{(k)}(g_a)+q_k^{(k)}(g_a)\\ q_l^{(k)}(g_a)+q_j^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_a)+q_k^{(j)}(g_a)+q_l^{(j)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_a)=\frac{2\alpha(\phi+1)+6T_j(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(k)}(g_a)+q_k^{(k)}(g_a)\\ q_l^{(k)}(g_a)+q_i^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_a)+q_k^{(j)}(g_a)+q_l^{(j)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_k(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(k)}(g_a)+q_j^{(k)}(g_a)\\ q_l^{(k)}(g_a)+q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_a)+q_j^{(j)}(g_a)+q_l^{(j)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_i^{(k)}(g_a)+q_j^{(k)}(g_a)\\ q_k^{(k)}(g_a)+q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_k^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_a)+q_j^{(j)}(g_a)+q_k^{(j)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_i(g)+\phi\begin{pmatrix}q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_j^{(j)}(g_a)+q_k^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_j^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_a)+q_k^{(k)}(g_a)+q_l^{(k)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_j(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(j)}(g_a)+q_k^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_i^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_a)+q_k^{(k)}(g_a)+q_l^{(k)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_a)=\frac{2\alpha(\phi+1)+6T_k(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(j)}(g_a)+q_j^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_l^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_a)+q_j^{(k)}(g_a)+q_l^{(k)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_i^{(j)}(g_a)+q_j^{(j)}(g_a)\\ q_k^{(j)}(g_a)+q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_k^{(l)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_a)+q_j^{(k)}(g_a)+q_k^{(k)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_i(g)+\phi\begin{pmatrix}q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_j^{(j)}(g_a)+q_k^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_j^{(k)}(g_a)+q_k^{(k)}(g_a)+q_l^{(k)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_i(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(j)}(g_a)+q_k^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_i^{(k)}(g_a)+q_k^{(k)}(g_a)+q_l^{(k)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_a)+q_k^{(l)}(g_a)+q_l^{(l)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_a)=\frac{2\alpha(\phi+1)-2(2+\phi)T_i(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_l^{(i)}(g_a)+q_i^{(j)}(g_a)+q_j^{(j)}(g_a)\\ q_l^{(j)}(g_a)+q_i^{(k)}(g_a)+q_j^{(k)}(g_a)+q_l^{(k)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_l^{(l)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_a)=\frac{2\alpha(\phi+1)+6T_k(g)+\phi\begin{pmatrix}q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_i^{(j)}(g_a)+q_j^{(j)}(g_a)\\ q_k^{(j)}(g_a)+q_i^{(k)}(g_a)+q_j^{(k)}(g_a)+q_k^{(k)}(g_a)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_a)+q_j^{(l)}(g_a)+q_k^{(l)}(g_a)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: $\alpha = 1$ and $\phi = 0$

From the generic equations presented above it is concluded that $q_i^{(i)}(g_a)=\frac{2+6T_i(g)}{10}$;

$q_j^{(i)}(g_a)=\frac{2-4T_j(g)}{10}$; $q_k^{(i)}(g_a)=\frac{2-4T_k(g)}{10}$; $q_l^{(i)}(g_a)=\frac{2-4T_l(g)}{10}$; $q_i^{(j)}(g_a)=\frac{2-4T_i(g)}{10}$;

$q_j^{(j)}(g_a) = \frac{2+6T_j(g)}{10}$; $q_k^{(j)}(g_a) = \frac{2-4T_k(g)}{10}$; $q_l^{(j)}(g_a) = \frac{2-4T_l(g)}{10}$; $q_i^{(k)}(g_a) = \frac{2-4T_i(g)}{10}$;

$q_j^{(k)}(g_a) = \frac{2-4T_j(g)}{10}$; $q_k^{(k)}(g_a) = \frac{2+6T_k(g)}{10}$; $q_l^{(k)}(g_a) = \frac{2-4T_l(g)}{10}$; $q_i^{(l)}(g_a) = \frac{2-4T_i(g)}{10}$;

$q_j^{(l)}(g_a) = \frac{2-4T_i(g)}{10}$; $q_k^{(l)}(g_a) = \frac{2-4T_i(g)}{10}$; and $q_l^{(l)}(g_a) = \frac{2+6T_k(g)}{10}$. Therefore, $CS_i(g_a) = \frac{1}{2}(q_i^{(i)}(g_a) + q_i^{(j)}(g_a) + q_i^{(k)}(g_a) + q_i^{(l)}(g_a))^2 = 0.5(0.8000 - 0.6000T_i(g_a))^2$; $\pi_i^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g)\right)^2 = \left(0.2000+0.6000T_i(g)\right)^2$; $\pi_j^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g)\right)^2 = \left(0.2000-0.4000T_j(g)\right)^2$; $\pi_k^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g)\right)^2 = \left(0.2000-0.4000T_k(g)\right)^2$; $\pi_l^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g)\right)^2 = \left(0.2000-0.4000T_l(g)\right)^2$; $\pi_i(g_a) = \pi_i^{(i)}(g_a) + \pi_j^{(i)}(g_a) + \pi_k^{(i)}(g_a) + \pi_l^{(i)}(g_a) = \left(0.2000+0.6000T_i(g)\right)^2 + \left(0.2000-0.4000T_j(g)\right)^2 + \left(0.2000-0.4000T_k(g)\right)^2 + \left(0.2000-0.4000T_l(g)\right)^2$; $PS_i(g_a) = \frac{\phi}{4}\left(q_i^{(i)}(g_a)+q_j^{(i)}(g_a)+q_k^{(i)}(g_a)+q_l^{(i)}(g_a)\right)=0$; $TR_i(g_a) = T_i(g_a)\left(q_i^{(j)}(g_a)+q_i^{(k)}(g_a)q_i^{(l)}(g_a)\right) = 0.6000T_i(g_a) - 1.2000T_i^2(g_a)$. Thus, the welfare function in this case is given by: $W_i(g_a) = CS_i(g_a) + \pi_i(g_a) + PS_i(g_a) + TR_i(g_a) = 0.5\left(0.8000-0.6000T_i(g_a)\right)^2 + \left(0.2000+0.6000T_i(g)\right)^2 + \left(0.2000-0.4000T_j(g)\right)^2 + \left(0.2000-0.4000T_k(g)\right)^2 + \left(0.2000-0.4000T_l(g)\right)^2 + 0.6000T_i(g_a) - 1.2000T_i^2(g_a)$. The first and second order conditions of the welfare function are $\frac{\partial W_i(g_a)}{\partial T_i(g_a)} = (-0.6000)\left(0.8000-0.6000T_i(g_a)\right) + (2)(0.6000)\left(0.2000+0.6000T_i(g)\right) + 0.6000 - 2.4000T_i^2(g_a) = 0.3600 - 1.3200T_i(g_a)$;

and $\frac{\partial^2 W_i(g_a)}{\partial T_i^2(g_a)} = -1.32$. Given symmetry and by considering these optimal conditions, the tariffs that maximise social welfare in countries *i*, *j*, *k*, and *l* are: $T_i^*(g_a) = T_j^*(g_a) = T_k^*(g_a) = T_l^*(g_a) = 0.2727$. Therefore, $CS_i(g_a) =$ $CS_j(g_a)$ = $CS_k(g_a)$ = $CS_l(g_a) = 0.2025$; $\pi_i(g_a) = \pi_j(g_a) = \pi_k(g_a) = \pi_l(g_a) = 0.1570$; $PS_i(g_a)$ = $PS_j(g_a)$ = $PS_k(g_a)$ = $PS_l(g_a) = 0$; $TR_i(g_a) = TR_j(g_a) = TR_k(g_a) = TR_l(g_a) = 0.0744$; $W_i(g_a)$ = $W_j(g_a) = W_k(g_a) = W_l(g_a) = 0.4339$.

Simulation 5: α = 1 and ϕ = 0.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_a) = 0.1600 + 0.3962T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)$$

$$q_j^{(i)}(g_a) = 0.1600 - 0.0610T_i(g_a) - 0.3149T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)$$

$$q_k^{(i)}(g_a) = 0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) - 0.3149T_k(g_a) + 0.0279T_l(g_a)$$

$$q_l^{(i)}(g_a) = 0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) - 0.3149T_l(g_a)$$

$$q_i^{(j)}(g_a) = 0.1600 - 0.3149T_i(g_a) - 0.0610T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)$$

$$q_j^{(j)}(g_a) = 0.1600 + 0.0279T_i(g_a) + 0.3962T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)$$

$$q_k^{(j)}(g_a) = 0.1600 + 0.0279T_i(g_a) - 0.0610T_j(g_a) - 0.3149T_k(g_a) + 0.0279T_l(g_a)$$

$$q_l^{(j)}(g_a) = 0.1600 + 0.0279T_i(g_a) - 0.0610T_j(g_a) + 0.0279T_k(g_a) - 0.3149T_l(g_a)$$

$$q_i^{(k)}(g_a) = 0.1600 - 0.3149T_i(g_a) + 0.0279T_j(g_a) - 0.0610T_k(g_a) + 0.0279T_l(g_a)$$

$$q_j^{(k)}(g_a) = 0.1600 + 0.0279T_i(g_a) - 0.3149T_j(g_a) - 0.0610T_k(g_a) + 0.0279T_l(g_a)$$

$$q_k^{(k)}(g_a) = 0.1600 + 0.0279T_i(g_a) + 0.0279T_j(g_a) + 0.3962T_k(g_a) + 0.0279T_l(g_a)$$

$$q_l^{(k)}(g_a) = 0.1600 + 0.0279T_i(g_a) + 0.0279T_j(g_a) - 0.0610T_k(g_a) - 0.3149T_l(g_a)$$

$$q_i^{(l)}(g_a) = 0.1600 - 0.3149T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) - 0.0610T_l(g_a)$$

$$q_j^{(l)}(g_a) = 0.1600 + 0.0279T_i(g_a) - 0.3149T_j(g_a) + 0.0279T_k(g_a) - 0.0610T_l(g_a)$$

$$q_k^{(l)}(g_a) = 0.1600 + 0.0279T_i(g_a) + 0.0279T_j(g_a) - 0.3149T_k(g_a) - 0.0610T_l(g_a)$$

$$q_l^{(l)}(g_a) = 0.1600 + 0.0279T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) + 0.3962T_l(g_a).$$

Solving by substitution, the following expressions are obtained: $CS_i(g_a)$ = $\frac{1}{2}\left(q_i^{(i)}(g_a) + q_i^{(j)}(g_a) + q_i^{(k)}(g_a) + q_i^{(l)}(g_a)\right)^2$ = 0.5(0.6400 – 0.5486$T_i(g_a)$ + 0.0229$T_j(g_a)$ + 0.0229$T_k(g_a)$ + 0.0229$T_l(g_a)$)^2; $\pi_i^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g)\right)^2$ = 1.25(0.1600 + $0.3962T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)$)^2; $\pi_j^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g)\right)^2$ = $1.25\left(0.1600 - 0.0610T_i(g_a) - 0.3149T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)\right)^2$; $\pi_k^{(i)}(g_a)$ = $\frac{(\phi+2)}{2}\left(q_k^{(i)}(g)\right)^2$ = 1.25(0.1600 – 0.0610$T_i(g_a)$ + 0.0279$T_j(g_a)$ – 0.3149$T_k(g_a)$ + 0.0279$T_l(g_a)$)^2; $\pi_l^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g)\right)^2$ = 1.25(0.1600 – 0.0610$T_i(g_a)$ + 0.0279$T_j(g_a)$ + 0.0279$T_k(g_a)$ – 0.3149$T_l(g_a)$)^2; $PS_i(g_a) = \frac{\phi}{4}\left(q_i^{(i)}(g_a) + q_j^{(i)}(g_a) + q_k^{(i)}(g_a) + q_l^{(i)}(g_a)\right)^2$ = $0.125\left(0.6400 + 0.2133T_i(g_a) - 0.2311T_j(g_a) - 0.2311T_k(g_a) - 0.2311T_l(g_a)\right)^2$; $TR_i(g_a)$ = $T_i(g_a)\left(q_i^{(j)}(g_a) + q_i^{(k)}(g_a) + q_i^{(l)}(g_a)\right)$ = $0.4800T_i(g_a) - 0.9488T_i^2(g_a) - 0.0051T_j(g_a)T_i(g_a)$

$-\ 0.0051T_k(g_a)T_i(g_a)\ -\ 0.0051T_l(g_a)T_i(g_a)$. Thus, the welfare function in this case is given by:

$$
\begin{aligned}
&W_i(g_a) = CS_i(g_a) + \pi_i(g_a) + PS_i(g_a) + TR_i(g_a) = \\
&0.5\left(0.6400 - 0.5486T_i(g_a) + 0.0229T_j(g_a) + 0.0229T_k(g_a) + 0.0229T_l(g_a)\right)^2 \\
&+1.25\left(0.1600 + 0.3962T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)\right)^2 \\
&+1.25\left(0.1600 - 0.0610T_i(g_a) - 0.3149T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)\right)^2 \\
&+1.25\left(0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) - 0.3149T_k(g_a) + 0.0279T_l(g_a)\right)^2 \\
&+1.25\left(0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) - 0.3149T_l(g_a)\right)^2 \\
&+0.125\left(0.6400 + 0.2133T_i(g_a) - 0.2311T_j(g_a) - 0.2311T_k(g_a) - 0.2311T_l(g_a)\right)^2 \\
&+0.4800T_i(g_a) - 0.9488T_i^2(g_a) - 0.0051T_j(g_a)T_i(g_a) \\
&-0.0051T_k(g_a)T_i(g_a) - 0.0051T_l(g_a)T_i(g_a)
\end{aligned}
$$

The first and second order conditions of the welfare function are:

$$
\begin{aligned}
&\frac{\partial W_i(g_a)}{\partial T_i(g_a)} = (-0.5486)\left(0.6400 - 0.5486T_i(g_a) + 0.0229T_j(g_a) + 0.0229T_k(g_a) + 0.0229T_l(g_a)\right) \\
&+(2.5)(0.3962)\left(0.1600 + 0.3962T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)\right) \\
&+(2.5)(-0.0610)\left(0.1600 - 0.0610T_i(g_a) - 0.3149T_j(g_a) + 0.0279T_k(g_a) + 0.0279T_l(g_a)\right) \\
&+(2.5)(-0.0610)\left(0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) - 0.3149T_k(g_a) + 0.0279T_l(g_a)\right) \\
&+(2.5)(-0.0610)\left(0.1600 - 0.0610T_i(g_a) + 0.0279T_j(g_a) + 0.0279T_k(g_a) - 0.3149T_l(g_a)\right) \\
&+(0.25)(0.2133)\left(0.6400 + 0.2133T_i(g_a) - 0.2311T_j(g_a) - 0.2311T_k(g_a) - 0.2311T_l(g_a)\right) \\
&+0.4800 - 1.8976T_i(g_a) - 0.0051T_j(g_a) - 0.0051T_k(g_a) - 0.0051T_l(g_a) \\
&= 0.2483 - 1.1569T_i(g_a) + 0.0372T_j(g_a) + 0.0372T_k(g_a) + 0.0372T_l(g_a);
\end{aligned}
$$

$$
\frac{\partial^2 W_i(g_a)}{\partial T_i^2(g_a)} = -1.1569
$$

Therefore the optimal tariff in country i is $T_i^*(g_a) =$ 0.2146 + $0.0322T_j(g_a)$ + $0.0322T_k(g_a)$ + $0.0322T_l(g_a)$. Given symmetry it is concluded that the optimal tariff that maximises welfare in countries i, j, k, and l are $T_i^*(g_a)$ = $T_j^*(g_a)$ = $T_k^*(g_a)$ = $T_l^*(g_a)$ = 0.2376. Therefore, $CS_i(g_a)$ = $CS_j(g_a)$ = $CS_k(g_a)$ = $CS_l(g_a)$ = 0.1383; $\pi_i(g_a)$ = $\pi_j(g_a)$ = $\pi_k(g_a)$ = $\pi_l(g_a)$ = 0.1203; $PS_i(g_a)$ = $PS_j(g_a)$ = $PS_k(g_a)$ = $PS_l(g_a)$ = 0.0346; $TR_i(g_a)$ = $TR_j(g_a)$ = $TR_k(g_a)$ = $TR_l(g_a)$ = 0.0599; $W_i(g_a) = W_j(g_a) = W_k(g_a) = W_l(g_a) = 0.3531$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_a) = 1143 + 0.2416T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)$$

$$q_j^{(i)}(g_a) = 1143 - 0.0565T_i(g_a) - 0.2396T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)$$

$$q_k^{(i)}(g_a) = 1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) - 0.2396T_k(g_a) + 0.0337T_l(g_a)$$

$$q_l^{(i)}(g_a) = 1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) - 0.2396T_l(g_a)$$

$$q_i^{(j)}(g_a) = 1143 - 0.2396T_i(g_a) - 0.0565T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)$$

$$q_j^{(j)}(g_a) = 1143 + 0.0337T_i(g_a) + 0.2416T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)$$

$$q_k^{(j)}(g_a) = 1143 + 0.0337T_i(g_a) - 0.0565T_j(g_a) - 0.2396T_k(g_a) + 0.0337T_l(g_a)$$

$$q_l^{(j)}(g_a) = 1143 + 0.0337T_i(g_a) - 0.0565T_j(g_a) + 0.0337T_k(g_a) - 0.2396T_l(g_a)$$

$$q_i^{(k)}(g_a) = 1143 - 0.2396T_i(g_a) + 0.0337T_j(g_a) - 0.0565T_k(g_a) + 0.0337T_l(g_a)$$

$$q_j^{(k)}(g_a) = 1143 + 0.0337T_i(g_a) - 0.2396T_j(g_a) - 0.0565T_k(g_a) + 0.0337T_l(g_a)$$

$$q_k^{(k)}(g_a) = 1143 + 0.0337T_i(g_a) + 0.0337T_j(g_a) + 0.2416T_k(g_a) + 0.0337T_l(g_a)$$

$$q_l^{(k)}(g_a) = 1143 + 0.0337T_i(g_a) + 0.0337T_j(g_a) - 0.0565T_k(g_a) - 0.2396T_l(g_a)$$

$$q_i^{(l)}(g_a) = 1143 - 0.2396T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) - 0.0565T_l(g_a)$$

$$q_j^{(l)}(g_a) = 1143 + 0.0337T_i(g_a) - 0.2396T_j(g_a) + 0.0337T_k(g_a) - 0.0565T_l(g_a)$$

$$q_k^{(l)}(g_a) = 1143 + 0.0337T_i(g_a) + 0.0337T_j(g_a) - 0.2396T_k(g_a) - 0.0565T_l(g_a)$$

$$q_l^{(l)}(g_a) = 1143 + 0.0337T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) + 0.2416T_l(g_a)$$

Solving by substitution, the following expressions are obtained: $CS_i(g_a) = \frac{1}{2}(q_i^{(i)}(g_a) + q_i^{(j)}(g_a) + q_i^{(k)}(g_a) + q_i^{(l)}(g_a))^2 = 0.5(0.4571 - 0.4770T_i(g_a) + 0.0447T_j(g_a) + 0.0447T_k(g_a) + 0.0447T_l(g_a))^2$; $\pi_i^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g)\right)^2 = 1.75(1143 + 0.2416T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a))^2$; $\pi_j^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g)\right)^2 = 1.75(1143 - 0.0565T_i(g_a) - 0.2396T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a))^2$; $\pi_k^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g)\right)^2 = 1.75(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) - 0.2396T_k(g_a) + 0.0337T_l(g_a))^2$; $\pi_l^{(i)}(g_a) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g)\right)^2 = 1.75(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) - 0.2396T_l(g_a))^2$; $PS_i(g_a) = \frac{\phi}{4}(q_i^{(i)}(g_a) + q_j^{(i)}(g_a) + q_k^{(i)}(g_a) + q_l^{(i)}(g_a))^2 = 0.375(0.4571 + 0.0722T_i(g_a) - 0.1383T_j(g_a) - 0.1383T_k(g_a) - 0.1383T_l(g_a))^2$; $TR_i(g_a) = T_i(g_a)(q_i^{(j)}(g_a) + q_i^{(k)}(g_a) + q_i^{(l)}(g_a)) = 0.3429T_i(g_a) -$

$0.7187T_i^2(g_a)\ +\ 0.0110T_j(g_a)T_i(g_a)\ +\ 0.0110T_k(g_a)T_i(g_a)\ +\ 0.0110T_l(g_a)T_i(g_a)$. Thus, the welfare function in this case is given by:

$$\begin{aligned}
&W_i(g_a) = CS_i(g_a) + \pi_i(g_a) + PS_i(g_a) + TR_i(g_a) \\
&= 0.5\big(0.4571 - 0.4770T_i(g_a) + 0.0447T_j(g_a) + 0.0447T_k(g_a) + 0.0447T_l(g_a)\big)^2 \\
&+ 1.75\big(1143 + 0.2416T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)\big)^2 \\
&+ 1.75\big(1143 - 0.0565T_i(g_a) - 0.2396T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)\big)^2 \\
&+ 1.75\big(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) - 0.2396T_k(g_a) + 0.0337T_l(g_a)\big)^2 \\
&+ 1.75\big(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) - 0.2396T_l(g_a)\big)^2 \\
&+ 0.375\big(0.4571 + 0.0722T_i(g_a) - 0.1383T_j(g_a) - 0.1383T_k(g_a) - 0.1383T_l(g_a)\big)^2 \\
&+ 0.3429T_i(g_a) - 0.7187T_i^2(g_a) + 0.0110T_j(g_a)T_i(g_a) \\
&+ 0.0110T_k(g_a)T_i(g_a) + 0.0110T_l(g_a)T_i(g_a)
\end{aligned}$$

The first and second order conditions of the welfare function are:

$$\begin{aligned}
&\frac{\partial W_i(g_a)}{\partial T_i(g_a)} = \\
&(-0.4770)\big(0.4571 - 0.4770T_i(g_a) + 0.0447T_j(g_a) + 0.0447T_k(g_a) + 0.0447T_l(g_a)\big) \\
&+ (3.5)(0.2416)\big(1143 + 0.2416T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)\big) \\
&+ (3.5)(-0.0565)\big(1143 - 0.0565T_i(g_a) - 0.2396T_j(g_a) + 0.0337T_k(g_a) + 0.0337T_l(g_a)\big) \\
&+ (3.5)(-0.0565)\big(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) - 0.2396T_k(g_a) + 0.0337T_l(g_a)\big) \\
&+ (3.5)(-0.0565)\big(1143 - 0.0565T_i(g_a) + 0.0337T_j(g_a) + 0.0337T_k(g_a) - 0.2396T_l(g_a)\big) \\
&+ (0.75)(0.0722)\big(0.4571 + 0.0722T_i(g_a) - 0.1383T_j(g_a) - 0.1383T_k(g_a) - 0.1383T_l(g_a)\big) \\
&+ 0.3429 - 1.4374T_i(g_a) + 0.0110T_j(g_a) + 0.0110T_k(g_a) + 0.0110T_l(g_a) \\
&= 0.1722 - 1.0498T_i(g_a) + 0.0528T_j(g_a) + 0.0528T_k(g_a) + 0.0528T_l(g_a)
\end{aligned}$$

$$\frac{\partial^2 W_i(g_a)}{\partial T_i^2(g_a)} = -1.0498$$

Therefore the optimal tariff that maximises the welfare function in country *i* is given by $T_i^*(g_a)$ = 0.1649 + $0.0503T_j(g_a)$ + $0.0503T_k(g_a)$ + $0.0503T_l(g_a)$. Given symmetry it holds that $T_i^*(g_a)$ = $T_j^*(g_a)$ = $T_k^*(g_a)$ = $T_l^*(g_a)$ = 0.1932. Using these tariffs it is concluded that: $CS_i(g_a)$ = $CS_j(g_a)$ = $CS_k(g_a)$ = $CS_l(g_a)$ = 0.0764; $\pi_i(g_a)=\pi_j(g_a)=\pi_k(g_a)=\pi_l(g_a)=0.0828$; $PS_i(g_a)$ = $PS_j(g_a)$ = $PS_k(g_a)$ = $PS_l(g_a)$ = 0.0573; $TR_i(g_a)$ = $TR_j(g_a)$ = $TR_k(g_a)$ = $TR_l(g_a)$ = 0.0406; $W_i(g_a)$ = $W_j(g_a)$ = $W_k(g_a)$ = $W_l(g_a)$ = 0.2572.

Network b

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_i(g)+\phi\left(q_j^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\right)\\+\phi\left(q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)\\-\phi(5+\phi)\left(q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(2+\phi)T_j(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_k^{(k)}(g_b)\\+q_l^{(k)}(g_b)+q_i^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_k(g)+\phi\left(q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_l^{(j)}(g_b)+q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_l^{(l)}(g_b)\right)\\+\phi\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_l^{(k)}(g_b)\right)\\-\phi(5+\phi)\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_l^{(i)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_b)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_i^{(j)}(g_b)+q_j^{(j)}(g_b)\\+q_k^{(j)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)\end{pmatrix}+\phi\left(q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_b)=\frac{2\alpha(\phi+1)-2(3+\phi)T_i(g)+\phi\begin{pmatrix}q_j^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\\+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}+\phi\left(q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)-\phi(5+\phi)\left(q_j^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_b)=\frac{2\alpha(\phi+1)+6T_j(g_b)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_k^{(k)}(g_b)\\+q_l^{(k)}(g_b)+q_i^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}+\phi\left(q_i^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_b)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_b)+\phi\begin{pmatrix}q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_l^{(j)}(g_b)\\+q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}+\phi\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_l^{(k)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_l^{(j)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_b)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_i^{(j)}(g_b)+q_j^{(j)}(g_b)\\+q_k^{(j)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)\end{pmatrix}+\phi\left(q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_k^{(j)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_b)=\frac{2\alpha(\phi+1)+4T_i(g)+\phi\left(q_j^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\right)+\phi\left(q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)-\phi(5+\phi)\left(q_j^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(2+\phi)T_j(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_k^{(k)}(g_b)\\+q_l^{(k)}(g_b)+q_i^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_k(g)+\phi\begin{pmatrix}q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_l^{(j)}(g_b)\\+q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(i)}(g)+q_j^{(i)}(g)+q_l^{(i)}(g)+q_i^{(k)}(g)+q_j^{(k)}(g)+q_l^{(k)}(g)\right)\\-\phi(5+\phi)\left(q_i^{(k)}(g)+q_j^{(k)}(g)+q_l^{(k)}(g)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(2+\phi)T_l(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_i^{(j)}(g_b)+q_j^{(j)}(g_b)\\+q_k^{(j)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_i(g)+\phi\begin{pmatrix}q_j^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\\+q_j^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_j^{(k)}(g_b)+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)\right)\\-\phi(5+\phi)\left(q_j^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(2+\phi)T_j(g)+\phi\begin{pmatrix}q_i^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)\\+q_k^{(k)}(g_b)+q_l^{(k)}(g_b)+q_i^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(j)}(g_b)+q_k^{(j)}(g_b)+q_l^{(j)}(g_b)\right)-\phi(5+\phi)\left(q_i^{(l)}(g_b)+q_k^{(l)}(g_b)+q_l^{(l)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_b)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_k(g_b)+\phi\begin{pmatrix}q_i^{(j)}(g_b)+q_j^{(j)}(g_b)+q_l^{(j)}(g_b)\\+q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_l^{(l)}(g_b)\end{pmatrix}\\+\phi\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_l^{(i)}(g_b)+q_i^{(k)}(g_b)+q_j^{(k)}(g_b)+q_l^{(k)}(g_b)\right)\\-\phi(5+\phi)\left(q_i^{(l)}(g_b)+q_j^{(l)}(g_b)+q_l^{(l)}(g_b)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_b) = \frac{2\alpha(\phi+1) + 6T_l(g) + \phi\begin{pmatrix} q_i^{(i)}(g_b) + q_j^{(i)}(g_b) + q_k^{(i)}(g_b) + q_i^{(j)}(g_b) \\ + q_j^{(j)}(g_b) + q_k^{(j)}(g_b) + q_i^{(k)}(g_b) + q_j^{(k)}(g_b) + q_k^{(k)}(g_b) \end{pmatrix} + \phi\left(q_i^{(l)}(g_b) + q_j^{(l)}(g_b) + q_k^{(l)}(g_b)\right) - \phi(5+\phi)\left(q_i^{(l)}(g_b) + q_j^{(l)}(g_b) + q_k^{(l)}(g_b)\right)}{2(\phi+1)(5+\phi)}$$

<u>*Simulation 4: α = 1 and ϕ = 0*</u>

From the generic equations presented above it is concluded that $q_i^{(i)}(g_b) = \frac{2+4T_i(g)}{10} = \frac{1+2T_i(g)}{5}$; $q_j^{(i)}(g_b) = \frac{2-4T_j(g)}{10} = \frac{1-2T_j(g)}{5}$; $q_k^{(i)}(g_b) = \frac{2+4T_k(g)}{10} = \frac{1+2T_k(g)}{5}$; $q_l^{(i)}(g_b) = \frac{2-4T_l(g)}{10} = \frac{1-2T_l(g)}{5}$; $q_i^{(j)}(g_b) = \frac{2-6T_i(g)}{10} = \frac{1-3T_i(g)}{5}$; $q_j^{(j)}(g_b) = \frac{2+6T_j(g_b)}{10} = \frac{1+3T_j(g_b)}{5}$; $q_k^{(j)}(g_b) = \frac{2-6T_k(g_b)}{10} = \frac{1-3T_k(g_b)}{5}$; $q_l^{(j)}(g_b) = \frac{2-4T_l(g)}{10} = \frac{1-2T_l(g)}{5}$; $q_i^{(k)}(g_b) = \frac{2+4T_i(g)}{10} = \frac{1+2T_i(g)}{5}$; $q_j^{(k)}(g_b) = \frac{2-4T_j(g)}{10} = \frac{1-2T_j(g)}{5}$; $q_k^{(k)}(g_b) = \frac{2+4T_k(g)}{10} = \frac{1+2T_k(g)}{5}$; $q_l^{(k)}(g_b) = \frac{2-4T_l(g)}{10} = \frac{1-2T_l(g)}{5}$; $q_i^{(l)}(g_b) = \frac{2-6T_i(g)}{10} = \frac{1-3T_i(g)}{5}$; $q_j^{(l)}(g_b) = \frac{2-4T_j(g)}{10} = \frac{1-2T_j(g)}{5}$; $q_k^{(l)}(g_b) = \frac{2-6T_k(g_b)}{10} = \frac{1-3T_k(g_b)}{5}$; and $q_l^{(l)}(g_b) = \frac{2+6T_l(g)}{10} = \frac{1+3T_l(g)}{5}$.

Using these outputs, the following results are obtained for country *i*:

$CS_i(g_b) = \frac{1}{2}\left(q_i^{(i)}(g_b) + q_i^{(j)}(g_b) + q_i^{(k)}(g_b) + q_i^{(l)}(g_b)\right)^2 = 0.5\left(0.8000 - 0.4000T_i(g_b)\right)^2$; $\pi_i^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_b)\right)^2 = \left(0.2000 + 0.4000T_i(g)\right)^2$; $\pi_j^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_b)\right)^2 = \left(0.2000 - 0.4000T_j(g)\right)^2$; $\pi_k^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_b)\right)^2 = \left(0.2000 + 0.4000T_k(g)\right)^2$; $\pi_l^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_b)\right)^2 = \left(0.2000 - 0.4000T_l(g)\right)^2$; $\pi_i(g_b) = \pi_i^{(i)}(g_b) + \pi_j^{(i)}(g_b) + \pi_k^{(i)}(g_b) + \pi_l^{(i)}(g_b) = \left(0.2000 + 0.4000T_i(g)\right)^2 + \left(0.2000 - 0.4000T_j(g)\right)^2 + \left(0.2000 + 0.4000T_k(g)\right)^2 + \left(0.2000 - 0.4000T_l(g)\right)^2$; $PS_i(g_b) = 0$; $TR_i(g_b) = T_i(g_b)\left(q_i^{(j)}(g_b) + q_i^{(l)}(g_b)\right) = 0.4000T_i(g_b) - 1.2000T_i^2(g_b)$. Therefore the welfare function in country *i* is given by:

$$
\begin{aligned}
W_i(g_b) &= CS_i(g_b) + \pi_i(g_b) + PS_i(g_b) + TR_i(g_b) = 0.5\left(0.8000 - 0.4000T_i(g_b)\right)^2 \\
&+ \left(0.2000 + 0.4000T_i(g)\right)^2 + \left(0.2000 - 0.4000T_j(g)\right)^2 + \left(0.2000 + 0.4000T_k(g)\right)^2 \\
&+ \left(0.2000 - 0.4000T_l(g)\right)^2 + 0.4000T_i(g_b) - 1.2000T_i^2(g_b)
\end{aligned}
$$

The first and second order conditions of the welfare function are:

$$
\frac{\partial W_i(g_b)}{\partial T_i(g_b)} = (-0.4000)\left(0.8000 - 0.4000T_i(g_b)\right) + (2)(0.4000)\left(0.2000 + 0.4000T_i(g)\right) + 0.4000
$$

$$
- \; 2.4000T_i(g_b) = 0.24 - 1.9200T_i(g_b)
$$

$$
\frac{\partial^2 W_i(g_b)}{\partial T_i^2(g_b)} = -1.9200
$$

Therefore the optimal tariff in countries *i* and *k* given symmetry are $T_i^*(g_b) = T_k^*(g_b) = 0.1250$.

On the other hand, the following expressions are obtained for country *j*: $CS_j(g_b)$ = $\frac{1}{2}\left(q_j^{(i)}(g_b)+q_j^{(j)}(g_b)+q_j^{(k)}(g_b)+q_j^{(l)}(g_b)\right)^2 = 0.5\left(0.8000-0.6000T_j(g_b)\right)^2$; $\pi_i^{(j)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_i^{(j)}(g_b)\right)^2 = \left(0.2000-0.6000T_i(g)\right)^2$; $\pi_j^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_b)\right)^2$ = $(0.2000 + 0.6000T_j(g_b))^2$; $\pi_k^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_b)\right)^2 = \left(0.2000-0.6000T_k(g_b)\right)^2$; $\pi_l^{(j)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_l^{(j)}(g_b)\right)^2 = \left(0.2000-0.4000T_l(g)\right)^2$; $\pi_j(g_b)$ = $\pi_i^{(j)}(g_b)$ + $\pi_j^{(j)}(g_b)$ + $\pi_k^{(j)}(g_b)$ + $\pi_l^{(j)}(g_b)$ = $\left(0.2000-0.6000T_i(g)\right)^2$ + $\left(0.2000+0.6000T_j(g_b)\right)^2$ + $\left(0.2000-0.6000T_k(g_b)\right)^2$ + $\left(0.2000-0.4000T_l(g)\right)^2$; $PS_j(g_b) = 0$; $TR_j(g_b) = T_j(g_b)\left(q_j^{(i)}(g_b)+q_j^{(k)}(g_b)+q_j^{(l)}(g_b)\right)$ = $0.6000T_j(g_b) - 1.2000T_j^2(g_b)$; Therefore welfare in country *j* is given by:

$$
\begin{aligned}
W_j(g_b) &= CS_j(g_b)+\pi_j(g_b)+PS_j(g_b)+TR_j(g_b) = 0.5\left(0.8000-0.6000T_j(g_b)\right)^2 \\
&+\left(0.2000-0.6000T_i(g)\right)^2+\left(0.2000+0.6000T_j(g_b)\right)^2+\left(0.2000-0.6000T_k(g_b)\right)^2 \\
&+\left(0.2000-0.4000T_l(g)\right)^2+0.6000T_j(g_b)-1.2000T_j^2(g_b)
\end{aligned}
$$

The first and second order conditions of the welfare function are:

$$
\frac{\partial W_j(g_b)}{\partial T_j(g_b)} = (-0.6000)\left(0.8000-0.6000T_j(g_b)\right) + (2)(0.6000)\left(0.2000+0.6000T_j(g_b)\right) + 0.6000 - 2.4000T_j(g_b) = 0.3600-1.3200T_j(g_b)
$$

$$\frac{\partial^2 W_j(g_b)}{\partial T_j^2(g_b)} = -1.3200$$

Given symmetry it is concluded that the optimal tariffs in countries *j* and *l* correspond to $T_j^*(g_b) = T_l^*(g_b) = 0.2727$. Therefore, $CS_i(g_b) = CS_k(g_b) = 0.2813$; $CS_j(g_b) = CS_l(g_b) = 0.2025$; $\pi_i(g_b) = \pi_k(g_b) = 0.1415$; $\pi_j(g_b) = \pi_l(g_b) = 0.1717$; $PS_i(g_b) = PS_j(g_b) = PS_k(g_b) = PS_l(g_b) =$; $PS_i(g_b) = PS_j(g_b) = PS_k(g_b) = PS_l(g_b) = 0$; $TR_i(g_b) = TR_k(g_b) = 0.0313$; $TR_j(g_b) = TR_l(g_b) = 0.0744$; $W_i(g_b) = W_k(g_b) = 0.4540$; and $W_j(g_b) = W_l(g_b) = 0.4486$.

<u>*Simulation 5: $\alpha = 1$ and $\phi = 0.5$*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_b) = 0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)$$

$$q_j^{(i)}(g_b) = 0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)$$

$$q_k^{(i)}(g_b) = 0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)$$

$$q_l^{(i)}(g_b) = 0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)$$

$$q_i^{(j)}(g_b) = 0.1600 - 0.4470T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)$$

$$q_j^{(j)}(g_b) = 0.1600 + 0.0483T_i(g_b) + 0.3962T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)$$

$$q_k^{(j)}(g_b) = 0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) - 0.4470T_k(g_b) + 0.0279T_l(g_b)$$

$$q_l^{(j)}(g_b) = 0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) - 0.3149T_l(g_b)$$

$$q_i^{(k)}(g_b) = 0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)$$

$$q_j^{(k)}(g_b) = 0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)$$

$$q_k^{(k)}(g_b) = 0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)$$

$$q_l^{(k)}(g_b) = 0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)$$

$$q_i^{(l)}(g_b) = 0.1600 - 0.4470T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)$$

$$q_j^{(l)}(g_b) = 0.1600 + 0.0483T_i(g_b) - 0.3149T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)$$

$$q_k^{(l)}(g_b) = 0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) - 0.4470T_k(g_b) - 0.0610T_l(g_b)$$

$$q_l^{(l)}(g_b) = 0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) + 0.3962T_l(g_b)$$

Solving by substitution, the following expressions are obtained for country *i*:

$CS_i(g_b) = \frac{1}{2}\left(q_i^{(i)}(g_b) + q_i^{(j)}(g_b) + q_i^{(k)}(g_b) + q_i^{(l)}(g_b)\right)^2 =$ 0.5(0.6400 – 0.3657$T_i(g_b)$ + 0.0229$T_j(g_b)$ + 0.0152$T_k(g_b)$ + 0.0229$T_l(g_b)$)^2; $\pi_i^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_b)\right)^2$ = $1.25\left(0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right)^2$; $\pi_j^{(i)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_b)\right)^2$ = 1.25(0.1600 – 0.0406$T_i(g_b)$ – 0.3149$T_j(g_b)$ – 0.0406$T_k(g_b)$ + 0.0279$T_l(g_b)$)^2; $\pi_k^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_b)\right)^2$ = 1.25(0.1600 – 0.0406$T_i(g_b)$ + 0.0279$T_j(g_b)$ + 0.2641$T_k(g_b)$ + 0.0279$T_l(g_b)$)^2; $\pi_l^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_b)\right)^2$ = 1.25(0.1600 – 0.0406$T_i(g_b)$ + 0.0279$T_j(g_b)$ – 0.0406$T_k(g_b)$ – 0.3149$T_l(g_b)$)^2; $PS_i(g_b)$

$= \quad \frac{\phi}{4}\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)\right)^2 \quad = \quad 0.125(0.6400 \quad + \quad 0.1422T_i(g_b) \quad -$ $0.2311T_j(g_b) \; + \; 0.1422T_k(g_b) \; - \; 0.2311T_l(g_b))^2$; and $TR_i(g_b) = T_i(g_b)\left(q_i^{(j)}(g_b)+q_i^{(l)}(g_b)\right)$ $= \quad 0.3200T_i(g_b) \quad - \quad 0.8940T_i^2(g_b) \quad - \quad 0.0330T_j(g_b)T_i(g_b) \quad + \quad 0.0965T_k(g_b)T_i(g_b) \quad -$ $0.0330T_l(g_b)T_i(g_b)$. Therefore welfare in country i is given by:

$$
\begin{aligned}
&W_i(g_b) = CS_i(g_b) + \pi_i(g_b) + PS_i(g_b) + TR_i(g_b) \\
&= 0.5\left(0.6400 - 0.3657T_i(g_b) + 0.0229T_j(g_b) + 0.0152T_k(g_b) + 0.0229T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)\right)^2 \\
&+ 0.125\left(0.6400 + 0.1422T_i(g_b) - 0.2311T_j(g_b) + 0.1422T_k(g_b) - 0.2311T_l(g_b)\right)^2 \\
&+ 0.3200T_i(g_b) - 0.8940T_i^2(g_b) - 0.0330T_j(g_b)T_i(g_b) \\
&+ 0.0965T_k(g_b)T_i(g_b) - 0.0330T_l(g_b)T_i(g_b)
\end{aligned}
$$

The first and second order conditions of the welfare function are:

$$
\begin{aligned}
&\frac{\partial W_i(g_b)}{\partial T_i(g_b)} = \\
&= (-0.3657)\left(0.6400 - 0.3657T_i(g_b) + 0.0229T_j(g_b) + 0.0152T_k(g_b) + 0.0229T_l(g_b)\right) \\
&+ (2.5)(0.2641)\left(0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)\right) \\
&+ (0.25)(0.1422)\left(0.6400 + 0.1422T_i(g_b) - 0.2311T_j(g_b) + 0.1422T_k(g_b) - 0.2311T_l(g_b)\right) \\
&+ 0.3200 - 1.7879T_i(g_b) - 0.0330T_j(g_b) + 0.0965T_k(g_b) - 0.0330T_l(g_b) \\
&= 0.1656 - 1.4624T_i(g_b) - 0.0049T_j(g_b) + 0.0506T_k(g_b) - 0.0049T_l(g_b)
\end{aligned}
$$

$$\frac{\partial^2 W_i(g_b)}{\partial T_i^2(g_b)} = -1.4624$$

This implies that the optimal tariff in country i is $T_i^*(g_b)$ = 0.1133 – $0.0033T_j(g_b) + 0.0346T_k(g_b) - 0.0033T_l(g_b)$.

On the other hand, the following information is obtained for country j:

$CS_j(g_b) = \frac{1}{2}\left(q_j^{(i)}(g_b) + q_j^{(j)}(g_b) + q_j^{(k)}(g_b) + q_j^{(l)}(g_b)\right)^2$ = 0.5(0.6400 + $0.0152T_i(g_b)$ – $0.5486T_j(g_b)$ + $0.0152T_k(g_b)$ + $0.0229T_l(g_b)$)²; $\pi_i^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_b)\right)^2$ = $1.25\left(0.1600 - 0.4470T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)\right)^2$; $\pi_j^{(j)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_j^{(j)}(g_b)\right)^2$ = 1.25(0.1600 + $0.0483T_i(g_b)$ + $0.3962T_j(g_b)$ + $0.0483T_k(g_b)$ + $0.0279T_l(g_b)$)²; $\pi_k^{(j)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_k^{(j)}(g_b)\right)^2$ = 1.25(0.1600 + $0.0483T_i(g_b)$ – $0.0610T_j(g_b)$ – $0.4470T_k(g_b)$ + $0.0279T_l(g_b)$)²; $\pi_l^{(j)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_l^{(j)}(g_b)\right)^2$ = $1.25\left(0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) - 0.3149T_l(g_b)\right)^2$; $PS_j(g_b)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_b) + q_j^{(j)}(g_b) + q_k^{(j)}(g_b) + q_l^{(j)}(g_b)\right)^2$ = 0.125(0.6400 – $0.3022T_i(g_b)$ + $0.2133T_j(g_b)$ – $0.3022T_k(g_b)$ – $0.2311T_l(g_b)$)²; and $TR_j(g_b)$ = $T_j(g_b)$ ($q_j^{(i)}(g_b)$ + $q_j^{(k)}(g_b)$ + $q_j^{(l)}(g_b)$)

$= 0.4800T_j(g_b) - 0.0330T_i(g_b)T_j(g_b) - 0.9448T_j^2(g_b) - 0.0330T_k(g_b)T_j(g_b) -$

$0.0051T_l(g_b)T_j(g_b)$. Therefore welfare in country *j* is given by:

$$
\begin{aligned}
&W_j(g_b) = CS_j(g_b) + \pi_j(g_b) + PS_j(g_b) + TR_j(g_b) \\
&= 0.5\left(0.6400 + 0.0152T_i(g_b) - 0.5486T_j(g_b) + 0.0152T_k(g_b) + 0.0229T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.4470T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) + 0.3962T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) - 0.4470T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) - 0.3149T_l(g_b)\right)^2 \\
&+ 0.125\left(0.6400 - 0.3022T_i(g_b) + 0.2133T_j(g_b) - 0.3022T_k(g_b) - 0.2311T_l(g_b)\right)^2 \\
&+ 0.4800T_j(g_b) - 0.0330T_i(g_b)T_j(g_b) - 0.9448T_j^2(g_b) \\
&- 0.0330T_k(g_b)T_j(g_b) - 0.0051T_l(g_b)T_j(g_b)
\end{aligned}
$$

The first and second conditions of this welfare function are:

$$
\begin{aligned}
&\frac{\partial W_j(g_b)}{\partial T_j(g_b)} = \\
&= (-0.5486)\left(0.6400 + 0.0152T_i(g_b) - 0.5486T_j(g_b) + 0.0152T_k(g_b) + 0.0229T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 - 0.4470T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(0.3962)\left(0.1600 + 0.0483T_i(g_b) + 0.3962T_j(g_b) + 0.0483T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) - 0.4470T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 + 0.0483T_i(g_b) - 0.0610T_j(g_b) + 0.0483T_k(g_b) - 0.3149T_l(g_b)\right) \\
&+ (0.25)(0.2133)\left(0.6400 - 0.3022T_i(g_b) + 0.2133T_j(g_b) - 0.3022T_k(g_b) - 0.2311T_l(g_b)\right) \\
&+ 0.4800 - 0.0330T_i(g_b) - 1.8895T_j(g_b) - 0.0330T_k(g_b) - 0.0051T_l(g_b) \\
&= 0.2483 + 0.0438T_i(g_b) - 1.1569T_j(g_b) + 0.0438T_k(g_b) + 0.0372T_l(g_b)
\end{aligned}
$$

$$\frac{\partial^2 W_j(g_b)}{\partial T_j^2(g_b)} = -1.1569$$

This implies that the optimal tariff in country j is $T_j^*(g_b)$ = $0.2146 + 0.0378T_i(g_b) + 0.0378T_k(g_b) + 0.0322T_l(g_b)$.

In relation to country k, the following information is obtained:

$CS_k(g_b) = \frac{1}{2}\left(q_k^{(i)}(g_b) + q_k^{(j)}(g_b) + q_k^{(k)}(g_b) + q_k^{(l)}(g_b)\right)^2$ = 0.5(0.6400 + $0.0152T_i(g_b)$ + $0.0229T_j(g_b)$ – $0.3657T_k(g_b)$ + $0.0229T_l(g_b))^2$; $\pi_i^{(k)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_i^{(k)}(g_b)\right)^2$ = 1.25(0.1600 + $0.2641T_i(g_b)$ + $0.0279T_j(g_b)$ – $0.0406T_k(g_b)$ + $0.0279T_l(g_b))^2$; $\pi_j^{(k)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_j^{(k)}(g_b)\right)^2$ = 1.25(0.1600 – $0.0406T_i(g_b)$ – $0.3149T_j(g_b)$ – $0.0406T_k(g_b)$ + $0.0279T_l(g_b))^2$; $\pi_k^{(k)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(k)}(g_b)\right)^2$ = 1.25(0.1600 – $0.0406T_i(g_b)$ + $0.0279T_j(g_b)$ + $0.2641T_k(g_b)$ + $0.0279T_l(g_b))^2$; $\pi_l^{(k)}(g_b) = \frac{(\phi+2)}{2}\left(q_l^{(k)}(g_b)\right)^2$ = $1.25\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)\right)^2$; $PS_k(g_b)$ = $\frac{\phi}{4}\left(q_i^{(k)}(g_b) + q_j^{(k)}(g_b) + q_k^{(k)}(g_b) + q_l^{(k)}(g_b)\right)^2$ = 0.125(0.6400 + $0.1422T_i(g_b)$ – $0.2311T_j(g_b)$ + $0.1422T_k(g_b)$ – $0.2311T_l(g_b))^2$; $TR_k(g_b) = T_k(g_b)\left(q_k^{(j)}(g_b) + q_k^{(l)}(g_b)\right)$ = $0.3200T_k(g_b)$ + $0.0965T_i(g_b)T_k(g_b)$ – $0.0330T_j(g_b)T_k(g_b)$ – $0.8940T_k^2(g_b)$ – $0.0330T_l(g_b)T_k(g_b)$. Therefore welfare in country k is given by:

$$
\begin{aligned}
W_k(g_b) &= CS_k(g_b) + \pi_k(g_b) + PS_k(g_b) + TR_k(g_b) \\
&= 0.5\left(0.6400 + 0.0152T_i(g_b) + 0.0229T_j(g_b) - 0.3657T_k(g_b) + 0.0229T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)\right)^2 \\
&+ 0.125\left(0.6400 + 0.1422T_i(g_b) - 0.2311T_j(g_b) + 0.1422T_k(g_b) - 0.2311T_l(g_b)\right)^2 \\
&+ 0.3200T_k(g_b) + 0.0965T_i(g_b)T_k(g_b) - 0.0330T_j(g_b)T_k(g_b) \\
&- 0.8940T_k^2(g_b) - 0.0330T_l(g_b)T_k(g_b)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_k(g_b)}{\partial T_k(g_b)} = \\
&= (-0.3657)\left(0.6400 + 0.0152T_i(g_b) + 0.0229T_j(g_b) - 0.3657T_k(g_b) + 0.0229T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 + 0.2641T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_b) - 0.3149T_j(g_b) - 0.0406T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(0.2641)\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) + 0.2641T_k(g_b) + 0.0279T_l(g_b)\right) \\
&+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_b) + 0.0279T_j(g_b) - 0.0406T_k(g_b) - 0.3149T_l(g_b)\right) \\
&+ (0.25)(0.1422)\left(0.6400 + 0.1422T_i(g_b) - 0.2311T_j(g_b) + 0.1422T_k(g_b) - 0.2311T_l(g_b)\right)^2 \\
&+ 0.3200 + 0.0965T_i(g_b) - 0.0330T_j(g_b) - 1.7879T_k(g_b) - 0.0330T_l(g_b) \\
&= 0.1656 + 0.0506T_i(g_b) - 0.0049T_j(g_b) - 1.4624T_k(g_b) - 0.0049T_l(g_b)
\end{aligned}
$$

$$
\frac{\partial^2 W_k(g_b)}{\partial T_k^2(g_b)} = -1.4624
$$

Therefore the optimal tariff in country k is given by $T_k^*(g_b)$ = $0.1133 + 0.0346T_i(g_b) - 0.0033T_j(g_b) - 0.0033T_l(g_b)$.

Finally, the following information is obtained for country l:

$CS_l(g_b) = \frac{1}{2}\left(q_l^{(i)}(g_b) + q_l^{(j)}(g_b) + q_l^{(k)}(g_b) + q_l^{(l)}(g_b)\right)^2$ = 0.5(0.6400 + 0.0152$T_i(g_b)$ + 0.0229$T_j(g_b)$ + 0.0152$T_k(g_b)$ – 0.5486$T_l(g_b)$)^2; $\pi_i^{(l)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_i^{(l)}(g_b)\right)^2$ = 1.25(0.1600 – 0.4470$T_i(g_b)$ + 0.0279$T_j(g_b)$ + 0.0483$T_k(g_b)$ – 0.0610$T_l(g_b)$)^2; $\pi_j^{(l)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_j^{(l)}(g_b)\right)^2$ = 1.25(0.1600 + 0.0483$T_i(g_b)$ – 0.3149$T_j(g_b)$ + 0.0483$T_k(g_b)$ – 0.0610$T_l(g_b)$)^2; $\pi_k^{(l)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_k^{(l)}(g_b)\right)^2$ = 1.25(0.1600 + 0.0483$T_i(g_b)$ + 0.0279$T_j(g_b)$ – 0.4470$T_k(g_b)$ – 0.0610$T_l(g_b)$)^2; $\pi_l^{(l)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_l^{(l)}(g_b)\right)^2$ = 1.25(0.1600 + 0.0483$T_i(g_b)$ + 0.0279$T_j(g_b)$ + 0.0483$T_k(g_b)$ + 0.3962$T_l(g_b)$)^2; $PS_l(g_b)$ = $\frac{\phi}{4}\left(q_i^{(l)}(g_b) + q_j^{(l)}(g_b) + q_k^{(l)}(g_b) + q_l^{(l)}(g_b)\right)^2$ = 0.125(0.6400 – 0.3022$T_i(g_b)$ – 0.2311$T_j(g_b)$ – 0.3022$T_k(g_b)$ + 0.2133$T_l(g_b)$)^2; and $TR_l(g_b)$ = $T_l(g_b)$($q_l^{(i)}(g_b)$ + $q_l^{(j)}(g_b)$ + $q_l^{(k)}(g_b)$) = 0.4800$T_l(g_b)$ – 0.0330$T_i(g_b)T_l(g_b)$ – 0.0051$T_j(g_b)T_l(g_b)$ – 0.0330$T_k(g_b)T_l(g_b)$ – 0.9448$T_l^2(g_b)$;. Therefore welfare in country l is given by:

$$
\begin{aligned}
W_l(g_b) &= CS_l(g_b) + \pi_l(g_b) + PS_l(g_b) + TR_l(g_b) \\
&= 0.5\left(0.6400 + 0.0152T_i(g_b) + 0.0229T_j(g_b) + 0.0152T_k(g_b) - 0.5486T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 - 0.4470T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) - 0.3149T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) - 0.4470T_k(g_b) - 0.0610T_l(g_b)\right)^2 \\
&+ 1.25\left(0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) + 0.3962T_l(g_b)\right)^2 \\
&+ 0.125\left(0.6400 - 0.3022T_i(g_b) - 0.2311T_j(g_b) - 0.3022T_k(g_b) + 0.2133T_l(g_b)\right)^2 \\
&+ 0.4800T_l(g_b) - 0.0330T_i(g_b)T_l(g_b) - 0.0051T_j(g_b)T_l(g_b) \\
&- 0.0330T_k(g_b)T_l(g_b) - 0.9448T_l^2(g_b)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_l(g_b)}{\partial T_l(g_b)} = \\
&= (-0.5486)\left(0.6400 + 0.0152T_i(g_b) + 0.0229T_j(g_b) + 0.0152T_k(g_b) - 0.5486T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 - 0.4470T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 + 0.0483T_i(g_b) - 0.3149T_j(g_b) + 0.0483T_k(g_b) - 0.0610T_l(g_b)\right) \\
&+ (2.5)(-0.0610)\left(0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) - 0.4470T_k(g_b) - 0.0610T_l(g_b)\right) \\
&+ (2.5)(0.3962)\left(0.1600 + 0.0483T_i(g_b) + 0.0279T_j(g_b) + 0.0483T_k(g_b) + 0.3962T_l(g_b)\right) \\
&+ (0.25)(0.2133)\left(0.6400 - 0.3022T_i(g_b) - 0.2311T_j(g_b) - 0.3022T_k(g_b) + 0.2133T_l(g_b)\right)^2 \\
&+ 0.4800 - 0.0330T_i(g_b) - 0.0051T_j(g_b) - 0.0330T_k(g_b) - 1.8895T_l(g_b) \\
&= 0.2483 + 0.0438T_i(g_b) + 0.0372T_j(g_b) + 0.0438T_k(g_b) - 1.1569T_l(g_b)
\end{aligned}
$$

$$
\frac{\partial^2 W_l(g_b)}{\partial T_l^2(g_b)} = -1.1569
$$

Therefore the optimal tariff in country l is $T_l^*(g_b)$ = 0.2146 + $0.0378T_i(g_b)$ + $0.0322T_j(g_b)$ + $0.0378T_k(g_b)$.

In considering the optimal tariff equations for countries *i*, *j*, *k* and *l*, the following equation system is obtained:

$$\begin{pmatrix} 1 & 0.0033 & -0.0346 & 0.0033 \\ -0.0378 & 1 & -0.0378 & -0.0322 \\ -0.0346 & 0.0033 & 1 & 0.0033 \\ -0.0378 & -0.0322 & -0.0378 & 1 \end{pmatrix} \begin{pmatrix} T_i(g_b) \\ T_j(g_b) \\ T_k(g_b) \\ T_l(g_b) \end{pmatrix} = \begin{pmatrix} 0.1133 \\ 0.2146 \\ 0.1133 \\ 0.2146 \end{pmatrix}$$

Optimal tariffs are obtained by solving this system. These tariffs are $T_i^*(g_b)$ = $T_k^*(g_b)$ = 0.1158; and $T_j^*(g_b) = T_l^*(g_b) = 0.2308$. Therefore, $CS_i(g_b) = CS_k(g_b) = 0.1860$; $CS_j(g_b)$ = $CS_l(g_b)$ = 0.1363; $\pi_i(g_b) = \pi_k(g_b) = 0.1166$; $\pi_j(g_b) = \pi_l(g_b) = 0.1276$; $PS_i(g_b)$ = $PS_k(g_b)$ = 0.0401; $PS_j(g_b) = PS_l(g_b) = 0.0400$; $TR_i(g_b) = TR_k(g_b) = 0.0246$; $TR_j(g_b)$ = $TR_l(g_b)$ = 0.0584 ; $W_i(g_b) = W_k(g_b) = 0.3673$; and $W_j(g_b) = W_l(g_b) = 0.3623$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_b) = 0.1143 + 0.1611T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)$$

$$q_j^{(i)}(g_b) = 0.1143 - 0.0377T_i(g_b) - 0.2396T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)$$

$$q_k^{(i)}(g_b) = 0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) + 0.1611T_k(g_b) + 0.0337T_l(g_b)$$

$$q_l^{(i)}(g_b) = 0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) - 0.2396T_l(g_b)$$

$$q_i^{(j)}(g_b) = 0.1143 - 0.3201T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)$$

$$q_j^{(j)}(g_b)=0.1143+0.0526T_i(g_b)+0.2416T_j(g_b)+0.0526T_k(g_b)+0.0337T_l(g_b)$$

$$q_k^{(j)}(g_b)=0.1143+0.0526T_i(g_b)-0.0565T_j(g_b)-0.3201T_k(g_b)+0.0337T_l(g_b)$$

$$q_l^{(j)}(g_b)=0.1143+0.0526T_i(g_b)-0.0565T_j(g_b)+0.0526T_k(g_b)-0.2396T_l(g_b)$$

$$q_i^{(k)}(g_b)=0.1143+0.1611T_i(g_b)+0.0337T_j(g_b)-0.0377T_k(g_b)+0.0337T_l(g_b)$$

$$q_j^{(k)}(g_b)=0.1143-0.0377T_i(g_b)-0.2396T_j(g_b)-0.0377T_k(g_b)+0.0337T_l(g_b)$$

$$q_k^{(k)}(g_b)=0.1143-0.0377T_i(g_b)+0.0337T_j(g_b)+0.1611T_k(g_b)+0.0337T_l(g_b)$$

$$q_l^{(k)}(g_b)=0.1143-0.0377T_i(g_b)+0.0337T_j(g_b)-0.0377T_k(g_b)-0.2396T_l(g_b)$$

$$q_i^{(l)}(g_b)=0.1143-0.3201T_i(g_b)+0.0337T_j(g_b)+0.0526T_k(g_b)-0.0565T_l(g_b)$$

$$q_j^{(l)}(g_b)=0.1143+0.0526T_i(g_b)-0.2396T_j(g_b)+0.0526T_k(g_b)-0.0565T_l(g_b)$$

$$q_k^{(l)}(g_b)=0.1143+0.0526T_i(g_b)+0.0337T_j(g_b)-0.3201T_k(g_b)-0.0565T_l(g_b)$$

$$q_l^{(l)}(g_b)=0.1143+0.0526T_i(g_b)+0.0337T_j(g_b)+0.0526T_k(g_b)+0.2416T_l(g_b)$$

Solving by substitution, the following expressions are obtained for country *i*:

$CS_i(g_b)=\frac{1}{2}\left(q_i^{(i)}(g_b)+q_i^{(j)}(g_b)+q_i^{(k)}(g_b)+q_i^{(l)}(g_b)\right)^2 = 0.5(0.4571 - 0.3180T_i(g_b) + 0.0447T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b))^2$; $\pi_i^{(i)}(g_b)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_b)\right)^2 = 1.75\left(0.1143+0.1611T_i(g_b)+0.0337T_j(g_b)-0.0377T_k(g_b)+0.0337T_l(g_b)\right)^2$; $\pi_j^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_b)\right)^2 = 1.75(0.1143 - 0.0377T_i(g_b) - 0.2396T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b))^2$; $\pi_k^{(i)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_b)\right)^2 = 1.75(0.1143 - 0.0377T_i(g_b) +$

$0.0337T_j(g_b)$ + $0.1611T_k(g_b)$ + $0.0337T_l(g_b))^2$; $\pi_l^{(i)}(g_b)$ = $\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_b)\right)^2$ = $1.75(0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) - 0.2396T_l(g_b))^2$; $PS_i(g_b)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)+q_k^{(i)}(g_b)+q_l^{(i)}(g_b)\right)^2$ = $0.375(0.4571 + 0.0481T_i(g_b) - 0.1383T_j(g_b) + 0.0481T_k(g_b) - 0.1383T_l(g_b))^2$; and $TR_i(g_b)$ = $T_i(g_b)\left(q_i^{(j)}(g_b)+q_i^{(l)}(g_b)\right)$ = $0.2286T_i(g_b) - 0.6402T_i^2(g_b) - 0.0227T_j(g_b)T_i(g_b) + 0.1051T_k(g_b)T_i(g_b) - 0.0227T_l(g_b)T_i(g_b)$. Therefore welfare in country i is given by:

$$
\begin{aligned}
&W_i(g_b) = CS_i(g_b) + \pi_i(g_b) + PS_i(g_b) + TR_i(g_b) \\
&= 0.5\left(0.4571 - 0.3180T_i(g_b) + 0.0447T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 + 0.1611T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_b) - 0.2396T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) + 0.1611T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) - 0.2396T_l(g_b)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0481T_i(g_b) - 0.1383T_j(g_b) + 0.0481T_k(g_b) - 0.1383T_l(g_b)\right)^2 \\
&+ 0.2286T_i(g_b) - 0.6402T_i^2(g_b) - 0.0227T_j(g_b)T_i(g_b) \\
&+ 0.1051T_k(g_b)T_i(g_b) - 0.0227T_l(g_b)T_i(g_b)
\end{aligned}
$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_b)}{\partial T_i(g_b)} =$$
$$= (-0.3180)\left(0.4571 - 0.3180T_i(g_b) + 0.0447T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b)\right)$$
$$+ (3.5)(0.1611)\left(0.1143 + 0.1611T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) - 0.2396T_j(g_b) - 0.0377T_k(g_b) + 0.0337T_l(g_b)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) + 0.1611T_k(g_b) + 0.0337T_l(g_b)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) + 0.0337T_j(g_b) - 0.0377T_k(g_b) - 0.2396T_l(g_b)\right)$$
$$+ (0.75)(0.0481)\left(0.4571 + 0.0481T_i(g_b) - 0.1383T_j(g_b) + 0.0481T_k(g_b) - 0.1383T_l(g_b)\right)$$
$$+ 0.2286 - 1.2804T_i(g_b) - 0.0227T_j(g_b) + 0.1051T_k(g_b) - 0.0227T_l(g_b)$$
$$= 0.1189 - 1.0718T_i(g_b) - 0.0002T_j(g_b) + 0.0648T_k(g_b) - 0.0002T_l(g_b)$$

$$\frac{\partial^2 W_i(g_b)}{\partial T_i^2(g_b)} = -1.0718$$

Therefore the optimal tariff in country i is $T_i^*(g_b)$ = 0.1109 − $0.0002T_j(g_b)$ + $0.0605T_k(g_b)$ − $0.0002T_l(g_b)$.

In relation to country j, the following results hold:

$CS_j(g_b) = \frac{1}{2}\left(q_j^{(i)}(g_b) + q_j^{(j)}(g_b) + q_j^{(k)}(g_b) + q_j^{(l)}(g_b)\right)^2$ = $0.5(0.4571 + 0.0298T_i(g_b) - 0.4771T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b))^2$; $\pi_i^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_b)\right)^2 = 1.75\left(0.1143 - 0.3201T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)\right)^2$; $\pi_j^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_b)\right)^2$ = $1.75(0.1143 + 0.0526T_i(g_b) + 0.2416T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b))^2$; $\pi_k^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_b)\right)^2$ = $1.75(0.1143 + 0.0526T_i(g_b) -$

$0.0565T_j(g_b) - 0.3201T_k(g_b) + 0.0337T_l(g_b))^2$; $\pi_l^{(j)}(g_b) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_b)\right)^2 =$ $1.75(0.1143 + 0.0526T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) - 0.2396T_l(g_b))^2$; $PS_j(g_b)$ $= \frac{\phi}{4}\left(q_i^{(j)}(g_b) + q_j^{(j)}(g_b) + q_k^{(j)}(g_b) + q_l^{(j)}(g_b)\right)^2 = 0.375(0.4571 - 0.1624T_i(g_b) + 0.0722T_j(g_b) - 0.1624T_k(g_b) - 0.1384T_l(g_b))^2$; $TR_j(g_b) = T_j(g_b)(q_j^{(i)}(g_b) + q_j^{(k)}(g_b) + q_j^{(l)}(g_b)) = 0.3429T_j(g_b) - 0.0228T_i(g_b)T_j(g_b) - 0.7187T_j^2(g_b) - 0.0228T_k(g_b)T_j(g_b) + 0.0110T_l(g_b)T_j(g_b)$. Therefore welfare in country *j* is given by:

$$
\begin{aligned}
W_j(g_b) &= CS_j(g_b) + \pi_j(g_b) + PS_j(g_b) + TR_j(g_b) \\
&= 0.5\left(0.4571 + 0.0298T_i(g_b) - 0.4771T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 - 0.3201T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 + 0.0526T_i(g_b) + 0.2416T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 + 0.0526T_i(g_b) - 0.0565T_j(g_b) - 0.3201T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ 1.75\left(0.1143 + 0.0526T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) - 0.2396T_l(g_b)\right)^2 \\
&+ 0.375\left(0.4571 - 0.1624T_i(g_b) + 0.0722T_j(g_b) - 0.1624T_k(g_b) - 0.1384T_l(g_b)\right)^2 \\
&+ 0.3429T_j(g_b) - 0.0228T_i(g_b)T_j(g_b) - 0.7187T_j^2(g_b) \\
&- 0.0228T_k(g_b)T_j(g_b) + 0.0110T_l(g_b)T_j(g_b)
\end{aligned}
$$

The first and second order conditions of the welfare function are:

$$\frac{\partial W_j(g_b)}{\partial T_j(g_b)} =$$

$$\begin{aligned}
&= (-0.4771)\left(0.4571 + 0.0298T_i(g_b) - 0.4771T_j(g_b) + 0.0298T_k(g_b) + 0.0447T_l(g_b)\right) \\
&+ (3.5)(-0.0565)\left(0.1143 - 0.3201T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)\right) \\
&+ (3.5)(0.2416)\left(0.1143 + 0.0526T_i(g_b) + 0.2416T_j(g_b) + 0.0526T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ (3.5)(-0.0565)\left(0.1143 + 0.0526T_i(g_b) - 0.0565T_j(g_b) - 0.3201T_k(g_b) + 0.0337T_l(g_b)\right)^2 \\
&+ (3.5)(-0.0565)\left(0.1143 + 0.0526T_i(g_b) - 0.0565T_j(g_b) + 0.0526T_k(g_b) - 0.2396T_l(g_b)\right)^2 \\
&+ (0.75)(0.0722)\left(0.4571 - 0.1624T_i(g_b) + 0.0722T_j(g_b) - 0.1624T_k(g_b) - 0.1384T_l(g_b)\right)^2 \\
&+ 0.3429 - 0.0228T_i(g_b) - 1.4374T_j(g_b) - 0.0228T_k(g_b) + 0.0110T_l(g_b) \\
&= 0.1783 + 0.0412T_i(g_b) - 0.9681T_j(g_b) + 0.0412T_k(g_b) + 0.0447T_l(g_b)
\end{aligned}$$

$$\frac{\partial^2 W_j(g_b)}{\partial T_j^2(g_b)} = -0.9681$$

Consequently the optimal tariff in country j is $T_j^*(g_b)$ = 0.1842 + $0.0425T_i(g_b)$ + $0.0425T_k(g_b)$ + $0.0462T_l(g_b)$. Using symmetry across countries and the optimal tariff of country i it is concluded that $T_i^*(g_b) = 0.1180 - 0.0004T_j(g_b)$ and $T_j^*(g_b)$ = 0.1931 + $0.0891T_i(g_b)$. Solving by substitution, the following optimal tariff of countries i, j, k and l are obtained: $T_i^*(g_b) = T_k^*(g_b) = 0.1180$; and $T_j^*(g_b) = T_l^*(g_b) = 0.2036$. Therefore, $CS_i(g_b) = CS_k(g_b) = 0.1369$; $CS_j(g_b) = CS_l(g_b) = 0.0707$; $\pi_i(g_b)$ = $\pi_k(g_b)$ = 0.0853; $\pi_j(g_b) = \pi_l(g_b) = 0.0875$; $PS_i(g_b) = PS_k(g_b) = 0.0637$; $PS_j(g_b)$ = $PS_l(g_b)$ = 0.0616; $TR_i(g_b) = TR_k(g_b) = 0.0184$; $TR_j(g_b) = TR_l(g_b) = 0.0394$; $W_i(g_b)$ = $W_k(g_b)$ = 0.3043; and $W_j(g_b) = W_l(g_b) = 0.0394$.

Network c

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_i(g)+\phi\left(q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)+q_j^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_j^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)-\phi(4+\phi)\left(q_j^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_j(g)+\phi\left(q_i^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)+q_i^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_i^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_k(g)+\phi\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_l^{(j)}(g_c)+q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_l^{(k)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_l^{(i)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_l(g)+\phi\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_k^{(l)}(g_c)\right)\\+\phi\left(q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_k^{(k)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_k^{(i)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_i(g)+\phi\left(q_j^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)+q_j^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_j^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)-\phi(4+\phi)\left(q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_j(g)+\phi\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)+q_i^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_i^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_k(g)+\phi\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_l^{(i)}(g_c)+q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_l^{(l)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_l^{(j)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_l(g)+\phi\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_k^{(i)}(g_c)+q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_k^{(k)}(g_c)\right)\\+\phi\left(q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_k^{(l)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_k^{(j)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_i(g)+\phi\left(q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)+q_j^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_j^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)\right)-\phi(4+\phi)\left(q_j^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_j(g)+\phi\left(q_i^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)+q_i^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_k(g)+\phi\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_l^{(j)}(g_c)+q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\\+\phi\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_l^{(i)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_l(g)+\phi\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_k^{(l)}(g_c)\right)\\+\phi\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_k^{(i)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_k^{(k)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_i(g)+\phi\left(q_j^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)+q_j^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_j^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)\right)-\phi(4+\phi)\left(q_j^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)+4T_j(g)+\phi\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)+q_l^{(i)}(g_c)+q_i^{(k)}(g_c)+q_k^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_i^{(j)}(g_c)+q_k^{(j)}(g_c)+q_l^{(j)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(l)}(g_c)+q_k^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_c)=\frac{\begin{array}{l}2\alpha(\phi+1)-2(3+\phi)T_k(g)+\phi\left(q_i^{(i)}(g_c)+q_j^{(i)}(g_c)+q_l^{(i)}(g_c)+q_i^{(k)}(g_c)+q_j^{(k)}(g_c)+q_l^{(k)}(g_c)\right)\\+\phi\left(q_i^{(j)}(g_c)+q_j^{(j)}(g_c)+q_l^{(j)}(g_c)\right)-\phi(4+\phi)\left(q_i^{(l)}(g_c)+q_j^{(l)}(g_c)+q_l^{(l)}(g_c)\right)\end{array}}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_c) = \frac{\begin{array}{c} 2\alpha(\phi+1) + 4T_l(g) + \phi\left(q_i^{(i)}(g_c) + q_j^{(i)}(g_c) + q_k^{(i)}(g_c) + q_i^{(k)}(g_c) + q_j^{(k)}(g_c) + q_k^{(k)}(g_c)\right) \\ + \phi\left(q_i^{(j)}(g_c) + q_j^{(j)}(g_c) + q_k^{(j)}(g_c)\right) - \phi(4+\phi)\left(q_i^{(l)}(g_c) + q_j^{(l)}(g_c) + q_k^{(l)}(g_c)\right) \end{array}}{2(\phi+1)(5+\phi)}$$

<u>*Simulation 4: $\alpha = 1$ and $\phi = 0$*</u>

From these generic equations it is concluded that $q_i^{(i)}(g_c) = \frac{2+4T_i(g)}{10}$; $q_j^{(i)}(g_c) = \frac{2-6T_j(g)}{10}$; $q_k^{(i)}(g_c) = \frac{2+4T_k(g)}{10}$; $q_l^{(i)}(g_c) = \frac{2-6T_l(g)}{10}$; $q_i^{(j)}(g_c) = \frac{2-6T_i(g)}{10}$; $q_j^{(j)}(g_c) = \frac{2+4T_j(g)}{10}$; $q_k^{(j)}(g_c) = \frac{2-6T_k(g)}{10}$; $q_l^{(j)}(g_c) = \frac{2+4T_l(g)}{10}$; $q_i^{(k)}(g_c) = \frac{2+4T_i(g)}{10}$; $q_j^{(k)}(g_c) = \frac{2-6T_j(g)}{10}$; $q_k^{(k)}(g_c) = \frac{2+4T_k(g)}{10}$; $q_l^{(k)}(g_c) = \frac{2-6T_l(g)}{10}$; $q_i^{(l)}(g_c) = \frac{2-6T_i(g)}{10}$; $q_j^{(l)}(g_c) = \frac{2+4T_j(g)}{10}$; $q_k^{(l)}(g_c) = \frac{2-6T_k(g)}{10}$; $q_l^{(l)}(g_c) = \frac{2+4T_l(g)}{10}$. Using these outputs it is concluded that $CS_i(g_c) = \frac{1}{2}\left(q_i^{(i)}(g_c) + q_i^{(j)}(g_c) + q_i^{(k)}(g_c) + q_i^{(l)}(g_c)\right)^2$ = $0.5\left(0.8000 - 0.4000T_i(g_c)\right)^2$; $\pi_i^{(i)}(g_c)$ = $\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_c)\right)^2$ = $(0.2000 + 0.4000T_i(g_c))^2$; $\pi_j^{(i)}(g_c)$ = $\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_c)\right)^2$ = $\left(0.2000 - 0.6000T_j(g_c)\right)^2$; $\pi_k^{(i)}(g_c)$ = $\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_c)\right)^2$ = $(0.2000 + 0.4000T_k(g_c))^2$; $\pi_l^{(i)}(g_c)$ = $\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_c)\right)^2$ = $(0.2000 - 0.6000T_l(g_c))^2$; $PS_i(g_c) = \frac{\phi}{4}\left(q_i^{(i)}(g_c) + q_j^{(i)}(g_c) + q_k^{(i)}(g_c) + q_l^{(i)}(g_c)\right)^2 = 0$; and $TR_i(g_c)$ = $T_i(g_c)\,(\,q_i^{(j)}(g_c)$ + $q_i^{(l)}(g_c)\,)$ = $0.4000T_i(g_c) - 1.2000T_i^2(g_c)$. Therefore welfare in country *i* is given by:

$$W_i(g_c) = CS_i(g_c) + \pi_i(g_c) + PS_i(g_c) + TR_i(g_c) = 0.5\left(0.8000 - 0.4000T_i(g_c)\right)^2$$
$$+\left(0.2000 + 0.4000T_i(g_c)\right)^2 + \left(0.2000 - 0.6000T_j(g_c)\right)^2 + \left(0.2000 + 0.4000T_k(g_c)\right)^2$$
$$+\left(0.2000 - 0.6000T_l(g_c)\right)^2 + 0.4000T_i(g_c) - 1.2000T_i^2(g_c)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_c)}{\partial T_i(g_c)} = (-0.4000)\left(0.8000 - 0.4000T_i(g_c)\right) + (2)(0.4000)\left(0.2000 + 0.4000T_i(g_c)\right) +$$

$0.4000 - 2.4000T_i(g_c) = 0.2400 - 1.9200T_i(g_c)$; and $\frac{\partial^2 W_i(g_c)}{\partial T_i^2(g_c)} = -1.9200$. By using symmetry across countries the following optimal tariffs for countries *i*, *j*, *k* and *l* are obtained: $T_i^*(g_c) = T_j^*(g_c) = T_k^*(g_c) = T_l^*(g_c) = 0.1250$. Using these tariffs it is concluded that $CS_i(g_c)$ = $CS_j(g_c)$ = $CS_k(g_c)$ = $CS_l(g_c)$ = 0.2813; $\pi_i(g_c)$ = $\pi_j(g_c)$ = $\pi_k(g_c)$ = $\pi_l(g_c)$ = 0.1563; $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = 0$; $TR_i(g_c)$ = $TR_j(g_c)$ = $TR_k(g_c)$ = $TR_l(g_c)$ = 0.0313; and $W_i(g_c) = W_j(g_c) = W_k(g_c) = W_l(g_c) = 0.4689$.

Simulation 5: α = 1 and ϕ = 0.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_c) = 0.1600 + 0.2641T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)$$

$$q_j^{(i)}(g_c) = 0.1600 - 0.0406T_i(g_c) - 0.4470T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)$$

$$q_k^{(i)}(g_c) = 0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) + 0.2641T_k(g_c) + 0.0483T_l(g_c)$$

$$q_l^{(i)}(g_c) = 0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) - 0.4470T_l(g_c)$$

$$q_i^{(j)}(g_c) = 0.1600 - 0.4470T_i(g_c) - 0.0406T_j(g_c) + 0.0483T_k(g_c) - 0.0406T_l(g_c)$$

$$q_j^{(j)}(g_c) = 0.1600 + 0.0483T_i(g_c) + 0.2641T_j(g_c) + 0.0483T_k(g_c) - 0.0406T_l(g_c)$$

$$q_k^{(j)}(g_c) = 0.1600 + 0.0483T_i(g_c) - 0.0406T_j(g_c) - 0.4470T_k(g_c) - 0.0406T_l(g_c)$$

$$q_l^{(j)}(g_c) = 0.1600 + 0.0483T_i(g_c) - 0.0406T_j(g_c) + 0.0483T_k(g_c) + 0.2141T_l(g_c)$$

$$q_i^{(k)}(g_c) = 0.1600 + 0.2641T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)$$

$$q_j^{(k)}(g_c) = 0.1600 - 0.0406T_i(g_c) - 0.4470T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)$$

$$q_k^{(k)}(g_c) = 0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) + 0.2641T_k(g_c) + 0.0483T_l(g_c)$$

$$q_l^{(k)}(g_c) = 0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) - 0.4470T_l(g_c)$$

$$q_i^{(l)}(g_c) = 0.1600 - 0.4470T_i(g_c) - 0.0406T_j(g_c) + 0.0483T_k(g_c) - 0.0406T_l(g_c)$$

$$q_j^{(l)}(g_c) = 0.1600 + 0.0483T_i(g_c) + 0.2641T_j(g_c) + 0.0483T_k(g_c) - 0.0406T_l(g_c)$$

$$q_k^{(l)}(g_c) = 0.1600 + 0.0483T_i(g_c) - 0.0406T_j(g_c) - 0.4470T_k(g_c) - 0.0406T_l(g_c)$$

$$q_l^{(l)}(g_c) = 0.1600 + 0.0483T_i(g_c) - 0.0406T_j(g_c) + 0.0483T_k(g_c) + 0.2141T_l(g_c)$$

Solving by substitution the following expressions are obtained: $CS_i(g_c) = \frac{1}{2}(q_i^{(i)}(g_c) + q_i^{(j)}(g_c) + q_i^{(k)}(g_c) + q_i^{(l)}(g_c))^2 = 0.5(0.6400 - 0.3657T_i(g_c) + 0.0152T_j(g_c) + 0.0152T_k(g_c) + 0.0152T_l(g_c))^2$; $\pi_i^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_c)\right)^2 = 1.25(0.1600 + 0.2641T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c))^2$; $\pi_j^{(i)}(g_c) =$

$\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_c)\right)^2 = 1.25(0.1600 - 0.0406T_i(g_c) - 0.4470T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c))^2$; $\pi_k^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_c)\right)^2 = 1.25(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) + 0.2641T_k(g_c) + 0.0483T_l(g_c))^2$; $\pi_l^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_c)\right)^2 = 1.25\left(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) - 0.4470T_l(g_c)\right)^2$; $PS_i(g_c) = \frac{\phi}{4}\left(q_i^{(i)}(g_c) + q_j^{(i)}(g_c) + q_k^{(i)}(g_c) + q_l^{(i)}(g_c)\right)^2 = 0.125(0.6400 + 0.1422T_i(g_c) - 0.3022T_j(g_c) + 0.1422T_k(g_c) - 0.3022T_l(g_c))^2$; and $TR_i(g_c) = T_i(g_c)\left(q_i^{(j)}(g_c) + q_i^{(l)}(g_c)\right) = 0.3200T_i(g_c) - 0.8940T_i^2(g_c) - 0.0813T_j(g_c)T_i(g_c) + 0.0965T_k(g_c)T_i(g_b) - 0.0813T_l(g_c)T_i(g_c)$. Therefore welfare in country *i* is given by:

$$
\begin{aligned}
&W_i(g_c) = CS_i(g_c) + \pi_i(g_c) + PS_i(g_c) + TR_i(g_c) \\
&= 0.5\left(0.6400 - 0.3657T_i(g_c) + 0.0152T_j(g_c) + 0.0152T_k(g_c) + 0.0152T_l(g_c)\right)^2 \\
&+ 1.25\left(0.1600 + 0.2641T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_c) - 0.4470T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) + 0.2641T_k(g_c) + 0.0483T_l(g_c)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) - 0.4470T_l(g_c)\right)^2 \\
&+ 0.125\left(0.6400 + 0.1422T_i(g_c) - 0.3022T_j(g_c) + 0.1422T_k(g_c) - 0.3022T_l(g_c)\right)^2 \\
&+ 0.3200T_i(g_c) - 0.8940T_i^2(g_c) - 0.0813T_j(g_c)T_i(g_c) \\
&+ 0.0965T_k(g_c)T_i(g_c) - 0.0813T_l(g_c)T_i(g_c)
\end{aligned}
$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_c)}{\partial T_i(g_c)} =$$
$$= (-0.3657)\left(0.6400 - 0.3657T_i(g_c) + 0.0152T_j(g_c) + 0.0152T_k(g_c) + 0.0152T_l(g_c)\right)$$
$$+ (2.5)(0.2641)\left(0.1600 + 0.2641T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)\right)$$
$$+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_c) - 0.4470T_j(g_c) - 0.0406T_k(g_c) + 0.0483T_l(g_c)\right)$$
$$+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) + 0.2641T_k(g_c) + 0.0483T_l(g_c)\right)$$
$$+ (2.5)(-0.0406)\left(0.1600 - 0.0406T_i(g_c) + 0.0483T_j(g_c) - 0.0406T_k(g_c) - 0.4470T_l(g_c)\right)$$
$$+ (0.25)(0.1422)\left(0.6400 + 0.1422T_i(g_c) - 0.3022T_j(g_c) + 0.1422T_k(g_c) - 0.3022T_l(g_c)\right)$$
$$+ 0.3200 - 1.7880T_i(g_c) - 0.0813T_j(g_c) + 0.0965T_k(g_c) - 0.0813T_l(g_c)$$
$$= 0.1656 - 1.4624T_i(g_c) - 0.0302T_j(g_c) + 0.0506T_k(g_c) - 0.0302T_l(g_c)$$

$$\frac{\partial^2 W_i(g_c)}{\partial T_i^2(g_c)} = -1.4624$$

Consequently the optimal tariff in country *I* is given by $T_i^*(g_c)$ = 0.1133 − $0.0206T_j(g_c)$ + $0.0346T_k(g_c)$ − $0.0206T_l(g_c)$. Given symmetry it is concluded that the optimal tariffs in countries *i*, *j*, *k* and *l* are $T_i^*(g_c)$ = $T_j^*(g_c)$ = $T_k^*(g_c)$ = $T_l^*(g_c)$ = 0.1125. Using these tariffs, he following expressions are obtained: $CS_i(g_c)$ = $CS_j(g_c)$ = $CS_k(g_c)$ = $CS_l(g_c)$ = 0.1824; $\pi_i(g_c) = \pi_j(g_c) = \pi_k(g_c) = \pi_l(g_c) = 0.1241$; $PS_i(g_c)$ = $PS_j(g_c)$ = $PS_k(g_c)$ = $PS_l(g_c)$ = 0.0456; $TR_i(g_c)$ = $TR_j(g_c)$ = $TR_k(g_c)$ = $TR_l(g_c)$ = 0.0239; and $W_i(g_c) = W_j(g_c) = W_k(g_c) = W_l(g_c) = 0.3760$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_c) = 0.1143 + 0.1611T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)$$

$$q_j^{(i)}(g_c) = 0.1143 - 0.0377T_i(g_c) - 0.3201T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)$$

$$q_k^{(i)}(g_c) = 0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) + 0.1611T_k(g_c) + 0.0526T_l(g_c)$$

$$q_l^{(i)}(g_c) = 0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) - 0.3101T_l(g_c)$$

$$q_i^{(j)}(g_c) = 0.1143 - 0.3201T_i(g_c) - 0.0377T_j(g_c) + 0.0526T_k(g_c) - 0.0377T_l(g_c)$$

$$q_j^{(j)}(g_c) = 0.1143 + 0.0526T_i(g_c) + 0.1611T_j(g_c) + 0.0526T_k(g_c) - 0.0377T_l(g_c)$$

$$q_k^{(j)}(g_c) = 0.1143 + 0.0526T_i(g_c) - 0.0377T_j(g_c) - 0.3201T_k(g_c) - 0.0377T_l(g_c)$$

$$q_l^{(j)}(g_c) = 0.1143 + 0.0526T_i(g_c) - 0.0377T_j(g_c) + 0.0526T_k(g_c) + 0.1611T_l(g_c)$$

$$q_i^{(k)}(g_c) = 0.1143 + 0.1611T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)$$

$$q_j^{(k)}(g_c) = 0.1143 - 0.0377T_i(g_c) - 0.3201T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)$$

$$q_k^{(k)}(g_c) = 0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) + 0.1611T_k(g_c) + 0.0526T_l(g_c)$$

$$q_l^{(k)}(g_c) = 0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) - 0.3201T_l(g_c)$$

$$q_i^{(l)}(g_c) = 0.1143 - 0.3201T_i(g_c) - 0.0377T_j(g_c) + 0.0526T_k(g_c) - 0.0377T_l(g_c)$$

$$q_j^{(l)}(g_c) = 0.1143 + 0.0526T_i(g_c) + 0.1611T_j(g_c) + 0.0526T_k(g_c) - 0.0377T_l(g_c)$$

$$q_k^{(l)}(g_c) = 0.1143 + 0.0526T_i(g_c) - 0.0377T_j(g_c) - 0.3201T_k(g_c) - 0.0377T_l(g_c)$$

$$q_l^{(l)}(g_c) = 0.1143 + 0.0526T_i(g_c) - 0.0377T_j(g_c) + 0.0526T_k(g_c) + 0.1611T_l(g_c)$$

Solving by substitution the following expressions are obtained

$$CS_i(g_c) = \frac{1}{2}\left(q_i^{(i)}(g_c) + q_i^{(j)}(g_c) + q_i^{(k)}(g_c) + q_i^{(l)}(g_c)\right)^2$$
$$= 0.5\left(0.4571 - 0.3180T_i(g_c) + 0.0298T_j(g_c) + 0.0298T_k(g_c) + 0.0298T_l(g_c)\right)^2$$

$$\pi_i^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_c)\right)^2$$
$$= 1.75\left(0.1143 + 0.1611T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)\right)^2$$

$$\pi_j^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_c)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_i(g_c) - 0.3201T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)\right)^2$$

$$\pi_k^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_c)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) + 0.1611T_k(g_c) + 0.0526T_l(g_c)\right)^2$$

$$\pi_l^{(i)}(g_c) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_c)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) - 0.3101T_l(g_c)\right)^2$$

$$PS_i(g_c) = \frac{\phi}{4}\left(q_i^{(i)}(g_c) + q_j^{(i)}(g_c) + q_k^{(i)}(g_c) + q_l^{(i)}(g_c)\right)^2$$
$$= 0.375\left(0.4571 + 0.0481T_i(g_c) - 0.1624T_j(g_c) + 0.0481T_k(g_c) - 0.1624T_l(g_c)\right)^2$$

$$TR_i(g_c) = T_i(g_c)\left(q_i^{(j)}(g_c) + q_i^{(l)}(g_c)\right)$$
$$= 0.2286T_i(g_c) - 0.6402T_i^2(g_c) - 0.0753T_j(g_c)T_i(g_c)$$
$$+ 0.1051T_k(g_c)T_i(g_c) - 0.0753T_l(g_c)T_i(g_c)$$

Therefor welfare in country *i* is given by:

$$
\begin{aligned}
W_i(g_c) &= CS_i(g_c) + \pi_i(g_c) + PS_i(g_c) + TR_i(g_c) \\
&= 0.5\left(0.4571 - 0.3180T_i(g_c) + 0.0298T_j(g_c) + 0.0298T_k(g_c) + 0.0298T_l(g_c)\right)^2 \\
&+ 1.75\left(0.1143 + 0.1611T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_c) - 0.3201T_j(g_c) - 0.0377T_k(g_c) + 0.0526T_l(g_c)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) + 0.1611T_k(g_c) + 0.0526T_l(g_c)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_i(g_c) + 0.0526T_j(g_c) - 0.0377T_k(g_c) - 0.3101T_l(g_c)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0481T_i(g_c) - 0.1624T_j(g_c) + 0.0481T_k(g_c) - 0.1624T_l(g_c)\right)^2 \\
&+ 0.2286T_i(g_c) - 0.6402T_i^2(g_c) - 0.0753T_j(g_c)T_i(g_c) \\
&+ 0.1051T_k(g_c)T_i(g_c) - 0.0753T_l(g_c)T_i(g_c)
\end{aligned}
$$

The first and second conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_i(g_c)}{\partial T_i(g_c)} = \\
&= (-0.3180)\left(0.4571 - 0.3180T_i(g_b) + 0.0298T_j(g_b) + 0.0298T_k(g_b) + 0.0298T_l(g_b)\right) \\
&+ (3.5)(0.1611)\left(0.1143 + 0.1611T_i(g_b) + 0.0526T_j(g_b) - 0.0377T_k(g_b) + 0.0526T_l(g_b)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) - 0.3201T_j(g_b) - 0.0377T_k(g_b) + 0.0526T_l(g_b)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) + 0.0526T_j(g_b) + 0.1611T_k(g_b) + 0.0526T_l(g_b)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_i(g_b) + 0.0526T_j(g_b) - 0.0377T_k(g_b) - 0.3101T_l(g_b)\right) \\
&+ (0.75)(0.0481)\left(0.4571 + 0.0481T_i(g_b) - 0.1624T_j(g_b) + 0.0481T_k(g_b) - 0.1624T_l(g_b)\right) \\
&+ 0.2286 - 1.2804T_i(g_b) - 0.0753T_j(g_b) + 0.1051T_k(g_b) - 0.0753T_l(g_b) \\
&= 0.1189 - 1.0718T_i(g_b) - 0.0327T_j(g_b) + 0.0648T_k(g_b) - 0.0327T_l(g_b)
\end{aligned}
$$

$$
\frac{\partial^2 W_i(g_c)}{\partial T_i^2(g_c)} = -1.0718
$$

Therefore the optimal tariff in country *i* is: $T_i^*(g_c) = 0.1109 - 0.0305T_j(g_c) + 0.0605T_k(g_c) - 0.0305T_l(g_c)$. Using symmetry, the optimal tariffs for countries *i*, *j*, *k* and *l* are: $T_i^*(g_c) = T_j^*(g_c) = T_k^*(g_c) = T_l^*(g_c) = 0.1109$. Using these tariffs, the following expressions are obtained: $CS_i(g_c) = CS_j(g_c) = CS_k(g_c) = CS_l(g_c) = 0.0932$; $\pi_i(g_c) = \pi_j(g_c) = \pi_k(g_c) = \pi_l(g_c) = 0.0886$; $PS_i(g_c)$ = $PS_j(g_c)$ = $PS_k(g_c)$ = $PS_l(g_c)$ = 0.0699; $TR_i(g_c) = TR_j(g_c) = TR_k(g_c) = TR_l(g_c) = 0.0169$; and $W_i(g_c)$ = $W_j(g_c)$ = $W_k(g_c)$ = $W_l(g_c)$ = 0.2687.

Network d

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_d) = \frac{2\alpha(\phi+1) + 2T_i(g_d) + \phi\begin{pmatrix} q_j^{(j)}(g_d) + q_k^{(j)}(g_d) + q_l^{(j)}(g_d) + q_j^{(k)}(g_d) + q_k^{(k)}(g_d) \\ q_l^{(k)}(g_d) + q_j^{(l)}(g_d) + q_k^{(l)}(g_d) + q_l^{(l)}(g_d) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_d) + q_k^{(i)}(g_d) + q_l^{(i)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_d) = \frac{2\alpha(\phi+1) + 2T_j(g_d) + \phi\begin{pmatrix} q_i^{(j)}(g_d) + q_k^{(j)}(g_d) + q_l^{(j)}(g_d) + q_i^{(k)}(g_d) + q_k^{(k)}(g_d) \\ q_l^{(k)}(g_d) + q_i^{(l)}(g_d) + q_k^{(l)}(g_d) + q_l^{(l)}(g_d) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_d) + q_k^{(i)}(g_d) + q_l^{(i)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_d) = \frac{2\alpha(\phi+1) + 4T_k(g_d) + \phi\begin{pmatrix} q_i^{(j)}(g_d) + q_j^{(j)}(g_d) + q_l^{(j)}(g_d) + q_i^{(k)}(g_d) + q_j^{(k)}(g_d) \\ q_l^{(k)}(g_d) + q_i^{(l)}(g_d) + q_j^{(l)}(g_d) + q_l^{(l)}(g_d) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_d) + q_j^{(i)}(g_d) + q_l^{(i)}(g_d)\right)}{2(\phi+1)(N+\phi+1)}$$

$$q_l^{(i)}(g_d)=\frac{2\alpha(\phi+1)-2(3+\phi)T_l(g_d)+\phi\begin{pmatrix}q_i^{(j)}(g_d)+q_j^{(j)}(g_d)+q_k^{(j)}(g_d)+q_i^{(k)}(g_d)+q_j^{(k)}(g_d)\\ q_k^{(k)}(g_d)+q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_k^{(i)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_d)=\frac{2\alpha(\phi+1)+2T_i(g_d)+\phi\begin{pmatrix}q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_j^{(k)}(g_d)+q_k^{(k)}(g_d)\\ q_l^{(k)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_d)+q_k^{(j)}(g_d)+q_l^{(j)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_d)=\frac{2\alpha(\phi+1)+2T_j(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(k)}(g_d)+q_k^{(k)}(g_d)\\ q_l^{(k)}(g_d)+q_i^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_d)+q_k^{(j)}(g_d)+q_l^{(j)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_d)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(k)}(g_d)+q_j^{(k)}(g_d)\\ q_l^{(k)}(g_d)+q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_d)+q_j^{(j)}(g_d)+q_l^{(j)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_d)=\frac{2\alpha(\phi+1)+4T_l(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_i^{(k)}(g_d)+q_j^{(k)}(g_d)\\ q_k^{(k)}(g_d)+q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_d)+q_j^{(j)}(g_d)+q_k^{(j)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_d)=\frac{2\alpha(\phi+1)+2T_i(g_d)+\phi\begin{pmatrix}q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_j^{(j)}(g_d)+q_k^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_d)+q_k^{(k)}(g_d)+q_l^{(k)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_d)=\frac{2\alpha(\phi+1)-2(4+\phi)T_j(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(j)}(g_d)+q_k^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_i^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_d)+q_k^{(k)}(g_d)+q_l^{(k)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_d)=\frac{2\alpha(\phi+1)+4T_k(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(j)}(g_d)+q_j^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_l^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_d)+q_j^{(k)}(g_d)+q_l^{(k)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_d)=\frac{2\alpha(\phi+1)-2(3+\phi)T_l(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_i^{(j)}(g_d)+q_j^{(j)}(g_d)\\ q_k^{(j)}(g_d)+q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_d)+q_j^{(k)}(g_d)+q_k^{(k)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_d)=\frac{2\alpha(\phi+1)-2(4+\phi)T_i(g_d)+\phi\begin{pmatrix}q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_j^{(j)}(g_d)+q_k^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_j^{(k)}(g_d)+q_k^{(k)}(g_d)+q_l^{(k)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_d)=\frac{2\alpha(\phi+1)+2T_j(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_k^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(j)}(g_d)+q_k^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_i^{(k)}(g_d)+q_k^{(k)}(g_d)+q_l^{(k)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_d)+q_k^{(l)}(g_d)+q_l^{(l)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_d)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_l^{(i)}(g_d)+q_i^{(j)}(g_d)+q_j^{(j)}(g_d)\\ q_l^{(j)}(g_d)+q_i^{(k)}(g_d)+q_j^{(k)}(g_d)+q_l^{(k)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_l^{(l)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_d)=\frac{2\alpha(\phi+1)+4T_l(g_d)+\phi\begin{pmatrix}q_i^{(i)}(g_d)+q_j^{(i)}(g_d)+q_k^{(i)}(g_d)+q_i^{(j)}(g_d)+q_j^{(j)}(g_d)\\ q_k^{(j)}(g_d)+q_i^{(k)}(g_d)+q_j^{(k)}(g_d)+q_k^{(k)}(g_d)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_d)+q_j^{(l)}(g_d)+q_k^{(l)}(g_d)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: $\alpha = 1$ and $\phi = 0$

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_d) = \dfrac{2+2T_i(g_d)}{10}$; $q_j^{(i)}(g_d) = \dfrac{2+2T_j(g_d)}{10}$; $q_k^{(i)}(g_d) = \dfrac{2+4T_k(g_d)}{10}$; $q_l^{(i)}(g_d) = \dfrac{2-6T_l(g_d)}{10}$; $q_i^{(j)}(g_d) = \dfrac{2+2T_i(g_d)}{10}$; $q_j^{(j)}(g_d) = \dfrac{2+2T_j(g_d)}{10}$; $q_k^{(j)}(g_d) = \dfrac{2-6T_k(g_d)}{10}$; $q_l^{(j)}(g_d) = \dfrac{2+4T_l(g_d)}{10}$; $q_i^{(k)}(g_d) = \dfrac{2+2T_i(g_d)}{10}$; $q_j^{(k)}(g_d) = \dfrac{2-8T_j(g_d)}{10}$; $q_k^{(k)}(g_d) = \dfrac{2+4T_k(g_d)}{10}$; $q_l^{(k)}(g_d) = \dfrac{2-6T_l(g_d)}{10}$; $q_i^{(l)}(g_d) = \dfrac{2-8T_i(g_d)}{10}$; $q_j^{(l)}(g_d) = \dfrac{2+2T_j(g_d)}{10}$; $q_k^{(l)}(g_d) = \dfrac{2-6T_k(g_d)}{10}$; and $q_l^{(l)}(g_d) = \dfrac{2+4T_l(g_d)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_d) = \frac{1}{2}\left(q_i^{(i)}(g_d) + q_i^{(j)}(g_d) + q_i^{(k)}(g_d) + q_i^{(l)}(g_d)\right)^2 = 0.5\left(0.8000 - 0.2000T_i(g_d)\right)^2$$

$$\pi_i^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_d)\right)^2 = \left(0.2000 + 0.2000T_i(g_d)\right)^2$$

$$\pi_j^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_d)\right)^2 = \left(0.2000 + 0.2000T_j(g_d)\right)^2$$

$$\pi_k^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_d)\right)^2 = \left(0.2000 + 0.4000T_k(g_d)\right)^2$$

$$\pi_l^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_d)\right)^2 = \left(0.2000 - 0.6000T_l(g_d)\right)^2$$

$$PS_i(g_d) = \frac{\phi}{4}\left(q_i^{(i)}(g_d) + q_j^{(i)}(g_d) + q_k^{(i)}(g_d) + q_l^{(i)}(g_d)\right)^2 = 0$$

$$TR_i(g_d) = T_i(g_d)\left(q_i^{(l)}(g_d)\right) = 0.2000T_i(g_d) - 0.8000T_i^2(g_d)$$

Therefore welfare in country *i* is given by:

$$W_i(g_d) = CS_i(g_d) + \pi_i(g_d) + PS_i(g_d) + TR_i(g_d) = 0.5\left(0.8000 - 0.2000T_i(g_d)\right)^2$$
$$+\left(0.2000 + 0.2000T_i(g_d)\right)^2 + \left(0.2000 + 0.2000T_j(g_d)\right)^2 + \left(0.2000 + 0.4000T_k(g_d)\right)^2$$
$$+\left(0.2000 - 0.6000T_l(g_d)\right)^2 + 0.2000T_i(g_d) - 0.8000T_i^2(g_d)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_d)}{\partial T_i(g_d)} = (-0.2000)\left(0.8000 - 0.2000T_i(g_d)\right) + (2)(0.2000)\left(0.2000 + 0.2000T_i(g_d)\right)$$
$$+0.2000 - 1.6000T_i(g_d) = 0.1200 - 1.4800T_i(g_d)$$

$$\frac{\partial^2 W_i(g_d)}{\partial T_i^2(g_d)} = -1.4800$$

Therefore, using symmetry it is inferred that $T_i^*(g_d) = T_j^*(g_d) = 0.0811$

On the other hand, the following expressions hold in country *k*:

$$CS_k(g_d) = \frac{1}{2}\left(q_k^{(i)}(g_d) + q_k^{(j)}(g_d) + q_k^{(k)}(g_d) + q_k^{(l)}(g_d)\right)^2 = 0.5\left(0.8000 - 0.4000T_k(g_d)\right)^2$$

$$\pi_i^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_i^{(k)}(g_d)\right)^2 = \left(0.2000 + 0.2000T_i(g_d)\right)^2$$

$$\pi_j^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_j^{(k)}(g_d)\right)^2 = \left(0.2000 - 0.8000T_j(g_d)\right)^2$$

$$\pi_k^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(k)}(g_d)\right)^2 = \left(0.2000 + 0.4000T_k(g_d)\right)^2$$

$$\pi_l^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(k)}(g_d)\right)^2 = \left(0.2000 - 0.6000T_l(g_d)\right)^2$$

$$PS_k(g_d) = \frac{\phi}{4}\left(q_i^{(k)}(g_d) + q_j^{(k)}(g_d) + q_k^{(k)}(g_d) + q_l^{(k)}(g_d)\right)^2 = 0$$

$$TR_k(g_d) = T_k(g_d)\left(q_k^{(j)}(g_d) + q_k^{(l)}(g_d)\right) = 0.4000T_k(g_d) - 1.2000T_k^2(g_d)$$

Therefore welfare in country *k* is given by:

$$W_k(g_d) = CS_k(g_d) + \pi_k(g_d) + PS_k(g_d) + TR_k(g_d) = 0.5\left(0.8000 - 0.4000T_k(g_d)\right)^2$$
$$+\left(0.2000 + 0.2000T_i(g_d)\right)^2 + \left(0.2000 - 0.8000T_j(g_d)\right)^2 + \left(0.2000 + 0.4000T_k(g_d)\right)^2$$
$$+\left(0.2000 - 0.6000T_l(g_d)\right)^2 + 0.4000T_k(g_d) - 1.2000T_k^2(g_d)$$

The first and second conditions of this function are:

$$\frac{\partial W_k(g_d)}{\partial T_k(g_d)} = (-0.4000)\left(0.8000 - 0.4000T_k(g_d)\right) + (2)(0.4000)\left(0.2000 + 0.4000T_k(g_d)\right)$$
$$+0.4000 - 2.4000T_k(g_d) = 0.2400 - 1.9200T_k(g_d)$$

$$\frac{\partial^2 W_k(g_d)}{\partial T_k^2(g_d)} = -1.9200$$

Therefore, using symmetry it is concluded that the optimal tariffs in country *k* and *l* are $T_k^*(g_d) = T_l^*(g_d) = 0.125$. Finally, using the optimal tariffs for countries *l*, *j*, *k* and *l* the following expressions are obtained: $CS_i(g_d) = CS_j(g_d) = 0.3072$; $CS_k(g_d)$ = $CS_l(g_d)$ = 0.2813; $\pi_i(g_d) = \pi_j(g_d) = 0.1716$; $\pi_k(g_d) = \pi_l(g_d) = 0.1431$; $PS_i(g_d)$ = $PS_j(g_d)$ =

$PS_k(g_d) = PS_l(g_d) = 0$; $TR_i(g_d) = TR_j(g_d) = 0.0110$; $TR_k(g_d) = TR_l(g_d) = 0.0313$;

$W_i(g_d) = W_j(g_d) = 0.4897$; $W_k(g_d) = W_l(g_d) = 0.4556$.

Simulation 5: α = 1 and ϕ = 0.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_d) = 0.1600 + 0.1321T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)$$

$$q_j^{(i)}(g_d) = 0.1600 - 0.0203T_i(g_d) + 0.1321T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)$$

$$q_k^{(i)}(g_d) = 0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)$$

$$q_l^{(i)}(g_d) = 0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)$$

$$q_i^{(j)}(g_d) = 0.1600 + 0.1321T_i(g_d) - 0.0203T_j(g_d) + 0.0483T_k(g_d) - 0.0406T_l(g_d)$$

$$q_j^{(j)}(g_d) = 0.1600 - 0.0203T_i(g_d) + 0.1321T_j(g_d) + 0.0483T_k(g_d) - 0.0406T_l(g_d)$$

$$q_k^{(j)}(g_d) = 0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) - 0.4470T_k(g_d) - 0.0406T_l(g_d)$$

$$q_l^{(j)}(g_d) = 0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) + 0.0483T_k(g_d) + 0.2641T_l(g_d)$$

$$q_i^{(k)}(g_d) = 0.1600 + 0.1321T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)$$

$$q_j^{(k)}(g_d) = 0.1600 - 0.0203T_i(g_d) - 0.5790T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)$$

$$q_k^{(k)}(g_d) = 0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)$$

$$q_l^{(k)}(g_d) = 0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)$$

$$q_i^{(l)}(g_d) = 0.1600 - 0.5790T_i(g_d) - 0.0203T_j(g_d) + 0.0483T_k(g_d) - 0.0406T_l(g_d)$$

$$q_j^{(l)}(g_d) = 0.1600 + 0.0686T_i(g_d) + 0.1321T_j(g_d) + 0.0483T_k(g_d) - 0.0406T_l(g_d)$$

$$q_k^{(l)}(g_d) = 0.1600 + 0.0686T_i(g_d) - 0.0203T_j(g_d) - 0.4470T_k(g_d) - 0.0406T_l(g_d)$$

$$q_l^{(l)}(g_d) = 0.1600 + 0.0686T_i(g_d) - 0.0203T_j(g_d) + 0.0483T_k(g_d) + 0.2641T_l(g_d)$$

Solving by substitution the following expressions hold for country *i*:

$$CS_i(g_d) = \frac{1}{2}\left(q_i^{(i)}(g_d) + q_i^{(j)}(g_d) + q_i^{(k)}(g_d) + q_i^{(l)}(g_d)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_d) + 0.0076T_j(g_d) + 0.0152T_k(g_d) + 0.0152T_l(g_d)\right)^2$$

$$\pi_i^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_j^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) + 0.1321T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_k^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_l^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right)^2$$

$$PS_i(g_d) = \frac{\phi}{4}\left(q_i^{(i)}(g_d) + q_j^{(i)}(g_d) + q_k^{(i)}(g_d) + q_l^{(i)}(g_d)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_d) + 0.0711T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right)^2$$

$$TR_i(g_d) = T_i(g_d)\left(q_i^{(l)}(g_d)\right) = 0.1600T_i(g_d) - 0.5790T_i^2(g_d)$$
$$- 0.0203T_j(g_d)T_i(g_d) + 0.0483T_k(g_d)T_i(g_d) - 0.0406T_l(g_d)T_i(g_d)$$

$$
\begin{aligned}
W_i(g_d) &= CS_i(g_d) + \pi_i(g_d) + PS_i(g_d) + TR_i(g_d) \\
&= 0.5\left(0.6400 - 0.1829T_i(g_d) + 0.0076T_j(g_d) + 0.0152T_k(g_d) + 0.0152T_l(g_d)\right)^2 \\
&+ 1.25\left(0.1600 + 0.1321T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_d) + 0.1321T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right)^2 \\
&+ 0.125\left(0.6400 + 0.0711T_i(g_d) + 0.0711T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right)^2 \\
&+ 0.1600T_i(g_d) - 0.5790T_i^2(g_d) \\
&- 0.0203T_j(g_d)T_i(g_d) + 0.0483T_k(g_d)T_i(g_d) - 0.0406T_l(g_d)T_i(g_d)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
\frac{\partial W_i(g_d)}{\partial T_i(g_d)} &= \\
&(-0.1829)\left(0.6400 - 0.1829T_i(g_d) + 0.0076T_j(g_d) + 0.0152T_k(g_d) + 0.0152T_l(g_d)\right) \\
&+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_d) + 0.1321T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_d) - 0.0203T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right) \\
&+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_d) + 0.0711T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right) \\
&+ 0.1600 - 1.1580T_i(g_d) - 0.0203T_j(g_d) + 0.0483T_k(g_d) - 0.0406T_l(g_d) \\
&= 0.0828 - 1.0767T_i(g_d) - 0.0318T_j(g_d) + 0.0253T_k(g_d) - 0.0151T_l(g_d)
\end{aligned}
$$

$$
\frac{\partial^2 W_i(g_d)}{\partial T_i^2(g_d)} = -1.0767
$$

Therefore the optimal tariff for country i is $T_i^*(g_d)$ = 0.0769 – $0.0295T_j(g_d)$ + $0.0235T_k(g_d)$ – $0.0140T_l(g_d)$. Using symmetry, this optimal tariff converges to the following: $T_i^*(g_d) = 0.0747 + 0.0092T_k(g_d)$.

On the other hand, the following expressions hold for country *k*:

$$CS_k(g_d) = \frac{1}{2}\left(q_k^{(i)}(g_d) + q_k^{(j)}(g_d) + q_k^{(k)}(g_d) + q_k^{(l)}(g_d)\right)^2$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_d) + 0.0076T_j(g_d) - 0.3657T_k(g_d) + 0.0152T_l(g_d)\right)^2$$

$$\pi_i^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_j^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) - 0.5790T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_k^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right)^2$$

$$\pi_l^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_d)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right)^2$$

$$PS_k(g_d) = \frac{\phi}{4}\left(q_i^{(k)}(g_d) + q_j^{(k)}(g_d) + q_k^{(k)}(g_d) + q_l^{(k)}(g_d)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_d) - 0.3733T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right)^2$$

$$TR_k(g_d) = T_k(g_d)\left(q_k^{(j)}(g_d) + q_k^{(l)}(g_d)\right)$$
$$= 0.3200T_k(g_d) + 0.0483T_i(g_d)T_k(g_d) - 0.0406T_j(g_d)T_k(g_d)$$
$$- 0.8940T_k^2(g_d) - 0.0813T_l(g_d)T_k(g_d)$$

Therefore welfare in country *k* is given by:

$$W_k(g_d) = CS_k(g_d) + \pi_k(g_d) + PS_k(g_d) + TR_k(g_d)$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_d) + 0.0076T_j(g_d) - 0.3657T_k(g_d) + 0.0152T_l(g_d)\right)^2$$
$$+1.25\left(0.1600 + 0.1321T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_d) - 0.5790T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right)^2$$
$$+0.125\left(0.6400 + 0.0711T_i(g_d) - 0.3733T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right)^2$$
$$+0.3200T_k(g_d) + 0.0483T_i(g_d)T_k(g_d) - 0.0406T_j(g_d)T_k(g_d)$$
$$-0.8940T_k^2(g_d) - 0.0813T_l(g_d)T_k(g_d)$$

The first and second conditions of this function are:

$$\frac{\partial W_k(g_d)}{\partial T_k(g_d)}$$
$$= (-0.3657)\left(0.6400 + 0.0076T_i(g_d) + 0.0076T_j(g_d) - 0.3657T_k(g_d) + 0.0152T_l(g_d)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 + 0.1321T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 - 0.0203T_i(g_d) - 0.5790T_j(g_d) - 0.0406T_k(g_d) + 0.0483T_l(g_d)\right)$$
$$+(2.5)(0.2641)\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) + 0.2641T_k(g_d) + 0.0483T_l(g_d)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 - 0.0203T_i(g_d) + 0.0686T_j(g_d) - 0.0406T_k(g_d) - 0.4470T_l(g_d)\right)$$
$$+(0.25)(0.1422)\left(0.6400 + 0.0711T_i(g_d) - 0.3733T_j(g_d) + 0.1422T_k(g_d) - 0.3022T_l(g_d)\right)$$
$$+0.3200 + 0.0483T_i(g_d) - 0.0406T_j(g_d) - 1.7880T_k(g_d) - 0.0813T_l(g_d)$$
$$= 0.1656 + 0.0253T_i(g_d) + 0.0334T_j(g_d) - 1.4624T_k(g_d) - 0.0302T_l(g_d)$$

$$\frac{\partial^2 W_k(g_d)}{\partial T_k^2(g_d)} = -1.4624$$

Therefore the optimal tariff in country k is: $T_k^*(g_d)$ = 0.1133 + $0.0173T_i(g_d)$ + $0.0229T_j(g_d) - 0.0206T_l(g_d)$. Given symmetry this expression converges to $T_k^*(g_d)$ = 0.1110 + $0.0394T_i(g_d)$. Using this equation, the expression for the optimal tariff of country, and symmetry across countries it was possible to calculate the following values of the optimal tariffs in countries *i*, *j*, *k* and *l*: $T_i^*(g_d) = T_j^*(g_d) = 0.0757$; and $T_k^*(g_d)$ = $T_l^*(g_d)$ = 0.1139. Using these tariffs, the following equalities hold: $CS_i(g_d)$ = $CS_j(g_d)$ = 0.1986; $CS_k(g_d) = CS_l(g_d) = 0.1807$; $\pi_i(g_d) = \pi_j(g_d) = 0.1308$; $\pi_k(g_d)$ = $\pi_l(g_d)$ = 0.1196; $PS_i(g_d) = PS_j(g_d) = 0.0500$; $PS_k(g_d) = PS_l(g_d) = 0.0448$; $TR_i(g_d)$ = $TR_j(g_d)$ = 0.0087; $TR_k(g_d) = TR_l(g_d) = 0.0239$; $W_i(g_d) = W_j(g_d) = 0.3882$; $W_k(g_d)$ = $W_j(g_d)$ = 0.3691.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_d) = 0.1143 + 0.0805T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)$$

$$q_j^{(i)}(g_d) = 0.1143 - 0.0188T_i(g_d) + 0.0805T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)$$

$$q_k^{(i)}(g_d) = 0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)$$

$$q_l^{(i)}(g_d) = 0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)$$

$$q_i^{(j)}(g_d) = 0.1143 + 0.0805T_i(g_d) - 0.0188T_j(g_d) + 0.0526T_k(g_d) - 0.0377T_l(g_d)$$

$$q_j^{(j)}(g_d) = 0.1143 - 0.0188T_i(g_d) + 0.0805T_j(g_d) + 0.0526T_k(g_d) - 0.0377T_l(g_d)$$

$$q_k^{(j)}(g_d) = 0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) - 0.3201T_k(g_d) - 0.0377T_l(g_d)$$

$$q_l^{(j)}(g_d) = 0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) + 0.0526T_k(g_d) + 0.1611T_l(g_d)$$

$$q_i^{(k)}(g_d) = 0.1143 + 0.0805T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)$$

$$q_j^{(k)}(g_d) = 0.1143 - 0.0188T_i(g_d) - 0.4007T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)$$

$$q_k^{(k)}(g_d) = 0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)$$

$$q_l^{(k)}(g_d) = 0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)$$

$$q_i^{(l)}(g_d) = 0.1143 - 0.4007T_i(g_d) - 0.0188T_j(g_d) + 0.0526T_k(g_d) - 0.0377T_l(g_d)$$

$$q_j^{(l)}(g_d) = 0.1143 + 0.0714T_i(g_d) + 0.0805T_j(g_d) + 0.0526T_k(g_d) - 0.0377T_l(g_d)$$

$$q_k^{(l)}(g_d) = 0.1143 + 0.0714T_i(g_d) - 0.0188T_j(g_d) - 0.3201T_k(g_d) - 0.0377T_l(g_d)$$

$$q_l^{(l)}(g_d) = 0.1143 + 0.0714T_i(g_d) - 0.0188T_j(g_d) + 0.0526T_k(g_d) + 0.1611T_l(g_d)$$

Solving by substitution the following expressions hold for country *i*:

$$CS_i(g_d) = \frac{1}{2}\left(q_i^{(i)}(g_d) + q_i^{(j)}(g_d) + q_i^{(k)}(g_d) + q_i^{(l)}(g_d)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_d) + 0.0149T_j(g_d) + 0.0298T_k(g_d) + 0.0298T_l(g_d)\right)^2$$

$$\pi_i^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$

$$\pi_j^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0805T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$

$$\pi_k^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\right)^2$$

$$\pi_l^{(i)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\right)^2$$

$$PS_i(g_d) = \frac{\phi}{4}\left(q_i^{(i)}(g_d) + q_j^{(i)}(g_d) + q_k^{(i)}(g_d) + q_l^{(i)}(g_d)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_d) + 0.0241T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\right)^2$$

$$TR_i(g_d) = T_i(g_d)\left(q_i^{(l)}(g_d)\right) = 0.1143T_i(g_d) - 0.4007T_i^2(g_d)$$
$$- 0.0188T_j(g_d)T_i(g_d) + 0.0526T_k(g_d)T_i(g_d) - 0.0377T_l(g_d)T_i(g_d)$$

Therefore welfare in country *i* is given by:

$$W_i(g_d) = CS_i(g_d) + \pi_i(g_d) + PS_i(g_d) + TR_i(g_d)$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_d) + 0.0149T_j(g_d) + 0.0298T_k(g_d) + 0.0298T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 + 0.0805T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0805T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\right)^2$$
$$+ 0.375\left(0.4571 + 0.0241T_i(g_d) + 0.0241T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\right)^2$$
$$+ 0.1143T_i(g_d) - 0.4007T_i^2(g_d) - 0.0188T_j(g_d)T_i(g_d) + 0.0526T_k(g_d)T_i(g_d)$$
$$- 0.0377T_l(g_d)T_i(g_d)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_d)}{\partial T_i(g_d)} =$$
$$(-0.1590)\big(0.4571 - 0.1590T_i(g_d) + 0.0149T_j(g_d) + 0.0298T_k(g_d) + 0.0298T_l(g_d)\big)$$
$$+ (3.5)(0.0805)\big(0.1143 + 0.0805T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\big)$$
$$+ (3.5)(-0.0188)\big(0.1143 - 0.0188T_i(g_d) + 0.0805T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\big)$$
$$+ (3.5)(-0.0188)\big(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\big)$$
$$+ (3.5)(-0.0188)\big(0.1143 - 0.0188T_i(g_d) - 0.0188T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\big)$$
$$+ (0.75)(0.0241)\big(0.4571 + 0.0241T_i(g_d) + 0.0241T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\big)$$
$$+ 0.1143 - 0.8014T_i(g_d) - 0.0188T_j(g_d) + 0.0526T_k(g_d) - 0.0377T_l(g_d)$$
$$= 0.0595 - 0.7492T_i(g_d) - 0.0289T_j(g_d) + 0.0324T_k(g_d) - 0.0164T_l(g_d)$$

$$\frac{\partial^2 W_i(g_d)}{\partial T_i^2(g_d)} = -0.7492$$

Therefore the optimal tariff in country i is: $T_i^*(g_d) = 0.0794 - 0.0386T_j(g_d) + 0.0433T_k(g_d) - 0.0219T_l(g_d)$. By symmetry across countries this expression converges to $T_i^*(g_d) = 0.0725 + 0.0206T_k(g_d)$.

On the other hand, the following expressions hold in country *k*:

$$CS_k(g_d) = \frac{1}{2}\big(q_k^{(i)}(g_d) + q_k^{(j)}(g_d) + q_k^{(k)}(g_d) + q_k^{(l)}(g_d)\big)^2$$
$$= 0.5\big(0.4571 + 0.0149T_i(g_d) + 0.0149T_j(g_d) - 0.3180T_k(g_d) + 0.0298T_l(g_d)\big)^2$$

$$\pi_i^{(k)}(g_d) = \frac{(\phi+2)}{2}\big(q_i^{(i)}(g_d)\big)^2$$
$$= 1.75\big(0.1143 + 0.0805T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\big)^2$$

$$\pi_j^{(k)}(g_d) = \frac{(\phi+2)}{2}\big(q_j^{(i)}(g_d)\big)^2$$
$$= 1.75\big(0.1143 - 0.0188T_i(g_d) - 0.4007T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\big)^2$$

$$\pi_k^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\right)^2$$

$$\pi_l^{(k)}(g_d) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_d)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\right)^2$$

$$PS_k(g_d) = \frac{\phi}{4}\left(q_i^{(k)}(g_d) + q_j^{(k)}(g_d) + q_k^{(k)}(g_d) + q_l^{(k)}(g_d)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_d) - 0.1865T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\right)^2$$

$$TR_k(g_d) = T_k(g_d)\left(q_k^{(j)}(g_d) + q_k^{(l)}(g_d)\right)$$
$$= 0.2286T_k(g_d) + 0.0526T_i(g_d)T_k(g_d) - 0.0377T_j(g_d)T_k(g_d)$$
$$- 0.6402T_k^2(g_d) - 0.0753T_l(g_d)T_k(g_d)$$

Therefore welfare in country k is given by:

$$W_k(g_d) = CS_k(g_d) + \pi_k(g_d) + PS_k(g_d) + TR_k(g_d)$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_d) + 0.0149T_j(g_d) - 0.3180T_k(g_d) + 0.0298T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 + 0.0805T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) - 0.4007T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\right)^2$$
$$+ 1.75\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\right)^2$$
$$+ 0.375\left(0.4571 + 0.0241T_i(g_d) - 0.1865T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\right)^2$$
$$+ 0.2286T_k(g_d) + 0.0526T_i(g_d)T_k(g_d) - 0.0377T_j(g_d)T_k(g_d)$$
$$- 0.6402T_k^2(g_d) - 0.0753T_l(g_d)T_k(g_d)$$

The first and second order conditions of this function are:

$$\frac{\partial W_k(g_d)}{\partial T_k(g_d)}$$
$$= (-0.3180)\left(0.4571 + 0.0149T_i(g_d) + 0.0149T_j(g_d) - 0.3180T_k(g_d) + 0.0298T_l(g_d)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 + 0.0805T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.0188T_i(g_d) - 0.4007T_j(g_d) - 0.0377T_k(g_d) + 0.0526T_l(g_d)\right)$$
$$+ (3.5)(0.1611)\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) + 0.1611T_k(g_d) + 0.0526T_l(g_d)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.0188T_i(g_d) + 0.0714T_j(g_d) - 0.0377T_k(g_d) - 0.3201T_l(g_d)\right)$$
$$+ (0.75)(0.0481)\left(0.4571 + 0.0241T_i(g_d) - 0.1865T_j(g_d) + 0.0481T_k(g_d) - 0.1624T_l(g_d)\right)$$
$$+ 0.2286 + 0.0526T_i(g_d) - 0.0377T_j(g_d) - 1.2804T_k^2(g_d) - 0.0753T_l(g_d)$$
$$= 0.1189 + 0.0324T_i(g_d) + 0.0252T_j(g_d) - 1.0718T_k(g_d) - 0.0327T_l(g_d)$$

$$\frac{\partial^2 W_k(g_d)}{\partial T_k^2(g_d)} = -1.0718$$

Therefore the optimal tariff in country k is: $T_k^*(g_d)$ = 0.1109 + $0.0302T_i(g_d)$ + $0.0235T_j(g_d)$ − $0.0305T_l(g_d)$. By symmetry across countries, this expression converges to $T_k^*(g_d) = 0.1076 + 0.0521T_i(g_d)$. Using the optimal tariff in country i, solving by substitution, and using symmetry, the following optimal tariffs are obtained for countries i, j, k and l: $T_i^*(g_d) = T_j^*(g_d) = 0.0788$; and $T_k^*(g_d) = T_l^*(g_d) = 0.1118$. In considering these tariffs, the following expressions hold: $CS_i(g_d) = CS_j(g_d) = 0.1024$; $CS_k(g_d)$ = $CS_l(g_d)$ = 0.0913 ; $\pi_i(g_d) = \pi_j(g_d) = 0.0920$; $\pi_k(g_d) = \pi_l(g_d) = 0.0869$; $PS_i(g_d)$ = $PS_j(g_d)$ = 0.0753; $PS_k(g_d) = PS_l(g_d) = 0.0698$; $TR_i(g_d) = TR_j(g_d) = 0.0065$; $TR_k(g_d)$ = $TR_l(g_d)$ = 0.0167; $W_i(g_d) = W_j(g_d) = 0.2762$; $W_k(g_d) = W_j(g_d) = 0.2647$.

<u>Network e</u>

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_e) = \frac{2\alpha(\phi+1) + 2T_i(g_e) + \phi\begin{pmatrix} q_j^{(j)}(g_e) + q_k^{(j)}(g_e) + q_l^{(j)}(g_e) + q_j^{(k)}(g_e) + q_k^{(k)}(g_e) \\ q_l^{(k)}(g_e) + q_j^{(l)}(g_e) + q_k^{(l)}(g_e) + q_l^{(l)}(g_e) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_e) + q_k^{(i)}(g_e) + q_l^{(i)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_e) = \frac{2\alpha(\phi+1) + 2T_j(g_e) + \phi\begin{pmatrix} q_i^{(j)}(g_e) + q_k^{(j)}(g_e) + q_l^{(j)}(g_e) + q_i^{(k)}(g_e) + q_k^{(k)}(g_e) \\ q_l^{(k)}(g_e) + q_i^{(l)}(g_e) + q_k^{(l)}(g_e) + q_l^{(l)}(g_e) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_e) + q_k^{(i)}(g_e) + q_l^{(i)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_e) = \frac{2\alpha(\phi+1) + 2T_k(g_e) + \phi\begin{pmatrix} q_i^{(j)}(g_e) + q_j^{(j)}(g_e) + q_l^{(j)}(g_e) + q_i^{(k)}(g_e) + q_j^{(k)}(g_e) \\ q_l^{(k)}(g_e) + q_i^{(l)}(g_e) + q_j^{(l)}(g_e) + q_l^{(l)}(g_e) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_e) + q_j^{(i)}(g_e) + q_l^{(i)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_e) = \frac{2\alpha(\phi+1) - 2(4+\phi)T_l(g_e) + \phi\begin{pmatrix} q_i^{(j)}(g_e) + q_j^{(j)}(g_e) + q_k^{(j)}(g_e) + q_i^{(k)}(g_e) + q_j^{(k)}(g_e) \\ q_k^{(k)}(g_e) + q_i^{(l)}(g_e) + q_j^{(l)}(g_e) + q_k^{(l)}(g_e) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_e) + q_j^{(i)}(g_e) + q_k^{(i)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_e) = \frac{2\alpha(\phi+1) + 2T_i(g_e) + \phi\begin{pmatrix} q_j^{(i)}(g_e) + q_k^{(i)}(g_e) + q_l^{(i)}(g_e) + q_j^{(k)}(g_e) + q_k^{(k)}(g_e) \\ q_l^{(k)}(g_e) + q_j^{(l)}(g_e) + q_k^{(l)}(g_e) + q_l^{(l)}(g_e) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(j)}(g_e) + q_k^{(j)}(g_e) + q_l^{(j)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_e)=\frac{2\alpha(\phi+1)+2T_j(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(k)}(g_e)+q_k^{(k)}(g_e)\\ q_l^{(k)}(g_e)+q_i^{(l)}(g_e)+q_k^{(l)}(g_e)+q_l^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_e)+q_k^{(j)}(g_e)+q_l^{(j)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_e)=\frac{2\alpha(\phi+1)-2(4+\phi)T_k(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(k)}(g_e)+q_j^{(k)}(g_e)\\ q_l^{(k)}(g_e)+q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_l^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_e)+q_j^{(j)}(g_e)+q_l^{(j)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_e)=\frac{2\alpha(\phi+1)+2T_l(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_i^{(k)}(g_e)+q_j^{(k)}(g_e)\\ q_k^{(k)}(g_e)+q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_k^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_e)+q_j^{(j)}(g_e)+q_k^{(j)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_e)=\frac{2\alpha(\phi+1)+2T_i(g_e)+\phi\begin{pmatrix}q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)+q_j^{(j)}(g_e)+q_k^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_j^{(l)}(g_e)+q_k^{(l)}(g_e)+q_l^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_e)+q_k^{(k)}(g_e)+q_l^{(k)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_e)=\frac{2\alpha(\phi+1)-2(4+\phi)T_j(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(j)}(g_e)+q_k^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_i^{(l)}(g_e)+q_k^{(l)}(g_e)+q_l^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_e)+q_k^{(k)}(g_e)+q_l^{(k)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_e)=\frac{2\alpha(\phi+1)+2T_k(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(j)}(g_e)+q_j^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_l^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_e)+q_j^{(k)}(g_e)+q_l^{(k)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_e)=\frac{2\alpha(\phi+1)+2T_l(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_i^{(j)}(g_e)+q_j^{(j)}(g_e)\\ q_k^{(j)}(g_e)+q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_k^{(l)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_e)+q_j^{(k)}(g_e)+q_k^{(k)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_e)=\frac{2\alpha(\phi+1)-2(4+\phi)T_i(g_e)+\phi\begin{pmatrix}q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)+q_j^{(j)}(g_e)+q_k^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_j^{(k)}(g_e)+q_k^{(k)}(g_e)+q_l^{(k)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_e)+q_k^{(l)}(g_e)+q_l^{(l)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_e)=\frac{2\alpha(\phi+1)+2T_j(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(j)}(g_e)+q_k^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_i^{(k)}(g_e)+q_k^{(k)}(g_e)+q_l^{(k)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_e)+q_k^{(l)}(g_e)+q_l^{(l)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_e)=\frac{2\alpha(\phi+1)+2T_k(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_l^{(i)}(g_e)+q_i^{(j)}(g_e)+q_j^{(j)}(g_e)\\ q_l^{(j)}(g_e)+q_i^{(k)}(g_e)+q_j^{(k)}(g_e)+q_l^{(k)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_l^{(l)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_e)=\frac{2\alpha(\phi+1)+2T_l(g_e)+\phi\begin{pmatrix}q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_i^{(j)}(g_e)+q_j^{(j)}(g_e)\\ q_k^{(j)}(g_e)+q_i^{(k)}(g_e)+q_j^{(k)}(g_e)+q_k^{(k)}(g_e)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_e)+q_j^{(l)}(g_e)+q_k^{(l)}(g_e)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: α = 1 and ϕ = 0

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_e)=\frac{2+2T_i(g_e)}{10}$; $q_j^{(i)}(g_e)=\frac{2+2T_j(g_e)}{10}$; $q_k^{(i)}(g_e)=\frac{2+2T_k(g_e)}{10}$; $q_l^{(i)}(g_e)=\frac{2-8T_l(g_e)}{10}$; $q_i^{(j)}(g_e)$ = $\frac{2+2T_i(g_e)}{10}$; $q_j^{(j)}(g_e)=\frac{2+2T_j(g_e)}{10}$; $q_k^{(j)}(g_e)=\frac{2-8T_k(g_e)}{10}$; $q_l^{(j)}(g_e)=\frac{2+2T_l(g_e)}{10}$; $q_i^{(k)}(g_e)=\frac{2+2T_i(g_e)}{10}$; $q_j^{(k)}(g_e)=\frac{2-8T_j(g_e)}{10}$; $q_k^{(k)}(g_e)=\frac{2+2T_k(g_e)}{10}$; $q_l^{(k)}(g_e)$ =

$\frac{2+2T_l(g_e)}{10}$; $q_i^{(l)}(g_e)=\frac{2-8T_i(g_e)}{10}$; $q_j^{(l)}(g_e)=\frac{2+2T_j(g_e)}{10}$; $q_k^{(l)}(g_e)=\frac{2+2T_k(g_e)}{10}$; and $q_l^{(l)}(g_e)=\frac{2+2T_l(g_e)}{10}$. From these outputs, the following expressions are obtained for country i:

$$CS_i(g_e)=\frac{1}{2}\left(q_i^{(i)}(g_e)+q_i^{(j)}(g_e)+q_i^{(k)}(g_e)+q_i^{(l)}(g_e)\right)^2=0.5\left(0.8000-0.2000T_i(g_e)\right)^2$$

$$\pi_i^{(i)}(g_e)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_e)\right)^2=\left(0.2000+0.2000T_i(g_e)\right)^2$$

$$\pi_j^{(i)}(g_e)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_e)\right)^2=\left(0.2000+0.2000T_j(g_e)\right)^2$$

$$\pi_k^{(i)}(g_e)=\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_e)\right)^2=\left(0.2000+0.2000T_k(g_e)\right)^2$$

$$\pi_l^{(i)}(g_e)=\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_e)\right)^2=\left(0.2000-0.8000T_l(g_e)\right)^2$$

$$PS_i(g_e)=\frac{\phi}{4}\left(q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)+q_l^{(i)}(g_e)\right)^2=0$$

$$TR_i(g_e)=T_i(g_e)\left(q_i^{(l)}(g_e)\right)=0.2000T_i(g_e)-0.8000T_i^2(g_e)$$

Therefore welfare in country i is given by:

$$\begin{aligned}W_i(g_e)&=CS_i(g_e)+\pi_i(g_e)+PS_i(g_e)+TR_i(g_e)=0.5\left(0.8000-0.2000T_i(g_e)\right)^2\\&+\left(0.2000+0.2000T_i(g_e)\right)^2+\left(0.2000+0.2000T_j(g_e)\right)^2+\left(0.2000+0.2000T_k(g_e)\right)^2\\&+\left(0.2000-0.8000T_l(g_e)\right)^2+0.2000T_i(g_e)-0.8000T_i^2(g_e)\end{aligned}$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_e)}{\partial T_i(g_e)} = (-0.2000)\big(0.8000 - 0.2000T_i(g_e)\big) + (2)(0.2000)\big(0.2000 + 0.2000T_i(g_e)\big)$$
$$+ 0.2000 - 1.6000T_i(g_e) = 0.1200 - 1.4800T_i(g_e)$$

$$\frac{\partial^2 W_i(g_e)}{\partial T_i^2(g_e)} = -1.4800$$

Given symmetry, it is concluded that the optimal tariffs in countries *i*, *j*, *k* and *l* are $T_i^*(g_e)$ = $T_j^*(g_e)$ = $T_k^*(g_e)$ = $T_l^*(g_e)$ = 0.0811. Using these tariffs, the following results are obtained: $CS_i(g_e) = CS_j(g_e) = CS_k(g_e) = CS_l(g_e) = 0.3072$; $\pi_i(g_e)$ = $\pi_j(g_e)$ = $\pi_k(g_e)$ = $\pi_l(g_e)$ = 0.1585; $PS_i(g_e) = PS_j(g_e) = PS_k(g_e) = PS_l(g_e) = 0$; $TR_i(g_e)$ = $TR_j(g_e)$ = $TR_k(g_e)$ = $TR_l(g_e)$ = 0.0110; $W_i(g_e) = W_j(g_e) = W_k(g_e) = W_l(g_e) = 0.4766$.

<u>*Simulation 5: α = 1 and ϕ = 0.5*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_e) = 0.1600 + 0.1321T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)$$

$$q_j^{(i)}(g_e) = 0.1600 - 0.0203T_i(g_e) + 0.1321T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)$$

$$q_k^{(i)}(g_e) = 0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) + 0.1321T_k(g_e) + 0.0686T_l(g_e)$$

$$q_l^{(i)}(g_e) = 0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.5790T_l(g_e)$$

$$q_i^{(j)}(g_e) = 0.1600 + 0.1321T_i(g_e) - 0.0203T_j(g_e) + 0.0686T_k(g_e) - 0.0203T_l(g_e)$$

$$q_j^{(j)}(g_e) = 0.1600 - 0.0203T_i(g_e) + 0.1321T_j(g_e) + 0.0686T_k(g_e) - 0.0203T_l(g_e)$$

$$q_k^{(j)}(g_e) = 0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) - 0.5790T_k(g_e) - 0.0203T_l(g_e)$$

$$q_l^{(j)}(g_e) = 0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) + 0.0686T_k(g_e) + 0.1321T_l(g_e)$$

$$q_i^{(k)}(g_e) = 0.1600 + 0.1321T_i(g_e) + 0.0686T_j(g_e) - 0.0203T_k(g_e) - 0.0203T_l(g_e)$$

$$q_j^{(k)}(g_e) = 0.1600 - 0.0203T_i(g_e) - 0.5790T_j(g_e) - 0.0203T_k(g_e) - 0.0203T_l(g_e)$$

$$q_k^{(k)}(g_e) = 0.1600 - 0.0203T_i(g_e) + 0.0686T_j(g_e) + 0.1321T_k(g_e) - 0.0203T_l(g_e)$$

$$q_l^{(k)}(g_e) = 0.1600 - 0.0203T_i(g_e) + 0.0686T_j(g_e) - 0.0203T_k(g_e) + 0.1321T_l(g_e)$$

$$q_i^{(l)}(g_e) = 0.1600 - 0.5790T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.0203T_l(g_e)$$

$$q_j^{(l)}(g_e) = 0.1600 + 0.0686T_i(g_e) + 0.1321T_j(g_e) - 0.0203T_k(g_e) - 0.0203T_l(g_e)$$

$$q_k^{(l)}(g_e) = 0.1600 + 0.0686T_i(g_e) - 0.0203T_j(g_e) + 0.1321T_k(g_e) - 0.0203T_l(g_e)$$

$$q_l^{(l)}(g_e) = 0.1600 + 0.0686T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) + 0.1321T_l(g_e)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_e) = \frac{1}{2}\left(q_i^{(i)}(g_e) + q_i^{(j)}(g_e) + q_i^{(k)}(g_e) + q_i^{(l)}(g_e)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_e) + 0.0076T_j(g_e) + 0.0076T_k(g_e) + 0.0076T_l(g_e)\right)^2$$

$$\pi_i^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_e)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)^2$$

$$\pi_j^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_e)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_e) + 0.1321T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)^2$$

$$\pi_k^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_e)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) + 0.1321T_k(g_e) + 0.0686T_l(g_e)\right)^2$$

$$\pi_l^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_e)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.5790T_l(g_e)\right)^2$$

$$PS_i(g_e) = \frac{\phi}{4}\left(q_i^{(i)}(g_e) + q_j^{(i)}(g_e) + q_k^{(i)}(g_e) + q_l^{(i)}(g_e)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_e) + 0.0711T_j(g_e) + 0.0711T_k(g_e) - 0.3733T_l(g_e)\right)^2$$

$$TR_i(g_e) = T_i(g_e)\left(q_i^{(l)}(g_e)\right)$$
$$= 0.1600T_i(g_e) - 0.5790T_i^2(g_e) - 0.0203T_j(g_e)T_i(g_e)$$
$$- 0.0203T_k(g_e)T_i(g_e) - 0.0203T_l(g_e)T_i(g_e)$$

Therefore welfare in country *i* is given by:

$$W_i(g_e) = CS_i(g_e) + \pi_i(g_e) + PS_i(g_e) + TR_i(g_e)$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_e) + 0.0076T_j(g_e) + 0.0076T_k(g_e) + 0.0076T_l(g_e)\right)^2$$
$$+ 1.25\left(0.1600 + 0.1321T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_e) + 0.1321T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) + 0.1321T_k(g_e) + 0.0686T_l(g_e)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.5790T_l(g_e)\right)^2$$
$$+ 0.125\left(0.6400 + 0.0711T_i(g_e) + 0.0711T_j(g_e) + 0.0711T_k(g_e) - 0.3733T_l(g_e)\right)^2$$
$$+ 0.1600T_i(g_e) - 0.5790T_i^2(g_e) - 0.0203T_j(g_e)T_i(g_e)$$
$$- 0.0203T_k(g_e)T_i(g_e) - 0.0203T_l(g_e)T_i(g_e)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_e)}{\partial T_i(g_e)} =$$

$$(-0.1829)\left(0.6400 - 0.1829T_i(g_e) + 0.0076T_j(g_e) + 0.0076T_k(g_e) + 0.0076T_l(g_e)\right)$$
$$+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_e) + 0.1321T_j(g_e) - 0.0203T_k(g_e) + 0.0686T_l(g_e)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) + 0.1321T_k(g_e) + 0.0686T_l(g_e)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.5790T_l(g_e)\right)$$
$$+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_e) + 0.0711T_j(g_e) + 0.0711T_k(g_e) - 0.3733T_l(g_e)\right)$$
$$+ 0.1600 - 1.158T_i(g_e) - 0.0203T_j(g_e) - 0.0203T_k(g_e) - 0.0203T_l(g_e)$$
$$= 0.0828 - 1.0767T_i(g_e) - 0.0318T_j(g_e) - 0.0318T_k(g_e) + 0.0167T_l(g_e)$$

$$\frac{\partial^2 W_i(g_e)}{\partial T_i^2(g_e)} = -1.0767$$

Consequently the optimal tariff in country i is $T_i^*(g_e)$ = 0.0769 – $0.0295T_j(g_e)$ – $0.0295T_k(g_e)$ + $0.0155T_l(g_e)$. Using symmetry it is inferred that the optimal tariffs in countries i, j, k and l are: $T_i^*(g_e) = T_j^*(g_e) = T_k^*(g_e) = T_l^*(g_e) = 0.0737$. Using these tariffs the following results were obtained: $CS_i(g_e) = CS_j(g_e) = CS_k(g_e) = CS_l(g_e) = 0.1973$; $\pi_i(g_e)$ = $\pi_j(g_e)$ = $\pi_k(g_e)$ = $\pi_l(g_e)$ = 0.1266; $PS_i(g_e)$ = $PS_j(g_e)$ = $PS_k(g_e)$ = $PS_l(g_e)$ = 0.0493; $TR_i(g_e) = TR_j(g_e) = TR_k(g_e) = TR_l(g_e) = 0.0083$; $W_i(g_e)$ = $W_j(g_e)$ = $W_k(g_e)$ = $W_l(g_e)$ = 0.3816.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_e) = 0.1143 + 0.0805T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)$$

$$q_j^{(i)}(g_e) = 0.1143 - 0.0188T_i(g_e) + 0.0805T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)$$

$$q_k^{(i)}(g_e) = 0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) + 0.0805T_k(g_e) + 0.0714T_l(g_e)$$

$$q_l^{(i)}(g_e) = 0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.4007T_l(g_e)$$

$$q_i^{(j)}(g_e) = 0.1143 + 0.0805T_i(g_e) - 0.0188T_j(g_e) + 0.0714T_k(g_e) - 0.0188T_l(g_e)$$

$$q_j^{(j)}(g_e) = 0.1143 - 0.0188T_i(g_e) + 0.0805T_j(g_e) + 0.0714T_k(g_e) - 0.0805T_l(g_e)$$

$$q_k^{(j)}(g_e) = 0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) - 0.4007T_k(g_e) - 0.0188T_l(g_e)$$

$$q_l^{(j)}(g_e) = 0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) + 0.0714T_k(g_e) + 0.0805T_l(g_e)$$

$$q_i^{(k)}(g_e) = 0.1143 + 0.0805T_i(g_e) + 0.0714T_j(g_e) - 0.0188T_k(g_e) - 0.0188T_l(g_e)$$

$$q_j^{(k)}(g_e) = 0.1143 - 0.0188T_i(g_e) - 0.4007T_j(g_e) - 0.0188T_k(g_e) - 0.0188T_l(g_e)$$

$$q_k^{(k)}(g_e) = 0.1143 - 0.0188T_i(g_e) + 0.0714T_j(g_e) + 0.0805T_k(g_e) - 0.0188T_l(g_e)$$

$$q_l^{(k)}(g_e) = 0.1143 - 0.0188T_i(g_e) + 0.0714T_j(g_e) - 0.0188T_k(g_e) + 0.0805T_l(g_e)$$

$$q_i^{(l)}(g_e) = 0.1143 - 0.4007T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.0188T_l(g_e)$$

$$q_j^{(l)}(g_e) = 0.1143 + 0.0714T_i(g_e) + 0.0805T_j(g_e) - 0.0188T_k(g_e) - 0.0188T_l(g_e)$$

$$q_k^{(l)}(g_e) = 0.1143 + 0.0714T_i(g_e) - 0.0188T_j(g_e) + 0.0805T_k(g_e) - 0.0188T_l(g_e)$$

$$q_l^{(l)}(g_e) = 0.1143 + 0.0714T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) + 0.0805T_l(g_e)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_e) = \frac{1}{2}\left(q_i^{(i)}(g_e) + q_i^{(j)}(g_e) + q_i^{(k)}(g_e) + q_i^{(l)}(g_e)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_e) + 0.0149T_j(g_e) + 0.0149T_k(g_e) + 0.0149T_l(g_e)\right)^2$$

$$\pi_i^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_e)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right)^2$$

$$\pi_j^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_e)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_e) + 0.0805T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right)^2$$

$$\pi_k^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_e)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) + 0.0805T_k(g_e) + 0.0714T_l(g_e)\right)^2$$

$$\pi_l^{(i)}(g_e) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_e)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.4007T_l(g_e)\right)^2$$

$$PS_i(g_e) = \frac{\phi}{4}\left(q_i^{(i)}(g_e) + q_j^{(i)}(g_e) + q_k^{(i)}(g_e) + q_l^{(i)}(g_e)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_e) + 0.0241T_j(g_e) + 0.0241T_k(g_e) - 0.1865T_l(g_e)\right)^2$$

$$TR_i(g_e) = T_i(g_e)\left(q_i^{(l)}(g_e)\right)$$
$$= 0.1143T_i(g_e) - 0.4007T_i^2(g_e) - 0.0188T_j(g_e)T_i(g_e)$$
$$- 0.0188T_k(g_e)T_i(g_e) - 0.0188T_l(g_e)T_i(g_e)$$

Therefore welfare in country *i* is given by:

$$
\begin{aligned}
&W_i(g_e) = CS_i(g_e) + \pi_i(g_e) + PS_i(g_e) + TR_i(g_e) \\
&= 0.5\left(0.4571 - 0.1590T_i(g_e) + 0.0149T_j(g_e) + 0.0149T_k(g_e) + 0.0149T_l(g_e)\right)^2 \\
&+ 1.75\left(0.1143 + 0.0805T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_e) + 0.0805T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) + 0.0805T_k(g_e) + 0.0714T_l(g_e)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.4007T_l(g_e)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0241T_i(g_e) + 0.0241T_j(g_e) + 0.0241T_k(g_e) - 0.1865T_l(g_e)\right)^2 \\
&+ 0.1143T_i(g_e) - 0.4007T_i^2(g_e) - 0.0188T_j(g_e)T_i(g_e) \\
&- 0.0188T_k(g_e)T_i(g_e) - 0.0188T_l(g_e)T_i(g_e)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_i(g_e)}{\partial T_i(g_e)} = \\
&= (-0.1590)\left(0.4571 - 0.1590T_i(g_e) + 0.0149T_j(g_e) + 0.0149T_k(g_e) + 0.0149T_l(g_e)\right) \\
&+ (3.5)(0.0805)\left(0.1143 + 0.0805T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right) \\
&+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_e) + 0.0805T_j(g_e) - 0.0188T_k(g_e) + 0.0714T_l(g_e)\right) \\
&+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) + 0.0805T_k(g_e) + 0.0714T_l(g_e)\right) \\
&+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.4007T_l(g_e)\right) \\
&+ (0.75)(0.0241)\left(0.4571 + 0.0241T_i(g_e) + 0.0241T_j(g_e) + 0.0241T_k(g_e) - 0.1865T_l(g_e)\right) \\
&+ 0.1143 - 0.8014T_i(g_e) - 0.0188T_j(g_e) - 0.0188T_k(g_e) - 0.0188T_l(g_e) \\
&= 0.0595 - 0.7492T_i(g_e) - 0.0289T_j(g_e) - 0.0289T_k(g_e) + 0.0125T_l(g_e)
\end{aligned}
$$

$$
\frac{\partial^2 W_i(g_e)}{\partial T_i^2(g_e)} = -0.7492
$$

Therefore the optimal tariff in country i is $T_i^*(g_e)$ = 0.0794 – $0.0386T_j(g_e)$ – $0.0386T_k(g_e)$ + $0.0167T_l(g_e)$. Using symmetry across countries it is concluded that the optimal tariffs in countries i, j, k and l are $T_i^*(g_e) = T_j^*(g_e) = T_k^*(g_e) = T_l^*(g_e) = 0.0749$.

Using these tariffs, the following results are obtained: $CS_i(g_e)$ = $CS_j(g_e)$ = $CS_k(g_e)$ = $CS_l(g_e)$ = 0.1006; $\pi_i(g_e)=\pi_j(g_e)=\pi_k(g_e)=\pi_l(g_e)=0.0904$; $PS_i(g_e)$ = $PS_j(g_e)$ = $PS_k(g_e)$ = $PS_l(g_e)$ = 0.0755; $TR_i(g_e)=TR_j(g_e)=TR_k(g_e)=TR_l(g_e)=0.0060$; and $W_i(g_e)=W_j(g_e)=W_k(g_e)=W_l(g_e)=0.2725$.

Network f

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_f)=\frac{2\alpha(\phi+1)+2T_i(g_f)+\phi\begin{pmatrix}q_j^{(j)}(g_f)+q_k^{(j)}(g_f)+q_l^{(j)}(g_f)+q_j^{(k)}(g_f)+q_k^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_f)=\frac{2\alpha(\phi+1)+4T_j(g_f)+\phi\begin{pmatrix}q_i^{(j)}(g_f)+q_k^{(j)}(g_f)+q_l^{(j)}(g_f)+q_i^{(k)}(g_f)+q_k^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_i^{(l)}(g_f)+q_k^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_f)=\frac{2\alpha(\phi+1)+4T_k(g_f)+\phi\begin{pmatrix}q_i^{(j)}(g_f)+q_j^{(j)}(g_f)+q_l^{(j)}(g_f)+q_i^{(k)}(g_f)+q_j^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_i^{(l)}(g_f)+q_j^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_l^{(i)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_f)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g_f)+\phi\begin{pmatrix}q_i^{(j)}(g_f)+q_j^{(j)}(g_f)+q_k^{(j)}(g_f)+q_i^{(k)}(g_f)+q_j^{(k)}(g_f)\\ q_k^{(k)}(g_f)+q_i^{(l)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_k^{(i)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_f)=\frac{2\alpha(\phi+1)+2T_i(g_f)+\phi\begin{pmatrix}q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)+q_j^{(k)}(g_f)+q_k^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_f)+q_k^{(j)}(g_f)+q_l^{(j)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_f)=\frac{2\alpha(\phi+1)+4T_j(g_f)+\phi\begin{pmatrix}q_i^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)+q_i^{(k)}(g_f)+q_k^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_i^{(l)}(g_f)+q_k^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_f)+q_k^{(j)}(g_f)+q_l^{(j)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_f)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_f)+\phi\begin{pmatrix}q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_l^{(i)}(g_f)+q_i^{(k)}(g_f)+q_j^{(k)}(g_f)\\ q_l^{(k)}(g_f)+q_i^{(l)}(g_f)+q_j^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_f)+q_j^{(j)}(g_f)+q_l^{(j)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_f)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g_f)+\phi\begin{pmatrix}q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_i^{(k)}(g_f)+q_j^{(k)}(g_f)\\ q_k^{(k)}(g_f)+q_i^{(l)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_f)+q_j^{(j)}(g_f)+q_k^{(j)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_f)=\frac{2\alpha(\phi+1)+2T_i(g_f)+\phi\begin{pmatrix}q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)+q_j^{(j)}(g_f)+q_k^{(j)}(g_f)\\ q_l^{(j)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)+q_l^{(l)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_f)+q_k^{(k)}(g_f)+q_l^{(k)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_f) = \frac{2\alpha(\phi+1) - 2(3+\phi)T_j(g_f) + \phi\begin{pmatrix} q_i^{(i)}(g_f) + q_k^{(i)}(g_f) + q_l^{(i)}(g_f) + q_i^{(j)}(g_f) + q_k^{(j)}(g_f) \\ q_l^{(j)}(g_f) + q_i^{(l)}(g_f) + q_k^{(l)}(g_f) + q_l^{(l)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_f) + q_k^{(k)}(g_f) + q_l^{(k)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_f) = \frac{2\alpha(\phi+1) + 4T_k(g_f) + \phi\begin{pmatrix} q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_l^{(i)}(g_f) + q_i^{(j)}(g_f) + q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) + q_i^{(l)}(g_f) + q_j^{(l)}(g_f) + q_l^{(l)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_f) + q_j^{(k)}(g_f) + q_l^{(k)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_f) = \frac{2\alpha(\phi+1) - 2(2+\phi)T_l(g_f) + \phi\begin{pmatrix} q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_k^{(i)}(g_f) + q_i^{(j)}(g_f) + q_j^{(j)}(g_f) \\ q_k^{(j)}(g_f) + q_i^{(l)}(g_f) + q_j^{(l)}(g_f) + q_k^{(l)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_f) + q_j^{(k)}(g_f) + q_k^{(k)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_f) = \frac{2\alpha(\phi+1) - 2(4+\phi)T_i(g_f) + \phi\begin{pmatrix} q_j^{(i)}(g_f) + q_k^{(i)}(g_f) + q_l^{(i)}(g_f) + q_j^{(j)}(g_f) + q_k^{(j)}(g_f) \\ q_l^{(j)}(g_f) + q_j^{(k)}(g_f) + q_k^{(k)}(g_f) + q_l^{(k)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(l)}(g_f) + q_k^{(l)}(g_f) + q_l^{(l)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_f) = \frac{2\alpha(\phi+1) - 2(3+\phi)T_j(g_f) + \phi\begin{pmatrix} q_i^{(i)}(g_f) + q_k^{(i)}(g_f) + q_l^{(i)}(g_f) + q_i^{(j)}(g_f) + q_k^{(j)}(g_f) \\ q_l^{(j)}(g_f) + q_i^{(k)}(g_f) + q_k^{(k)}(g_f) + q_l^{(k)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(l)}(g_f) + q_k^{(l)}(g_f) + q_l^{(l)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_f) = \frac{2\alpha(\phi+1) - 2(3+\phi)T_k(g_f) + \phi\begin{pmatrix} q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_l^{(i)}(g_f) + q_i^{(j)}(g_f) + q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) + q_i^{(k)}(g_f) + q_j^{(k)}(g_f) + q_l^{(k)}(g_f) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(l)}(g_f) + q_j^{(l)}(g_f) + q_l^{(l)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_f)=\frac{2\alpha(\phi+1)+6T_l(g_f)+\phi\begin{pmatrix}q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_i^{(j)}(g_f)+q_j^{(j)}(g_f)\\ q_k^{(j)}(g_f)+q_i^{(k)}(g_f)+q_j^{(k)}(g_f)+q_k^{(k)}(g_f)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_f)+q_j^{(l)}(g_f)+q_k^{(l)}(g_f)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: α = 1 and ϕ = 0

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_f)=\frac{2+2T_i(g_f)}{10}$; $q_j^{(i)}(g_f)=\frac{2+4T_j(g_f)}{10}$; $q_k^{(i)}(g_f)=\frac{2+4T_k(g_f)}{10}$; $q_l^{(i)}(g_f)=\frac{2-4T_l(g_f)}{10}$; $q_i^{(j)}(g_f)=\frac{2+2T_i(g_f)}{10}$; $q_j^{(j)}(g_f)=\frac{2+4T_j(g_f)}{10}$; $q_k^{(j)}(g_f)=\frac{2-6T_k(g_f)}{10}$; $q_l^{(j)}(g_f)=\frac{2-4T_l(g_f)}{10}$; $q_i^{(k)}(g_f)=\frac{2+2T_i(g_f)}{10}$; $q_j^{(k)}(g_f)=\frac{2-6T_j(g_f)}{10}$; $q_k^{(k)}(g_f)=\frac{2+4T_k(g_f)}{10}$; $q_l^{(k)}(g_f)=\frac{2-4T_l(g_f)}{10}$; $q_i^{(l)}(g_f)=\frac{2-8T_i(g_f)}{10}$; $q_j^{(l)}(g_f)=\frac{2-6T_j(g_f)}{10}$; $q_k^{(l)}(g_f)=\frac{2-6T_k(g_f)}{10}$; $q_l^{(l)}(g_f)=\frac{2+6T_l(g_f)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_f)=\frac{1}{2}\left(q_i^{(i)}(g_f)+q_i^{(j)}(g_f)+q_i^{(k)}(g_f)+q_i^{(l)}(g_f)\right)^2=0.5\left(0.8000-0.2000T_i(g_f)\right)^2$$

$$\pi_i^{(i)}(g_f)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_f)\right)^2=\left(0.2000+0.2000T_i(g_f)\right)^2$$

$$\pi_j^{(i)}(g_f)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_f)\right)^2=\left(0.2000+0.4000T_j(g_f)\right)^2$$

$$\pi_k^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_f)\right)^2 = \left(0.2000 + 0.4000T_k(g_f)\right)^2$$

$$\pi_l^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_f)\right)^2 = \left(0.2000 - 0.4000T_l(g_f)\right)^2$$

$$PS_i(g_f) = \frac{\phi}{4}\left(q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_k^{(i)}(g_f) + q_l^{(i)}(g_f)\right)^2 = 0$$

$$TR_i(g_f) = T_i(g_f)\left(q_i^{(l)}(g_f)\right) = 0.2000T_i(g_f) - 0.8000T_i^2(g_f)$$

Therefore welfare in country *i* is given by:

$$\begin{aligned} W_i(g_f) &= CS_i(g_f) + \pi_i(g_f) + PS_i(g_f) + TR_i(g_f) = 0.5\left(0.8000 - 0.2000T_i(g_f)\right)^2 \\ &+ \left(0.2000 + 0.2000T_i(g_f)\right)^2 + \left(0.2000 + 0.4000T_j(g_f)\right)^2 + \left(0.2000 + 0.4000T_k(g_f)\right)^2 \\ &+ \left(0.2000 - 0.4000T_l(g_f)\right)^2 + 0.2000T_i(g_f) - 0.8000T_i^2(g_f) \end{aligned}$$

The first and second order conditions of this function are:

$$\begin{aligned} \frac{\partial W_i(g_f)}{\partial T_i(g_f)} &= (-0.2000)\left(0.8000 - 0.2000T_i(g_f)\right) + (2)(0.2000)\left(0.2000 + 0.2000T_i(g_f)\right) \\ &+ 0.2000 - 1.6000T_i(g_f) = 0.1200 - 1.4800T_i(g_f) \end{aligned}$$

$$\frac{\partial^2 W_i(g_f)}{\partial T_i^2(g_f)} = -1.48$$

Therefore the optimal tariff in country *i* is $T_i^*(g_f) = 0.0811$.

On the other hand, the following expressions are obtained for country *j*:

$$CS_j(g_f) = \frac{1}{2}\left(q_j^{(i)}(g_f) + q_j^{(j)}(g_f) + q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right)^2 = 0.5\left(0.8000 - 0.4000T_j(g_f)\right)^2$$

$$\pi_i^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_f)\right)^2 = \left(0.2000 + 0.2000T_i(g_f)\right)^2$$

$$\pi_j^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_f)\right)^2 = \left(0.2000 + 0.4000T_j(g_f)\right)^2$$

$$\pi_k^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_f)\right)^2 = \left(0.2000 - 0.6000T_k(g_f)\right)^2$$

$$\pi_l^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_f)\right)^2 = \left(0.2000 - 0.4000T_l(g_f)\right)^2$$

$$PS_j(g_f) = 0$$

$$TR_j(g_f) = T_j(g_f)\left(q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right) = 0.4000T_j(g_f) - 1.2000T_j^2(g_f)$$

Therefore welfare in country j is given by:

$$W_j(g_f) = CS_j(g_f) + \pi_j(g_f) + PS_j(g_f) + TR_j(g_f) = 0.5\left(0.8000 - 0.4000T_j(g_f)\right)^2$$
$$+\left(0.2000 + 0.2000T_i(g_f)\right)^2 + \left(0.2000 + 0.4000T_j(g_f)\right)^2 + \left(0.2000 - 0.6000T_k(g_f)\right)^2$$
$$+\left(0.2000 - 0.4000T_l(g_f)\right)^2 + 0.4000T_j(g_f) - 1.2000T_j^2(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_j(g_f)}{\partial T_j(g_f)} = (-0.4000)\left(0.8000 - 0.4000T_j(g_f)\right) + (2)(0.4000)\left(0.2000 + 0.4000T_j(g_f)\right)$$
$$+ 0.4000 - 2.4000T_j(g_f) = 0.2400 - 1.9200T_j(g_f)$$

$$\frac{\partial^2 W_j(g_f)}{\partial T_j^2(g_f)} = -1.9200$$

Therefore the optimal tariff in country j is $T_j^*(g_f) = 0.1250$. Given symmetry across countries, it is concluded that $T_j^*(g_f) = T_k^*(g_f) = 0.1250$.

Finally, the following expressions are obtained for country l:

$$CS_l(g_f) = \frac{1}{2}\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f) + q_l^{(l)}(g_f)\right)^2 = 0.5\left(0.8000 - 0.6000T_l(g_f)\right)^2$$

$$\pi_i^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_f)\right)^2 = \left(0.2000 - 0.8000T_i(g_f)\right)^2$$

$$\pi_j^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_f)\right)^2 = \left(0.2000 - 0.6000T_j(g_f)\right)^2$$

$$\pi_k^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_f)\right)^2 = \left(0.2000 - 0.6000T_k(g_f)\right)^2$$

$$\pi_l^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_f)\right)^2 = \left(0.2000 + 0.6000T_l(g_f)\right)^2$$

$$PS_l(g_f) = 0$$

$$TR_l(g_f) = T_l(g_f)\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f)\right)$$
$$= 0.6000T_l(g_f) - 1.2000T_l^2(g_f)$$

Therefore welfare in country l is given by:

$$W_l(g_f) = CS_l(g_f) + \pi_l(g_f) + PS_l(g_f) + TR_l(g_f) = 0.5\left(0.8000 - 0.6000T_l(g_f)\right)^2$$
$$+\left(0.2000 - 0.8000T_i(g_f)\right)^2 + \left(0.2000 - 0.6000T_j(g_f)\right)^2 + \left(0.2000 - 0.6000T_k(g_f)\right)^2$$
$$+\left(0.2000 + 0.6000T_l(g_f)\right)^2 + 0.6000T_l(g_f) - 1.2000T_l^2(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_f)}{\partial T_l(g_f)} = (-0.6000)\left(0.8000 - 0.6000T_l(g_f)\right) + (2)(0.6000)\left(0.2000 + 0.6000T_l(g_f)\right)$$
$$+0.6000 - 2.4000T_l(g_f) = 0.3600 - 1.3200T_l(g_f)$$

$$\frac{\partial^2 W_l(g_f)}{\partial T_l^2(g_f)} = -1.3200$$

Therefore the optimal tariff in country *l* is $T_l^*(g_f) = 0.2727$.

In considering the optimal tariffs in countries *i*, *j*, *k* and *l*, the following results are obtained: $CS_i(g_f) = 0.3072$; $CS_j(g_f) = CS_k(g_f) = 0.2813$; $CS_l(g_f) = 0.2025$; $\pi_i(g_f) = 0.1800$; $\pi_j(g_f)$ = $\pi_k(g_f)$ = 0.1331; $\pi_l(g_f) = 0.1817$; $PS_i(g_f)$ = $PS_j(g_f)$ = $PS_k(g_f)$ = $PS_l(g_f)$ = 0; $TR_i(g_f) = 0.0110$; $TR_j(g_f) = TR_k(g_f) = 0.0313$; and $TR_l(g_f) = 0.0744$; $W_i(g_f)$ = 0.4981 ; $W_j(g_f) = W_k(g_f) = 0.4456$; $W_l(g_f) = 0.4586$.

<u>Simulation 5: α = 1 and ϕ = 0.5</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_f) = 0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)$$

$$q_j^{(i)}(g_f) = 0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)$$

$$q_k^{(i)}(g_f) = 0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.2641T_k(g_f) + 0.0279T_l(g_f)$$

$$q_l^{(i)}(g_f) = 0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.0406T_k(g_f) - 0.3149T_l(g_f)$$

$$q_i^{(j)}(g_f) = 0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)$$

$$q_j^{(j)}(g_f) = 0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)$$

$$q_k^{(j)}(g_f) = 0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.4470T_k(g_f) + 0.0279T_l(g_f)$$

$$q_l^{(j)}(g_f) = 0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) - 0.3149T_l(g_f)$$

$$q_i^{(k)}(g_f) = 0.1600 + 0.1321T_i(g_f) + 0.0483T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)$$

$$q_j^{(k)}(g_f) = 0.1600 - 0.0203T_i(g_f) - 0.4470T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)$$

$$q_k^{(k)}(g_f) = 0.1600 - 0.0203T_i(g_f) + 0.0483T_j(g_f) + 0.2641T_k(g_f) + 0.0279T_l(g_f)$$

$$q_l^{(k)}(g_f) = 0.1600 - 0.0203T_i(g_f) + 0.0483T_j(g_f) - 0.0406T_k(g_f) - 0.3149T_l(g_f)$$

$$q_i^{(l)}(g_f) = 0.1600 - 0.5790T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)$$

$$q_j^{(l)}(g_f) = 0.1600 + 0.686T_i(g_f) - 0.4470T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)$$

$$q_k^{(l)}(g_f) = 0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) - 0.4470T_k(g_f) - 0.0610T_l(g_f)$$

$$q_l^{(l)}(g_f) = 0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) + 0.3962T_l(g_f)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_f) = \frac{1}{2}\left(q_i^{(i)}(g_f) + q_i^{(j)}(g_f) + q_i^{(k)}(g_f) + q_i^{(l)}(g_f)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_f) + 0.0152T_j(g_f) + 0.0152T_k(g_f) + 0.0229T_l(g_f)\right)^2$$

$$\pi_i^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)\right)^2$$

$$\pi_j^{(i)}(g_f)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_f)\right)^2$$
$$=1.25\left(0.1600-0.0203T_i(g_f)+0.2641T_j(g_f)-0.0406T_k(g_f)+0.0279T_l(g_f)\right)^2$$

$$\pi_k^{(i)}(g_f)=\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_f)\right)^2$$
$$=1.25\left(0.1600-0.0203T_i(g_f)-0.0406T_j(g_f)+0.2641T_k(g_f)+0.0279T_l(g_f)\right)^2$$

$$\pi_l^{(i)}(g_f)=\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_f)\right)^2$$
$$=1.25\left(0.1600-0.0203T_i(g_f)-0.0406T_j(g_f)-0.0406T_k(g_f)-0.3149T_l(g_f)\right)^2$$

$$PS_i(g_f)=\frac{\phi}{4}\left(q_i^{(i)}(g_f)+q_j^{(i)}(g_f)+q_k^{(i)}(g_f)+q_l^{(i)}(g_f)\right)^2$$
$$=0.125\left(0.6400+0.0711T_i(g_f)+0.1422T_j(g_f)+0.1422T_k(g_f)-0.2311T_l(g_f)\right)^2$$

$$TR_i(g_f)=T_i(g_f)\left(q_i^{(l)}(g_f)\right)=0.1600T_i(g_f)-0.5790T_i^2(g_f)+0.0483T_j(g_f)T_i(g_f)$$
$$+0.0483T_k(g_f)T_i(g_f)-0.0610T_l(g_f)T_i(g_f)$$

Therefore welfare in country *i* is given by:

$$W_i(g_f)=CS_i(g_f)+\pi_i(g_f)+PS_i(g_f)+TR_i(g_f)$$
$$=0.5\left(0.6400-0.1829T_i(g_f)+0.0152T_j(g_f)+0.0152T_k(g_f)+0.0229T_l(g_f)\right)^2$$
$$+1.25\left(0.1600+0.1321T_i(g_f)-0.0406T_j(g_f)-0.0406T_k(g_f)+0.0279T_l(g_f)\right)^2$$
$$+1.25\left(0.1600-0.0203T_i(g_f)+0.2641T_j(g_f)-0.0406T_k(g_f)+0.0279T_l(g_f)\right)^2$$
$$+1.25\left(0.1600-0.0203T_i(g_f)-0.0406T_j(g_f)+0.2641T_k(g_f)+0.0279T_l(g_f)\right)^2$$
$$+1.25\left(0.1600-0.0203T_i(g_f)-0.0406T_j(g_f)-0.0406T_k(g_f)-0.3149T_l(g_f)\right)^2$$
$$+0.125\left(0.6400+0.0711T_i(g_f)+0.1422T_j(g_f)+0.1422T_k(g_f)-0.2311T_l(g_f)\right)^2$$
$$+0.1600T_i(g_f)-0.5790T_i^2(g_f)+0.0483T_j(g_f)T_i(g_f)$$
$$+0.0483T_k(g_f)T_i(g_f)-0.0610T_l(g_f)T_i(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_f)}{\partial T_i(g_f)} =$$
$$(-0.1829)\left(0.6400 - 0.1829T_i(g_f) + 0.0152T_j(g_f) + 0.0152T_k(g_f) + 0.0229T_l(g_f)\right)$$
$$+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) - 0.0406T_k(g_f) + 0.0279T_l(g_f)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.2641T_k(g_f) + 0.0279T_l(g_f)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.0406T_k(g_f) - 0.3149T_l(g_f)\right)$$
$$+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_f) + 0.1422T_j(g_f) + 0.1422T_k(g_f) - 0.2311T_l(g_f)\right)$$
$$+ 0.1600 - 1.1580T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)$$
$$= 0.0828 - 1.0767T_i(g_f) + 0.0253T_j(g_f) + 0.0253T_k(g_f) - 0.0469T_l(g_f)$$

$$\frac{\partial^2 W_i(g_f)}{\partial T_i^2(g_f)} = -1.0767$$

Therefore the optimal tariff in country i is: $T_i^*(g_f) = 0.0769 + 0.0235T_j(g_f) + 0.0235T_k(g_f) - 0.0435T_l(g_f)$. Given symmetry across counties, this expression converges to $T_i^*(g_f) = 0.0769 + 0.0470T_j(g_f) - 0.0435T_l(g_f)$.

On the other hand, the following expressions are obtained for country j:

$$CS_j(g_f) = \frac{1}{2}\left(q_j^{(i)}(g_f) + q_j^{(j)}(g_f) + q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right)^2$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_f) - 0.3657T_j(g_f) + 0.0152T_k(g_f) + 0.0229T_l(g_f)\right)^2$$

$$\pi_i^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\right)^2$$

$$\pi_j^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\right)^2$$

$$\pi_k^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.4470T_k(g_f) + 0.0279T_l(g_f)\right)^2$$

$$\pi_l^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) - 0.3149T_l(g_f)\right)^2$$

$$PS_j(g_f) = \frac{\phi}{4}\left(q_i^j(g_f) + q_j^j(g_f) + q_k^j(g_f) + q_l^j(g_f)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_f) + 0.1422T_j(g_f) - 0.3022T_k(g_f) - 0.2311T_l(g_f)\right)^2$$

$$TR_j(g_f) = T_j(g_f)\left(q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right)$$
$$= 0.3200T_j(g_f) + 0.0483T_i(g_f)T_j(g_f) - 0.8940T_j^2(g_f)$$
$$+ 0.0076T_k(g_f)T_j(g_f) - 0.0330T_l(g_f)T_j(g_f)$$

Therefore welfare in country *j* is given by:

$$W_j(g_f) = CS_j(g_f) + \pi_j(g_f) + PS_j(g_f) + TR_j(g_f)$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_f) - 0.3657T_j(g_f) + 0.0152T_k(g_f) + 0.0229T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.4470T_k(g_f) + 0.0279T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) - 0.3149T_l(g_f)\right)^2$$
$$+ 0.125\left(0.6400 + 0.0711T_i(g_f) + 0.1422T_j(g_f) - 0.3022T_k(g_f) - 0.2311T_l(g_f)\right)^2$$
$$+ 0.3200T_j(g_f) + 0.0483T_i(g_f)T_j(g_f) - 0.8940T_j^2(g_f)$$
$$+ 0.0076T_k(g_f)T_j(g_f) - 0.0330T_l(g_f)T_j(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_j(g_f)}{\partial T_j(g_f)} =$$
$$= (-0.3657)\big(0.6400 + 0.0076T_i(g_f) - 0.3657T_j(g_f) + 0.0152T_k(g_f) + 0.0229T_l(g_f)\big)$$
$$+ (2.5)(-0.0406)\big(0.1600 + 0.1321T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\big)$$
$$+ (2.5)(0.2641)\big(0.1600 - 0.0203T_i(g_f) + 0.2641T_j(g_f) + 0.0483T_k(g_f) + 0.0279T_l(g_f)\big)$$
$$+ (2.5)(-0.0406)\big(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) - 0.4470T_k(g_f) + 0.0279T_l(g_f)\big)($$
$$+ (2.5)(-0.0406)\big(0.1600 - 0.0203T_i(g_f) - 0.0406T_j(g_f) + 0.0483T_k(g_f) - 0.3149T_l(g_f)\big)$$
$$+ (0.25)(0.1422)\big(0.6400 + 0.0711T_i(g_f) + 0.1422T_j(g_f) - 0.3022T_k(g_f) - 0.2311T_l(g_f)\big)$$
$$+ 0.3200 + 0.0483T_i(g_f) - 1.7880T_j(g_f) + 0.0076T_k(g_f) - 0.0330T_l(g_f)$$
$$= 0.1656 + 0.0253T_i(g_f) - 1.4624T_j(g_f) + 0.0587T_k(g_f) - 0.0049T_l(g_f)$$

$$\frac{\partial^2 W_j(g_f)}{\partial T_j^2(g_f)} = -1.4624$$

Therefore the optimal tariff in country j is: $T_j^*(g_f)$ = 0.1133 + $0.0173T_i(g_f)$ + $0.0402T_k(g_f)$ – $0.0033T_l(g_f)$. Given symmetry this expression converges to $T_j^*(g_f)$ = $T_k^*(g_f)$ = 0.1180 + $0.0180T_i(g_f)$ – $0.0035T_l(g_f)$.

On the other hand, the following expressions are obtained for country l:

$$CS_l(g_f) = \frac{1}{2}\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f) + q_l^{(l)}(g_f)\right)^2$$
$$= 0.5\big(0.6400 + 0.0076T_i(g_f) + 0.0152T_j(g_f) + 0.0152T_k(g_f) - 0.5486T_l(g_f)\big)^2$$

$$\pi_i^{(l)}(g_f) = \frac{(\phi + 2)}{2}\left(q_i^{(l)}(g_f)\right)^2$$
$$= 1.25\big(0.1600 - 0.5790T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\big)^2$$

$$\pi_j^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 + 0.686T_i(g_f) - 0.4470T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\right)^2$$

$$\pi_k^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) - 0.4470T_k(g_f) - 0.0610T_l(g_f)\right)^2$$

$$\pi_l^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_f)\right)^2$$
$$= 1.25\left(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) + 0.3962T_l(g_f)\right)^2$$

$$PS_l(g_f) = \frac{\phi}{4}\left(q_i^{(l)}(g_f) + q_j^{(l)}(g_f) + q_k^{(l)}(g_f) + q_l^{(l)}(g_f)\right)^2$$
$$= 0.125\left(0.6400 - 0.3733T_i(g_f) - 0.3022T_j(g_f) - 0.3022T_k(g_f) + 0.2133T_l(g_f)\right)^2$$

$$TR_l(g_f) = T_l(g_f)\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f)\right)$$
$$= 0.4800T_l(g_f) - 0.0610T_i(g_f)T_l(g_f) - 0.0330T_j(g_f)T_l(g_f)$$
$$- 0.0330T_k(g_f)T_l(g_f) - 0.9448T_l^2(g_f)$$

Therefore welfare in country *l* is:

$$W_l(g_f) = CS_l(g_f) + \pi_l(g_f) + PS_l(g_f) + TR_l(g_f)$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_f) + 0.0152T_j(g_f) + 0.0152T_k(g_f) - 0.5486T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 - 0.5790T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 + 0.686T_i(g_f) - 0.4470T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) - 0.4470T_k(g_f) - 0.0610T_l(g_f)\right)^2$$
$$+ 1.25\left(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) + 0.3962T_l(g_f)\right)^2$$
$$+ 0.125\left(0.6400 - 0.3733T_i(g_f) - 0.3022T_j(g_f) - 0.3022T_k(g_f) + 0.2133T_l(g_f)\right)^2$$
$$+ 0.4800T_l(g_f) - 0.0610T_i(g_f)T_l(g_f) - 0.0330T_j(g_f)T_l(g_f)$$
$$- 0.0330T_k(g_f)T_l(g_f) - 0.9448T_l^2(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_f)}{\partial T_l(g_f)} =$$
$$(-0.5486)\big(0.6400 + 0.0076T_i(g_f) + 0.0152T_j(g_f) + 0.0152T_k(g_f) - 0.5486T_l(g_f)\big)$$
$$+ (2.5)(-0.0610)\big(0.1600 - 0.5790T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\big)$$
$$+ (2.5)(-0.0610)\big(0.1600 + 0.686T_i(g_f) - 0.4470T_j(g_f) + 0.0483T_k(g_f) - 0.0610T_l(g_f)\big)$$
$$+ (2.5)(-0.0610)\big(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) - 0.4470T_k(g_f) - 0.0610T_l(g_f)\big)$$
$$+ (2.5)(0.3962)\big(0.1600 + 0.0686T_i(g_f) + 0.0483T_j(g_f) + 0.0483T_k(g_f) + 0.3962T_l(g_f)\big)$$
$$+ (0.25)(0.2133)\big(0.6400 - 0.3733T_i(g_f) - 0.3022T_j(g_f) - 0.3022T_k(g_f) + 0.2133T_l(g_f)\big)$$
$$+ 0.4800 - 0.0610T_i(g_f) - 0.0330T_j(g_f) - 0.0330T_k(g_f) - 1.8896T_l(g_f)$$
$$= 0.2483 + 0.0503T_i(g_f) + 0.0438T_j(g_f) + 0.0438T_k(g_f) - 1.1569T_l(g_f)$$

$$\frac{\partial^2 W_l(g_f)}{\partial T_l^2(g_f)} = -1.1569$$

Therefore the optimal tariff in country *l* is: $T_l^*(g_f)$ = 0.2146 + $0.0435T_i(g_f)$ + $0.0378T_j(g_f)$ + $0.0378T_k(g_f)$. Given symmetry across countries, this expression converges to: $T_l^*(g_f) = 0.2146 + 0.0435T_i(g_f) + 0.0756T_j(g_f)$. Thus, in considering the optimal tariff equations for countries *i*, *j* and *l*, the following matrix system is obtained:

$$\begin{pmatrix} 1 & -0.0470 & 0.0435 \\ -0.0180 & 1 & 0.0035 \\ -0.0435 & -0.0756 & 1 \end{pmatrix} \begin{pmatrix} T_i(g_f) \\ T_j(g_f) \\ T_l(g_f) \end{pmatrix} = \begin{pmatrix} 0.0769 \\ 0.1180 \\ 0.2146 \end{pmatrix}$$

By solving this system and by using symmetry across countries, the following optimal tariffs for countries *i*, *j*, *k* and *l* are obtained: $T_i^*(g_f) = 0.0726$; $T_j^*(g_f) = T_k^*(g_f) = 0.1185$;

$T_l^*(g_f) = 0.2267$. Using these tariffs it is concluded that: $CS_i(g_f) = 0.2019$; $CS_j(g_f)$ = $CS_k(g_f)$ = 0.1825; $CS_l(g_f) = 0.1351$; $\pi_i(g_f) = 0.1336$; $\pi_j(g_f) = \pi_k(g_f) = 0.1140$; $\pi_l(g_f)$ = 0.1323; $PS_i(g_f) = 0.0491$; $PS_j(g_f) = PS_k(g_f) = 0.0412$; $PS_l(g_f) = 0.0435$; $TR_i(g_f)$ = 0.0084; $TR_j(g_f) = TR_k(g_f) = 0.0250$; $TR_l(g_f) = 0.0575$; $W_i(g_f) = 0.3930$; $W_j(g_f) = W_k(g_f) = 0.3627$; and $W_l(g_f) = 0.3683$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_f) = 0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)$$

$$q_j^{(i)}(g_f) = 0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)$$

$$q_k^{(i)}(g_f) = 0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.1611T_k(g_f) + 0.0337T_l(g_f)$$

$$q_l^{(i)}(g_f) = 0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) - 0.2396T_l(g_f)$$

$$q_i^{(j)}(g_f) = 0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)$$

$$q_j^{(j)}(g_f) = 0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)$$

$$q_k^{(j)}(g_f) = 0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.3201T_k(g_f) + 0.0337T_l(g_f)$$

$$q_l^{(j)}(g_f) = 0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) - 0.2396T_l(g_f)$$

$$q_i^{(k)}(g_f) = 0.1143 + 0.0805T_i(g_f) + 0.0526T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)$$

$$q_j^{(k)}(g_f) = 0.1143 - 0.0188T_i(g_f) - 0.3201T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)$$

$$q_k^{(k)}(g_f) = 0.1143 - 0.0188T_i(g_f) + 0.0526T_j(g_f) + 0.1611T_k(g_f) + 0.0337T_l(g_f)$$

$$q_l^{(k)}(g_f) = 0.1143 - 0.0188T_i(g_f) + 0.0526T_j(g_f) - 0.0377T_k(g_f) - 0.2396T_l(g_f)$$

$$q_i^{(l)}(g_f) = 0.1143 - 0.4007T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)$$

$$q_j^{(l)}(g_f) = 0.1143 + 0.0714T_i(g_f) - 0.3201T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)$$

$$q_k^{(l)}(g_f) = 0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) - 0.3201T_k(g_f) - 0.0565T_l(g_f)$$

$$q_l^{(l)}(g_f) = 0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) + 0.2416T_l(g_f)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_f) = \frac{1}{2}\left(q_i^{(i)}(g_f) + q_i^{(j)}(g_f) + q_i^{(k)}(g_f) + q_i^{(l)}(g_f)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\right)^2$$

$$\pi_i^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_j^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_k^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.1611T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_l^{(i)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) - 0.2396T_l(g_f)\right)^2$$

$$PS_i(g_f) = \frac{\phi}{4}\left(q_i^{(i)}(g_f) + q_j^{(i)}(g_f) + q_k^{(i)}(g_f) + q_l^{(i)}(g_f)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) + 0.0481T_k(g_f) - 0.1383T_l(g_f)\right)^2$$

$$TR_i(g_f) = T_i(g_f)\left(q_i^{(I)}(g_f)\right) = 0.1143T_i(g_f) - 0.4007T_i^2(g_f) + 0.0526T_j(g_f)T_i(g_f)$$
$$+0.0526T_k(g_f)T_i(g_f) - 0.0565T_l(g_f)T_i(g_f)$$

Therefore welfare in country *i* is given by:

$$W_i(g_f) = CS_i(g_f) + \pi_i(g_f) + PS_i(g_f) + TR_i(g_f)$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\right)^2$$
$$+1.75\left(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.1611T_k(g_f) + 0.0337T_l(g_f)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) - 0.2396T_l(g_f)\right)^2$$
$$+0.375\left(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) + 0.0481T_k(g_f) - 0.1383T_l(g_f)\right)^2$$
$$+0.1143T_i(g_f) - 0.4007T_i^2(g_f) + 0.0526T_j(g_f)T_i(g_f)$$
$$+0.0526T_k(g_f)T_i(g_f) - 0.0565T_l(g_f)T_i(g_f)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_f)}{\partial T_i(g_f)} =$$
$$(-0.1590)\left(0.4571 - 0.1590T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\right)$$
$$+(3.5)(0.0805)\left(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)$$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) - 0.0377T_k(g_f) + 0.0337T_l(g_f)\right)$$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.1611T_k(g_f) + 0.0337T_l(g_f)\right)($$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.0377T_k(g_f) - 0.2396T_l(g_f)\right)$$
$$+(0.75)(0.0241)\left(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) + 0.0481T_k(g_f) - 0.1383T_l(g_f)\right)^2$$
$$+0.1143 - 0.8014T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)$$
$$= 0.0595 - 0.7492T_i(g_f) + 0.0324T_j(g_f) + 0.0324T_k(g_f) - 0.0453T_l(g_f)$$

$$\frac{\partial^2 W_i(g_f)}{\partial T_i^2(g_f)} = -0.7492$$

Therefore the optimal tariff in country i is: $T_i^*(g_f)$ = 0.0794 + $0.0433T_j(g_f)$ + $0.0433T_k(g_f)$ − $0.0604T_l(g_f)$. Given symmetry across countries this expression converges to: $T_i^*(g_f) = 0.0794 + 0.0866T_j(g_f) - 0.0604T_l(g_f)$.

On the other hand, the following expressions are obtained for country j:

$$CS_j(g_f) = \frac{1}{2}\left(q_j^{(i)}(g_f) + q_j^{(j)}(g_f) + q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right)^2$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_f) - 0.3180T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\right)^2$$

$$\pi_i^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_j^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_k^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.3201T_k(g_f) + 0.0337T_l(g_f)\right)^2$$

$$\pi_l^{(j)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) - 0.2396T_l(g_f)\right)^2$$

$$PS_j(g_f) = \frac{\phi}{4}\left(q_i^{(j)}(g_f) + q_j^{(j)}(g_f) + q_k^{(j)}(g_f) + q_l^{(j)}(g_f)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) - 0.1624T_k(g_f) - 0.1383T_l(g_f)\right)^2$$

$$TR_j(g_f) = T_j(g_f)\left(q_j^{(k)}(g_f) + q_j^{(l)}(g_f)\right) = 0.2286T_j(g_f) + 0.0526T_i(g_f)T_j(g_f)$$
$$- 0.6402T_j^2(g_f) + 0.0149T_k(g_f)T_j(g_f) - 0.0228T_l(g_f)T_j(g_f)$$

Therefore welfare in country *j* is given by:

$$
\begin{aligned}
W_j(g_f) &= CS_j(g_f) + \pi_j(g_f) + PS_j(g_f) + TR_j(g_f) \\
&= 0.5\big(0.4571 + 0.0149T_i(g_f) - 0.3180T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.3201T_k(g_f) + 0.0337T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) - 0.2396T_l(g_f)\big)^2 \\
&+ 0.375\big(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) - 0.1624T_k(g_f) - 0.1383T_l(g_f)\big)^2 \\
&+ 0.2286T_j(g_f) + 0.0526T_i(g_f)T_j(g_f) - 0.6402T_j^2(g_f) \\
&+ 0.0149T_k(g_f)T_j(g_f) - 0.0228T_l(g_f)T_j(g_f)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_j(g_f)}{\partial T_j(g_f)} = \\
&(-0.3180)\big(0.4571 + 0.0149T_i(g_f) - 0.3180T_j(g_f) + 0.0298T_k(g_f) + 0.0447T_l(g_f)\big) \\
&+ (3.5)(-0.0377)\big(0.1143 + 0.0805T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\big) \\
&+ (3.5)(0.1611)\big(0.1143 - 0.0188T_i(g_f) + 0.1611T_j(g_f) + 0.0526T_k(g_f) + 0.0337T_l(g_f)\big) \\
&+ (3.5)(-0.0377)\big(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) - 0.3201T_k(g_f) + 0.0337T_l(g_f)\big) \\
&+ (3.5)(-0.0377)\big(0.1143 - 0.0188T_i(g_f) - 0.0377T_j(g_f) + 0.0526T_k(g_f) - 0.2396T_l(g_f)\big) \\
&+ (0.75)(0.0481)\big(0.4571 + 0.0241T_i(g_f) + 0.0481T_j(g_f) - 0.1624T_k(g_f) - 0.1383T_l(g_f)\big) \\
&+ 0.2286 + 0.0526T_i(g_f) - 1.2804T_j(g_f) + 0.0149T_k(g_f) - 0.0228T_l(g_f) \\
&= 0.1189 + 0.0324T_i(g_f) - 1.0718T_j(g_f) + 0.0576T_k(g_f) - 0.0002T_l(g_f)
\end{aligned}
$$

$$
\frac{\partial^2 W_j(g_f)}{\partial T_j^2(g_f)} = -1.0718
$$

Therefore the optimal tariff in country j is: $T_j^*(g_f)$ = 0.1109 + $0.0302T_i(g_f)$ + $0.0537T_k(g_f)$ − $0.0002T_l(g_f)$. Given symmetry across countries this expression converges to: $T_j^*(g_f) = T_k^*(g_f) = 0.1172 + 0.0320T_i(g_f) - 0.0002T_l(g_f)$.

Finally, the following expressions are obtained for country l:

$$CS_l(g_f) = \frac{1}{2}\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f) + q_l^{(l)}(g_f)\right)^2$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) - 0.4770T_l(g_f)\right)^2$$

$$\pi_i^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 - 0.4007T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\right)^2$$

$$\pi_j^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_f) - 0.3201T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\right)^2$$

$$\pi_k^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) - 0.3201T_k(g_f) - 0.0565T_l(g_f)\right)^2$$

$$\pi_l^{(l)}(g_f) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_f)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) + 0.2416T_l(g_f)\right)^2$$

$$PS_l(g_f) = \frac{\phi}{4}\left(q_i^{(l)}(g_f) + q_j^{(l)}(g_f) + q_k^{(l)}(g_f) + q_l^{(l)}(g_f)\right)^2$$
$$= 0.375\left(0.4571 - 0.1865T_i(g_f) - 0.1624T_j(g_f) - 0.1624T_k(g_f) + 0.0722T_l(g_f)\right)^2$$

$$TR_l(g_f) = T_l(g_f)\left(q_l^{(i)}(g_f) + q_l^{(j)}(g_f) + q_l^{(k)}(g_f)\right) = 0.3429T_l(g_f)$$
$$-0.0565T_i(g_f)T_l(g_f) - 0.0228T_j(g_f)T_l(g_f) - 0.0228T_k(g_f)T_l(g_f) - 0.7187T_l^2(g_f)$$

Therefore welfare in country *l* is given by:

$$
\begin{aligned}
W_l(g_f) &= CS_l(g_f) + \pi_l(g_f) + PS_l(g_f) + TR_l(g_f) \\
&= 0.5\big(0.4571 + 0.0149T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) - 0.4770T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 - 0.4007T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_f) - 0.3201T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) - 0.3201T_k(g_f) - 0.0565T_l(g_f)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) + 0.2416T_l(g_f)\big)^2 \\
&+ 0.375\big(0.4571 - 0.1865T_i(g_f) - 0.1624T_j(g_f) - 0.1624T_k(g_f) + 0.0722T_l(g_f)\big)^2 \\
&+ 0.3429T_l(g_f) - 0.0565T_i(g_f)T_l(g_f) - 0.0228T_j(g_f)T_l(g_f) \\
&- 0.0228T_k(g_f)T_l(g_f) - 0.7187T_l^2(g_f)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_l(g_f)}{\partial T_l(g_f)} = \\
&(-0.4770)\big(0.4571 + 0.0149T_i(g_f) + 0.0298T_j(g_f) + 0.0298T_k(g_f) - 0.4770T_l(g_f)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 - 0.4007T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 + 0.0714T_i(g_f) - 0.3201T_j(g_f) + 0.0526T_k(g_f) - 0.0565T_l(g_f)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) - 0.3201T_k(g_f) - 0.0565T_l(g_f)\big) \\
&+ (3.5)(0.2416)\big(0.1143 + 0.0714T_i(g_f) + 0.0526T_j(g_f) + 0.0526T_k(g_f) + 0.2416T_l(g_f)\big) \\
&+ (0.75)(0.0722)\big(0.4571 - 0.1865T_i(g_f) - 0.1624T_j(g_f) - 0.1624T_k(g_f) + 0.0722T_l(g_f)\big) \\
&+ 0.3429 - 0.0565T_i(g_f) - 0.0228T_j(g_f) - 0.0228T_k(g_f) - 1.4374T_l(g_f) \\
&= 0.1784 + 0.0377T_i(g_f) + 0.0412T_j(g_f) + 0.0412T_k(g_f) - 0.9680T_l(g_f)
\end{aligned}
$$

$$
\frac{\partial^2 W_l(g_f)}{\partial T_l^2(g_f)} = -0.9680
$$

Therefore the optimal tariff in country l is: $T_l^*(g_f)$ = 0.1843 + $0.0389T_i(g_f)$ + $0.0426T_j(g_f)$ + $0.0426T_k(g_f)$. Given symmetry across countries, this expression converges to: $T_l^*(g_f) = 0.1843 + 0.0389T_i(g_f) + 0.0851T_j(g_f)$. Thus, in considering the optimal tariff functions of countries *i*, *j* and *l*, the following matrix system is obtained:

$$\begin{pmatrix} 1 & -0.0866 & 0.0604 \\ -0.0320 & 1 & 0.0002 \\ -0.0389 & -0.0851 & 1 \end{pmatrix} \begin{pmatrix} T_i(g_f) \\ T_j(g_f) \\ T_l(g_f) \end{pmatrix} = \begin{pmatrix} 0.0794 \\ 0.1172 \\ 0.1843 \end{pmatrix}$$

By solving this system and using symmetry across countries, the following optimal tariffs for countries *i*, *j*, *k* and *l* are obtained: $T_i^*(g_f) = 0.0778$; $T_j^*(g_f) = T_k^*(g_f) = 0.1197$; and $T_l^*(g_f) = 0.1975$. Using these tariffs it is concluded that: $CS_i(g_f) = 0.1061$; $CS_j(g_f)$ = $CS_k(g_f)$ = 0.0936; $CS_l(g_f) = 0.0689$; $\pi_i(g_f) = 0.0931$; $\pi_j(g_f) = \pi_k(g_f) = 0.0842$; $\pi_l(g_f)$ = 0.0899; $PS_i(g_f) = 0.0737$; $PS_j(g_f) = PS_k(g_f) = 0.0655$; $PS_l(g_f) = 0.0655$; $TR_i(g_f) = 0.0066$; $TR_j(g_f) = TR_k(g_f) = 0.0184$; $TR_l(g_f) = 0.0377$; $W_i(g_f) = 0.2795$; $W_j(g_f) = W_k(g_f) = 0.2616$; $W_l(g_f) = 0.2620$.

Network g

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_g)=\frac{2\alpha(\phi+1)+2T_i(g_g)+\phi\begin{pmatrix}q_j^{(j)}(g_g)+q_k^{(j)}(g_g)+q_l^{(j)}(g_g)+q_j^{(k)}(g_g)+q_k^{(k)}(g_g)\\q_l^{(k)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_g)=\frac{2\alpha(\phi+1)+2T_j(g_g)+\phi\begin{pmatrix}q_i^{(j)}(g_g)+q_k^{(j)}(g_g)+q_l^{(j)}(g_g)+q_i^{(k)}(g_g)+q_k^{(k)}(g_g)\\q_l^{(k)}(g_g)+q_i^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_g)=\frac{2\alpha(\phi+1)+2T_k(g_g)+\phi\begin{pmatrix}q_i^{(j)}(g_g)+q_j^{(j)}(g_g)+q_l^{(j)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)\\q_l^{(k)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_l^{(i)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_g)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g_g)+\phi\begin{pmatrix}q_i^{(j)}(g_g)+q_j^{(j)}(g_g)+q_k^{(j)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)\\q_k^{(k)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_g)=\frac{2\alpha(\phi+1)+2T_i(g_g)+\phi\begin{pmatrix}q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_j^{(k)}(g_g)+q_k^{(k)}(g_g)\\q_l^{(k)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_g)+q_k^{(j)}(g_g)+q_l^{(j)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_g)=\frac{2\alpha(\phi+1)+2T_j(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(k)}(g_g)+q_k^{(k)}(g_g)\\ q_l^{(k)}(g_g)+q_i^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_g)+q_k^{(j)}(g_g)+q_l^{(j)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_g)=\frac{2\alpha(\phi+1)+2T_k(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)\\ q_l^{(k)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_g)+q_j^{(j)}(g_g)+q_l^{(j)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_g)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)\\ q_k^{(k)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_g)+q_j^{(j)}(g_g)+q_k^{(j)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_g)=\frac{2\alpha(\phi+1)+2T_i(g_g)+\phi\begin{pmatrix}q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_j^{(j)}(g_g)+q_k^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_g)+q_k^{(k)}(g_g)+q_l^{(k)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_g)=\frac{2\alpha(\phi+1)+2T_j(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(j)}(g_g)+q_k^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_i^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_g)+q_k^{(k)}(g_g)+q_l^{(k)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_g)=\frac{2\alpha(\phi+1)+2T_k(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(j)}(g_g)+q_j^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_l^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_g)+q_j^{(k)}(g_g)+q_l^{(k)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_g)=\frac{2\alpha(\phi+1)-2(2+\phi)T_l(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_i^{(j)}(g_g)+q_j^{(j)}(g_g)\\ q_k^{(j)}(g_g)+q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_g)+q_j^{(k)}(g_g)+q_k^{(k)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_g)=\frac{2\alpha(\phi+1)-2(4+\phi)T_i(g_g)+\phi\begin{pmatrix}q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_j^{(j)}(g_g)+q_k^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_j^{(k)}(g_g)+q_k^{(k)}(g_g)+q_l^{(k)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_g)=\frac{2\alpha(\phi+1)-2(4+\phi)T_j(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(j)}(g_g)+q_k^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_i^{(k)}(g_g)+q_k^{(k)}(g_g)+q_l^{(k)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_g)+q_k^{(l)}(g_g)+q_l^{(l)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_g)=\frac{2\alpha(\phi+1)-2(4+\phi)T_k(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_l^{(i)}(g_g)+q_i^{(j)}(g_g)+q_j^{(j)}(g_g)\\ q_l^{(j)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)+q_l^{(k)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_l^{(l)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_g)=\frac{2\alpha(\phi+1)+6T_l(g_g)+\phi\begin{pmatrix}q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_i^{(j)}(g_g)+q_j^{(j)}(g_g)\\ q_k^{(j)}(g_g)+q_i^{(k)}(g_g)+q_j^{(k)}(g_g)+q_k^{(k)}(g_g)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_g)+q_j^{(l)}(g_g)+q_k^{(l)}(g_g)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: α = 1 and ϕ = 0

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_g)=\frac{2+2T_i(g_g)}{10}$; $q_j^{(i)}(g_g)=\frac{2+2T_j(g_g)}{10}$; $q_k^{(i)}(g_g)=\frac{2+2T_k(g_g)}{10}$; $q_l^{(i)}(g_g)=\frac{2-4T_l(g_g)}{10}$; $q_i^{(j)}(g_g)$ = $\frac{2+2T_i(g_g)}{10}$; $q_j^{(j)}(g_g)=\frac{2+2T_j(g_g)}{10}$; $q_k^{(j)}(g_g)=\frac{2+2T_k(g_g)}{10}$; $q_l^{(j)}(g_g)=\frac{2-4T_l(g_g)}{10}$; $q_i^{(k)}(g_g)=\frac{2+2T_i(g_g)}{10}$; $q_j^{(k)}(g_g)=\frac{2+2T_j(g_g)}{10}$; $q_k^{(k)}(g_g)=\frac{2+2T_k(g_g)}{10}$; $q_l^{(k)}(g_g)$ =

$\dfrac{2-4T_l(g_g)}{10}$; $\quad q_i^{(l)}(g_g)=\dfrac{2-8T_i(g_g)}{10}$; $\quad q_j^{(l)}(g_g)=\dfrac{2-8T_j(g_g)}{10}$; $\quad q_k^{(l)}(g_g)=\dfrac{2-8T_k(g_g)}{10}$;

and $q_l^{(l)}(g_g)=\dfrac{2+6T_l(g_g)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_g)=\frac{1}{2}\left(q_i^{(i)}(g_g)+q_i^{(j)}(g_g)+q_i^{(k)}(g_g)+q_i^{(l)}(g_g)\right)^2=0.5\left(0.8000-0.2000T_i(g_g)\right)^2$$

$$\pi_i^{(i)}(g_g)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_g)\right)^2=\left(0.2000+0.2000T_i(g_g)\right)^2$$

$$\pi_j^{(i)}(g_g)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_g)\right)^2=\left(0.2000+0.2000T_j(g_g)\right)^2$$

$$\pi_k^{(i)}(g_g)=\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_g)\right)^2=\left(0.2000+0.2000T_k(g_g)\right)^2$$

$$\pi_l^{(i)}(g_g)=\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_g)\right)^2=\left(0.2000-0.4000T_l(g_g)\right)^2$$

$$PS_i(g_g)=\frac{\phi}{4}\left(q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)\right)^2=0$$

$$TR_i(g_g)=T_i(g_g)\left(q_i^{(l)}(g_g)\right)=0.2000T_i(g_g)-0.8000T_i^2(g_g)$$

Therefore welfare in country *i* is:

$$\begin{aligned}W_i(g_g)&=CS_i(g_g)+\pi_i(g_g)+PS_i(g_g)+TR_i(g_g)=0.5\left(0.8000-0.2000T_i(g_g)\right)^2\\&+\left(0.2000+0.2000T_i(g_g)\right)^2+\left(0.2000+0.2000T_j(g_g)\right)^2+\left(0.2000+0.2000T_k(g_g)\right)^2\\&+\left(0.2000-0.4000T_l(g_g)\right)^2+0.2000T_i(g_g)-0.8000T_i^2(g_g)\end{aligned}$$

The first and second conditions of this function are:

$$\begin{aligned}\frac{\partial W_i(g_g)}{\partial T_i(g_g)}&=(-0.2000)\left(0.8000-0.2000T_i(g_g)\right)+(2)(0.2000)\left(0.2000+0.2000T_i(g_g)\right)\\&+0.2000-1.6000T_i(g_g)=0.1200-1.4800T_i(g_g)\end{aligned}$$

$$\frac{\partial^2 W_i(g_g)}{\partial T_i^2(g_g)} = -1.48$$

Therefore the optimal tariff in country *i* is: $T_i^*(g_g) = 0.0811$. Given symmetry across countries it is concluded that: $T_i^*(g_g) = T_j^*(g_g) = T_k^*(g_g) = 0.0811$:

On the other hand, the following expressions are obtained for country *l*:

$$CS_l(g_g) = \frac{1}{2}\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g) + q_l^{(l)}(g_g)\right)^2 = 0.5\left(0.8000 - 0.6000T_l(g_g)\right)^2$$

$$\pi_i^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_g)\right)^2 = \left(0.2000 - 0.8000T_i(g_g)\right)^2$$

$$\pi_j^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_g)\right)^2 = \left(0.2000 - 0.8000T_j(g_g)\right)^2$$

$$\pi_k^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_g)\right)^2 = \left(0.2000 - 0.8000T_k(g_g)\right)^2$$

$$\pi_l^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_g)\right)^2 = \left(0.2000 + 0.6000T_l(g_g)\right)^2$$

$$PS_l(g_g) = 0$$

$$TR_l(g_g) = T_l(g_g)\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g)\right) = 0.6000T_l(g_g) - 1.2000T_l^2(g_g)$$

Therefore welfare in country *l* is:

$$W_l(g_g) = CS_l(g_g) + \pi_l(g_g) + PS_l(g_g) + TR_l(g_g) = 0.5\left(0.8000 - 0.6000T_l(g_g)\right)^2$$
$$+\left(0.2000 - 0.8000T_i(g_g)\right)^2 + \left(0.2000 - 0.8000T_j(g_g)\right)^2 + \left(0.2000 - 0.8000T_k(g_g)\right)^2$$
$$+\left(0.2000 + 0.6000T_l(g_g)\right)^2 + 0.6000T_l(g_g) - 1.2000T_l^2(g_g)$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_g)}{\partial T_l(g_g)} = (-0.6000)\big(0.8000 - 0.6000T_l(g_g)\big) + (2)(0.6000)\big(0.2000 + 0.6000T_l(g_g)\big)$$
$$+0.6000 - 2.4000T_l(g_g) = 0.36 - 1.3200T_l(g_g)$$

$$\frac{\partial^2 W_l(g_g)}{\partial T_l^2(g_g)} = -1.3200$$

Therefore the optimal tariff in country *l* is: $T_l^*(g_g) = 0.2727$. In considering the optimal tariff in countries *i*, *j*, *k* and *l*, the following results are obtained: $CS_i(g_g)$ = $CS_j(g_g)$ = $CS_k(g_g)$ = 0.3072; $CS_l(g_g) = 0.2025$; $\pi_i(g_g) = \pi_j(g_g) = \pi_k(g_g) = 0.1485$; $\pi_l(g_g) = 0.1870$; $PS_i(g_g)$ = = $PS_j(g_g)$ = $PS_k(g_g)$ = $PS_l(g_g)$ = 0; $TR_i(g_g) = TR_j(g_g) = TR_k(g_g) = 0.0110$; $TR_l(g_g)$ = 0.0744; $W_i(g_g) = W_j(g_g) = W_k(g_g) = 0.4666$; and $W_l(g_g) = 0.4639$.

<u>*Simulation 5: α = 1 and ϕ = 0.5*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_g) = 0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_j^{(i)}(g_g) = 0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_k^{(i)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)$$

$$q_l^{(i)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)$$

$$q_i^{(j)}(g_g) = 0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_j^{(j)}(g_g) = 0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_k^{(j)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)$$

$$q_l^{(j)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)$$

$$q_i^{(k)}(g_g) = 0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_j^{(k)}(g_g) = 0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)$$

$$q_k^{(k)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)$$

$$q_l^{(k)}(g_g) = 0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)$$

$$q_i^{(l)}(g_g) = 0.1600 - 0.5790T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)$$

$$q_j^{(l)}(g_g) = 0.1600 + 0.0686T_i(g_g) - 0.5790T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)$$

$$q_k^{(l)}(g_g) = 0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) - 0.5790T_k(g_g) - 0.0610T_l(g_g)$$

$$q_l^{(l)}(g_g) = 0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) + 0.3962T_l(g_g)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_g) = \frac{1}{2}\left(q_i^{(i)}(g_g) + q_i^{(j)}(g_g) + q_i^{(k)}(g_g) + q_i^{(l)}(g_g)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) + 0.0229T_l(g_g)\right)^2$$

$$\pi_i^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)^2$$

$$\pi_j^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_g)\right)^2 =$$
$$1.25\left(0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)^2$$

$$\pi_k^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_g)\right)^2 =$$
$$1.25\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)\right)^2$$

$$\pi_l^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)\right)^2$$

$$PS_i(g_g) = \frac{\phi}{4}\left(q_i^{(i)}(g_g) + q_j^{(i)}(g_g) + q_k^{(i)}(g_g) + q_l^{(i)}(g_g)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_g) + 0.0711T_j(g_g) + 0.0711T_k(g_g) - 0.2311T_l(g_g)\right)^2$$

$$TR_i(g_g) = T_i(g_g)\left(q_i^{(l)}(g_g)\right) = 0.1600T_i(g_g) - 0.5790T_i^2(g_g)$$
$$+ 0.0686T_j(g_g)T_i(g_g) + 0.0686T_k(g_g)T_i(g_g) - 0.0610T_l(g_g)T_i(g_g)$$

Therefore welfare in country *i* is:

$$W_i(g_g) = CS_i(g_g) + \pi_i(g_g) + PS_i(g_g) + TR_i(g_g)$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) + 0.0229T_l(g_g)\right)^2$$
$$+ 1.25\left(0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)\right)^2$$
$$+ 0.125\left(0.6400 + 0.0711T_i(g_g) + 0.0711T_j(g_g) + 0.0711T_k(g_g) - 0.2311T_l(g_g)\right)^2$$
$$+ 0.1600T_i(g_g) - 0.5790T_i^2(g_g) + 0.0686T_j(g_g)T_i(g_g)$$
$$+ 0.0686T_k(g_g)T_i(g_g) - 0.0610T_l(g_g)T_i(g_g)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_g)}{\partial T_i(g_g)} =$$
$$(-0.1829)\left(0.6400 - 0.1829T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) + 0.0229T_l(g_g)\right)$$
$$+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_g) + 0.1321T_j(g_g) - 0.0203T_k(g_g) + 0.0279T_l(g_g)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) + 0.1321T_k(g_g) + 0.0279T_l(g_g)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_g) - 0.0203T_j(g_g) - 0.0203T_k(g_g) - 0.3149T_l(g_g)\right)$$
$$+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_g) + 0.0711T_j(g_g) + 0.0711T_k(g_g) - 0.2311T_l(g_g)\right)$$
$$+ 0.1600 - 1.1580T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)$$
$$= 0.0828 - 1.0767T_i(g_g) + 0.0571T_j(g_g) + 0.0571T_k(g_g) - 0.0469T_l(g_g)$$

$$\frac{\partial^2 W_i(g_g)}{\partial T_i^2(g_g)} = -1.0767$$

Therefore the optimal tariff in country i is: $T_i^*(g_g)$ = 0.0769 + $0.0530T_j(g_g)$ + $0.0530T_k(g_g)$ – $0.0435T_l(g_g)$. Given symmetry across countries this expression converges to: $T_i^*(g_g) = T_j^*(g_g) = T_k^*(g_g) = 0.0860 - 0.0487T_l(g_g)$.

On the other hand, the following expressions are obtained for country l:

$$CS_l(g_g) = \frac{1}{2}\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g) + q_l^{(l)}(g_g)\right)^2$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) - 0.5486T_l(g_g)\right)^2$$

$$\pi_i^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 - 0.5790T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\right)^2$$

$$\pi_j^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 + 0.0686T_i(g_g) - 0.5790T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\right)^2$$

$$\pi_k^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) - 0.5790T_k(g_g) - 0.0610T_l(g_g)\right)^2$$

$$\pi_l^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_g)\right)^2$$
$$= 1.25\left(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) + 0.3962T_l(g_g)\right)^2$$

$$PS_l(g_g) = \frac{\phi}{4}\left(q_i^{(l)}(g_g) + q_j^{(l)}(g_g) + q_k^{(l)}(g_g) + q_l^{(l)}(g_g)\right)^2$$
$$= 0.125\left(0.6400 - 0.3733T_i(g_g) - 0.3733T_j(g_g) - 0.3733T_k(g_g) + 0.2133T_l(g_g)\right)^2$$

$$TR_l(g_g) = T_l(g_g)\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g)\right) = 0.4800T_l(g_g) - 0.0610T_i(g_g)T_l(g_g)$$
$$- 0.0610T_j(g_g)T_l(g_g) - 0.0610T_k(g_g)T_l(g_g) - 0.9448T_l^2(g_g)$$

Therefore welfare in country l is given by:

$$
\begin{aligned}
W_l(g_g) &= CS_l(g_g) + \pi_l(g_g) + PS_l(g_g) + TR_l(g_g) \\
&= 0.5\big(0.6400 + 0.0076T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) - 0.5486T_l(g_g)\big)^2 \\
&+ 1.25\big(0.1600 - 0.5790T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\big)^2 \\
&+ 1.25\big(0.1600 + 0.0686T_i(g_g) - 0.5790T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\big)^2 \\
&+ 1.25\big(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) - 0.5790T_k(g_g) - 0.0610T_l(g_g)\big)^2 \\
&+ 1.25\big(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) + 0.3962T_l(g_g)\big)^2 \\
&+ 0.125\big(0.6400 - 0.3733T_i(g_g) - 0.3733T_j(g_g) - 0.3733T_k(g_g) + 0.2133T_l(g_g)\big)^2 \\
&+ 0.4800T_l(g_g) - 0.0610T_i(g_g)T_l(g_g) - 0.0610T_j(g_g)T_l(g_g) \\
&- 0.0610T_k(g_g)T_l(g_g) - 0.9448T_l^2(g_g)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
\frac{\partial W_l(g_g)}{\partial T_l(g_g)} &= \\
&(-0.5486)\big(0.6400 + 0.0076T_i(g_g) + 0.0076T_j(g_g) + 0.0076T_k(g_g) - 0.5486T_l(g_g)\big) \\
&+ (2.5)(-0.0610)\big(0.1600 - 0.5790T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\big) \\
&+ (2.5)(-0.0610)\big(0.1600 + 0.0686T_i(g_g) - 0.5790T_j(g_g) + 0.0686T_k(g_g) - 0.0610T_l(g_g)\big) \\
&+ (2.5)(-0.0610)\big(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) - 0.5790T_k(g_g) - 0.0610T_l(g_g)\big) \\
&+ (2.5)(0.3962)\big(0.1600 + 0.0686T_i(g_g) + 0.0686T_j(g_g) + 0.0686T_k(g_g) + 0.3962T_l(g_g)\big) \\
&+ (0.25)(0.2133)\big(0.6400 - 0.3733T_i(g_g) - 0.3733T_j(g_g) - 0.3733T_k(g_g) + 0.2133T_l(g_g)\big) \\
&+ 0.4800 - 0.0610T_i(g_g) - 0.0610T_j(g_g) - 0.0610T_k(g_g) - 1.8896T_l(g_g) \\
&= 0.2484 + 0.0502T_i(g_g) + 0.0502T_j(g_g) + 0.0502T_k(g_g) - 1.1569T_l(g_g)
\end{aligned}
$$

$$
\frac{\partial^2 W_l(g_g)}{\partial T_l^2(g_g)} = -1.1569
$$

Therefore the optimal tariff in country l is: $T_l^*(g_g)$ = 0.2147 + $0.0434T_i(g_g)$ + $0.0434T_j(g_g)$ + $0.0434T_k(g_g)$. Given symmetry across countries this expression converges to: $T_l^*(g_g) = 0.2147 + 0.1302T_i(g_g)$. In considering the tariff functions of countries *i* and *l* and by using symmetry across countries, it is concluded that: $T_i^*(g_g)$ = $T_j^*(g_g)$ = $T_k^*(g_g)$ = 0.0751; and $T_l^*(g_g) = 0.2245$. Using these tariffs, the following results are obtained: $CS_i(g_g)$ = $CS_j(g_g)$ = $CS_k(g_g)$ = 0.2001; $CS_l(g_g) = 0.1345$; $\pi_i(g_g) = \pi_j(g_g) = \pi_k(g_g) = 0.1214$; $\pi_l(g_g) = 0.1354$; $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) =$ $PS_l(g_g) = 0.0456$; $TR_i(g_g)$ = $TR_j(g_g)$ = $TR_k(g_g)$ = 0.0085; $TR_l(g_g) = 0.0571$; $W_i(g_g) = W_j(g_g) = W_k(g_g) = 0.3756$; $W_l(g_g) = 0.3725$.

<u>*Simulation 6: α = 1 and ϕ = 1.5*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_g) = 0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_j^{(i)}(g_g) = 0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_k^{(i)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)$$

$$q_l^{(i)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)$$

$$q_i^{(j)}(g_g) = 0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_j^{(j)}(g_g) = 0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_k^{(j)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)$$

$$q_l^{(j)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)$$

$$q_i^{(k)}(g_g) = 0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_j^{(k)}(g_g) = 0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)$$

$$q_k^{(k)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)$$

$$q_l^{(k)}(g_g) = 0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)$$

$$q_i^{(l)}(g_g) = 0.1143 - 0.4007T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)$$

$$q_j^{(l)}(g_g) = 0.1143 + 0.0714T_i(g_g) - 0.4007T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)$$

$$q_k^{(l)}(g_g) = 0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) - 0.4007T_k(g_g) - 0.0565T_l(g_g)$$

$$q_l^{(l)}(g_g) = 0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) + 0.2416T_l(g_g)$$

Solving by substitution, the following expressions are obtained:

$$CS_i(g_g) = \frac{1}{2}\left(q_i^{(i)}(g_g) + q_i^{(j)}(g_g) + q_i^{(k)}(g_g) + q_i^{(l)}(g_g)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) + 0.0447T_l(g_g)\right)^2$$

$$\pi_i^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)^2$$

$$\pi_j^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_g)\right)^2 =$$
$$1.75\left(0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)^2$$

$$\pi_k^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_g)\right)^2 =$$
$$1.75\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)\right)^2$$

$$\pi_l^{(i)}(g_g) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)\right)^2$$

$$PS_i(g_g) = \frac{\phi}{4}\left(q_i^{(i)}(g_g) + q_j^{(i)}(g_g) + q_k^{(i)}(g_g) + q_l^{(i)}(g_g)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_g) + 0.0241T_j(g_g) + 0.0241T_k(g_g) - 0.1383T_l(g_g)\right)^2$$

$$TR_i(g_g) = T_i(g_g)\left(q_i^{(I)}(g_g)\right) = 0.1143T_i(g_g) - 0.4007T_i^2(g_g)$$
$$+0.0714T_j(g_g)T_i(g_g) + 0.0714T_k(g_g)T_i(g_g) - 0.0565T_l(g_g)T_i(g_g)$$

Therefore welfare in country *i* is given by:

$$W_i(g_g) = CS_i(g_g) + \pi_i(g_g) + PS_i(g_g) + TR_i(g_g)$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) + 0.0447T_l(g_g)\right)^2$$
$$+1.75\left(0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)\right)^2$$
$$+0.375\left(0.4571 + 0.0241T_i(g_g) + 0.0241T_j(g_g) + 0.0241T_k(g_g) - 0.1383T_l(g_g)\right)^2$$
$$+0.1143T_i(g_g) - 0.4007T_i^2(g_g) + 0.0714T_j(g_g)T_i(g_g)$$
$$+0.0714T_k(g_g)T_i(g_g) - 0.0565T_l(g_g)T_i(g_g)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_g)}{\partial T_i(g_g)} =$$
$$(-0.1590)\left(0.4571 - 0.1590T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) + 0.0447T_l(g_g)\right)$$
$$+(3.5)(0.0805)\left(0.1143 + 0.0805T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)$$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_g) + 0.0805T_j(g_g) - 0.0188T_k(g_g) + 0.0337T_l(g_g)\right)$$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) + 0.0805T_k(g_g) + 0.0337T_l(g_g)\right)$$
$$+(3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_g) - 0.0188T_j(g_g) - 0.0188T_k(g_g) - 0.2396T_l(g_g)\right)$$
$$+(0.75)(0.0241)\left(0.4571 + 0.0241T_i(g_g) + 0.0241T_j(g_g) + 0.0241T_k(g_g) - 0.1383T_l(g_g)\right)$$
$$+0.1143 - 0.8014T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)$$
$$= 0.0595 - 0.7492T_i(g_g) + 0.0613T_j(g_g) + 0.0613T_k(g_g) - 0.0452T_l(g_g)$$

$$\frac{\partial^2 W_i(g_g)}{\partial T_i^2(g_g)} = -0.7492$$

Therefore the optimal tariff in country i is: $T_i^*(g_g)$ = 0.0794 + $0.0819T_j(g_g)$ + $0.0819T_k(g_g)$ – $0.0604T_l(g_g)$. Given symmetry across countries this expression converges to: $T_i^*(g_g) = T_j^*(g_g) = T_k^*(g_g) = 0.0949 - 0.0722T_l(g_g)$.

On the other hand, the following expressions are obtained for country l:

$$CS_l(g_g) = \frac{1}{2}\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g) + q_l^{(l)}(g_g)\right)^2$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) - 0.4770T_l(g_g)\right)^2$$

$$\pi_i^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 - 0.4007T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\right)^2$$

$$\pi_j^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_g) - 0.4007T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\right)^2$$

$$\pi_k^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) - 0.4007T_k(g_g) - 0.0565T_l(g_g)\right)^2$$

$$\pi_l^{(l)}(g_g) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_g)\right)^2$$
$$= 1.75\left(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) + 0.2416T_l(g_g)\right)^2$$

$$PS_l(g_g) = \frac{\phi}{4}\left(q_i^{(l)}(g_g) + q_j^{(l)}(g_g) + q_k^{(l)}(g_g) + q_l^{(l)}(g_g)\right)^2$$
$$= 0.375\left(0.4571 - 0.1865T_i(g_g) - 0.1865T_j(g_g) - 0.1865T_k(g_g) + 0.0722T_l(g_g)\right)^2$$

$$TR_l(g_g) = T_l(g_g)\left(q_l^{(i)}(g_g) + q_l^{(j)}(g_g) + q_l^{(k)}(g_g)\right) = 0.3429T_l(g_g) - 0.0565T_i(g_g)T_l(g_g)$$
$$- 0.0565T_j(g_g)T_l(g_g) - 0.0565T_k(g_g)T_l(g_g) - 0.7187T_l^2(g_g)$$

Therefore welfare in country *l* is given by:

$$
\begin{aligned}
W_l(g_g) &= CS_l(g_g) + \pi_l(g_g) + PS_l(g_g) + TR_l(g_g) \\
&= 0.5\big(0.4571 + 0.0149T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) - 0.4770T_l(g_g)\big)^2 \\
&+ 1.75\big(0.1143 - 0.4007T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_g) - 0.4007T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) - 0.4007T_k(g_g) - 0.0565T_l(g_g)\big)^2 \\
&+ 1.75\big(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) + 0.2416T_l(g_g)\big)^2 \\
&+ 0.375\big(0.4571 - 0.1865T_i(g_g) - 0.1865T_j(g_g) - 0.1865T_k(g_g) + 0.0722T_l(g_g)\big)^2 \\
&+ 0.3429T_l(g_g) - 0.0565T_i(g_g)T_l(g_g) - 0.0565T_j(g_g)T_l(g_g) \\
&- 0.0565T_k(g_g)T_l(g_g) - 0.7187T_l^2(g_g)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
&\frac{\partial W_l(g_g)}{\partial T_l(g_g)} = \\
&(-0.4770)\big(0.4571 + 0.0149T_i(g_g) + 0.0149T_j(g_g) + 0.0149T_k(g_g) - 0.4770T_l(g_g)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 - 0.4007T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 + 0.0714T_i(g_g) - 0.4007T_j(g_g) + 0.0714T_k(g_g) - 0.0565T_l(g_g)\big) \\
&+ (3.5)(-0.0565)\big(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) - 0.4007T_k(g_g) - 0.0565T_l(g_g)\big) \\
&+ (3.5)(0.2416)\big(0.1143 + 0.0714T_i(g_g) + 0.0714T_j(g_g) + 0.0714T_k(g_g) + 0.2416T_l(g_g)\big) \\
&+ (0.75)(0.0722)\big(0.4571 - 0.1865T_i(g_g) - 0.1865T_j(g_g) - 0.1865T_k(g_g) + 0.0722T_l(g_g)\big) \\
&+ 0.3429 - 0.0565T_i(g_g) - 0.0565T_j(g_g) - 0.0565T_k(g_g) - 1.4374T_l(g_g) \\
&= 0.1784 + 0.0377T_i(g_g) + 0.0377T_j(g_g) + 0.0377T_k(g_g) - 0.9680T_l(g_g)
\end{aligned}
$$

$$
\frac{\partial^2 W_l(g_g)}{\partial T_l^2(g_g)} = -0.9680
$$

Therefore the optimal tariff in country *l* is: $T_l^*(g_g)$ = 0.1843 + $0.0389T_i(g_g)$ + $0.0389T_j(g_g)$ + $0.0389T_k(g_g)$. Given symmetry across countries it is concluded that: $T_l^*(g_g) = 0.1843 + 0.1168T_i(g_g)$. In considering the optimal tariff equations of countries *i* and *l* and using symmetry across countries, the following optimal tariffs for countries *i*, *j*, *k* and *l* are obtained: $T_i^*(g_g) = T_j^*(g_g) = T_k^*(g_g) = 0.0809$; and $T_l^*(g_g) = 0.1938$. Using these tariffs it is concluded that: $CS_i(g_g) = CS_j(g_g) = CS_k(g_g) = 0.1037$; $CS_l(g_g) = 0.0678$; $\pi_i(g_g)$ = $\pi_j(g_g)$ = $\pi_k(g_g)$ = 0.0881; $\pi_l(g_g) = 0.0914$; $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) = 0.0713$; $PS_l(g_g) = 0.0680$; $TR_i(g_g)$ = $TR_j(g_g)$ = $TR_k(g_g)$ = 0.0067; $TR_l(g_g) = 0.0368$; $W_i(g_g)$ = $W_j(g_g)$ = $W_k(g_g)$ = 0.2698; $W_l(g_g) = 0.2641$.

Network h

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_h) = \frac{2\alpha(\phi+1) + \phi\begin{pmatrix} q_j^{(j)}(g_h) + q_k^{(j)}(g_h) + q_l^{(j)}(g_h) + q_j^{(k)}(g_h) + q_k^{(k)}(g_h) \\ q_l^{(k)}(g_h) + q_j^{(l)}(g_h) + q_k^{(l)}(g_h) + q_l^{(l)}(g_h) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_h) + q_k^{(i)}(g_h) + q_l^{(i)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_h) = \frac{2\alpha(\phi+1) + 4T_j(g_h) + \phi\begin{pmatrix} q_i^{(j)}(g_h) + q_k^{(j)}(g_h) + q_l^{(j)}(g_h) + q_i^{(k)}(g_h) + q_k^{(k)}(g_h) \\ q_l^{(k)}(g_h) + q_i^{(l)}(g_h) + q_k^{(l)}(g_h) + q_l^{(l)}(g_h) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_h) + q_k^{(i)}(g_h) + q_l^{(i)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_h) = \frac{2\alpha(\phi+1) + 4T_k(g_h) + \phi\begin{pmatrix} q_i^{(j)}(g_h) + q_j^{(j)}(g_h) + q_l^{(j)}(g_h) + q_i^{(k)}(g_h) + q_j^{(k)}(g_h) \\ q_l^{(k)}(g_h) + q_i^{(l)}(g_h) + q_j^{(l)}(g_h) + q_l^{(l)}(g_h) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_h) + q_j^{(i)}(g_h) + q_l^{(i)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_h)=\frac{2\alpha(\phi+1)+4T_l(g_h)+\phi\begin{pmatrix}q_i^{(j)}(g_h)+q_j^{(j)}(g_h)+q_k^{(j)}(g_h)+q_i^{(k)}(g_h)+q_j^{(k)}(g_h)\\ q_k^{(k)}(g_h)+q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_k^{(i)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_h)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_j^{(k)}(g_h)+q_k^{(k)}(g_h)\\ q_l^{(k)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_h)+q_k^{(j)}(g_h)+q_l^{(j)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_h)=\frac{2\alpha(\phi+1)+4T_j(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(k)}(g_h)+q_k^{(k)}(g_h)\\ q_l^{(k)}(g_h)+q_i^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_h)+q_k^{(j)}(g_h)+q_l^{(j)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(k)}(g_h)+q_j^{(k)}(g_h)\\ q_l^{(k)}(g_h)+q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_h)+q_j^{(j)}(g_h)+q_l^{(j)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_l(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_i^{(k)}(g_h)+q_j^{(k)}(g_h)\\ q_k^{(k)}(g_h)+q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_h)+q_j^{(j)}(g_h)+q_k^{(j)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_h)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_j^{(j)}(g_h)+q_k^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_h)+q_k^{(k)}(g_h)+q_l^{(k)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_j(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(j)}(g_h)+q_k^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_i^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_h)+q_k^{(k)}(g_h)+q_l^{(k)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_h)=\frac{2\alpha(\phi+1)+4T_k(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_l^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_h)+q_j^{(k)}(g_h)+q_l^{(k)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_l(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\\ q_k^{(j)}(g_h)+q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_h)+q_j^{(k)}(g_h)+q_k^{(k)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_h)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_j^{(j)}(g_h)+q_k^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_j^{(k)}(g_h)+q_k^{(k)}(g_h)+q_l^{(k)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_j(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_k^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(j)}(g_h)+q_k^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_i^{(k)}(g_h)+q_k^{(k)}(g_h)+q_l^{(k)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_h)+q_k^{(l)}(g_h)+q_l^{(l)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_h)=\frac{2\alpha(\phi+1)-2(3+\phi)T_k(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_l^{(i)}(g_h)+q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\\ q_l^{(j)}(g_h)+q_i^{(k)}(g_h)+q_j^{(k)}(g_h)+q_l^{(k)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_l^{(l)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_h)=\frac{2\alpha(\phi+1)+4T_l(g_h)+\phi\begin{pmatrix}q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_k^{(i)}(g_h)+q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\\ q_k^{(j)}(g_h)+q_i^{(k)}(g_h)+q_j^{(k)}(g_h)+q_k^{(k)}(g_h)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_h)+q_j^{(l)}(g_h)+q_k^{(l)}(g_h)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: $\alpha = 1$ and $\phi = 0$

From the generic equations presented above it is concluded that $q_i^{(i)}(g_h) = \frac{2}{10}$; $q_j^{(i)}(g_h)$ $= \frac{2+4T_j(g_h)}{10}$; $q_k^{(i)}(g_h) = \frac{2+4T_k(g_h)}{10}$; $q_l^{(i)}(g_h) = \frac{2+4T_l(g_h)}{10}$; $q_i^{(j)}(g_h) = \frac{2}{10}$; $q_j^{(j)}(g_h)$ $= \frac{2+4T_j(g_h)}{10}$; $q_k^{(j)}(g_h) = \frac{2-6T_k(g_h)}{10}$; $q_l^{(j)}(g_h) = \frac{2-6T_l(g_h)}{10}$; $q_i^{(k)}(g_h) = \frac{2}{10}$; $q_j^{(k)}(g_h) = \frac{2-6T_j(g_h)}{10}$; $q_k^{(k)}(g_h) = \frac{2+4T_k(g_h)}{10}$; $q_l^{(k)}(g_h) = \frac{2-6T_l(g_h)}{10}$; $q_i^{(l)}(g_h) = \frac{2}{10}$; $q_j^{(l)}(g_h) = \frac{2-6T_j(g_h)}{10}$; $q_k^{(l)}(g_h) = \frac{2-6T_k(g_h)}{10}$; and $q_l^{(l)}(g_h) = \frac{2+4T_l(g_h)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_h) = \frac{1}{2}\left(q_i^{(i)}(g_h) + q_i^{(j)}(g_h) + q_i^{(k)}(g_h) + q_i^{(l)}(g_h)\right)^2 = 0.5(0.8000)^2 = 0.3200$$

$$\pi_i^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_h)\right)^2 = (0.2000)^2 = 0.0400$$

$$\pi_j^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_h)\right)^2 = \left(0.2000 + 0.4000T_j(g_h)\right)^2$$

$$\pi_k^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_h)\right)^2 = \left(0.2000 + 0.4000T_k(g_h)\right)^2$$

$$\pi_l^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_h)\right)^2 = \left(0.2000 + 0.4000T_l(g_h)\right)^2$$

$$PS_i(g_h) = \frac{\phi}{4}\left(q_i^{(i)}(g_h) + q_j^{(i)}(g_h) + q_k^{(i)}(g_h) + q_l^{(i)}(g_h)\right)^2 = 0$$

Therefore welfare in country *i* is given by:

$$W_i(g_h) = CS_i(g_h) + \pi_i(g_h) + PS_i(g_h) + TR_i(g_h) = 0.3600$$
$$+ \left(0.2000 + 0.4000T_j(g_h)\right)^2 + \left(0.2000 + 0.4000T_k(g_h)\right)^2 + \left(0.2000 + 0.4000T_l(g_h)\right)^2$$

Because this country has an agreement with all countries of the world, it holds that $T_i^*(g_h) = 0$.

On the other hand, the following expressions are obtained for country *j*:

$$CS_j(g_h) = \frac{1}{2}\left(q_j^{(i)}(g_h) + q_j^{(j)}(g_h) + q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right)^2 = 0.5\left(0.8000 - 0.4000T_j(g_h)\right)^2$$

$$\pi_i^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_h)\right)^2 = (0.2000)^2 = 0.0400$$

$$\pi_j^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_h)\right)^2 = \left(0.2000 + 0.4000T_j(g_h)\right)^2$$

$$\pi_k^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_h)\right)^2 = \left(0.2000 - 0.6000T_k(g_h)\right)^2$$

$$\pi_l^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_h)\right)^2 = \left(0.2000 - 0.6000T_l(g_h)\right)^2$$

$$PS_j(g_h) = 0$$

$$TR_j(g_h) = T_j(g_h)\left(q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right) = 0.4000T_j(g_h) - 1.2000T_j^2(g_h)$$

Therefore welfare in country *j* is given by:

$$W_j(g_h) = CS_j(g_h) + \pi_j(g_h) + PS_j(g_h) + TR_j(g_h) = 0.5\left(0.8000 - 0.4000T_j(g_h)\right)^2$$
$$+ 0.0400 + \left(0.2000 + 0.4000T_j(g_h)\right)^2 + \left(0.2000 - 0.6000T_k(g_h)\right)^2$$
$$+ \left(0.2000 - 0.6000T_l(g_h)\right)^2 + 0.4000T_j(g_h) - 1.2000T_j^2(g_h)$$

The first and second order conditions of this function are:

$$\frac{\partial W_j(g_h)}{\partial T_j(g_h)} = (-0.4000)\big(0.8000 - 0.4000T_j(g_h)\big) + (2)(0.4000)\big(0.2000 + 0.4000T_j(g_h)\big)$$
$$+ 0.4000 - 2.4000T_j(g_h) = 0.2400 - 1.9200T_j(g_h)$$

$$\frac{\partial^2 W_j(g_h)}{\partial T_j^2(g_h)} = -1.9200$$

Therefore the optimal tariff in country j is: $T_j^*(g_h) = 0.125$. Given symmetry across countries it is concluded that: $T_j^*(g_h) = T_k^*(g_h) = T_l^*(g_h) = 0.125$. Using these tariffs, the following results are obtained: $CS_i(g_h) = 0.3200$; $CS_j(g_h) = CS_k(g_h) = CS_l(g_h) = 0.2813$; $\pi_i(g_h)$ = 0.2275; $\pi_j(g_h) = \pi_k(g_h) = \pi_l(g_h) = 0.1338$; $PS_i(g_h) = PS_j(g_h) = PS_k(g_h) = PS_l(g_h) = 0$; $TR_i(g_h) = 0$; $TR_j(g_h) = TR_k(g_h) = TR_l(g_h) = 0.0313$; $W_i(g_h) = 0.5475$; and $W_j(g_h)$ = $W_k(g_h)$ = $W_l(g_h)$ = 0.4463; ; .

Simulation 5: α = 1 and ϕ = 0.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_h) = 0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)$$

$$q_j^{(i)}(g_h) = 0.1600 + 0.2641T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)$$

$$q_k^{(i)}(g_h) = 0.1600 - 0.0406T_j(g_h) + 0.2641T_k(g_h) - 0.0406T_l(g_h)$$

$$q_l^{(i)}(g_h) = 0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) + 0.2641T_l(g_h)$$

$$q_i^{(j)}(g_h) = 0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)$$

$$q_j^{(j)}(g_h) = 0.1600 + 0.2641T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)$$

$$q_k^{(j)}(g_h) = 0.1600 - 0.0406T_j(g_h) - 0.4470T_k(g_h) + 0.0483T_l(g_h)$$

$$q_l^{(j)}(g_h) = 0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) - 0.4470T_l(g_h)$$

$$q_i^{(k)}(g_h) = 0.1600 + 0.0483T_j(g_h) - 0.0406T_k(g_h) + 0.0483T_l(g_h)$$

$$q_j^{(k)}(g_h) = 0.1600 - 0.4470T_j(g_h) - 0.0406T_k(g_h) + 0.0483T_l(g_h)$$

$$q_k^{(k)}(g_h) = 0.1600 + 0.0483T_j(g_h) + 0.2641T_k(g_h) + 0.0483T_l(g_h)$$

$$q_l^{(k)}(g_h) = 0.1600 + 0.0483T_j(g_h) - 0.0406T_k(g_h) - 0.4470T_l(g_h)$$

$$q_i^{(l)}(g_h) = 0.1600 + 0.0483T_j(g_h) + 0.0483T_k(g_h) - 0.0406T_l(g_h)$$

$$q_j^{(l)}(g_h) = 0.1600 - 0.4470T_j(g_h) + 0.0483T_k(g_h) - 0.0406T_l(g_h)$$

$$q_k^{(l)}(g_h) = 0.1600 + 0.0483T_j(g_h) - 0.4470T_k(g_h) - 0.0406T_l(g_h)$$

$$q_l^{(l)}(g_h) = 0.1600 + 0.0483T_j(g_h) + 0.0483T_k(g_h) + 0.2641T_l(g_h)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_h) = \frac{1}{2}\left(q_i^{(i)}(g_h) + q_i^{(j)}(g_h) + q_i^{(k)}(g_h) + q_i^{(l)}(g_h)\right)^2$$
$$= 0.5\left(0.6400 + 0.0152T_j(g_h) + 0.0152T_k(g_h) + 0.0152T_l(g_h)\right)^2$$

$$\pi_i^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)\right)^2$$

$$\pi_j^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 + 0.2641T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)\right)^2$$

$$\pi_k^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) + 0.2641T_k(g_h) - 0.0406T_l(g_h)\right)^2$$

$$\pi_l^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) + 0.2641T_l(g_h)\right)^2$$

$$PS_i(g_h) = \frac{\phi}{4}\left(q_i^{(i)}(g_h) + q_j^{(i)}(g_h) + q_k^{(i)}(g_h) + q_l^{(i)}(g_h)\right)^2$$
$$= 0.125\left(0.6400 + 0.1422T_j(g_h) + 0.1422T_k(g_h) + 0.1422T_l(g_h)\right)^2$$

Therefore welfare in country *i* is:

$$W_i(g_h) = CS_i(g_h) + \pi_i(g_h) + PS_i(g_h) + TR_i(g_h)$$
$$= 0.5\left(0.6400 + 0.0152T_j(g_h) + 0.0152T_k(g_h) + 0.0152T_l(g_h)\right)^2$$
$$+1.25\left(0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)\right)^2$$
$$+1.25\left(0.1600 + 0.2641T_j(g_h) - 0.0406T_k(g_h) - 0.0406T_l(g_h)\right)^2$$
$$+1.25\left(0.1600 - 0.0406T_j(g_h) + 0.2641T_k(g_h) - 0.0406T_l(g_h)\right)^2$$
$$+1.25\left(0.1600 - 0.0406T_j(g_h) - 0.0406T_k(g_h) + 0.2641T_l(g_h)\right)^2$$
$$+0.125\left(0.6400 + 0.1422T_j(g_h) + 0.1422T_k(g_h) + 0.1422T_l(g_h)\right)^2$$

Because this country has agreements with all countries of the world, tariff in country *i* is: $T_i^*(g_h) = 0$.

On the other hand, the following expressions are obtained for country *j*:

$$CS_j(g_h) = \frac{1}{2}\left(q_j^{(i)}(g_h) + q_j^{(j)}(g_h) + q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right)^2$$
$$= 0.5\left(0.6400 - 0.3657T_j(g_h) + 0.0152T_k(g_h) + 0.0152T_l(g_h)\right)^2$$

$$\pi_i^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\right)^2$$

$$\pi_j^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 + 0.2641T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\right)^2$$

$$\pi_k^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) - 0.4470T_k(g_h) + 0.0483T_l(g_h)\right)^2$$

$$\pi_l^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_h)\right)^2$$
$$= 1.25\left(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) - 0.4470T_l(g_h)\right)^2$$

$$PS_j(g_h) = \frac{\phi}{4}\left(q_i^{(j)}(g_h) + q_j^{(j)}(g_h) + q_k^{(j)}(g_h) + q_l^{(j)}(g_h)\right)^2$$
$$= 0.125\left(0.6400 + 0.1422T_j(g_h) - 0.3022T_k(g_h) - 0.3022T_l(g_h)\right)^2$$

$$TR_j(g_h) = T_j(g_h)\left(q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right)$$
$$= 0.3200T_j(g_h) - 0.8940T_j^2(g_h) + 0.0076T_k(g_h)T_j(g_h) + 0.0076T_l(g_h)T_j(g_h)$$

Therefore welfare in country *j* is:

$$W_j(g_h) = CS_j(g_h) + \pi_j(g_h) + PS_j(g_h) + TR_j(g_h)$$
$$= 0.5\left(0.6400 - 0.3657T_j(g_h) + 0.0152T_k(g_h) + 0.0152T_l(g_h)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\right)^2$$
$$+ 1.25\left(0.1600 + 0.2641T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0406T_j(g_h) - 0.4470T_k(g_h) + 0.0483T_l(g_h)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) - 0.4470T_l(g_h)\right)^2$$
$$+ 0.125\left(0.6400 + 0.1422T_j(g_h) - 0.3022T_k(g_h) - 0.3022T_l(g_h)\right)^2$$
$$+ 0.3200T_j(g_h) - 0.8940T_j^2(g_h) + 0.0076T_k(g_h)T_j(g_h) + 0.0076T_l(g_h)T_j(g_h)$$

The first and second order conditions of this function are:

$$\frac{\partial W_j(g_h)}{\partial T_j(g_h)} =$$
$$= (-0.3657)\big(0.6400 - 0.3657T_j(g_h) + 0.0152T_k(g_h) + 0.0152T_l(g_h)\big)$$
$$+ (2.5)(-0.0406)\big(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\big)$$
$$+ (2.5)(0.2641)\big(0.1600 + 0.2641T_j(g_h) + 0.0483T_k(g_h) + 0.0483T_l(g_h)\big)$$
$$+ (2.5)(-0.0406)\big(0.1600 - 0.0406T_j(g_h) - 0.4470T_k(g_h) + 0.0483T_l(g_h)\big)$$
$$+ (2.5)(-0.0406)\big(0.1600 - 0.0406T_j(g_h) + 0.0483T_k(g_h) - 0.4470T_l(g_h)\big)$$
$$+ (0.25)(0.1422)\big(0.6400 + 0.1422T_j(g_h) - 0.3022T_k(g_h) - 0.3022T_l(g_h)\big)$$
$$+ 0.3200 - 1.7880T_j(g_h) + 0.0076T_k(g_h) + 0.0076T_l(g_h)$$
$$= 0.1656 - 1.4623T_j(g_h) + 0.0588T_k(g_h) + 0.0588T_l(g_h)$$

$$\frac{\partial^2 W_j(g_h)}{\partial T_j^2(g_h)} = -1.4623$$

Therefore the optimal tariff in country *j* is: $T_j^*(g_h) = 0.1132 + 0.0402T_k(g_h) + 0.0402T_l(g_h)$. Given symmetry across countries in is concluded that $T_j^*(g_h) = T_k^*(g_h) = T_l^*(g_h) = 0.1231$. Using these tariffs, the following results are obtained: $CS_i(g_h) = 0.2084$; $CS_j(g_h)$ = $CS_k(g_h)$ = $CS_l(g_h)$ = 0.1792; $\pi_i(g_h) = 0.1512$; $\pi_j(g_h) = \pi_k(g_h) = \pi_l(g_h) = 0.1151$; $PS_i(g_h)$ = 0.0599; $PS_j(g_h) = PS_k(g_h) = PS_l(g_h) = 0.0425$; $TR_i(g_h) = 0$; $TR_j(g_h)$ = $TR_k(g_h)$ = $TR_l(g_h)$ = 0.0261; $W_i(g_h) = 0.4196$; and $W_j(g_h)$ = $W_k(g_h)$ = $W_l(g_h)$ = 0.3629.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_h) = 0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)$$

$$q_j^{(i)}(g_h) = 0.1143 + 0.1611T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)$$

$$q_k^{(i)}(g_h) = 0.1143 - 0.0377T_j(g_h) + 0.1611T_k(g_h) - 0.0377T_l(g_h)$$

$$q_l^{(i)}(g_h) = 0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) + 0.1611T_l(g_h)$$

$$q_i^{(j)}(g_h) = 0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)$$

$$q_j^{(j)}(g_h) = 0.1143 + 0.1611T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)$$

$$q_k^{(j)}(g_h) = 0.1143 - 0.0377T_j(g_h) - 0.3201T_k(g_h) + 0.0526T_l(g_h)$$

$$q_l^{(j)}(g_h) = 0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) - 0.3201T_l(g_h)$$

$$q_i^{(k)}(g_h) = 0.1143 + 0.0526T_j(g_h) - 0.0377T_k(g_h) + 0.0526T_l(g_h)$$

$$q_j^{(k)}(g_h) = 0.1143 - 0.3201T_j(g_h) - 0.0377T_k(g_h) + 0.0526T_l(g_h)$$

$$q_k^{(k)}(g_h) = 0.1143 + 0.0526T_j(g_h) + 0.1611T_k(g_h) + 0.0526T_l(g_h)$$

$$q_l^{(k)}(g_h) = 0.1143 + 0.0526T_j(g_h) - 0.0377T_k(g_h) - 0.3201T_l(g_h)$$

$$q_i^{(l)}(g_h) = 0.1143 + 0.0526T_j(g_h) + 0.0526T_k(g_h) - 0.0377T_l(g_h)$$

$$q_j^{(l)}(g_h) = 0.1143 - 0.3201T_j(g_h) + 0.0526T_k(g_h) - 0.0377T_l(g_h)$$

$$q_k^{(l)}(g_h) = 0.1143 + 0.0526T_j(g_h) - 0.3201T_k(g_h) - 0.0377T_l(g_h)$$

$$q_l^{(l)}(g_h) = 0.1143 + 0.0526T_j(g_h) + 0.0526T_k(g_h) + 0.1611T_l(g_h)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_h) = \frac{1}{2}\left(q_i^{(i)}(g_h) + q_i^{(j)}(g_h) + q_i^{(k)}(g_h) + q_i^{(l)}(g_h)\right)^2$$
$$= 0.5\left(0.4571 + 0.0298T_j(g_h) + 0.0298T_k(g_h) + 0.0298T_l(g_h)\right)^2$$

$$\pi_i^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)\right)^2$$

$$\pi_j^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 + 0.1611T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)\right)^2$$

$$\pi_k^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.1611T_k(g_h) - 0.0377T_l(g_h)\right)^2$$

$$\pi_l^{(i)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) + 0.1611T_l(g_h)\right)^2$$

$$PS_i(g_h) = \frac{\phi}{4}\left(q_i^{(i)}(g_h) + q_j^{(i)}(g_h) + q_k^{(i)}(g_h) + q_l^{(i)}(g_h)\right)^2$$
$$= 0.375\left(0.4571 + 0.0481T_j(g_h) + 0.0481T_k(g_h) + 0.0481T_l(g_h)\right)^2$$

Therefore welfare in country *i* is:

$$\begin{aligned}
W_i(g_h) &= CS_i(g_h) + \pi_i(g_h) + PS_i(g_h) + TR_i(g_h) \\
&= 0.5\left(0.4571 + 0.0298T_j(g_h) + 0.0298T_k(g_h) + 0.0298T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 + 0.1611T_j(g_h) - 0.0377T_k(g_h) - 0.0377T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.1611T_k(g_h) - 0.0377T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.0377T_k(g_h) + 0.1611T_l(g_h)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0481T_j(g_h) + 0.0481T_k(g_h) + 0.0481T_l(g_h)\right)^2
\end{aligned}$$

Because this country has an agreement with all countries of the world, tariff in this country is: $T_i^*(g_h) = 0$.

On the other hand, the following expressions are obtained for country *j*:

$$CS_j(g_h) = \frac{1}{2}\left(q_j^{(i)}(g_h) + q_j^{(j)}(g_h) + q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right)^2$$
$$= 0.5\left(0.4571 - 0.3180T_j(g_h) + 0.0298T_k(g_h) + 0.0298T_l(g_h)\right)^2$$

$$\pi_i^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right)^2$$

$$\pi_j^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 + 0.1611T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right)^2$$

$$\pi_k^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.3201T_k(g_h) + 0.0526T_l(g_h)\right)^2$$

$$\pi_l^{(j)}(g_h) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_h)\right)^2$$
$$= 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) - 0.3201T_l(g_h)\right)^2$$

$$PS_j(g_h) = \frac{\phi}{4}\left(q_i^{(j)}(g_h) + q_j^{(j)}(g_h) + q_k^{(j)}(g_h) + q_l^{(j)}(g_h)\right)^2$$
$$= 0.375\left(0.4571 + 0.0481T_j(g_h) - 0.1624T_k(g_h) - 0.1624T_l(g_h)\right)^2$$

$$TR_j(g_h) = T_j(g_h)\left(q_j^{(k)}(g_h) + q_j^{(l)}(g_h)\right)$$
$$= 0.2286T_j(g_h) - 0.6402T_j^2(g_h) + 0.0149T_k(g_h)T_j(g_h) + 0.0149T_l(g_h)T_j(g_h)$$

Therefore welfare in country *j* is:

$$
\begin{aligned}
W_j(g_h) &= CS_j(g_h) + \pi_j(g_h) + PS_j(g_h) + TR_j(g_h) \\
&= 0.5\left(0.4571 - 0.3180T_j(g_h) + 0.0298T_k(g_h) + 0.0298T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 + 0.1611T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) - 0.3201T_k(g_h) + 0.0526T_l(g_h)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) - 0.3201T_l(g_h)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0481T_j(g_h) - 0.1624T_k(g_h) - 0.1624T_l(g_h)\right)^2 \\
&+ 0.2286T_j(g_h) - 0.6402T_j^2(g_h) + 0.0149T_k(g_h)T_j(g_h) + 0.0149T_l(g_h)T_j(g_h)
\end{aligned}
$$

The first and second order conditions of this function are:

$$
\begin{aligned}
\frac{\partial W_j(g_h)}{\partial T_j(g_h)} &= \\
&= (-0.3180)\left(0.4571 - 0.3180T_j(g_h) + 0.0298T_k(g_h) + 0.0298T_l(g_h)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right) \\
&+ (3.5)(0.1611)\left(0.1143 + 0.1611T_j(g_h) + 0.0526T_k(g_h) + 0.0526T_l(g_h)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_j(g_h) - 0.3201T_k(g_h) + 0.0526T_l(g_h)\right) \\
&+ (3.5)(-0.0377)\left(0.1143 - 0.0377T_j(g_h) + 0.0526T_k(g_h) - 0.3201T_l(g_h)\right) \\
&+ (0.75)(0.0481)\left(0.4571 + 0.0481T_j(g_h) - 0.1624T_k(g_h) - 0.1624T_l(g_h)\right) \\
&+ 0.2286 - 1.2804T_j(g_h) + 0.0149T_k(g_h) + 0.0149T_l(g_h) \\
&= 0.1189 - 1.0718T_j(g_h) + 0.0575T_k(g_h) + 0.0575T_l(g_h)
\end{aligned}
$$

$$
\frac{\partial^2 W_j(g_h)}{\partial T_j^2(g_h)} = -1.0718
$$

Therefore the optimal tariff in country j is: $T_j^*(g_h) = 0.1110 + 0.0537T_k(g_h) + 0.0537T_l(g_h)$. Given symmetry across countries it is concluded that: $T_j^*(g_h) = T_k^*(g_h) = T_l^*(g_h) = 0.1243$. Using these tariffs, the following results are obtained: $CS_i(g_h) = 0.1096$; $CS_j(g_h)$ = $CS_k(g_h)$ = $CS_l(g_h)$ = 0.0903; $\pi_i(g_h) = 0.0995$; $\pi_j(g_h) = \pi_k(g_h) = \pi_l(g_h) = 0.0847$; $PS_i(g_h)$ = 0.0846;

$PS_j(g_h) = PS_k(g_h) = PS_l(g_h) = 0.0670$; $TR_i(g_h) = 0$; $TR_j(g_h)$ = $TR_k(g_h)$ = $TR_l(g_h)$ = 0.0190; $W_i(g_h) = 0.2938$; and $W_j(g_h) = W_k(g_h) = W_l(g_h) = 0.2611$.

Network i

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_i) = \frac{2\alpha(\phi+1) + 2T_i(g_i) + \phi\begin{pmatrix} q_j^{(j)}(g_i) + q_k^{(j)}(g_i) + q_l^{(j)}(g_i) + q_j^{(k)}(g_i) + q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) + q_j^{(l)}(g_i) + q_k^{(l)}(g_i) + q_l^{(l)}(g_i) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_i) + q_k^{(i)}(g_i) + q_l^{(i)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_i) = \frac{2\alpha(\phi+1) + 2T_j(g_i) + \phi\begin{pmatrix} q_i^{(j)}(g_i) + q_k^{(j)}(g_i) + q_l^{(j)}(g_i) + q_i^{(k)}(g_i) + q_k^{(k)}(g_i) \\ q_l^{(k)}(g_i) + q_i^{(l)}(g_i) + q_k^{(l)}(g_i) + q_l^{(l)}(g_i) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_i) + q_k^{(i)}(g_i) + q_l^{(i)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_i) = \frac{2\alpha(\phi+1) + \phi\begin{pmatrix} q_i^{(j)}(g_i) + q_j^{(j)}(g_i) + q_l^{(j)}(g_i) + q_i^{(k)}(g_i) + q_j^{(k)}(g_i) \\ q_l^{(k)}(g_i) + q_i^{(l)}(g_i) + q_j^{(l)}(g_i) + q_l^{(l)}(g_i) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_i) + q_j^{(i)}(g_i) + q_l^{(i)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_i) = \frac{2\alpha(\phi+1) - 2(3+\phi)T_l(g_i) + \phi\begin{pmatrix} q_i^{(j)}(g_i) + q_j^{(j)}(g_i) + q_k^{(j)}(g_i) + q_i^{(k)}(g_i) + q_j^{(k)}(g_i) \\ q_k^{(k)}(g_i) + q_i^{(l)}(g_i) + q_j^{(l)}(g_i) + q_k^{(l)}(g_i) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_i) + q_j^{(i)}(g_i) + q_k^{(i)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_i)=\frac{2\alpha(\phi+1)+2T_i(g_i)+\phi\left(\begin{array}{l}q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_j^{(k)}(g_i)+q_k^{(k)}(g_i)\\ q_l^{(k)}(g_i)+q_j^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_j^{(j)}(g_i)+q_k^{(j)}(g_i)+q_l^{(j)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_i)=\frac{2\alpha(\phi+1)+2T_j(g_i)+\phi\left(\begin{array}{l}q_i^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(k)}(g_i)+q_k^{(k)}(g_i)\\ q_l^{(k)}(g_i)+q_i^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_i^{(j)}(g_i)+q_k^{(j)}(g_i)+q_l^{(j)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_i)=\frac{2\alpha(\phi+1)+\phi\left(\begin{array}{l}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(k)}(g_i)+q_j^{(k)}(g_i)\\ q_l^{(k)}(g_i)+q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_i^{(j)}(g_i)+q_j^{(j)}(g_i)+q_l^{(j)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_i)=\frac{2\alpha(\phi+1)-2(3+\phi)T_l(g_i)+\phi\left(\begin{array}{l}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_i^{(k)}(g_i)+q_j^{(k)}(g_i)\\ q_k^{(k)}(g_i)+q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_k^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_i^{(j)}(g_i)+q_j^{(j)}(g_i)+q_k^{(j)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_i)=\frac{2\alpha(\phi+1)+2T_i(g_i)+\phi\left(\begin{array}{l}q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_j^{(j)}(g_i)+q_k^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_j^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_j^{(k)}(g_i)+q_k^{(k)}(g_i)+q_l^{(k)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_i)=\frac{2\alpha(\phi+1)+2T_j(g_i)+\phi\left(\begin{array}{l}q_i^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(j)}(g_i)+q_k^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_i^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_i^{(k)}(g_i)+q_k^{(k)}(g_i)+q_l^{(k)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_i)=\frac{2\alpha(\phi+1)+\phi\left(\begin{array}{l}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(j)}(g_i)+q_j^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_l^{(l)}(g_i)\end{array}\right)-\phi(4+\phi)\left(q_i^{(k)}(g_i)+q_j^{(k)}(g_i)+q_l^{(k)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_i)=\frac{2\alpha(\phi+1)+4T_l(g_i)+\phi\begin{pmatrix}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_i^{(j)}(g_i)+q_j^{(j)}(g_i)\\ q_k^{(j)}(g_i)+q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_k^{(l)}(g_i)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_i)+q_j^{(k)}(g_i)+q_k^{(k)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_i)=\frac{2\alpha(\phi+1)-2(4+\phi)T_i(g_i)+\phi\begin{pmatrix}q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_j^{(j)}(g_i)+q_k^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_j^{(k)}(g_i)+q_k^{(k)}(g_i)+q_l^{(k)}(g_i)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_i)=\frac{2\alpha(\phi+1)-2(4+\phi)T_j(g_i)+\phi\begin{pmatrix}q_i^{(i)}(g_i)+q_k^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(j)}(g_i)+q_k^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_i^{(k)}(g_i)+q_k^{(k)}(g_i)+q_l^{(k)}(g_i)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_i)+q_k^{(l)}(g_i)+q_l^{(l)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_i)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_l^{(i)}(g_i)+q_i^{(j)}(g_i)+q_j^{(j)}(g_i)\\ q_l^{(j)}(g_i)+q_i^{(k)}(g_i)+q_j^{(k)}(g_i)+q_l^{(k)}(g_i)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_l^{(l)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_i)=\frac{2\alpha(\phi+1)+4T_l(g_i)+\phi\begin{pmatrix}q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_k^{(i)}(g_i)+q_i^{(j)}(g_i)+q_j^{(j)}(g_i)\\ q_k^{(j)}(g_i)+q_i^{(k)}(g_i)+q_j^{(k)}(g_i)+q_k^{(k)}(g_i)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_i)+q_j^{(l)}(g_i)+q_k^{(l)}(g_i)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: $\alpha = 1$ and $\phi = 0$

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_i)=\frac{2+2T_i(g_i)}{10}$;

$q_j^{(i)}(g_i) = \frac{2+2T_j(g_i)}{10}$; $q_k^{(i)}(g_i)=\frac{2}{10}$; $q_l^{(i)}(g_i)=\frac{2-6T_l(g_i)}{10}$; $q_i^{(j)}(g_i)=\frac{2+2T_i(g_i)}{10}$;

$q_j^{(j)}(g_i)=\frac{2+2T_j(g_i)}{10}$; $q_k^{(j)}(g_i)=\frac{2}{10}$; $q_l^{(j)}(g_i)=\frac{2-6T_l(g_i)}{10}$; $q_i^{(k)}(g_i)=\frac{2+2T_i(g_i)}{10}$;

$q_j^{(k)}(g_i) = \frac{2 + 2T_j(g_i)}{10}$; $q_k^{(k)}(g_i) = \frac{2}{10}$; $q_l^{(k)}(g_i) = \frac{2 + 4T_l(g_i)}{10}$; $q_i^{(l)}(g_i) = \frac{2 - 8T_i(g_i)}{10}$;

$q_j^{(l)}(g_i) = \frac{2 - 8T_j(g_i)}{10}$; $q_k^{(l)}(g_i) = \frac{2}{10}$; and $q_l^{(l)}(g_i) = \frac{2 + 4T_l(g_i)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_i) = \frac{1}{2}\left(q_i^{(i)}(g_i) + q_i^{(j)}(g_i) + q_i^{(k)}(g_i) + q_i^{(l)}(g_i)\right)^2 = 0.5\left(0.8000 - 0.2000T_i(g_i)\right)^2$$

$$\pi_i^{(i)}(g_i) = \frac{(\phi + 2)}{2}\left(q_i^{(i)}(g_i)\right)^2 = \left(0.2000 + 0.2000T_i(g_i)\right)^2$$

$$\pi_j^{(i)}(g_i) = \frac{(\phi + 2)}{2}\left(q_j^{(i)}(g_i)\right)^2 = \left(0.2000 + 0.2000T_j(g_i)\right)^2$$

$$\pi_k^{(i)}(g_i) = \frac{(\phi + 2)}{2}\left(q_k^{(i)}(g_i)\right)^2 = \left(0.2000\right)^2 = 0.0400$$

$$\pi_l^{(i)}(g_i) = \frac{(\phi + 2)}{2}\left(q_l^{(i)}(g_i)\right)^2 = \left(0.2000 - 0.6000T_l(g_i)\right)^2$$

$$PS_i(g_i) = \frac{\phi}{4}\left(q_i^{(i)}(g_i) + q_j^{(i)}(g_i) + q_k^{(i)}(g_i) + q_l^{(i)}(g_i)\right)^2 = 0$$

$$TR_i(g_i) = T_i(g_i)\left(q_i^{(l)}(g_i)\right) = 0.2000T_i(g_i) - 0.8000T_i^2(g_i)$$

Therefore welfare in country *i* is:

$$\begin{aligned} W_i(g_i) &= CS_i(g_i) + \pi_i(g_i) + PS_i(g_i) + TR_i(g_i) \\ &= 0.5\left(0.8000 - 0.2000T_i(g_i)\right)^2 + \left(0.2000 + 0.2000T_i(g_i)\right)^2 + \left(0.2000 + 0.2000T_j(g_i)\right)^2 \\ &\quad + 0.0400 + \left(0.2000 - 0.6000T_l(g_i)\right)^2 + 0.2000T_i(g_i) - 0.8000T_i^2(g_i) \end{aligned}$$

The first and second conditions of this function are:

$$\frac{\partial W_i(g_i)}{\partial T_i(g_i)} = (-0.2000)\big(0.8000 - 0.2000T_i(g_i)\big)$$
$$+(2)(0.2000)\big(0.2000 + 0.2000T_i(g_i)\big) + 0.2000 - 1.6000T_i(g_i)$$
$$= 0.1200 - 1.4800T_i(g_i)$$

$$\frac{\partial^2 W_i(g_i)}{\partial T_i^2(g_i)} = -1.4800$$

Therefore the optimal tariff in country *i* is: $T_i^*(g_i) = 0.0811$. Given symmetry across countries it is concluded that: $T_i^*(g_i) = T_j^*(g_i) = 0.0811$

On the other hand, the following information is obtained for country *k*:

$$CS_k(g_i) = \frac{1}{2}\big(q_k^{(i)}(g_i) + q_k^{(j)}(g_i) + q_k^{(k)}(g_i) + q_k^{(l)}(g_i)\big)^2 = 0.5(0.8000)^2 = 0.3200$$

$$\pi_i^{(k)}(g_i) = \frac{(\phi+2)}{2}\big(q_i^{(k)}(g_i)\big)^2 = \big(0.2000 + 0.2000T_i(g_i)\big)^2$$

$$\pi_j^{(k)}(g_i) = \frac{(\phi+2)}{2}\big(q_j^{(k)}(g_i)\big)^2 = \big(0.2000 + 0.2000T_j(g_i)\big)^2$$

$$\pi_k^{(k)}(g_i) = \frac{(\phi+2)}{2}\big(q_k^{(k)}(g_i)\big)^2 = (0.2000)^2 = 0.0400$$

$$\pi_l^{(k)}(g_i) = \frac{(\phi+2)}{2}\big(q_l^{(k)}(g_i)\big)^2 = \big(0.2000 + 0.4000T_l(g_i)\big)^2$$

$$PS_k(g_i) = \frac{\phi}{4}\big(q_i^{(k)}(g_i) + q_j^{(k)}(g_i) + q_k^{(k)}(g_i) + q_l^{(k)}(g_i)\big)^2 = 0$$

Therefore welfare in country *k* is:

$$W_k(g_i) = CS_k(g_i) + \pi_k(g_i) + PS_k(g_i) + TR_k(g_i) = 0.3600 + \big(0.2000 + 0.2000T_i(g_i)\big)^2$$
$$+\big(0.2000 + 0.2000T_j(g_i)\big)^2 + \big(0.2000 + 0.4000T_l(g_i)\big)^2$$

Because country i has an agreement with all countries in the world, the tariff in this country is: $TR_k(g_i) = 0$.

Finally, the following information is obtained for country l:

$$CS_l(g_i) = \frac{1}{2}\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i) + q_l^{(k)}(g_i) + q_l^{(l)}(g_i)\right)^2 = 0.5\left(0.8000 - 0.4000T_l(g_i)\right)^2$$

$$\pi_i^{(l)}(g_i) = \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_i)\right)^2 = \left(0.2000 - 0.8000T_i(g_i)\right)^2$$

$$\pi_j^{(l)}(g_i) = \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_i)\right)^2 = \left(0.2000 - 0.8000T_j(g_i)\right)^2$$

$$\pi_k^{(l)}(g_i) = \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_i)\right)^2 = \left(0.2000\right)^2 = 0.0400$$

$$\pi_l^{(l)}(g_i) = \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_i)\right)^2 = \left(0.2000 + 0.4000T_l(g_i)\right)^2$$

$$PS_l(g_i) = \frac{\phi}{4}\left(q_i^{(l)}(g_i) + q_j^{(l)}(g_i) + q_k^{(l)}(g_i) + q_l^{(l)}(g_i)\right)^2 = 0$$

$$TR_l(g_i) = T_l(g_i)\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i)\right) = 0.4000T_l(g_i) - 1.2000T_l^2(g_i)$$

Therefore welfare in country l is:

$$\begin{aligned} W_l(g_i) &= CS_l(g_i) + \pi_l(g_i) + PS_l(g_i) + TR_l(g_i) \\ &= 0.5\left(0.8000 - 0.4000T_l(g_i)\right)^2 + \left(0.2000 - 0.8000T_i(g_i)\right)^2 + \left(0.2000 - 0.8000T_j(g_i)\right)^2 \\ &+ 0.0400 + \left(0.2000 + 0.4000T_l(g_i)\right)^2 + 0.4000T_l(g_i) - 1.2000T_l^2(g_i) \end{aligned}$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_i)}{\partial T_l(g_i)} = (-0.4000)\big(0.8000 - 0.4000T_l(g_i)\big) + (2)(0.4000)\big(0.2000 + 0.4000T_l(g_i)\big)$$
$$+ 0.4000 - 2.4000T_l(g_i) = 0.2400 - 1.9200T_l(g_i)$$

$$\frac{\partial^2 W_l(g_i)}{\partial T_l^2(g_i)} = -1.9200$$

Therefore the optimal tariff in country *l* is $T_l^*(g_i) = 0.125$. In considering this tariff and the tariffs in countries *i*, *j* and *k*, the following results are obtained: $CS_i(g_i) = CS_j(g_i) = 0.3072$; $CS_k(g_i) = 0.3200$; $CS_l(g_i) = 0.2813$; $\pi_i(g_i) = \pi_j(g_i) = 0.1491$; $\pi_k(g_i) = 0.1960$; $\pi_l(g_i)$ = 0.1390; $PS_i(g_i) = PS_j(g_i) = PS_k(g_i) = PS_l(g_i) = 0$; $TR_i(g_i) = TR_j(g_i) = 0.0110$; $TR_k(g_i)$ = 0; $TR_l(g_i) = 0.0313$; $W_i(g_i) = W_j(g_i) = 0.4672$; $W_k(g_i) = 0.5160$; and $W_l(g_i) = 0.4515$.

<u>*Simulation 5: α = 1 and φ = 0.5*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_i) = 0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)$$

$$q_j^{(i)}(g_i) = 0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) + 0.0483T_l(g_i)$$

$$q_k^{(i)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)$$

$$q_l^{(i)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.4470T_l(g_i)$$

$$q_i^{(j)}(g_i) = 0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)$$

$$q_j^{(j)}(g_i) = 0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) + 0.0483T_l(g_i)$$

$$q_k^{(j)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)$$

$$q_l^{(j)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.4470T_l(g_i)$$

$$q_i^{(k)}(g_i) = 0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)$$

$$q_j^{(k)}(g_i) = 0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) - 0.0406T_l(g_i)$$

$$q_k^{(k)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)$$

$$q_l^{(k)}(g_i) = 0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.2641T_l(g_i)$$

$$q_i^{(l)}(g_i) = 0.1600 - 0.5790T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)$$

$$q_j^{(l)}(g_i) = 0.1600 + 0.0686T_i(g_i) - 0.5790T_j(g_i) - 0.0406T_l(g_i)$$

$$q_k^{(l)}(g_i) = 0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)$$

$$q_l^{(l)}(g_i) = 0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) + 0.2641T_l(g_i)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_i) = \frac{1}{2}\left(q_i^{(i)}(g_i) + q_i^{(j)}(g_i) + q_i^{(k)}(g_i) + q_i^{(l)}(g_i)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_i) + 0.0076T_j(g_i) + 0.0152T_l(g_i)\right)^2$$

$$\pi_i^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)^2$$

$$\pi_j^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) + 0.0483T_l(g_i)\right)^2$$

$$\pi_k^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)^2$$

$$\pi_l^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.4470T_l(g_i)\right)^2$$

$$PS_i(g_i) = \frac{\phi}{4}\left(q_i^{(i)}(g_i) + q_j^{(i)}(g_i) + q_k^{(i)}(g_i) + q_l^{(i)}(g_i)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_i) + 0.0711T_j(g_i) - 0.3022T_l(g_i)\right)^2$$

$$TR_i(g_i) = T_i(g_i)\left(q_i^{(l)}(g_i)\right)$$
$$= 0.1600T_i(g_i) - 0.5790T_i^2(g_i) + 0.0686T_j(g_i)T_i(g_i) - 0.0406T_l(g_i)T_i(g_i)$$

Therefore welfare in country *i* is given by:

$$W_i(g_i) = CS_i(g_i) + \pi_i(g_i) + PS_i(g_i) + TR_i(g_i)$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_i) + 0.0076T_j(g_i) + 0.0152T_l(g_i)\right)^2$$
$$+ 1.25\left(0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) + 0.0483T_l(g_i)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)^2$$
$$+ 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.4470T_l(g_i)\right)^2$$
$$+ 0.125\left(0.6400 + 0.0711T_i(g_i) + 0.0711T_j(g_i) - 0.3022T_l(g_i)\right)^2$$
$$+ 0.1600T_i(g_i) - 0.5790T_i^2(g_i) + 0.0686T_j(g_i)T_i(g_i) - 0.0406T_l(g_i)T_i(g_i)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_i)}{\partial T_i(g_i)} =$$
$$(-0.1829)\left(0.6400 - 0.1829T_i(g_i) + 0.0076T_j(g_i) + 0.0152T_l(g_i)\right)$$
$$+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) + 0.0483T_l(g_i)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.0483T_l(g_i)\right)$$
$$+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.4470T_l(g_i)\right)$$
$$+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_i) + 0.0711T_j(g_i) - 0.3022T_l(g_i)\right)$$
$$+ 0.1600 - 1.158T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)$$
$$= 0.0828 - 1.0767T_i(g_i) + 0.0571T_j(g_i) - 0.0151T_l(g_i)$$

$$\frac{\partial^2 W_i(g_i)}{\partial T_i^2(g_i)} = -1.0767$$

Therefore the optimal tariff in country *i* is: $T_i^*(g_i) = 0.0769 + 0.0530T_j(g_i) - 0.0140T_l(g_i)$. Given symmetry across countries it is concluded that: $T_i^*(g_i) = T_j^*(g_i) = 0.0812 - 0.0148T_l(g_i)$.

On the other hand, the following expressions are obtained for country *k*:

$$CS_k(g_i) = \frac{1}{2}\left(q_k^{(i)}(g_i) + q_k^{(j)}(g_i) + q_k^{(k)}(g_i) + q_k^{(l)}(g_i)\right)^2$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_i) + 0.0076T_j(g_i) + 0.0152T_l(g_i)\right)^2$$

$$\pi_i^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_i^{(k)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)\right)^2$$

$$\pi_j^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_j^{(k)}(g_i)\right)^2 =$$
$$1.25\left(0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) - 0.0406T_l(g_i)\right)^2$$

$$\pi_k^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_k^{(k)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)\right)^2$$

$$\pi_l^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_l^{(k)}(g_i)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.2641T_l(g_i)\right)^2$$

$$PS_k(g_i) = \frac{\phi}{4}\left(q_i^{(k)}(g_i) + q_j^{(k)}(g_i) + q_k^{(k)}(g_i) + q_l^{(k)}(g_i)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_i) + 0.0711T_j(g_i) + 0.1422T_l(g_i)\right)^2$$

Therefore welfare in country *k* is given by:

$$
\begin{aligned}
W_k(g_i) &= CS_k(g_i) + \pi_k(g_i) + PS_k(g_i) + TR_k(g_i) \\
&= 0.5\left(0.6400 + 0.0076T_i(g_i) + 0.0076T_j(g_i) + 0.0152T_l(g_i)\right)^2 \\
&+ 1.25\left(0.1600 + 0.1321T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_i) + 0.1321T_j(g_i) - 0.0406T_l(g_i)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) - 0.0406T_l(g_i)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_i) - 0.0203T_j(g_i) + 0.2641T_l(g_i)\right)^2 \\
&+ 0.125\left(0.6400 + 0.0711T_i(g_i) + 0.0711T_j(g_i) + 0.1422T_l(g_i)\right)^2
\end{aligned}
$$

Because this country has agreements with all countries of the world, the tariff in this country is $T_k(g_i) = 0$.

Finally, the following expressions are obtained for country *l*:

$$
\begin{aligned}
CS_l(g_i) &= \frac{1}{2}\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i) + q_l^{(k)}(g_i) + q_l^{(l)}(g_i)\right)^2 \\
&= 0.5\left(0.6400 + 0.0076T_i(g_i) + 0.0076T_j(g_i) - 0.3657T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_i^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_i)\right)^2 \\
&= 1.25\left(0.1600 - 0.5790T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_j^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_i)\right)^2 \\
&= 1.25\left(0.1600 + 0.0686T_i(g_i) - 0.5790T_j(g_i) - 0.0406T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_k^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_i)\right)^2 \\
&= 1.25\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_l^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_i)\right)^2 \\
&= 1.25\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) + 0.2641T_l(g_i)\right)^2
\end{aligned}
$$

$$PS_l(g_i) = \frac{\phi}{4}\left(q_i^{(l)}(g_i) + q_j^{(l)}(g_i) + q_k^{(l)}(g_i) + q_l^{(l)}(g_i)\right)^2$$
$$= 0.125\left(0.6400 - 0.3733T_i(g_i) - 0.3733T_j(g_i) + 0.1422T_l(g_i)\right)^2$$

$$TR_l(g_i) = T_l(g_i)\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i)\right)$$
$$= 0.3200T_l(g_i) - 0.0406T_i(g_i)T_l(g_i) - 0.0406T_j(g_i)T_l(g_i) - 0.8940T_l^2(g_i)$$

Therefore welfare in country l is given by:

$$W_l(g_i) = CS_l(g_i) + \pi_l(g_i) + PS_l(g_i) + TR_l(g_i)$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_i) + 0.0076T_j(g_i) - 0.3657T_l(g_i)\right)^2$$
$$+1.25\left(0.1600 - 0.5790T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)^2$$
$$+1.25\left(0.1600 + 0.0686T_i(g_i) - 0.5790T_j(g_i) - 0.0406T_l(g_i)\right)^2$$
$$+1.25\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)^2$$
$$+1.25\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) + 0.2641T_l(g_i)\right)^2$$
$$+0.125\left(0.6400 - 0.3733T_i(g_i) - 0.3733T_j(g_i) + 0.1422T_l(g_i)\right)^2$$
$$+0.3200T_l(g_i) - 0.0406T_i(g_i)T_l(g_i) - 0.0406T_j(g_i)T_l(g_i) - 0.8940T_l^2(g_i)$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_i)}{\partial T_l(g_i)} =$$
$$= (-0.3657)\left(0.6400 + 0.0076T_i(g_i) + 0.0076T_j(g_i) - 0.3657T_l(g_i)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 - 0.5790T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 + 0.0686T_i(g_i) - 0.5790T_j(g_i) - 0.0406T_l(g_i)\right)$$
$$+(2.5)(-0.0406)\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) - 0.0406T_l(g_i)\right)$$
$$+(2.5)(0.2641)\left(0.1600 + 0.0686T_i(g_i) + 0.0686T_j(g_i) + 0.2641T_l(g_i)\right)$$
$$+(0.25)(0.1422)\left(0.6400 - 0.3733T_i(g_i) - 0.3733T_j(g_i) + 0.1422T_l(g_i)\right)$$
$$+0.3200 - 0.0406T_i(g_i) - 0.0406T_j(g_i) - 1.7880T_l(g_i)$$
$$= 0.1656 + 0.0335T_i(g_i) + 0.0335T_j(g_i) - 1.4623T_l(g_i)$$

$$\frac{\partial^2 W_l(g_i)}{\partial T_l^2(g_i)} = -1.4623$$

Therefore the optimal tariff in country *l* is: $T_l^*(g_i) = 0.1132 + 0.0229T_i(g_i) + 0.0229T_j(g_i)$. Using symmetry across countries, it is concluded that: $T_l^*(g_i) = 0.1132 + 0.0458T_i(g_i)$. Using the tariff function of country *i*, the following tariffs for countries *i*, *j*, *k* and *l* are obtained: $T_i^*(g_i) = T_j^*(g_i) = 0.0795$; $T_k^*(g_i) = 0$; and $T_l^*(g_i) = 0.1169$. Using these tariffs it is concluded that: $CS_i(g_i) = CS_j(g_i) = 0.1971$; $CS_k(g_i) = 0.2067$; $CS_l(g_i) = 0.1791$; $\pi_i(g_i)$ = $\pi_j(g_i)$ = 0.1228; $\pi_k(g_i) = 0.1403$; $\pi_l(g_i) = 0.1183$; $PS_i(g_i) = PS_j(g_i) = 0.0474$; $PS_k(g_i)$ = 0.0558; $PS_l(g_i) = 0.0446$; $TR_i(g_i) = TR_j(g_i) = 0.0091$; $TR_k(g_i) = 0$; $TR_l(g_i) = 0.0244$; $W_i(g_i) = W_j(g_i) = 0.3764$; $W_k(g_i) = 0.4027$; and $W_l(g_i) = 0.3664$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_i) = 0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)$$

$$q_j^{(i)}(g_i) = 0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) + 0.0526T_l(g_i)$$

$$q_k^{(i)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)$$

$$q_l^{(i)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.3201T_l(g_i)$$

$$q_i^{(j)}(g_i) = 0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)$$

$$q_j^{(j)}(g_i) = 0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) + 0.0526T_l(g_i)$$

$$q_k^{(j)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)$$

$$q_l^{(j)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.3201T_l(g_i)$$

$$q_i^{(k)}(g_i) = 0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)$$

$$q_j^{(k)}(g_i) = 0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) - 0.0377T_l(g_i)$$

$$q_k^{(k)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)$$

$$q_l^{(k)}(g_i) = 0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.1611T_l(g_i)$$

$$q_i^{(l)}(g_i) = 0.1143 - 0.4007T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)$$

$$q_j^{(l)}(g_i) = 0.1143 + 0.0714T_i(g_i) - 0.4007T_j(g_i) - 0.0377T_l(g_i)$$

$$q_k^{(l)}(g_i) = 0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)$$

$$q_l^{(l)}(g_i) = 0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) + 0.1611T_l(g_i)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_i) = \frac{1}{2}\left(q_i^{(i)}(g_i) + q_i^{(j)}(g_i) + q_i^{(k)}(g_i) + q_i^{(l)}(g_i)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_i) + 0.0149T_j(g_i) + 0.0298T_l(g_i)\right)^2$$

$$\pi_i^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)^2$$

$$\pi_j^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) + 0.0526T_l(g_i)\right)^2$$

$$\pi_k^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)^2$$

$$\pi_l^{(i)}(g_i) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.3201T_l(g_i)\right)^2$$

$$PS_i(g_i) = \frac{\phi}{4}\left(q_i^{(i)}(g_i) + q_j^{(i)}(g_i) + q_k^{(i)}(g_i) + q_l^{(i)}(g_i)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_i) + 0.0241T_j(g_i) - 0.1624T_l(g_i)\right)^2$$

$$TR_i(g_i) = T_i(g_i)\left(q_i^{(l)}(g_i)\right)$$
$$= 0.1143T_i(g_i) - 0.4007T_i^2(g_i) + 0.0714T_j(g_i)T_i(g_i) - 0.0377T_l(g_i)T_i(g_i)$$

Therefore welfare in country *i* is given by:

$$W_i(g_i) = CS_i(g_i) + \pi_i(g_i) + PS_i(g_i) + TR_i(g_i)$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_i) + 0.0149T_j(g_i) + 0.0298T_l(g_i)\right)^2$$
$$+1.75\left(0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) + 0.0526T_l(g_i)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.3201T_l(g_i)\right)^2$$
$$+0.375\left(0.4571 + 0.0241T_i(g_i) + 0.0241T_j(g_i) - 0.1624T_l(g_i)\right)^2$$
$$+0.1143T_i(g_i) - 0.4007T_i^2(g_i) + 0.0714T_j(g_i)T_i(g_i) - 0.0377T_l(g_i)T_i(g_i)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_i)}{\partial T_i(g_i)} =$$
$$= (-0.1590)\left(0.4571 - 0.1590T_i(g_i) + 0.0149T_j(g_i) + 0.0298T_l(g_i)\right)$$
$$+ (3.5)(0.0805)\left(0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) + 0.0526T_l(g_i)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.0526T_l(g_i)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.3201T_l(g_i)\right)$$
$$+ (0.75)(0.0241)\left(0.4571 + 0.0241T_i(g_i) + 0.0241T_j(g_i) - 0.1624T_l(g_i)\right)$$
$$+ 0.1143 - 0.8014T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)$$
$$= 0.0595 - 0.7492T_i(g_i) + 0.0613T_j(g_i) - 0.0163T_l(g_i)$$

$$\frac{\partial^2 W_i(g_i)}{\partial T_i^2(g_i)} = -0.7492$$

Therefore the optimal tariff in country i is: $T_i^*(g_i) = 0.0794 + 0.0819T_j(g_i) - 0.0218T_l(g_i)$. Using symmetry across countries it is concluded that: $T_i^*(g_i) = T_j^*(g_i) = 0.0865 - 0.0238T_l(g_i)$.

On the other hand, the following expressions are obtained for country k:

$$CS_k(g_i) = \frac{1}{2}\left(q_k^{(i)}(g_i) + q_k^{(j)}(g_i) + q_k^{(k)}(g_i) + q_k^{(l)}(g_i)\right)^2$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_i) + 0.0149T_j(g_i) + 0.0298T_l(g_i)\right)^2$$

$$\pi_i^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_i^{(k)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)\right)^2$$

$$\pi_j^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_j^{(k)}(g_i)\right)^2 =$$
$$1.75\left(0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) - 0.0377T_l(g_i)\right)^2$$

$$\pi_k^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_k^{(k)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)\right)^2$$

$$\pi_l^{(k)}(g_i) = \frac{(\phi+2)}{2}\left(q_l^{(k)}(g_i)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.1611T_l(g_i)\right)^2$$

$$PS_k(g_i) = \frac{\phi}{4}\left(q_i^{(k)}(g_i) + q_j^{(k)}(g_i) + q_k^{(k)}(g_i) + q_l^{(k)}(g_i)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_i) + 0.0241T_j(g_i) + 0.0481T_l(g_i)\right)^2$$

$$TR_k(g_i) = 0$$

Therefore welfare in country k is:

$$
\begin{aligned}
W_k(g_i) &= CS_k(g_i) + \pi_k(g_i) + PS_k(g_i) + TR_k(g_i) \\
&= 0.5\left(0.4571 + 0.0149T_i(g_i) + 0.0149T_j(g_i) + 0.0298T_l(g_i)\right)^2 \\
&+ 1.75\left(0.1143 + 0.0805T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_i) + 0.0805T_j(g_i) - 0.0377T_l(g_i)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) - 0.0377T_l(g_i)\right)^2 \\
&+ 1.75\left(0.1143 - 0.0188T_i(g_i) - 0.0188T_j(g_i) + 0.1611T_l(g_i)\right)^2 \\
&+ 0.375\left(0.4571 + 0.0241T_i(g_i) + 0.0241T_j(g_i) + 0.0481T_l(g_i)\right)^2
\end{aligned}
$$

Because this country has agreements with all countries of the world, the tariff in this country is $TR_k(g_i) = 0$.

Finally, the following expressions are obtained for country *l*:

$$
\begin{aligned}
CS_l(g_i) &= \frac{1}{2}\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i) + q_l^{(k)}(g_i) + q_l^{(l)}(g_i)\right)^2 \\
&= 0.5\left(0.4571 + 0.0149T_i(g_i) + 0.0149T_j(g_i) - 0.3180T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_i^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_i^{(l)}(g_i)\right)^2 \\
&= 1.75\left(0.1143 - 0.4007T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_j^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_j^{(l)}(g_i)\right)^2 \\
&= 1.75\left(0.1143 + 0.0714T_i(g_i) - 0.4007T_j(g_i) - 0.0377T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_k^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_k^{(l)}(g_i)\right)^2 \\
&= 1.75\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\pi_l^{(l)}(g_i) &= \frac{(\phi+2)}{2}\left(q_l^{(l)}(g_i)\right)^2 \\
&= 1.75\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) + 0.1611T_l(g_i)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
PS_l(g_i) &= \frac{\phi}{4}\left(q_i^{(l)}(g_i) + q_j^{(l)}(g_i) + q_k^{(l)}(g_i) + q_l^{(l)}(g_i)\right)^2 \\
&= 0.375\left(0.4571 - 0.1865T_i(g_i) - 0.1865T_j(g_i) + 0.0481T_l(g_i)\right)^2
\end{aligned}
$$

$$TR_l(g_i) = T_l(g_i)\left(q_l^{(i)}(g_i) + q_l^{(j)}(g_i)\right)$$
$$= 0.2286T_l(g_i) - 0.0377T_i(g_i)T_l(g_i) - 0.0377T_j(g_i)T_l(g_i) - 0.6402T_l^2(g_i)$$

Therefore welfare in country *l* is given by:

$$W_l(g_i) = CS_l(g_i) + \pi_l(g_i) + PS_l(g_i) + TR_l(g_i)$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_i) + 0.0149T_j(g_i) - 0.3180T_l(g_i)\right)^2$$
$$+ 1.75\left(0.1143 - 0.4007T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)^2$$
$$+ 1.75\left(0.1143 + 0.0714T_i(g_i) - 0.4007T_j(g_i) - 0.0377T_l(g_i)\right)^2$$
$$+ 1.75\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)^2$$
$$+ 1.75\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) + 0.1611T_l(g_i)\right)^2$$
$$+ 0.375\left(0.4571 - 0.1865T_i(g_i) - 0.1865T_j(g_i) + 0.0481T_l(g_i)\right)^2$$
$$+ 0.2286T_l(g_i) - 0.0377T_i(g_i)T_l(g_i) - 0.0377T_j(g_i)T_l(g_i) - 0.6402T_l^2(g_i)$$

The first and second order conditions of this function are:

$$\frac{\partial W_l(g_i)}{\partial T_l(g_i)} =$$
$$= (-0.3180)\left(0.4571 + 0.0149T_i(g_i) + 0.0149T_j(g_i) - 0.3180T_l(g_i)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 - 0.4007T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 + 0.0714T_i(g_i) - 0.4007T_j(g_i) - 0.0377T_l(g_i)\right)$$
$$+ (3.5)(-0.0377)\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) - 0.0377T_l(g_i)\right)$$
$$+ (3.5)(0.1611)\left(0.1143 + 0.0714T_i(g_i) + 0.0714T_j(g_i) + 0.1611T_l(g_i)\right)$$
$$+ (0.75)(0.0481)\left(0.4571 - 0.1865T_i(g_i) - 0.1865T_j(g_i) + 0.0481T_l(g_i)\right)$$
$$+ 0.2286 - 0.0377T_i(g_i) - 0.0377T_j(g_i) - 1.2804T_l(g_i)$$
$$= 0.1189 + 0.0251T_i(g_i) + 0.0251T_j(g_i) - 1.0718T_l(g_i)$$

$$\frac{\partial^2 W_l(g_i)}{\partial T_l^2(g_i)} = -1.0718$$

Therefore the optimal tariff in country l is: $T_l^*(g_i) = 0.1110 + 0.0234T_i(g_i) + 0.0234T_j(g_i)$. Using symmetry across countries it is concluded that: $T_l^*(g_i) = 0.1110 + 0.0469T_i(g_i)$. Finally, using the tariff function of country i it is concluded that: $T_i^*(g_i)$ = $T_j^*(g_i)$ = 0.0837; $T_k^*(g_i) = 0$; and $T_l^*(g_i) = 0.1149$. Using these tariffs, the following results are obtained: $CS_i(g_i) = CS_j(g_i) = 0.1006$; $CS_k(g_i) = 0.1072$; $CS_l(g_i) = 0.0895$; $\pi_i(g_i) = \pi_j(g_i) =$ 0.0888; $\pi_k(g_i) = 0.0958$; $\pi_l(g_i) = 0.0864$; $PS_i(g_i) = PS_j(g_i) = 0.0734$; $PS_k(g_i) = 0.0817$; $PS_l(g_i)$ = 0.0698; $TR_i(g_i) = TR_j(g_i) = 0.0069$; $TR_k(g_i) = 0$; $TR_l(g_i) = 0.0171$; $W_i(g_i)$ = $W_j(g_i)$ = 0.2697; $W_k(g_i) = 0.2847$; and $W_l(g_i) = 0.2628$.

Network j

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_j) = \frac{2\alpha(\phi+1) + 2T_i(g_j) + \phi\begin{pmatrix} q_j^{(j)}(g_j) + q_k^{(j)}(g_j) + q_l^{(j)}(g_j) + q_j^{(k)}(g_j) + q_k^{(k)}(g_j) \\ q_l^{(k)}(g_j) + q_j^{(l)}(g_j) + q_k^{(l)}(g_j) + q_l^{(l)}(g_j) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_j) + q_k^{(i)}(g_j) + q_l^{(i)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_j) = \frac{2\alpha(\phi+1) + \phi\begin{pmatrix} q_i^{(j)}(g_j) + q_k^{(j)}(g_j) + q_l^{(j)}(g_j) + q_i^{(k)}(g_j) + q_k^{(k)}(g_j) \\ q_l^{(k)}(g_j) + q_i^{(l)}(g_j) + q_k^{(l)}(g_j) + q_l^{(l)}(g_j) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(i)}(g_j) + q_k^{(i)}(g_j) + q_l^{(i)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_l^{(j)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)\\ q_l^{(k)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_l^{(i)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_j)=\frac{2\alpha(\phi+1)-2(4+\phi)T_l(g_j)+\phi\begin{pmatrix}q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)\\ q_k^{(k)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_j)=\frac{2\alpha(\phi+1)+2T_i(g_j)+\phi\begin{pmatrix}q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_j^{(k)}(g_j)+q_k^{(k)}(g_j)\\ q_l^{(k)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_j)+q_k^{(j)}(g_j)+q_l^{(j)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(k)}(g_j)+q_k^{(k)}(g_j)\\ q_l^{(k)}(g_j)+q_i^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_j)+q_k^{(j)}(g_j)+q_l^{(j)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)\\ q_l^{(k)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_l^{(j)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_j)=\frac{2\alpha(\phi+1)+2T_l(g_j)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)\\ q_k^{(k)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_j)=\frac{2\alpha(\phi+1)+2T_i(g_j)+\phi\begin{pmatrix}q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(k)}(g_j)+q_k^{(k)}(g_j)+q_l^{(k)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(j)}(g_j)+q_k^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_i^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_j)+q_k^{(k)}(g_j)+q_l^{(k)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(j)}(g_j)+q_j^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_l^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_j)+q_j^{(k)}(g_j)+q_l^{(k)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_j)=\frac{2\alpha(\phi+1)+2T_l(g_j)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_i^{(j)}(g_j)+q_j^{(j)}(g_j)\\ q_k^{(j)}(g_j)+q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(k)}(g_j)+q_j^{(k)}(g_j)+q_k^{(k)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_j)=\frac{2\alpha(\phi+1)-2(4+\phi)T_i(g_j)+\phi\begin{pmatrix}q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_j^{(k)}(g_j)+q_k^{(k)}(g_j)+q_l^{(k)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(j)}(g_j)+q_k^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_i^{(k)}(g_j)+q_k^{(k)}(g_j)+q_l^{(k)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_j)+q_k^{(l)}(g_j)+q_l^{(l)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_j)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_l^{(i)}(g_j)+q_i^{(j)}(g_j)+q_j^{(j)}(g_j)\\ q_l^{(j)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)+q_l^{(k)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_l^{(l)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_j)=\frac{2\alpha(\phi+1)+2T_l(g_j)+\phi\begin{pmatrix}q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_i^{(j)}(g_j)+q_j^{(j)}(g_j)\\ q_k^{(j)}(g_j)+q_i^{(k)}(g_j)+q_j^{(k)}(g_j)+q_k^{(k)}(g_j)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_j)+q_j^{(l)}(g_j)+q_k^{(l)}(g_j)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: α = 1 and ϕ = 0

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_j)=\dfrac{2+2T_i(g_j)}{10}$;

$q_j^{(i)}(g_j)=\dfrac{2}{10}$; $q_k^{(i)}(g_j)=\dfrac{2}{10}$; $q_l^{(i)}(g_j)=\dfrac{2-8T_l(g_j)}{10}$; $q_i^{(j)}(g_j)=\dfrac{2+2T_i(g_j)}{10}$;

$q_j^{(j)}(g_j)=\dfrac{2}{10}$; $q_k^{(j)}(g_j)=\dfrac{2}{10}$; $q_l^{(j)}(g_j)=\dfrac{2+2T_l(g_j)}{10}$; $q_i^{(k)}(g_j)=\dfrac{2+2T_i(g_j)}{10}$;

$q_j^{(k)}(g_j)=\dfrac{2}{10}$; $q_k^{(k)}(g_j)=\dfrac{2}{10}$; $q_l^{(k)}(g_j)=\dfrac{2+2T_l(g_j)}{10}$; $q_i^{(l)}(g_j)=\dfrac{2-8T_i(g_j)}{10}$;

$q_j^{(l)}(g_j)=\dfrac{2}{10}$; $q_k^{(l)}(g_j)=\dfrac{2}{10}$; $q_l^{(l)}(g_j) = \dfrac{2+2T_l(g_j)}{10}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_j)=\frac{1}{2}\left(q_i^{(i)}(g_j)+q_i^{(j)}(g_j)+q_i^{(k)}(g_j)+q_i^{(l)}(g_j)\right)^2=0.5\left(0.8000-0.2000T_i(g_j)\right)^2$$

$$\pi_i^{(i)}(g_j)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_j)\right)^2=\left(0.2000+0.2000T_i(g_j)\right)^2$$

$$\pi_j^{(i)}(g_j)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_j)\right)^2=\left(0.2000\right)^2=0.0400$$

$$\pi_k^{(i)}(g_j)=\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_j)\right)^2=\left(0.2000\right)^2=0.0400$$

$$\pi_l^{(i)}(g_j)=\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_j)\right)^2=\left(0.2000-0.8000T_l(g_j)\right)^2$$

$$PS_i(g_j)=\frac{\phi}{4}\left(q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)+q_l^{(i)}(g_j)\right)^2=0$$

$$TR_i(g_j)=T_i(g_j)\left(q_i^{(l)}(g_j)\right)=0.2000T_i(g_j)-0.8000T_i^2(g_j)$$

Therefore welfare in country *i* is given by:

$$W_i(g_j) = CS_i(g_j) + \pi_i(g_j) + PS_i(g_j) + TR_i(g_j) = 0.5\left(0.8000 - 0.2000T_i(g_j)\right)^2$$
$$+ 0.0800 + \left(0.2000 + 0.2000T_i(g_j)\right)^2 + \left(0.2000 - 0.8000T_l(g_j)\right)^2$$
$$+ 0.2000T_i(g_j) - 0.8000T_i^2(g_j)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_j)}{\partial T_i(g_j)} = (-0.2000)\left(0.8000 - 0.2000T_i(g_j)\right) + (2)(0.2000)\left(0.2000 + 0.2000T_i(g_h)\right)$$
$$+ 0.2000 - 1.6000T_i(g_h) = 0.1200 - 1.4800T_j(g_h)$$

$$\frac{\partial^2 W_i(g_j)}{\partial T_i^2(g_j)} = -1.4800$$

Therefore the optimal tariff in country *i* is $T_i^*(g_j) = 0.0811$. Using symmetry across countries it is concluded that $T_i^*(g_j) = T_l^*(g_j) = 0.0811$.

On the other hand, the following equations are obtained for country *j*:

$$CS_j(g_j) = \frac{1}{2}\left(q_j^{(i)}(g_j) + q_j^{(j)}(g_j) + q_j^{(k)}(g_j) + q_j^{(l)}(g_j)\right)^2 = 0.5(0.8000)^2 = 0.3200$$

$$\pi_i^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_j)\right)^2 = \left(0.2000 + 0.2000T_i(g_j)\right)^2$$

$$\pi_j^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_j)\right)^2 = (0.2000)^2 = 0.0400$$

$$\pi_k^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_j)\right)^2 = (0.2000)^2 = 0.0400$$

$$\pi_l^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_j)\right)^2 = \left(0.2000 + 0.2000T_l(g_j)\right)^2$$

$$PS_j(g_j) = 0$$

Therefore welfare in country *j* is given by:

$$W_j(g_j) = CS_j(g_j) + \pi_j(g_j) + PS_j(g_j) + TR_j(g_j) = 0.4000$$
$$+ \left(0.2000 + 0.2000 T_i(g_j)\right)^2 + \left(0.2000 + 0.2000 T_l(g_j)\right)^2$$

Because this country has agreements with all countries of the world, it is concluded that $TR_j(g_j) = 0$. The same holds in country *k* which implies that $TR_k(g_j) = 0$. Thus, in considering the tariffs in countries *i*, *j*, *k* and *l* it is concluded that: $CS_i(g_j)$ = $CS_l(g_j)$ = 0.3072; $CS_j(g_j) = CS_k(g_j) = 0.3200$; $\pi_i(g_j) = \pi_l(g_j) = 0.1450$; $\pi_j(g_j) = \pi_k(g_j) = 0.1735$; $PS_i(g_j) = PS_j(g_j) = PS_k(g_j) = PS_l(g_j) = 0$; $TR_i(g_j) = TR_l(g_j) = 0.0110$; $TR_j(g_j)$ = $TR_k(g_j)$ = 0; $W_i(g_j) = W_l(g_j) = 0.4631$; and $W_j(g_j) = W_k(g_j) = 4935$.

<u>*Simulation 5: α = 1 and ϕ = 0.5*</u>

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_j) = 0.1600 + 0.1321 T_i(g_j) + 0.0686 T_l(g_j)$$

$$q_j^{(i)}(g_j) = 0.1600 - 0.0203 T_i(g_j) + 0.0686 T_l(g_j)$$

$$q_k^{(i)}(g_j) = 0.1600 - 0.0203 T_i(g_j) + 0.0686 T_l(g_j)$$

$$q_l^{(i)}(g_j) = 0.1600 - 0.0203 T_i(g_j) - 0.5790 T_l(g_j)$$

$$q_i^{(j)}(g_j) = 0.1600 + 0.1321 T_i(g_j) - 0.0203 T_l(g_j)$$

$$q_j^{(j)}(g_j) = 0.1600 - 0.0203 T_i(g_j) - 0.0203 T_l(g_j)$$

$$q_k^{(j)}(g_j) = 0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)$$

$$q_l^{(j)}(g_j) = 0.1600 - 0.0203T_i(g_j) + 0.1321T_l(g_j)$$

$$q_i^{(k)}(g_j) = 0.1600 + 0.1321T_i(g_j) - 0.0203T_l(g_j)$$

$$q_j^{(k)}(g_j) = 0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)$$

$$q_k^{(k)}(g_j) = 0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)$$

$$q_l^{(k)}(g_j) = 0.1600 - 0.0203T_i(g_j) + 0.1321T_l(g_j)$$

$$q_i^{(l)}(g_j) = 0.1600 - 0.5790T_i(g_j) - 0.0203T_l(g_j)$$

$$q_j^{(l)}(g_j) = 0.1600 + 0.686T_i(g_j) - 0.0203T_l(g_j)$$

$$q_k^{(l)}(g_j) = 0.1600 + 0.0686T_i(g_j) - 0.0203T_l(g_j)$$

$$q_l^{(l)}(g_j) = 0.1600 + 0.0686T_i(g_j) + 0.1321T_l(g_j)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_j) = \frac{1}{2}\left(q_i^{(i)}(g_j) + q_i^{(j)}(g_j) + q_i^{(k)}(g_j) + q_i^{(l)}(g_j)\right)^2$$
$$= 0.5\left(0.6400 - 0.1829T_i(g_j) + 0.0076T_l(g_j)\right)^2$$

$$\pi_i^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_j)\right)^2 = 1.25\left(0.1600 + 0.1321T_i(g_j) + 0.0686T_l(g_j)\right)^2$$

$$\pi_j^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_j)\right)^2 = 1.25\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right)^2$$

$$\pi_k^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_j)\right)^2 = 1.25\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right)^2$$

$$\pi_l^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_j)\right)^2 = 1.25\left(0.1600 - 0.0203T_i(g_j) - 0.5790T_l(g_j)\right)^2$$

$$PS_i(g_j) = \frac{\phi}{4}\left(q_i^{(i)}(g_j) + q_j^{(i)}(g_j) + q_k^{(i)}(g_j) + q_l^{(i)}(g_j)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_j) - 0.3733T_l(g_j)\right)^2$$

$$TR_i(g_j) = T_i(g_j)\left(q_i^{(l)}(g_j)\right) = 0.1600T_i(g_j) - 0.5790T_i^2(g_j) - 0.0203T_l(g_j)T_i(g_j)$$

Therefore welfare in country *i* is given by:

$$\begin{aligned}
W_i(g_j) &= CS_i(g_j) + \pi_i(g_j) + PS_i(g_j) + TR_i(g_j) \\
&= 0.5\left(0.6400 - 0.1829T_i(g_j) + 0.0076T_l(g_j)\right)^2 \\
&+ 1.25\left(0.1600 + 0.1321T_i(g_j) + 0.0686T_l(g_j)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right)^2 \\
&+ 1.25\left(0.1600 - 0.0203T_i(g_j) - 0.5790T_l(g_j)\right)^2 \\
&+ 0.125\left(0.6400 + 0.0711T_i(g_j) - 0.3733T_l(g_j)\right)^2 \\
&+ 0.1600T_i(g_j) - 0.5790T_i^2(g_j) - 0.0203T_l(g_j)T_i(g_j)
\end{aligned}$$

The first and second order conditions of this function are:

$$\begin{aligned}
\frac{\partial W_i(g_j)}{\partial T_i(g_j)} &= \\
&= (-0.1829)\left(0.6400 - 0.1829T_i(g_j) + 0.0076T_l(g_j)\right) \\
&+ (2.5)(0.1321)\left(0.1600 + 0.1321T_i(g_j) + 0.0686T_l(g_j)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_j) + 0.0686T_l(g_j)\right) \\
&+ (2.5)(-0.0203)\left(0.1600 - 0.0203T_i(g_j) - 0.5790T_l(g_j)\right) \\
&+ (0.25)(0.0711)\left(0.6400 + 0.0711T_i(g_j) - 0.3733T_l(g_j)\right) \\
&+ 0.1600 - 1.1580T_i(g_j) - 0.0203T_l(g_j) \\
&= 0.0828 - 1.0767T_i(g_j) + 0.0167T_l(g_j)
\end{aligned}$$

$$\frac{\partial^2 W_i(g_j)}{\partial T_i^2(g_j)} = -1.0767$$

Therefore the optimal tariff in country i is: $T_i^*(g_j) = 0.0769 + 0.0155T_l(g_j)$. Using symmetry across countries it is concluded that: $T_i^*(g_j) = T_l^*(g_j) = 0.0781$.

On the other hand, the following expressions are obtained for country j:

$$CS_j(g_j) = \frac{1}{2}\left(q_j^{(i)}(g_j) + q_j^{(j)}(g_j) + q_j^{(k)}(g_j) + q_j^{(l)}(g_j)\right)^2$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_j) + 0.0076T_l(g_j)\right)^2$$

$$\pi_i^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_j)\right)^2$$
$$= 1.25\left(0.1600 + 0.1321T_i(g_j) - 0.0203T_l(g_j)\right)^2$$

$$\pi_j^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_j)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)\right)^2$$

$$\pi_k^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_j)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)\right)^2$$

$$\pi_l^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_j)\right)^2$$
$$= 1.25\left(0.1600 - 0.0203T_i(g_j) + 0.1321T_l(g_j)\right)^2$$

$$PS_j(g_j) = \frac{\phi}{4}\left(q_i^{(j)}(g_j) + q_j^{(j)}(g_j) + q_k^{(j)}(g_j) + q_l^{(j)}(g_j)\right)^2$$
$$= 0.125\left(0.6400 + 0.0711T_i(g_j) + 0.0711T_l(g_j)\right)^2$$

Therefore welfare in country j is given by:

$$W_j(g_j) = CS_j(g_j) + \pi_j(g_j) + PS_j(g_j) + TR_j(g_j)$$
$$= 0.5\left(0.6400 + 0.0076T_i(g_j) + 0.0076T_l(g_j)\right)^2$$
$$+1.25\left(0.1600 + 0.1321T_i(g_j) - 0.0203T_l(g_j)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_j) - 0.0203T_l(g_j)\right)^2$$
$$+1.25\left(0.1600 - 0.0203T_i(g_j) + 0.1321T_l(g_j)\right)^2$$
$$+0.125\left(0.6400 + 0.0711T_i(g_j) + 0.0711T_l(g_j)\right)^2$$

Because this country has agreements with all countries of the world, it is concluded that $TR_j(g_j) = 0$. The same holds in country *k* which implies that $TR_k(g_j) = 0$. Thus, in considering the tariffs in countries *i*, *j*, *k* and *l* it is concluded that: $CS_i(g_j)$ = $CS_l(g_j)$ = 0.1961; $CS_j(g_j) = CS_k(g_j) = 0.2056$; $\pi_i(g_j) = \pi_j(g_j) = 0.1216$; $\pi_j(g_j) = \pi_k(g_j) = 0.1327$; $PS_i(g_j) = PS_l(g_j) = 0.0475$; $PS_j(g_j) = PS_k(g_j) = 0.0530$; $TR_i(g_j) = TR_l(g_j) = 0.0088$; $TR_j(g_j) = TR_k(g_j) = 0$; $W_i(g_j) = W_l(g_j) = 0.3741$; and $W_j(g_j) = W_k(g_j) = 0.3912$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that:

$$q_i^{(i)}(g_j) = 0.1143 + 0.0805T_i(g_j) + 0.0714T_l(g_j)$$

$$q_j^{(i)}(g_j) = 0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)$$

$$q_k^{(i)}(g_j) = 0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)$$

$$q_l^{(i)}(g_j) = 0.1143 - 0.0188T_i(g_j) - 0.4007T_l(g_j)$$

$$q_i^{(j)}(g_j) = 0.1143 + 0.0805T_i(g_j) - 0.0188T_l(g_j)$$

$$q_j^{(j)}(g_j) = 0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)$$

$$q_k^{(j)}(g_j) = 0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)$$

$$q_l^{(j)}(g_j) = 0.1143 - 0.0188T_i(g_j) + 0.0805T_l(g_j)$$

$$q_i^{(k)}(g_j) = 0.1143 + 0.0805T_i(g_j) - 0.0188T_l(g_j)$$

$$q_j^{(k)}(g_j) = 0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)$$

$$q_k^{(k)}(g_j) = 0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)$$

$$q_l^{(k)}(g_j) = 0.1143 - 0.0188T_i(g_j) + 0.0805T_l(g_j)$$

$$q_i^{(l)}(g_j) = 0.1143 - 0.4007T_i(g_j) - 0.0188T_l(g_j)$$

$$q_j^{(l)}(g_j) = 0.1143 + 0.0714T_i(g_j) - 0.0188T_l(g_j)$$

$$q_k^{(l)}(g_j) = 0.1143 + 0.0714T_i(g_j) - 0.0188T_l(g_j)$$

$$q_l^{(l)}(g_j) = 0.1143 + 0.0714T_i(g_j) + 0.0805T_l(g_j)$$

Solving by substitution, the following expressions are obtained for country *i*:

$$CS_i(g_j) = \frac{1}{2}\left(q_i^{(i)}(g_j) + q_i^{(j)}(g_j) + q_i^{(k)}(g_j) + q_i^{(l)}(g_j)\right)^2$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_j) + 0.0149T_l(g_j)\right)^2$$

$$\pi_i^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_j)\right)^2 = 1.75\left(0.1143 + 0.0805T_i(g_j) + 0.0714T_l(g_j)\right)^2$$

$$\pi_j^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_j)\right)^2 = 1.75\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)^2$$

$$\pi_k^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_j)\right)^2 = 1.75\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)^2$$

$$\pi_l^{(i)}(g_j) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_j)\right)^2 = 1.75\left(0.1143 - 0.0188T_i(g_j) - 0.4007T_l(g_j)\right)^2$$

$$PS_i(g_j) = \frac{\phi}{4}\left(q_i^{(i)}(g_j) + q_j^{(i)}(g_j) + q_k^{(i)}(g_j) + q_l^{(i)}(g_j)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241T_i(g_j) - 0.1865T_l(g_j)\right)^2$$

$$TR_i(g_j) = T_i(g_j)\left(q_i^{(l)}(g_j)\right) = 0.1143T_i(g_j) - 0.4007T_i^2(g_j) - 0.0188T_l(g_j)T_i(g_j)$$

Therefore welfare in country *i* is given by:

$$W_i(g_j) = CS_i(g_j) + \pi_i(g_j) + PS_i(g_j) + TR_i(g_j)$$
$$= 0.5\left(0.4571 - 0.1590T_i(g_j) + 0.0149T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 + 0.0805T_i(g_j) + 0.0714T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) - 0.4007T_l(g_j)\right)^2$$
$$+0.375\left(0.4571 + 0.0241T_i(g_j) - 0.1865T_l(g_j)\right)^2$$
$$+0.1143T_i(g_j) - 0.4007T_i^2(g_j) - 0.0188T_l(g_j)T_i(g_j)$$

The first and second order conditions of this function are:

$$\frac{\partial W_i(g_j)}{\partial T_i(g_j)} =$$
$$= (-0.1590)\left(0.4571 - 0.1590T_i(g_j) + 0.0149T_l(g_j)\right)$$
$$+ (3.5)(0.0805)\left(0.1143 + 0.0805T_i(g_j) + 0.0714T_l(g_j)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_j) + 0.0714T_l(g_j)\right)$$
$$+ (3.5)(-0.0188)\left(0.1143 - 0.0188T_i(g_j) - 0.4007T_l(g_j)\right)$$
$$+ (0.75)(0.0241)\left(0.4571 + 0.0241T_i(g_j) - 0.1865T_l(g_j)\right)$$
$$+ 0.1143 - 0.8014T_i(g_j) - 0.0188T_l(g_j)$$
$$= 0.0595 - 0.7492T_i(g_j) + 0.0126T_l(g_j)$$

$$\frac{\partial^2 W_i(g_j)}{\partial T_i^2(g_j)} = -0.7492$$

Therefore the optimal tariff in country i is: $T_i^*(g_j) = 0.0794 + 0.0168 T_l(g_j)$. Using symmetry across countries it is concluded that: $T_i^*(g_j) = T_l^*(g_j) = 0.0807$.

On the other hand, the following expressions are obtained for country j:

$$CS_j(g_j) = \frac{1}{2}\left(q_j^{(i)}(g_j) + q_j^{(j)}(g_j) + q_j^{(k)}(g_j) + q_j^{(l)}(g_j)\right)^2$$
$$= 0.5\left(0.4571 + 0.0149 T_i(g_j) + 0.0149 T_l(g_j)\right)^2$$

$$\pi_i^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_i^{(j)}(g_j)\right)^2$$
$$= 1.75\left(0.1143 + 0.0805 T_i(g_j) - 0.0188 T_l(g_j)\right)^2$$

$$\pi_j^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_j^{(j)}(g_j)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188 T_i(g_j) - 0.0188 T_l(g_j)\right)^2$$

$$\pi_k^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_k^{(j)}(g_j)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188 T_i(g_j) - 0.0188 T_l(g_j)\right)^2$$

$$\pi_l^{(j)}(g_j) = \frac{(\phi+2)}{2}\left(q_l^{(j)}(g_j)\right)^2$$
$$= 1.75\left(0.1143 - 0.0188 T_i(g_j) + 0.0805 T_l(g_j)\right)^2$$

$$PS_j(g_j) = \frac{\phi}{4}\left(q_i^{(j)}(g_j) + q_j^{(j)}(g_j) + q_k^{(j)}(g_j) + q_j^{(j)}(g_j)\right)^2$$
$$= 0.375\left(0.4571 + 0.0241 T_i(g_j) + 0.0241 T_l(g_j)\right)^2$$

Therefore welfare in country j is given by:

$$W_j(g_j) = CS_j(g_j) + \pi_j(g_j) + PS_j(g_j) + TR_j(g_j)$$
$$= 0.5\left(0.4571 + 0.0149T_i(g_j) + 0.0149T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 + 0.0805T_i(g_j) - 0.0188T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) - 0.0188T_l(g_j)\right)^2$$
$$+1.75\left(0.1143 - 0.0188T_i(g_j) + 0.0805T_l(g_j)\right)^2$$
$$+0.375\left(0.4571 + 0.0241T_i(g_j) + 0.0241T_l(g_j)\right)^2$$

Because this country has agreements with all countries of the world, it is concluded that $TR_j(g_j) = 0$. The same holds in country *k* which implies that $TR_k(g_j) = 0$. Thus, in considering the tariffs in countries *i*, *j*, *k* and *l* it is concluded that: $CS_i(g_j)$ = $CS_l(g_j)$ = 0.0992; $CS_j(g_j) = CS_k(g_j) = 0.1056$; $\pi_i(g_j) = \pi_l(g_j) = 0.0885$; $\pi_j(g_j) = \pi_k(g_j) = 0.0931$; $PS_i(g_j) = PS_l(g_j) = 0.0739$; $PS_j(g_j) = PS_k(g_j) = 0.0797$; $TR_i(g_j) = TR_l(g_j) = 0.0065$; $TR_j(g_j) = TR_k(g_j) = 0$; $W_i(g_j) = W_l(g_j) = 0.2682$; and $W_j(g_j) = W_k(g_j) = 0.2784$.

Network k

In considering the equations presented in Section 4.2.1.4, the following generic expressions are obtained:

$$q_i^{(i)}(g_k) = \frac{2\alpha(\phi+1) + \phi\begin{pmatrix} q_j^{(j)}(g_k) + q_k^{(j)}(g_k) + q_l^{(j)}(g_k) + q_j^{(k)}(g_k) + q_k^{(k)}(g_k) \\ q_l^{(k)}(g_k) + q_j^{(l)}(g_k) + q_k^{(l)}(g_k) + q_l^{(l)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(i)}(g_k) + q_k^{(i)}(g_k) + q_l^{(i)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(i)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(j)}(g_k)+q_k^{(j)}(g_k)+q_l^{(j)}(g_k)+q_i^{(k)}(g_k)+q_k^{(k)}(g_k)\\ q_l^{(k)}(g_k)+q_i^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(i)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(j)}(g_k)+q_j^{(j)}(g_k)+q_l^{(j)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)\\ q_l^{(k)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_l^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_l^{(i)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(i)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(j)}(g_k)+q_j^{(j)}(g_k)+q_k^{(j)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)\\ q_k^{(k)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_k^{(i)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(j)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_j^{(k)}(g_k)+q_k^{(k)}(g_k)\\ q_l^{(k)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_j^{(j)}(g_k)+q_k^{(j)}(g_k)+q_l^{(j)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(j)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(k)}(g_k)+q_k^{(k)}(g_k)\\ q_l^{(k)}(g_k)+q_i^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_k)+q_k^{(j)}(g_k)+q_l^{(j)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(j)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)\\ q_l^{(k)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_l^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_k)+q_j^{(j)}(g_k)+q_l^{(j)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(j)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)\\ q_k^{(k)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(j)}(g_k)+q_j^{(j)}(g_k)+q_k^{(j)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(k)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_j^{(j)}(g_k)+q_k^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(k)}(g_k)+q_k^{(k)}(g_k)+q_l^{(k)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(k)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_i^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(j)}(g_k)+q_k^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_i^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_k)+q_k^{(k)}(g_k)+q_l^{(k)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(k)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(j)}(g_k)+q_j^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_l^{(l)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_k)+q_j^{(k)}(g_k)+q_l^{(k)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(k)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_i^{(j)}(g_k)+q_j^{(j)}(g_k) \\ q_k^{(j)}(g_k)+q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(k)}(g_k)+q_j^{(k)}(g_k)+q_k^{(k)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_i^{(l)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_j^{(j)}(g_k)+q_k^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_j^{(k)}(g_k)+q_k^{(k)}(g_k)+q_l^{(k)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_j^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_j^{(l)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_i^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(j)}(g_k)+q_k^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_i^{(k)}(g_k)+q_k^{(k)}(g_k)+q_l^{(k)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(l)}(g_k)+q_k^{(l)}(g_k)+q_l^{(l)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_k^{(l)}(g_k) = \frac{2\alpha(\phi+1)+\phi\begin{pmatrix} q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_l^{(i)}(g_k)+q_i^{(j)}(g_k)+q_j^{(j)}(g_k) \\ q_l^{(j)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)+q_l^{(k)}(g_k) \end{pmatrix} - \phi(4+\phi)\left(q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_l^{(l)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

$$q_l^{(l)}(g_k)=\frac{2\alpha(\phi+1)+\phi\begin{pmatrix}q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_i^{(j)}(g_k)+q_j^{(j)}(g_k)\\ q_k^{(j)}(g_k)+q_i^{(k)}(g_k)+q_j^{(k)}(g_k)+q_k^{(k)}(g_k)\end{pmatrix}-\phi(4+\phi)\left(q_i^{(l)}(g_k)+q_j^{(l)}(g_k)+q_k^{(l)}(g_k)\right)}{2(\phi+1)(5+\phi)}$$

Simulation 4: α = 1 and ϕ = 0

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_k)$ = $q_j^{(i)}(g_k)$ = $q_k^{(i)}(g_k)$ = $q_l^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_k^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_i^{(k)}(g_k)$ = $q_j^{(k)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_i^{(l)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_k^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = $\frac{1}{5}$. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_k)=\frac{1}{2}\left(q_i^{(i)}(g_k)+q_i^{(j)}(g_k)+q_i^{(k)}(g_k)+q_i^{(l)}(g_k)\right)^2=0.5(0.8000)^2=0.3200$$

$$\pi_i^{(i)}(g_k)=\frac{(\phi+2)}{2}\left(q_i^{(i)}(g_k)\right)^2=(0.2000)^2=0.0400$$

$$\pi_j^{(i)}(g_k)=\frac{(\phi+2)}{2}\left(q_j^{(i)}(g_k)\right)^2=(0.2000)^2=0.0400$$

$$\pi_k^{(i)}(g_k)=\frac{(\phi+2)}{2}\left(q_k^{(i)}(g_k)\right)^2=(0.2000)^2=0.0400$$

$$\pi_l^{(i)}(g_k)=\frac{(\phi+2)}{2}\left(q_l^{(i)}(g_k)\right)^2=(0.2000)^2=0.0400$$

$$PS_i(g_k)=\frac{\phi}{4}\left(q_i^{(i)}(g_k)+q_j^{(i)}(g_k)+q_k^{(i)}(g_k)+q_l^{(i)}(g_k)\right)^2=0$$

$$TR_i(g_k)=0$$

Therefore welfare in country *i* is $W_i(g_k) = 0.4800$. Using symmetry across countries it is concluded that: $CS_i(g_k)$ = $CS_j(g_k)$ = $CS_k(g_k)$ = $CS_l(g_k)$ = 0.3200; $\pi_i(g_k)$ = $\pi_j(g_k)$ = $\pi_k(g_k)$ = $\pi_l(g_k)$ = 0.1600; $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k) = 0$; $TR_i(g_k)$ = $TR_j(g_k)$ = $TR_k(g_k)$ = $TR_l(g_k)$ = 0; and $W_i(g_k) = W_j(g_k) = W_k(g_k) = W_l(g_k) = 0.4800$.

<u>*Simulation 5: α = 1 and ϕ = 0.5*</u>

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_k)$ = $q_j^{(i)}(g_k)$ = $q_k^{(i)}(g_k)$ = $q_l^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_k^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_i^{(k)}(g_k)$ = $q_j^{(k)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_i^{(l)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_k^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = 0.1600. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_k) = \frac{1}{2}\left(q_i^{(i)}(g_k) + q_i^{(j)}(g_k) + q_i^{(k)}(g_k) + q_i^{(l)}(g_k)\right)^2 = 0.5(0.6400)^2 = 0.2048$$

$$\pi_i^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_k)\right)^2 = 1.25(0.1600)^2 = 0.0320$$

$$\pi_j^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_k)\right)^2 = 1.25(0.1600)^2 = 0.0320$$

$$\pi_k^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_k)\right)^2 = 1.25(0.1600)^2 = 0.0320$$

$$\pi_l^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_k)\right)^2 = 1.25(0.1600)^2 = 0.0320$$

$$PS_i(g_k) = \frac{\phi}{4}\left(q_i^{(i)}(g_k) + q_j^{(i)}(g_k) + q_k^{(i)}(g_k) + q_l^{(i)}(g_k)\right)^2 = 0.125(0.6400)^2 = 0.0512$$

$$TR_i(g_k) = 0$$

Therefore welfare in country *i* is $W_i(g_k) = 0.3840$. Using symmetry across countries it is concluded that: $CS_i(g_k)$ = $CS_j(g_k)$ = $CS_k(g_k)$ = $CS_l(g_k)$ = 0.2048; $\pi_i(g_k)$ = $\pi_j(g_k)$ = $\pi_k(g_k)$ = $\pi_l(g_k)$ = 0.1280; $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k)$ = 0.0512; $TR_i(g_k)$ = $TR_j(g_k)$ = $TR_k(g_k)$ = $TR_l(g_k)$ = 0; and $W_i(g_k) = W_j(g_k) = W_k(g_k) = W_l(g_k) = 0.3840$.

Simulation 6: α = 1 and ϕ = 1.5

From the generic equations presented above it is concluded that: $q_i^{(i)}(g_k)$ = $q_j^{(i)}(g_k)$ = $q_k^{(i)}(g_k)$ = $q_l^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_k^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_i^{(k)}(g_k)$ = $q_j^{(k)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_i^{(l)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_k^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = 0.1143. From these outputs, the following expressions are obtained for country *i*:

$$CS_i(g_k) = \frac{1}{2}\left(q_i^{(i)}(g_k) + q_i^{(j)}(g_k) + q_i^{(k)}(g_k) + q_i^{(l)}(g_k)\right)^2 = 0.5(0.4571)^2 = 0.1045$$

$$\pi_i^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_i^{(i)}(g_k)\right)^2 = 1.75(0.1143)^2 = 0.0229$$

$$\pi_j^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_j^{(i)}(g_k)\right)^2 = 1.75(0.1143)^2 = 0.0229$$

$$\pi_k^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_k^{(i)}(g_k)\right)^2 = 1.75(0.1143)^2 = 0.0229$$

$$\pi_l^{(i)}(g_k) = \frac{(\phi+2)}{2}\left(q_l^{(i)}(g_k)\right)^2 = 1.75(0.1143)^2 = 0.0229$$

$$PS_i(g_k) = \frac{\phi}{4}\left(q_i^{(i)}(g_k) + q_j^{(i)}(g_k) + q_k^{(i)}(g_k) + q_l^{(i)}(g_k)\right)^2 = 0.375(0.4571)^2 = 0.0784$$

$$TR_i(g_k) = 0$$

Therefore welfare in country i is $W_i(g_k) = 0.2743$. Using symmetry across countries it is concluded that: $CS_i(g_k)$ = $CS_j(g_k)$ = $CS_k(g_k)$ = $CS_l(g_k)$ = 0.1045; $\pi_i(g_k)$ = $\pi_j(g_k)$ = $\pi_k(g_k)$ = $\pi_l(g_k)$ = 0.0914; $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k)$ = 0.0784; $TR_i(g_k)$ = $TR_j(g_k)$ = $TR_k(g_k)$ = $TR_l(g_k)$ = 0; and $W_i(g_k) = W_j(g_k) = W_k(g_k) = W_l(g_k) = 0.2743$.

APPENDIX C

Simulations for the case of asymmetry in market size

Network a

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_a)$ = $q_k^{(k)}(g_a)$ = $\frac{\alpha}{\phi+2}$; $q_j^{(j)}(g_a)$ = $q_l^{(l)}(g_a)$ = $\frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_a)$ = $CS_k(g_a)$ = $\frac{1}{2}\left(q_i^{(i)}(g_a)\right)^2$; $CS_j(g_a)$ = $CS_l(g_a)$ = $\frac{1}{2}\left(q_j^{(j)}(g_a)\right)^2$; $\pi_i(g_a)$ = $\pi_k(g_a)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_a)\right)^2$; $\pi_j(g_a)$ = $\pi_l(g_a)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_a)\right)^2$; $PS_i(g_a)$ = $PS_k(g_a)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_a)\right)^2$; and $PS_j(g_a)$ = $PS_l(g_a)$ = $\frac{\phi}{4}\left(q_j^{(j)}(g_a)\right)^2$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

$q_i^{(i)}(g_a)$ = $q_k^{(k)}(g_a)$ = $\frac{1}{2}$ = 0.5000; $q_j^{(j)}(g_a)$ = $q_l^{(l)}(g_a)$ = 0; $CS_i(g_a)$ = $CS_k(g_a)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $CS_j(g_a)$ = $CS_l(g_a)$ = 0; $\pi_i(g_a)$ = $\pi_k(g_a)$ = $(0.5000)^2$ = 0.2500; $\pi_j(g_a)$ = $\pi_l(g_a)$ = 0 ; $PS_i(g_a)$ = $PS_j(g_a)$ = $PS_k(g_a)$ = $PS_l(g_a)$ = 0.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

$q_i^{(i)}(g_a) = q_k^{(k)}(g_a) = \frac{1}{2.5} = 0.4000$; $q_j^{(j)}(g_a) = q_l^{(l)}(g_a) = 0$; $CS_i(g_a) = CS_k(g_a) = \frac{1}{2}(0.4000)^2 = 0.0800$; $CS_j(g_a) = CS_l(g_a) = 0$; $\pi_i(g_a) = \pi_k(g_a) = \frac{(2.5)}{2}(0.4000)^2 = 0.2000$; $\pi_j(g_a) = \pi_l(g_a) = 0$; $PS_i(g_a) = PS_k(g_a) = \frac{0.5}{4}(0.4000)^2 = 0.0200$; and $PS_j(g_a) = PS_l(g_a) = 0$.

Simulation 13: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 1.5$

$q_i^{(i)}(g_a) = q_k^{(k)}(g_a) = \frac{1}{3.5} = 0.2857$; $q_j^{(j)}(g_a) = q_l^{(l)}(g_a) = 0$; $CS_i(g_a) = CS_k(g_a) = \frac{1}{2}(0.2857)^2 = 0.0408$; $CS_j(g_a) = CS_l(g_a) = 0$; $\pi_i(g_a) = \pi_k(g_a) = \frac{(3.5)}{2}(0.2857)^2 = 0.1428$; $\pi_j(g_a) = \pi_l(g_a) = 0$; $PS_i(g_a) = PS_k(g_a) = \frac{1.5}{4}(0.2857)^2 = 0.0306$; and $PS_j(g_a) = PS_l(g_a) = 0$.

Simulation 14: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0$

$q_i^{(i)}(g_a) = q_k^{(k)}(g_a) = \frac{1}{2} = 0.5000$; $q_j^{(j)}(g_a) = q_l^{(l)}(g_a) = \frac{0.5}{2} = 0.2500$; $CS_i(g_a) = CS_k(g_a) = \frac{1}{2}(0.5000)^2 = 0.1250$; $CS_j(g_a) = CS_l(g_a) = \frac{1}{2}(0.2500)^2 = 0.0313$; $\pi_i(g_a) = \pi_k(g_a) = (0.5000)^2 = 0.2500$; $\pi_j(g_a) = \pi_l(g_a) = (0.2500)^2 = 0.0625$; and $PS_i(g_a) = PS_j(g_a) = PS_k(g_a) = PS_l(g_a) = 0$.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

$q_i^{(i)}(g_a)$ = $q_k^{(k)}(g_a)$ = $\frac{1}{2.5}$ = 0.4000 $q_j^{(j)}(g_a)$ = $q_l^{(l)}(g_a)$ = $\frac{0.5}{2.5}$ = 0.2000; $CS_i(g_a)$ = $CS_k(g_a)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $CS_j(g_a)$ = $CS_l(g_a)$ = $\frac{1}{2}(0.2000)^2$ = 0.0200; $\pi_i(g_a)$ = $\pi_k(g_a)$ = $\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_j(g_a)$ = $\pi_l(g_a)$ = $\frac{(2.5)}{2}(0.2000)^2$ = 0.0500; $PS_i(g_a)$ = $PS_k(g_a)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; and $PS_j(g_a)$ = $PS_l(g_a)$ = $\frac{0.5}{4}(0.2000)^2$ = 0.0050.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

$q_i^{(i)}(g_a)$ = $q_k^{(k)}(g_a)$ = $\frac{1}{3.5}$ = 0.2857; $q_j^{(j)}(g_a)$ = $q_l^{(l)}(g_a)$ = $\frac{0.5}{3.5}$ = 0.1429; $CS_i(g_a)$ = $CS_k(g_a)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_j(g_a)$ = $CS_l(g_a)$ = $\frac{1}{2}(0.1429)^2$ = 0.0102; $\pi_i(g_a)$ = $\pi_k(g_a)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $\pi_j(g_a)$ = $\pi_l(g_a)$ = $\frac{(3.5)}{2}(0.1429)^2$ = 0.0357; $PS_i(g_a)$ = $PS_k(g_a)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; and $PS_j(g_a)$ = $PS_l(g_a)$ = $\frac{1.5}{4}(0.1429)^2$ = 0.0077.

Network b

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_b)$ = $q_i^{(j)}(g_b)$ = $\frac{2\alpha(\phi+1)-\phi(\phi+1)q_j^{(i)}(g_b)}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_b)$ = $q_j^{(j)}(g_b)$ =

$\frac{2\tilde{\alpha}(\phi+1)-\phi(\phi+1)q_i^{(i)}(g_b)}{2(\phi+3)(\phi+1)}$; $q_k^{(k)}(g_b) = \frac{\alpha}{\phi+2}$; $q_l^{(l)}(g_b) = \frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_b) = \frac{1}{2}\left(q_i^{(i)}(g_b)+q_i^{(j)}(g_b)\right)^2$; $CS_j(g_b) = \frac{1}{2}\left(q_j^{(i)}(g_b)+q_j^{(j)}(g_b)\right)^2$; $CS_k(g_b) = \frac{1}{2}\left(q_k^{(k)}(g_b)\right)^2$; $CS_l(g_b) = \frac{1}{2}\left(q_l^{(l)}(g_b)\right)^2$; $\pi_i^{(i)}(g_b) = \pi_i^{(j)}(g_b) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_b)\right)^2$; $\pi_j^{(i)}(g_b) = \pi_j^{(j)}(g_b) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g_b)\right)^2$; $\pi_k^{(k)}(g_b) = \frac{(2+\phi)}{2}\left(q_k^{(k)}(g_b)\right)^2$; $\pi_l^{(l)}(g_b) = \frac{(2+\phi)}{2}\left(q_l^{(l)}(g_b)\right)^2$; $PS_i(g_b) = \frac{\phi}{4}\left(q_i^{(i)}(g_b)+q_j^{(i)}(g_b)\right)^2$; $PS_j(g_b) = \frac{\phi}{4}\left(q_i^{(j)}(g_b)+q_j^{(j)}(g_b)\right)^2$; $PS_k(g_b) = \frac{\phi}{4}\left(q_k^{(k)}(g_b)\right)^2$; and $PS_l(g_b) = \frac{\phi}{4}\left(q_l^{(l)}(g_b)\right)^2$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

$q_i^{(i)}(g_b) = q_i^{(j)}(g_b) = \frac{1}{3} = 0.3333$; $q_j^{(i)}(g_b) = q_j^{(j)}(g_b) = 0$; $q_k^{(k)}(g_b) = \frac{1}{2} = 0.5000$; $q_l^{(l)}(g_b) = 0$; $CS_i(g_b) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_b) = 0$; $CS_k(g_b) = \frac{1}{2}(0.5000)^2 = 0.1250$; $CS_l(g_b) = 0$; $\pi_i^{(i)}(g_b) = \pi_i^{(j)}(g_b) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_b) = \pi_j^{(j)}(g_b) = 0$; $\pi_k^{(k)}(g_b) = (0.5000)^2$;= 0.2500; $\pi_l^{(l)}(g_b) = 0$; and $PS_i(g_b) = PS_j(g_b) = PS_k(g_b) = PS_l(g_b) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

$q_i^{(i)}(g_b) = q_i^{(j)}(g_b) = \frac{3}{10.5} = 0.2857$; $q_j^{(i)}(g_b) = q_j^{(j)}(g_b) = 0$; $q_k^{(k)}(g_b) = \frac{1}{2.5} = 0.4000$; $q_l^{(l)}(g_b) = 0$; $CS_i(g_b) = \frac{1}{2}(0.2857+0.2857)^2 = 0.1632$; $CS_j(g_b) = 0$; $CS_k(g_b) =$

$\frac{1}{2}(0.4000)^2$ = 0.0800; $CS_l(g_b)$ = 0; $\pi_i^{(i)}(g_b)$ = $\pi_i^{(j)}(g_b)$ = $\frac{(2.5)}{2}(0.2857)^2$ = 0.1020; $\pi_j^{(i)}(g_b)$ = $\pi_j^{(j)}(g_b)$ = 0; $\pi_k^{(k)}(g_b)$ = $\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_l^{(l)}(g_b)$ = 0; $PS_i(g_b)$ = $PS_j(g_b)$ = $\frac{0.5}{4}(0.2857)^2$ = 0.0102; $PS_k(g_b)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; and $PS_l(g_b)$ = 0.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

$q_i^{(i)}(g_b)$ = $q_i^{(j)}(g_b)$ = $\frac{1}{4.5}$ = 0.2222; $q_j^{(i)}(g_b)$ = $q_j^{(j)}(g_b)$ = 0; $q_k^{(k)}(g_b)$ = $\frac{1}{3.5}$ = 0.2857; $q_l^{(l)}(g_b)$ = 0; $CS_i(g_b)$ = $\frac{1}{2}(0.2222+0.2222)^2$ = 0.0987; $CS_j(g_b)$ = 0; $CS_k(g_b)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_l(g_b)$ = 0; $\pi_i^{(i)}(g_b)$ = $\pi_i^{(j)}(g_b)$ = $\frac{(3.5)}{2}(0.2222)^2$ = 0.0864; $\pi_j^{(i)}(g_b)$ = $\pi_j^{(j)}(g_b)$ = 0; $\pi_k^{(k)}(g_b)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $\pi_l^{(l)}(g_b)$ = 0; $PS_i(g_b)$ = $PS_j(g_b)$ = $\frac{1.5}{4}(0.2222)^2$ = 0.0185; $PS_k(g_b)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; and $PS_l(g_b)$ = 0.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

$q_i^{(i)}(g_b)$ = $q_i^{(j)}(g_b)$ = $\frac{1}{3}$ = 0.3333; $q_j^{(i)}(g_b)$ = $q_j^{(j)}(g_b)$ = $\frac{0.5}{3}$ = 0.1667; $q_k^{(k)}(g_b)$ = $\frac{1}{2}$ = 0.5000; $q_l^{(l)}(g_b)$ = $\frac{0.5}{2}$ = 0.2500; $CS_i(g_b)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_j(g_b)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $CS_k(g_b)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $CS_l(g_b)$ = $\frac{1}{2}(0.2500)^2$ = 0.0313; $\pi_i^{(i)}(g_b)$ = $\pi_i^{(j)}(g_b)$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(i)}(g_b)$ = $\pi_j^{(j)}(g_b)$ = $(0.1667)^2$

= 0.0278; $\pi_k^{(k)}(g_b)$ = $(0.5000)^2$ = 0.2500; $\pi_l^{(l)}(g_b)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_b)$ = $PS_j(g_b)$ = $PS_k(g_b)$ = $PS_l(g_b)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

$q_i^{(i)}(g_b)$ = $q_i^{(j)}(g_b)$ = $\dfrac{3-0.75q_j^{(i)}(g_b)}{10.5}$ = 0.2769; $q_j^{(i)}(g_b)$ = $q_j^{(j)}(g_b)$ = $\dfrac{1.5-0.75q_i^{(i)}(g_b)}{10.5}$ = 0.1231; $q_k^{(k)}(g_b)$ = $\dfrac{1}{2.5}$ = 0.4000; $q_l^{(l)}(g_b)$ = $\dfrac{0.5}{2.5}$ = 0.2000; $CS_i(g_b)$ = $\dfrac{1}{2}(0.2769+0.2769)^2$ = 0.1533; $CS_j(g_b)$ = $\dfrac{1}{2}(0.1231+0.1231)^2$ = 0.0303; $CS_k(g_b)$ = $\dfrac{1}{2}(0.4000)^2$ = 0.0800; $CS_l(g_b)$ = $\dfrac{1}{2}(0.2000)^2$ = 0.0200; $\pi_i^{(i)}(g_b)$ = $\pi_i^{(j)}(g_b)$ = $\dfrac{(2.5)}{2}(0.2769)^2$ = 0.0958; $\pi_j^{(i)}(g_b)$ = $\pi_j^{(j)}(g_b)$ = $\dfrac{(2.5)}{2}(0.1231)^2$ = 0.0189; $\pi_k^{(k)}(g_b)$ = $\dfrac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_l^{(l)}(g_b)$ = $\dfrac{(2.5)}{2}(0.2000)^2$ = 0.0500; $PS_i(g_b)$ = $PS_j(g_b)$ = $PS_k(g_b)$ = $\dfrac{0.5}{4}(0.2769+0.1231)^2$;= 0.0200; and $PS_l(g_b)$ = $\dfrac{0.5}{4}(0.2000)^2$ = 0.0050.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

$q_i^{(i)}(g_b)$ = $q_i^{(j)}(g_b)$ = $\dfrac{5-3.75q_j^{(i)}(g_b)}{22.5}$ = 0.2095; $q_j^{(i)}(g_b)$ = $q_j^{(j)}(g_b)$ = $\dfrac{2.5-3.75q_i^{(i)}(g_b)}{22.5}$ = 0.0762; $q_k^{(k)}(g_b)$ = $\dfrac{1}{3.5}$ = 0.2857; $q_l^{(l)}(g_b)$ = $\dfrac{0.5}{3.5}$ = 0.1429; $CS_i(g_b)$ = $\dfrac{1}{2}(0.2095+0.2095)^2$ = 0.0878; $CS_j(g_b)$ = $\dfrac{1}{2}(0.0762+0.0762)^2$ = 0.0116; $CS_k(g_b)$ = $\dfrac{1}{2}(0.2857)^2$ = 0.0408; $CS_l(g_b)$ = $\dfrac{1}{2}(0.1429)^2$ = 0.0102; $\pi_i^{(i)}(g_b)$ = $\pi_i^{(j)}(g_b)$

$= \frac{(3.5)}{2}(0.2095)^2 = 0.0768$; $\pi_j^{(i)}(g_b) = \pi_j^{(j)}(g_b) = \frac{(3.5)}{2}(0.0762)^2 = 0.0102$; $\pi_k^{(k)}(g_b) = \frac{(3.5)}{2}(0.2857)^2 = 0.1428$; $\pi_l^{(l)}(g_b) = \frac{(3.5)}{2}(0.1429)^2 = 0.0357$; $PS_i(g_b) = PS_j(g_b) = PS_k(g_b) = \frac{1.5}{4}(0.2095+0.0762)^2 = 0.0306$; and $PS_l(g_b) = \frac{1.5}{4}(0.1429)^2 = 0.0077$.

Network c

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = \frac{2\alpha}{3\phi+6}$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = \frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_c) = CS_k(g_c) = \frac{1}{2}\left(q_i^{(i)}(g_c)+q_i^{(k)}(g_c)\right)^2$; $CS_j(g_c) = CS_l(g_c) = \frac{1}{2}\left(q_j^{(j)}(g_c)\right)^2$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_i^{(k)}(g_c) = \pi_k^{(k)}(g_c) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_c)\right)^2$; $\pi_j^{(j)}(g_c) = \pi_l^{(l)}(g_c) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_c)\right)^2$; $PS_i(g_c) = PS_k(g_c) = \frac{\phi}{4}\left(q_i^{(i)}(g_c)+q_k^{(i)}(g_c)\right)^2$; and $PS_j(g_c) = PS_l(g_c) = \frac{\phi}{4}\left(q_j^{(j)}(g_c)\right)^2$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = \frac{2}{6} = 0.3333$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = 0$; $CS_i(g_c) = CS_k(g_c) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_c) = CS_l(g_c) = 0$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_i^{(k)}(g_c) = \pi_k^{(k)}(g_c) = (0.3333)^2 = 0.1111$; $\pi_j^{(j)}(g_c) = \pi_l^{(l)}(g_c) = 0$; and $PS_i(g_c) = PS_j(g_c) = PS_k(g_c) = PS_l(g_c) = 0$.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = \frac{2}{7.5} = 0.2667$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = 0$;

$CS_i(g_c) = CS_k(g_c) = \frac{1}{2}(0.2667 + 0.2667)^2 = 0.1423$; $CS_j(g_c) = CS_l(g_c) = 0$; $\pi_i^{(i)}(g_c) =$

$\pi_k^{(i)}(g_c) = \pi_i^{(k)}(g_c) = \pi_k^{(k)}(g_c) = \frac{(2.5)}{2}(0.2667)^2 = 0.0889$; $\pi_j^{(j)}(g_c) = \pi_l^{(l)}(g_c) = 0$;

$PS_i(g_c) = PS_k(g_c) = \frac{0.5}{4}(0.2667 + 0.2667)^2 = 0.0356$; and $PS_j(g_c) = PS_l(g_c) = 0$.

Simulation 13: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 1.5$

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = \frac{2}{10.5} = 0.1905$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = 0$;

$CS_i(g_c) = CS_k(g_c) = \frac{1}{2}(0.1905 + 0.1905)^2 = 0.0726$; $CS_j(g_c) = CS_l(g_c) = 0$; $\pi_i^{(i)}(g_c) =$

$\pi_k^{(i)}(g_c) = \pi_i^{(k)}(g_c) = \pi_k^{(k)}(g_c) = \frac{(3.5)}{2}(0.1905)^2 = 0.0635$; $\pi_j^{(j)}(g_c) = \pi_l^{(l)}(g_c) = 0$;

$PS_i(g_c) = PS_k(g_c) = \frac{1.5}{4}(0.1905 + 0.1905)^2 = 0.0544$; and $PS_j(g_c) = PS_l(g_c) = 0$.

Simulation 14: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0$

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = \frac{2}{6} = 0.3333$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = \frac{0.5}{2} =$ 0.2500; $CS_i(g_c) = CS_k(g_c) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_j(g_c) = CS_l(g_c) =$

$\frac{1}{2}(0.2500)^2$ = 0.0313; $\pi_i^{(i)}(g_c)$ = $\pi_k^{(i)}(g_c)$ = $\pi_i^{(k)}(g_c)$ = $\pi_k^{(k)}(g_c)$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(j)}(g_c)$ = $\pi_l^{(l)}(g_c)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_c)$ = $PS_j(g_c)$ = $PS_k(g_c)$ = $PS_l(g_c)$ = 0.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

$q_i^{(i)}(g_c)$ = $q_k^{(i)}(g_c)$ = $q_i^{(k)}(g_c)$ = $q_k^{(k)}(g_c)$ = $\frac{2}{7.5}$ = 0.2667; $q_j^{(j)}(g_c)$ = $q_l^{(l)}(g_c)$ = $\frac{0.5}{2.5}$ = 0.2000; $CS_i(g_c)$ = $CS_k(g_c)$ = $\frac{1}{2}(0.2667+0.2667)^2$ = 0.1422; $CS_j(g_c)$ = $CS_l(g_c)$ = $\frac{1}{2}(0.2000)^2$ = 0.0200; $\pi_i^{(i)}(g_c)$ = $\pi_k^{(i)}(g_c)$ = $\pi_i^{(k)}(g_c)$ = $\pi_k^{(k)}(g_c)$ = $\frac{(2.5)}{2}(0.2667)^2$ = 0.0889; $\pi_j^{(j)}(g_c)$ = $\pi_l^{(l)}(g_c)$ = $\frac{(2.5)}{2}(0.2000)^2$ = 0.0500; $PS_i(g_c)$ = $PS_k(g_c)$ = $\frac{0.5}{4}(0.2667+0.2667)^2$ = 0.0356; and $PS_j(g_c)$ = $PS_l(g_c)$ = $\frac{0.5}{4}(0.2000)^2$ = 0.0050.

Simulation 16: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 1.5$

$q_i^{(i)}(g_c)$ = $q_k^{(i)}(g_c)$ = $q_i^{(k)}(g_c)$ = $q_k^{(k)}(g_c)$ = $\frac{2}{10.5}$ = 0.1905; $q_j^{(j)}(g_c)$ = $q_l^{(l)}(g_c)$ = $\frac{0.5}{3.5}$ = 0.1429; $CS_i(g_c)$ = $CS_k(g_c)$ = $\frac{1}{2}(0.1905+0.1905)^2$ = 0.0726; $CS_j(g_c)$ = $CS_l(g_c)$ = $\frac{1}{2}(0.1429)^2$ = 0.0102; $\pi_i^{(i)}(g_c)$ = $\pi_k^{(i)}(g_c)$ = $\pi_i^{(k)}(g_c)$ = $\pi_k^{(k)}(g_c)$ = $\frac{(3.5)}{2}(0.1905)^2$ = 0.0635; $\pi_j^{(j)}(g_c)$ = $\pi_l^{(l)}(g_c)$ = $\frac{(3.5)}{2}(0.1429)^2$ = 0.0357; $PS_i(g_c)$ = $PS_k(g_c)$ = $\frac{1.5}{4}(0.1905+0.1905)^2$ = 0.0544; and $PS_j(g_c)$ = $PS_l(g_c)$ = $\frac{1.5}{4}(0.1429)^2$ = 0.0077.

Network d

In considering the equations presented in Section 4.2.1.2 it holds: $q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{\alpha}{\phi+2}$; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = $\frac{2\tilde{\alpha}}{3\phi+6}$; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}\left(q_i^{(i)}(g_d)\right)^2$; $CS_j(g_d)$ = $CS_l(g_d)$ = $\frac{1}{2}\left(q_j^{(j)}(g_d)+q_j^{(l)}(g_d)\right)^2$; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_d)\right)^2$; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_d)\right)^2$; $PS_i(g_d)$ = $PS_k(g_d)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_d)\right)^2$; and $PS_j(g_d)$ = $PS_l(g_d)$ = $\frac{\phi}{4}\left(q_j^{(j)}(g_d)+q_l^{(j)}(g_d)\right)^2$.

Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{2}$ = 0.5000; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = 0; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.5000)^2$ = 0.125; $CS_j(g_d)$ = $CS_l(g_d)$ = 0; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $(0.5000)^2$ = 0.2500; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = 0; and $PS_i(g_d)$ = $PS_j(g_d)$ = $PS_k(g_d)$ = $PS_l(g_d)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{2.5}$ = 0.4000; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = 0; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $CS_j(g_d)$ = $CS_l(g_d)$ = 0; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ =

$\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = 0; $PS_i(g_d)$ = $PS_k(g_d)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; and $PS_j(g_d)$ = $PS_l(g_d)$ = 0.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{3.5}$ = 0.2857; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = 0; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_j(g_d)$ = $CS_l(g_d)$ = 0; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = 0; $PS_i(g_d)$ = $PS_k(g_d)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; and $PS_j(g_d)$ = $PS_l(g_d)$ = 0.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{2}$ = 0.5000; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = $\frac{1}{6}$ = 0.1667; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $CS_j(g_d)$ = $CS_l(g_d)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $(0.5000)^2$ = 0.2500; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = $(0.1667)^2$ = 0.0278; and $PS_i(g_d)$ = $PS_j(g_d)$ = $PS_k(g_d)$ = $PS_l(g_d)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{2.5}$ = 0.4000; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = $\frac{1}{7.5}$ = 0.1333; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $CS_j(g_d)$ = $CS_l(g_d)$ = $\frac{1}{2}(0.1333+0.1333)^2$ = 0.0355; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = $\frac{(2.5)}{2}(0.1333)^2$ = 0.0222; $PS_i(g_d)$ = $PS_k(g_d)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; and $PS_j(g_d)$ = $PS_l(g_d)$ = $\frac{0.5}{4}(0.1333+0.1333)^2$ = 0.0089.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = $\frac{1}{3.5}$ = 0.2857; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = $\frac{1}{10.5}$ = 0.0952; $CS_i(g_d)$ = $CS_k(g_d)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_j(g_d)$ = $CS_l(g_d)$ = $\frac{1}{2}(0.0952+0.0952)^2$ = 0.0181; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = $\frac{(3.5)}{2}(0.0952)^2$ = 0.0159; $PS_i(g_d)$ = $PS_k(g_d)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; and $PS_j(g_d)$ = $PS_l(g_d)$ = $\frac{1.5}{4}(0.0952+0.0952)^2$ = 0.0136.

Network e

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_e)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_e)+\phi q_k^{(k)}(g_e)-\phi(\phi+3)q_j^{(i)}(g_e)-\phi(\phi+3)q_k^{(i)}(g_e)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_e)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_e)-\phi(\phi+2)q_i^{(i)}(g_e)-\phi(\phi+2)q_k^{(i)}(g_e)}{2(\phi+3)(\phi+1)}$; $q_k^{(i)}(g_e)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_e)-\phi(\phi+2)q_i^{(i)}(g_e)-\phi(\phi+2)q_j^{(i)}(g_e)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_e)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_e)+\phi q_k^{(i)}(g_e)+\phi q_k^{(k)}(g_e)-\phi(\phi+3)q_j^{(j)}(g_e)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_e)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_e)+\phi q_k^{(i)}(g_e)-\phi(\phi+2)q_i^{(j)}(g_e)}{2(\phi+3)(\phi+1)}$; $q_i^{(k)}(g_e)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_e)+\phi q_k^{(i)}(g_e)+\phi q_j^{(j)}(g_e)-\phi(\phi+3)q_k^{(k)}(g_e)}{2(\phi+4)(\phi+1)}$; $q_k^{(k)}(g_e)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_e)+\phi q_j^{(i)}(g_e)-\phi(\phi+2)q_i^{(k)}(g_e)}{2(\phi+3)(\phi+1)}$; $q_l^{(l)}(g_e)$ = $\frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_e)$ = $\frac{1}{2}\left(q_i^{(i)}(g_e)+q_i^{(j)}(g_e)+q_i^{(k)}(g_e)\right)^2$; $CS_j(g_e)$ = $\frac{1}{2}\left(q_j^{(i)}(g_e)+q_j^{(j)}(g_e)\right)^2$; $CS_k(g_e)$ = $\frac{1}{2}\left(q_k^{(i)}(g_e)+q_k^{(k)}(g_e)\right)^2$; $CS_l(g_e)$ = $\frac{1}{2}\left(q_l^{(l)}(g_e)\right)^2$; $\pi_i^{(i)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_e)\right)^2$; $\pi_j^{(i)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_e)\right)^2$; $\pi_k^{(i)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_k^{(i)}(g_e)\right)^2$; $\pi_i^{(j)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_e)\right)^2$; $\pi_j^{(j)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_e)\right)^2$; $\pi_i^{(k)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_i^{(k)}(g_e)\right)^2$; $\pi_k^{(k)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_e)\right)^2$; $\pi_l^{(l)}(g_e)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_e)\right)^2$; $PS_i(g_e)$ =

$\frac{\phi}{4}\left(q_i^{(i)}(g_e)+q_j^{(i)}(g_e)+q_k^{(i)}(g_e)\right)^2$; $PS_j(g_e) = \frac{\phi}{4}\left(q_i^{(j)}(g_e)+q_j^{(j)}(g_e)\right)^2$; $PS_k(g_e) = \frac{\phi}{4}\left(q_i^{(k)}(g_e)+q_k^{(k)}(g_e)\right)^2$; and $PS_l(g_e) = \frac{\phi}{4}\left(q_l^{(l)}(g_e)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} \\ \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & 0 & 0 \\ \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & 0 & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & -\frac{\phi}{2(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & 0 & 0 \\ 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & 0 & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_j^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_j^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\alpha}{\phi+4} \\ \frac{\alpha}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.3333 \\ 0.2500 \\ 0.2500 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e) = q_i^{(j)}(g_e) = q_i^{(k)}(g_e) = 0.2500$; $q_k^{(i)}(g_e) = q_k^{(k)}(g_e) = 0.3333$; $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$; $CS_i(g_e) = \frac{1}{2}\left(0.2500+0.2500+0.2500\right)^2 = 0.2813$; $CS_j(g_e) = CS_l(g_e) = 0$; $CS_k(g_e) = \frac{1}{2}\left(0.3333+0.3333\right)^2 = 0.2222$; $\pi_i^{(i)}(g_e) = \pi_i^{(j)}(g_e) =$

$\pi_i^{(k)}(g_e) = (0.2500)^2 = 0.0625$; $\pi_k^{(i)}(g_e) = \pi_k^{(k)}(g_e) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_e) = \pi_j^{(j)}(g_e) = \pi_l^{(l)}(g_e) = 0$; and $PS_i(g_e) = PS_j(g_e) = PS_k(g_e) = PS_l(g_e) = 0$.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.1296 & 0 & 0 & -0.0370 \\ 0.1190 & 1 & 0 & -0.0476 & 0 \\ 0 & -0.0370 & 1 & 0 & -0.0370 \\ 0 & -0.0370 & 0 & 1 & 0.1296 \\ -0.0476 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.2857 \\ 0.2222 \\ 0.2222 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e) = q_i^{(k)}(g_e) = 0.1970$; $q_k^{(i)}(g_e) = q_k^{(k)}(g_e) = 0.2716$; $q_i^{(j)}(g_e) = 0.2423$; $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$; $CS_i(g_e) = \frac{1}{2}(0.1970 + 0.2423 + 0.1970)^2 = 0.2024$; $CS_j(g_e) = CS_l(g_e) = 0$; $CS_k(g_e) = \frac{1}{2}(0.2716 + 0.2716)^2 = 0.1475$; $\pi_i^{(i)}(g_e) = \pi_i^{(k)}(g_e) = \frac{(2.5)}{2}(0.1970)^2 = 0.0485$; $\pi_k^{(i)}(g_e) = \pi_k^{(k)}(g_e) = \frac{(2.5)}{2}(0.2716)^2 = 0.0922$; $\pi_i^{(j)}(g_e) = \frac{(2.5)}{2}(0.2423)^2 = 0.0734$; $\pi_j^{(i)}(g_e) = \pi_j^{(j)}(g_e) = \pi_l^{(l)}(g_e) = 0$; $PS_i(g_e) = PS_k(g_e) = \frac{0.5}{4}(0.1970 + 0.2716)^2 = 0.0274$; $PS_j(g_e) = \frac{0.5}{4}(0.2423)^2 = 0.0073$; and $PS_l(g_e) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.2455 & 0 & 0 & -0.0545 \\ 0.2333 & 1 & 0 & -0.0667 & 0 \\ 0 & -0.0545 & 1 & 0 & -0.0545 \\ 0 & -0.0545 & 0 & 1 & 0.2455 \\ -0.0667 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e) = q_i^{(k)}(g_e) = 0.1439$; $q_k^{(i)}(g_e) = q_k^{(k)}(g_e) = 0.1982$; $q_i^{(j)}(g_e) = 0.2034$; $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = q_l^{(l)}(g_e) = 0$; $CS_i(g_e) = \frac{1}{2}(0.1439 + 0.2034 + 0.1439)^2 = 0.1206$; $CS_j(g_e) = CS_l(g_e) = 0$; $CS_k(g_e) = \frac{1}{2}(0.1982 + 0.1982)^2 = 0.0786$; $\pi_i^{(i)}(g_e) = \pi_i^{(k)}(g_e) = \frac{(3.5)}{2}(0.1439)^2 = 0.0362$; $\pi_k^{(i)}(g_e) = \pi_k^{(k)}(g_e) = \frac{(3.5)}{2}(0.1982)^2 = 0.0687$; $\pi_i^{(j)}(g_e) = \frac{(3.5)}{2}(0.2034)^2 = 0.0724$; $\pi_j^{(i)}(g_e) = \pi_j^{(j)}(g_e) = \pi_l^{(l)}(g_e) = 0$; $PS_i(g_e) = PS_k(g_e) = \frac{1.5}{4}(0.1439 + 0.1982)^2 = 0.0439$; $PS_j(g_e) = \frac{1.5}{4}(0.2034)^2 = 0.0155$; and $PS_l(g_e) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_j^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_j^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1667 \\ 0.3333 \\ 0.2500 \\ 0.1667 \\ 0.2500 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e) = q_i^{(j)}(g_e) = q_i^{(k)}(g_e) = q_l^{(l)}(g_e) = 0.2500$; $q_j^{(i)}(g_e) = q_j^{(j)}(g_e) = 0.1667$; $q_k^{(i)}(g_e) = q_k^{(k)}(g_e) = 0.3333$; $CS_i(g_e) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_e) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $CS_k(g_e) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_l(g_e) = \frac{1}{2}(0.2500)^2 = 0.0313$; $\pi_i^{(i)}(g_e) = \pi_i^{(j)}(g_e) = \pi_i^{(k)}(g_e) = \pi_l^{(l)}(g_e) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_e) = \pi_j^{(j)}(g_e) = (0.1667)^2 = 0.0278$; $\pi_k^{(i)}(g_e) = \pi_k^{(k)}(g_e) = (0.3333)^2 = 0.1111$; $PS_i(g_e) = PS_j(g_e) = PS_k(g_e) = PS_l(g_e) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1296 & 0.1296 & 0 & -0.0370 & 0 & -0.0370 \\ 0.1190 & 1 & 0.1190 & -0.0476 & 0 & 0 & 0 \\ 0.1190 & 0.1190 & 1 & 0 & 0 & -0.0476 & 0 \\ 0 & -0.0370 & -0.0370 & 1 & 0.1296 & 0 & -0.0370 \\ -0.0476 & 0 & -0.0476 & 0.1190 & 1 & 0 & 0 \\ 0 & -0.0370 & -0.0370 & 0 & -0.0370 & 1 & 0.1296 \\ -0.0476 & -0.0476 & 0 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_j^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_j^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.1429 \\ 0.2857 \\ 0.2222 \\ 0.1429 \\ 0.2222 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e)$ = 0.1907; $q_j^{(i)}(g_e)$ = 0.1000; $q_k^{(i)}(g_e)$ = 0.2609; $q_i^{(j)}(g_e)$ = 0.2279; $q_j^{(j)}(g_e)$ = 0.1373; $q_i^{(k)}(g_e)$ = 0.2050; $q_k^{(k)}(g_e)$ = 0.2751; $q_l^{(l)}(g_e)$ = 0.2000;

$CS_i(g_e)$ = $\frac{1}{2}(0.1907+0.2279+0.2050)^2$ = 0.1944; $CS_j(g_e)$ = $\frac{1}{2}(0.1000+0.1373)^2$ = 0.0282; $CS_k(g_e)$ = $\frac{1}{2}(0.2609+0.2751)^2$ = 0.1436; $CS_l(g_e)$ = $\frac{1}{2}(0.2000)^2$ = 0.0200;

$\pi_i^{(i)}(g_e)$ = $\frac{(2.5)}{2}(0.1907)^2$ = 0.0455; $\pi_j^{(i)}(g_e)$ = $\frac{(2.5)}{2}(0.1000)^2$ = 0.0125; $\pi_k^{(i)}(g_e)$ = $\frac{(2.5)}{2}(0.2609)^2$ = 0.0851; $\pi_i^{(j)}(g_e)$ = $\frac{(2.5)}{2}(0.2279)^2$ = 0.0649; $\pi_j^{(j)}(g_e)$ = $\frac{(2.5)}{2}(0.1373)^2$ = 0.0236; $\pi_i^{(k)}(g_e)$ = $\frac{(2.5)}{2}(0.2050)^2$ = 0.0525; $\pi_k^{(k)}(g_e)$ = $\frac{(2.5)}{2}(0.2751)^2$ = 0.0946; $\pi_l^{(l)}(g_e)$ = $\frac{(2.5)}{2}(0.2000)^2$ = 0.0500; $PS_i(g_e)$ = $\frac{0.5}{4}(0.1907+0.1000+0.2609)^2$ = 0.0380; $PS_j(g_e)$ = $\frac{0.5}{4}(0.2279+0.1373)^2$ = 0.0167; $PS_k(g_e)$ = $\frac{0.5}{4}(0.2050+0.2751)^2$ = 0.0288; and $PS_l(g_e)$ = $\frac{0.5}{4}(0.2000)^2$ = 0.0050.

Simulation 16: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2455 & 0.2455 & 0 & -0.0545 & 0 & -0.0545 \\ 0.2333 & 1 & 0.2333 & -0.0667 & 0 & 0 & 0 \\ 0.2333 & 0.2333 & 1 & 0 & 0 & -0.0667 & 0 \\ 0 & -0.0545 & -0.0545 & 1 & 0.2455 & 0 & -0.0545 \\ -0.0667 & 0 & -0.0667 & 0.2333 & 1 & 0 & 0 \\ 0 & -0.0545 & -0.0545 & 0 & -0.0545 & 1 & 0.2455 \\ -0.0667 & -0.0667 & 0 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_e) \\ q_j^{(i)}(g_e) \\ q_k^{(i)}(g_e) \\ q_i^{(j)}(g_e) \\ q_j^{(j)}(g_e) \\ q_i^{(k)}(g_e) \\ q_k^{(k)}(g_e) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1111 \\ 0.2222 \\ 0.1818 \\ 0.1111 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_e)$ = 0.1398; $q_j^{(i)}(g_e)$ = 0.0467; $q_k^{(i)}(g_e)$ = 0.1887; $q_i^{(j)}(g_e)$ = 0.1834; $q_j^{(j)}(g_e)$ = 0.0902; $q_i^{(k)}(g_e)$ = 0.1506; $q_k^{(k)}(g_e)$ = 0.1995; $q_l^{(l)}(g_e)$ = 0.1429; $CS_i(g_e)$ = $\frac{1}{2}(0.1398+0.1834+0.1506)^2$; = 0.1122; $CS_j(g_e)$ = $\frac{1}{2}(0.0467+0.0902)^2$ = 0.0094; $CS_k(g_e)$ = $\frac{1}{2}(0.1887+0.1995)^2$ = 0.0753; $CS_l(g_e)$ = $\frac{1}{2}(0.1429)^2$ = 0.0102; $\pi_i^{(i)}(g_e)$ = $\frac{(3.5)}{2}(0.1398)^2$ = 0.0342; $\pi_j^{(i)}(g_e)$ = $\frac{(3.5)}{2}(0.0467)^2$ = 0.0038; $\pi_k^{(i)}(g_e)$ = $\frac{(3.5)}{2}(0.1887)^2$ = 0.0623; $\pi_i^{(j)}(g_e)$ = $\frac{(3.5)}{2}(0.1834)^2$ = 0.0589; $\pi_j^{(j)}(g_e)$ = $\frac{(3.5)}{2}(0.0902)^2$ = 0.0142; $\pi_i^{(k)}(g_e)$ = $\frac{(3.5)}{2}(0.1506)^2$ = 0.0397; $\pi_k^{(k)}(g_e)$ = $\frac{(3.5)}{2}(0.1995)^2$ = 0.0697; $\pi_l^{(l)}(g_e)$ = $\frac{(3.5)}{2}(0.1429)^2$ = 0.0357; $PS_i(g_e)$ = $\frac{1.5}{4}(0.1398+0.0467+0.1887)^2$ = 0.0528; $PS_j(g_e)$ = $\frac{1.5}{4}(0.1834+0.0902)^2$ = 0.0281; $PS_k(g_e)$ = $\frac{1.5}{4}(0.1506+0.1995)^2$ = 0.0460; and $PS_l(g_e)$ = $\frac{1.5}{4}(0.1429)^2$ = 0.0077.

<u>Network f</u>

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_f)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_f)+\phi q_l^{(j)}(g_f)-\phi(\phi+2)q_j^{(i)}(g_f)}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_f)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_f)+\phi q_l^{(j)}(g_f)+\phi q_l^{(l)}(g_f)-\phi(\phi+3)q_i^{(i)}(g_f)}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_f)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_f)-\phi(\phi+2)q_j^{(j)}(g_f)-\phi(\phi+2)q_l^{(j)}(g_f)}{2(\phi+3)(\phi+1)}$; $q_j^{(j)}(g_f)$ =

$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_f)+\phi q_l^{(l)}(g_f)-\phi(\phi+3)q_i^{(j)}(g_f)-\phi(\phi+3)q_l^{(j)}(g_f)}{2(\phi+4)(\phi+1)}$; $q_l^{(j)}(g_f)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_j^{(l)}(g_f)-\phi(\phi+2)q_i^{(j)}(g_f)-\phi(\phi+2)q_j^{(j)}(g_f)}{2(\phi+3)(\phi+1)}$; $q_k^{(k)}(g_f)$ = $\frac{\alpha}{\phi+2}$; $q_j^{(l)}(g_f)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_f)+\phi q_i^{(j)}(g_f)+\phi q_l^{(j)}(g_f)-\phi(\phi+3)q_l^{(l)}(g_f)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_f)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_f)+\phi q_j^{(j)}(g_f)-\phi(\phi+2)q_j^{(l)}(g_f)}{2(\phi+3)(\phi+1)}$; $CS_i(g_f)$ = $\frac{1}{2}\left(q_i^{(i)}(g_f)+q_i^{(j)}(g_f)\right)^2$; $CS_j(g_f)$ = $\frac{1}{2}\left(q_j^{(i)}(g_f)+q_j^{(j)}(g_f)+q_j^{(l)}(g_f)\right)^2$; $CS_k(g_f)$ = $\frac{1}{2}\left(q_k^{(k)}(g_f)\right)^2$; $CS_l(g_f)$ = $\frac{1}{2}\left(q_l^{(j)}(g_f)+q_l^{(l)}(g_f)\right)^2$; $\pi_i^{(i)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_f)\right)^2$; $\pi_j^{(i)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_f)\right)^2$; $\pi_i^{(j)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_f)\right)^2$; $\pi_j^{(j)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_f)\right)^2$; $\pi_l^{(j)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_l^{(j)}(g_f)\right)^2$; $\pi_k^{(k)}(g_k)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_f)\right)^2$; $\pi_j^{(l)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_j^{(l)}(g_f)\right)^2$; $\pi_l^{(l)}(g_f)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_f)\right)^2$; $PS_i(g_f)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_f)+q_j^{(i)}(g_f)\right)^2$; $PS_j(g_f)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_f)+q_j^{(j)}(g_f)+q_l^{(j)}(g_f)\right)^2$; $PS_k(g_f)$ = $\frac{\phi}{4}\left(q_k^{(k)}(g_f)\right)^2$; and $PS_l(g_f)$ = $\frac{\phi}{4}\left(q_j^{(l)}(g_f)+q_l^{(l)}(g_f)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & 0 \\ \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} \\ 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 0 & 0 \\ -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} \\ 0 & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} \\ 0 & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_i^{(j)}(g_f) \\ q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) \\ q_j^{(l)}(g_f) \\ q_l^{(l)}(g_f) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_f) = q_j^{(j)}(g_j) = q_j^{(l)}(g_f) = q_l^{(j)}(g_j) = q_l^{(l)}(g_f) = 0$. Therefore, $q_i^{(i)}(g_f) = q_i^{(j)}(g_f) = \frac{1}{3} = 0.3333$; $q_k^{(k)}(g_f) = \frac{1}{2} = 0.5000$; $CS_i(g_f) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_j(g_f) = CS_l(g_f) = 0$; $CS_k(g_f) = \frac{1}{2}(0.5000)^2 = 0.1250$; $\pi_i^{(i)}(g_f) = \pi_i^{(j)}(g_f) = (0.3333)^2 = 0.1111$; $\pi_k^{(k)}(g_k) = (0.5000)^2 = 0.2500$; and $PS_i(g_f) = PS_j(g_f) = PS_k(g_f) = PS_l(g_f) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_f) = q_j^{(j)}(g_j) = q_j^{(l)}(g_f) = q_l^{(j)}(g_j) = q_l^{(l)}(g_f) = 0$. Therefore, $q_i^{(i)}(g_f) = q_i^{(j)}(g_f) = \frac{1}{3.5} = 0.2857$; $q_k^{(k)}(g_f) = \frac{1}{2.5} = 0.4000$; $CS_i(g_f) = \frac{1}{2}(0.2857 + 0.2857)^2 = 0.1632$; $CS_j(g_f) = CS_l(g_f) = 0$; $CS_k(g_f) = \frac{1}{2}(0.4000)^2 = 0.0800$; $\pi_i^{(i)}(g_f) = \pi_i^{(j)}(g_f) = \frac{(2.5)}{2}(0.2857)^2 = 0.1020$; $\pi_k^{(k)}(g_k) = \frac{(2.5)}{2}(0.4000)^2 = 0.2000$; $PS_i(g_f) = PS_j(g_f) = \frac{0.5}{4}(0.2857)^2 = 0.0102$; $PS_k(g_f) = \frac{0.5}{4}(0.4000)^2 = 0.0200$; and $PS_l(g_f) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_f) = q_j^{(j)}(g_j) = q_j^{(l)}(g_f) = q_l^{(j)}(g_j) = q_l^{(l)}(g_f) = 0$. Therefore, $q_i^{(i)}(g_f) = q_i^{(j)}(g_f) = \frac{1}{4.5} = 0.2222$; $q_k^{(k)}(g_f) = \frac{1}{3.5} = 0.2857$; $CS_i(g_f) = \frac{1}{2}(0.2222+0.2222)^2 = 0.0988$; $CS_j(g_f) = CS_l(g_f) = 0$; $CS_k(g_f) = \frac{1}{2}(0.2857)^2 = 0.0408$; $\pi_i^{(i)}(g_f) = \pi_i^{(j)}(g_f) = \frac{(3.5)}{2}(0.2222)^2 = 0.0864$; $\pi_k^{(k)}(g_k) = \frac{(3.5)}{2}(0.2857)^2 = 0.1428$; $PS_i(g_f) = PS_j(g_f) = \frac{1.5}{4}(0.2222)^2 = 0.0185$; $PS_k(g_f) = \frac{1.5}{4}(0.2857)^2 = 0.0306$; and $PS_l(g_f) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_i^{(j)}(g_f) \\ q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) \\ q_j^{(l)}(g_f) \\ q_l^{(l)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.3333 \\ 0.1250 \\ 0.3333 \\ 0.1250 \\ 0.1667 \\ 0.1250 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f) = q_i^{(j)}(g_f) = 0.3333$; $q_j^{(i)}(g_f) = q_j^{(j)}(g_f) = q_j^{(l)}(g_f) = 0.1250$; $q_k^{(k)}(g_f) = \frac{1}{2} = 0.5000$; $q_l^{(j)}(g_f) = q_l^{(l)}(g_f) = 0.1667$; $CS_i(g_f) = \frac{1}{2}(0.3333+0.3333)^2$

= 0.2222; $CS_j(g_f)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $CS_k(g_f)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $CS_l(g_f)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $\pi_i^{(i)}(g_f)$ = $\pi_i^{(j)}(g_f)$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(i)}(g_j)$ = $\pi_j^{(j)}(g_f)$ = $\pi_j^{(l)}(g_f)$ = $(0.1250)^2$ = 0.0156; $\pi_k^{(k)}(g_k)$ = $(0.5000)^2$ = 0.2500; $\pi_l^{(j)}(g_f)$ = $\pi_l^{(l)}(g_f)$ = $(0.1667)^2$ = 0.0278; and $PS_i(g_f)$ = $PS_j(g_f)$ = $PS_k(g_f)$ = $PS_l(g_f)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1190 & 0 & -0.0476 & -0.0476 & 0 & 0 \\ 0.1296 & 1 & -0.0370 & 0 & -0.0370 & 0 & -0.0370 \\ 0 & -0.0476 & 1 & 0.1190 & 0.1190 & 0 & 0 \\ -0.0370 & 0 & 0.1296 & 1 & 0.1296 & 0 & -0.0370 \\ 0 & 0 & 0.1190 & 0.1190 & 1 & -0.0476 & 0 \\ -0.0370 & 0 & -0.0370 & 0 & -0.0370 & 1 & 0.1296 \\ 0 & 0 & -0.1190 & -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_i^{(j)}(g_f) \\ q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) \\ q_j^{(l)}(g_f) \\ q_l^{(l)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.2857 \\ 0.1111 \\ 0.2857 \\ 0.1111 \\ 0.1429 \\ 0.1111 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f)$ = 0.2833; $q_j^{(i)}(g_f)$ = 0.0944; $q_i^{(j)}(g_f)$ = 0.2680; $q_j^{(j)}(g_f)$ = 0.0791; $q_l^{(j)}(g_f)$ = 0.1070; $q_k^{(k)}(g_f)$ = $\frac{1}{2.5}$ = 0.4000; $q_j^{(l)}(g_f)$ = 0.1141; $q_l^{(l)}(g_f)$ = 0.1650; $CS_i(g_f)$ = $\frac{1}{2}(0.2833+0.2680)^2$ = 0.1520; $CS_j(g_f)$ = $\frac{1}{2}(0.0944+0.0791+0.1141)^2$ = 0.0414; $CS_k(g_f)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $CS_l(g_f)$ = $\frac{1}{2}(0.1070+0.1650)^2$; = 0.0370; $\pi_i^{(i)}(g_f)$ = $\frac{(2.5)}{2}(0.2833)^2$ = 0.1003; $\pi_j^{(i)}(g_j)$ =

$\frac{(2.5)}{2}(0.0944)^2$ = 0.0111; $\pi_i^{(j)}(g_f)$ = $\frac{(2.5)}{2}(0.2680)^2$ = 0.0898; $\pi_j^{(j)}(g_f)$ = $\frac{(2.5)}{2}(0.0791)^2$ = 0.0078; $\pi_l^{(j)}(g_f)$ = $\frac{(2.5)}{2}(0.1070)^2$ = 0.0143; $\pi_k^{(k)}(g_k)$ = $\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $\pi_j^{(l)}(g_f)$ = $\frac{(2.5)}{2}(0.1141)^2$ = 0.0163; $\pi_l^{(l)}(g_f)$ = $\frac{(2.5)}{2}(0.1650)^2$ = 0.0340; $PS_i(g_f)$ = $\frac{0.5}{4}(0.2833+0.0944)^2$ = 0.0178; $PS_j(g_f)$ = $\frac{0.5}{4}(0.2680+0.0791+0.1070)^2$ = 0.0258; $PS_k(g_f)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; and $PS_l(g_f)$ = $\frac{0.5}{4}(0.1141+0.1650)^2$ = 0.0097.

Simulation 16: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2333 & 0 & -0.0667 & -0.0667 & 0 & 0 \\ 0.2455 & 1 & -0.0545 & 0 & -0.0545 & 0 & -0.0545 \\ 0 & -0.0667 & 1 & 0.2333 & 0.2333 & 0 & 0 \\ -0.0545 & 0 & 0.2455 & 1 & 0.2455 & 0 & -0.0545 \\ 0 & 0 & 0.2333 & 0.2333 & 1 & -0.0667 & 0 \\ -0.0545 & 0 & -0.0545 & 0 & -0.0545 & 1 & 0.2455 \\ 0 & 0 & -0.0667 & -0.0667 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_f) \\ q_j^{(i)}(g_f) \\ q_i^{(j)}(g_f) \\ q_j^{(j)}(g_f) \\ q_l^{(j)}(g_f) \\ q_j^{(l)}(g_f) \\ q_l^{(l)}(g_f) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.0909 \\ 0.2222 \\ 0.0909 \\ 0.1111 \\ 0.0909 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_f)$ = 0.2156; $q_j^{(i)}(g_f)$ = 0.0580; $q_i^{(j)}(g_f)$ = 0.2018; $q_j^{(j)}(g_f)$ = 0.0442; $q_l^{(j)}(g_f)$ = 0.0598; $q_k^{(k)}(g_f)$ = $\frac{1}{3.5}$ = 0.2857; $q_j^{(l)}(g_f)$ = 0.0908; $q_l^{(l)}(g_f)$ = 0.1063; $CS_i(g_f)$ = $\frac{1}{2}(0.2156+0.2018)^2$ = 0.0871; $CS_j(g_f)$ = $\frac{1}{2}(0.0580+0.0442+0.0908)^2$ = 0.0186; $CS_k(g_f)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_l(g_f)$ =

$\frac{1}{2}(0.0598+0.1063)^2$ = 0.0138; $\pi_i^{(i)}(g_f)$ = $\frac{(3.5)}{2}(0.2156)^2$ = 0.0813; $\pi_j^{(i)}(g_j)$ = $\frac{(3.5)}{2}(0.0580)^2$ = 0.0059; $\pi_i^{(j)}(g_f)$ = $\frac{(3.5)}{2}(0.2018)^2$ = 0.0713; $\pi_j^{(j)}(g_f)$ = $\frac{(3.5)}{2}(0.0442)^2$ = 0.0034; $\pi_l^{(j)}(g_f)$ = $\frac{(3.5)}{2}(0.0598)^2$ = 0.0063; $\pi_k^{(k)}(g_k)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $\pi_j^{(l)}(g_f)$ = $\frac{(3.5)}{2}(0.0908)^2$ = 0.0144; $\pi_l^{(l)}(g_f)$ = $\frac{(3.5)}{2}(0.1063)^2$ = 0.0198; $PS_i(g_f)$ = $\frac{1.5}{4}(0.2156+0.0580)^2$ = 0.0281; $PS_j(g_f)$ = $\frac{1.5}{4}(0.2018+0.0442+0.0598)^2$ = 0.0351; $PS_k(g_f)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; and $PS_l(g_f)$ = $\frac{1.5}{4}(0.0908+0.1063)^2$ = 0.0146.

Network g

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_g)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_g)+\phi q_k^{(k)}(g_g)-2\phi(\phi+4)q_j^{(i)}(g_g)-\phi(\phi+4)q_k^{(i)}(g_g)}{2(\phi+5)(\phi+1)}$; $q_j^{(i)}(g_g)$ = $q_l^{(i)}(g_g)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_g)-\phi(\phi+2)q_i^{(i)}(g_g)-\phi(\phi+2)q_k^{(i)}(g_g)}{3\phi^2+10\phi+6}$; $q_k^{(i)}(g_g)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_g)-\phi(\phi+2)q_i^{(i)}(g_g)-2\phi(\phi+2)q_j^{(i)}(g_g)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_g)$ = $q_i^{(l)}(g_g)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_g)+\phi q_k^{(i)}(g_g)+\phi q_k^{(k)}(g_g)-\phi(\phi+3)q_j^{(j)}(g_g)}{2(\phi+5)(\phi+1)}$; $q_j^{(j)}(g_g)$ = $q_l^{(l)}(g_g)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_g)+\phi q_k^{(i)}(g_g)+\phi q_j^{(i)}(g_g)-\phi(\phi+2)q_i^{(j)}(g_g)}{2(\phi+3)(\phi+1)}$; $q_i^{(k)}(g_g)$ =

$\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_g)+\phi q_k^{(i)}(g_g)+2\phi q_j^{(j)}(g_g)-\phi(\phi+4)q_k^{(k)}(g_g)}{2(\phi+5)(\phi+1)}$; $q_k^{(k)}(g_g)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_g)+2\phi q_j^{(i)}(g_g)-\phi(\phi+2)q_i^{(k)}(g_g)}{2(\phi+3)(\phi+1)}$; $CS_i(g_g)$ = $\frac{1}{2}\left(q_i^{(i)}(g_g)+q_i^{(j)}(g_g)+q_i^{(k)}(g_g)+q_i^{(l)}(g_g)\right)^2$; $CS_j(g_g)$ = $\frac{1}{2}\left(q_j^{(i)}(g_g)+q_j^{(j)}(g_g)\right)^2$; $CS_k(g_g)$ = $\frac{1}{2}\left(q_k^{(i)}(g_g)+q_k^{(k)}(g_g)\right)^2$; $CS_l(g_g)$ = $\frac{1}{2}\left(q_l^{(i)}(g_e)+q_l^{(l)}(g_e)\right)^2$; $\pi_i^{(i)}(g_g) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_g)\right)^2$; $\pi_j^{(i)}(g_g) = \pi_l^{(i)}(g_g) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g_g)\right)^2$; $\pi_k^{(i)}(g_g) = \frac{(2+\phi)}{2}\left(q_k^{(i)}(g_g)\right)^2$; $\pi_i^{(j)}(g_g)$ = $\pi_i^{(l)}(g_g) = \frac{(2+\phi)}{2}\left(q_i^{(j)}(g_g)\right)^2$; $\pi_j^{(j)}(g_g) = \pi_l^{(l)}(g_g) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_g)\right)^2$; $\pi_i^{(k)}(g_g)$ = $\frac{(2+\phi)}{2}\left(q_i^{(k)}(g_g)\right)^2$; $\pi_k^{(k)}(g_g) = \frac{(2+\phi)}{2}\left(q_k^{(k)}(g_g)\right)^2$; $PS_i(g_g)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_g)+q_j^{(i)}(g_g)+q_k^{(i)}(g_g)+q_l^{(i)}(g_g)\right)^2$; $PS_j(g_g) = \frac{\phi}{4}\left(q_i^{(j)}(g_g)+q_j^{(j)}(g_g)\right)^2$; $PS_k(g_g)$ = $\frac{\phi}{4}\left(q_i^{(k)}(g_g)+q_k^{(k)}(g_g)\right)^2$; and $PS_l(g_g) = \frac{\phi}{4}\left(q_i^{(l)}(g_g)+q_l^{(l)}(g_g)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+4)}{(\phi+5)(\phi+1)} & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & 1 & \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & -\frac{\phi}{3\phi^2+10\phi+6} & 0 & 0 & 0 \\ \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & 1 & 0 & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & 0 & 0 \\ 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{(\phi+3)(\phi+1)} & 0 & 0 & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_j^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_j^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+5} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+10\phi+6} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+5} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\alpha}{\phi+5} \\ \frac{\alpha}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_g) = q_l^{(i)}(g_g) = q_j^{(j)}(g_g) = q_l^{(l)}(g_g) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.3333 \\ 0.2000 \\ 0.2000 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g) = q_i^{(j)}(g_g) = q_i^{(l)}(g_g) = q_i^{(k)}(g_g) = 0.2000$; $q_k^{(i)}(g_g) = q_k^{(k)}(g_g) = 0.3333$; $CS_i(g_g) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.32$; $CS_j(g_g) = CS_l(g_g) = 0$; $CS_k(g_g) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $\pi_i^{(i)}(g_g) = \pi_i^{(j)}(g_g) = \pi_i^{(l)}(g_g) = \pi_i^{(k)}(g_g) = (0.2000)^2 = 0.0400$; $\pi_k^{(i)}(g_g) = \pi_k^{(k)}(g_g) = (0.3333)^2 = 0.1111$; and $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) = PS_l(g_g) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_g) = q_l^{(i)}(g_g) = q_j^{(j)}(g_g) = q_l^{(l)}(g_g) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.1364 & 0 & 0 & -0.0303 \\ 0.1190 & 1 & 0 & -0.0476 & 0 \\ 0 & -0.0303 & 1 & 0 & -0.0303 \\ 0 & -0.0303 & 0 & 1 & 0.1364 \\ -0.0476 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2857 \\ 0.1818 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g)$ = $q_i^{(k)}(g_g)$ = 0.1526; $q_k^{(i)}(g_g)$ = $q_k^{(k)}(g_g)$ = 0.2748; $q_i^{(j)}(g_g)$ = $q_i^{(l)}(g_g)$ = 0.1985; $CS_i(g_g)$ = $\frac{1}{2}(0.1526+0.1985+0.1526+0.1985)^2$ = 0.2465; $CS_j(g_g)$ = $CS_l(g_g)$ = 0; $CS_k(g_g)$ = $\frac{1}{2}(0.2748+0.2748)^2$ = 0.1510; $\pi_i^{(i)}(g_g)$ = $\pi_i^{(k)}(g_g)$ = $\frac{(2.5)}{2}(0.1526)^2$ = 0.0291; $\pi_k^{(i)}(g_g)$ = $\pi_k^{(k)}(g_g)$ = $\frac{(2.5)}{2}(0.2748)^2$ = 0.0944; $\pi_i^{(j)}(g_g)$ = $\pi_i^{(l)}(g_g)$ = $\frac{(2.5)}{2}(0.1985)^2$ = 0.0493; $PS_i(g_g)$ = $PS_k(g_g)$ = $\frac{0.5}{4}(0.1526+0.2748)^2$ = 0.0228; and $PS_j(g_g)$ = $PS_l(g_g)$ = $\frac{0.5}{4}(0.1985)^2$ = 0.0049.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

Because in this case $q_j^{(i)}(g_g)$ = $q_l^{(i)}(g_g)$ = $q_j^{(j)}(g_g)$ = $q_l^{(l)}(g_g)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.2538 & 0 & 0 & -0.0462 \\ 0.2333 & 1 & 0 & -0.0667 & 0 \\ 0 & -0.0462 & 1 & 0 & -0.0462 \\ 0 & -0.0462 & 0 & 1 & 0.2538 \\ -0.0667 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.2222 \\ 0.1538 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g)$ = $q_i^{(k)}(g_g)$ = 0.1115; $q_k^{(i)}(g_g)$ = $q_k^{(k)}(g_g)$ = 0.2036; $q_i^{(j)}(g_g)$ = $q_i^{(l)}(g_g)$ = 0.1726; $CS_i(g_g)$ = $\frac{1}{2}(0.1115+0.1726+0.1115+0.1726)^2$ = 0.1614; $CS_j(g_g)$ = $CS_l(g_g)$ = 0; $CS_k(g_g)$ = $\frac{1}{2}(0.2036+0.2036)^2$ = 0.0829; $\pi_i^{(i)}(g_g)$ = $\pi_i^{(k)}(g_g)$ = $\frac{(3.5)}{2}(0.1115)^2$ = 0.0218; $\pi_k^{(i)}(g_g)$ = $\pi_k^{(k)}(g_g)$ = $\frac{(3.5)}{2}(0.2036)^2$ = 0.0725; $\pi_i^{(j)}(g_g)$ =

$\pi_i^{(l)}(g_g) = \frac{(3.5)}{2}(0.1726)^2 = 0.0521$; $PS_i(g_g) = PS_k(g_g) = \frac{1.5}{4}(0.1115 + 0.2036)^2 =$ 0.0372; and $PS_j(g_g) = PS_l(g_g) = \frac{1.5}{4}(0.1726)^2 = 0.0112$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_j^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_j^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.1667 \\ 0.3333 \\ 0.2000 \\ 0.1667 \\ 0.2000 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g) = q_i^{(j)}(g_g) = q_i^{(l)}(g_g) = q_i^{(k)}(g_g) = 0.2000$; $q_j^{(i)}(g_g) = q_l^{(i)}(g_g) = q_j^{(j)}(g_g) = q_l^{(l)}(g_g) = 0.1667$; $q_k^{(i)}(g_g) = q_k^{(k)}(g_g) = 0.3333$; $CS_i(g_g) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.3200$; $CS_j(g_g) = CS_l(g_g) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $CS_k(g_g) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $\pi_i^{(i)}(g_g) = \pi_i^{(j)}(g_g) = \pi_i^{(k)}(g_g) = \pi_i^{(l)}(g_g) = (0.2000)^2 = 0.0400$; $\pi_j^{(i)}(g_g) = \pi_j^{(j)}(g_g) = \pi_l^{(i)}(g_g) = \pi_l^{(l)}(g_g) = (0.1667)^2 = 0.0278$; $\pi_k^{(i)}(g_g) = \pi_k^{(k)}(g_g) = (0.3333)^2 = 0.1111$; and $PS_i(g_g) = PS_j(g_g) = PS_k(g_g) = PS_l(g_g) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2727 & 0.1364 & 0 & -0.0606 & 0 & -0.0303 \\ 0.1064 & 1 & 0.1064 & -0.0426 & 0 & 0 & 0 \\ 0.1190 & 0.2381 & 1 & 0 & 0 & -0.0476 & 0 \\ 0 & -0.0606 & -0.0303 & 1 & 0.1061 & 0 & -0.0303 \\ -0.0476 & -0.0476 & -0.0476 & 0.1190 & 1 & 0 & 0 \\ 0 & -0.0606 & -0.0303 & 0 & -0.0606 & 1 & 0.1364 \\ -0.0476 & -0.0952 & 0 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_j^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_j^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1277 \\ 0.2857 \\ 0.1818 \\ 0.1429 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g) = 0.1387$; $q_j^{(i)}(g_g) = q_l^{(i)}(g_g) = 0.0939$; $q_k^{(i)}(g_g) = 0.2547$; $q_i^{(j)}(g_g) = q_i^{(l)}(g_g) = 0.1885$; $q_j^{(j)}(g_g) = q_l^{(l)}(g_g) = 0.1437$; $q_i^{(k)}(g_g) = 0.1655$; $q_k^{(k)}(g_g) = 0.2815$; $CS_i(g_g) = \frac{1}{2}(0.1387 + 0.1885 + 0.1655 + 0.1885)^2 = 0.2320$; $CS_j(g_g) = CS_l(g_g) = \frac{1}{2}(0.0939 + 0.1437)^2 = 0.0282$; $CS_k(g_g) = \frac{1}{2}(0.2547 + 0.2815)^2 = 0.1438$; $\pi_i^{(i)}(g_g) = \frac{(2.5)}{2}(0.1387)^2 = 0.0240$; $\pi_j^{(i)}(g_g) = \pi_l^{(i)}(g_g) = \frac{(2.5)}{2}(0.0939)^2 = 0.0110$; $\pi_k^{(i)}(g_g) = \frac{(2.5)}{2}(0.2547)^2 = 0.0811$; $\pi_i^{(j)}(g_g) = \pi_i^{(l)}(g_g) = \frac{(2.5)}{2}(0.1885)^2 = 0.0444$; $\pi_j^{(j)}(g_g) = \pi_l^{(l)}(g_g) = \frac{(2.5)}{2}(0.1437)^2 = 0.0258$; $\pi_i^{(k)}(g_g) = \frac{(2.5)}{2}(0.1655)^2 = 0.0342$; $\pi_k^{(k)}(g_g) = \frac{(2.5)}{2}(0.2815)^2 = 0.0991$; $PS_i(g_g) = \frac{0.5}{4}(0.1387 + 0.0939 + 0.2547 + 0.0939)^2 = 0.0422$; $PS_j(g_g) = PS_l(g_g) = \frac{0.5}{4}(0.1885 + 0.1437)^2 = 0.0138$; and $PS_k(g_g) = \frac{0.5}{4}(0.1655 + 0.2815)^2 = 0.0250$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.5077 & 0.2538 & 0 & -0.0923 & 0 & -0.0462 \\ 0.1892 & 1 & 0.1892 & -0.0541 & 0 & 0 & 0 \\ 0.2333 & 0.4667 & 1 & 0 & 0 & -0.0667 & 0 \\ 0 & -0.0923 & -0.0462 & 1 & 0.2077 & 0 & -0.0462 \\ -0.0667 & -0.0667 & -0.0667 & 0.2333 & 1 & 0 & 0 \\ 0 & -0.0923 & -0.0462 & 0 & -0.0923 & 1 & 0.2538 \\ -0.0667 & -0.1333 & 0 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_g) \\ q_j^{(i)}(g_g) \\ q_k^{(i)}(g_g) \\ q_i^{(j)}(g_g) \\ q_j^{(j)}(g_g) \\ q_i^{(k)}(g_g) \\ q_k^{(k)}(g_g) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.0901 \\ 0.2222 \\ 0.1538 \\ 0.1111 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_g) = 0.1028$; $q_j^{(i)}(g_g) = q_l^{(i)}(g_g) = 0.0439$; $q_k^{(i)}(g_g) = 0.1859$; $q_i^{(j)}(g_g) = q_i^{(l)}(g_g) = 0.1558$; $q_j^{(j)}(g_g) = q_l^{(l)}(g_g) = 0.0969$; $q_i^{(k)}(g_g) = 0.1231$; $q_k^{(k)}(g_g) = 0.2062$; $CS_i(g_g) = \frac{1}{2}(0.1028+0.1558+0.1231+0.1558)^2 = 0.1445$; $CS_j(g_g) = CS_l(g_g) = \frac{1}{2}(0.0439+0.0969)^2 = 0.0099$; $CS_k(g_g) = \frac{1}{2}(0.1859+0.2062)^2 = 0.0769$; $\pi_i^{(i)}(g_g) = \frac{(3.5)}{2}(0.1028)^2 = 0.0185$; $\pi_j^{(i)}(g_g) = \pi_l^{(i)}(g_g) = \frac{(3.5)}{2}(0.0439)^2 = 0.0034$; $\pi_k^{(i)}(g_g) = \frac{(3.5)}{2}(0.1859)^2 = 0.0605$; $\pi_i^{(j)}(g_g) = \pi_i^{(l)}(g_g) = \frac{(3.5)}{2}(0.1558)^2 = 0.0425$; $\pi_j^{(j)}(g_g) = \pi_l^{(l)}(g_g) = \frac{(3.5)}{2}(0.0969)^2 = 0.0164$; $\pi_i^{(k)}(g_g) = \frac{(3.5)}{2}(0.1231)^2 = 0.0265$; $\pi_k^{(k)}(g_g) = \frac{(3.5)}{2}(0.2062)^2 = 0.0744$; $PS_i(g_g) = \frac{1.5}{4}(0.1028+0.0439+0.1859+0.0439)^2 = 0.0532$; $PS_j(g_g) = PS_l(g_g) = \frac{1.5}{4}(0.1558+0.0969)^2 = 0.0239$; and $PS_k(g_g) = \frac{1.5}{4}(0.1231+0.2062)^2 = 0.0407$.

Network h

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $\frac{2\alpha(1+\phi)+\phi q_j^{(j)}(g_h)-\phi(\phi+2)q_j^{(i)}(g_h)}{3\phi^2+12\phi+8}$; $q_j^{(i)}(g_h)$ = $q_l^{(k)}(g_h)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_h)-2\phi(\phi+2)q_i^{(i)}(g_h)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_h)+2\phi q_i^{(i)}(g_h)-\phi(\phi+3)q_j^{(j)}(g_h)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_h)$ = $q_l^{(l)}(g_h)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_h)-\phi(\phi+2)q_i^{(j)}(g_h)}{2(\phi+3)(\phi+1)}$; *CS*$_i$(*g*$_h$) = *CS*$_k$(*g*$_h$) = $\frac{1}{2}\left(q_i^{(i)}(g_h)+q_i^{(j)}(g_h)+q_i^{(k)}(g_h)\right)^2$; *CS*$_j$(*g*$_h$) = *CS*$_l$(*g*$_h$) = $\frac{1}{2}\left(q_j^{(i)}(g_h)+q_j^{(j)}(g_h)\right)^2$; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_h)\right)^2$; $\pi_j^{(i)}(g_h)$ = $\pi_l^{(k)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_h)\right)^2$; $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_h)\right)^2$; $\pi_j^{(j)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_h)\right)^2$; *PS*$_i$(*g*$_h$) = *PS*$_k$(*g*$_h$) = $\frac{\phi}{4}\left(q_i^{(i)}(g_h)+q_j^{(i)}(g_h)+q_k^{(i)}(g_h)\right)^2$; and *PS*$_j$(*g*$_h$) = *PS*$_l$(*g*$_h$) = $\frac{\phi}{4}\left(q_i^{(j)}(g_h)+q_j^{(j)}(g_h)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 0 & -\frac{\phi}{3\phi^2+12\phi+8} \\ \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\alpha}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0

Because in this case $q_j^{(i)}(g_h)$ = $q_j^{(j)}(g_h)$ = $q_l^{(k)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0; $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $\frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8}$; and $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = $\frac{\alpha(\phi+1)+\phi q_i^{(i)}(g_h)}{(\phi+4)(\phi+1)}$, it holds that $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.2500 ; $CS_i(g_h)$ = $CS_k(g_h)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_h)$ = $CS_l(g_h)$ = 0; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_h)$ = $PS_j(g_h)$ = $PS_k(g_h)$ = $PS_l(g_h)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

Because in this case $q_j^{(i)}(g_h)$ = $q_j^{(j)}(g_h)$ = $q_l^{(k)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0; $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $\frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8}$; and $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = $\frac{\alpha(\phi+1)+\phi q_i^{(i)}(g_h)}{(\phi+4)(\phi+1)}$, it holds that $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = 0.2034; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.2373; $CS_i(g_h)$ = $CS_k(g_h)$ = $\frac{1}{2}(0.2034+0.2373+0.2034)^2$ = 0.2074; $CS_j(g_h)$ = $CS_l(g_h)$ = 0; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\frac{2.5}{2}(0.2034)^2$ = 0.0517; $\pi_i^{(j)}(g_h)$ =

$\pi_k^{(l)}(g_h)$ = $\frac{2.5}{2}(0.2373)^2$ = 0.0704; $PS_i(g_h)$ = $PS_k(g_h)$ = $\frac{0.5}{4}(0.2034 + 0.2034)^2$ = 0.0207; and $PS_j(g_h) = PS_l(g_h) = \frac{0.5}{4}(0.2373)^2$ = 0.0070.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

Because in this case $q_j^{(i)}(g_h)$ = $q_j^{(j)}(g_h)$ = $q_l^{(k)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0; $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $\frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8}$; and $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = $\frac{\alpha(\phi+1)+\phi q_i^{(i)}(g_h)}{(\phi+4)(\phi+1)}$, it holds that $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = 0.1527; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.1985; $CS_i(g_h) = CS_k(g_h) = \frac{1}{2}(0.1527 + 0.1985 + 0.1527)^2$ = 0.1270; $CS_j(g_h) = CS_l(g_h)$ = 0; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\frac{3.5}{2}(0.1527)^2$ = 0.0408; $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $\frac{3.5}{2}(0.1985)^2$ = 0.0690; $PS_i(g_h) = PS_k(g_h) = \frac{1.5}{4}(0.1527 + 0.1527)^2$ = 0.0350; and $PS_j(g_h) = PS_l(g_h) = \frac{1.5}{4}(0.1985)^2$ = 0.0148.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1667 \\ 0.2500 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.2500; $q_j^{(i)}(g_h)$ = $q_l^{(k)}(g_h)$ = $q_j^{(j)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0.1667; $CS_i(g_h)$ = $CS_k(g_h)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_h)$ = $CS_l(g_h)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_h)$ = $\pi_l^{(k)}(g_h)$ = $\pi_j^{(j)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $(0.1667)^2$ = 0.0278; and $PS_i(g_h)$ = $PS_j(g_h)$ = $PS_k(g_h)$ =$PS_l(g_h)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.0847 & 0 & -0.0339 \\ 0.2381 & 1 & -0.0476 & 0 \\ -0.0741 & -0.0741 & 1 & 0.1296 \\ -0.0952 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.2034 \\ 0.1429 \\ 0.2222 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = 0.1990; $q_j^{(i)}(g_h)$ = $q_l^{(k)}(g_h)$ = 0.1063; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.2274; $q_j^{(j)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0.1348; $CS_i(g_h)$ = $CS_k(g_h)$ = $\frac{1}{2}(0.1990+0.2274+0.1990)^2$ = 0.1956; $CS_j(g_h)$ = $CS_l(g_h)$ = $\frac{1}{2}(0.1063+0.1348)^2$ = 0.0291; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\frac{2.5}{2}(0.1990)^2$ = 0.0500; $\pi_j^{(i)}(g_h)$ = $\pi_l^{(k)}(g_h)$ = $\frac{2.5}{2}(0.1063)^2$ = 0.0141; $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $\frac{2.5}{2}(0.2274)^2$ = 0.0646; $\pi_j^{(j)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $\frac{2.5}{2}(0.1348)^2$ = 0.0227; $PS_i(g_h)$ = $PS_k(g_h)$ =

$\frac{0.5}{4}(0.1990+0.1063+0.1990)^2$ = 0.0318; and $PS_j(g_h)$ = $PS_l(g_h)$ = $\frac{0.5}{4}(0.2274+0.1348)^2$ = 0.0164.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1603 & 0 & -0.0458 \\ 0.4667 & 1 & -0.0667 & 0 \\ -0.1091 & -0.1091 & 1 & 0.2455 \\ -0.1333 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_h) \\ q_j^{(i)}(g_h) \\ q_i^{(j)}(g_h) \\ q_j^{(j)}(g_h) \end{pmatrix} = \begin{pmatrix} 0.1527 \\ 0.1111 \\ 0.1818 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = 0.1481; $q_j^{(i)}(g_h)$ = $q_l^{(k)}(g_h)$ = 0.0541; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.1822; $q_j^{(j)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0.0883; $CS_i(g_h)$ = $CS_k(g_h)$ = $\frac{1}{2}(0.1481+0.1822+0.1481)^2$ = 0.1144; $CS_j(g_h)$ = $CS_l(g_h)$ = $\frac{1}{2}(0.0541+0.0883)^2$ = 0.0101; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = $\frac{3.5}{2}(0.1481)^2$ = 0.0384; $\pi_j^{(i)}(g_h)$ = $\pi_l^{(k)}(g_h)$ = $\frac{3.5}{2}(0.0541)^2$ = 0.0051; $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = $\frac{3.5}{2}(0.1822)^2$ = 0.0581; $\pi_j^{(j)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = $\frac{3.5}{2}(0.0883)^2$ = 0.0136; $PS_i(g_h)$ = $PS_k(g_h)$ = $\frac{1.5}{4}(0.1481+0.0541+0.1481)^2$ = 0.0460; and $PS_j(g_h)$ = $PS_l(g_h)$ = $\frac{1.5}{4}(0.1822+0.0883)^2$ = 0.0274.

Network i

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_i)+\phi q_l^{(j)}(g_i)+\phi q_k^{(k)}(g_i)-\phi(\phi+3)q_j^{(i)}(g_i)-\phi(\phi+3)q_k^{(i)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_i)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_i)+\phi q_l^{(j)}(g_i)+\phi q_l^{(l)}(g_i)-\phi(\phi+3)q_i^{(i)}(g_i)-\phi(\phi+3)q_k^{(i)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_k^{(i)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_i)-\phi(\phi+2)q_i^{(i)}(g_i)-\phi(\phi+2)q_j^{(i)}(g_i)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_i)+\phi q_k^{(i)}(g_i)+\phi q_k^{(k)}(g_i)-\phi(\phi+3)q_j^{(j)}(g_i)-\phi(\phi+3)q_l^{(j)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_i)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_i)+\phi q_k^{(i)}(g_i)+\phi q_l^{(l)}(g_i)-\phi(\phi+3)q_i^{(j)}(g_i)-\phi(\phi+3)q_l^{(j)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_l^{(j)}(g_i)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_j^{(l)}(g_i)-\phi(\phi+2)q_i^{(j)}(g_i)-\phi(\phi+2)q_j^{(j)}(g_i)}{2(\phi+3)(\phi+1)}$; $q_i^{(k)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_i)+\phi q_k^{(i)}(g_i)+\phi q_j^{(j)}(g_i)+\phi q_l^{(j)}(g_i)-\phi(\phi+3)q_k^{(k)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_k^{(k)}(g_i)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_i)+\phi q_j^{(i)}(g_i)-\phi(\phi+2)q_i^{(k)}(g_i)}{2(\phi+3)(\phi+1)}$; $q_j^{(l)}(g_i)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_i)+\phi q_k^{(i)}(g_i)+\phi q_i^{(j)}(g_i)+\phi q_l^{(j)}(g_i)-\phi(\phi+3)q_l^{(l)}(g_i)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_i)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_i)+\phi q_j^{(j)}(g_i)-\phi(\phi+2)q_j^{(l)}(g_i)}{2(\phi+3)(\phi+1)}$; $CS_i(g_i)$ = $\frac{1}{2}\left(q_i^{(i)}(g_i)+q_i^{(j)}(g_i)+q_i^{(k)}(g_i)\right)^2$; $CS_j(g_i)$ = $\frac{1}{2}\left(q_j^{(i)}(g_i)+q_j^{(j)}(g_i)+q_j^{(l)}(g_i)\right)^2$; $CS_k(g_i)$ = $\frac{1}{2}\left(q_k^{(i)}(g_i)+q_k^{(k)}(g_i)\right)^2$; $CS_l(g_i)$ = $\frac{1}{2}\left(q_l^{(j)}(g_i)+q_l^{(l)}(g_i)\right)^2$; $\pi_i^{(i)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_i)\right)^2$; $\pi_j^{(i)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_i)\right)^2$; $\pi_k^{(i)}(g_i)$ = $\frac{(2+\phi)}{2}\left(q_k^{(i)}(g_i)\right)^2$; $\pi_i^{(j)}(g_i)$ =

$\dfrac{(2+\phi)}{2}\left(q_i^{(j)}(g_i)\right)^2$; $\pi_j^{(j)}(g_i) = \dfrac{(2+\phi)}{2}\left(q_j^{(j)}(g_i)\right)^2$; $\pi_l^{(j)}(g_i) = \dfrac{(2+\phi)}{2}\left(q_l^{(j)}(g_i)\right)^2$; $\pi_i^{(k)}(g_i)$ $= \dfrac{(2+\phi)}{2}\left(q_i^{(k)}(g_i)\right)^2$; $\pi_k^{(k)}(g_i) = \dfrac{(2+\phi)}{2}\left(q_k^{(k)}(g_i)\right)^2$; $\pi_j^{(l)}(g_i) = \dfrac{(2+\phi)}{2}\left(q_j^{(l)}(g_i)\right)^2$; $\pi_l^{(l)}(g_i) = \dfrac{(2+\phi)}{2}\left(q_l^{(l)}(g_i)\right)^2$; $PS_i(g_i) = \dfrac{\phi}{4}\left(q_i^{(i)}(g_i)+q_j^{(i)}(g_i)+q_k^{(i)}(g_i)\right)^2$; $PS_j(g_i) = \dfrac{\phi}{4}\left(q_i^{(j)}(g_i)+q_j^{(j)}(g_i)+q_l^{(j)}(g_i)\right)^2$; $PS_k(g_i) = \dfrac{\phi}{4}\left(q_i^{(k)}(g_i)+q_k^{(k)}(g_i)\right)^2$; and $PS_l(g_i) = \dfrac{\phi}{4}\left(q_j^{(l)}(g_i)+q_l^{(l)}(g_i)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix}
1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & 0 & 0 \\
\beta_0 & 1 & \beta_0 & -\beta_1 & 0 & -\beta_1 & 0 & 0 & 0 & -\beta_1 \\
\beta_2 & \beta_2 & 1 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 \\
0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & 0 & 0 \\
-\beta_1 & 0 & -\beta_1 & \beta_0 & 1 & \beta_0 & 0 & 0 & 0 & -\beta_1 \\
0 & 0 & 0 & \beta_2 & \beta_2 & 1 & 0 & 0 & -\beta_3 & 0 \\
0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & 0 & 0 \\
-\beta_3 & -\beta_3 & 0 & 0 & 0 & 0 & \beta_2 & 1 & 0 & 0 \\
-\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & 0 & 0 & 1 & \beta_0 \\
0 & 0 & 0 & -\beta_3 & -\beta_3 & 0 & 0 & 0 & \beta_2 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_j^{(j)}(g_i) \\ q_l^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_j^{(l)}(g_i) \\ q_l^{(l)}(g_i)
\end{pmatrix}
=
\begin{pmatrix}
\beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_4 \\ \beta_5 \\ \beta_7 \\ \beta_4 \\ \beta_6 \\ \beta_5 \\ \beta_7
\end{pmatrix}$$

Where $\beta_0 = \dfrac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; $\beta_1 = \dfrac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_2 = \dfrac{\phi(\phi+2)}{2(\phi+3)(\phi+1)}$; $\beta_3 = \dfrac{\phi}{2(\phi+3)(\phi+1)}$; $\beta_4 = \dfrac{\alpha}{\phi+4}$; $\beta_5 = \dfrac{\tilde{\alpha}}{\phi+4}$; $\beta_6 = \dfrac{\alpha}{\phi+3}$; $\beta_7 = \dfrac{\tilde{\alpha}}{\phi+3}$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_l^{(j)}(g_i)$ = $q_j^{(l)}(g_i)$ = $q_l^{(l)}(g_i)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.3333 \\ 0.2500 \\ 0.2500 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_i^{(j)}(g_i)$ = $q_i^{(k)}(g_i)$ = 0.2500; $q_k^{(i)}(g_i)$ = $q_k^{(k)}(g_i)$ = 0.3333; $CS_i(g_i)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_i)$ = $CS_l(g_i)$ = 0; $CS_k(g_i)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $\pi_i^{(i)}(g_i)$ = $\pi_i^{(j)}(g_i)$ = $\pi_i^{(k)}(g_i)$ = $(0.2500)^2$ = 0.0625; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(k)}(g_i)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_i) = PS_j(g_i) = PS_k(g_i) = PS_l(g_i) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_l^{(j)}(g_i)$ = $q_j^{(l)}(g_i)$ = $q_l^{(l)}(g_i)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.1296 & 0 & 0 & -0.0370 \\ 0.1190 & 1 & 0 & -0.0476 & 0 \\ 0 & -0.0370 & 1 & 0 & -0.0370 \\ 0 & -0.0370 & 0 & 1 & 0.1296 \\ -0.0476 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.2857 \\ 0.2222 \\ 0.2222 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_i^{(k)}(g_i)$ = 0.1970; $q_k^{(i)}(g_i)$ = $q_k^{(k)}(g_i)$ = 0.2716; $q_i^{(j)}(g_i)$ = 0.2423; $CS_i(g_i) = \frac{1}{2}(0.1970+0.2423+0.1970)^2$ = 0.2024; $CS_j(g_i) = CS_l(g_i) = 0$; $CS_k(g_i)$ = $\frac{1}{2}(0.2716+0.2716)^2$ = 0.1475; $\pi_i^{(i)}(g_i)$ = $\pi_i^{(k)}(g_i)$ = $\frac{(2.5)}{2}(0.1970)^2$ = 0.0485; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(k)}(g_i)$ = $\frac{(2.5)}{2}(0.2716)^2$ = 0.0922; $\pi_i^{(j)}(g_i)$ = $\frac{(2.5)}{2}(0.2423)^2$ = 0.0734; $PS_i(g_i) = PS_k(g_i) = \frac{0.5}{4}(0.1970+0.2716)^2$ = 0.0274; $PS_j(g_i) = \frac{0.5}{4}(0.2423)^2$ = 0.0073; and $PS_l(g_i) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_i)$ = $q_j^{(j)}(g_i)$ = $q_l^{(j)}(g_i)$ = $q_j^{(l)}(g_i)$ = $q_l^{(l)}(g_i)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.2455 & 0 & 0 & -0.0545 \\ 0.2333 & 1 & 0 & -0.0667 & 0 \\ 0 & -0.0545 & 1 & 0 & -0.0545 \\ 0 & -0.0545 & 0 & 1 & 0.2455 \\ -0.0667 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = $q_i^{(k)}(g_i)$ = 0.1439; $q_k^{(i)}(g_i)$ = $q_k^{(k)}(g_i)$ = 0.1982; $q_i^{(j)}(g_i)$ = 0.2034; $CS_i(g_i) = \frac{1}{2}(0.1439+0.2034+0.1439)^2$ = 0.1206; $CS_j(g_i) = CS_l(g_i) = 0$; $CS_k(g_i)$ = $\frac{1}{2}(0.1982+0.1982)^2$ = 0.0786; $\pi_i^{(i)}(g_i)$ = $\pi_i^{(k)}(g_i)$ = $\frac{(3.5)}{2}(0.1439)^2$ = 0.0362; $\pi_k^{(i)}(g_i)$ = $\pi_k^{(k)}(g_i)$ = $\frac{(3.5)}{2}(0.1982)^2$ = 0.0687; $\pi_i^{(j)}(g_i)$ = $\frac{(3.5)}{2}(0.2034)^2$ = 0.0724;

$PS_i(g_i) = PS_k(g_i) = \frac{1.5}{4}(0.1439 + 0.1982)^2 = 0.0439$; $PS_j(g_i) = \frac{1.5}{4}(0.2034)^2 = 0.0155$; and $PS_l(g_i) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_j^{(j)}(g_i) \\ q_l^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_j^{(l)}(g_i) \\ q_l^{(l)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.3333 \\ 0.2500 \\ 0.1250 \\ 0.1667 \\ 0.2500 \\ 0.3333 \\ 0.1250 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i) = q_i^{(j)}(g_i) = q_i^{(k)}(g_i) = 0.2500$; $q_j^{(i)}(g_i) = q_j^{(j)}(g_i) = q_j^{(l)}(g_i) = 0.1250$; $q_k^{(i)}(g_i) = q_k^{(k)}(g_i) = 0.3333$; $q_l^{(j)}(g_i) = q_l^{(l)}(g_i) = 0.1667$; $CS_i(g_i) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_i) = \frac{1}{2}(0.1250 + 0.1250 + 0.1250)^2 = 0.0703$; $CS_k(g_i) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_l(g_i) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $\pi_i^{(i)}(g_i) = \pi_i^{(j)}(g_i) = \pi_i^{(k)}(g_i) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_i) = \pi_j^{(j)}(g_i) = \pi_j^{(l)}(g_i) = (0.1250)^2 = 0.0156$; $\pi_k^{(i)}(g_i) = \pi_k^{(k)}(g_i) = (0.3333)^2 = 0.1111$; $\pi_l^{(j)}(g_i) = \pi_l^{(l)}(g_i) = (0.1667)^2 = 0.0278$; and $PS_i(g_i) = PS_j(g_i) = PS_k(g_i) = PS_l(g_i) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1296 & 0.1296 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 \\ 0.1296 & 1 & 0.1296 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0 & -0.0370 \\ 0.1190 & 0.1190 & 1 & 0 & 0 & 0 & -0.0476 & 0 & 0 & 0 \\ 0 & -0.0370 & -0.0370 & 1 & 0.1296 & 0.1296 & 0 & -0.0370 & 0 & 0 \\ -0.0370 & 0 & -0.0370 & 0.1296 & 1 & 0.1296 & 0 & 0 & 0 & -0.0370 \\ 0 & 0 & 0 & 0.1190 & 0.1190 & 1 & 0 & 0 & -0.0476 & 0 \\ 0 & -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & 1 & 0.1296 & 0 & 0 \\ -0.0476 & -0.0476 & 0 & 0 & 0 & 0 & 0.1190 & 1 & 0 & 0 \\ -0.0370 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 1 & 0.1296 \\ 0 & 0 & 0 & -0.0476 & -0.0476 & 0 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_j^{(j)}(g_i) \\ q_l^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_j^{(l)}(g_i) \\ q_l^{(l)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.1111 \\ 0.2857 \\ 0.2222 \\ 0.1111 \\ 0.1429 \\ 0.2222 \\ 0.2857 \\ 0.1111 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = 0.1967; $q_j^{(i)}(g_i)$ = 0.0689; $q_k^{(i)}(g_i)$ = 0.2639; $q_i^{(j)}(g_i)$ = 0.2184; $q_j^{(j)}(g_i)$ = 0.0906; $q_l^{(j)}(g_i)$ = 0.1119; $q_i^{(k)}(g_i)$ = 0.2065; $q_k^{(k)}(g_i)$ = 0.2738; $q_j^{(l)}(g_i)$ = 0.1218; $q_l^{(l)}(g_i)$ = 0.1431; $CS_i(g_i) = \frac{1}{2}(0.1967+0.2184+0.2065)^2$ = 0.1932; $CS_j(g_i) = \frac{1}{2}(0.0689+0.0906+0.1218)^2$ = 0.0396; $CS_k(g_i) = \frac{1}{2}(0.2639+0.2738)^2$ = 0.1446; $CS_l(g_i) = \frac{1}{2}(0.1119+0.1431)^2$ = 0.0325; $\pi_i^{(i)}(g_i) = \frac{(2.5)}{2}(01967)^2$ = 0.0484; $\pi_j^{(i)}(g_i) = \frac{(2.5)}{2}(0.0689)^2$ = 0.0059; $\pi_k^{(i)}(g_i) = \frac{(2.5)}{2}(0.2639)^2$ = 0.0871; $\pi_i^{(j)}(g_i) = \frac{(2.5)}{2}(0.2184)^2$ = 0.0596; $\pi_j^{(j)}(g_i) = \frac{(2.5)}{2}(0.0906)^2$ = 0.0103; $\pi_l^{(j)}(g_i) = \frac{(2.5)}{2}(0.1119)^2$ = 0.0157; $\pi_i^{(k)}(g_i) = \frac{(2.5)}{2}(0.2065)^2$ = 0.0533; $\pi_k^{(k)}(g_i) = \frac{(2.5)}{2}(0.2738)^2$ = 0.0937; $\pi_j^{(l)}(g_i) = \frac{(2.5)}{2}(0.1218)^2$ = 0.0185; $\pi_l^{(l)}(g_i) = \frac{(2.5)}{2}(0.1431)^2$ = 0.0256; $PS_i(g_i) = \frac{0.5}{4}(0.1967+0.0689+0.2639)^2$ = 0.0350; $PS_j(g_i)$ =

$\frac{0.5}{4}(0.2184+0.0906+0.1119)^2$ = 0.0221; $PS_k(g_i)$ = $\frac{0.5}{4}(0.2065+0.2738)^2$ = 0.0288;

and $PS_l(g_i)$ = $\frac{0.5}{4}(0.1218+0.1431)^2$ = 0.0088.

Simulation 16: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2455 & 0.2455 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 \\ 0.2455 & 1 & 0.2455 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0 & -0.0545 \\ 0.2333 & 0.2333 & 1 & 0 & 0 & 0 & -0.0667 & 0 & 0 & 0 \\ 0 & -0.0545 & -0.0545 & 1 & 0.2455 & 0.2455 & 0 & -0.0545 & 0 & 0 \\ -0.0545 & 0 & -0.0545 & 0.2455 & 1 & 0.2455 & 0 & 0 & 0 & -0.0545 \\ 0 & 0 & 0 & 0.2333 & 0.2333 & 1 & 0 & 0 & -0.0667 & 0 \\ 0 & -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & 1 & 0.2455 & 0 & 0 \\ -0.0667 & -0.0667 & 0 & 0 & 0 & 0 & 0.2333 & 1 & 0 & 0 \\ -0.0545 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 1 & 0.2455 \\ 0 & 0 & 0 & -0.0667 & -0.0667 & 0 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_i) \\ q_j^{(i)}(g_i) \\ q_k^{(i)}(g_i) \\ q_i^{(j)}(g_i) \\ q_j^{(j)}(g_i) \\ q_l^{(j)}(g_i) \\ q_i^{(k)}(g_i) \\ q_k^{(k)}(g_i) \\ q_j^{(l)}(g_i) \\ q_l^{(l)}(g_i) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.0909 \\ 0.2222 \\ 0.1818 \\ 0.0909 \\ 0.1111 \\ 0.1818 \\ 0.2222 \\ 0.0909 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_i)$ = 0.1454; $q_j^{(i)}(g_i)$ = 0.0267; $q_k^{(i)}(g_i)$ = 0.1922; $q_i^{(j)}(g_i)$ = 0.1751; $q_j^{(j)}(g_i)$ = 0.0564; $q_l^{(j)}(g_i)$ = 0.0636; $q_i^{(k)}(g_i)$ = 0.1516; $q_k^{(k)}(g_i)$ = 0.1983; $q_j^{(l)}(g_i)$ = 0.0968; $q_l^{(l)}(g_i)$ = 0.1040; $CS_i(g_i)$ = $\frac{1}{2}(0.1454+0.1751+0.1516)^2$ = 0.1114; $CS_j(g_i)$ = $\frac{1}{2}(0.0267+0.0564+0.0968)^2$ = 0.0162; $CS_k(g_i)$ = $\frac{1}{2}(0.1922+0.1983)^2$ = 0.0762; $CS_l(g_i)$ = $\frac{1}{2}(0.0636+0.1040)^2$ = 0.0140; $\pi_i^{(i)}(g_i)$ = $\frac{(3.5)}{2}(0.1454)^2$ = 0.0370; $\pi_j^{(i)}(g_i)$ = $\frac{(3.5)}{2}(0.0267)^2$ = 0.0012; $\pi_k^{(i)}(g_i)$ = $\frac{(3.5)}{2}(0.1922)^2$ = 0.0646; $\pi_i^{(j)}(g_i)$ = $\frac{(3.5)}{2}(0.1751)^2$ = 0.0537; $\pi_j^{(j)}(g_i)$ = $\frac{(3.5)}{2}(0.0564)^2$ = 0.0056; $\pi_l^{(j)}(g_i)$ = $\frac{(3.5)}{2}(0.0636)^2$ = 0.0071; $\pi_i^{(k)}(g_i)$ = $\frac{(3.5)}{2}(0.1516)^2$ = 0.0402; $\pi_k^{(k)}(g_i)$ =

$\frac{(3.5)}{2}(0.1983)^2$ = 0.0688; $\pi_j^{(l)}(g_i)$ = $\frac{(3.5)}{2}(0.0968)^2$ = 0.0164; $\pi_l^{(l)}(g_i)$ = $\frac{(3.5)}{2}(0.1040)^2$ = 0.0189; $PS_i(g_i)$ = $\frac{1.5}{4}(0.1454+0.0267+0.1922)^2$ = 0.0498; $PS_j(g_i)$ = $\frac{1.5}{4}(0.1751+0.0564+0.0636)^2$ = 0.0327; $PS_k(g_i)$ = $\frac{1.5}{4}(0.1516+0.1983)^2$ = 0.0459; and $PS_l(g_i)$ = $\frac{1.5}{4}(0.0968+0.1040)^2$ = 0.0151.

Network j

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_k^{(k)}(g_j)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_j)+\phi q_i^{(j)}(g_j)-\phi(\phi+2)q_j^{(i)}(g_j)}{3\phi^2+12\phi+8}$; $q_j^{(i)}(g_j)$ = $q_j^{(k)}(g_j)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_j)-2\phi(\phi+2)q_i^{(i)}(g_j)}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_j)$ = $q_k^{(j)}(g_j)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_j)+2\phi q_i^{(i)}(g_j)-\phi(\phi+3)q_j^{(j)}(g_j)}{3\phi^2+13\phi+8}$; $q_j^{(j)}(g_j)$ = $\frac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(i)}(g_j)-2\phi(\phi+3)q_i^{(j)}(g_j)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_j)$ = $\frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}\left(q_i^{(i)}(g_j)+q_i^{(j)}(g_j)+q_i^{(k)}(g_j)\right)^2$; $CS_j(g_j)$ = $\frac{1}{2}\left(q_j^{(i)}(g_j)+q_j^{(j)}(g_j)+q_j^{(k)}(g_j)\right)^2$; $CS_l(g_j)$ = $\frac{1}{2}\left(q_l^{(l)}(g_j)\right)^2$; $\pi_i^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_j)\right)^2$; $\pi_j^{(i)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_j)\right)^2$; $\pi_i^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_j)\right)^2$; $\pi_j^{(j)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_j)\right)^2$; $\pi_l^{(l)}(g_j)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_j)\right)^2$; $PS_i(g_j)$ = $PS_k(g_j)$ =

$\frac{\phi}{4}\left(q_i^{(i)}(g_j)+q_j^{(i)}(g_j)+q_k^{(i)}(g_j)\right)^2$; $PS_j(g_j)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_j)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)\right)^2$; and $PS_l(g_j)$ = $\frac{\phi}{4}\left(q_l^{(l)}(g_j)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} \\ \frac{\phi(\phi+2)}{(\phi+4)(\phi+1)} & 1 & -\frac{\phi}{(\phi+4)(\phi+1)} & 0 \\ -\frac{2\phi}{3\phi^2+13\phi+8} & -\frac{2\phi}{3\phi^2+13\phi+8} & 1 & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} \\ -\frac{2\phi}{(\phi+4)(\phi+1)} & 0 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_i^{(j)}(g_j) \\ q_j^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{2\alpha(\phi+1)}{3\phi^2+13\phi+8} \\ \frac{\tilde{\alpha}}{\phi+4} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_j)$ = $q_j^{(k)}(g_j)$ = $q_j^{(j)}(g_j)$ = $q_l^{(l)}(g_j)$ = 0, it holds that $q_i^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_i^{(j)}(g_j)$ = $q_k^{(j)}(g_j)$ = 0.2500; $CS_i(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}\left(0.2500+0.2500+0.2500\right)^2$ = 0.2813; $CS_j(g_j)$ = $CS_l(g_j)$ = 0; $\pi_i^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_i^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\left(0.2500\right)^2$ = 0.0625; and $PS_i(g_j) = PS_k(g_j) = PS_j(g_j) = PS_l(g_j) = 0$.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_j) = q_j^{(k)}(g_j) = q_j^{(j)}(g_j) = q_l^{(l)}(g_j) = 0$, it holds that $q_i^{(i)}(g_j) = q_k^{(i)}(g_j) = q_i^{(k)}(g_j) = q_k^{(k)}(g_j) = q_i^{(j)}(g_j) = q_k^{(j)}(g_j) = 0.2105$; $CS_i(g_j) = CS_k(g_j) = \frac{1}{2}(0.2105 + 0.2105 + 0.2105)^2 = 0.1994$; $CS_j(g_j) = CS_l(g_j) = 0$; $\pi_i^{(i)}(g_j) = \pi_k^{(i)}(g_j) = \pi_i^{(k)}(g_j) = \pi_k^{(k)}(g_j) = \pi_i^{(j)}(g_j) = \pi_k^{(j)}(g_j) = \frac{(2.5)}{2}(0.2105)^2 = 0.0554$; $PS_i(g_j) = PS_j(g_j) = PS_k(g_j) = \frac{0.5}{4}(0.2105 + 0.2105)^2 = 0.0222$; and $PS_l(g_j) = 0$.

Simulation 13: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_j) = q_j^{(k)}(g_j) = q_j^{(j)}(g_j) = q_l^{(l)}(g_j) = 0$, it holds that $q_i^{(i)}(g_j) = q_k^{(i)}(g_j) = q_i^{(k)}(g_j) = q_k^{(k)}(g_j) = q_i^{(j)}(g_j) = q_k^{(j)}(g_j) = 0.1600$; $CS_i(g_j) = CS_k(g_j) = \frac{1}{2}(0.1600 + 0.1600 + 0.1600)^2 = 0.1152$; $CS_j(g_j) = CS_l(g_j) = 0$; $\pi_i^{(i)}(g_j) = \pi_k^{(i)}(g_j) = \pi_i^{(k)}(g_j) = \pi_k^{(k)}(g_j) = \pi_i^{(j)}(g_j) = \pi_k^{(j)}(g_j) = \frac{(3.5)}{2}(0.1600)^2 = 0.0448$; $PS_i(g_j) = PS_k(g_j) = PS_j(g_j) = \frac{1.5}{4}(0.1600 + 0.1600)^2 = 0.0384$; and $PS_l(g_j) = 0$.

Simulation 14: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_i^{(j)}(g_j) \\ q_j^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.2500 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_i^{(j)}(g_j)$ = $q_k^{(j)}(g_j)$ = $q_l^{(l)}(g_j)$ = 0.2500; $q_j^{(i)}(g_j)$ = $q_j^{(k)}(g_j)$ = $q_j^{(j)}(g_j)$ = 0.1250; $CS_i(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_j)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $CS_l(g_j)$ = $\frac{1}{2}(0.2500)^2$ = 0.0313; $\pi_i^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_i^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\pi_l^{(l)}(g_j)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\pi_j^{(j)}(g_j)$ = $(0.1250)^2$ = 0.0156; and $PS_i(g_j) = PS_j(g_j) = PS_k(g_j) = PS_l(g_j) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.0847 & -0.0339 & -0.0339 \\ 0.1852 & 1 & -0.0741 & 0 \\ -0.0656 & -0.0656 & 1 & 0.1148 \\ -0.1481 & 0 & 0.2593 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_i^{(j)}(g_j) \\ q_j^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.2034 \\ 0.1111 \\ 0.1967 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_i^{(j)}(g_j)$ = $q_k^{(j)}(g_j)$ = 0.2059; $q_j^{(i)}(g_j)$ = $q_j^{(k)}(g_j)$ = $q_j^{(j)}(g_j)$ = 0.0882; $q_l^{(l)}(g_j)$ = 0.2000; $CS_i(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}(0.2059+0.2059+0.2059)^2$ = 0.1908; $CS_j(g_j)$ = $\frac{1}{2}(0.0882+0.0882+0.0882)^2$ =

0.0350; $CS_l(g_j)$ = $\frac{1}{2}(0.2000)^2$ = 0.0200; $\pi_i^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_i^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\frac{(2.5)}{2}(0.2059)^2$ = 0.0530; $\pi_j^{(i)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\pi_j^{(j)}(g_j)$ = $\frac{(2.5)}{2}(0.0882)^2$ = 0.0097; $\pi_l^{(l)}(g_j)$ = $\frac{(2.5)}{2}(0.2000)^2$ = 0.0500; $PS_i(g_j)$ = $PS_j(g_j)$ = $PS_k(g_j)$ = $\frac{0.5}{4}(0.2059+0.0882+0.2059)^2$ = 0.0313; and $PS_l(g_j)$ = $\frac{0.5}{4}(0.2000)^2$ = 0.0050.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1603 & -0.0458 & -0.0458 \\ 0.3818 & 1 & -0.1091 & 0 \\ -0.0876 & -0.0876 & 1 & 0.1971 \\ -0.2182 & 0 & 0.4910 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_j) \\ q_j^{(i)}(g_j) \\ q_i^{(j)}(g_j) \\ q_j^{(j)}(g_j) \end{pmatrix} = \begin{pmatrix} 0.1527 \\ 0.0909 \\ 0.1460 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_j)$ = $q_k^{(i)}(g_j)$ = $q_i^{(k)}(g_j)$ = $q_k^{(k)}(g_j)$ = $q_i^{(j)}(g_j)$ = $q_k^{(j)}(g_j)$ = 0.1542; $q_j^{(i)}(g_j)$ = $q_j^{(k)}(g_j)$ = 0.0489; $q_j^{(j)}(g_j)$ = 0.0488; $q_l^{(l)}(g_j)$ = 0.1429; $CS_i(g_j)$ = $CS_k(g_j)$ = $\frac{1}{2}(0.1542+0.1542+0.1542)^2$ = 0.1070; $CS_j(g_j)$ = $\frac{1}{2}(0.0489+0.0488+0.0489)^2$ = 0.0107; $CS_l(g_j)$ = $\frac{1}{2}(0.1429)^2$ = 0.0102; $\pi_i^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = $\pi_i^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = $\frac{(3.5)}{2}(0.1542)^2$ = 0.0416; $\pi_j^{(i)}(g_j)$ = $\pi_j^{(j)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\frac{(3.5)}{2}(0.0489)^2$ = 0.0042; $\pi_l^{(l)}(g_j)$ = $\frac{(3.5)}{2}(0.1429)^2$ = 0.0357; $PS_i(g_j)$ = $PS_k(g_j)$ =

$\frac{1.5}{4}(0.1542+0.0489+0.1542)^2$ = 0.0479; $PS_j(g_j)$ = $\frac{1.5}{4}(0.1542+0.0488+0.1542)^2$ = 0.0478; and $PS_l(g_j)$ = $\frac{1.5}{4}(0.1429)^2$ = 0.0077.

Network k

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_k)$ = $q_k^{(k)}(g_k)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_k)-\phi(\phi+2)q_j^{(i)}(g_k)}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_k)$ = $q_l^{(k)}(g_k)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_k)+2\phi q_j^{(j)}(g_k)-\phi(\phi+3)q_i^{(i)}(g_k)}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_k)$ = $q_k^{(l)}(g_k)$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_k)-\phi(\phi+2)q_j^{(j)}(g_k)}{2(\phi+3)(\phi+1)}$; $q_j^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_k)-\phi(\phi+2)q_i^{(j)}(g_k)}{3\phi^2+12\phi+8}$; $CS_i(g_k)$ = $CS_k(g_k)$ = $\frac{1}{2}\left(q_i^{(i)}(g_k)+q_i^{(j)}(g_k)\right)^2$; $CS_j(g_k)$ = $CS_l(g_k)$ = $\frac{1}{2}\left(q_j^{(i)}(g_k)+q_j^{(j)}(g_k)+q_j^{(l)}(g_k)\right)^2$; $\pi_i^{(i)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_k)\right)^2$; $\pi_j^{(i)}(g_k)$ = $\pi_l^{(k)}(g_k)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_k)\right)^2$; $\pi_i^{(j)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_k)\right)^2$; $\pi_j^{(j)}(g_k)$ = $\pi_l^{(j)}(g_k)$ = $\pi_j^{(l)}(g_k)$ = $\pi_l^{(l)}(g_k)$; $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_k)\right)^2$; $PS_i(g_k)$ = $PS_k(g_k)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_k)+q_j^{(i)}(g_k)\right)^2$; and $PS_j(g_k)$ = $PS_l(g_k)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_k)+q_j^{(j)}(g_k)+q_l^{(j)}(g_k)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 0 & -\frac{\phi}{(\phi+3)(\phi+1)} \\ \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)} & 1 & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} \\ 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} \\ -\frac{\phi}{3\phi^2+12\phi+8} & 0 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_k) \\ q_j^{(i)}(g_k) \\ q_i^{(j)}(g_k) \\ q_j^{(j)}(g_k) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{\alpha}{\phi+3} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+12\phi+8} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_l^{(l)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_j^{(l)}(g_k)$ =0, it holds that $q_i^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_k^{(l)}(g_k)$ = $\frac{1}{3}$ = 0.3333; $CS_i(g_k)$ = $CS_k(g_k)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_j(g_k)$ = $CS_l(g_k)$ = 0; $\pi_i^{(i)}(g_k)$ = $\pi_i^{(j)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_k)$ = $PS_j(g_k)$ = $PS_k(g_k)$ = $PS_l(g_k)$ = 0.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_k)$ = $q_l^{(k)}(g_k)$ = $q_j^{(j)}(g_k)$ = $q_l^{(l)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_j^{(l)}(g_k)$ =0, it holds that $q_i^{(i)}(g_k)$ = $q_i^{(j)}(g_k)$ = $q_k^{(k)}(g_k)$ = $q_k^{(l)}(g_k)$ = $\frac{1}{3.5}$ = 0.2857; $CS_i(g_k)$ = $CS_k(g_k)$ = $\frac{1}{2}(0.2857+0.2857)^2$ = 0.1632; $CS_j(g_k)$ = $CS_l(g_k)$ = 0; $\pi_i^{(i)}(g_k)$ = $\pi_i^{(j)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = $\frac{(2.5)}{2}(0.2857)^2$ = 0.1020; and $PS_i(g_k)$ = $PS_j(g_k)$ = $PS_k(g_k)$ = $PS_l(g_k)$ = $\frac{0.5}{4}(0.2857)^2$ = 0.0102.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_k) = q_l^{(k)}(g_k) = q_j^{(j)}(g_k) = q_l^{(l)}(g_k) = q_l^{(j)}(g_k) = q_j^{(l)}(g_k)$ =0, it holds that $q_i^{(i)}(g_k) = q_i^{(j)}(g_k) = q_k^{(k)}(g_k) = q_k^{(l)}(g_k) = \frac{1}{4.5} = 0.2222$; $CS_i(g_k) = CS_k(g_k) = \frac{1}{2}(0.2222+0.2222)^2 = 0.0988$; $CS_j(g_k) = CS_l(g_k) = 0$; $\pi_i^{(i)}(g_k) = \pi_i^{(j)}(g_k) = \pi_k^{(k)}(g_k) = \pi_k^{(l)}(g_k) = \frac{(3.5)}{2}(0.2222)^2 = 0.0864$; and $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k) = \frac{1.5}{4}(0.2222)^2 = 0.0185$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_k) \\ q_j^{(i)}(g_k) \\ q_i^{(j)}(g_k) \\ q_j^{(j)}(g_k) \end{pmatrix} = \begin{pmatrix} 0.3333 \\ 0.1250 \\ 0.3333 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_k) = q_k^{(k)}(g_k) = q_i^{(j)}(g_k) = q_k^{(l)}(g_k) = 0.3333$; $q_j^{(i)}(g_k) = q_l^{(k)}(g_k) = q_j^{(j)}(g_k) = q_l^{(j)}(g_k) = q_j^{(l)}(g_k) = q_l^{(l)}(g_k) = 0.1250$; $CS_i(g_k) = CS_k(g_k) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_k) = CS_l(g_k) = \frac{1}{2}(0.1250+0.1250+0.1250)^2 = 0.0703$; $\pi_i^{(i)}(g_k) = \pi_k^{(k)}(g_k) = \pi_i^{(j)}(g_k) = \pi_k^{(l)}(g_k) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_k) = \pi_l^{(k)}(g_k) = \pi_j^{(j)}(g_k) = \pi_l^{(j)}(g_k) = \pi_j^{(l)}(g_k) = \pi_l^{(l)}(g_k) = (0.1250)^2 = 0.0156$; and $PS_i(g_k) = PS_j(g_k) = PS_k(g_k) = PS_l(g_k) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1190 & 0 & -0.0952 \\ 0.1296 & 1 & -0.0741 & -0.0741 \\ 0 & -0.0476 & 1 & 0.1190 \\ -0.0339 & 0 & 0.0847 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_k) \\ q_j^{(i)}(g_k) \\ q_i^{(j)}(g_k) \\ q_j^{(j)}(g_k) \end{pmatrix} = \begin{pmatrix} 0.2857 \\ 0.1111 \\ 0.2857 \\ 0.1017 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_k) = q_k^{(k)}(g_k) = 0.2819$; $q_j^{(i)}(g_k) = q_l^{(k)}(g_k) = 0.1018$; $q_i^{(j)}(g_k) = q_k^{(l)}(g_k) = 0.2801$; $q_j^{(j)}(g_k) = q_l^{(j)}(g_k) = q_j^{(l)}(g_k) = q_l^{(l)}(g_k) = 0.0875$; $CS_i(g_k) = CS_k(g_k) = \frac{1}{2}(0.2819 + 0.2801)^2 = 0.1579$; $CS_j(g_k) = CS_l(g_k) = \frac{1}{2}(0.1018 + 0.0875 + 0.0875)^2 = 0.0383$; $\pi_i^{(i)}(g_k) = \pi_k^{(k)}(g_k) = \frac{(2.5)}{2}(0.2819)^2 = 0.0993$; $\pi_j^{(i)}(g_k) = \pi_l^{(k)}(g_k) = \frac{(2.5)}{2}(0.1018)^2 = 0.0130$; $\pi_i^{(j)}(g_k) = \pi_k^{(l)}(g_k) = \frac{(2.5)}{2}(0.2801)^2 = 0.0981$; $\pi_j^{(j)}(g_k) = \pi_l^{(j)}(g_k) = \pi_j^{(l)}(g_k) = \pi_l^{(l)}(g_k) = \frac{(2.5)}{2}(0.0875)^2 = 0.0096$; $PS_i(g_k) = PS_k(g_k) = \frac{0.5}{4}(0.2819 + 0.1018)^2 = 0.0184$; and $PS_j(g_k) = PS_l(g_k) = \frac{0.5}{4}(0.2801 + 0.0875 + 0.0875)^2 = 0.0259$.

Simulation 16: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2333 & 0 & -0.1333 \\ 0.2455 & 1 & -0.1091 & -0.1091 \\ 0 & -0.0667 & 1 & 0.2333 \\ -0.0458 & 0 & 0.1603 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_k) \\ q_j^{(i)}(g_k) \\ q_i^{(j)}(g_k) \\ q_j^{(j)}(g_k) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.0909 \\ 0.2222 \\ 0.0763 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_k) = q_k^{(k)}(g_k) = 0.2133$; $q_j^{(i)}(g_k) = q_l^{(k)}(g_k) = 0.0676$; $q_i^{(j)}(g_k) = q_k^{(l)}(g_k) = 0.2147$; $q_j^{(j)}(g_k) = q_l^{(j)}(g_k) = q_j^{(l)}(g_k) = q_l^{(l)}(g_k) = 0.0517$; $CS_i(g_k) = CS_k(g_k) = \frac{1}{2}(0.2133+0.2147)^2 = 0.0916$; $CS_j(g_k) = CS_l(g_k) = \frac{1}{2}(0.0676+0.0517+0.0517)^2 = 0.0146$; $\pi_i^{(i)}(g_k) = \pi_k^{(k)}(g_k) = \frac{(3.5)}{2}(0.2133)^2 = 0.0796$; $\pi_j^{(i)}(g_k) = \pi_l^{(k)}(g_k) = \frac{(3.5)}{2}(0.0676)^2 = 0.0080$; $\pi_i^{(j)}(g_k) = \pi_k^{(l)}(g_k) = \frac{(3.5)}{2}(0.2147)^2 = 0.0807$; $\pi_j^{(j)}(g_k) = \pi_l^{(j)}(g_k) = \pi_j^{(l)}(g_k) = \pi_l^{(l)}(g_k) = \frac{(3.5)}{2}(0.0517)^2 = 0.0047$; $PS_i(g_k) = PS_k(g_k) = \frac{1.5}{4}(0.2133+0.0676)^2 = 0.0296$; and $PS_j(g_k) = PS_l(g_k) = \frac{1.5}{4}(0.2147+0.0517+0.0517)^2 = 0.0379$.

Network l

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_l) = q_k^{(k)}(g_l) = \frac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_l)+\phi q_j^{(j)}(g_l)+\phi q_l^{(j)}(g_l)-\phi(\phi+2)q_j^{(i)}(g_l)}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_l) = q_j^{(k)}(g_l) = \frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_l)+\phi q_l^{(j)}(g_l)+\phi q_l^{(l)}(g_l)-\phi(\phi+3)q_i^{(i)}(g_l)}{2(\phi+5)(\phi+1)}$; $q_i^{(j)}(g_l) = q_k^{(j)}(g_l) = \frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_l)-\phi(\phi+2)q_j^{(j)}(g_l)-\phi(\phi+2)q_l^{(j)}(g_l)}{3\phi^2+10\phi+6}$; $q_j^{(j)}(g_l) =$

$\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_l)+\phi q_l^{(l)}(g_l)-2\phi(\phi+4)q_i^{(j)}(g_l)-\phi(\phi+4)q_l^{(j)}(g_l)}{2(\phi+5)(\phi+1)}$; $q_l^{(j)}(g_l)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+\phi q_j^{(l)}(g_l)-2\phi(\phi+2)q_i^{(j)}(g_l)-\phi(\phi+2)q_j^{(j)}(g_l)}{2(\phi+3)(\phi+1)}$; $q_j^{(l)}(g_l)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_l)+\phi q_l^{(j)}(g_l)-\phi(\phi+4)q_l^{(l)}(g_l)}{2(\phi+5)(\phi+1)}$; $q_l^{(l)}(g_l)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_l)+\phi q_j^{(j)}(g_l)-\phi(\phi+2)q_j^{(l)}(g_l)}{2(\phi+3)(\phi+1)}$; $CS_i(g_l)$ = $CS_k(g_l)$ = $\dfrac{1}{2}\left(q_i^{(i)}(g_l)+q_i^{(j)}(g_l)\right)^2$; $CS_j(g_l)$ = $\dfrac{1}{2}\left(q_j^{(i)}(g_l)+q_j^{(j)}(g_l)+q_j^{(k)}(g_l)+q_j^{(l)}(g_l)\right)^2$; $CS_l(g_l)$ = $\dfrac{1}{2}\left(q_l^{(j)}(g_l)+q_l^{(l)}(g_l)\right)^2$; $\pi_i^{(i)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_i^{(i)}(g_l)\right)^2$; $\pi_j^{(i)}(g_l)$ = $\pi_j^{(k)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_j^{(i)}(g_l)\right)^2$; $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_i^{(j)}(g_l)\right)^2$; $\pi_j^{(j)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_j^{(j)}(g_l)\right)^2$; $\pi_l^{(j)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_l^{(j)}(g_l)\right)^2$; $\pi_j^{(l)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_j^{(l)}(g_l)\right)^2$; $\pi_l^{(l)}(g_l)$ = $\dfrac{(2+\phi)}{2}\left(q_l^{(l)}(g_l)\right)^2$; $PS_i(g_l)$ = $PS_k(g_l)$ = $\dfrac{\phi}{4}\left(q_i^{(i)}(g_l)+q_j^{(i)}(g_l)\right)^2$; $PS_j(g_l)$ = $\dfrac{\phi}{4}\left(q_i^{(j)}(g_l)+q_j^{(j)}(g_j)+q_k^{(j)}(g_j)+q_l^{(j)}(g_l)\right)^2$; and $PS_l(g_l)$ = $\dfrac{\phi}{4}\left(q_j^{(l)}(g_l)+q_l^{(l)}(g_l)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix}
1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & 0 \\
\frac{\phi(\phi+3)}{2(\phi+5)(\phi+1)} & 1 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\
0 & -\frac{\phi}{3\phi^2+10\phi+6} & 1 & \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & 0 & 0 \\
-\frac{\phi}{(\phi+5)(\phi+1)} & 0 & \frac{\phi(\phi+4)}{(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\
0 & 0 & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\
0 & 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} \\
0 & 0 & -\frac{\phi}{(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1
\end{pmatrix}
\begin{pmatrix} q_i^{(i)}(g_l) \\ q_j^{(i)}(g_l) \\ q_i^{(j)}(g_l) \\ q_j^{(j)}(g_l) \\ q_l^{(j)}(g_l) \\ q_j^{(l)}(g_l) \\ q_l^{(l)}(g_l) \end{pmatrix}
=
\begin{pmatrix} \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{2\alpha(\phi+1)}{3\phi^2+10\phi+6} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_l) = q_j^{(k)}(g_l) = q_j^{(j)}(g_l) = q_l^{(j)}(g_l) = q_j^{(l)}(g_l) = q_l^{(l)}(g_l) = 0$, it holds that $q_i^{(i)}(g_l) = q_i^{(j)}(g_l) = q_k^{(j)}(g_l) = q_k^{(k)}(g_l) = \frac{1}{3} = 0.3333$; $CS_i(g_l) = CS_k(g_l) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_l) = CS_l(g_l) = 0$; $\pi_i^{(i)}(g_l) = \pi_i^{(j)}(g_l) = \pi_k^{(j)}(g_l) = \pi_k^{(k)}(g_l) = (0.3333)^2 = 0.1111$; and $PS_i(g_l) = PS_j(g_l) = PS_k(g_l) = PS_l(g_l) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_l) = q_j^{(k)}(g_l) = q_j^{(j)}(g_l) = q_l^{(j)}(g_l) = q_j^{(l)}(g_l) = q_l^{(l)}(g_l) = 0$, it holds that $q_i^{(i)}(g_l) = q_k^{(k)}(g_l) = 0.2979$; $q_i^{(j)}(g_l) = q_k^{(j)}(g_l) = 0.2553$; $CS_i(g_l) = CS_k(g_l) = \frac{1}{2}(0.2979+0.2553)^2 = 0.1530$; $CS_j(g_l) = CS_l(g_l) = 0$; $\pi_i^{(i)}(g_l) = \pi_k^{(k)}(g_l) = \frac{(2.5)}{2}(0.2979)^2 = 0.1109$; $\pi_i^{(j)}(g_l) = \pi_k^{(j)}(g_l) = \frac{(2.5)}{2}(0.2553)^2 = 0.0815$; $PS_i(g_l) = PS_k(g_l) = \frac{0.5}{4}(0.2979)^2 = 0.0111$; $PS_j(g_l) = \frac{0.5}{4}(0.2553+0.2553)^2 = 0.0326$; and $PS_l(g_l) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_l) = q_j^{(k)}(g_l) = q_j^{(j)}(g_l) = q_l^{(j)}(g_l) = q_j^{(l)}(g_l) = q_l^{(l)}(g_l) = 0$, it holds that $q_i^{(i)}(g_l) = q_k^{(k)}(g_l) = 0.2342$; $q_i^{(j)}(g_l) = q_k^{(j)}(g_l) = 0.1802$; $CS_i(g_l) =$

$CS_k(g_l)$ = $\frac{1}{2}(0.2342+0.1802)^2$ = 0.0859; $CS_j(g_l)$ = $CS_l(g_l)$ = 0; $\pi_i^{(i)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = $\frac{(3.5)}{2}(0.2342)^2$ = 0.0960; $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = $\frac{(3.5)}{2}(0.1802)^2$ = 0.0568; $PS_i(g_l)$ = $PS_k(g_l)$ = $\frac{1.5}{4}(0.2342)^2$ = 0.0206; $PS_j(g_l)$ = $\frac{1.5}{4}(0.1802+0.1802)^2$ = 0.0487; and $PS_l(g_l)$ = 0.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_l) \\ q_j^{(i)}(g_l) \\ q_i^{(j)}(g_l) \\ q_j^{(j)}(g_l) \\ q_l^{(j)}(g_l) \\ q_j^{(l)}(g_l) \\ q_l^{(l)}(g_l) \end{pmatrix} = \begin{pmatrix} 0.3333 \\ 0.1000 \\ 0.3333 \\ 0.1000 \\ 0.1667 \\ 0.1000 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_l)$ = $q_i^{(j)}(g_l)$ = $q_k^{(j)}(g_l)$ = $q_k^{(k)}(g_l)$ = 0.3333; $q_j^{(i)}(g_l)$ = $q_j^{(j)}(g_l)$ = $q_j^{(k)}(g_l)$ = $q_j^{(l)}(g_l)$ = 0.1000; $q_l^{(j)}(g_l)$ = $q_l^{(l)}(g_l)$ = 0.1667; $CS_i(g_l)$ = $CS_k(g_l)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_j(g_l)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $CS_l(g_l)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $\pi_i^{(i)}(g_l)$ = $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(i)}(g_l)$ = $\pi_j^{(j)}(g_l)$ = $\pi_j^{(k)}(g_l)$ = $\pi_j^{(l)}(g_l)$ = $(0.1000)^2$ = 0.0100; $\pi_l^{(j)}(g_l)$ = $\pi_l^{(l)}(g_l)$ = $(0.1667)^2$ = 0.0278; and $PS_i(g_l)$ = $PS_j(g_l)$ = $PS_k(g_l)$ = $PS_l(g_l)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1190 & -0.0476 & -0.0476 & -0.0476 & 0 & 0 \\ 0.1061 & 1 & -0.0606 & 0 & -0.0303 & 0 & -0.0303 \\ 0 & -0.0426 & 1 & 0.1064 & 0.1064 & 0 & 0 \\ -0.0606 & 0 & 0.2727 & 1 & 0.1364 & 0 & -0.0303 \\ 0 & 0 & 0.2381 & 0.1190 & 1 & -0.0476 & 0 \\ 0 & 0 & -0.0606 & 0 & -0.0303 & 1 & 0.1364 \\ 0 & 0 & -0.0952 & -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_l) \\ q_j^{(i)}(g_l) \\ q_i^{(j)}(g_l) \\ q_j^{(j)}(g_l) \\ q_l^{(j)}(g_l) \\ q_j^{(l)}(g_l) \\ q_l^{(l)}(g_l) \end{pmatrix} = \begin{pmatrix} 0.2857 \\ 0.0909 \\ 0.2553 \\ 0.0909 \\ 0.1429 \\ 0.0909 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_l)$ = $q_k^{(k)}(g_l)$ = 0.2933; $q_j^{(i)}(g_l)$ = $q_j^{(k)}(g_l)$ = 0.0820; $q_i^{(j)}(g_l)$ = $q_k^{(j)}(g_l)$ = 0.2461; $q_j^{(j)}(g_l)$ = 0.0348; $q_l^{(j)}(g_l)$ = 0.0843; $q_j^{(l)}(g_l)$ = 0.0869; $q_l^{(l)}(g_l)$ = 0.1577; $CS_i(g_l)$ = $CS_k(g_l)$ = $\frac{1}{2}(0.2933+0.2461)^2$ = 0.1455; $CS_j(g_l)$ = $\frac{1}{2}(0.0820+0.0348+0.0820+0.0869)^2$ = 0.0408; $CS_l(g_l)$ = $\frac{1}{2}(0.0843+0.1577)^2$ = 0.0293; $\pi_i^{(i)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = $\frac{(2.5)}{2}(0.2933)^2$ = 0.1075; $\pi_j^{(i)}(g_l)$ = $\pi_j^{(k)}(g_l)$ = $\frac{(2.5)}{2}(0.0820)^2$ = 0.0084; $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = $\frac{(2.5)}{2}(0.2461)^2$ = 0.0757; $\pi_j^{(j)}(g_l)$ = $\frac{(2.5)}{2}(0.0348)^2$ = 0.0015; $\pi_l^{(j)}(g_l)$ = $\frac{(2.5)}{2}(0.0843)^2$ = 0.0089; $\pi_j^{(l)}(g_l)$ = $\frac{(2.5)}{2}(0.0869)^2$ = 0.0094; $\pi_l^{(l)}(g_l)$ = $\frac{(2.5)}{2}(0.1577)^2$ = 0.0311; $PS_i(g_l)$ = $PS_k(g_l)$ = $\frac{0.5}{4}(0.2933+0.0820)^2$ = 0.0176; $PS_j(g_l)$ = $\frac{0.5}{4}(0.2461+0.0348+0.2461+0.0843)^2$ = 0.0467; and $PS_l(g_l)$ = $\frac{0.5}{4}(0.0869+0.1577)^2$ = 0.0075.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2333 & -0.0667 & -0.0667 & -0.0667 & 0 & 0 \\ 0.2077 & 1 & -0.0923 & 0 & -0.0462 & 0 & -0.0462 \\ 0 & -0.0541 & 1 & 0.1892 & 0.1892 & 0 & 0 \\ -0.0923 & 0 & 0.5077 & 1 & 0.2538 & 0 & -0.0462 \\ 0 & 0 & 0.4667 & 0.2333 & 1 & -0.0667 & 0 \\ 0 & 0 & -0.0923 & 0 & -0.0462 & 1 & 0.2538 \\ 0 & 0 & -0.1333 & -0.0667 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_l) \\ q_j^{(i)}(g_l) \\ q_i^{(j)}(g_l) \\ q_j^{(j)}(g_l) \\ q_l^{(j)}(g_l) \\ q_j^{(l)}(g_l) \\ q_l^{(l)}(g_l) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.0769 \\ 0.1802 \\ 0.0769 \\ 0.1111 \\ 0.0769 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_l)$ = $q_k^{(k)}(g_l)$ = 0.2239; $q_j^{(i)}(g_l)$ = $q_j^{(k)}(g_l)$ = 0.0537; $q_i^{(j)}(g_l)$ = $q_k^{(j)}(g_l)$ = 0.1760; $q_j^{(j)}(g_l)$ = 0.0057; $q_l^{(j)}(g_l)$ = 0.0319; $q_j^{(l)}(g_l)$ = 0.0642; $q_l^{(l)}(g_l)$ = 0.1200; $CS_i(g_l)$ = $CS_k(g_l)$ = $\frac{1}{2}(0.2239+0.1760)^2$ = 0.0800; $CS_j(g_l)$ = $\frac{1}{2}(0.0537+0.0057+0.0537+0.0642)^2$ = 0.0157; $CS_l(g_l)$ = $\frac{1}{2}(0.0319+0.1200)^2$ = 0.0115; $\pi_i^{(i)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = $\frac{(3.5)}{2}(0.2239)^2$ = 0.0877; $\pi_j^{(i)}(g_l)$ = $\pi_j^{(k)}(g_l)$ = $\frac{(3.5)}{2}(0.0537)^2$ = 0.0050; $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = $\frac{(3.5)}{2}(0.1760)^2$ = 0.0542; $\pi_j^{(j)}(g_l)$ = $\frac{(3.5)}{2}(0.0057)^2$ = 0.0001; $\pi_l^{(j)}(g_l)$ = $\frac{(3.5)}{2}(0.0319)^2$ = 0.0018; $\pi_j^{(l)}(g_l)$ = $\frac{(3.5)}{2}(0.0642)^2$ = 0.0072; $\pi_l^{(l)}(g_l)$ = $\frac{(3.5)}{2}(0.1200)^2$ = 0.0252; $PS_i(g_l)$ = $PS_k(g_l)$ = $\frac{1.5}{4}(0.2239+0.0537)^2$ = 0.0289; $PS_j(g_l)$ = $\frac{1.5}{4}(0.1760+0.0057+0.1760+0.0319)^2$ = 0.0569; and $PS_l(g_l)$ = $\frac{1.5}{4}(0.0642+0.1200)^2$ = 0.0127.

Network m

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_m)$ = $\dfrac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_m)+2\phi q_l^{(j)}(g_m)-2\phi(\phi+3)q_j^{(i)}(g_m)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_m)$ = $q_l^{(i)}(g_m)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_m)+\phi q_l^{(j)}(g_m)+\phi q_j^{(j)}(g_m)-\phi(\phi+3)q_i^{(i)}(g_m)}{3\phi^2+13\phi+8}$; $q_i^{(j)}(g_m)$ = $q_i^{(l)}(g_m)$ = $\dfrac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_m)-\phi(\phi+2)q_j^{(j)}(g_m)-\phi(\phi+2)q_l^{(j)}(g_m)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_m)$ = $q_l^{(l)}(g_m)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_m)+\phi q_j^{(i)}(g_m)-\phi(\phi+2)q_i^{(j)}(g_m)-\phi(\phi+3)q_l^{(j)}(g_m)}{2\phi^2+9\phi+8}$; $q_l^{(j)}(g_m)$ = $q_j^{(l)}(g_m)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_m)+\phi q_j^{(i)}(g_m)-\phi(\phi+2)q_i^{(j)}(g_m)-\phi(\phi+3)q_j^{(j)}(g_m)}{2\phi^2+9\phi+8}$;

$q_k^{(k)}(g_m)$ = $\dfrac{\alpha}{\phi+2}$; $CS_i(g_m)$ = $\dfrac{1}{2}\left(q_i^{(i)}(g_m)+q_i^{(j)}(g_m)+q_i^{(l)}(g_m)\right)^2$; $CS_j(g_m)$ = $CS_l(g_m)$ = $\dfrac{1}{2}\left(q_j^{(i)}(g_m)+q_j^{(j)}(g_m)+q_j^{(l)}(g_m)\right)^2$; $CS_k(g_m)$ = $\dfrac{1}{2}\left(q_k^{(k)}(g_m)\right)^2$; $\pi_i^{(i)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_i^{(i)}(g_m)\right)^2$; $\pi_j^{(i)}(g_m)$ = $\pi_l^{(i)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_j^{(i)}(g_m)\right)^2$; $\pi_i^{(j)}(g_m)$ = $\pi_i^{(l)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_i^{(j)}(g_m)\right)^2$; $\pi_j^{(j)}(g_m)$ = $\pi_l^{(l)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_j^{(j)}(g_m)\right)^2$; $\pi_l^{(j)}(g_m)$ = $\pi_j^{(l)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_l^{(j)}(g_m)\right)^2$; $\pi_k^{(k)}(g_m)$ = $\dfrac{(2+\phi)}{2}\left(q_k^{(k)}(g_m)\right)^2$; $PS_i(g_m)$ = $\dfrac{\phi}{4}\left(q_i^{(i)}(g_m)+q_j^{(i)}(g_m)+q_l^{(i)}(g_m)\right)^2$; $PS_j(g_m)$ = $PS_l(g_m)$ = $\dfrac{\phi}{4}\left(q_i^{(j)}(g_m)+q_j^{(j)}(g_m)+q_l^{(j)}(g_m)\right)^2$

; and $PS_k(g_m)$ = $\dfrac{\phi}{4}\left(q_k^{(k)}(g_m)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} \\ \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & 1 & -\frac{2\phi}{3\phi^2+13\phi+8} & -\frac{\phi}{3\phi^2+13\phi+8} & -\frac{\phi}{3\phi^2+13\phi+8} \\ 0 & -\frac{\phi}{(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+4)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & \frac{\phi(\phi+2)}{2\phi^2+9\phi+8} & 1 & \frac{\phi(\phi+3)}{2\phi^2+9\phi+8} \\ -\frac{\phi}{2\phi^2+9\phi+8} & -\frac{\phi}{2\phi^2+9\phi+8} & \frac{\phi(\phi+2)}{2\phi^2+9\phi+8} & \frac{\phi(\phi+3)}{2\phi^2+9\phi+8} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_m) \\ q_j^{(i)}(g_m) \\ q_i^{(j)}(g_m) \\ q_j^{(j)}(g_m) \\ q_l^{(j)}(g_m) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+13\phi+8} \\ \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{2\phi^2+9\phi+8} \\ \frac{2\tilde{\alpha}(\phi+1)}{2\phi^2+9\phi+8} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_m) = q_l^{(i)}(g_m) = q_j^{(j)}(g_m) = q_l^{(l)}(g_m) = q_l^{(j)}(g_m) = q_j^{(l)}(g_m) = 0$, it holds that $q_i^{(i)}(g_m) = q_i^{(j)}(g_m) = q_i^{(l)}(g_m) = \frac{1}{4} = 0.2500$; $q_k^{(k)}(g_m) = \frac{1}{2} = 0.5000$; $CS_i(g_m) = \frac{1}{2}(0.2500+0.2500+0.2500)^2 = 0.2813$; $CS_j(g_m) = CS_l(g_m) = 0$; $CS_k(g_m) = \frac{1}{2}(0.5000)^2 = 0.1250$; $\pi_i^{(i)}(g_m) = \pi_i^{(j)}(g_m) = \pi_i^{(l)}(g_m) = (0.2500)^2 = 0.0625$; $\pi_k^{(k)}(g_m) = (0.5000)^2 = 0.2500$; and $PS_i(g_m) = PS_j(g_m) = PS_k(g_m) = PS_l(g_m) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_m) = q_l^{(i)}(g_m) = q_j^{(j)}(g_m) = q_l^{(l)}(g_m) = q_l^{(j)}(g_m) = q_j^{(l)}(g_m) = 0$, it holds that $q_i^{(i)}(g_m) = q_i^{(j)}(g_m) = q_i^{(l)}(g_m) = \frac{1}{4.5} = 0.2222$; $q_k^{(k)}(g_m) = \frac{1}{2.5} = 0.4000$; $CS_i(g_m) = \frac{1}{2}(0.2222+0.2222+0.2222)^2 = 0.2222$; $CS_j(g_m) = CS_l(g_m) = 0$; $CS_k(g_m) = \frac{1}{2}(0.4000)^2 = 0.0800$; $\pi_i^{(i)}(g_m) = \pi_i^{(j)}(g_m) = \pi_i^{(l)}(g_m) = \frac{(2.5)}{2}(0.2222)^2 =$

0.0617; $\pi_k^{(k)}(g_m)$ = $\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $PS_i(g_m)$ = $PS_j(g_m)$ = $PS_l(g_m)$ = $\frac{0.5}{4}(0.2222)^2$ = 0.0062; and $PS_k(g_m)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_m) = q_l^{(i)}(g_m) = q_j^{(j)}(g_m) = q_l^{(l)}(g_m) = q_l^{(j)}(g_m) = q_j^{(l)}(g_m) = 0$, it holds that $q_i^{(i)}(g_m) = q_i^{(j)}(g_m) = q_i^{(l)}(g_m) = \frac{1}{5.5}$ = 0.1818; $q_k^{(k)}(g_m) = \frac{1}{3.5}$ = 0.2857; $CS_i(g_m)$ = $\frac{1}{2}(0.1818+0.1818+0.1818)^2$ = 0.1487; $CS_j(g_m)$ = $CS_l(g_m)$ = 0; $CS_k(g_m)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $\pi_i^{(i)}(g_m) = \pi_i^{(j)}(g_m) = \pi_i^{(l)}(g_m) = \frac{(3.5)}{2}(0.1818)^2$ = 0.0578; $\pi_k^{(k)}(g_m)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $PS_i(g_m)$ = $PS_j(g_m)$ = $PS_l(g_m)$ = $\frac{1.5}{4}(0.1818)^2$ = 0.0124; and $PS_k(g_m)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_m) \\ q_j^{(i)}(g_m) \\ q_i^{(j)}(g_m) \\ q_j^{(j)}(g_m) \\ q_l^{(j)}(g_m) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.2500 \\ 0.1250 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_m) = q_i^{(j)}(g_m) = q_i^{(l)}(g_m)$ = 0.2500; $q_j^{(i)}(g_m) = q_l^{(i)}(g_m) = q_j^{(j)}(g_m) = q_l^{(l)}(g_m) = q_l^{(j)}(g_m) = q_j^{(l)}(g_m)$ = 0.1250; $q_k^{(k)}(g_m) = \frac{1}{2}$ = 0.5000; $CS_i(g_m)$ =

$\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_m)$ = $CS_l(g_m)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $CS_k(g_m)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $\pi_i^{(i)}(g_m)$ = $\pi_i^{(j)}(g_m)$ = $\pi_i^{(l)}(g_m)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_m)$ = $\pi_l^{(i)}(g_m)$ = $\pi_j^{(j)}(g_m)$ = $\pi_l^{(l)}(g_m)$ = $\pi_l^{(j)}(g_m)$ = $\pi_j^{(l)}(g_m)$ = $(0.1250)^2$ = 0.0156; $\pi_k^{(k)}(g_m)$ = $(0.5000)^2$ = 0.2500; and $PS_i(g_m) = PS_j(g_m) = PS_l(g_m) = PS_k(g_m) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2593 & 0 & -0.0741 & -0.0741 \\ 0.1148 & 1 & -0.0656 & -0.0328 & -0.0328 \\ 0 & -0.0741 & 1 & 0.0926 & 0.0926 \\ -0.0385 & -0.0385 & 0.0962 & 1 & 0.1346 \\ -0.0385 & -0.0385 & 0.0962 & 0.1346 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_m) \\ q_j^{(i)}(g_m) \\ q_i^{(j)}(g_m) \\ q_j^{(j)}(g_m) \\ q_l^{(j)}(g_m) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.0984 \\ 0.2222 \\ 0.1154 \\ 0.1154 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_m)$ = $q_i^{(j)}(g_m)$ = $q_i^{(l)}(g_m)$ = 0.2117; $q_j^{(i)}(g_m)$ = $q_l^{(i)}(g_m)$ = 0.0942; $q_j^{(j)}(g_m)$ = $q_l^{(l)}(g_m)$ = $q_l^{(j)}(g_m)$ = $q_j^{(l)}(g_m)$ = 0.0941; $q_k^{(k)}(g_m)$ = $\frac{1}{2.5}$ = 0.4000; $CS_i(g_m)$ = $\frac{1}{2}(0.2117+0.2117+0.2117)^2$ = 0.2017; $CS_j(g_m)$ = $CS_l(g_m)$ = $\frac{1}{2}(0.0942+0.0941+0.0941)^2$ = 0.0399; $CS_k(g_m)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $\pi_i^{(i)}(g_m)$ = $\pi_i^{(j)}(g_m)$ = $\pi_i^{(l)}(g_m)$ = $\frac{(2.5)}{2}(0.2117)^2$ = 0.0560; $\pi_j^{(i)}(g_m)$ = $\pi_l^{(i)}(g_m)$ = $\pi_j^{(j)}(g_m)$ = $\pi_l^{(l)}(g_m)$ = $\pi_l^{(j)}(g_m)$ = $\pi_j^{(l)}(g_m)$ = $\frac{(2.5)}{2}(0.0942)^2$ = 0.0111; $\pi_k^{(k)}(g_m)$ =

$\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; and $PS_i(g_m)$ = $PS_j(g_m)$ = $PS_k(g_m)$ = $PS_l(g_m)$ = $\frac{0.5}{4}(0.2117+0.0942+0.0942)^2$ = 0.0200.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.4909 & 0 & -0.1091 & -0.1091 \\ 0.1971 & 1 & -0.0876 & -0.0438 & -0.0438 \\ 0 & -0.1091 & 1 & 0.1909 & 0.1909 \\ -0.0577 & -0.0577 & 0.2019 & 1 & 0.2596 \\ -0.0577 & -0.0577 & 0.2019 & 0.2596 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_m) \\ q_j^{(i)}(g_m) \\ q_i^{(j)}(g_m) \\ q_j^{(j)}(g_m) \\ q_l^{(j)}(g_m) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.0730 \\ 0.1818 \\ 0.0962 \\ 0.0962 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_m)$ = $q_i^{(j)}(g_m)$ = $q_i^{(l)}(g_m)$ = 0.1654; $q_j^{(i)}(g_m)$ = $q_l^{(i)}(g_m)$ = $q_j^{(j)}(g_m)$ = $q_l^{(l)}(g_m)$ = $q_l^{(j)}(g_m)$ = $q_j^{(l)}(g_m)$ = 0.0602; $q_k^{(k)}(g_m)$ = $\frac{1}{3.5}$ = 0.2857; $CS_i(g_m)$ = $\frac{1}{2}(0.1654+0.1654+0.1654)^2$ = 0.1231; $CS_j(g_m)$ = $CS_l(g_m)$ = $\frac{1}{2}(0.0602+0.0602+0.0602)^2$ = 0.0163; $CS_k(g_m)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $\pi_i^{(i)}(g_m)$ = $\pi_i^{(j)}(g_m)$ = $\pi_i^{(l)}(g_m)$ = $\frac{(3.5)}{2}(0.1654)^2$ = 0.0479; $\pi_j^{(i)}(g_m)$ = $\pi_l^{(i)}(g_m)$ = $\pi_j^{(j)}(g_m)$ = $\pi_l^{(l)}(g_m)$ = $\pi_l^{(j)}(g_m)$ = $\pi_j^{(l)}(g_m)$ = $\frac{(3.5)}{2}(0.0602)^2$ = 0.0063; $\pi_k^{(k)}(g_m)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; and $PS_i(g_m)$ = $PS_j(g_m)$ = $PS_k(g_m)$ = $PS_l(g_m)$ = $\frac{1.5}{4}(0.1654+0.0602+0.0602)^2$ = 0.0306.

<u>Network n</u>

In considering the equations presented in Section 4.2.1.2 it holds that

$$q_i^{(i)}(g_n) = \frac{\begin{array}{l}2\alpha(\phi+1)+\phi q_j^{(j)}(g_n)+\phi q_k^{(k)}(g_n)+\phi q_l^{(k)}(g_n)+\phi q_k^{(l)}(g_n)+\phi q_l^{(l)}(g_n)\\ -\phi(\phi+4)q_j^{(i)}(g_n)-\phi(\phi+4)q_k^{(i)}(g_n)-\phi(\phi+4)q_l^{(i)}(g_n)\end{array}}{2(\phi+5)(\phi+1)};$$

$$q_j^{(i)}(g_n) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_n)-\phi(\phi+2)q_i^{(i)}(g_n)-\phi(\phi+2)q_k^{(i)}(g_n)-\phi(\phi+2)q_l^{(i)}(g_n)}{2(\phi+3)(\phi+1)};$$

$$q_k^{(i)}(g_n) = \frac{\begin{array}{l}2\alpha(\phi+1)+\phi q_i^{(k)}(g_n)+\phi q_l^{(k)}(g_n)+\phi q_i^{(l)}(g_n)+\phi q_l^{(l)}(g_n)\\ -\phi(\phi+3)q_i^{(i)}(g_n)-\phi(\phi+3)q_j^{(i)}(g_n)-\phi(\phi+3)q_l^{(i)}(g_n)\end{array}}{2(\phi+4)(\phi+1)};$$

$$q_l^{(i)}(g_n) = \frac{\begin{array}{l}2\tilde{\alpha}(\phi+1)+\phi q_i^{(k)}(g_n)+\phi q_k^{(k)}(g_n)+\phi q_i^{(l)}(g_n)+\phi q_k^{(l)}(g_n)\\ -\phi(\phi+3)q_i^{(i)}(g_n)-\phi(\phi+3)q_j^{(i)}(g_n)-\phi(\phi+3)q_k^{(i)}(g_n)\end{array}}{2(\phi+4)(\phi+1)};$$

$$q_i^{(j)}(g_n) = \frac{\begin{array}{l}2\alpha(\phi+1)+\phi q_j^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_l^{(i)}(g_n)+\phi q_k^{(k)}(g_n)\\ +\phi q_l^{(k)}(g_n)+\phi q_k^{(l)}(g_n)+\phi q_l^{(l)}(g_n)-\phi(\phi+4)q_j^{(j)}(g_n)\end{array}}{2(\phi+5)(\phi+1)};$$

$$q_j^{(j)}(g_n) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_l^{(i)}(g_n)-\phi(\phi+2)q_i^{(j)}(g_n)}{2(\phi+3)(\phi+1)};$$

$$q_i^{(k)}(g_n) = \frac{\begin{array}{l}2\alpha(\phi+1)+\phi q_j^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_l^{(i)}(g_n)+\phi q_j^{(j)}(g_n)\\ +\phi q_k^{(l)}(g_n)+\phi q_l^{(l)}(g_n)-\phi(\phi+4)q_k^{(k)}(g_n)-\phi(\phi+4)q_l^{(k)}(g_n)\end{array}}{2(\phi+5)(\phi+1)};$$

$$q_k^{(k)}(g_n) = \frac{\begin{array}{l}2\alpha(\phi+1)+\phi q_i^{(i)}(g_n)+\phi q_j^{(i)}(g_n)+\phi q_l^{(i)}(g_n)+\phi q_i^{(l)}(g_n)\\ +\phi q_l^{(l)}(g_n)-\phi(\phi+3)q_i^{(k)}(g_n)-\phi(\phi+3)q_l^{(k)}(g_n)\end{array}}{2(\phi+4)(\phi+1)};$$

$$q_l^{(k)}(g_n) = \frac{\begin{array}{l}2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_n)+\phi q_j^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_i^{(l)}(g_n)\\ +\phi q_k^{(l)}(g_n)-\phi(\phi+3)q_i^{(k)}(g_n)-\phi(\phi+3)q_k^{(k)}(g_n)\end{array}}{2(\phi+4)(\phi+1)};$$

$q_i^{(l)}(g_n) =$

$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_l^{(i)}(g_n)+\phi q_j^{(j)}(g_n) +\phi q_k^{(k)}(g_n)+\phi q_l^{(k)}(g_n)-\phi(\phi+4)q_k^{(l)}(g_n)-\phi(\phi+4)q_l^{(l)}(g_n)}{2(\phi+5)(\phi+1)}$; $q_k^{(l)}(g_n)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_n)+\phi q_j^{(i)}(g_n)+\phi q_l^{(i)}(g_n)+\phi q_i^{(k)}(g_n) +\phi q_l^{(k)}(g_n)-\phi(\phi+3)q_i^{(l)}(g_n)-\phi(\phi+3)q_l^{(l)}(g_n)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_n)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_n)+\phi q_j^{(i)}(g_n)+\phi q_k^{(i)}(g_n)+\phi q_i^{(k)}(g_n) +\phi q_k^{(k)}(g_n)-\phi(\phi+3)q_i^{(l)}(g_n)-\phi(\phi+3)q_k^{(l)}(g_n)}{2(\phi+4)(\phi+1)}$; $CS_i(g_n)$ = $\frac{1}{2}\left(q_i^{(i)}(g_n)+q_i^{(j)}(g_n)+q_i^{(k)}(g_n)+q_i^{(l)}(g_n)\right)^2$; $CS_j(g_n)$ = $\frac{1}{2}\left(q_j^{(i)}(g_n)+q_j^{(j)}(g_n)\right)^2$; $CS_k(g_n)$ = $\frac{1}{2}\left(q_k^{(i)}(g_n)+q_k^{(k)}(g_n)+q_k^{(l)}(g_n)\right)^2$; $CS_l(g_n)$ = $\frac{1}{2}\left(q_l^{(i)}(g_n)+q_l^{(k)}(g_n)+q_l^{(l)}(g_n)\right)^2$; $\pi_i^{(i)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_n)\right)^2$; $\pi_j^{(i)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_n)\right)^2$; $\pi_k^{(i)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_k^{(i)}(g_n)\right)^2$; $\pi_l^{(i)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_l^{(i)}(g_n)\right)^2$; $\pi_i^{(j)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_n)\right)^2$; $\pi_j^{(j)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_n)\right)^2$; $\pi_i^{(k)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_i^{(k)}(g_n)\right)^2$; $\pi_k^{(k)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_n)\right)^2$; $\pi_l^{(k)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_l^{(k)}(g_n)\right)^2$; $\pi_i^{(l)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_i^{(l)}(g_n)\right)^2$; $\pi_k^{(l)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_k^{(l)}(g_n)\right)^2$; $\pi_l^{(l)}(g_n)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_n)\right)^2$; $PS_i(g_n)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_n)+q_j^{(i)}(g_n)+q_k^{(i)}(g_n)+q_l^{(i)}(g_n)\right)^2$; $PS_j(g_n)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_n)+q_j^{(j)}(g_n)\right)^2$; $PS_k(g_n)$ = $\frac{\phi}{4}\left(q_i^{(k)}(g_n)+q_k^{(k)}(g_n)+q_l^{(k)}(g_n)\right)^2$; and $PS_l(g_n)$ = $\frac{\phi}{4}\left(q_i^{(l)}(g_n)+q_k^{(l)}(g_n)+q_l^{(l)}(g_n)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \beta_0 & \beta_0 & \beta_0 & 0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\ \beta_2 & 1 & \beta_2 & \beta_2 & -\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_4 & \beta_4 & 1 & \beta_4 & 0 & 0 & -\beta_5 & 0 & -\beta_5 & -\beta_5 & 0 & -\beta_5 \\ \beta_4 & \beta_4 & \beta_4 & 1 & 0 & 0 & -\beta_5 & -\beta_5 & 0 & -\beta_5 & -\beta_5 & 0 \\ 0 & -\beta_1 & -\beta_1 & -\beta_1 & 1 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\ -\beta_3 & 0 & -\beta_3 & -\beta_3 & \beta_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 \\ -\beta_5 & -\beta_5 & 0 & -\beta_5 & 0 & 0 & \beta_4 & 1 & \beta_4 & -\beta_5 & 0 & -\beta_5 \\ -\beta_5 & -\beta_5 & -\beta_5 & 0 & 0 & 0 & \beta_4 & \beta_4 & 1 & -\beta_5 & -\beta_5 & 0 \\ 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 \\ -\beta_5 & -\beta_5 & 0 & -\beta_5 & 0 & 0 & -\beta_5 & 0 & -\beta_5 & \beta_4 & 1 & \beta_4 \\ -\beta_5 & -\beta_5 & -\beta_5 & 0 & 0 & 0 & -\beta_5 & -\beta_5 & 0 & \beta_4 & \beta_4 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_j^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_l^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_j^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_l^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \\ q_l^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_6 \\ \beta_7 \\ \beta_6 \\ \beta_8 \\ \beta_9 \\ \beta_6 \\ \beta_8 \\ \beta_9 \end{pmatrix}$$

Where $\beta_0 = \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)}$; $\beta_1 = \frac{\phi}{2(\phi+5)(\phi+1)}$; $\beta_2 = \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)}$; $\beta_3 = \frac{\phi}{2(\phi+3)(\phi+1)}$; $\beta_4 = \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; $\beta_5 = \frac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_6 = \frac{\alpha}{\phi+5}$; $\beta_7 = \frac{\tilde{\alpha}}{\phi+3}$; $\beta_8 = \frac{\alpha}{\phi+4}$; $\beta_9 = \frac{\tilde{\alpha}}{\phi+4}$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_n) = q_l^{(i)}(g_n) = q_j^{(j)}(g_n) = q_l^{(k)}(g_n) = q_l^{(l)}(g_n) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2500 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n)$ = $q_i^{(j)}(g_n)$ = $q_i^{(k)}(g_n)$ = $q_i^{(l)}(g_n)$ = 0.2000; $q_k^{(i)}(g_n)$ = $q_k^{(k)}(g_n)$ = $q_k^{(l)}(g_n)$ = 0.2500; $CS_i(g_n)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_n)$ = $CS_l(g_n)$ = 0; $CS_k(g_n)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $\pi_i^{(i)}(g_n)$ = $\pi_i^{(j)}(g_n)$ = $\pi_i^{(k)}(g_n)$ = $\pi_i^{(l)}(g_n)$ = $(0.2000)^2$ = 0.0400; $\pi_k^{(i)}(g_n)$ = $\pi_k^{(k)}(g_n)$ = $\pi_k^{(l)}(g_n)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_n)$ = $PS_j(g_n)$ = $PS_k(g_n)$ = $PS_l(g_n)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

In this case $q_j^{(i)}(g_n)$ = $q_l^{(i)}(g_n)$ = $q_j^{(j)}(g_n)$ = $q_l^{(k)}(g_n)$ = $q_l^{(l)}(g_n)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1364 & 0 & 0 & -0.0303 & 0 & -0.0303 \\ 0.1296 & 1 & 0 & -0.0370 & 0 & -0.0370 & 0 \\ 0 & -0.0303 & 1 & 0 & -0.0303 & 0 & -0.0303 \\ 0 & -0.0303 & 0 & 1 & 0.1364 & 0 & -0.0303 \\ -0.0370 & 0 & 0 & 0.1296 & 1 & -0.0370 & 0 \\ 0 & -0.0303 & 0 & 0 & -0.0303 & 1 & 0.1364 \\ -0.0370 & 0 & 0 & -0.0370 & 0 & 0.1296 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n)$ = $q_i^{(k)}(g_n)$ = $q_i^{(l)}(g_n)$ = 0.1657; $q_k^{(i)}(g_n)$ = $q_k^{(k)}(g_n)$ = $q_k^{(l)}(g_n)$ = 0.2130; $q_i^{(j)}(g_n)$ = 0.2012; $CS_i(g_n)$ = $\frac{1}{2}(0.1657+0.2012+0.1657+0.1657)^2$ = 0.2438; $CS_j(g_n)$ = $CS_l(g_n)$ = 0; $CS_k(g_n)$ = $\frac{1}{2}(0.2130+0.2130+0.2130)^2$ = 0.2042; $\pi_i^{(i)}(g_n)$ = $\pi_i^{(k)}(g_n)$ = $\pi_i^{(l)}(g_n)$ = $\frac{(2.5)}{2}(0.1657)^2$ = 0.0343; $\pi_k^{(i)}(g_n)$ = $\pi_k^{(k)}(g_n)$ = $\pi_k^{(l)}(g_n)$ =

$\frac{(2.5)}{2}(0.2130)^2$ = 0.0567; $\pi_i^{(j)}(g_n)$ = $\frac{(2.5)}{2}(0.2012)^2$ = 0.0506; $PS_i(g_n)$ = $PS_k(g_n)$ = $PS_l(g_n)$ = $\frac{0.5}{4}(0.1657+0.2130)^2$ = 0.0179; and $PS_j(g_n)$ = $\frac{0.5}{4}(0.2012)^2$ = 0.0051.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_n)$ = $q_l^{(i)}(g_n)$ = $q_j^{(j)}(g_n)$ = $q_l^{(k)}(g_n)$ = $q_l^{(l)}(g_n)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2538 & 0 & 0 & -0.0462 & 0 & -0.0462 \\ 0.2455 & 1 & 0 & -0.0545 & 0 & -0.0545 & 0 \\ 0 & -0.0462 & 1 & 0 & -0.0462 & 0 & -0.0462 \\ 0 & -0.0462 & 0 & 1 & 0.2538 & 0 & -0.0462 \\ -0.0545 & 0 & 0 & 0.2455 & 1 & -0.0545 & 0 \\ 0 & -0.0462 & 0 & 0 & -0.0462 & 1 & 0.2538 \\ -0.0545 & 0 & 0 & -0.0545 & 0 & 0.2455 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1818 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n)$ = $q_i^{(k)}(g_n)$ = $q_i^{(l)}(g_n)$ = 0.1273; $q_k^{(i)}(g_n)$ = $q_k^{(k)}(g_n)$ = $q_k^{(l)}(g_n)$ = 0.1644; $q_i^{(j)}(g_n)$ = 0.1766; $CS_i(g_n)$ = $\frac{1}{2}(0.1273+0.1766+0.1273+0.1273)^2$ = 0.1560; $CS_j(g_n)$ = $CS_l(g_n)$ = 0; $CS_k(g_n)$ = $\frac{1}{2}(0.1644+0.1644+0.1644)^2$ = 0.1216; $\pi_i^{(i)}(g_n)$ = $\pi_i^{(k)}(g_n)$ = $\pi_i^{(l)}(g_n)$ = $\frac{(3.5)}{2}(0.1273)^2$ = 0.0284; $\pi_k^{(i)}(g_n)$ = $\pi_k^{(k)}(g_n)$ = $\pi_k^{(l)}(g_n)$ = $\frac{(3.5)}{2}(0.1644)^2$ = 0.0473; $\pi_i^{(j)}(g_n)$ = $\frac{(3.5)}{2}(0.1766)^2$ = 0.0546; $PS_i(g_n)$ = $PS_k(g_n)$ = $PS_l(g_n)$ = $\frac{1.5}{4}(0.1273+0.1644)^2$ = 0.0319 and $PS_j(g_n)$ = $\frac{1.5}{4}(0.1766)^2$ = 0.0117.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_j^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_l^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_j^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_l^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \\ q_l^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.1667 \\ 0.2500 \\ 0.1250 \\ 0.2000 \\ 0.1667 \\ 0.2000 \\ 0.2500 \\ 0.1250 \\ 0.2000 \\ 0.2500 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n) = q_i^{(j)}(g_n) = q_i^{(k)}(g_n) = q_i^{(l)}(g_n) = 0.2000$; $q_j^{(i)}(g_n) = q_j^{(j)}(g_n) = 0.1667$; $q_k^{(i)}(g_n) = q_k^{(k)}(g_n) = q_k^{(l)}(g_n) = 0.2500$; $q_l^{(i)}(g_n) = q_l^{(k)}(g_n) = q_l^{(l)}(g_n) = 0.1250$; $CS_i(g_n) = \frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2 = 0.3200$; $CS_j(g_n) = \frac{1}{2}(0.1667+0.1667)^2 = 0.0556$; $CS_k(g_n) = \frac{1}{2}(0.2500+0.2500+0.2500)^2 = 0.2813$; $CS_l(g_n) = \frac{1}{2}(0.1250+0.1250+0.1250)^2 = 0.0703$; $\pi_i^{(i)}(g_n) = \pi_i^{(j)}(g_n) = \pi_i^{(k)}(g_n) = \pi_i^{(l)}(g_n) = (0.2000)^2 = 0.0400$; $\pi_j^{(i)}(g_n) = \pi_j^{(j)}(g_n) = (0.1667)^2 = 0.0278$; $\pi_k^{(i)}(g_n) = \pi_k^{(k)}(g_n) = \pi_k^{(l)}(g_n) = (0.2500)^2 = 0.0625$; $\pi_l^{(i)}(g_n) = \pi_l^{(k)}(g_n) = \pi_l^{(l)}(g_n) = (0.1250)^2 = 0.0156$; and $PS_i(g_n) = PS_j(g_n) = PS_k(g_n) = PS_l(g_n) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ *and* $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1364 & 0.1364 & 0.1364 & 0 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 \\ 0.1190 & 1 & 0.1190 & 0.1190 & -0.0476 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1296 & 0.1296 & 1 & 0.1296 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 \\ 0.1296 & 0.1296 & 0.1296 & 1 & 0 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 1 & 0.1364 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 \\ -0.0476 & 0 & -0.0476 & -0.0476 & 0.1190 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & 1 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 \\ -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0.1296 & 1 & 0.1296 & -0.0370 & 0 & -0.0370 \\ -0.0370 & -0.0370 & -0.0370 & 0 & 0 & 0 & 0.1296 & 0.1296 & 1 & -0.0370 & -0.0370 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & 0 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 \\ -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & -0.0370 & 0 & -0.0370 & 0.1296 & 1 & 0.1296 \\ -0.0370 & -0.0370 & -0.0370 & 0 & 0 & 0 & -0.0370 & -0.0370 & 0 & 0.1296 & 0.1296 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_j^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_l^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_j^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_l^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \\ q_l^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1429 \\ 0.2222 \\ 0.1111 \\ 0.1818 \\ 0.1429 \\ 0.1818 \\ 0.2222 \\ 0.1111 \\ 0.1818 \\ 0.2222 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n) = 0.1527$; $q_j^{(i)}(g_n) = 0.1008$; $q_k^{(i)}(g_n) = 0.1980$; $q_l^{(i)}(g_n) = 0.0804$; $q_i^{(j)}(g_n) = 0.1925$; $q_j^{(j)}(g_n) = 0.1405$; $q_i^{(k)}(g_n) = q_i^{(l)}(g_n) = 0.1653$; $q_k^{(k)}(g_n) = q_k^{(l)}(g_n) = 0.2106$; $q_l^{(k)}(g_n) = q_l^{(l)}(g_n) = 0.0930$; $CS_i(g_n) = \frac{1}{2}(0.1527 + 0.1925 + 0.1653 + 0.1653)^2 = 0.2284$; $CS_j(g_n) = \frac{1}{2}(0.1008 + 0.1405)^2 = 0.0291$; $CS_k(g_n) = \frac{1}{2}(0.1980 + 0.2106 + 0.2106)^2 = 0.1917$; $CS_l(g_n) = \frac{1}{2}(0.0804 + 0.0930 + 0.0930)^2 = 0.0355$; $\pi_i^{(i)}(g_n) = \frac{(2.5)}{2}(0.1527)^2 = 0.0291$; $\pi_j^{(i)}(g_n) = \frac{(2.5)}{2}(0.1008)^2 = 0.0127$; $\pi_k^{(i)}(g_n) = \frac{(2.5)}{2}(0.1980)^2 = 0.0490$; $\pi_l^{(i)}(g_n) = \frac{(2.5)}{2}(0.0804)^2 = 0.0081$; $\pi_i^{(j)}(g_n) = \frac{(2.5)}{2}(0.1925)^2 = 0.0463$; $\pi_j^{(j)}(g_n) = \frac{(2.5)}{2}(0.1405)^2 = 0.0247$; $\pi_i^{(k)}(g_n) = \pi_i^{(l)}(g_n) = \frac{(2.5)}{2}(0.1653)^2 = 0.0342$; $\pi_k^{(k)}(g_n) = \pi_k^{(l)}(g_n) = \frac{(2.5)}{2}(0.2106)^2 = 0.0554$; $\pi_l^{(k)}(g_n) = \pi_l^{(l)}(g_n) = \frac{(2.5)}{2}(0.0930)^2 = 0.0108$; $PS_i(g_n) = \frac{0.5}{4}(0.1527 + 0.1008 + 0.1980 + 0.0804)^2 = 0.0354$; $PS_j(g_n) =$

$\frac{0.5}{4}(0.1925+0.1405)^2$ = 0.0139; and $PS_k(g_n)$ = $PS_l(g_n)$ = $\frac{0.5}{4}(0.1653+0.2106+0.0930)^2$ = 0.0275.

Simulation 16: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2538 & 0.2538 & 0.2538 & 0 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 \\ 0.2333 & 1 & 0.2333 & 0.2333 & -0.0667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2455 & 0.2455 & 1 & 0.2455 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 \\ 0.2455 & 0.2455 & 0.2455 & 1 & 0 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 1 & 0.2538 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 \\ -0.0667 & 0 & -0.0667 & -0.0667 & 0.2333 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & 1 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 \\ -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0.2455 & 1 & 0.2455 & -0.0545 & 0 & -0.0545 \\ -0.0545 & -0.0545 & -0.0545 & 0 & 0 & 0 & 0.2455 & 0.2455 & 1 & -0.0545 & -0.0545 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & 0 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 \\ -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & -0.0545 & 0 & -0.0545 & 0.2455 & 1 & 0.2455 \\ -0.0545 & -0.0545 & -0.0545 & 0 & 0 & 0 & -0.0545 & -0.0545 & 0 & 0.2455 & 0.2455 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_n) \\ q_j^{(i)}(g_n) \\ q_k^{(i)}(g_n) \\ q_l^{(i)}(g_n) \\ q_i^{(j)}(g_n) \\ q_j^{(j)}(g_n) \\ q_i^{(k)}(g_n) \\ q_k^{(k)}(g_n) \\ q_l^{(k)}(g_n) \\ q_i^{(l)}(g_n) \\ q_k^{(l)}(g_n) \\ q_l^{(l)}(g_n) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.1111 \\ 0.1818 \\ 0.0909 \\ 0.1538 \\ 0.1111 \\ 0.1538 \\ 0.1818 \\ 0.0909 \\ 0.1538 \\ 0.1818 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_n)$ = 0.1160; $q_j^{(i)}(g_n)$ = 0.0494; $q_k^{(i)}(g_n)$ = 0.1498; $q_l^{(i)}(g_n)$ = 0.0445; $q_i^{(j)}(g_n)$ = 0.1608; $q_j^{(j)}(g_n)$ = 0.0943; $q_i^{(k)}(g_n)$ = $q_i^{(l)}(g_n)$ = 0.1252; $q_k^{(k)}(g_n)$ = $q_k^{(l)}(g_n)$ = 0.1590; $q_l^{(k)}(g_n)$ = $q_l^{(l)}(g_n)$ = 0.0538; $CS_i(g_n)$ = $\frac{1}{2}(0.1160+0.1608+0.1252+0.1252)^2$ = 0.1390; $CS_j(g_n)$ = $\frac{1}{2}(0.0494+0.0943)^2$ = 0.0103; $CS_k(g_n)$ = $\frac{1}{2}(0.1498+0.1590+0.1590)^2$ = 0.1094; $CS_l(g_n)$ = $\frac{1}{2}(0.0445+0.0538+0.0538)^2$ = 0.0116; $\pi_i^{(i)}(g_n)$ = $\frac{(3.5)}{2}(0.1160)^2$ = 0.0235; $\pi_j^{(i)}(g_n)$ = $\frac{(3.5)}{2}(0.0494)^2$ = 0.0043; $\pi_k^{(i)}(g_n)$ = $\frac{(3.5)}{2}(0.1498)^2$ = 0.0393; $\pi_l^{(i)}(g_n)$ = $\frac{(3.5)}{2}(0.0445)^2$ = 0.0035; $\pi_i^{(j)}(g_n)$ = $\frac{(3.5)}{2}(0.1608)^2$ = 0.0452; $\pi_j^{(j)}(g_n)$ =

$\frac{(3.5)}{2}(0.0943)^2$ = 0.0156; $\pi_i^{(k)}(g_n)$ = $\pi_i^{(l)}(g_n)$ = $\frac{(3.5)}{2}(0.1252)^2$ = 0.0274; $\pi_k^{(k)}(g_n)$ = $\pi_k^{(l)}(g_n)$ = $\frac{(3.5)}{2}(0.1590)^2$ = 0.0442; $\pi_l^{(k)}(g_n)$ = $\pi_l^{(l)}(g_n)$ = $\frac{(3.5)}{2}(0.0538)^2$ = 0.0051;

$PS_i(g_n)$ = $\frac{1.5}{4}(0.1160+0.0494+0.1498+0.0445)^2$ = 0.0485; $PS_j(g_n)$ = $\frac{1.5}{4}(0.1608+0.0943)^2$ = 0.0244; and $PS_k(g_n)$ = $PS_l(g_n)$ = $\frac{1.5}{4}(0.1252+0.1590+0.0538)^2$ = 0.0428.

Network o

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_o)$ = $\frac{2\alpha(\phi+1)+4\phi q_j^{(j)}(g_o)+\phi q_k^{(k)}(g_o)-2\phi(\phi+4)q_j^{(i)}(g_o)-\phi(\phi+4)q_k^{(i)}(g_o)}{2(\phi+5)(\phi+1)}$; $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_o)+2\phi q_j^{(j)}(g_o)-\phi(\phi+3)q_i^{(i)}(g_o)-\phi(\phi+3)q_k^{(i)}(g_o)}{3\phi^2+13\phi+8}$;

$q_k^{(i)}(g_o)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_o)-\phi(\phi+2)q_i^{(i)}(g_o)-2\phi(\phi+2)q_j^{(i)}(g_o)}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_o)+\phi q_k^{(i)}(g_o)+\phi q_k^{(k)}(g_o)-\phi(\phi+3)q_j^{(j)}(g_o)}{2(\phi+5)(\phi+1)}$; $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_0)+\phi q_k^{(i)}(g_o)+\phi q_j^{(i)}(g_o)-\phi(\phi+2)q_i^{(j)}(g_o)}{3\phi^2+12\phi+8}$; $q_i^{(k)}(g_o)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_o)+\phi q_k^{(i)}(g_o)+4\phi q_j^{(j)}(g_o)-\phi(\phi+4)q_k^{(k)}(g_o)}{2(\phi+5)(\phi+1)}$; $q_k^{(k)}(g_o)$ = $\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_o)+2\phi q_j^{(i)}(g_o)-\phi(\phi+2)q_i^{(k)}(g_o)}{2(\phi+3)(\phi+1)}$; $CS_i(g_o)$ =

$\frac{1}{2}\left(q_i^{(i)}(g_o)+q_i^{(j)}(g_o)+q_i^{(k)}(g_o)+q_i^{(l)}(g_o)\right)^2$; $CS_j(g_o)$ = $CS_l(g_o)$ = $\frac{1}{2}\left(q_j^{(i)}(g_o)+q_j^{(j)}(g_o)+q_j^{(l)}(g_o)\right)^2$; $CS_k(g_o)$ = $\frac{1}{2}\left(q_k^{(i)}(g_o)+q_k^{(k)}(g_o)\right)^2$; $\pi_i^{(i)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_o)\right)^2$; $\pi_j^{(i)}(g_o)$ = $\pi_l^{(i)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_o)\right)^2$; $\pi_k^{(i)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_k^{(i)}(g_o)\right)^2$; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_o)\right)^2$; $\pi_j^{(j)}(g_o)$ = $\pi_l^{(j)}(g_o)$ = $\pi_j^{(l)}(g_o)$ = $\pi_l^{(l)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_o)\right)^2$; $\pi_i^{(k)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_i^{(k)}(g_o)\right)^2$; $\pi_k^{(k)}(g_o)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_o)\right)^2$; $PS_i(g_o)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_o)+q_j^{(i)}(g_o)+q_k^{(i)}(g_o)+q_l^{(i)}(g_o)\right)^2$; $PS_j(g_o)$ = $PS_l(g_o) = \frac{\phi}{4}\left(q_i^{(j)}(g_o)+q_j^{(j)}(g_o)+q_l^{(j)}(g_o)\right)^2$; and $PS_k(g_o) = \frac{\phi}{4}\left(q_i^{(k)}(g_o)+q_k^{(k)}(g_o)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+4)}{(\phi+5)(\phi+1)} & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{2\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & 1 & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & -\frac{2\phi}{3\phi^2+13\phi+8} & -\frac{2\phi}{3\phi^2+13\phi+8} & 0 & 0 \\ \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & 1 & 0 & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ -\frac{\phi}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 & 0 & 0 \\ 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & -\frac{\phi}{2(\phi+5)(\phi+1)} & 0 & -\frac{2\phi}{(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{(\phi+3)(\phi+1)} & 0 & 0 & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_j^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_j^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\phi+5} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+13\phi+8} \\ \frac{\alpha}{\phi+3} \\ \frac{\alpha}{\phi+5} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\alpha}{\phi+5} \\ \frac{\alpha}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.3333 \\ 0.2000 \\ 0.2000 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = $q_i^{(j)}(g_o)$ = $q_i^{(k)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.2000; $q_k^{(i)}(g_o)$ = $q_k^{(k)}(g_o)$ = 0.3333; $CS_i(g_o)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_o)$ = $CS_l(g_o)$ = 0; $CS_k(g_o)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $\pi_i^{(i)}(g_o)$ = $\pi_i^{(j)}(g_o)$ = $\pi_i^{(k)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $(0.2000)^2$ = 0.0400; $\pi_k^{(i)}(g_o)$ = $\pi_k^{(k)}(g_o)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_o)$ = $PS_j(g_o)$ = $PS_k(g_o)$ = $PS_l(g_o)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

Because in this case $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.1364 & 0 & 0 & -0.0303 \\ 0.1190 & 1 & 0 & -0.0476 & 0 \\ 0 & -0.0303 & 1 & 0 & -0.0303 \\ 0 & -0.0303 & 0 & 1 & 0.1364 \\ -0.0476 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.2857 \\ 0.1818 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = $q_i^{(k)}(g_o)$ = 0.1526; $q_k^{(i)}(g_o)$ = $q_k^{(k)}(g_o)$ = 0.2748; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.1985; $CS_i(g_o)$ = $\frac{1}{2}(0.1526+0.1985+0.1526+0.1985)^2$ = 0.2465; $CS_j(g_o)$ =

$CS_l(g_o)$ = 0; $CS_k(g_o)$ = $\frac{1}{2}(0.2748 + 0.2748)^2$ = 0.1510; $\pi_i^{(i)}(g_o)$ = $\pi_i^{(k)}(g_o)$ = $\frac{(2.5)}{2}(0.1526)^2$ = 0.0291; $\pi_k^{(i)}(g_o)$ = $\pi_k^{(k)}(g_o)$ = $\frac{(2.5)}{2}(0.2748)^2$ = 0.0944; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $\frac{(2.5)}{2}(0.1985)^2$ = 0.0493; $PS_i(g_o)$ = $PS_k(g_o)$ = $\frac{0.5}{4}(0.1526 + 0.2748)^2$ = 0.0228; and $PS_j(g_o) = PS_l(g_o) = \frac{0.5}{4}(0.1985)^2$ = 0.0049.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_o) = q_l^{(i)}(g_o) = q_j^{(j)}(g_o) = q_l^{(j)}(g_o) = q_j^{(l)}(g_o) = q_l^{(l)}(g_o) = 0$, the output matrix becomes:

$$\begin{pmatrix} 1 & 0.2538 & 0 & 0 & -0.0462 \\ 0.2333 & 1 & 0 & -0.0667 & 0 \\ 0 & -0.0462 & 1 & 0 & -0.0462 \\ 0 & -0.0462 & 0 & 1 & 0.2538 \\ -0.0667 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.2222 \\ 0.1538 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = $q_i^{(k)}(g_o)$ = 0.1115; $q_k^{(i)}(g_o)$ = $q_k^{(k)}(g_o)$ = 0.2036; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.1726; $CS_i(g_o) = \frac{1}{2}(0.1115 + 0.1726 + 0.1115 + 0.1726)^2$ = 0.1614; $CS_j(g_o)$ = $CS_l(g_o)$ = 0; $CS_k(g_o)$ = $\frac{1}{2}(0.2036 + 0.2036)^2$ = 0.0829; $\pi_i^{(i)}(g_o)$ = $\pi_i^{(k)}(g_o)$ = $\frac{(3.5)}{2}(0.1115)^2$ = 0.0218; $\pi_k^{(i)}(g_o)$ = $\pi_k^{(k)}(g_o)$ = $\frac{(3.5)}{2}(0.2036)^2$ = 0.0725; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $\frac{(3.5)}{2}(0.1726)^2$ = 0.0521; $PS_i(g_o)$ = $PS_k(g_o)$ = $\frac{1.5}{4}(0.1115 + 0.2036)^2$ = 0.0372; and $PS_j(g_o) = PS_l(g_o) = \frac{1.5}{4}(0.1726)^2$ = 0.0112.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_j^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_j^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.1250 \\ 0.3333 \\ 0.2000 \\ 0.1250 \\ 0.2000 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = $q_i^{(j)}(g_o)$ = $q_i^{(k)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.2000; $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0.1250; $q_k^{(i)}(g_o)$ = $q_k^{(k)}(g_o)$ = 0.3333; $CS_i(g_o)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_o)$ = $CS_l(g_o)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $CS_k(g_o)$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $\pi_i^{(i)}(g_o)$ = $\pi_i^{(j)}(g_o)$ = $\pi_i^{(k)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $(0.2000)^2$ = 0.0400; $\pi_j^{(i)}(g_o)$ = $\pi_l^{(i)}(g_o)$ = $\pi_j^{(j)}(g_o)$ = $\pi_l^{(j)}(g_o)$ = $\pi_j^{(l)}(g_o)$ = $\pi_l^{(l)}(g_o)$ = $(0.1250)^2$ = 0.0156; $\pi_k^{(i)}(g_o)$ = $\pi_k^{(k)}(g_o)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_o) = PS_j(g_o) = PS_k(g_o) = PS_l(g_o) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.2727 & 0.1364 & 0 & -0.1212 & 0 & -0.0303 \\ 0.1148 & 1 & 0.1148 & -0.0656 & -0.0656 & 0 & 0 \\ 0.1190 & 0.2381 & 1 & 0 & 0 & -0.0476 & 0 \\ 0 & -0.0606 & -0.0303 & 1 & 0.2121 & 0 & -0.0303 \\ -0.0339 & -0.0339 & -0.0339 & 0.0847 & 1 & 0 & 0 \\ 0 & -0.0606 & -0.0303 & 0 & -0.1212 & 1 & 0.1364 \\ -0.0476 & -0.0952 & 0 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_j^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_j^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.0984 \\ 0.2857 \\ 0.1818 \\ 0.1017 \\ 0.1818 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = 0.1482; $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = 0.0702; $q_k^{(i)}(g_o)$ = 0.2594; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.1806; $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0.1026; $q_i^{(k)}(g_o)$ = 0.1682; $q_k^{(k)}(g_o)$ = 0.2794; $CS_i(g_o)$ = $\frac{1}{2}(0.1482+0.1806+0.1682+0.1806)^2$ = 0.2296; $CS_j(g_o)$ = $CS_l(g_o)$ = $\frac{1}{2}(0.0702+0.1026+0.1026)^2$ = 0.0379; $CS_k(g_o)$ = $\frac{1}{2}(0.2594+0.2794)^2$ = 0.1452; $\pi_i^{(i)}(g_o)$ = $\frac{(2.5)}{2}(0.1482)^2$ = 0.0275; $\pi_j^{(i)}(g_o)$ = $\pi_l^{(i)}(g_o)$ = $\frac{(2.5)}{2}(0.0702)^2$ = 0.0062; $\pi_k^{(i)}(g_o)$ = $\frac{(2.5)}{2}(0.2594)^2$ = 0.0841; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $\frac{(2.5)}{2}(0.1806)^2$ = 0.0408; $\pi_j^{(j)}(g_o)$ = $\pi_l^{(j)}(g_o)$ = $\pi_j^{(l)}(g_o)$ = $\pi_l^{(l)}(g_o)$ = $\frac{(2.5)}{2}(0.1026)^2$ = 0.0132; $\pi_i^{(k)}(g_o)$ = $\frac{(2.5)}{2}(0.1682)^2$ = 0.0354; $\pi_k^{(k)}(g_o)$ = $\frac{(2.5)}{2}(0.2794)^2$ = 0.0976; $PS_i(g_o)$ = $\frac{0.5}{4}(0.1482+0.0702+0.2594+0.0702)^2$ = 0.0375; $PS_j(g_o)$ = $PS_l(g_o)$ = $\frac{0.5}{4}(0.1806+0.1026+0.1026)^2$ = 0.0186; and $PS_k(g_o)$ = $\frac{0.5}{4}(0.1682+0.2794)^2$ = 0.0250.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.5077 & 0.2538 & 0 & -0.1846 & 0 & -0.0462 \\ 0.1971 & 1 & 0.1971 & -0.0876 & -0.0876 & 0 & 0 \\ 0.2333 & 0.4667 & 1 & 0 & 0 & -0.0667 & 0 \\ 0 & -0.0923 & -0.0462 & 1 & 0.4154 & 0 & -0.0462 \\ -0.0458 & -0.0458 & -0.0458 & 0.1603 & 1 & 0 & 0 \\ 0 & -0.0923 & -0.0462 & 0 & -0.1846 & 1 & 0.2538 \\ -0.0667 & -0.1333 & 0 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_o) \\ q_j^{(i)}(g_o) \\ q_k^{(i)}(g_o) \\ q_i^{(j)}(g_o) \\ q_j^{(j)}(g_o) \\ q_i^{(k)}(g_o) \\ q_k^{(k)}(g_o) \end{pmatrix} = \begin{pmatrix} 0.1538 \\ 0.0730 \\ 0.2222 \\ 0.1538 \\ 0.0763 \\ 0.1538 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_o)$ = 0.1112; $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = 0.0326; $q_k^{(i)}(g_o)$ = 0.1895; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.1467; $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0.0680; $q_i^{(k)}(g_o)$ = 0.1262; $q_k^{(k)}(g_o)$ = 0.2045; $CS_i(g_o)$ = $\frac{1}{2}(0.1112+0.1467+0.1262+0.1467)^2$ = 0.1409; $CS_j(g_o)$ = $CS_l(g_o)$ = $\frac{1}{2}(0.0326+0.0680+0.0680)^2$ = 0.0142; $CS_k(g_o)$ = $\frac{1}{2}(0.1895+0.2045)^2$ = 0.0776; $\pi_i^{(i)}(g_o)$ = $\frac{(3.5)}{2}(0.1112)^2$ = 0.0216; $\pi_j^{(i)}(g_o)$ = $\pi_l^{(i)}(g_o)$ = $\frac{(3.5)}{2}(0.0326)^2$ = 0.0019; $\pi_k^{(i)}(g_o)$ = $\frac{(3.5)}{2}(0.1895)^2$ = 0.0628; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = $\frac{(3.5)}{2}(0.1467)^2$ = 0.0377; $\pi_j^{(j)}(g_o)$ = $\pi_l^{(j)}(g_o)$ = $\pi_j^{(l)}(g_o)$ = $\pi_l^{(l)}(g_o)$ = $\frac{(3.5)}{2}(0.0680)^2$ = 0.0081; $\pi_i^{(k)}(g_o)$ = $\frac{(3.5)}{2}(0.1262)^2$ = 0.0279; $\pi_k^{(k)}(g_o)$ = $\frac{(3.5)}{2}(0.2045)^2$ = 0.0732; $PS_i(g_o)$ = $\frac{1.5}{4}(0.1112+0.0326+0.1895+0.0326)^2$ = 0.0502; $PS_j(g_o)$ = $PS_l(g_o)$ = $\frac{1.5}{4}(0.1467+0.0680+0.0680)^2$ = 0.0300; and $PS_k(g_o)$ = $\frac{1.5}{4}(0.1262+0.2045)^2$ = 0.0410.

Network p

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_p)$ = $q_k^{(i)}(g_p)$ = $q_i^{(k)}(g_p)$ = $q_k^{(k)}(g_p)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_p)-\phi(\phi+2)q_j^{(i)}(g_p)}{3\phi^2+12\phi+8}$; $q_j^{(i)}(g_p)$ = $q_l^{(k)}(g_p)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_p)+2\phi q_j^{(j)}(g_p)-2\phi(\phi+3)q_i^{(i)}(g_p)}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_p)$ = $q_k^{(l)}(g_p)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_p)+2\phi q_i^{(i)}(g_p)-2\phi(\phi+3)q_j^{(j)}(g_p)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_p)$ = $q_l^{(j)}(g_p)$ = $q_j^{(l)}(g_p)$ = $q_l^{(l)}(g_p)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_p)-\phi(\phi+2)q_i^{(j)}(g_p)}{3\phi^2+12\phi+8}$; $CS_i(g_p)$ = $CS_k(g_p)$ = $\frac{1}{2}\left(q_i^{(i)}(g_p)+q_i^{(j)}(g_p)+q_i^{(k)}(g_p)\right)^2$; $CS_j(g_p)$ = $CS_l(g_p)$ = $\frac{1}{2}\left(q_j^{(i)}(g_p)+q_j^{(j)}(g_p)+q_j^{(l)}(g_p)\right)^2$; $\pi_i^{(i)}(g_p)$ = $\pi_k^{(i)}(g_p)$ = $\pi_i^{(k)}(g_p)$ = $\pi_k^{(k)}(g_p)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_p)\right)^2$; $\pi_j^{(i)}(g_p)$ = $\pi_l^{(k)}(g_p)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_p)\right)^2$; $\pi_i^{(j)}(g_p)$ = $\pi_k^{(l)}(g_p)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_p)\right)^2$; $\pi_j^{(j)}(g_p)$ = $\pi_l^{(j)}(g_p)$ = $\pi_j^{(l)}(g_p)$ = $\pi_l^{(l)}(g_p)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_p)\right)^2$; $PS_i(g_p)$ = $PS_k(g_p)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_p)+q_j^{(i)}(g_p)+q_k^{(i)}(g_p)\right)^2$; and $PS_j(g_p)$ = $PS_l(g_p)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_p)+q_j^{(j)}(g_p)+q_l^{(j)}(g_p)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 0 & -\frac{2\phi}{3\phi^2+12\phi+8} \\ \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 1 & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} \\ -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} \\ -\frac{2\phi}{3\phi^2+12\phi+8} & 0 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_p) \\ q_j^{(i)}(g_p) \\ q_i^{(j)}(g_p) \\ q_j^{(j)}(g_p) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+12\phi+8} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_p) = q_j^{(j)}(g_p) = q_l^{(j)}(g_p) = q_l^{(k)}(g_p) = q_j^{(l)}(g_p) = q_l^{(l)}(g_p) = 0$, it holds that $q_i^{(i)}(g_p) = q_k^{(i)}(g_p) = q_i^{(j)}(g_p) = q_i^{(k)}(g_p) = q_k^{(k)}(g_p) = q_k^{(l)}(g_p) = \frac{1}{4} = 0.2500$; $CS_i(g_p) = CS_k(g_p) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$ $CS_j(g_p) = CS_l(g_p) = 0$; $\pi_i^{(i)}(g_p) = \pi_k^{(i)}(g_p) = \pi_i^{(j)}(g_p) = \pi_i^{(k)}(g_p) = \pi_k^{(k)}(g_p) = \pi_k^{(l)}(g_p) = (0.2500)^2 = 0.0625$; and $PS_i(g_p) = PS_j(g_p) = PS_k(g_p) = PS_l(g_p) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_p) = q_j^{(j)}(g_p) = q_l^{(j)}(g_p) = q_l^{(k)}(g_p) = q_j^{(l)}(g_p) = q_l^{(l)}(g_p) = 0$, it holds that $q_i^{(i)}(g_p) = q_k^{(i)}(g_p) = q_i^{(k)}(g_p) = q_k^{(k)}(g_p) = 0.2034$; $q_i^{(j)}(g_p) = q_k^{(l)}(g_p) = 0.2373$; $CS_i(g_p) = CS_k(g_p) = \frac{1}{2}(0.2034 + 0.2373 + 0.2034)^2 = 0.2074$; $CS_j(g_p) = CS_l(g_p) = 0$; $\pi_i^{(i)}(g_p) = \pi_k^{(i)}(g_p) = \pi_i^{(k)}(g_p) = \pi_k^{(k)}(g_p) = \frac{(2.5)}{2}(0.2034)^2 = 0.0517$; $\pi_i^{(j)}(g_p) = \pi_k^{(l)}(g_p) = \frac{(2.5)}{2}(0.2373)^2 = 0.0704$; $PS_i(g_p) = PS_k(g_p) = \frac{0.5}{4}(0.2034 + 0.2034)^2 = 0.0207$; and $PS_j(g_p) = PS_l(g_p) = \frac{0.5}{4}(0.2373)^2 = 0.0070$.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

Because in this case $q_j^{(i)}(g_p)$ = $q_j^{(j)}(g_p)$ = $q_l^{(j)}(g_p)$ = $q_l^{(k)}(g_p)$ = $q_j^{(l)}(g_p)$ = $q_l^{(l)}(g_p)$ = 0, it holds that $q_i^{(i)}(g_p)$ = $q_k^{(i)}(g_p)$ = $q_i^{(k)}(g_p)$ = $q_k^{(k)}(g_p)$ = 0.1527; $q_i^{(j)}(g_p)$ = $q_k^{(l)}(g_p)$ = 0.1985; $CS_i(g_p)$ = $CS_k(g_p)$ = $\frac{1}{2}(0.1527+0.1985+0.1527)^2$ = 0.1270; $CS_j(g_p)$ = $CS_l(g_p)$ = 0; $\pi_i^{(i)}(g_p)$ = $\pi_k^{(i)}(g_p)$ = $\pi_i^{(k)}(g_p)$ = $\pi_k^{(k)}(g_p)$ = $\frac{(3.5)}{2}(0.1527)^2$ = 0.0408; $\pi_i^{(j)}(g_p)$ = $\pi_k^{(l)}(g_p)$ = $\frac{(3.5)}{2}(0.1985)^2$ = 0.0690; $PS_i(g_p)$ = $PS_k(g_p)$ = $\frac{1.5}{4}(0.1527+0.1527)^2$ = 0.0350; and $PS_j(g_p)$ = $PS_l(g_p)$ = $\frac{1.5}{4}(0.1985)^2$ = 0.0148.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_p) \\ q_j^{(i)}(g_p) \\ q_i^{(j)}(g_p) \\ q_j^{(j)}(g_p) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.2500 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_p)$ = $q_k^{(i)}(g_p)$ = $q_i^{(j)}(g_p)$ = $q_i^{(k)}(g_p)$ = $q_k^{(k)}(g_p)$ = $q_k^{(l)}(g_p)$ = 0.2500; $q_j^{(i)}(g_p)$ = $q_j^{(j)}(g_p)$ = $q_l^{(j)}(g_p)$ = $q_l^{(k)}(g_p)$ = $q_j^{(l)}(g_p)$ = $q_l^{(l)}(g_p)$ = 0.1250; $CS_i(g_p)$ = $CS_k(g_p)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_p)$ = $CS_l(g_p)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $\pi_i^{(i)}(g_p)$ = $\pi_k^{(i)}(g_p)$ = $\pi_i^{(j)}(g_p)$ = $\pi_i^{(k)}(g_p)$ =

$\pi_k^{(k)}(g_p) = \pi_k^{(l)}(g_p) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_p) = \pi_j^{(j)}(g_p) = \pi_l^{(j)}(g_p) = \pi_l^{(k)}(g_p) = \pi_j^{(l)}(g_p) = \pi_l^{(l)}(g_p) = (0.1250)^2 = 0.0156$; and $PS_i(g_p) = PS_j(g_p) = PS_k(g_p) = PS_l(g_p) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.0847 & 0 & -0.0678 \\ 0.2593 & 1 & -0.0741 & -0.0741 \\ -0.0741 & -0.0741 & 1 & 0.2593 \\ -0.0678 & 0 & 0.0847 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_p) \\ q_j^{(i)}(g_p) \\ q_i^{(j)}(g_p) \\ q_j^{(j)}(g_p) \end{pmatrix} = \begin{pmatrix} 0.2034 \\ 0.1111 \\ 0.2222 \\ 0.1017 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_p) = q_k^{(i)}(g_p) = q_i^{(k)}(g_p) = q_k^{(k)}(g_p) = 0.2030$; $q_j^{(i)}(g_p) = q_l^{(k)}(g_p) = 0.0818$; $q_i^{(j)}(g_p) = q_k^{(l)}(g_p) = 0.2182$; $q_j^{(j)}(g_p) = q_l^{(j)}(g_p) = q_j^{(l)}(g_p) = q_l^{(l)}(g_p) = 0.0970$; $CS_i(g_p) = CS_k(g_p) = \frac{1}{2}(0.2030 + 0.2182 + 0.2030)^2 = 0.1948$; $CS_j(g_p) = CS_l(g_p) = \frac{1}{2}(0.0818 + 0.0970 + 0.0970)^2 = 0.0380$; $\pi_i^{(i)}(g_p) = \pi_k^{(i)}(g_p) = \pi_i^{(k)}(g_p) = \pi_k^{(k)}(g_p) = \frac{(2.5)}{2}(0.2030)^2 = 0.0515$; $\pi_j^{(i)}(g_p) = \pi_l^{(k)}(g_p) = \frac{(2.5)}{2}(0.0818)^2 = 0.0084$; $\pi_i^{(j)}(g_p) = \pi_k^{(l)}(g_p) = \frac{(2.5)}{2}(0.2182)^2 = 0.0595$; $\pi_j^{(j)}(g_p) = \pi_l^{(j)}(g_p) = \pi_j^{(l)}(g_p) = \pi_l^{(l)}(g_p) = \frac{(2.5)}{2}(0.0970)^2 = 0.0118$; $PS_i(g_p) = PS_k(g_p) = \frac{0.5}{4}(0.2030 + 0.0818 + 0.2030)^2 = 0.0297$; and $PS_j(g_p) = PS_l(g_p) = \frac{0.5}{4}(0.2182 + 0.0970 + 0.0970)^2 = 0.0212$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1603 & 0 & -0.0916 \\ 0.4909 & 1 & -0.1091 & -0.1091 \\ -0.1091 & -0.1091 & 1 & 0.4909 \\ -0.0916 & 0 & 0.1603 & 1 \end{pmatrix}\begin{pmatrix} q_i^{(i)}(g_p) \\ q_j^{(i)}(g_p) \\ q_i^{(j)}(g_p) \\ q_j^{(j)}(g_p) \end{pmatrix} = \begin{pmatrix} 0.1527 \\ 0.0909 \\ 0.1818 \\ 0.0763 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_p) = q_k^{(i)}(g_p) = q_i^{(k)}(g_p) = q_k^{(k)}(g_p) = 0.1517$; $q_j^{(i)}(g_p) = q_l^{(k)}(g_p) = 0.0421$; $q_i^{(j)}(g_p) = q_k^{(l)}(g_p) = 0.1722$; $q_j^{(j)}(g_p) = q_l^{(j)}(g_p) = q_j^{(l)}(g_p) = q_l^{(l)}(g_p) = 0.0626$; $CS_i(g_p) = CS_k(g_p) = \frac{1}{2}(0.1517 + 0.1722 + 0.1517)^2 = 0.1131$; $CS_j(g_p) = CS_l(g_p) = \frac{1}{2}(0.0421 + 0.0626 + 0.0626)^2 = 0.0140$; $\pi_i^{(i)}(g_p) = \pi_k^{(i)}(g_p) = \pi_i^{(k)}(g_p) = \pi_k^{(k)}(g_p) = \frac{(3.5)}{2}(0.1517)^2 = 0.0403$; $\pi_j^{(i)}(g_p) = \pi_l^{(k)}(g_p) = \frac{(3.5)}{2}(0.0421)^2 = 0.0031$; $\pi_i^{(j)}(g_p) = \pi_k^{(l)}(g_p) = \frac{(3.5)}{2}(0.1722)^2 = 0.0519$; $\pi_j^{(j)}(g_p) = \pi_l^{(j)}(g_p) = \pi_j^{(l)}(g_p) = \pi_l^{(l)}(g_p) = \frac{(3.5)}{2}(0.0626)^2 = 0.0069$; $PS_i(g_p) = PS_k(g_p) = \frac{1.5}{4}(0.1517 + 0.0421 + 0.1517)^2 = 0.0448$; and $PS_j(g_p) = PS_l(g_p) = \frac{1.5}{4}(0.1722 + 0.0626 + 0.0626)^2 = 0.0332$.

Network q

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_q) = q_k^{(i)}(g_q) = q_i^{(k)}(g_q) = q_k^{(k)}(g_q) =$

$$\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_g)+\phi q_i^{(j)}(g_q)+\phi q_l^{(j)}(g_q)-\phi(\phi+2)q_j^{(i)}(g_q)}{3\phi^2+12\phi+8}; \quad q_j^{(i)}(g_q) = q_j^{(k)}(g_q) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_q)+\phi q_l^{(j)}(g_q)+\phi q_l^{(l)}(g_q)-2\phi(\phi+3)q_i^{(i)}(g_q)}{2(\phi+5)(\phi+1)}; \quad q_i^{(j)}(g_q) = q_k^{(j)}(g_q) =$$

$$\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_q)+2\phi q_i^{(i)}(g_q)-\phi(\phi+3)q_j^{(j)}(g_q)-\phi(\phi+3)q_l^{(j)}(g_q)}{3\phi^2+13\phi+8}; \quad q_j^{(j)}(g_q) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(i)}(g_q)+\phi q_l^{(l)}(g_q)-2\phi(\phi+4)q_i^{(j)}(g_q)-\phi(\phi+4)q_l^{(j)}(g_q)}{2(\phi+5)(\phi+1)}; \quad q_l^{(j)}(g_q) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_j^{(l)}(g_q)-2\phi(\phi+2)q_i^{(j)}(g_q)-\phi(\phi+2)q_j^{(j)}(g_q)}{2(\phi+3)(\phi+1)}; \quad q_j^{(l)}(g_q) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(i)}(g_q)+2\phi q_i^{(j)}(g_q)+\phi q_l^{(j)}(g_q)-\phi(\phi+4)q_l^{(l)}(g_q)}{2(\phi+5)(\phi+1)}; \quad q_l^{(l)}(g_q) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_q)+\phi q_j^{(j)}(g_q)-\phi(\phi+2)q_j^{(l)}(g_q)}{2(\phi+3)(\phi+1)}; \quad CS_i(g_q) = CS_k(g_q) =$$

$$\frac{1}{2}\left(q_i^{(i)}(g_q)+q_i^{(j)}(g_q)+q_i^{(k)}(g_q)\right)^2; \quad CS_j(g_q) = \frac{1}{2}\left(q_j^{(i)}(g_q)+q_j^{(j)}(g_q)+q_j^{(k)}(g_q)+q_j^{(l)}(g_q)\right)^2;$$

$$CS_l(g_q) = \frac{1}{2}\left(q_l^{(j)}(g_q)+q_l^{(l)}(g_q)\right)^2; \quad \pi_i^{(i)}(g_q) = \pi_k^{(i)}(g_q) = \pi_i^{(k)}(g_q) = \pi_k^{(k)}(g_q) =$$

$$\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_q)\right)^2; \quad \pi_j^{(i)}(g_q) = \pi_j^{(k)}(g_q) = \frac{(2+\phi)}{2}\left(q_j^{(i)}(g_q)\right)^2; \quad \pi_i^{(j)}(g_q) = \pi_k^{(j)}(g_q) =$$

$$\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_q)\right)^2; \quad \pi_j^{(j)}(g_q) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_q)\right)^2; \quad \pi_l^{(j)}(g_q) = \frac{(2+\phi)}{2}\left(q_l^{(j)}(g_q)\right)^2;$$

$$\pi_j^{(l)}(g_q) = \frac{(2+\phi)}{2}\left(q_j^{(l)}(g_q)\right)^2; \quad \pi_l^{(l)}(g_q) = \frac{(2+\phi)}{2}\left(q_l^{(l)}(g_q)\right)^2; \quad PS_i(g_q) = PS_k(g_q) =$$

$$\frac{\phi}{4}\left(q_i^{(i)}(g_q)+q_j^{(i)}(g_q)+q_k^{(i)}(g_q)\right)^2; \quad PS_j(g_q) = \frac{\phi}{4}\left(q_i^{(j)}(g_q)+q_j^{(j)}(g_q)+q_k^{(j)}(g_q)+q_l^{(j)}(g_q)\right)^2;$$

and $PS_l(g_q) = \frac{\phi}{4}\left(q_j^{(l)}(g_q)+q_l^{(l)}(g_q)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & -\frac{\phi}{3\phi^2+12\phi+8} & 0 & 0 \\ \frac{\phi(\phi+3)}{(\phi+5)(\phi+1)} & 1 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ -\frac{2\phi}{3\phi^2+13\phi+8} & -\frac{2\phi}{3\phi^2+13\phi+8} & 1 & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & \frac{\phi(\phi+3)}{3\phi^2+13\phi+8} & 0 & 0 \\ -\frac{2\phi}{(\phi+5)(\phi+1)} & 0 & \frac{\phi(\phi+4)}{(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} \\ 0 & 0 & \frac{\phi(\phi+2)}{(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ -\frac{2\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{(\phi+5)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+5)(\phi+1)} & 1 & \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)} \\ 0 & 0 & -\frac{\phi}{(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_q) \\ q_j^{(i)}(g_q) \\ q_i^{(j)}(g_q) \\ q_j^{(j)}(g_q) \\ q_l^{(j)}(g_q) \\ q_j^{(l)}(g_q) \\ q_l^{(l)}(g_q) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{2\alpha(\phi+1)}{3\phi^2+13\phi+8} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+5} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_q) = q_j^{(j)}(g_q) = q_l^{(j)}(g_q) = q_j^{(k)}(g_q) = q_j^{(l)}(g_q) = q_l^{(l)}(g_q) = 0$. Therefore it holds that $q_i^{(i)}(g_q) = q_k^{(i)}(g_q) = q_i^{(j)}(g_q) = q_k^{(j)}(g_q) = q_i^{(k)}(g_q) = q_k^{(k)}(g_q)$ $= 0.2500$; $CS_i(g_q) = CS_k(g_q) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_q) = CS_l(g_q)$ $= 0$; $\pi_i^{(i)}(g_q) = \pi_k^{(i)}(g_q) = \pi_i^{(j)}(g_q) = \pi_k^{(j)}(g_q) = \pi_i^{(k)}(g_q) = \pi_k^{(k)}(g_q) = (0.2500)^2 =$ 0.0625; $PS_i(g_q) = PS_j(g_q) = PS_k(g_q) = PS_l(g_q) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_q) = q_j^{(j)}(g_q) = q_l^{(j)}(g_q) = q_j^{(k)}(g_q) = q_j^{(l)}(g_q) = q_l^{(l)}(g_q) = 0$. Therefore it holds that $q_i^{(i)}(g_q) = q_k^{(i)}(g_q) = q_i^{(j)}(g_q) = q_k^{(j)}(g_q) = q_i^{(k)}(g_q) = q_k^{(k)}(g_q)$ $= 0.2105$; $CS_i(g_q) = CS_k(g_q) = \frac{1}{2}(0.2105 + 0.2105 + 0.2105)^2 = 0.1994$; $CS_j(g_q) = CS_l(g_q)$ $= 0$; $\pi_i^{(i)}(g_q) = \pi_k^{(i)}(g_q) = \pi_i^{(j)}(g_q) = \pi_k^{(j)}(g_q) = \pi_i^{(k)}(g_q) = \pi_k^{(k)}(g_q) = \frac{(2.5)}{2}(0.2105)^2$

= 0.0554; $PS_i(g_q) = PS_j(g_q) = PS_k(g_q) = \frac{0.5}{4}(0.2105 + 0.2105)^2 = 0.0222$; and $PS_l(g_q) =$ 0.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_q) = q_j^{(j)}(g_q) = q_l^{(j)}(g_q) = q_j^{(k)}(g_q) = q_j^{(l)}(g_q) = q_l^{(l)}(g_q) = 0$. Therefore it holds that $q_i^{(i)}(g_q) = q_k^{(i)}(g_q) = q_i^{(j)}(g_q) = q_k^{(j)}(g_q) = q_i^{(k)}(g_q) = q_k^{(k)}(g_q) = 0.1600$; $CS_i(g_q) = CS_k(g_q) = \frac{1}{2}(0.1600 + 0.1600 + 0.1600)^2 = 0.1152$; $CS_j(g_q) = CS_l(g_q) = 0$; $\pi_i^{(i)}(g_q) = \pi_k^{(i)}(g_q) = \pi_i^{(j)}(g_q) = \pi_k^{(j)}(g_q) = \pi_i^{(k)}(g_q) = \pi_k^{(k)}(g_q) = \frac{(3.5)}{2}(0.1600)^2 = 0.0448$; $PS_i(g_q) = PS_j(g_q) = PS_k(g_q) = \frac{1.5}{4}(0.1600 + 0.1600)^2 = 0.0384$; and $PS_l(g_q) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_q) \\ q_j^{(i)}(g_q) \\ q_i^{(j)}(g_q) \\ q_j^{(j)}(g_q) \\ q_l^{(j)}(g_q) \\ q_j^{(l)}(g_q) \\ q_l^{(l)}(g_q) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1000 \\ 0.2500 \\ 0.1000 \\ 0.1667 \\ 0.1000 \\ 0.1667 \end{pmatrix}$$

Therefore, Therefore, $q_i^{(i)}(g_q)$ = $q_k^{(i)}(g_q)$ = $q_i^{(j)}(g_q)$ = $q_k^{(j)}(g_q)$ = $q_i^{(k)}(g_q)$ = $q_k^{(k)}(g_q)$ = 0.2500; $q_j^{(i)}(g_q)$ = $q_j^{(j)}(g_q)$ = $q_j^{(k)}(g_q)$ = $q_j^{(l)}(g_q)$ = 0.1000; $q_l^{(j)}(g_q)$ = $q_l^{(l)}(g_q)$ = 0.1667; $CS_i(g_q)$ = $CS_k(g_q)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_q)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $CS_l(g_q)$ = $\frac{1}{2}(0.1667+0.1667)^2$ = 0.0556; $\pi_i^{(i)}(g_q)$ = $\pi_k^{(i)}(g_q)$ = $\pi_i^{(j)}(g_q)$ = $\pi_k^{(j)}(g_q)$ = $\pi_i^{(k)}(g_q)$ = $\pi_k^{(k)}(g_q)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_q)$ = $\pi_j^{(j)}(g_q)$ = $\pi_j^{(k)}(g_q)$ = $\pi_j^{(l)}(g_q)$ = $(0.1000)^2$ = 0.0100; $\pi_l^{(j)}(g_q)$ = $\pi_l^{(l)}(g_q)$ = $(0.1667)^2$ = 0.0278; and $PS_i(g_q) = PS_j(g_q) = PS_k(g_q) = PS_l(g_q) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.0847 & -0.0339 & -0.0339 & -0.0339 & 0 & 0 \\ 0.2121 & 1 & -0.0606 & 0 & -0.0303 & 0 & -0.0303 \\ -0.0656 & -0.0656 & 1 & 0.1148 & 0.1148 & 0 & 0 \\ -0.1212 & 0 & 0.2727 & 1 & 0.1364 & 0 & -0.0303 \\ 0 & 0 & 0.2381 & 0.1190 & 1 & -0.0476 & 0 \\ -0.1212 & 0 & -0.0606 & 0 & -0.0303 & 1 & 0.1364 \\ 0 & 0 & -0.0952 & -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_q) \\ q_j^{(i)}(g_q) \\ q_i^{(j)}(g_q) \\ q_j^{(j)}(g_q) \\ q_l^{(j)}(g_q) \\ q_j^{(l)}(g_q) \\ q_l^{(l)}(g_q) \end{pmatrix} = \begin{pmatrix} 0.2034 \\ 0.0909 \\ 0.1967 \\ 0.0909 \\ 0.1429 \\ 0.0909 \\ 0.1429 \end{pmatrix}$$

Therefore, Therefore, $q_i^{(i)}(g_q)$ = $q_k^{(i)}(g_q)$ = $q_i^{(k)}(g_q)$ = $q_k^{(k)}(g_q)$ = 0.2096; $q_j^{(i)}(g_q)$ = $q_j^{(k)}(g_q)$ = 0.0659; $q_i^{(j)}(g_q)$ = $q_k^{(j)}(g_q)$ = 0.1977; $q_j^{(j)}(g_q)$ = 0.0541; $q_l^{(j)}(g_q)$ = 0.0947; $q_j^{(l)}(g_q)$ = 0.1105; $q_l^{(l)}(g_q)$ = 0.1511; $CS_i(g_q)$ = $CS_k(g_q)$ = $\frac{1}{2}(0.2096+0.1977+0.2096)^2$ = 0.1903; $CS_j(g_q)$ =

$\frac{1}{2}(0.0659+0.0541+0.0659+0.1105)^2$ = 0.0439; $CS_l(g_q)$ = $\frac{1}{2}(0.0947+0.1511)^2$ = 0.0302; $\pi_i^{(i)}(g_q)$ = $\pi_k^{(i)}(g_q)$ = $\pi_i^{(k)}(g_q)$ = $\pi_k^{(k)}(g_q)$ = $\frac{(2.5)}{2}(0.2096)^2$ = 0.0549; $\pi_j^{(i)}(g_q)$ = $\pi_j^{(k)}(g_q)$ = $\frac{(2.5)}{2}(0.0659)^2$ = 0.0054; $\pi_i^{(j)}(g_q)$ = $\pi_k^{(j)}(g_q)$ = $\frac{(2.5)}{2}(0.1977)^2$ = 0.0489; $\pi_j^{(j)}(g_q)$ = $\frac{(2.5)}{2}(0.0541)^2$ = 0.0037; $\pi_l^{(j)}(g_q)$ = $\frac{(2.5)}{2}(0.0947)^2$ = 0.0112; $\pi_j^{(l)}(g_q)$ = $\frac{(2.5)}{2}(0.1105)^2$ = 0.0153; $\pi_l^{(l)}(g_q)$ = $\frac{(2.5)}{2}(0.1511)^2$ = 0.0285; $PS_i(g_q)$ = $PS_k(g_q)$ = $\frac{0.5}{4}(0.2096+0.0659+0.2096)^2$ = 0.0294; $PS_j(g_q)$ = $\frac{0.5}{4}(0.1977+0.0541+0.1977+0.0947)^2$ = 0.0370; and $PS_l(g_q)$ = $\frac{0.5}{4}(0.1105+0.1511)^2$ = 0.0086.

Simulation 16: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1603 & -0.0458 & -0.0458 & -0.0458 & 0 & 0 \\ 0.4154 & 1 & -0.0923 & 0 & -0.0462 & 0 & -0.0462 \\ -0.0876 & -0.0876 & 1 & 0.1971 & 0.1971 & 0 & 0 \\ -0.1846 & 0 & 0.5077 & 1 & 0.2538 & 0 & -0.0462 \\ 0 & 0 & 0.4667 & 0.2333 & 1 & -0.0667 & 0 \\ -0.1846 & 0 & -0.0923 & 0 & -0.0462 & 1 & 0.2538 \\ 0 & 0 & -0.1333 & -0.0667 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_q) \\ q_j^{(i)}(g_q) \\ q_i^{(j)}(g_q) \\ q_j^{(j)}(g_q) \\ q_l^{(j)}(g_q) \\ q_j^{(l)}(g_q) \\ q_l^{(l)}(g_q) \end{pmatrix} = \begin{pmatrix} 0.1527 \\ 0.0769 \\ 0.1460 \\ 0.0769 \\ 0.1111 \\ 0.0769 \\ 0.1111 \end{pmatrix}$$

Therefore, Therefore, $q_i^{(i)}(g_q)$ = $q_k^{(i)}(g_q)$ = $q_i^{(k)}(g_q)$ = $q_k^{(k)}(g_q)$ = 0.1574; $q_j^{(i)}(g_q)$ = $q_j^{(k)}(g_q)$ = 0.0324; $q_i^{(j)}(g_q)$ = $q_k^{(j)}(g_q)$ = 0.1495; $q_j^{(j)}(g_q)$ = 0.0245; $q_l^{(j)}(g_q)$ =

0.0418; $q_j^{(l)}(g_q)$ = 0.0936; $q_l^{(l)}(g_q)$ = 0.1108; $CS_i(g_q)$ = $CS_k(g_q)$ = $\frac{1}{2}(0.1575+0.1495+0.1574)^2$ = 0.1078; $CS_j(g_q)$ = $\frac{1}{2}(0.0324+0.0245+0.0324+0.0936)^2$ = 0.0167; $CS_l(g_q)$ = $\frac{1}{2}(0.0418+0.1108)^2$ = 0.0116; $\pi_i^{(i)}(g_q) = \pi_k^{(i)}(g_q) = \pi_i^{(k)}(g_q) = \pi_k^{(k)}(g_q) = \frac{(3.5)}{2}(0.1574)^2$ = 0.0434; $\pi_j^{(i)}(g_q)$ = $\pi_j^{(k)}(g_q)$ = $\frac{(3.5)}{2}(0.0324)^2$ = 0.0018; $\pi_i^{(j)}(g_q)$ = $\pi_k^{(j)}(g_q)$ = $\frac{(3.5)}{2}(0.1495)^2$ = 0.0391; $\pi_j^{(j)}(g_q)$ = $\frac{(3.5)}{2}(0.0245)^2$ = 0.0011; $\pi_l^{(j)}(g_q)$ = $\frac{(3.5)}{2}(0.0418)^2$ = 0.0031; $\pi_j^{(l)}(g_q)$ = $\frac{(3.5)}{2}(0.0936)^2$ = 0.0153; $\pi_l^{(l)}(g_q)$ = $\frac{(3.5)}{2}(0.1108)^2$ = 0.0215; $PS_i(g_q)$ = $PS_k(g_q)$ = $\frac{1.5}{4}(0.1574+0.0324+0.1574)^2$ = 0.0452; $PS_j(g_q)$ = $\frac{1.5}{4}(0.1495+0.0245+0.1495+0.0418)^2$ = 0.0500; and $PS_l(g_q)$ = $\frac{1.5}{4}(0.0936+0.1108)^2$ = 0.0157.

Network r

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_r)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_r)+\phi q_l^{(j)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_k^{(l)}(g_r)+\phi q_l^{(l)}(g_r)-\phi(\phi+3)q_j^{(i)}(g_r)-\phi(\phi+3)q_l^{(i)}(g_r)}{2(\phi+4)(\phi+1)};$$

$q_j^{(i)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_r)+\phi q_l^{(j)}(g_r)+\phi q_i^{(l)}(g_r)+\phi q_k^{(l)}(g_r)+\phi q_l^{(l)}(g_r)-\phi(\phi+3)q_i^{(i)}(g_r)-\phi(\phi+3)q_l^{(i)}(g_n)}{2(\phi+4)(\phi+1)};$$

$q_l^{(i)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_r)+\phi q_j^{(j)}(g_r)+\phi q_k^{(k)}(g_r)+\phi q_i^{(l)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_k^{(l)}(g_r)-\phi(\phi+4)q_i^{(i)}(g_r)-\phi(\phi+4)q_j^{(i)}(g_r)}{2(\phi+5)(\phi+1)};$$

$q_i^{(j)}(g_r)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_r)+\phi q_l^{(i)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_k^{(l)}(g_r)+\phi q_l^{(l)}(g_r)-\phi(\phi+3)q_j^{(j)}(g_r)-\phi(\phi+3)q_l^{(j)}(g_r)}{2(\phi+4)(\phi+1)};$$

$q_j^{(j)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_r)+\phi q_l^{(i)}(g_r)+\phi q_i^{(l)}(g_r)+\phi q_k^{(l)}(g_r)+\phi q_l^{(l)}(g_r)-\phi(\phi+3)q_i^{(j)}(g_r)-\phi(\phi+3)q_l^{(j)}(g_r)}{2(\phi+4)(\phi+1)};$$

$q_l^{(j)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_r)+\phi q_j^{(i)}(g_r)+\phi q_k^{(k)}(g_r)+\phi q_i^{(l)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_k^{(l)}(g_r)-\phi(\phi+4)q_i^{(j)}(g_r)-\phi(\phi+4)q_j^{(j)}(g_r)}{2(\phi+5)(\phi+1)};$$

$q_k^{(k)}(g_r)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(l)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_l^{(l)}(g_r)-\phi(\phi+2)q_l^{(k)}(g_r)}{2(\phi+3)(\phi+1)};$$

$q_l^{(k)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_r)+\phi q_j^{(i)}(g_r)+\phi q_i^{(j)}(g_r)+\phi q_j^{(j)}(g_r)+\phi q_i^{(l)}(g_r)+\phi q_j^{(l)}(g_r)+\phi q_k^{(l)}(g_r)-\phi(\phi+4)q_k^{(k)}(g_r)}{2(\phi+5)(\phi+1)};$$

$q_i^{(l)}(g_r)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_r)+\phi q_l^{(i)}(g_r)+\phi q_j^{(j)}(g_r)+\phi q_l^{(j)}(g_r)-\phi(\phi+3)q_j^{(l)}(g_r)-\phi(\phi+3)q_k^{(l)}(g_r)-\phi(\phi+3)q_l^{(l)}(g_r)}{2(\phi+4)(\phi+1)};$$

$q_j^{(l)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_r)+\phi q_l^{(i)}(g_r)+\phi q_i^{(j)}(g_r)+\phi q_l^{(j)}(g_r) -\phi(\phi+3)q_i^{(l)}(g_r)-\phi(\phi+3)q_k^{(l)}(g_r)-\phi(\phi+3)q_l^{(l)}(g_r)}{2(\phi+4)(\phi+1)};$$

$q_k^{(l)}(g_r)$ =

$$\frac{2\alpha(\phi+1)+\phi q_l^{(k)}(g_r)-\phi(\phi+2)q_i^{(l)}(g_r)-\phi(\phi+2)q_j^{(l)}(g_r)-\phi(\phi+2)q_l^{(l)}(g_r)}{2(\phi+3)(\phi+1)};$$

$q_l^{(l)}(g_r)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_r)+\phi q_j^{(i)}(g_r)+\phi q_i^{(j)}(g_r)+\phi q_j^{(j)}(g_r) +\phi q_k^{(k)}(g_r)-\phi(\phi+4)q_i^{(l)}(g_r)-\phi(\phi+4)q_j^{(l)}(g_r)-\phi(\phi+4)q_k^{(l)}(g_r)}{2(\phi+5)(\phi+1)};$$

$CS_i(g_r)$ = $\frac{1}{2}\left(q_i^{(i)}(g_r)+q_i^{(j)}(g_r)+q_i^{(l)}(g_r)\right)^2$; $CS_j(g_r)$ = $\frac{1}{2}\left(q_j^{(i)}(g_r)+q_j^{(j)}(g_r)+q_j^{(l)}(g_r)\right)^2$; $CS_k(g_r)$ = $\frac{1}{2}\left(q_k^{(k)}(g_r)+q_k^{(l)}(g_r)\right)^2$; $CS_l(g_r)$ = $\frac{1}{2}\left(q_l^{(i)}(g_r)+q_l^{(j)}(g_r)+q_l^{(k)}(g_r)+q_l^{(l)}(g_r)\right)^2$; $\pi_i^{(i)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_r)\right)^2$; $\pi_j^{(i)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_r)\right)^2$; $\pi_l^{(i)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_l^{(i)}(g_r)\right)^2$; $\pi_i^{(j)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_r)\right)^2$; $\pi_j^{(j)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_r)\right)^2$; $\pi_l^{(j)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_l^{(j)}(g_r)\right)^2$; $\pi_k^{(k)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_r)\right)^2$; $\pi_l^{(k)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_l^{(k)}(g_r)\right)^2$; $\pi_i^{(l)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_i^{(l)}(g_r)\right)^2$; $\pi_j^{(l)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_j^{(l)}(g_r)\right)^2$; $\pi_k^{(l)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_k^{(l)}(g_r)\right)^2$; $\pi_l^{(l)}(g_r)$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_r)\right)^2$; $PS_i(g_r)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_r)+q_j^{(i)}(g_r)+q_l^{(i)}(g_r)\right)^2$; $PS_j(g_r)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_r)+q_j^{(j)}(g_r)+q_l^{(j)}(g_r)\right)^2$; $PS_k(g_r)$ = $\frac{\phi}{4}\left(q_k^{(k)}(g_r)+q_l^{(k)}(g_r)\right)^2$; and $PS_l(g_r)$ = $\frac{\phi}{4}\left(q_i^{(l)}(g_r)+q_j^{(l)}(g_r)+q_k^{(l)}(g_r)+q_l^{(l)}(g_r)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & 0 & 0 & -\beta_1 & -\beta_1 & -\beta_1 \\ \beta_0 & 1 & \beta_0 & -\beta_1 & 0 & -\beta_1 & 0 & 0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\ \beta_2 & \beta_2 & 1 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 \\ 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & 0 & 0 & -\beta_1 & -\beta_1 & -\beta_1 \\ -\beta_1 & 0 & -\beta_1 & \beta_0 & 1 & \beta_0 & 0 & 0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\ -\beta_3 & -\beta_3 & 0 & \beta_2 & \beta_2 & 1 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \beta_4 & -\beta_5 & -\beta_5 & 0 & -\beta_5 \\ -\beta_3 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & 0 & \beta_2 & 1 & -\beta_3 & -\beta_3 & -\beta_3 & 0 \\ 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & 0 & 1 & \beta_0 & \beta_0 & \beta_0 \\ -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & 0 & 0 & \beta_0 & 1 & \beta_0 & \beta_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_5 & \beta_4 & \beta_4 & 1 & \beta_4 \\ -\beta_3 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & \beta_2 & \beta_2 & \beta_2 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_j^{(i)}(g_r) \\ q_l^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_j^{(j)}(g_r) \\ q_l^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_l^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_j^{(l)}(g_r) \\ q_k^{(l)}(g_r) \\ q_l^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_8 \\ \beta_6 \\ \beta_7 \\ \beta_9 \\ \beta_8 \end{pmatrix}$$

Where $\beta_0 = \dfrac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; $\beta_1 = \dfrac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_2 = \dfrac{\phi(\phi+4)}{2(\phi+5)(\phi+1)}$; $\beta_3 = \dfrac{\phi}{2(\phi+5)(\phi+1)}$; $\beta_4 = \dfrac{\phi(\phi+2)}{2(\phi+3)(\phi+1)}$; $\beta_5 = \dfrac{\phi}{2(\phi+3)(\phi+1)}$; $\beta_6 = \dfrac{\alpha}{\phi+4}$; $\beta_7 = \dfrac{\tilde{\alpha}}{\phi+4}$; $\beta_8 = \dfrac{\tilde{\alpha}}{\phi+5}$; $\beta_9 = \dfrac{\alpha}{\phi+3}$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_r) = q_l^{(i)}(g_r) = q_j^{(j)}(g_r) = q_l^{(j)}(g_r) = q_l^{(k)}(g_r) = q_j^{(l)}(g_r) = q_l^{(l)}(g_r) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_k^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.2500 \\ 0.3333 \\ 0.2500 \\ 0.3333 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r)$ = $q_i^{(j)}(g_r)$ = $q_i^{(l)}(g_r)$ = 0.2500; $q_k^{(k)}(g_r)$ = $q_k^{(l)}(g_r)$ = 0.3333; $CS_i(g_r)$ = $\frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2$ = 0.2813; $CS_j(g_r)$ = $CS_l(g_r)$ = 0; $CS_k(g_r)$ = $\frac{1}{2}(0.3333 + 0.3333)^2$ = 0.2222; $\pi_i^{(i)}(g_r)$ = $\pi_i^{(j)}(g_r)$ = $\pi_i^{(l)}(g_r)$ = $(0.2500)^2$ = 0.0625; $\pi_k^{(k)}(g_r)$ = $\pi_k^{(l)}(g_r)$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_r) = PS_j(g_r) = PS_k(g_r) = PS_l(g_r) = 0$.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_r)$ = $q_l^{(i)}(g_r)$ = $q_j^{(j)}(g_r)$ = $q_l^{(j)}(g_r)$ = $q_l^{(k)}(g_r)$ = $q_j^{(l)}(g_r)$ = $q_l^{(l)}(g_r)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -0.0370 \\ 0 & 1 & 0 & 0 & -0.0370 \\ 0 & 0 & 1 & -0.0476 & 0 \\ 0 & 0 & 0 & 1 & 0.1296 \\ 0 & 0 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_k^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.2222 \\ 0.2857 \\ 0.2222 \\ 0.2857 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r)$ = $q_i^{(j)}(g_r)$ = 0.2319; $q_k^{(k)}(g_r)$ = 0.2947; $q_i^{(l)}(g_r)$ = 0.1881; $q_k^{(l)}(g_r)$ = 0.2633; $CS_i(g_r)$ = $\frac{1}{2}(0.2319 + 0.2319 + 0.1881)^2$ = 0.2125; $CS_j(g_r)$ = $CS_l(g_r)$ = 0; $CS_k(g_r)$ = $\frac{1}{2}(0.2947 + 0.2633)^2$ = 0.1557; $\pi_i^{(i)}(g_r)$ = $\pi_i^{(j)}(g_r)$ = $\frac{(2.5)}{2}(0.2319)^2$ = 0.0672; $\pi_k^{(k)}(g_r)$ = $\frac{(2.5)}{2}(0.2947)^2$ = 0.1086; $\pi_i^{(l)}(g_r)$ = $\frac{(2.5)}{2}(0.1881)^2$ = 0.0442; $\pi_k^{(l)}(g_r)$ = $\frac{(2.5)}{2}(0.2633)^2$ = 0.0867; $PS_i(g_r) = PS_j(g_r)$ = $\frac{0.5}{4}(0.2319)^2$ = 0.0067; $PS_k(g_r)$ = $\frac{0.5}{4}(0.2947)^2$ = 0.0109; and $PS_l(g_r)$ = $\frac{0.5}{4}(0.1881 + 0.2633)^2$ = 0.0255.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

In this case $q_j^{(i)}(g_r)$ = $q_l^{(i)}(g_r)$ = $q_j^{(j)}(g_r)$ = $q_l^{(j)}(g_r)$ = $q_l^{(k)}(g_r)$ = $q_j^{(l)}(g_r)$ = $q_l^{(l)}(g_r)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -0.0545 \\ 0 & 1 & 0 & 0 & -0.0545 \\ 0 & 0 & 1 & -0.0667 & 0 \\ 0 & 0 & 0 & 1 & 0.2455 \\ 0 & 0 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_k^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r)$ = $q_i^{(j)}(g_r)$ = 0.1922; $q_k^{(k)}(g_r)$ = 0.2312; $q_i^{(l)}(g_r)$ = 0.1350; $q_k^{(l)}(g_r)$ = 0.1907; $CS_i(g_r)$ = $\frac{1}{2}(0.1922 + 0.1922 + 0.1350)^2$ = 0.1349; $CS_j(g_r)$ = $CS_l(g_r)$ = 0; $CS_k(g_r)$ = $\frac{1}{2}(0.2312 + 0.1907)^2$ = 0.0890; $\pi_i^{(i)}(g_r)$ = $\pi_i^{(j)}(g_r)$ = $\frac{(3.5)}{2}(0.1922)^2$ = 0.0646; $\pi_k^{(k)}(g_r)$ = $\frac{(3.5)}{2}(0.2312)^2$ = 0.0935; $\pi_i^{(l)}(g_r)$ = $\frac{(3.5)}{2}(0.1350)^2$ = 0.0319; $\pi_k^{(l)}(g_r)$ = $\frac{(3.5)}{2}(0.1907)^2$ = 0.0636; $PS_i(g_r)$ = $PS_j(g_r)$ = $\frac{1.5}{4}(0.1922)^2$ = 0.0139; $PS_k(g_r)$ = $\frac{1.5}{4}(0.2312)^2$ = 0.0200; and $PS_l(g_r)$ = $\frac{1.5}{4}(0.1350 + 0.1907)^2$ = 0.0398.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_j^{(i)}(g_r) \\ q_l^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_j^{(j)}(g_r) \\ q_l^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_l^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_j^{(l)}(g_r) \\ q_k^{(l)}(g_r) \\ q_l^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.1000 \\ 0.2500 \\ 0.1250 \\ 0.1000 \\ 0.3333 \\ 0.1000 \\ 0.2500 \\ 0.1250 \\ 0.3333 \\ 0.1000 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r) = q_i^{(j)}(g_r) = q_i^{(l)}(g_r) = 0.2500$; $q_j^{(i)}(g_r) = q_j^{(j)}(g_r) = q_j^{(l)}(g_r) = 0.1250$; $q_l^{(i)}(g_r) = q_l^{(j)}(g_r) = q_l^{(k)}(g_r) = q_l^{(l)}(g_r) = 0.1000$; $q_k^{(k)}(g_r) = q_k^{(l)}(g_r) = 0.3333$; $CS_i(g_r) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_r) = \frac{1}{2}(0.1250 + 0.1250 + 0.1250)^2 = 0.0703$; $CS_k(g_r) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_l(g_r) = \frac{1}{2}(0.1000 + 0.1000 + 0.1000 + 0.1000)^2 = 0.0800$; $\pi_i^{(i)}(g_r) = \pi_i^{(j)}(g_r) = \pi_i^{(l)}(g_r) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_r) = \pi_j^{(j)}(g_r) = \pi_j^{(l)}(g_r) = (0.1250)^2 = 0.0156$; $\pi_l^{(i)}(g_r) = \pi_l^{(j)}(g_r) = \pi_l^{(k)}(g_r) = \pi_l^{(l)}(g_r) = (0.1000)^2 = 0.0100$; $\pi_k^{(k)}(g_r) = \pi_k^{(l)}(g_r) = (0.3333)^2 = 0.1111$; and $PS_i(g_r) = PS_j(g_r) = PS_k(g_r) = PS_l(g_r) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1296 & 0.1296 & 0 & -0.0370 & -0.0370 & 0 & 0 & 0 & -0.0370 & -0.0370 & -0.0370 \\ 0.1296 & 1 & 0.1296 & -0.0370 & 0 & -0.0370 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 \\ 0.1364 & 0.1364 & 1 & -0.0303 & -0.0303 & 0 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 \\ 0 & -0.0370 & -0.0370 & 1 & 0.1296 & 0.1296 & 0 & 0 & 0 & -0.0370 & -0.0370 & -0.0370 \\ -0.0370 & 0 & -0.0370 & 0.1296 & 1 & 0.1296 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 \\ -0.0303 & -0.0303 & 0 & 0.1364 & 0.1364 & 1 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.1190 & -0.0476 & -0.0476 & 0 & -0.0476 \\ -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & 0.1364 & 1 & -0.0303 & -0.0303 & -0.0303 & 0 \\ 0 & -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & 0 & 0 & 1 & 0.1296 & 0.1296 & 0.1296 \\ -0.0370 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0.1296 & 1 & 0.1296 & 0.1296 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0476 & 0.1190 & 0.1190 & 1 & 0.1190 \\ -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & 0 & 0.1364 & 0.1364 & 0.1364 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_j^{(i)}(g_r) \\ q_l^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_j^{(j)}(g_r) \\ q_l^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_l^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_j^{(l)}(g_r) \\ q_k^{(l)}(g_r) \\ q_l^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.2222 \\ 0.1111 \\ 0.0909 \\ 0.2222 \\ 0.1111 \\ 0.0909 \\ 0.2857 \\ 0.0909 \\ 0.2222 \\ 0.1111 \\ 0.2857 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r) = q_i^{(j)}(g_r) = 0.2191$; $q_j^{(i)}(g_r) = q_j^{(j)}(g_r) = 0.1014$; $q_l^{(i)}(g_r) = q_l^{(j)}(g_r) = 0.0812$; $q_k^{(k)}(g_r) = 0.2900$; $q_l^{(k)}(g_r) = 0.0862$; $q_i^{(l)}(g_r) = 0.1874$; $q_j^{(l)}(g_r) = 0.0698$; $q_k^{(l)}(g_r) = 0.2533$; $q_l^{(l)}(g_r) = 0.0495$; $CS_i(g_r) = \frac{1}{2}(0.2191 + 0.2191 + 0.1874)^2 = 0.1957$; $CS_j(g_r) = \frac{1}{2}(0.1014 + 0.1014 + 0.0698)^2 = 0.0372$; $CS_k(g_r) = \frac{1}{2}(0.2900 + 0.2533)^2 = 0.1476$; $CS_l(g_r) = \frac{1}{2}(0.0812 + 0.0812 + 0.0862 + 0.0495)^2 = 0.0444$; $\pi_i^{(i)}(g_r) = \pi_i^{(j)}(g_r) = \frac{(2.5)}{2}(0.2191)^2 = 0.0600$; $\pi_j^{(i)}(g_r) = \pi_j^{(j)}(g_r) = \frac{(2.5)}{2}(0.1014)^2 = 0.0129$; $\pi_l^{(i)}(g_r) = \pi_l^{(j)}(g_r) = \frac{(2.5)}{2}(0.0812)^2 = 0.0082$; $\pi_k^{(k)}(g_r) = \frac{(2.5)}{2}(0.2900)^2 = 0.1051$; $\pi_l^{(k)}(g_r) = \frac{(2.5)}{2}(0.0862)^2 = 0.0093$; $\pi_i^{(l)}(g_r) = \frac{(2.5)}{2}(0.1874)^2 = 0.0439$; $\pi_j^{(l)}(g_r) = \frac{(2.5)}{2}(0.0698)^2 = 0.0061$; $\pi_k^{(l)}(g_r) = \frac{(2.5)}{2}(0.2533)^2 = 0.0802$; $\pi_l^{(l)}(g_r) = \frac{(2.5)}{2}(0.0495)^2 = 0.0031$; $PS_i(g_r) = PS_j(g_r) = \frac{0.5}{4}(0.2191 + 0.1014 + 0.0812)^2 = 0.0202$; $PS_k(g_r) = \frac{0.5}{4}(0.2900 + 0.0862)^2 = 0.0177$; and $PS_l(g_r) = \frac{0.5}{4}(0.1874 + 0.0698 + 0.2533 + 0.0495)^2 = 0.0392$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2455 & 0.2455 & 0 & -0.0545 & -0.0545 & 0 & 0 & 0 & -0.0545 & -0.0545 & -0.0545 \\ 0.2455 & 1 & 0.2455 & -0.0545 & 0 & -0.0545 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 \\ 0.2538 & 0.2538 & 1 & -0.0462 & -0.0462 & 0 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 \\ 0 & -0.0545 & -0.0545 & 1 & 0.2455 & 0.2455 & 0 & 0 & 0 & -0.0545 & -0.0545 & -0.0545 \\ -0.0545 & 0 & -0.0545 & 0.2455 & 1 & 0.2455 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 \\ -0.0462 & -0.0462 & 0 & 0.2538 & 0.2538 & 1 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.2333 & -0.0667 & -0.0667 & 0 & -0.0667 \\ -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & 0.2538 & 1 & -0.0462 & -0.0462 & -0.0462 & 0 \\ 0 & -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & 0 & 0 & 1 & 0.2455 & 0.2455 & 0.2455 \\ -0.0545 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0.2455 & 1 & 0.2455 & 0.2455 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0667 & 0.2333 & 0.2333 & 1 & 0.2333 \\ -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & 0 & 0.2538 & 0.2538 & 0.2538 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_r) \\ q_j^{(i)}(g_r) \\ q_l^{(i)}(g_r) \\ q_i^{(j)}(g_r) \\ q_j^{(j)}(g_r) \\ q_l^{(j)}(g_r) \\ q_k^{(k)}(g_r) \\ q_l^{(k)}(g_r) \\ q_i^{(l)}(g_r) \\ q_j^{(l)}(g_r) \\ q_k^{(l)}(g_r) \\ q_l^{(l)}(g_r) \end{pmatrix} = \begin{pmatrix} 0.1818 \\ 0.0909 \\ 0.0769 \\ 0.1818 \\ 0.0909 \\ 0.0769 \\ 0.2222 \\ 0.0769 \\ 0.1818 \\ 0.0909 \\ 0.2222 \\ 0.0769 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_r) = q_i^{(j)}(g_r) = 0.1716$; $q_j^{(i)}(g_r) = q_j^{(j)}(g_r) = 0.0663$; $q_l^{(i)}(g_r) = q_l^{(j)}(g_r) = 0.0540$; $q_k^{(k)}(g_r) = 0.2211$; $q_l^{(k)}(g_r) = 0.0590$; $q_i^{(l)}(g_r) = 0.1375$; $q_j^{(l)}(g_r) = 0.0322$; $q_k^{(l)}(g_r) = 0.1819$; $q_l^{(l)}(g_r) = 0.0199$; $CS_i(g_r) = \frac{1}{2}(0.1716+0.1716+0.1375)^2 = 0.1155$; $CS_j(g_r) = \frac{1}{2}(0.0663+0.0663+0.0322)^2 = 0.0136$; $CS_k(g_r) = \frac{1}{2}(0.2211+0.1819)^2 = 0.0812$; $CS_l(g_r) = \frac{1}{2}(0.0540+0.0540+0.0590+0.0199)^2 = 0.0175$; $\pi_i^{(i)}(g_r) = \pi_i^{(j)}(g_r) = \frac{(3.5)}{2}(0.1716)^2 = 0.0515$; $\pi_j^{(i)}(g_r) = \pi_j^{(j)}(g_r) = \frac{(3.5)}{2}(0.0663)^2 = 0.0077$; $\pi_l^{(i)}(g_r) = \pi_l^{(j)}(g_r) = \frac{(3.5)}{2}(0.0540)^2 = 0.0051$; $\pi_k^{(k)}(g_r) = \frac{(3.5)}{2}(0.2211)^2 = 0.0855$; $\pi_l^{(k)}(g_r) = \frac{(3.5)}{2}(0.0590)^2 = 0.0061$; $\pi_i^{(l)}(g_r) = \frac{(3.5)}{2}(0.1375)^2 = 0.0331$; $\pi_j^{(l)}(g_r) = \frac{(3.5)}{2}(0.0322)^2 = 0.0018$; $\pi_k^{(l)}(g_r) = \frac{(3.5)}{2}(0.1819)^2 = 0.0579$; $\pi_l^{(l)}(g_r) = \frac{(3.5)}{2}(0.0199)^2 = 0.0007$; $PS_i(g_r) = PS_j(g_r) =$

$\frac{1.5}{4}\left(0.1716+0.0663+0.0540\right)^2$ = 0.0320; $PS_k(g_r)$ = $\frac{1.5}{4}\left(0.2211+0.0590\right)^2$ = 0.0294;

and $PS_l(g_r)$ = $\frac{1.5}{4}\left(0.1375+0.0322+0.1819+0.0199\right)^2$ = 0.0518.

Network s

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_s)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_s)+\phi q_l^{(j)}(g_s)+\phi q_k^{(k)}(g_s)+\phi q_l^{(k)}(g_s)+\phi q_j^{(l)}(g_s)+\phi q_k^{(l)}(g_s) +\phi q_l^{(l)}(g_s)-\phi(\phi+4)q_j^{(i)}(g_s)-\phi(\phi+4)q_k^{(i)}(g_s)-\phi(\phi+4)q_l^{(i)}(g_s)}{2(\phi+5)(\phi+1)};$$

$q_j^{(i)}(g_s)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_s)+\phi q_l^{(j)}(g_s)+\phi q_i^{(l)}(g_s)+\phi q_k^{(l)}(g_s)+\phi q_l^{(l)}(g_s) -\phi(\phi+3)q_i^{(i)}(g_s)-\phi(\phi+3)q_k^{(i)}(g_s)-\phi(\phi+3)q_l^{(i)}(g_s)}{2(\phi+4)(\phi+1)};$$

$q_k^{(i)}(g_s)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(k)}(g_s)+\phi q_l^{(k)}(g_s)+\phi q_i^{(l)}(g_s)+\phi q_j^{(l)}(g_s)+\phi q_l^{(l)}(g_s) -\phi(\phi+3)q_i^{(i)}(g_s)-\phi(\phi+3)q_j^{(i)}(g_s)-\phi(\phi+3)q_l^{(i)}(g_s)}{2(\phi+4)(\phi+1)};$$

$q_l^{(i)}(g_s)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_s)+\phi q_j^{(j)}(g_s)+\phi q_i^{(k)}(g_s)+\phi q_k^{(k)}(g_s)+\phi q_i^{(l)}(g_s)+\phi q_j^{(l)}(g_s) +\phi q_k^{(l)}(g_s)-\phi(\phi+4)q_i^{(i)}(g_s)-\phi(\phi+4)q_j^{(i)}(g_s)-\phi(\phi+4)q_k^{(i)}(g_s)}{2(\phi+5)(\phi+1)};$$

$q_i^{(j)}(g_s)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_k^{(k)}(g_s)+\phi q_l^{(k)}(g_s) +\phi q_j^{(l)}(g_s)+\phi q_k^{(l)}(g_s)+\phi q_l^{(l)}(g_s)-\phi(\phi+4)q_j^{(j)}(g_s)-\phi(\phi+4)q_l^{(j)}(g_s)}{2(\phi+5)(\phi+1)};$$

$q_j^{(j)}(g_s)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_i^{(l)}(g_s) +\phi q_k^{(l)}(g_s)+\phi q_l^{(l)}(g_s)-\phi(\phi+3)q_i^{(j)}(g_s)-\phi(\phi+3)q_l^{(j)}(g_s)}{2(\phi+4)(\phi+1)};$$

$q_l^{(j)}(g_s)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_i^{(k)}(g_s)+\phi q_k^{(k)}(g_s) +\phi q_i^{(l)}(g_s)+\phi q_j^{(l)}(g_s)+\phi q_k^{(l)}(g_s)-\phi(\phi+4)q_i^{(j)}(g_s)-\phi(\phi+4)q_j^{(j)}(g_s)}{2(\phi+5)(\phi+1)};$$

$q_i^{(k)}(g_s)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_j^{(j)}(g_s)+\phi q_l^{(j)}(g_s) + \phi q_j^{(l)}(g_s)+\phi q_k^{(l)}(g_s)+\phi q_l^{(l)}(g_s)-\phi(\phi+4)q_k^{(k)}(g_s)-\phi(\phi+4)q_l^{(k)}(g_s)}{2(\phi+5)(\phi+1)}; \quad q_k^{(k)}(g_s) =$$

$$\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_j^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_i^{(l)}(g_s) + \phi q_j^{(l)}(g_s)+\phi q_l^{(l)}(g_s)-\phi(\phi+3)q_i^{(k)}(g_s)-\phi(\phi+3)q_l^{(k)}(g_s)}{2(\phi+4)(\phi+1)}; \quad q_l^{(k)}(g_s) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_i^{(j)}(g_s)+\phi q_j^{(j)}(g_s) + \phi q_i^{(l)}(g_s)+\phi q_j^{(l)}(g_s)+\phi q_k^{(l)}(g_s)-\phi(\phi+4)q_i^{(k)}(g_s)-\phi(\phi+4)q_k^{(k)}(g_s)}{2(\phi+5)(\phi+1)}; \quad q_i^{(l)}(g_s) =$$

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_j^{(j)}(g_s)+\phi q_l^{(j)}(g_s) + \phi q_k^{(k)}(g_s)+\phi q_l^{(k)}(g_s)-\phi(\phi+4)q_j^{(l)}(g_s)-\phi(\phi+4)q_k^{(l)}(g_s)-\phi(\phi+4)q_l^{(l)}(g_s)}{2(\phi+5)(\phi+1)}; \quad q_j^{(l)}(g_s) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_i^{(j)}(g_s) + \phi q_l^{(j)}(g_s)-\phi(\phi+3)q_i^{(l)}(g_s)-\phi(\phi+3)q_k^{(l)}(g_s)-\phi(\phi+3)q_l^{(l)}(g_s)}{2(\phi+4)(\phi+1)}; \quad q_k^{(l)}(g_s) =$$

$$\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_j^{(i)}(g_s)+\phi q_l^{(i)}(g_s)+\phi q_i^{(k)}(g_s) + \phi q_l^{(k)}(g_s)-\phi(\phi+3)q_i^{(l)}(g_s)-\phi(\phi+3)q_j^{(l)}(g_s)-\phi(\phi+3)q_l^{(l)}(g_s)}{2(\phi+4)(\phi+1)}; \quad q_l^{(l)}(g_s) =$$

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_s)+\phi q_j^{(i)}(g_s)+\phi q_k^{(i)}(g_s)+\phi q_i^{(j)}(g_s)+\phi q_j^{(j)}(g_s)+\phi q_i^{(k)}(g_s) + \phi q_k^{(k)}(g_s)-\phi(\phi+4)q_i^{(l)}(g_s)-\phi(\phi+4)q_j^{(l)}(g_s)-\phi(\phi+4)q_k^{(l)}(g_s)}{2(\phi+5)(\phi+1)}; \quad CS_i(g_s) =$$

$$\frac{1}{2}\left(q_i^{(i)}(g_s)+q_i^{(j)}(g_s)+q_i^{(k)}(g_s)+q_i^{(l)}(g_s)\right)^2; \quad CS_j(g_s) = \frac{1}{2}\left(q_j^{(i)}(g_s)+q_j^{(j)}(g_s)+q_j^{(l)}(g_s)\right)^2;$$

$$CS_k(g_s) = \frac{1}{2}\left(q_k^{(i)}(g_s)+q_k^{(k)}(g_s)+q_k^{(l)}(g_s)\right)^2; \quad CS_l(g_s) =$$

$$\frac{1}{2}\left(q_l^{(i)}(g_s)+q_l^{(j)}(g_s)+q_l^{(k)}(g_s)+q_l^{(l)}(g_s)\right)^2; \quad \pi_i^{(i)}(g_s) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_s)\right)^2; \quad \pi_j^{(i)}(g_s) =$$

$$\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_s)\right)^2; \quad \pi_k^{(i)}(g_s) = \frac{(2+\phi)}{2}\left(q_k^{(i)}(g_s)\right)^2; \quad \pi_l^{(i)}(g_s) = \frac{(2+\phi)}{2}\left(q_l^{(i)}(g_s)\right)^2; \quad \pi_i^{(j)}(g_s)$$

$$= \frac{(2+\phi)}{2}\left(q_i^{(j)}(g_s)\right)^2; \quad \pi_j^{(j)}(g_s) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_s)\right)^2; \quad \pi_l^{(j)}(g_s) = \frac{(2+\phi)}{2}\left(q_l^{(j)}(g_s)\right)^2;$$

$\pi_i^{(k)}(g_s) = \frac{(2+\phi)}{2}\left(q_i^{(k)}(g_s)\right)^2$; $\pi_k^{(k)}(g_s) = \frac{(2+\phi)}{2}\left(q_k^{(k)}(g_s)\right)^2$; $\pi_l^{(k)}(g_s) = \frac{(2+\phi)}{2}\left(q_l^{(k)}(g_s)\right)^2$; $\pi_i^{(l)}(g_s) = \frac{(2+\phi)}{2}\left(q_i^{(l)}(g_s)\right)^2$; $\pi_j^{(l)}(g_s) = \frac{(2+\phi)}{2}\left(q_j^{(l)}(g_s)\right)^2$; $\pi_k^{(l)}(g_s) = \frac{(2+\phi)}{2}\left(q_k^{(l)}(g_s)\right)^2$; $\pi_l^{(l)}(g_s) = \frac{(2+\phi)}{2}\left(q_l^{(l)}(g_s)\right)^2$; $PS_i(g_s) = \frac{\phi}{4}\left(q_i^{(i)}(g_s)+q_j^{(i)}(g_s)+q_k^{(i)}(g_s)+q_l^{(i)}(g_s)\right)^2$; $PS_j(g_s) = \frac{\phi}{4}\left(q_i^{(j)}(g_s)+q_j^{(j)}(g_s)+q_l^{(j)}(g_s)\right)^2$; $PS_k(g_s) = \frac{\phi}{4}\left(q_i^{(k)}(g_s)+q_k^{(k)}(g_s)+q_l^{(k)}(g_s)\right)^2$; and $PS_l(g_s) = \frac{\phi}{4}\left(q_i^{(l)}(g_s)+q_j^{(l)}(g_s)+q_k^{(l)}(g_s)+q_l^{(l)}(g_s)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix}
1 & \beta_0 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 \\
\beta_2 & 1 & \beta_2 & \beta_2 & -\beta_3 & 0 & -\beta_3 & 0 & 0 & 0 & -\beta_3 & 0 & -\beta_3 & -\beta_3 \\
\beta_2 & \beta_2 & 1 & \beta_2 & 0 & 0 & 0 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & -\beta_3 \\
\beta_0 & \beta_0 & \beta_0 & 1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 \\
-\beta_3 & 0 & -\beta_3 & -\beta_3 & \beta_2 & 1 & \beta_2 & 0 & 0 & 0 & -\beta_3 & 0 & -\beta_3 & -\beta_3 \\
-\beta_1 & -\beta_1 & -\beta_1 & 0 & \beta_0 & \beta_0 & 1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & -\beta_1 \\
-\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & 0 & 0 & \beta_2 & 1 & \beta_2 & -\beta_3 & -\beta_3 & 0 & -\beta_3 \\
-\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & \beta_0 & \beta_0 & 1 & -\beta_1 & -\beta_1 & -\beta_1 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & \beta_0 \\
-\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & 0 & 0 & \beta_2 & 1 & \beta_2 & \beta_2 \\
-\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & 0 & 0 & -\beta_3 & 0 & -\beta_3 & \beta_2 & \beta_2 & 1 & \beta_2 \\
-\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & \beta_0 & \beta_0 & \beta_0 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_s) \\ q_j^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_l^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_j^{(j)}(g_s) \\ q_l^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_l^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_j^{(l)}(g_s) \\ q_k^{(l)}(g_s) \\ q_l^{(l)}(g_s)
\end{pmatrix}
=
\begin{pmatrix}
\beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_4 \\ \beta_5 \\ \beta_7 \\ \beta_4 \\ \beta_6 \\ \beta_7 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7
\end{pmatrix}$$

Where $\beta_0 = \frac{\phi(\phi+4)}{2(\phi+5)(\phi+1)}$; $\beta_1 = \frac{\phi}{2(\phi+5)(\phi+1)}$; $\beta_2 = \frac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; $\beta_3 = \frac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_4 = \frac{\alpha}{\phi+5}$; $\beta_5 = \frac{\tilde{\alpha}}{\phi+4}$; $\beta_6 = \frac{\alpha}{\phi+4}$; $\beta_7 = \frac{\tilde{\alpha}}{\phi+5}$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_s) = q_l^{(i)}(g_s) = q_j^{(j)}(g_s) = q_l^{(j)}(g_s) = q_l^{(k)}(g_s) = q_j^{(l)}(g_s) = q_l^{(l)}(g_s) =$ 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_k^{(l)}(g_s) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2500 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_s) = q_i^{(j)}(g_s) = q_i^{(k)}(g_s) = q_i^{(l)}(g_s) = 0.2000$; $q_k^{(i)}(g_s) = q_k^{(k)}(g_s) = q_k^{(l)}(g_s) = 0.2500$; $CS_i(g_s) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.3200$; $CS_j(g_s) = CS_l(g_s) = 0$; $CS_k(g_s) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $\pi_i^{(i)}(g_s) = \pi_i^{(j)}(g_s) = \pi_i^{(k)}(g_s) = \pi_i^{(l)}(g_s) = (0.2000)^2 = 0.0400$; $\pi_k^{(i)}(g_s) = \pi_k^{(k)}(g_s) = \pi_k^{(l)}(g_s) = (0.2500)^2 =$ 0.0625; and $PS_i(g_s) = PS_j(g_s) = PS_k(g_s) = PS_l(g_s) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_s) = q_l^{(i)}(g_s) = q_j^{(j)}(g_s) = q_l^{(j)}(g_s) = q_l^{(k)}(g_s) = q_j^{(l)}(g_s) = q_l^{(l)}(g_s) =$ 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1364 & 0 & 0 & -0.0303 & 0 & -0.0303 \\ 0.1296 & 1 & 0 & -0.0370 & 0 & -0.0370 & 0 \\ 0 & -0.0303 & 1 & 0 & -0.0303 & 0 & -0.0303 \\ 0 & -0.0303 & 0 & 1 & 0.1364 & 0 & -0.0303 \\ -0.0370 & 0 & 0 & 0.1296 & 1 & -0.0370 & 0 \\ 0 & -0.0303 & 0 & 0 & -0.0303 & 1 & 0.1364 \\ -0.0370 & 0 & 0 & -0.0370 & 0 & 0.1296 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_k^{(l)}(g_s) \end{pmatrix} \begin{pmatrix} 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_s) = q_i^{(k)}(g_s) = q_i^{(l)}(g_s) = 0.1657$; $q_k^{(i)}(g_s) = q_k^{(k)}(g_s) = q_k^{(l)}(g_s) = 0.2130$; $q_i^{(j)}(g_s) = 0.2012$; $CS_i(g_s) = \frac{1}{2}(0.1657 + 0.2012 + 0.1657 + 0.1657)^2 = 0.2438$; $CS_j(g_s) = CS_l(g_s) = 0$; $CS_k(g_s) = \frac{1}{2}(0.2130 + 0.2130 + 0.2130)^2 = 0.2042$; $\pi_i^{(i)}(g_s) = \pi_i^{(k)}(g_s) = \pi_i^{(l)}(g_s) = \frac{(2.5)}{2}(0.1657)^2 = 0.0343$; $\pi_k^{(i)}(g_s) = \pi_k^{(k)}(g_s) = \pi_k^{(l)}(g_s) = \frac{(2.5)}{2}(0.2130)^2 = 0.0567$; $\pi_i^{(j)}(g_s) = \frac{(2.5)}{2}(0.2012)^2 = 0.0506$; $PS_i(g_s) = PS_k(g_s) = PS_l(g_s) = \frac{0.5}{4}(0.1657 + 0.2130)^2 = 0.0179$; and $PS_j(g_s) = \frac{0.5}{4}(0.2012)^2 = 0.0051$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_s) = q_l^{(i)}(g_s) = q_j^{(j)}(g_s) = q_l^{(j)}(g_s) = q_l^{(k)}(g_s) = q_j^{(l)}(g_s) = q_l^{(l)}(g_s) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2538 & 0 & 0 & -0.0462 & 0 & -0.0462 \\ 0.2455 & 1 & 0 & -0.0545 & 0 & -0.0545 & 0 \\ 0 & -0.0462 & 1 & 0 & -0.0462 & 0 & -0.0462 \\ 0 & -0.0462 & 0 & 1 & 0.2538 & 0 & -0.0462 \\ -0.0545 & 0 & 0 & 0.2455 & 1 & -0.0545 & 0 \\ 0 & -0.0462 & 0 & 0 & -0.0462 & 1 & 0.2538 \\ -0.0545 & 0 & 0 & -0.0545 & 0 & 0.2455 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_k^{(l)}(g_s) \end{pmatrix} \begin{pmatrix} 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1818 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_s) = q_i^{(k)}(g_s) = q_i^{(l)}(g_s)$ = 0.1273; $q_k^{(i)}(g_s) = q_k^{(k)}(g_s) = q_k^{(l)}(g_s)$ = 0.1644; $q_i^{(j)}(g_s)$ = 0.1766; $CS_i(g_s)$ = $\frac{1}{2}(0.1273 + 0.1273 + 0.1273 + 0.1766)^2$ = 0.1560; $CS_j(g_s)$ = $CS_l(g_s)$ = 0; $CS_k(g_s)$ = $\frac{1}{2}(0.1644 + 0.1644 + 0.1644)^2$ = 0.1216; $\pi_i^{(i)}(g_s)$ = $\pi_i^{(k)}(g_s)$ = $\pi_i^{(l)}(g_s)$ = $\frac{3.5}{2}(0.1273)^2$ = 0.0284; $\pi_k^{(i)}(g_s)$ = $\pi_k^{(k)}(g_s)$ = $\pi_k^{(l)}(g_s)$ = $\frac{3.5}{2}(0.1644)^2$ = 0.0473; $\pi_i^{(j)}(g_s)$ = $\frac{3.5}{2}(0.1766)^2$ = 0.0546; $PS_i(g_s) = PS_k(g_s) = PS_l(g_s)$ = $\frac{1.5}{4}(0.1273 + 0.1644)^2$ = 0.0319; $PS_j(g_s)$ = $\frac{1.5}{4}(0.1766)^2$ = 0.0117.

Simulation 14: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_s) \\ q_j^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_l^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_j^{(j)}(g_s) \\ q_l^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_l^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_j^{(l)}(g_s) \\ q_k^{(l)}(g_s) \\ q_l^{(l)}(g_s) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.1250 \\ 0.2500 \\ 0.1000 \\ 0.2000 \\ 0.1250 \\ 0.1000 \\ 0.2000 \\ 0.2500 \\ 0.1000 \\ 0.2000 \\ 0.1250 \\ 0.2500 \\ 0.1000 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_s)$ = $q_i^{(j)}(g_s)$ = $q_i^{(k)}(g_s)$ = $q_i^{(l)}(g_s)$ = 0.2000; $q_j^{(i)}(g_s)$ = $q_j^{(j)}(g_s)$ = $q_j^{(l)}(g_s)$ = 0.1250; $q_k^{(i)}(g_s)$ = $q_k^{(k)}(g_s)$ = $q_k^{(l)}(g_s)$ = 0.2500; $q_l^{(i)}(g_s)$ = $q_l^{(j)}(g_s)$ = $q_l^{(k)}(g_s)$ = $q_l^{(l)}(g_s)$ = 0.1000; $CS_i(g_s)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_s)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $CS_k(g_s)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_l(g_s)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $\pi_i^{(i)}(g_s)$ = $\pi_i^{(j)}(g_s)$ = $\pi_i^{(k)}(g_s)$ = $\pi_i^{(l)}(g_s)$ = $(0.2000)^2$ = 0.0400; $\pi_j^{(i)}(g_s)$ = $\pi_j^{(j)}(g_s)$ = $\pi_j^{(l)}(g_s)$ = $(0.1250)^2$ = 0.0156; $\pi_k^{(i)}(g_s)$ = $\pi_k^{(k)}(g_s)$ = $\pi_k^{(l)}(g_s)$ = $(0.2500)^2$ = 0.0625; $\pi_l^{(i)}(g_s)$ = $\pi_l^{(j)}(g_s)$ = $\pi_l^{(k)}(g_s)$ = $\pi_l^{(l)}(g_s)$ = $(0.1000)^2$ = 0.0100; and $PS_i(g_s) = PS_j(g_s) = PS_k(g_s) = PS_l(g_s) = 0$.

Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix}
1 & 0.1364 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 \\
0.1296 & 1 & 0.1296 & 0.1296 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 \\
0.1296 & 0.1296 & 1 & 0.1296 & 0 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 & -0.0370 & 0 & -0.0370 \\
0.1364 & 0.1364 & 0.1364 & 1 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 \\
0 & -0.0303 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 \\
-0.0370 & 0 & -0.0370 & -0.0370 & 0.1296 & 1 & 0.1296 & 0 & 0 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 \\
-0.0303 & -0.0303 & -0.0303 & 0 & 0.1364 & 0.1364 & 1 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 \\
0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 & -0.0303 \\
-0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0 & 0.1296 & 1 & 0.1296 & -0.0370 & -0.0370 & 0 & -0.0370 \\
-0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & 0.1364 & 0.1364 & 1 & -0.0303 & -0.0303 & -0.0303 & 0 \\
0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 & 0.1364 \\
-0.0370 & 0 & -0.0370 & -0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0 & 0.1296 & 1 & 0.1296 & 0.1296 \\
-0.0370 & -0.0370 & 0 & -0.0370 & 0 & 0 & 0 & -0.0370 & 0 & -0.0370 & 0.1296 & 0.1296 & 1 & 0.1296 \\
-0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & 0.1364 & 0.1364 & 0.1364 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_s) \\ q_j^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_l^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_j^{(j)}(g_s) \\ q_l^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_l^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_j^{(l)}(g_s) \\ q_k^{(l)}(g_s) \\ q_l^{(l)}(g_s)
\end{pmatrix}
=
\begin{pmatrix}
0.1818 \\ 0.1111 \\ 0.2222 \\ 0.0909 \\ 0.1818 \\ 0.1111 \\ 0.0909 \\ 0.1818 \\ 0.2222 \\ 0.0909 \\ 0.1818 \\ 0.1111 \\ 0.2222 \\ 0.0909
\end{pmatrix}$$

Therefore, $q_i^{(i)}(g_s)$ = $q_i^{(l)}(g_s)$ = 0.1595; $q_j^{(i)}(g_s)$ = $q_j^{(l)}(g_s)$ = 0.0817; $q_k^{(i)}(g_s)$ = $q_k^{(l)}(g_s)$ = 0.2030; $q_l^{(i)}(g_s)$ = $q_l^{(l)}(g_s)$ = 0.0642; $q_i^{(j)}(g_s)$ = 0.1848; $q_j^{(j)}(g_s)$ = 0.1071;

$q_l^{(j)}(g_s)$ = 0.0896; $q_i^{(k)}(g_s)$ = 0.1697; $q_k^{(k)}(g_s)$ = 0.2132; $q_l^{(k)}(g_s)$ = 0.0744; $CS_i(g_s)$ = $\frac{1}{2}(0.1595+0.1848+0.1697+0.1595)^2$ = 0.2268; $CS_j(g_s)$ = $\frac{1}{2}(0.0817+0.1071+0.0817)^2$ = 0.0366; $CS_k(g_s)$ = $\frac{1}{2}(0.2030+0.2132+0.2030)^2$ = 0.1917; $CS_l(g_s)$ = $\frac{1}{2}(0.0642+0.0896+0.0744+0.0642)^2$ = 0.0427; $\pi_i^{(i)}(g_s)$ = $\pi_i^{(l)}(g_s)$ = $\frac{(2.5)}{2}(0.1595)^2$ = 0.0318; $\pi_j^{(i)}(g_s)$ = $\pi_j^{(l)}(g_s)$ = $\frac{(2.5)}{2}(0.0817)^2$ = 0.0083; $\pi_k^{(i)}(g_s)$ = $\pi_k^{(l)}(g_s)$ = $\frac{(2.5)}{2}(0.2030)^2$ = 0.0515; $\pi_l^{(i)}(g_s)$ = $\pi_l^{(l)}(g_s)$ = $\frac{(2.5)}{2}(0.0642)^2$ = 0.0052; $\pi_i^{(j)}(g_s)$ = $\frac{(2.5)}{2}(0.1848)^2$ = 0.0427; $\pi_j^{(j)}(g_s)$ = $\frac{(2.5)}{2}(0.1071)^2$ = 0.0143; $\pi_l^{(j)}(g_s)$ = $\frac{(2.5)}{2}(0.0896)^2$ = 0.0100; $\pi_i^{(k)}(g_s)$ = $\frac{(2.5)}{2}(0.1697)^2$ = 0.0360; $\pi_k^{(k)}(g_s)$ = $\frac{(2.5)}{2}(0.2132)^2$ = 0.0568; $\pi_l^{(k)}(g_s)$ = $\frac{(2.5)}{2}(0.0744)^2$ = 0.0069; $PS_i(g_s)$ = $PS_l(g_s)$ = $\frac{0.5}{4}(0.1595+0.0817+0.2030+0.0642)^2$ = 0.0323; $PS_j(g_s)$ = $\frac{0.5}{4}(0.1848+0.1071+0.0896)^2$ = 0.0182; and $PS_k(g_s)$ = $\frac{0.5}{4}(0.1697+0.2132+0.0744)^2$ = 0.0261.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

In this case the output matrix is given by:

$$
\begin{pmatrix}
1 & 0.2538 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 \\
0.2455 & 1 & 0.2455 & 0.2455 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 \\
0.2455 & 0.2455 & 1 & 0.2455 & 0 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 & -0.0545 & 0 & -0.0545 \\
0.2538 & 0.2538 & 0.2538 & 1 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 \\
0 & -0.0462 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 \\
-0.0545 & 0 & -0.0545 & -0.0545 & 0.2455 & 1 & 0.2455 & 0 & 0 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 \\
-0.0462 & -0.0462 & -0.0462 & 0 & 0.2538 & 0.2538 & 1 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 \\
0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 & -0.0462 \\
-0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0 & 0.2455 & 1 & 0.2455 & -0.0545 & -0.0545 & 0 & -0.0545 \\
-0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & 0.2538 & 0.2538 & 1 & -0.0462 & -0.0462 & -0.0462 & 0 \\
0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 & 0.2538 \\
-0.0545 & 0 & -0.0545 & -0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0 & 0.2455 & 1 & 0.2455 & 0.2455 \\
-0.0545 & -0.0545 & 0 & -0.0545 & 0 & 0 & 0 & -0.0545 & 0 & -0.0545 & 0.2455 & 0.2455 & 1 & 0.2455 \\
-0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & 0.2538 & 0.2538 & 0.2538 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_s) \\ q_j^{(i)}(g_s) \\ q_k^{(i)}(g_s) \\ q_l^{(i)}(g_s) \\ q_i^{(j)}(g_s) \\ q_j^{(j)}(g_s) \\ q_l^{(j)}(g_s) \\ q_i^{(k)}(g_s) \\ q_k^{(k)}(g_s) \\ q_l^{(k)}(g_s) \\ q_i^{(l)}(g_s) \\ q_j^{(l)}(g_s) \\ q_k^{(l)}(g_s) \\ q_l^{(l)}(g_s)
\end{pmatrix}
=
\begin{pmatrix}
0.1538 \\ 0.0909 \\ 0.1818 \\ 0.0769 \\ 0.1538 \\ 0.0909 \\ 0.0769 \\ 0.1538 \\ 0.1818 \\ 0.0769 \\ 0.1538 \\ 0.0909 \\ 0.1818 \\ 0.0769
\end{pmatrix}
$$

Therefore, $q_i^{(i)}(g_s) = q_i^{(l)}(g_s) = 0.1215$; $q_j^{(i)}(g_s) = q_j^{(l)}(g_s) = 0.0435$; $q_k^{(i)}(g_s) = q_k^{(l)}(g_s) = 0.1531$; $q_l^{(i)}(g_s) = q_l^{(l)}(g_s) = 0.0346$; $q_i^{(j)}(g_s) = 0.1502$; $q_j^{(j)}(g_s) = 0.0722$; $q_l^{(j)}(g_s) = 0.0633$; $q_i^{(k)}(g_s) = 0.1296$; $q_k^{(k)}(g_s) = 0.1612$; $q_l^{(k)}(g_s) = 0.0427$; $CS_i(g_s) = \frac{1}{2}(0.1215+0.1502+0.1296+0.1215)^2 = 0.1367$; $CS_j(g_s) = \frac{1}{2}(0.0435+0.0722+0.0435)^2 = 0.0127$; $CS_k(g_s) = \frac{1}{2}(0.1531+0.1612+0.1531)^2 = 0.1092$; $CS_l(g_s) = \frac{1}{2}(0.0346+0.0633+0.0427+0.0346)^2 = 0.0153$; $\pi_i^{(i)}(g_s) = \pi_i^{(l)}(g_s) = \frac{(3.5)}{2}(0.1215)^2 = 0.0258$; $\pi_j^{(i)}(g_s) = \pi_j^{(l)}(g_s) = \frac{(3.5)}{2}(0.0435)^2 = 0.0033$; $\pi_k^{(i)}(g_s) = \pi_k^{(l)}(g_s) = \frac{(3.5)}{2}(0.1531)^2 = 0.0410$; $\pi_l^{(i)}(g_s) = \pi_l^{(l)}(g_s) = \frac{(3.5)}{2}(0.0346)^2 = 0.0021$; $\pi_i^{(j)}(g_s) = \frac{(3.5)}{2}(0.1502)^2 = 0.0395$; $\pi_j^{(j)}(g_s) = \frac{(3.5)}{2}(0.0722)^2 = 0.0091$; $\pi_l^{(j)}(g_s) = \frac{(3.5)}{2}(0.0633)^2 = 0.0070$; $\pi_i^{(k)}(g_s) = \frac{(3.5)}{2}(0.1296)^2 = 0.0294$; $\pi_k^{(k)}(g_s) = \frac{(3.5)}{2}(0.1612)^2 = 0.0455$; $\pi_l^{(k)}(g_s) = \frac{(3.5)}{2}(0.0427)^2 = 0.0032$; $PS_i(g_s) = PS_l(g_s) = \frac{1.5}{4}(0.1215+0.0435+0.1531+0.0346)^2 = 0.0466$; $PS_j(g_s) = \frac{1.5}{4}(0.1502+0.0722+0.0633)^2 = 0.0306$; and $PS_k(g_s) = \frac{1.5}{4}(0.1296+0.1612+0.0427)^2 = 0.0417$.

Network t

In considering the equations presented in Sections Section 4.2.1.2 it holds that $q_i^{(i)}(g_t)$ = $q_k^{(i)}(g_t)$ = $q_i^{(k)}(g_t)$ = $q_k^{(k)}(g_t)$ = $\dfrac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_t)+2\phi q_i^{(j)}(g_t)-2\phi(\phi+3)q_j^{(i)}(g_t)}{3\phi^2+15\phi+10}$; $q_j^{(i)}(g_t)$ = $q_l^{(i)}(g_t)$ = $q_j^{(k)}(g_t)$ = $q_l^{(k)}(g_t)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_t)-2\phi(\phi+2)q_i^{(i)}(g_t)}{3\phi^2+12\phi+8}$; $q_i^{(j)}(g_t)$ = $q_k^{(j)}(g_t)$ = $q_i^{(l)}(g_t)$ = $q_k^{(l)}(g_t)$ = $\dfrac{2\alpha(\phi+1)+4\phi q_j^{(i)}(g_t)+2\phi q_i^{(i)}(g_t)-\phi(\phi+3)q_j^{(j)}(g_t)}{3\phi^2+15\phi+10}$; $q_j^{(j)}(g_t)$ = $q_l^{(l)}(g_t)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(i)}(g_t)+2\phi q_j^{(i)}(g_t)-2\phi(\phi+3)q_i^{(j)}(g_t)}{2(\phi+4)(\phi+1)}$; $CS_i(g_t)$ = $CS_k(g_t)$ = $\frac{1}{2}\left(q_i^{(i)}(g_t)+q_i^{(j)}(g_t)+q_i^{(k)}(g_t)+q_i^{(l)}(g_t)\right)^2$; $CS_j(g_t)$ = $CS_l(g_t)$ = $\frac{1}{2}\left(q_j^{(i)}(g_t)+q_j^{(j)}(g_t)+q_j^{(k)}(g_t)\right)^2$; $\pi_i^{(i)}(g_t)$ = $\pi_k^{(i)}(g_t)$ = $\pi_i^{(k)}(g_t)$ = $\pi_k^{(k)}(g_t)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_t)\right)^2$; $\pi_j^{(i)}(g_t)$ = $\pi_l^{(i)}(g_t)$ = $\pi_j^{(k)}(g_t)$ = $\pi_l^{(k)}(g_t)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_t)\right)^2$; $\pi_i^{(j)}(g_t)$ = $\pi_k^{(j)}(g_t)$ = $\pi_i^{(l)}(g_t)$ = $\pi_k^{(l)}(g_t)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_t)\right)^2$; $\pi_j^{(j)}(g_t)$ = $\pi_l^{(l)}(g_t)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_t)\right)^2$; $PS_i(g_t)$ = $PS_k(g_t)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_t)+q_j^{(i)}(g_t)+q_k^{(i)}(g_t)+q_l^{(i)}(g_t)\right)^2$; and $PS_j(g_t)$ = $PS_l(g_t)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_t)+q_j^{(j)}(g_t)+q_k^{(j)}(g_t)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{2\phi(\phi+3)}{3\phi^2+15\phi+10} & -\frac{2\phi}{3\phi^2+15\phi+10} & -\frac{2\phi}{3\phi^2+15\phi+10} \\ \frac{2\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 & -\frac{2\phi}{3\phi^2+12\phi+8} & 0 \\ -\frac{2\phi}{3\phi^2+15\phi+10} & -\frac{4\phi}{3\phi^2+15\phi+10} & 1 & \frac{\phi(\phi+3)}{3\phi^2+15\phi+10} \\ -\frac{2\phi}{(\phi+4)(\phi+1)} & -\frac{2\phi}{(\phi+4)(\phi+1)} & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_t) \\ q_j^{(i)}(g_t) \\ q_i^{(j)}(g_t) \\ q_j^{(j)}(g_t) \end{pmatrix} = \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{\tilde{\alpha}}{\phi+4} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

Because in this case $q_j^{(i)}(g_t) = q_l^{(i)}(g_t) = q_j^{(k)}(g_t) = q_l^{(k)}(g_t) = q_j^{(j)}(g_t) = q_l^{(l)}(g_t) = 0$, it holds: $q_i^{(i)}(g_t) = q_k^{(i)}(g_t) = q_i^{(j)}(g_t) = q_k^{(j)}(g_t) = q_i^{(k)}(g_t) = q_k^{(k)}(g_t) = q_i^{(l)}(g_t) = q_k^{(l)}(g_t) = 0.2000$; $CS_i(g_t) = CS_k(g_t) = \frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2 = 0.3200$; $CS_j(g_t) = CS_l(g_t) = 0$; $\pi_i^{(i)}(g_t) = \pi_k^{(i)}(g_t) = \pi_i^{(j)}(g_t) = \pi_k^{(j)}(g_t) = \pi_i^{(k)}(g_t) = \pi_k^{(k)}(g_t) = \pi_i^{(l)}(g_t) = \pi_k^{(l)}(g_t) = (0.2000)^2 = 0.0400$; and $PS_i(g_t) = PS_j(g_t) = PS_k(g_t) = PS_l(g_t) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

Because in this case $q_j^{(i)}(g_t) = q_l^{(i)}(g_t) = q_j^{(k)}(g_t) = q_l^{(k)}(g_t) = q_j^{(j)}(g_t) = q_l^{(l)}(g_t) = 0$, it holds: $q_i^{(i)}(g_t) = q_k^{(i)}(g_t) = q_i^{(j)}(g_t) = q_k^{(j)}(g_t) = q_i^{(k)}(g_t) = q_k^{(k)}(g_t) = q_i^{(l)}(g_t) = q_k^{(l)}(g_t) = 0.1739$; $CS_i(g_t) = CS_k(g_t) = \frac{1}{2}(0.1739+0.1739+0.1739+0.1739)^2 = 0.2419$; $CS_j(g_t) = CS_l(g_t) = 0$; $\pi_i^{(i)}(g_t) = \pi_k^{(i)}(g_t) = \pi_i^{(j)}(g_t) = \pi_k^{(j)}(g_t) = \pi_i^{(k)}(g_t) = \pi_k^{(k)}(g_t) = \pi_i^{(l)}(g_t) = \pi_k^{(l)}(g_t) = \frac{(2.5)}{2}(0.1739)^2 = 0.0378$; and $PS_i(g_t) = PS_j(g_t) = PS_k(g_t) = PS_l(g_t) = \frac{0.5}{4}(0.1739+0.1739)^2 = 0.0151$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

Because in this case $q_j^{(i)}(g_t)$ = $q_l^{(i)}(g_t)$ = $q_j^{(k)}(g_t)$ = $q_l^{(k)}(g_t)$ = $q_j^{(j)}(g_t)$ = $q_l^{(l)}(g_t)$ =0, it holds: $q_i^{(i)}(g_t)$ = $q_k^{(i)}(g_t)$ = $q_i^{(j)}(g_t)$ = $q_k^{(j)}(g_t)$ = $q_i^{(k)}(g_t)$ = $q_k^{(k)}(g_t)$ = $q_i^{(l)}(g_t)$ = $q_k^{(l)}(g_t)$ = 0.1379; $CS_i(g_t)$ = $CS_k(g_t)$ = $\frac{1}{2}(0.1379+0.1379+0.1379+0.1379)^2$ = 0.1521; $CS_j(g_t)$ = $CS_l(g_t)$ = 0; $\pi_i^{(i)}(g_t)$ = $\pi_k^{(i)}(g_t)$ = $\pi_i^{(j)}(g_t)$ = $\pi_k^{(j)}(g_t)$ = $\pi_i^{(k)}(g_t)$ = $\pi_k^{(k)}(g_t)$ = $\pi_i^{(l)}(g_t)$ = $\pi_k^{(l)}(g_t)$ = $\frac{(3.5)}{2}(0.1379)^2$ = 0.0333; and $PS_i(g_t)$ = $PS_j(g_t)$ = $PS_k(g_t)$ = $PS_l(g_t)$ = $\frac{1.5}{4}(0.1379+0.1379)^2$ = 0.0285.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_t) \\ q_j^{(i)}(g_t) \\ q_i^{(j)}(g_t) \\ q_j^{(j)}(g_t) \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.1250 \\ 0.2000 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_t)$ = $q_k^{(i)}(g_t)$ = $q_i^{(j)}(g_t)$ = $q_k^{(j)}(g_t)$ = $q_i^{(k)}(g_t)$ = $q_k^{(k)}(g_t)$ = $q_i^{(l)}(g_t)$ = $q_k^{(l)}(g_t)$ = 0.2000; $q_j^{(i)}(g_t)$ = $q_l^{(i)}(g_t)$ = $q_j^{(j)}(g_t)$ = $q_j^{(k)}(g_t)$ = $q_l^{(k)}(g_t)$ = $q_l^{(l)}(g_t)$ = 0.1250; $CS_i(g_t)$ = $CS_k(g_t)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_t)$ = $CS_l(g_t)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $\pi_i^{(i)}(g_t)$ = $\pi_k^{(i)}(g_t)$ = $\pi_i^{(j)}(g_t)$ = $\pi_k^{(j)}(g_t)$ = $\pi_i^{(k)}(g_t)$ = $\pi_k^{(k)}(g_t)$ = $\pi_i^{(l)}(g_t)$ = $\pi_k^{(l)}(g_t)$ = $(0.2000)^2$ = 0.0400; $\pi_j^{(i)}(g_t)$ =

$\pi_l^{(i)}(g_t)$ = $\pi_j^{(j)}(g_t)$ = $\pi_j^{(k)}(g_t)$ = $\pi_l^{(k)}(g_t)$ = $\pi_l^{(l)}(g_t)$ = $(0.1250)^2$ = 0.0156; and $PS_i(g_t)$ = $PS_j(g_t)$ = $PS_k(g_t)$ = $PS_l(g_t)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.1918 & -0.0548 & -0.0548 \\ 0.1695 & 1 & -0.0678 & 0 \\ -0.0548 & -0.1096 & 1 & 0.0959 \\ -0.1481 & -0.1481 & 0.2593 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_t) \\ q_j^{(i)}(g_t) \\ q_i^{(j)}(g_t) \\ q_j^{(j)}(g_t) \end{pmatrix} = \begin{pmatrix} 0.1644 \\ 0.1017 \\ 0.1644 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_t)$ = $q_k^{(i)}(g_t)$ = $q_i^{(k)}(g_t)$ = $q_k^{(k)}(g_t)$ = 0.1631; $q_j^{(i)}(g_t)$ = $q_l^{(i)}(g_t)$ = $q_j^{(k)}(g_t)$ = $q_l^{(k)}(g_t)$ = 0.0858; $q_i^{(j)}(g_t)$ = $q_k^{(j)}(g_t)$ = $q_i^{(l)}(g_t)$ = $q_k^{(l)}(g_t)$ = 0.1728; $q_j^{(j)}(g_t)$ = $q_l^{(l)}(g_t)$ = 0.1031; $CS_i(g_t)$ = $CS_k(g_t)$ = $\frac{1}{2}(0.1631+0.1728+0.1631+0.1728)^2$ = 0.2257; $CS_j(g_t)$ = $CS_l(g_t)$ = $\frac{1}{2}(0.0858+0.1031+0.0858)^2$ = 0.0377; $\pi_i^{(i)}(g_t)$ = $\pi_k^{(i)}(g_t)$ = $\pi_i^{(k)}(g_t)$ = $\pi_k^{(k)}(g_t)$ = $\frac{(2.5)}{2}(0.1631)^2$ = 0.0333; $\pi_j^{(i)}(g_t)$ = $\pi_l^{(i)}(g_t)$ = $\pi_j^{(k)}(g_t)$ = $\pi_l^{(k)}(g_t)$ = $\frac{(2.5)}{2}(0.0858)^2$ = 0.0092; $\pi_i^{(j)}(g_t)$ = $\pi_k^{(j)}(g_t)$ = $\pi_i^{(l)}(g_t)$ = $\pi_k^{(l)}(g_t)$ = $\frac{(2.5)}{2}(0.1728)^2$ = 0.0373; $\pi_j^{(j)}(g_t)$ = $\pi_l^{(l)}(g_t)$ = $\frac{(2.5)}{2}(0.1031)^2$ = 0.0133; $PS_i(g_t)$ = $PS_k(g_t)$ = $\frac{0.5}{4}(0.1631+0.0858+0.1631+0.0858)^2$ = 0.0310; and $PS_j(g_t)$ = $PS_l(g_t)$ = $\frac{0.5}{4}(0.1728+0.1031+0.1728)^2$ = 0.0252.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

The output matrix in this case is given by:

$$\begin{pmatrix} 1 & 0.3439 & -0.0764 & -0.0764 \\ 0.3206 & 1 & -0.0916 & 0 \\ -0.0764 & -0.1529 & 1 & 0.1720 \\ -0.2182 & -0.2182 & 0.4909 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_t) \\ q_j^{(i)}(g_t) \\ q_i^{(j)}(g_t) \\ q_j^{(j)}(g_t) \end{pmatrix} = \begin{pmatrix} 0.1274 \\ 0.0763 \\ 0.1274 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_t) = q_k^{(i)}(g_t) = q_i^{(k)}(g_t) = q_k^{(k)}(g_t) = 0.1259$; $q_j^{(i)}(g_t) = q_l^{(i)}(g_t) = q_j^{(k)}(g_t) = q_l^{(k)}(g_t) = 0.0482$; $q_i^{(j)}(g_t) = q_k^{(j)}(g_t) = q_i^{(l)}(g_t) = q_k^{(l)}(g_t) = 0.1335$; $q_j^{(j)}(g_t) = q_l^{(l)}(g_t) = 0.0633$; $CS_i(g_t) = CS_k(g_t) = \frac{1}{2}(0.1259 + 0.1335 + 0.1259 + 0.1335)^2 = 0.1346$; $CS_j(g_t) = CS_l(g_t) = \frac{1}{2}(0.0482 + 0.0633 + 0.0482)^2 = 0.0128$; $\pi_i^{(i)}(g_t) = \pi_k^{(i)}(g_t) = \pi_i^{(k)}(g_t) = \pi_k^{(k)}(g_t) = \frac{(3.5)}{2}(0.1259)^2 = 0.0277$; $\pi_j^{(i)}(g_t) = \pi_l^{(i)}(g_t) = \pi_j^{(k)}(g_t) = \pi_l^{(k)}(g_t) = \frac{(3.5)}{2}(0.0482)^2 = 0.0041$; $\pi_i^{(j)}(g_t) = \pi_k^{(j)}(g_t) = \pi_i^{(l)}(g_t) = \pi_k^{(l)}(g_t) = \frac{(3.5)}{2}(0.1335)^2 = 0.0312$; $\pi_j^{(j)}(g_t) = \pi_l^{(l)}(g_t) = \frac{(3.5)}{2}(0.0633)^2 = 0.0070$; $PS_i(g_t) = PS_k(g_t) = \frac{1.5}{4}(0.1259 + 0.0482 + 0.1259 + 0.0482)^2 = 0.0455$; and $PS_j(g_t) = PS_l(g_t) = \frac{1.5}{4}(0.1335 + 0.0633 + 0.1335)^2 = 0.0409$.

Network u

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_u)+\phi q_k^{(j)}(g_u)+\phi q_l^{(j)}(g_u)+\phi q_j^{(k)}(g_u)+\phi q_k^{(k)}(g_u) +\phi q_j^{(l)}(g_u)+\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_j^{(i)}(g_u)-\phi(\phi+4)q_k^{(i)}(g_u)-\phi(\phi+4)q_l^{(i)}(g_u)}{2(\phi+5)(\phi+1)};$$

$q_j^{(i)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_u)+\phi q_k^{(j)}(g_u)+\phi q_l^{(j)}(g_u)+\phi q_i^{(k)}(g_u)+\phi q_k^{(k)}(g_u) +\phi q_i^{(l)}(g_u)+\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_i^{(i)}(g_u)-\phi(\phi+4)q_k^{(i)}(g_u)-\phi(\phi+4)q_l^{(i)}(g_u)}{2(\phi+5)(\phi+1)};$$

$q_k^{(i)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_u)+\phi q_j^{(j)}(g_u)+\phi q_l^{(j)}(g_u)+\phi q_i^{(k)}(g_u)+\phi q_j^{(k)}(g_u) -\phi(\phi+3)q_i^{(i)}(g_u)-\phi(\phi+3)q_j^{(i)}(g_u)-\phi(\phi+3)q_l^{(i)}(g_u)}{2(\phi+4)(\phi+1)};$$

$q_l^{(i)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_u)+\phi q_j^{(j)}(g_u)+\phi q_k^{(j)}(g_u)+\phi q_i^{(l)}(g_u)+\phi q_j^{(l)}(g_u) -\phi(\phi+3)q_i^{(i)}(g_u)-\phi(\phi+3)q_j^{(i)}(g_u)-\phi(\phi+3)q_k^{(i)}(g_u)}{2(\phi+4)(\phi+1)};$$

$q_i^{(j)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_j^{(k)}(g_u)+\phi q_k^{(k)}(g_u)+\phi q_j^{(l)}(g_u) +\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_j^{(j)}(g_u)-\phi(\phi+4)q_k^{(j)}(g_u)-\phi(\phi+4)q_l^{(j)}(g_u)}{2(\phi+5)(\phi+1)};$$

$q_j^{(j)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(k)}(g_u)+\phi q_k^{(k)}(g_u)+\phi q_i^{(l)}(g_u) +\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_i^{(j)}(g_u)-\phi(\phi+4)q_k^{(j)}(g_u)-\phi(\phi+4)q_l^{(j)}(g_u)}{2(\phi+5)(\phi+1)};$$

$q_k^{(j)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_j^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(k)}(g_u)+\phi q_j^{(k)}(g_u) -\phi(\phi+3)q_i^{(j)}(g_u)-\phi(\phi+3)q_j^{(j)}(g_u)-\phi(\phi+3)q_l^{(j)}(g_u)}{2(\phi+4)(\phi+1)};$$

$q_l^{(j)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_j^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_i^{(l)}(g_u)+\phi q_j^{(l)}(g_u) -\phi(\phi+3)q_i^{(j)}(g_u)-\phi(\phi+3)q_j^{(j)}(g_u)-\phi(\phi+3)q_k^{(j)}(g_u)}{2(\phi+4)(\phi+1)};$$

$q_i^{(k)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(j)}(g_u)+\phi q_k^{(j)}(g_u) +\phi q_l^{(j)}(g_u)+\phi q_j^{(l)}(g_u)+\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_j^{(k)}(g_u)-\phi(\phi+4)q_k^{(k)}(g_u)}{2(\phi+5)(\phi+1)};$$

$q_j^{(k)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(j)}(g_u)+\phi q_k^{(j)}(g_u) + \phi q_l^{(j)}(g_u)+\phi q_i^{(l)}(g_u)+\phi q_l^{(l)}(g_u)-\phi(\phi+4)q_i^{(k)}(g_u)-\phi(\phi+4)q_k^{(k)}(g_u)}{2(\phi+5)(\phi+1)};$$ $q_k^{(k)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_j^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(j)}(g_u) + \phi q_j^{(j)}(g_u)+\phi q_l^{(j)}(g_u)-\phi(\phi+3)q_i^{(k)}(g_u)-\phi(\phi+3)q_j^{(k)}(g_u)}{2(\phi+4)(\phi+1)};$$ $q_i^{(l)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_j^{(j)}(g_u)+\phi q_k^{(j)}(g_u) + \phi q_l^{(j)}(g_u)+\phi q_j^{(k)}(g_u)+\phi q_k^{(k)}(g_u)-\phi(\phi+4)q_j^{(l)}(g_u)-\phi(\phi+4)q_l^{(l)}(g_u)}{2(\phi+5)(\phi+1)};$$ $q_j^{(l)}(g_u)$ =

$$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_l^{(i)}(g_u)+\phi q_i^{(j)}(g_u)+\phi q_k^{(j)}(g_u) + \phi q_l^{(j)}(g_u)+\phi q_i^{(k)}(g_u)+\phi q_k^{(k)}(g_u)-\phi(\phi+4)q_i^{(l)}(g_u)-\phi(\phi+4)q_l^{(l)}(g_u)}{2(\phi+5)(\phi+1)};$$ $q_l^{(l)}(g_u)$ =

$$\frac{2\alpha(\phi+1)+\phi q_i^{(i)}(g_u)+\phi q_j^{(i)}(g_u)+\phi q_k^{(i)}(g_u)+\phi q_i^{(j)}(g_u) + \phi q_j^{(j)}(g_u)+\phi q_k^{(j)}(g_u)-\phi(\phi+3)q_i^{(l)}(g_u)-\phi(\phi+3)q_j^{(l)}(g_u)}{2(\phi+4)(\phi+1)};$$ $CS_i(g_u)$ =

$\frac{1}{2}\left(q_i^{(i)}(g_u)+q_i^{(j)}(g_u)+q_i^{(k)}(g_u)+q_i^{(l)}(g_u)\right)^2$; $CS_j(g_u)$ =

$\frac{1}{2}\left(q_j^{(i)}(g_u)+q_j^{(j)}(g_u)+q_j^{(k)}(g_u)+q_j^{(l)}(g_u)\right)^2$; $CS_k(g_u) = \frac{1}{2}\left(q_k^{(i)}(g_u)+q_k^{(j)}(g_u)+q_k^{(k)}(g_u)\right)^2$;

$CS_l(g_u) = \frac{1}{2}\left(q_l^{(i)}(g_u)+q_l^{(j)}(g_u)+q_l^{(l)}(g_u)\right)^2$; $\pi_i^{(i)}(g_u) = \frac{(2+\phi)}{2}\left(q_i^{(i)}(g_u)\right)^2$; $\pi_j^{(i)}(g_u)$ =

$\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_u)\right)^2$; $\pi_k^{(i)}(g_u) = \frac{(2+\phi)}{2}\left(q_k^{(i)}(g_u)\right)^2$; $\pi_l^{(i)}(g_u) = \frac{(2+\phi)}{2}\left(q_l^{(i)}(g_u)\right)^2$; $\pi_i^{(j)}(g_u)$

= $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_u)\right)^2$; $\pi_j^{(j)}(g_u) = \frac{(2+\phi)}{2}\left(q_j^{(j)}(g_u)\right)^2$; $\pi_k^{(j)}(g_u) = \frac{(2+\phi)}{2}\left(q_k^{(j)}(g_u)\right)^2$;

$\pi_l^{(j)}(g_u) = \frac{(2+\phi)}{2}\left(q_l^{(j)}(g_u)\right)^2$; $\pi_i^{(k)}(g_u) = \frac{(2+\phi)}{2}\left(q_i^{(k)}(g_u)\right)^2$; $\pi_j^{(k)}(g_u)$ =

$\frac{(2+\phi)}{2}\left(q_j^{(k)}(g_u)\right)^2$; $\pi_k^{(k)}(g_u) = \frac{(2+\phi)}{2}\left(q_k^{(k)}(g_u)\right)^2$; $\pi_i^{(l)}(g_u) = \frac{(2+\phi)}{2}\left(q_i^{(l)}(g_u)\right)^2$;

$\pi_j^{(l)}(g_u) = \frac{(2+\phi)}{2}\left(q_j^{(l)}(g_u)\right)^2$; $\pi_l^{(l)}(g_u) = \frac{(2+\phi)}{2}\left(q_l^{(l)}(g_u)\right)^2$; $PS_i(g_u)$ =

$\dfrac{\phi}{4}\left(q_i^{(i)}(g_u)+q_j^{(i)}(g_u)+q_k^{(i)}(g_u)+q_l^{(i)}(g_u)\right)^2$; $PS_j(g_u)$ = $\dfrac{\phi}{4}(q_i^{(j)}(g_u)$ + $q_j^{(j)}(g_u)$ + $q_k^{(j)}(g_u)$ + $q_l^{(j)}(g_u))^2$; $PS_k(g_u)$ $=\dfrac{\phi}{4}\left(q_i^{(k)}(g_u)+q_j^{(k)}(g_u)+q_k^{(k)}(g_u)\right)^2$; and $PS_l(g_u)$ = $\dfrac{\phi}{4}\left(q_i^{(l)}(g_u)+q_j^{(l)}(g_u)+q_l^{(l)}(g_u)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$
\begin{pmatrix}
1 & \beta_0 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\
\beta_0 & 1 & \beta_0 & \beta_0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 \\
\beta_2 & \beta_2 & 1 & \beta_2 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & 0 & 0 & 0 \\
\beta_2 & \beta_2 & \beta_2 & 1 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & 0 & 0 & 0 & -\beta_3 & -\beta_3 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 \\
-\beta_1 & 0 & -\beta_1 & -\beta_1 & \beta_0 & 1 & \beta_0 & \beta_0 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 0 & -\beta_1 \\
-\beta_3 & -\beta_3 & 0 & -\beta_3 & \beta_2 & \beta_2 & 1 & \beta_2 & -\beta_3 & -\beta_3 & 0 & 0 & 0 & 0 \\
-\beta_3 & -\beta_3 & -\beta_3 & 0 & \beta_2 & \beta_2 & \beta_2 & 1 & 0 & 0 & 0 & -\beta_3 & -\beta_3 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & -\beta_1 & -\beta_1 \\
-\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & \beta_0 & 1 & \beta_0 & -\beta_1 & 0 & -\beta_1 \\
-\beta_3 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & \beta_2 & \beta_2 & 1 & 0 & 0 & 0 \\
0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & 1 & \beta_0 & \beta_0 \\
-\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & -\beta_1 & -\beta_1 & 0 & -\beta_1 & \beta_0 & 1 & \beta_0 \\
-\beta_3 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & -\beta_3 & 0 & 0 & 0 & 0 & \beta_2 & \beta_2 & 1
\end{pmatrix}
\begin{pmatrix}
q_i^{(i)}(g_u) \\ q_j^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_l^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_j^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_l^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_j^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \\ q_j^{(l)}(g_u) \\ q_l^{(l)}(g_u)
\end{pmatrix}
\begin{pmatrix}
\beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_4 \\ \beta_5 \\ \beta_7
\end{pmatrix}
$$

Where β_0 = $\dfrac{\phi(\phi+4)}{2(\phi+5)(\phi+1)}$; β_1 = $\dfrac{\phi}{2(\phi+5)(\phi+1)}$; β_2 = $\dfrac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; β_3 = $\dfrac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_4 = \dfrac{\alpha}{\phi+5}$; $\beta_5 = \dfrac{\tilde{\alpha}}{\phi+5}$; $\beta_6 = \dfrac{\alpha}{\phi+4}$; $\beta_7 = \dfrac{\tilde{\alpha}}{\phi+4}$.

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_u)$ = $q_l^{(i)}(g_u)$ = $q_j^{(j)}(g_u)$ = $q_l^{(j)}(g_u)$ = $q_j^{(k)}(g_u)$ = $q_j^{(l)}(g_u)$ = $q_l^{(l)}(g_u)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2500 \\ 0.2000 \\ 0.2500 \\ 0.2000 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u) = q_i^{(j)}(g_u) = q_i^{(k)}(g_u) = q_i^{(l)}(g_u) = 0.2000$; $q_k^{(i)}(g_u) = q_k^{(j)}(g_u) = q_k^{(k)}(g_u) = 0.2500$; $CS_i(g_u) = \frac{1}{2}(0.2000 + 0.2000 + 0.2000 + 0.2000)^2 = 0.3200$; $CS_j(g_u) = CS_l(g_u) = 0$; $CS_k(g_u) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $\pi_i^{(i)}(g_u) = \pi_i^{(j)}(g_u) = \pi_i^{(k)}(g_u) = \pi_i^{(l)}(g_u) = (0.2000)^2 = 0.0400$; $\pi_k^{(i)}(g_u) = \pi_k^{(j)}(g_u) = \pi_k^{(k)}(g_u) = (0.2500)^2 = 0.0625$; and $PS_i(g_u) = PS_j(g_u) = PS_k(g_u) = PS_l(g_u) = 0$.

Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_u) = q_l^{(i)}(g_u) = q_j^{(j)}(g_u) = q_l^{(j)}(g_u) = q_j^{(k)}(g_u) = q_j^{(l)}(g_u) = q_l^{(l)}(g_u) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1364 & 0 & -0.0303 & 0 & -0.0303 & 0 \\ 0.1296 & 1 & -0.0370 & 0 & -0.0370 & 0 & 0 \\ 0 & -0.0303 & 1 & 0.1364 & 0 & -0.0303 & 0 \\ -0.0370 & 0 & 0.1296 & 1 & -0.0370 & 0 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 1 & 0.1364 & 0 \\ -0.0370 & 0 & -0.0370 & 0 & 0.1296 & 1 & 0 \\ 0 & -0.0303 & 0 & -0.0303 & 0 & -0.0303 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.2222 \\ 0.1818 \\ 0.2222 \\ 0.1818 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u)$ = $q_i^{(j)}(g_u)$ = 0.1656; $q_k^{(i)}(g_u)$ = $q_k^{(j)}(g_u)$ = 0.2132; $q_i^{(k)}(g_u)$ = 0.1708; $q_k^{(k)}(g_u)$ = 0.2123; $q_i^{(l)}(g_u)$ = 0.2012; $CS_i(g_u)$ = $\frac{1}{2}(0.1656+0.1656+0.1708+0.2012)^2$ = 0.2472; $CS_j(g_u)$ = $CS_l(g_u)$ = 0; $CS_k(g_u)$ = $\frac{1}{2}(0.2132+0.2132+0.2123)^2$ = 0.2040; $\pi_i^{(i)}(g_u)$ = $\pi_i^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.1656)^2$ = 0.0343; $\pi_k^{(i)}(g_u)$ = $\pi_k^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.2132)^2$ = 0.0568; $\pi_i^{(k)}(g_u)$ = $\frac{(2.5)}{2}(0.1708)^2$ = 0.0365; $\pi_k^{(k)}(g_u)$ = $\frac{(2.5)}{2}(0.2123)^2$ = 0.0563; $\pi_i^{(l)}(g_u)$ = $\frac{(2.5)}{2}(0.2012)^2$ = 0.0506; $PS_i(g_u)$ = $PS_j(g_u)$ = $\frac{0.5}{4}(0.1656+0.2132)^2$ = 0.0179; $PS_k(g_u)$ = $\frac{0.5}{4}(0.1708+0.2123)^2$ = 0.0183; and $PS_l(g_u)$ = $\frac{0.5}{4}(0.2012)^2$ = 0.0051.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

In this case $q_j^{(i)}(g_u)$ = $q_l^{(i)}(g_u)$ = $q_j^{(j)}(g_u)$ = $q_l^{(j)}(g_u)$ = $q_j^{(k)}(g_u)$ = $q_j^{(l)}(g_u)$ = $q_l^{(l)}(g_u)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2538 & 0 & -0.0462 & 0 & -0.0462 & 0 \\ 0.2455 & 1 & -0.0545 & 0 & -0.0545 & 0 & 0 \\ 0 & -0.0462 & 1 & 0.2538 & 0 & -0.0462 & 0 \\ -0.0545 & 0 & 0.2455 & 1 & -0.0545 & 0 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 1 & 0.2538 & 0 \\ -0.0545 & 0 & -0.0545 & 0 & 0.2455 & 1 & 0 \\ 0 & -0.0462 & 0 & -0.0462 & 0 & -0.0462 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1818 \\ 0.1538 \\ 0.1818 \\ 0.1538 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u)$ = $q_i^{(j)}(g_u)$ = 0.1271; $q_k^{(i)}(g_u)$ = $q_k^{(j)}(g_u)$ = 0.1648; $q_i^{(k)}(g_u)$ = 0.1336; $q_k^{(k)}(g_u)$ = 0.1629; $q_i^{(l)}(g_u)$ = 0.1766; $CS_i(g_u)$ = $\frac{1}{2}(0.1271+0.1271+0.1336+0.1766)^2$ = 0.1593; $CS_j(g_u)$ = $CS_l(g_u)$ = 0; $CS_k(g_u)$ = $\frac{1}{2}(0.1648+0.1648+0.1629)^2$ = 0.1213; $\pi_i^{(i)}(g_u)$ = $\pi_i^{(j)}(g_u)$ = $\frac{(3.5)}{2}(0.1271)^2$ = 0.0283; $\pi_k^{(i)}(g_u)$ = $\pi_k^{(j)}(g_u)$ = $\frac{(3.5)}{2}(0.1648)^2$ = 0.0475; $\pi_i^{(k)}(g_u)$ = $\frac{(3.5)}{2}(0.1336)^2$ = 0.0312; $\pi_k^{(k)}(g_u)$ = $\frac{(3.5)}{2}(0.1629)^2$ = 0.0464; $\pi_i^{(l)}(g_u)$ = $\frac{(3.5)}{2}(0.1766)^2$ = 0.0546; $PS_i(g_u)$ = $PS_j(g_u)$ = $\frac{1.5}{4}(0.1271+0.1648)^2$ = 0.0320; $PS_k(g_u)$ = $\frac{1.5}{4}(0.1336+0.1629)^2$ = 0.0330; and $PS_l(g_u)$ = $\frac{1.5}{4}(0.1766)^2$ = 0.0117.

Simulation 14: $\alpha = 1$, $\tilde{\alpha}$ = 0.5 and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u)\\ q_j^{(i)}(g_u)\\ q_k^{(i)}(g_u)\\ q_l^{(i)}(g_u)\\ q_i^{(j)}(g_u)\\ q_j^{(j)}(g_u)\\ q_k^{(j)}(g_u)\\ q_l^{(j)}(g_u)\\ q_i^{(k)}(g_u)\\ q_j^{(k)}(g_u)\\ q_k^{(k)}(g_u)\\ q_i^{(l)}(g_u)\\ q_j^{(l)}(g_u)\\ q_l^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.2000\\ 0.1000\\ 0.2500\\ 0.1250\\ 0.2000\\ 0.1000\\ 0.2500\\ 0.1250\\ 0.2000\\ 0.1000\\ 0.2500\\ 0.2000\\ 0.1000\\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u)$ = $q_i^{(j)}(g_u)$ = $q_i^{(k)}(g_u)$ = $q_i^{(l)}(g_u)$ = 0.2000; $q_j^{(i)}(g_u)$ = $q_j^{(j)}(g_u)$ = $q_j^{(k)}(g_u)$ = $q_j^{(l)}(g_u)$ = 0.1000; $q_k^{(i)}(g_u)$ = $q_k^{(j)}(g_u)$ = $q_k^{(k)}(g_u)$ = 0.2500; $q_l^{(i)}(g_u)$ = $q_l^{(j)}(g_u)$ = $q_l^{(l)}(g_u)$ = 0.1250; $CS_i(g_u)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_u)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $CS_k(g_u)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_l(g_u)$ = $\frac{1}{2}(0.1250+0.1250+0.1250)^2$ = 0.0703; $\pi_i^{(i)}(g_u)$ = $\pi_i^{(j)}(g_u)$ = $\pi_i^{(k)}(g_u)$ = $\pi_i^{(l)}(g_u)$ = $(0.2000)^2$ = 0.0400; $\pi_j^{(i)}(g_u)$ = $\pi_j^{(j)}(g_u)$ = $\pi_j^{(k)}(g_u)$ = $\pi_j^{(l)}(g_u)$ = $(0.1000)^2$ = 0.0100; $\pi_k^{(i)}(g_u)$ = $\pi_k^{(j)}(g_u)$ = $\pi_k^{(k)}(g_u)$ = $(0.2500)^2$ = 0.0625; $\pi_l^{(i)}(g_u)$ = $\pi_l^{(j)}(g_u)$ = $\pi_l^{(l)}(g_u)$ = $(0.1250)^2$ = 0.0156; and $PS_i(g_u)$ = $PS_j(g_u)$ = $PS_k(g_u)$ = $PS_l(g_u)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1364 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 \\ 0.1364 & 1 & 0.1364 & 0.1364 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 \\ 0.1296 & 0.1296 & 1 & 0.1296 & -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & -0.0370 & 0 & 0 & 0 & 0 \\ 0.1296 & 0.1296 & 0.1296 & 1 & -0.0370 & -0.0370 & -0.0370 & 0 & 0 & 0 & 0 & -0.0370 & -0.0370 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 \\ -0.0303 & 0 & -0.0303 & -0.0303 & 0.1364 & 1 & 0.1364 & 0.1364 & -0.0303 & 0 & -0.0303 & -0.0303 & 0 & -0.0303 \\ -0.0370 & -0.0370 & 0 & -0.0370 & 0.1296 & 0.1296 & 1 & 0.1296 & -0.0370 & -0.0370 & 0 & 0 & 0 & 0 \\ -0.0370 & -0.0370 & -0.0370 & 0 & 0.1296 & 0.1296 & 0.1296 & 1 & 0 & 0 & 0 & -0.0370 & -0.0370 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 & 0 & -0.0303 & -0.0303 \\ -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 0.1364 & 1 & 0.1364 & -0.0303 & 0 & -0.0303 \\ -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & -0.0370 & 0 & -0.0370 & 0.1296 & 0.1296 & 1 & 0 & 0 & 0 \\ 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & 1 & 0.1364 & 0.1364 \\ -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & -0.0303 & -0.0303 & 0 & -0.0303 & 0.1364 & 1 & 0.1364 \\ -0.0370 & -0.0370 & -0.0370 & 0 & -0.0370 & -0.0370 & -0.0370 & 0 & 0 & 0 & 0 & 0.1296 & 0.1296 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u) \\ q_j^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_l^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_j^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_l^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_j^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \\ q_j^{(l)}(g_u) \\ q_l^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.1818 \\ 0.0909 \\ 0.2222 \\ 0.1111 \\ 0.1818 \\ 0.0909 \\ 0.2222 \\ 0.1111 \\ 0.1818 \\ 0.0909 \\ 0.2222 \\ 0.1818 \\ 0.0909 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u)$ = $q_i^{(j)}(g_u)$ = 0.1595; $q_j^{(i)}(g_u)$ = $q_j^{(j)}(g_u)$ = 0.0642; $q_k^{(i)}(g_u)$ = $q_k^{(j)}(g_u)$ = 0.2030; $q_l^{(i)}(g_u)$ = $q_l^{(j)}(g_u)$ = 0.0817; $q_i^{(k)}(g_u)$ = 0.1697; $q_j^{(k)}(g_u)$ = 0.0744;

$q_k^{(k)}(g_u)$ = 0.2132; $q_i^{(l)}(g_u)$ = 0.1848; $q_j^{(l)}(g_u)$ = 0.0896; $q_l^{(l)}(g_u)$ = 0.1071; $CS_i(g_u)$ = $\frac{1}{2}(0.1595+0.1595+0.1697+0.1848)^2$ = 0.2268; $CS_j(g_u)$ = $\frac{1}{2}(0.0642+0.0642+0.0744+0.0896)^2$ = 0.0427; $CS_k(g_u)$ = $\frac{1}{2}(0.2030+0.2030+0.2132)^2$ = 0.1917; $CS_l(g_u)$ = $\frac{1}{2}(0.0817+0.0817+0.1071)^2$ = 0.0366; $\pi_i^{(i)}(g_u)$ = $\pi_i^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.1595)^2$ = 0.0318; $\pi_j^{(i)}(g_u)$ = $\pi_j^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.0642)^2$ = 0.0052; $\pi_k^{(i)}(g_u)$ = $\pi_k^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.2030)^2$ = 0.0515; $\pi_l^{(i)}(g_u)$ = $\pi_l^{(j)}(g_u)$ = $\frac{(2.5)}{2}(0.0817)^2$ = 0.0083; $\pi_i^{(k)}(g_u)$ = $\frac{(2.5)}{2}(0.1697)^2$ = 0.0360; $\pi_j^{(k)}(g_u)$ = $\frac{(2.5)}{2}(0.0744)^2$ = 0.0069; $\pi_k^{(k)}(g_u)$ = $\frac{(2.5)}{2}(0.2132)^2$ = 0.0568; $\pi_i^{(l)}(g_u)$ = $\frac{(2.5)}{2}(0.1848)^2$ = 0.0427; $\pi_j^{(l)}(g_u)$ = $\frac{(2.5)}{2}(0.0896)^2$ = 0.0100; $\pi_l^{(l)}(g_u)$ = $\frac{(2.5)}{2}(0.1071)^2$ = 0.0143; $PS_i(g_u) = PS_j(g_u)$ = $\frac{0.5}{4}(0.1595+0.0642+0.2030+0.0817)^2$ = 0.0323; $PS_k(g_u)$ = $\frac{0.5}{4}(0.1697+0.0744+0.2132)^2$ = 0.0261; and $PS_l(g_u)$ = $\frac{0.5}{4}(0.1848+0.0896+0.1071)^2$ = 0.0182.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2538 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 \\ 0.2538 & 1 & 0.2538 & 0.2538 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 \\ 0.2455 & 0.2455 & 1 & 0.2455 & -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & -0.0545 & 0 & 0 & 0 & 0 \\ 0.2455 & 0.2455 & 0.2455 & 1 & -0.0545 & -0.0545 & -0.0545 & 0 & 0 & 0 & 0 & -0.0545 & -0.0545 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 \\ -0.0462 & 0 & -0.0462 & -0.0462 & 0.2538 & 1 & 0.2538 & 0.2538 & -0.0462 & 0 & -0.0462 & -0.0462 & 0 & -0.0462 \\ -0.0545 & -0.0545 & 0 & -0.0545 & 0.2455 & 0.2455 & 1 & 0.2455 & -0.0545 & -0.0545 & 0 & 0 & 0 & 0 \\ -0.0545 & -0.0545 & -0.0545 & 0 & 0.2455 & 0.2455 & 0.2455 & 1 & 0 & 0 & 0 & -0.0545 & -0.0545 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 & 0 & -0.0462 & -0.0462 \\ -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 0.2538 & 1 & 0.2538 & -0.0462 & 0 & -0.0462 \\ -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & -0.0545 & 0 & -0.0545 & 0.2455 & 0.2455 & 1 & 0 & 0 & 0 \\ 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & 1 & 0.2538 & 0.2538 \\ -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & -0.0462 & -0.0462 & 0 & -0.0462 & 0.2538 & 1 & 0.2538 \\ -0.0545 & -0.0545 & -0.0545 & 0 & -0.0545 & -0.0545 & -0.0545 & 0 & 0 & 0 & 0 & 0.2455 & 0.2455 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_u) \\ q_j^{(i)}(g_u) \\ q_k^{(i)}(g_u) \\ q_l^{(i)}(g_u) \\ q_i^{(j)}(g_u) \\ q_j^{(j)}(g_u) \\ q_k^{(j)}(g_u) \\ q_l^{(j)}(g_u) \\ q_i^{(k)}(g_u) \\ q_j^{(k)}(g_u) \\ q_k^{(k)}(g_u) \\ q_i^{(l)}(g_u) \\ q_j^{(l)}(g_u) \\ q_l^{(l)}(g_u) \end{pmatrix} \begin{pmatrix} 0.1538 \\ 0.0769 \\ 0.1818 \\ 0.0909 \\ 0.1538 \\ 0.0769 \\ 0.1818 \\ 0.0909 \\ 0.1538 \\ 0.0769 \\ 0.1818 \\ 0.1538 \\ 0.0769 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_u) = q_i^{(j)}(g_u) = 0.1215$; $q_j^{(i)}(g_u) = q_j^{(j)}(g_u) = 0.0346$; $q_k^{(i)}(g_u) = q_k^{(j)}(g_u) = 0.1531$; $q_l^{(i)}(g_u) = q_l^{(j)}(g_u) = 0.0435$; $q_i^{(k)}(g_u) = 0.1296$; $q_j^{(k)}(g_u) = 0.0427$; $q_k^{(k)}(g_u) = 0.1612$; $q_i^{(l)}(g_u) = 0.1502$; $q_j^{(l)}(g_u) = 0.0633$; $q_l^{(l)}(g_u) = 0.0722$; $CS_i(g_u) = \frac{1}{2}(0.1215+0.1215+0.1296+0.1502)^2 = 0.1367$; $CS_j(g_u) = \frac{1}{2}(0.0346+0.0346+0.0427+0.0633)^2 = 0.0153$; $CS_k(g_u) = \frac{1}{2}(0.1531+0.1531+0.1612)^2 = 0.1092$; $CS_l(g_u) = \frac{1}{2}(0.0435+0.0435+0.0722)^2 = 0.0127$; $\pi_i^{(i)}(g_u) = \pi_i^{(j)}(g_u) = \frac{(3.5)}{2}(0.1215)^2 = 0.0258$; $\pi_j^{(i)}(g_u) = \pi_j^{(j)}(g_u) = \frac{(3.5)}{2}(0.0346)^2 = 0.0021$; $\pi_k^{(i)}(g_u) = \pi_k^{(j)}(g_u) = \frac{(3.5)}{2}(0.1531)^2 = 0.0410$; $\pi_l^{(i)}(g_u) = \pi_l^{(j)}(g_u) = \frac{(3.5)}{2}(0.0435)^2 = 0.0033$; $\pi_i^{(k)}(g_u) = \frac{(3.5)}{2}(0.1296)^2 = 0.0294$; $\pi_j^{(k)}(g_u) = \frac{(3.5)}{2}(0.0427)^2 = 0.0032$; $\pi_k^{(k)}(g_u) = \frac{(3.5)}{2}(0.1612)^2 = 0.0455$; $\pi_i^{(l)}(g_u) = \frac{(3.5)}{2}(0.1502)^2 = 0.0395$; $\pi_j^{(l)}(g_u) = \frac{(3.5)}{2}(0.0633)^2 = 0.0070$; $\pi_l^{(l)}(g_u) = \frac{(3.5)}{2}(0.0722)^2 = 0.0091$; $PS_i(g_u) = PS_j(g_u) = \frac{1.5}{4}(0.1215+0.0346+0.1531+0.0435)^2 = 0.0466$; $PS_k(g_u) = \frac{1.5}{4}(0.1296+0.0427+0.1612)^2 = 0.0417$; and $PS_l(g_u) = \frac{1.5}{4}(0.1502+0.0633+0.0722)^2 = 0.0306$.

Network v

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_v)$ = $q_k^{(k)}(g_v)$ = $\dfrac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_v)+2\phi q_i^{(j)}(g_v)+2\phi q_l^{(j)}(g_v)-2\phi(\phi+3)q_j^{(i)}(g_v)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_v)$ = $q_l^{(i)}(g_v)$ = $q_j^{(k)}(g_v)$ = $q_l^{(k)}(g_v)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(j)}(g_v)+\phi q_l^{(j)}(g_v)+\phi q_j^{(j)}(g_v)-\phi(\phi+3)q_i^{(i)}(g_v)}{3\phi^2+15\phi+10}$; $q_i^{(j)}(g_v)$ = $q_k^{(j)}(g_v)$ = $q_i^{(l)}(g_v)$ = $q_k^{(l)}(g_v)$ = $\dfrac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_v)-\phi(\phi+2)q_j^{(j)}(g_v)-\phi(\phi+2)q_l^{(j)}(g_v)}{3\phi^2+12\phi+8}$; $q_j^{(j)}(g_v)$ = $q_l^{(l)}(g_v)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_v)+2\phi q_j^{(i)}(g_v)-2\phi(\phi+3)q_i^{(j)}(g_v)-\phi(\phi+4)q_l^{(j)}(g_v)}{2\phi^2+11\phi+10}$; $q_l^{(j)}(g_v)$ = $q_j^{(l)}(g_v)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_v)+2\phi q_j^{(i)}(g_v)-2\phi(\phi+3)q_i^{(j)}(g_v)-\phi(\phi+4)q_j^{(j)}(g_v)}{2\phi^2+11\phi+10}$; $CS_i(g_v)$ = $CS_k(g_v)$ = $\frac{1}{2}\left(q_i^{(i)}(g_v)+q_i^{(j)}(g_v)+q_i^{(l)}(g_v)\right)^2$; $CS_j(g_v)$ = $CS_l(g_v)$ = $\frac{1}{2}\left(q_j^{(i)}(g_v)+q_j^{(j)}(g_v)+q_j^{(k)}(g_v)+q_j^{(l)}(g_v)\right)^2$; $\pi_i^{(i)}(g_v)$ = $\pi_k^{(k)}(g_v)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_v)\right)^2$; $\pi_j^{(i)}(g_v)$ = $\pi_l^{(i)}(g_v)$ = $\pi_j^{(k)}(g_v)$ = $\pi_l^{(k)}(g_v)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_v)\right)^2$; $\pi_i^{(j)}(g_v)$ = $\pi_k^{(j)}(g_v)$ = $\pi_i^{(l)}(g_v)$ = $\pi_k^{(l)}(g_v)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_v)\right)^2$; $\pi_j^{(j)}(g_v)$ = $\pi_l^{(l)}(g_v)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_v)\right)^2$; $\pi_l^{(j)}(g_v)$ = $\pi_j^{(l)}(g_v)$ = $\frac{(2+\phi)}{2}\left(q_l^{(j)}(g_v)\right)^2$; $PS_i(g_v)$ = $PS_k(g_v)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_v)+q_j^{(i)}(g_v)+q_l^{(i)}(g_v)\right)^2$; and $PS_j(g_v)$ = $PS_l(g_v)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_v)+q_j^{(j)}(g_v)+q_k^{(j)}(g_v)+q_l^{(j)}(g_v)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} \\ \frac{\phi(\phi+3)}{3\phi^2+15\phi+10} & 1 & -\frac{4\phi}{3\phi^2+15\phi+10} & -\frac{\phi}{3\phi^2+15\phi+10} & -\frac{\phi}{3\phi^2+15\phi+10} \\ 0 & -\frac{2\phi}{3\phi^2+12\phi+8} & 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} \\ -\frac{2\phi}{2\phi^2+11\phi+10} & -\frac{2\phi}{2\phi^2+11\phi+10} & \frac{2\phi(\phi+3)}{2\phi^2+11\phi+10} & 1 & \frac{\phi(\phi+4)}{2\phi^2+11\phi+10} \\ -\frac{2\phi}{2\phi^2+11\phi+10} & -\frac{2\phi}{2\phi^2+11\phi+10} & \frac{2\phi(\phi+3)}{2\phi^2+11\phi+10} & \frac{\phi(\phi+4)}{2\phi^2+11\phi+10} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_v) \\ q_j^{(i)}(g_v) \\ q_i^{(j)}(g_v) \\ q_j^{(j)}(g_v) \\ q_l^{(j)}(g_v) \end{pmatrix} \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{2\tilde{\alpha}(\phi+1)}{2\phi^2+11\phi+10} \\ \frac{2\tilde{\alpha}(\phi+1)}{2\phi^2+11\phi+10} \end{pmatrix}$$

Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0

In this case $q_j^{(i)}(g_v)$ = $q_l^{(i)}(g_v)$= $q_j^{(j)}(g_v)$ = $q_l^{(j)}(g_v)$ = $q_j^{(k)}(g_v)$ = $q_l^{(k)}(g_v)$ = $q_j^{(l)}(g_v)$ = $q_l^{(l)}(g_v)$ = 0. Therefore it holds that $q_i^{(i)}(g_v)$ = $q_i^{(j)}(g_v)$ = $q_k^{(j)}(g_v)$ = $q_k^{(k)}(g_v)$ = $q_i^{(l)}(g_v)$ = $q_k^{(l)}(g_v)$ = 0.2500; $CS_i(g_v)$ = $CS_k(g_v)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_v)$ = $CS_l(g_v)$ = 0; $\pi_i^{(i)}(g_v)$ = $\pi_i^{(j)}(g_v)$ = $\pi_k^{(j)}(g_v)$ = $\pi_k^{(k)}(g_v)$ = $\pi_i^{(l)}(g_v)$ = $\pi_k^{(l)}(g_v)$ = $(0.2500)^2$ = 0.0625; and $PS_i(g_v)$ = $PS_j(g_v)$ = $PS_k(g_v)$ = $PS_l(g_v)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

In this case $q_j^{(i)}(g_v)$ = $q_l^{(i)}(g_v)$= $q_j^{(j)}(g_v)$ = $q_l^{(j)}(g_v)$ = $q_j^{(k)}(g_v)$ = $q_l^{(k)}(g_v)$ = $q_j^{(l)}(g_v)$ = $q_l^{(l)}(g_v)$ = 0. Therefore it holds that $q_i^{(i)}(g_v)$ = $q_k^{(k)}(g_v)$ = 0.2373; $q_i^{(j)}(g_v)$ = $q_k^{(j)}(g_v)$ = $q_i^{(l)}(g_v)$ = $q_k^{(l)}(g_v)$ = 0.2034 $CS_i(g_v)$ = $CS_k(g_v)$ = $\frac{1}{2}(0.2373+0.2034+0.2034)^2$ =

0.2074; $CS_j(g_v) = CS_l(g_v) = 0$; $\pi_i^{(i)}(g_v) = \pi_k^{(k)}(g_v) = \frac{(2.5)}{2}(0.2373)^2 = 0.0704$; $\pi_i^{(j)}(g_v) = \pi_k^{(j)}(g_v) = \pi_i^{(l)}(g_v) = \pi_k^{(l)}(g_v) = \frac{(2.5)}{2}(0.2034)^2 = 0.0517$; $PS_i(g_v) = PS_k(g_v) = \frac{0.5}{4}(0.2373)^2 = 0.0070$; and $PS_j(g_v) = PS_l(g_v) = \frac{0.5}{4}(0.2034 + 0.2034)^2 = 0.0207$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_v) = q_l^{(i)}(g_v) = q_j^{(j)}(g_v) = q_l^{(j)}(g_v) = q_j^{(k)}(g_v) = q_l^{(k)}(g_v) = q_j^{(l)}(g_v) = q_l^{(l)}(g_v) = 0$. Therefore it holds that $q_i^{(i)}(g_v) = q_k^{(k)}(g_v) = 0.1985$; $q_i^{(j)}(g_v) = q_k^{(j)}(g_v) = q_i^{(l)}(g_v) = q_k^{(l)}(g_v) = 0.1527$; $CS_i(g_v) = CS_k(g_v) = \frac{1}{2}(0.1985 + 0.1527 + 0.1527)^2 = 0.1270$; $CS_j(g_v) = CS_l(g_v) = 0$; $\pi_i^{(i)}(g_v) = \pi_k^{(k)}(g_v) = \frac{(3.5)}{2}(0.1985)^2 = 0.0690$; $\pi_i^{(j)}(g_v) = \pi_k^{(j)}(g_v) = \pi_i^{(l)}(g_v) = \pi_k^{(l)}(g_v) = \frac{(3.5)}{2}(0.1527)^2 = 0.0408$; $PS_i(g_v) = PS_k(g_v) = \frac{1.5}{4}(0.1985)^2 = 0.0148$; and $PS_j(g_v) = PS_l(g_v) = \frac{1.5}{4}(0.1527 + 0.1527)^2 = 0.0350$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_v) \\ q_j^{(i)}(g_v) \\ q_i^{(j)}(g_v) \\ q_j^{(j)}(g_v) \\ q_l^{(j)}(g_v) \end{pmatrix} \begin{pmatrix} 0.2500 \\ 0.1000 \\ 0.2500 \\ 0.1000 \\ 0.1000 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_v)$ = $q_k^{(k)}(g_v)$ = $q_i^{(j)}(g_v)$ = $q_k^{(j)}(g_v)$ = $q_i^{(l)}(g_v)$ = $q_k^{(l)}(g_v)$ = 0.2500; $q_j^{(i)}(g_v)$ = $q_l^{(i)}(g_v)$ = $q_j^{(k)}(g_v)$ = $q_l^{(k)}(g_v)$ = $q_j^{(j)}(g_v)$ = $q_l^{(l)}(g_v)$ = $q_l^{(j)}(g_v)$ = $q_j^{(l)}(g_v)$ = 0.1000; $CS_i(g_v)$ = $CS_k(g_v)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_v)$ = $CS_l(g_v)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $\pi_i^{(i)}(g_v)$ = $\pi_k^{(k)}(g_v)$ = $\pi_i^{(j)}(g_v)$ = $\pi_k^{(j)}(g_v)$ = $\pi_i^{(l)}(g_v)$ = $\pi_k^{(l)}(g_v)$ = $(0.2500)^2$ = 0.0625; $\pi_j^{(i)}(g_v)$ = $\pi_l^{(i)}(g_v)$ = $\pi_j^{(k)}(g_v)$ = $\pi_l^{(k)}(g_v)$ = $\pi_j^{(j)}(g_v)$ = $\pi_l^{(l)}(g_v)$ = $\pi_l^{(j)}(g_v)$ = $\pi_j^{(l)}(g_v)$ = $(0.1000)^2$ = 0.0100; and $PS_i(g_v)$ = $PS_j(g_v)$ = $PS_k(g_v)$ = $PS_l(g_v)$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2593 & -0.0741 & -0.0741 & -0.0741 \\ 0.0959 & 1 & -0.1096 & -0.0274 & -0.0274 \\ 0 & -0.0678 & 1 & 0.0847 & 0.0847 \\ -0.0625 & -0.0625 & 0.2188 & 1 & 0.1406 \\ -0.0625 & -0.0625 & 0.2188 & 0.1406 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_v) \\ q_j^{(i)}(g_v) \\ q_i^{(j)}(g_v) \\ q_j^{(j)}(g_v) \\ q_l^{(j)}(g_v) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.0822 \\ 0.2034 \\ 0.0938 \\ 0.0938 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_v)$ = $q_k^{(k)}(g_v)$ = 0.2237; $q_j^{(i)}(g_v)$ = $q_l^{(i)}(g_v)$ = $q_j^{(k)}(g_v)$ = $q_l^{(k)}(g_v)$ = 0.0859; $q_i^{(j)}(g_v)$ = $q_k^{(j)}(g_v)$ = $q_i^{(l)}(g_v)$ = $q_k^{(l)}(g_v)$ = 0.1989; $q_j^{(j)}(g_v)$ = $q_l^{(j)}(g_v)$ = $q_j^{(l)}(g_v)$ = $q_l^{(l)}(g_v)$ = 0.0611; $CS_i(g_v)$ = $CS_k(g_v)$ = $\frac{1}{2}(0.2237+0.1989+0.1989)^2$ = 0.1931; $CS_j(g_v)$ = $CS_l(g_v)$ = $\frac{1}{2}(0.0859+0.0611+0.0859+0.0611)^2$ = 0.0432; $\pi_i^{(i)}(g_v)$ =

$\pi_k^{(k)}(g_v) = \frac{(2.5)}{2}(0.2237)^2 = 0.0626$; $\pi_j^{(i)}(g_v) = \pi_l^{(i)}(g_v) = \pi_j^{(k)}(g_v) = \pi_l^{(k)}(g_v) = \frac{(2.5)}{2}(0.0859)^2 = 0.0092$; $\pi_i^{(j)}(g_v) = \pi_k^{(j)}(g_v) = \pi_i^{(l)}(g_v) = \pi_k^{(l)}(g_v) = \frac{(2.5)}{2}(0.1989)^2 = 0.0495$; $\pi_j^{(j)}(g_v) = \pi_l^{(j)}(g_v) = \pi_j^{(l)}(g_v) = \pi_l^{(l)}(g_v) = \frac{(2.5)}{2}(0.0611)^2 = 0.0047$; $PS_i(g_v) = PS_k(g_v) = \frac{0.5}{4}(0.2237 + 0.0859 + 0.0859)^2 = 0.0196$; and $PS_j(g_v) = PS_l(g_v) = \frac{0.5}{4}(0.1989 + 0.0611 + 0.1989 + 0.0611)^2 = 0.0338$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.4909 & -0.1091 & -0.1091 & -0.1091 \\ 0.1720 & 1 & -0.1529 & -0.0382 & -0.0382 \\ 0 & -0.0916 & 1 & 0.1603 & 0.1603 \\ -0.0968 & -0.0968 & 0.4355 & 1 & 0.2661 \\ -0.0968 & -0.0968 & 0.4355 & 0.2661 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_v) \\ q_j^{(i)}(g_v) \\ q_i^{(j)}(g_v) \\ q_j^{(j)}(g_v) \\ q_l^{(j)}(g_v) \end{pmatrix} \begin{pmatrix} 0.1818 \\ 0.0637 \\ 0.1527 \\ 0.0806 \\ 0.0806 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_v) = q_k^{(k)}(g_v) = 0.1760$; $q_j^{(i)}(g_v) = q_l^{(i)}(g_v) = q_j^{(k)}(g_v) = q_l^{(k)}(g_v) = 0.0584$; $q_i^{(j)}(g_v) = q_k^{(j)}(g_v) = q_i^{(l)}(g_v) = q_k^{(l)}(g_v) = 0.1482$; $q_j^{(j)}(g_v) = q_l^{(j)}(g_v) = q_j^{(l)}(g_v) = q_l^{(l)}(g_v) = 0.0306$; $CS_i(g_v) = CS_k(g_v) = \frac{1}{2}(0.1760 + 0.1482 + 0.1482)^2 = 0.1116$; $CS_j(g_v) = CS_l(g_v) = \frac{1}{2}(0.0584 + 0.0306 + 0.0584 + 0.0306)^2 = 0.0158$; $\pi_i^{(i)}(g_v) = \pi_k^{(k)}(g_v) = \frac{(3.5)}{2}(0.1760)^2 = 0.0542$; $\pi_j^{(i)}(g_v) = \pi_l^{(i)}(g_v) = \pi_j^{(k)}(g_v) = \pi_l^{(k)}(g_v) = \frac{(3.5)}{2}(0.0584)^2 = 0.0060$; $\pi_i^{(j)}(g_v) = \pi_k^{(j)}(g_v) = \pi_i^{(l)}(g_v) = \pi_k^{(l)}(g_v) = \frac{(3.5)}{2}(0.1482)^2 = 0.0384$; $\pi_j^{(j)}(g_v) = \pi_l^{(j)}(g_v) = \pi_j^{(l)}(g_v) = \pi_l^{(l)}(g_v) = \frac{(3.5)}{2}(0.0306)^2 = 0.0016$; $PS_i(g_v) =$

$PS_k(g_v)$ = $\frac{1.5}{4}(0.1760+0.0584+0.0584)^2$ = 0.0321; and $PS_j(g_v)$ = $PS_l(g_v)$ = $\frac{1.5}{4}(0.1482+0.0306+0.1482+0.0306)^2$ = 0.0480.

Network w

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_w)$ = $q_k^{(k)}(g_w)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_w)+2\phi q_i^{(j)}(g_w)-2\phi(\phi+3)q_j^{(i)}(g_w)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_w)$ = $q_l^{(i)}(g_w)$ = $q_j^{(k)}(g_w)$ = $q_l^{(k)}(g_w)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_w)-\phi(\phi+2)q_i^{(i)}(g_w)}{3\phi^2+12\phi+8}$; $q_i^{(j)}(g_w)$ = $q_k^{(j)}(g_w)$ = $q_i^{(l)}(g_w)$ = $q_k^{(l)}(g_w)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_w)-\phi(\phi+2)q_j^{(j)}(g_w)}{3\phi^2+12\phi+8}$; $q_j^{(j)}(g_w)$ = $q_l^{(l)}(g_w)$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_w)+2\phi q_j^{(i)}(g_w)-2\phi(\phi+3)q_i^{(j)}(g_w)}{2(\phi+4)(\phi+1)}$; $CS_i(g_w)$ = $CS_k(g_w)$ = $\frac{1}{2}\left(q_i^{(i)}(g_w)+q_i^{(j)}(g_w)+q_i^{(l)}(g_w)\right)^2$; $CS_j(g_w) = CS_l(g_w) = \frac{1}{2}\left(q_j^{(i)}(g_w)+q_j^{(j)}(g_w)+q_j^{(k)}(g_w)\right)^2$; $\pi_i^{(i)}(g_w)$ = $\pi_k^{(k)}(g_w)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_w)\right)^2$; $\pi_j^{(i)}(g_w)$ = $\pi_l^{(i)}(g_w)$ = $\pi_j^{(k)}(g_w)$ = $\pi_l^{(k)}(g_w)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_w)\right)^2$; $\pi_i^{(j)}(g_w)$ = $\pi_k^{(j)}(g_w)$ = $\pi_i^{(l)}(g_w)$ = $\pi_k^{(l)}(g_w)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_w)\right)^2$; $\pi_j^{(j)}(g_w)$ = $\pi_l^{(l)}(g_w)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_w)\right)^2$; $PS_i(g_w)$ = $PS_k(g_w)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_w)+q_j^{(i)}(g_w)+q_l^{(i)}(g_w)\right)^2$; and $PS_j(g_w)$ = $PS_l(g_w)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_w)+q_j^{(j)}(g_w)+q_k^{(j)}(g_w)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} \\ \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} & 1 & -\frac{2\phi}{3\phi^2+12\phi+8} & 0 \\ 0 & -\frac{2\phi}{3\phi^2+12\phi+8} & 1 & \frac{\phi(\phi+2)}{3\phi^2+12\phi+8} \\ -\frac{\phi}{(\phi+4)(\phi+1)} & -\frac{\phi}{(\phi+4)(\phi+1)} & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_w) \\ q_j^{(i)}(g_w) \\ q_i^{(j)}(g_w) \\ q_j^{(j)}(g_w) \end{pmatrix} \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{2\alpha(\phi+1)}{3\phi^2+12\phi+8} \\ \frac{\tilde{\alpha}}{\phi+4} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(j)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = q_l^{(l)}(g_w) = 0$. Therefore it holds that $q_i^{(i)}(g_w) = q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_k^{(k)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w)$ $= 0.2500$; $CS_i(g_w) = CS_k(g_w) = \frac{1}{2}(0.2500+0.2500+0.2500)^2 = 0.2813$; $CS_j(g_w) = CS_l(g_w) = 0$; $\pi_i^{(i)}(g_w) = \pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) = \pi_k^{(k)}(g_w) = \pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = (0.2500)^2 = 0.0625$; and $PS_i(g_w) = PS_j(g_w) = PS_k(g_w) = PS_l(g_w) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(j)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = q_l^{(l)}(g_w) = 0$. Therefore it holds that $q_i^{(i)}(g_w) = q_k^{(k)}(g_w) = 0.2373$; $q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w) = 0.2034$; $CS_i(g_w) = CS_k(g_w) = \frac{1}{2}(0.2373+0.2034+0.2034)^2 = 0.2074$; $CS_j(g_w) = CS_l(g_w) = 0$; $\pi_i^{(i)}(g_w) = \pi_k^{(k)}(g_w) = \frac{(2.5)}{2}(0.2373)^2 = 0.0704$; $\pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) =$

$\pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = \frac{(2.5)}{2}(0.2034)^2 = 0.0517$; $PS_i(g_w) = PS_k(g_w) = \frac{0.5}{4}(0.2373)^2 =$ 0.0070; and $PS_j(g_w) = PS_l(g_w) = \frac{0.5}{4}(0.2034+0.2034)^2 = 0.0207$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(j)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = q_l^{(l)}(g_w) = 0$. Therefore it holds that $q_i^{(i)}(g_w) = q_k^{(k)}(g_w) = 0.1985$; $q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w) = 0.1527$; $CS_i(g_w) = CS_k(g_w) = \frac{1}{2}(0.1985+0.1527+0.1527)^2 = 0.1270$; $CS_j(g_w) = CS_l(g_w) = 0$; $\pi_i^{(i)}(g_w) = \pi_k^{(k)}(g_w) = \frac{(3.5)}{2}(0.1985)^2 = 0.0690$; $\pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) = \pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = \frac{(3.5)}{2}(0.1527)^2 = 0.0408$; $PS_i(g_w) = PS_k(g_w) = \frac{1.5}{4}(0.1985)^2 =$ 0.0148; and $PS_j(g_w) = PS_l(g_w) = \frac{1.5}{4}(0.1527+0.1527)^2 = 0.0350$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_w) \\ q_j^{(i)}(g_w) \\ q_i^{(j)}(g_w) \\ q_j^{(j)}(g_w) \end{pmatrix} \begin{pmatrix} 0.2500 \\ 0.1250 \\ 0.2500 \\ 0.1250 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_w) = q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_k^{(k)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w) = 0.2500$; $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(j)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = q_l^{(l)}(g_w) = 0.1250$; $CS_i(g_w) =$

$CS_k(g_w) = \frac{1}{2}(0.2500+0.2500+0.2500)^2 = 0.2813$; $CS_j(g_w) = CS_l(g_w) = \frac{1}{2}(0.1250+0.1250+0.1250)^2 = 0.0703$; $\pi_i^{(i)}(g_w) = \pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) = \pi_k^{(k)}(g_w) = \pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_w) = \pi_l^{(i)}(g_w) = \pi_j^{(j)}(g_w) = \pi_j^{(k)}(g_w) = \pi_l^{(k)}(g_w) = \pi_l^{(l)}(g_w) = (0.1250)^2 = 0.0156$; and $PS_i(g_w) = PS_j(g_w) = PS_k(g_w) = PS_l(g_w) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2593 & -0.0741 & -0.0741 \\ 0.0847 & 1 & -0.0678 & 0 \\ 0 & -0.0678 & 1 & 0.0847 \\ -0.0741 & -0.0741 & 0.2593 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_w) \\ q_j^{(i)}(g_w) \\ q_i^{(j)}(g_w) \\ q_j^{(j)}(g_w) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.1017 \\ 0.2034 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_w) = q_k^{(k)}(g_w) = 0.2182$; $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = 0.0970$; $q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w) = 0.2030$; $q_j^{(j)}(g_w) = q_l^{(l)}(g_w) = 0.0818$; $CS_i(g_w) = CS_k(g_w) = \frac{1}{2}(0.2182+0.2030+0.2030)^2 = 0.1948$; $CS_j(g_w) = CS_l(g_w) = \frac{1}{2}(0.0970+0.0818+0.0970)^2 = 0.0380$; $\pi_i^{(i)}(g_w) = \pi_k^{(k)}(g_w) = \frac{(2.5)}{2}(0.2182)^2 = 0.0595$; $\pi_j^{(i)}(g_w) = \pi_l^{(i)}(g_w) = \pi_j^{(k)}(g_w) = \pi_l^{(k)}(g_w) = \frac{(2.5)}{2}(0.0970)^2 = 0.0118$; $\pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) = \pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = \frac{(2.5)}{2}(0.2030)^2 = 0.0515$; $\pi_j^{(j)}(g_w) = \pi_l^{(l)}(g_w) = \frac{(2.5)}{2}(0.0818)^2 = 0.0084$; $PS_i(g_w) = PS_k(g_w) =$

$\frac{0.5}{4}(0.2182+0.0970+0.0970)^2$ = 0.0212; and $PS_j(g_w)$ = $PS_l(g_w)$ = $\frac{0.5}{4}(0.2030+0.0818+0.2030)^2$ = 0.0297.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.4909 & -0.1091 & -0.1091 \\ 0.1603 & 1 & -0.0916 & 0 \\ 0 & -0.0916 & 1 & 0.1603 \\ -0.1091 & -0.1091 & 0.4909 & 1 \end{pmatrix}\begin{pmatrix} q_i^{(i)}(g_w) \\ q_j^{(i)}(g_w) \\ q_i^{(j)}(g_w) \\ q_j^{(j)}(g_w) \end{pmatrix}\begin{pmatrix} 0.1818 \\ 0.0763 \\ 0.1527 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_w)$ = $q_k^{(k)}(g_w)$ = 0.1722; $q_j^{(i)}(g_w)$ = $q_l^{(i)}(g_w)$ = $q_j^{(k)}(g_w)$ = $q_l^{(k)}(g_w)$ = 0.0626; $q_i^{(j)}(g_w)$ = $q_k^{(j)}(g_w)$ = $q_i^{(l)}(g_w)$ = $q_k^{(l)}(g_w)$ = 0.1517; $q_j^{(j)}(g_w)$ = $q_l^{(l)}(g_w)$ = 0.0421; $CS_i(g_w)$ = $CS_k(g_w)$ = $\frac{1}{2}(0.1722+0.1517+0.1517)^2$ = 0.1131; $CS_j(g_w)$ = $CS_l(g_w)$ = $\frac{1}{2}(0.0626+0.0421+0.0626)^2$ = 0.0140; $\pi_i^{(i)}(g_w)$ = $\pi_k^{(k)}(g_w)$ = $\frac{(3.5)}{2}(0.1722)^2$ = 0.0519; $\pi_j^{(i)}(g_w)$ = $\pi_l^{(i)}(g_w)$ = $\pi_j^{(k)}(g_w)$ = $\pi_l^{(k)}(g_w)$ = $\frac{(3.5)}{2}(0.0626)^2$ = 0.0069; $\pi_i^{(j)}(g_w)$ = $\pi_k^{(j)}(g_w)$ = $\pi_i^{(l)}(g_w)$ = $\pi_k^{(l)}(g_w)$ = $\frac{(3.5)}{2}(0.1517)^2$ = 0.0403; $\pi_j^{(j)}(g_w)$ = $\pi_l^{(l)}(g_w)$ = $\frac{(3.5)}{2}(0.0421)^2$ = 0.0031; $PS_i(g_w)$ = $PS_k(g_w)$ = $\frac{1.5}{4}(0.1722+0.0626+0.0626)^2$ = 0.0332; and $PS_j(g_w)$ = $PS_l(g_w)$ = $\frac{1.5}{4}(0.1517+0.0421+0.1517)^2$ = 0.0448.

Network x

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_x)$ = $q_k^{(i)}(g_x)$ = $q_i^{(k)}(g_x)$ = $q_k^{(k)}(g_x)$ = $\frac{2\alpha(\phi+1)+4\phi q_j^{(j)}(g_x)+2\phi q_i^{(j)}(g_x)-2\phi(\phi+3)q_j^{(i)}(g_x)}{3\phi^2+15\phi+10}$; $q_j^{(i)}(g_x)$ = $q_l^{(i)}(g_x)$ = $q_j^{(k)}(g_x)$ = $q_l^{(k)}(g_x)$ = $\frac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(j)}(g_x)+2\phi q_j^{(j)}(g_x)-2\phi(\phi+3)q_i^{(i)}(g_x)}{3\phi^2+15\phi+10}$; $q_i^{(j)}(g_x)$ = $q_k^{(j)}(g_x)$ = $q_i^{(l)}(g_x)$ = $q_k^{(l)}(g_x)$ = $\frac{2\alpha(\phi+1)+4\phi q_j^{(i)}(g_x)+2\phi q_i^{(i)}(g_x)-2\phi(\phi+3)q_j^{(j)}(g_x)}{3\phi^2+15\phi+10}$; $q_j^{(j)}(g_x)$ = $q_l^{(j)}(g_x)$ = $q_j^{(l)}(g_x)$ = $q_l^{(l)}(g_x)$ = $\frac{2\tilde{\alpha}(\phi+1)+4\phi q_i^{(i)}(g_x)+2\phi q_j^{(i)}(g_x)-2\phi(\phi+3)q_i^{(j)}(g_x)}{3\phi^2+15\phi+10}$; $CS_i(g_x)$ = $CS_k(g_x)$ = $\frac{1}{2}\left(q_i^{(i)}(g_x)+q_i^{(j)}(g_x)+q_i^{(k)}(g_x)+q_i^{(l)}(g_x)\right)^2$; $CS_j(g_x)$ = $CS_l(g_x)$ = $\frac{1}{2}\left(q_j^{(i)}(g_x)+q_j^{(j)}(g_x)+q_j^{(k)}(g_x)+q_j^{(l)}(g_x)\right)^2$; $\pi_i^{(i)}(g_x)$ = $\pi_k^{(i)}(g_x)$ = $\pi_i^{(k)}(g_x)$ = $\pi_k^{(k)}(g_x)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_x)\right)^2$; $\pi_j^{(i)}(g_x)$ = $\pi_l^{(i)}(g_x)$ = $\pi_j^{(k)}(g_x)$ = $\pi_l^{(k)}(g_x)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_x)\right)^2$; $\pi_i^{(j)}(g_x)$ = $\pi_k^{(j)}(g_x)$ = $\pi_i^{(l)}(g_x)$ = $\pi_k^{(l)}(g_x)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_x)\right)^2$; $\pi_j^{(j)}(g_x)$ = $\pi_l^{(j)}(g_x)$ = $\pi_j^{(l)}(g_x)$ = $\pi_l^{(l)}(g_x)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_x)\right)^2$; $PS_i(g_x)$ = $PS_k(g_x)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_x)+q_j^{(i)}(g_x)+q_k^{(i)}(g_x)+q_l^{(i)}(g_x)\right)^2$; and $PS_j(g_x)$ = $PS_l(g_x)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_x)+q_j^{(j)}(g_x)+q_k^{(j)}(g_x)+q_l^{(j)}(g_x)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{2\phi(\phi+3)}{3\phi^2+15\phi+10} & -\frac{2\phi}{3\phi^2+15\phi+10} & -\frac{4\phi}{3\phi^2+15\phi+10} \\ \frac{2\phi(\phi+3)}{3\phi^2+15\phi+10} & 1 & -\frac{4\phi}{3\phi^2+15\phi+10} & -\frac{2\phi}{3\phi^2+15\phi+10} \\ -\frac{2\phi}{3\phi^2+15\phi+10} & -\frac{4\phi}{3\phi^2+15\phi+10} & 1 & \frac{2\phi(\phi+3)}{3\phi^2+15\phi+10} \\ -\frac{4\phi}{3\phi^2+15\phi+10} & -\frac{2\phi}{3\phi^2+15\phi+10} & \frac{2\phi(\phi+3)}{3\phi^2+15\phi+10} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_x) \\ q_j^{(i)}(g_x) \\ q_i^{(j)}(g_x) \\ q_j^{(j)}(g_x) \end{pmatrix} \begin{pmatrix} \frac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{2\alpha(\phi+1)}{3\phi^2+15\phi+10} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+15\phi+10} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_x) = q_j^{(j)}(g_x) = q_j^{(k)}(g_x) = q_j^{(l)}(g_x) = q_l^{(i)}(g_x) = q_l^{(j)}(g_x) = q_l^{(k)}(g_x) = q_l^{(l)}(g_x) = 0$. Therefore it holds that $q_i^{(i)}(g_x) = q_k^{(i)}(g_x) = q_i^{(j)}(g_x) = q_k^{(j)}(g_x) = q_i^{(k)}(g_x) = q_k^{(k)}(g_x) = q_i^{(l)}(g_x) = q_k^{(l)}(g_x) = 0.2000$; $CS_i(g_x) = CS_k(g_x) = \frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2 = 0.3200$; $CS_j(g_x) = CS_l(g_x) = 0$; $\pi_i^{(i)}(g_x) = \pi_k^{(i)}(g_x) = \pi_i^{(j)}(g_x) = \pi_k^{(j)}(g_x) = \pi_i^{(k)}(g_x) = \pi_k^{(k)}(g_x) = \pi_i^{(l)}(g_x) = \pi_k^{(l)}(g_x) = (0.2000)^2 = 0.0400$; and $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_x) = q_j^{(j)}(g_x) = q_j^{(k)}(g_x) = q_j^{(l)}(g_x) = q_l^{(i)}(g_x) = q_l^{(j)}(g_x) = q_l^{(k)}(g_x) = q_l^{(l)}(g_x) = 0$. Therefore it holds that $q_i^{(i)}(g_x) = q_k^{(i)}(g_x) = q_i^{(j)}(g_x) = q_k^{(j)}(g_x) = q_i^{(k)}(g_x) = q_k^{(k)}(g_x) = q_i^{(l)}(g_x) = q_k^{(l)}(g_x) = 0.1739$; $CS_i(g_x) = CS_k(g_x) = \frac{1}{2}(0.1739+0.1739+0.1739+0.1739)^2 = 0.2419$; $CS_j(g_x) = CS_l(g_x) = 0$; $\pi_i^{(i)}(g_x) = \pi_k^{(i)}(g_x) = \pi_i^{(j)}(g_x) = \pi_k^{(j)}(g_x) = \pi_i^{(k)}(g_x) = \pi_k^{(k)}(g_x) = \pi_i^{(l)}(g_x) = \pi_k^{(l)}(g_x) =$

$\frac{(2.5)}{2}(0.1739)^2 = 0.0378$; and $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) = \frac{0.5}{4}(0.1739 + 0.1739)^2 = 0.0151$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_x) = q_j^{(j)}(g_x) = q_j^{(k)}(g_x) = q_j^{(l)}(g_x) = q_l^{(i)}(g_x) = q_l^{(j)}(g_x) = q_l^{(k)}(g_x) = q_l^{(l)}(g_x) = 0$. Therefore it holds that $q_i^{(i)}(g_x) = q_k^{(i)}(g_x) = q_i^{(j)}(g_x) = q_k^{(j)}(g_x) = q_i^{(k)}(g_x) = q_k^{(k)}(g_x) = q_i^{(l)}(g_x) = q_k^{(l)}(g_x) = 0.1379$; $CS_i(g_x) = CS_k(g_x) = \frac{1}{2}(0.1379 + 0.1379 + 0.1379 + 0.1379)^2 = 0.1521$; $CS_j(g_x) = CS_l(g_x) = 0$; $\pi_i^{(i)}(g_x) = \pi_k^{(i)}(g_x) = \pi_i^{(j)}(g_x) = \pi_k^{(j)}(g_x) = \pi_i^{(k)}(g_x) = \pi_k^{(k)}(g_x) = \pi_i^{(l)}(g_x) = \pi_k^{(l)}(g_x) = \frac{(3.5)}{2}(0.1379)^2 = 0.0333$; and $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) = \frac{1.5}{4}(0.1379 + 0.1379)^2 = 0.0285$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_x) \\ q_j^{(i)}(g_x) \\ q_i^{(j)}(g_x) \\ q_j^{(j)}(g_x) \end{pmatrix} \begin{pmatrix} 0.2000 \\ 0.1000 \\ 0.2000 \\ 0.1000 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_x)$ = $q_k^{(i)}(g_x)$ = $q_i^{(j)}(g_x)$ = $q_k^{(j)}(g_x)$ = $q_i^{(k)}(g_x)$ = $q_k^{(k)}(g_x)$ = $q_i^{(l)}(g_x)$ = $q_k^{(l)}(g_x)$ = 0.2000; $q_j^{(i)}(g_x)$ = $q_l^{(i)}(g_x)$ = $q_j^{(j)}(g_x)$ = $q_l^{(j)}(g_x)$ = $q_j^{(k)}(g_x)$ = $q_l^{(k)}(g_x)$ = $q_j^{(l)}(g_x)$ = $q_l^{(l)}(g_x)$ = 0.1000; $CS_i(g_x)$ = $CS_k(g_x)$ = $\frac{1}{2}(0.2000+0.2000+0.2000+0.2000)^2$ = 0.3200; $CS_j(g_x)$ = $CS_l(g_x)$ = $\frac{1}{2}(0.1000+0.1000+0.1000+0.1000)^2$ = 0.0800; $\pi_i^{(i)}(g_x)$ = $\pi_k^{(i)}(g_x)$ = $\pi_i^{(j)}(g_x)$ = $\pi_k^{(j)}(g_x)$ = $\pi_i^{(k)}(g_x)$ = $\pi_k^{(k)}(g_x)$ = $\pi_i^{(l)}(g_x)$ = $\pi_k^{(l)}(g_x)$ = $(0.2000)^2$ = 0.0400; $\pi_j^{(i)}(g_x)$ = $\pi_l^{(i)}(g_x)$ = $\pi_j^{(j)}(g_x)$ = $\pi_l^{(j)}(g_x)$ = $\pi_j^{(k)}(g_x)$ = $\pi_l^{(k)}(g_x)$ = $\pi_j^{(l)}(g_x)$ = $\pi_l^{(l)}(g_x)$ = $(0.1000)^2$ = 0.0100; and $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1918 & -0.0548 & -0.1096 \\ 0.1918 & 1 & -0.1096 & -0.0548 \\ -0.0548 & -0.1096 & 1 & 0.1918 \\ -0.1096 & -0.0548 & 0.1918 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_x) \\ q_j^{(i)}(g_x) \\ q_i^{(j)}(g_x) \\ q_j^{(j)}(g_x) \end{pmatrix} \begin{pmatrix} 0.1644 \\ 0.0822 \\ 0.1644 \\ 0.0822 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_x)$ = $q_k^{(i)}(g_x)$ = $q_i^{(j)}(g_x)$ = $q_k^{(j)}(g_x)$ = $q_i^{(k)}(g_x)$ = $q_k^{(k)}(g_x)$ = $q_i^{(l)}(g_x)$ = $q_k^{(l)}(g_x)$ = 0.1676; $q_j^{(i)}(g_x)$ = $q_l^{(i)}(g_x)$ = $q_j^{(j)}(g_x)$ = $q_l^{(j)}(g_x)$ = $q_j^{(k)}(g_x)$ = $q_l^{(k)}(g_x)$ = $q_j^{(l)}(g_x)$ = $q_l^{(l)}(g_x)$ = 0.0724; $CS_i(g_x)$ = $CS_k(g_x)$ = $\frac{1}{2}(0.1676+0.1676+0.1676+0.1676)^2$ = 0.2247; $CS_j(g_x)$ = $CS_l(g_x)$ = $\frac{1}{2}(0.0724+0.0724+0.0724+0.0724)^2$ = 0.0419; $\pi_i^{(i)}(g_x)$ = $\pi_k^{(i)}(g_x)$ = $\pi_i^{(j)}(g_x)$ =

$\pi_k^{(j)}(g_x) = \pi_i^{(k)}(g_x) = \pi_k^{(k)}(g_x) = \pi_i^{(l)}(g_x) = \pi_k^{(l)}(g_x) = \frac{(2.5)}{2}(0.1676)^2 = 0.0351;$

$\pi_j^{(i)}(g_x) = \pi_l^{(i)}(g_x) = \pi_j^{(j)}(g_x) = \pi_l^{(j)}(g_x) = \pi_j^{(k)}(g_x) = \pi_l^{(k)}(g_x) = \pi_j^{(l)}(g_x) = \pi_l^{(l)}(g_x)$

$= \frac{(2.5)}{2}(0.0724)^2 = 0.0066$; and $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) =$

$\frac{0.5}{4}(0.1676 + 0.0724 + 0.1676 + 0.0724)^2 = 0.0288.$

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.3439 & -0.0764 & -0.1529 \\ 0.3439 & 1 & -0.1529 & -0.0764 \\ -0.0764 & -0.1529 & 1 & 0.3439 \\ -0.1529 & -0.0764 & 0.3439 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_x) \\ q_j^{(i)}(g_x) \\ q_i^{(j)}(g_x) \\ q_j^{(j)}(g_x) \end{pmatrix} \begin{pmatrix} 0.1274 \\ 0.0637 \\ 0.1274 \\ 0.0637 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_x) = q_k^{(i)}(g_x) = q_i^{(j)}(g_x) = q_k^{(j)}(g_x) = q_i^{(k)}(g_x) = q_k^{(k)}(g_x) = q_i^{(l)}(g_x) = q_k^{(l)}(g_x) = 0.1292$; $q_j^{(i)}(g_x) = q_l^{(i)}(g_x) = q_j^{(j)}(g_x) = q_l^{(j)}(g_x) = q_j^{(k)}(g_x) = q_l^{(k)}(g_x) = q_j^{(l)}(g_x) = q_l^{(l)}(g_x) = 0.0423$; $CS_i(g_x) = CS_k(g_x) = \frac{1}{2}(0.1292 + 0.1292 + 0.1292 + 0.1292)^2 = 0.1335$; $CS_j(g_x) = CS_l(g_x) = \frac{1}{2}(0.0423 + 0.0423 + 0.0423 + 0.0423)^2 = 0.0143$; $\pi_i^{(i)}(g_x) = \pi_k^{(i)}(g_x) = \pi_i^{(j)}(g_x) =$

$\pi_k^{(j)}(g_x) = \pi_i^{(k)}(g_x) = \pi_k^{(k)}(g_x) = \pi_i^{(l)}(g_x) = \pi_k^{(l)}(g_x) = \frac{(3.5)}{2}(0.1292)^2 = 0.0292;$

$\pi_j^{(i)}(g_x) = \pi_l^{(i)}(g_x) = \pi_j^{(j)}(g_x) = \pi_l^{(j)}(g_x) = \pi_j^{(k)}(g_x) = \pi_l^{(k)}(g_x) = \pi_j^{(l)}(g_x) = \pi_l^{(l)}(g_x)$

$= \frac{(3.5)}{2}(0.0423)^2 = 0.0031$; $PS_i(g_x) = PS_j(g_x) = PS_k(g_x) = PS_l(g_x) =$ $\frac{1.5}{4}(0.1292+0.0423+0.1292+0.0423)^2 = 0.0441$.

Network y

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_y) = \frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_y)+\phi q_k^{(j)}(g_y)-\phi(\phi+2)q_j^{(i)}(g_y)}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_y) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_y)+\phi q_k^{(j)}(g_y)+\phi q_k^{(k)}(g_y)+\phi q_l^{(k)}(g_y)-\phi(\phi+3)q_i^{(i)}(g_y)}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_y) = \frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_y)-\phi(\phi+2)q_j^{(j)}(g_y)-\phi(\phi+2)q_k^{(j)}(g_y)}{2(\phi+3)(\phi+1)}$; $q_j^{(j)}(g_y) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_y)+\phi q_k^{(k)}(g_y)+\phi q_l^{(k)}(g_y)-\phi(\phi+3)q_i^{(j)}(g_y)-\phi(\phi+3)q_k^{(j)}(g_y)}{2(\phi+4)(\phi+1)}$;

$q_k^{(j)}(g_y) = \frac{2\alpha(\phi+1)+\phi q_j^{(k)}(g_y)+\phi q_l^{(k)}(g_y)+\phi q_l^{(l)}(g_y)-\phi(\phi+3)q_i^{(j)}(g_y)-\phi(\phi+3)q_j^{(j)}(g_y)}{2(\phi+4)(\phi+1)}$;

$q_j^{(k)}(g_y) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_y)+\phi q_k^{(j)}(g_y)+\phi q_i^{(i)}(g_y)-\phi(\phi+3)q_k^{(k)}(g_y)-\phi(\phi+3)q_l^{(k)}(g_y)}{2(\phi+4)(\phi+1)}$;

$q_k^{(k)}(g_y) = \frac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_y)+\phi q_j^{(j)}(g_y)+\phi q_l^{(l)}(g_y)-\phi(\phi+3)q_j^{(k)}(g_y)-\phi(\phi+3)q_l^{(k)}(g_y)}{2(\phi+4)(\phi+1)}$;

$q_l^{(k)}(g_y) = \frac{2\tilde{\alpha}(\phi+1)+\phi q_k^{(l)}(g_y)-\phi(\phi+2)q_j^{(k)}(g_y)-\phi(\phi+2)q_k^{(k)}(g_y)}{2(\phi+3)(\phi+1)}$; $q_k^{(l)}(g_y) =$

$\dfrac{2\alpha(\phi+1)+\phi q_i^{(j)}(g_y)+\phi q_j^{(j)}(g_y)+\phi q_j^{(k)}(g_y)+\phi q_l^{(k)}(g_y)-\phi(\phi+3)q_l^{(l)}(g_y)}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_y)$ = $\dfrac{2\tilde{\alpha}(\phi+1)+\phi q_j^{(k)}(g_y)+\phi q_k^{(k)}(g_y)-\phi(\phi+2)q_k^{(l)}(g_y)}{2(\phi+3)(\phi+1)}$; $CS_i(g_y) = \dfrac{1}{2}\left(q_i^{(i)}(g_y)+q_i^{(j)}(g_y)\right)^2$; $CS_j(g_y) = \dfrac{1}{2}\left(q_j^{(i)}(g_y)+q_j^{(j)}(g_y)+q_j^{(k)}(g_y)\right)^2$; $CS_k(g_y) = \dfrac{1}{2}\left(q_k^{(j)}(g_y)+q_k^{(k)}(g_y)+q_k^{(l)}(g_y)\right)^2$; $CS_l(g_y) = \dfrac{1}{2}\left(q_l^{(k)}(g_y)+q_l^{(l)}(g_y)\right)^2$; $\pi_i^{(i)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_i^{(i)}(g_y)\right)^2$; $\pi_j^{(i)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_j^{(i)}(g_y)\right)^2$; $\pi_i^{(j)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_i^{(j)}(g_y)\right)^2$; $\pi_j^{(j)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_j^{(j)}(g_y)\right)^2$; $\pi_k^{(j)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_k^{(j)}(g_y)\right)^2$; $\pi_j^{(k)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_j^{(k)}(g_y)\right)^2$; $\pi_k^{(k)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_k^{(k)}(g_y)\right)^2$; $\pi_l^{(k)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_l^{(k)}(g_y)\right)^2$; $\pi_k^{(l)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_k^{(l)}(g_y)\right)^2$; $\pi_l^{(l)}(g_y) = \dfrac{(2+\phi)}{2}\left(q_l^{(l)}(g_y)\right)^2$; $PS_i(g_y) = \dfrac{\phi}{4}\left(q_i^{(i)}(g_y)+q_j^{(i)}(g_y)\right)^2$; $PS_j(g_y) = \dfrac{\phi}{4}\left(q_i^{(j)}(g_y)+q_j^{(j)}(g_y)+q_k^{(j)}(g_y)\right)^2$; $PS_k(g_y) = \dfrac{\phi}{4}\left(q_j^{(k)}(g_y)+q_k^{(k)}(g_y)+q_l^{(k)}(g_y)\right)^2$; and $PS_l(g_y) = \dfrac{\phi}{4}\left(q_k^{(l)}(g_y)+q_l^{(l)}(g_y)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \beta_0 & 0 & -\beta_1 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 1 & -\beta_3 & 0 & -\beta_3 & 0 & -\beta_3 & -\beta_3 & 0 & 0 \\ 0 & -\beta_1 & 1 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 \\ -\beta_3 & 0 & \beta_2 & 1 & \beta_2 & 0 & -\beta_3 & -\beta_3 & 0 & 0 \\ 0 & 0 & \beta_2 & \beta_2 & 1 & -\beta_3 & 0 & -\beta_3 & 0 & -\beta_3 \\ -\beta_3 & 0 & -\beta_3 & 0 & -\beta_3 & 1 & \beta_2 & \beta_2 & 0 & 0 \\ 0 & 0 & -\beta_3 & -\beta_3 & 0 & \beta_2 & 1 & \beta_2 & 0 & -\beta_3 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 & 1 & -\beta_1 & 0 \\ 0 & 0 & -\beta_3 & -\beta_3 & 0 & -\beta_3 & 0 & -\beta_3 & 1 & \beta_2 \\ 0 & 0 & 0 & 0 & 0 & -\beta_1 & -\beta_1 & 0 & \beta_0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_j^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_j^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_j^{(k)}(g_y) \\ q_k^{(k)}(g_y) \\ q_l^{(k)}(g_y) \\ q_k^{(l)}(g_y) \\ q_l^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} \beta_4 \\ \beta_5 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_6 \\ \beta_7 \end{pmatrix}$$

Where $\beta_0 = \dfrac{\phi(\phi+2)}{2(\phi+3)(\phi+1)}$; $\beta_1 = \dfrac{\phi}{2(\phi+3)(\phi+1)}$; $\beta_2 = \dfrac{\phi(\phi+3)}{2(\phi+4)(\phi+1)}$; $\beta_3 = \dfrac{\phi}{2(\phi+4)(\phi+1)}$; $\beta_4 = \dfrac{\alpha}{\phi+3}$; $\beta_5 = \dfrac{\tilde{\alpha}}{\phi+4}$; $\beta_6 = \dfrac{\alpha}{\phi+4}$; $\beta_7 = \dfrac{\tilde{\alpha}}{\phi+3}$;

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_y) = q_j^{(j)}(g_y) = q_j^{(k)}(g_y) = q_l^{(k)}(g_y) = q_l^{(l)}(g_y) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_k^{(k)}(g_y) \\ q_k^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.3333 \\ 0.3333 \\ 0.2500 \\ 0.2500 \\ 0.2500 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y) = q_i^{(j)}(g_y) = 0.3333$; $q_k^{(j)}(g_y) = q_k^{(k)}(g_y) = q_k^{(l)}(g_y) = 0.2500$; $CS_i(g_y) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_y) = CS_l(g_y) = 0$; $CS_k(g_y) = \frac{1}{2}(0.2500+0.2500+0.2500)^2 = 0.2813$; $\pi_i^{(i)}(g_y) = \pi_i^{(j)}(g_y) = (0.3333)^2 = 0.1111$;

$\pi_k^{(j)}(g_y) = \pi_k^{(k)}(g_y) = \pi_k^{(l)}(g_y) = (0.2500)^2 = 0.0625$; and $PS_i(g_y) = PS_j(g_y) = PS_k(g_y) =$ $PS_l(g_y) = \frac{\phi}{4}\left(q_k^{(l)}(g_y)\right)^2 = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_y) = q_j^{(j)}(g_y) = q_j^{(k)}(g_y) = q_l^{(k)}(g_y) = q_l^{(l)}(g_y) = 0$. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & -0.0476 & 0 & 0 \\ 0 & 1 & 0.1190 & 0 & 0 \\ 0 & 0.1296 & 1 & 0 & 0 \\ 0 & -0.0370 & 0 & 1 & 0 \\ 0 & -0.0370 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_k^{(k)}(g_y) \\ q_k^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.2857 \\ 0.2857 \\ 0.2222 \\ 0.2222 \\ 0.2222 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y) = 0.2947$; $q_i^{(j)}(g_y) = 0.2633$; $q_k^{(j)}(g_y) = 0.1881$; $q_k^{(k)}(g_y) =$ $q_k^{(l)}(g_y) = 0.2319$; $CS_i(g_y) = \frac{1}{2}(0.2947 + 0.2633)^2 = 0.1557$; $CS_j(g_y) = CS_l(g_y) = 0$; $CS_k(g_y) = \frac{1}{2}(0.1881 + 0.2319 + 0.2319)^2 = 0.2125$; $\pi_i^{(i)}(g_y) = \frac{(2.5)}{2}(0.2947)^2 = 0.1086$; $\pi_i^{(j)}(g_y) = \frac{(2.5)}{2}(0.2633)^2 = 0.0867$; $\pi_k^{(j)}(g_y) = \frac{(2.5)}{2}(0.1881)^2 = 0.0442$; $\pi_k^{(k)}(g_y) =$ $\pi_k^{(l)}(g_y) = \frac{(2.5)}{2}(0.2319)^2 = 0.0672$; $PS_i(g_y) = \frac{0.5}{4}(0.2947)^2 = 0.0109$; $PS_j(g_y) =$ $\frac{0.5}{4}(0.2633 + 0.1881)^2 = 0.0255$; and $PS_k(g_y) = PS_l(g_y) = \frac{0.5}{4}(0.2319)^2 = 0.0067$.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

In this case $q_j^{(i)}(g_y)$ = $q_j^{(j)}(g_y)$ = $q_j^{(k)}(g_y)$ = $q_l^{(k)}(g_y)$ = $q_l^{(l)}(g_y)$ = 0. Therefore, the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & -0.0667 & 0 & 0 \\ 0 & 1 & 0.2333 & 0 & 0 \\ 0 & 0.2455 & 1 & 0 & 0 \\ 0 & -0.0545 & 0 & 1 & 0 \\ 0 & -0.0545 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_k^{(k)}(g_y) \\ q_k^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.2222 \\ 0.1818 \\ 0.1818 \\ 0.1818 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y)$ = 0.2312; $q_i^{(j)}(g_y)$ = 0.1907; $q_k^{(j)}(g_y)$ = 0.1350; $q_k^{(k)}(g_y)$ = $q_k^{(l)}(g_y)$ = 0.1922; $CS_i(g_y)$ = $\frac{1}{2}(0.2312 + 0.1907)^2$ = 0.0890; $CS_j(g_y)$ = $CS_l(g_y)$ = 0; $CS_k(g_y)$ = $\frac{1}{2}(0.1350 + 0.1922 + 0.1922)^2$ = 0.1349; $\pi_i^{(i)}(g_y)$ = $\frac{(3.5)}{2}(0.2312)^2$ = 0.0935; $\pi_i^{(j)}(g_y)$ = $\frac{(3.5)}{2}(0.1907)^2$ = 0.0636; $\pi_k^{(j)}(g_y)$ = $\frac{(3.5)}{2}(0.1350)^2$ = 0.0319; $\pi_k^{(k)}(g_y)$ = $\pi_k^{(l)}(g_y)$ = $\frac{(3.5)}{2}(0.1922)^2$ = 0.0646; $PS_i(g_y)$ = $\frac{1.5}{4}(0.2312)^2$ = 0.0200; $PS_j(g_y)$ = $\frac{1.5}{4}(0.1907 + 0.1350)^2$ = 0.0398; and $PS_k(g_y)$ = $PS_l(g_y)$ = $\frac{1.5}{4}(0.1922)^2$ = 0.0139.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_j^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_j^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_j^{(k)}(g_y) \\ q_k^{(k)}(g_y) \\ q_l^{(k)}(g_y) \\ q_k^{(l)}(g_y) \\ q_l^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.3333 \\ 0.1250 \\ 0.3333 \\ 0.1250 \\ 0.2500 \\ 0.1250 \\ 0.2500 \\ 0.1667 \\ 0.2500 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y) = q_i^{(j)}(g_y) = 0.3333$; $q_j^{(i)}(g_y) = q_j^{(j)}(g_y) = q_j^{(k)}(g_y) = 0.1250$; $q_k^{(j)}(g_y) = q_k^{(k)}(g_y) = q_k^{(l)}(g_y) = 0.2500$; $q_l^{(k)}(g_y) = q_l^{(l)}(g_y) = 0.1667$; $CS_i(g_y) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_j(g_y) = \frac{1}{2}(0.1250 + 0.1250 + 0.1250)^2 = 0.0703$; $CS_k(g_y) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_l(g_y) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $\pi_i^{(i)}(g_y) = \pi_i^{(j)}(g_y) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_y) = \pi_j^{(j)}(g_y) = \pi_j^{(k)}(g_y) = (0.1250)^2 = 0.0156$; $\pi_k^{(j)}(g_y) = \pi_k^{(k)}(g_y) = \pi_k^{(l)}(g_y) = (0.2500)^2 = 0.0625$; $\pi_l^{(k)}(g_y) = \pi_l^{(l)}(g_y) = (0.1667)^2 = 0.0278$; and $PS_i(g_y) = PS_j(g_y) = PS_k(g_y) = PS_l(g_y) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1190 & 0 & -0.0476 & -0.0476 & 0 & 0 & 0 & 0 & 0 \\ 0.1296 & 1 & -0.0370 & 0 & -0.0370 & 0 & -0.0370 & -0.0370 & 0 & 0 \\ 0 & -0.0476 & 1 & 0.1190 & 0.1190 & 0 & 0 & 0 & 0 & 0 \\ -0.0370 & 0 & 0.1296 & 1 & 0.1296 & 0 & -0.0370 & -0.0370 & 0 & 0 \\ 0 & 0 & 0.1296 & 0.1296 & 1 & -0.0370 & 0 & -0.0370 & 0 & -0.0370 \\ -0.0370 & 0 & -0.0370 & 0 & -0.0370 & 1 & 0.1296 & 0.1296 & 0 & 0 \\ 0 & 0 & -0.0370 & -0.0370 & 0 & 0.1296 & 1 & 0.1296 & 0 & -0.0370 \\ 0 & 0 & 0 & 0 & 0 & 0.1190 & 0.1190 & 1 & -0.0476 & 0 \\ 0 & 0 & -0.0370 & -0.0370 & 0 & -0.0370 & 0 & -0.0370 & 1 & 0.1296 \\ 0 & 0 & 0 & 0 & 0 & -0.0476 & -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_j^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_j^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_j^{(k)}(g_y) \\ q_k^{(k)}(g_y) \\ q_l^{(k)}(g_y) \\ q_k^{(l)}(g_y) \\ q_l^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.2857 \\ 0.1111 \\ 0.2857 \\ 0.1111 \\ 0.2222 \\ 0.1111 \\ 0.2222 \\ 0.1429 \\ 0.2222 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y)$ = 0.2862; $q_j^{(i)}(g_y)$ = 0.1028; $q_i^{(j)}(g_y)$ = 0.2588; $q_j^{(j)}(g_y)$ = 0.0755; $q_k^{(j)}(g_y)$ = 0.1916; $q_j^{(k)}(g_y)$ = 0.0957; $q_k^{(k)}(g_y)$ = 0.2118; $q_l^{(k)}(g_y)$ = 0.1170; $q_k^{(l)}(g_y)$ = 0.2255; $q_l^{(l)}(g_y)$ = 0.1307; $CS_i(g_y)$ = $\frac{1}{2}(0.2862+0.2588)^2$ = 0.1485; $CS_j(g_y)$ = $\frac{1}{2}(0.1028+0.0755+0.0957)^2$ = 0.0375; $CS_k(g_y)$ = $\frac{1}{2}(0.1916+0.2118+0.2255)^2$ = 0.1978; $CS_l(g_y)$ = $\frac{1}{2}(0.1170+0.1307)^2$ = 0.0307; $\pi_i^{(i)}(g_y)$ = $\frac{(2.5)}{2}(0.2862)^2$ = 0.1024; $\pi_j^{(i)}(g_y)$ = $\frac{(2.5)}{2}(0.1028)^2$ = 0.0132; $\pi_i^{(j)}(g_y)$ = $\frac{(2.5)}{2}(0.2588)^2$ = 0.0837; $\pi_j^{(j)}(g_y)$ = $\frac{(2.5)}{2}(0.0755)^2$ = 0.0071; $\pi_k^{(j)}(g_y)$ = $\frac{(2.5)}{2}(0.1916)^2$ = 0.0459; $\pi_j^{(k)}(g_y)$ = $\frac{(2.5)}{2}(0.0957)^2$ = 0.0114; $\pi_k^{(k)}(g_y)$ = $\frac{(2.5)}{2}(0.2118)^2$ = 0.0561; $\pi_l^{(k)}(g_y)$ = $\frac{(2.5)}{2}(0.1170)^2$ = 0.0171; $\pi_k^{(l)}(g_y)$ = $\frac{(2.5)}{2}(0.2255)^2$ = 0.0636; $\pi_l^{(l)}(g_y)$ = $\frac{(2.5)}{2}(0.1307)^2$ = 0.0214; $PS_i(g_y)$ = $\frac{0.5}{4}(0.2862+0.1028)^2$ = 0.0189; $PS_j(g_y)$ = $\frac{0.5}{4}(0.2588+0.0755+0.1916)^2$ = 0.0346; $PS_k(g_y)$ = $\frac{0.5}{4}(0.0957+0.2118+0.1170)^2$ = 0.0225; and $PS_l(g_y)$ = $\frac{0.5}{4}(0.2255+0.1307)^2$ = 0.0159.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ *and* $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2333 & 0 & -0.0667 & -0.0667 & 0 & 0 & 0 & 0 & 0 \\ 0.2455 & 1 & -0.0545 & 0 & -0.0545 & 0 & -0.0545 & -0.0545 & 0 & 0 \\ 0 & -0.0667 & 1 & 0.2333 & 0.2333 & 0 & 0 & 0 & 0 & 0 \\ -0.0545 & 0 & 0.2455 & 1 & 0.2455 & 0 & -0.0545 & -0.0545 & 0 & 0 \\ 0 & 0 & 0.2455 & 0.2455 & 1 & -0.0545 & 0 & -0.0545 & 0 & -0.0545 \\ -0.0545 & 0 & -0.0545 & 0 & -0.0545 & 1 & 0.2455 & 0.2455 & 0 & 0 \\ 0 & 0 & -0.0545 & -0.0545 & 0 & 0.2455 & 1 & 0.2455 & 0 & -0.0545 \\ 0 & 0 & 0 & 0 & 0 & 0.2333 & 0.2333 & 1 & -0.0667 & 0 \\ 0 & 0 & -0.0545 & -0.0545 & 0 & -0.0545 & 0 & -0.0545 & 1 & 0.2455 \\ 0 & 0 & 0 & 0 & 0 & -0.0667 & -0.0667 & 0 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_y) \\ q_j^{(i)}(g_y) \\ q_i^{(j)}(g_y) \\ q_j^{(j)}(g_y) \\ q_k^{(j)}(g_y) \\ q_j^{(k)}(g_y) \\ q_k^{(k)}(g_y) \\ q_l^{(k)}(g_y) \\ q_k^{(l)}(g_y) \\ q_l^{(l)}(g_y) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.0909 \\ 0.2222 \\ 0.0909 \\ 0.1818 \\ 0.0909 \\ 0.1818 \\ 0.1111 \\ 0.1818 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_y)$ = 0.2180; $q_j^{(i)}(g_y)$ = 0.0679; $q_i^{(j)}(g_y)$ = 0.1859; $q_j^{(j)}(g_y)$ = 0.0358; $q_k^{(j)}(g_y)$ = 0.1392; $q_j^{(k)}(g_y)$ = 0.0626; $q_k^{(k)}(g_y)$ = 0.1660; $q_l^{(k)}(g_y)$ = 0.0698; $q_k^{(l)}(g_y)$ = 0.1804; $q_l^{(l)}(g_y)$ = 0.0843; $CS_i(g_y) = \frac{1}{2}(0.2180 + 0.1859)^2 = 0.0816$; $CS_j(g_y) = \frac{1}{2}(0.0679 + 0.0358 + 0.0626)^2 = 0.0138$; $CS_k(g_y) = \frac{1}{2}(0.1392 + 0.1660 + 0.1804)^2 = 0.1179$; $CS_l(g_y) = \frac{1}{2}(0.0698 + 0.0843)^2 = 0.0119$; $\pi_i^{(i)}(g_y) = \frac{(3.5)}{2}(0.2180)^2 = 0.0832$; $\pi_j^{(i)}(g_y) = \frac{(3.5)}{2}(0.0679)^2 = 0.0081$; $\pi_i^{(j)}(g_y) = \frac{(3.5)}{2}(0.1859)^2 = 0.0605$; $\pi_j^{(j)}(g_y) = \frac{(3.5)}{2}(0.0358)^2 = 0.0022$; $\pi_k^{(j)}(g_y) = \frac{(3.5)}{2}(0.1392)^2 = 0.0339$; $\pi_j^{(k)}(g_y) = \frac{(3.5)}{2}(0.0626)^2 = 0.0069$; $\pi_k^{(k)}(g_y) = \frac{(3.5)}{2}(0.1660)^2 = 0.0482$; $\pi_l^{(k)}(g_y) = \frac{(3.5)}{2}(0.0698)^2 = 0.0085$; $\pi_k^{(l)}(g_y) = \frac{(3.5)}{2}(0.1804)^2 = 0.0570$; $\pi_l^{(l)}(g_y) = \frac{(3.5)}{2}(0.0843)^2 = 0.0124$; $PS_i(g_y) = \frac{1.5}{4}(0.2180 + 0.0679)^2 = 0.0307$; $PS_j(g_y)$ =

$\frac{1.5}{4}(0.1859+0.0358+0.1392)^2$ = 0.0488; $PS_k(g_y)$ = $\frac{1.5}{4}(0.0626+0.1660+0.0698)^2$ = 0.0334; and $PS_l(g_y)$ = $\frac{1.5}{4}(0.1804+0.0843)^2$ = 0.0263.

Network z

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_z)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(j)}(g_z)-2\phi(\phi+3)q_j^{(i)}(g_z)}{2(\phi+4)(\phi+1)}$; $q_j^{(i)}(g_z)$ = $q_l^{(i)}(g_z)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_z)-\phi(\phi+2)q_i^{(i)}(g_z)}{3\phi^2+10\phi+6}$; $q_i^{(j)}(g_z)$ = $q_i^{(l)}(g_z)$ = $\frac{2\alpha(\phi+1)+2\phi q_j^{(i)}(g_2)-\phi(\phi+2)q_j^{(j)}(g_z)}{2(\phi+4)(\phi+1)}$; $q_j^{(j)}(g_z)$ = $q_l^{(l)}(g_z)$ = $\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_z)+\phi q_j^{(i)}(g_z)-\phi(\phi+2)q_i^{(j)}(g_z)}{2(\phi+3)(\phi+1)}$; $q_k^{(k)}(g_z)$ = $\frac{\alpha}{\phi+2}$; $CS_i(g_z)$ = $\frac{1}{2}\left(q_i^{(i)}(g_z)+q_i^{(j)}(g_z)+q_i^{(l)}(g_z)\right)^2$; $CS_j(g_z)$ = $CS_l(g_z)$ = $\frac{1}{2}\left(q_j^{(i)}(g_z)+q_j^{(j)}(g_z)\right)^2$; $CS_k(g_z)$ = $\frac{1}{2}\left(q_k^{(k)}(g_z)\right)^2$; $\pi_i^{(i)}(g_z)$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_z)\right)^2$; $\pi_j^{(i)}(g_z)$ = $\pi_l^{(i)}(g_z)$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_z)\right)^2$; $\pi_i^{(j)}(g_z)$ = $\pi_i^{(l)}(g_z)$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_z)\right)^2$; $\pi_j^{(j)}(g_z)$ = $\pi_l^{(l)}(g_z)$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_z)\right)^2$; $\pi_k^{(k)}(g_z)$ = $\frac{(2+\phi)}{2}\left(q_k^{(k)}(g_z)\right)^2$; $PS_i(g_z)$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_z)+q_j^{(i)}(g_z)+q_l^{(i)}(g_z)\right)^2$; $PS_j(g_z)$ = $PS_l(g_z)$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_z)+q_j^{(j)}(g_z)\right)^2$; and $PS_k(g_z)$ = $\frac{\phi}{4}\left(q_k^{(k)}(g_z)\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 0 & -\frac{\phi}{(\phi+4)(\phi+1)} \\ \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} & 1 & -\frac{\phi}{3\phi^2+10\phi+6} & 0 \\ 0 & -\frac{\phi}{(\phi+4)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+4)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_z) \\ q_j^{(i)}(g_z) \\ q_i^{(j)}(g_z) \\ q_j^{(j)}(g_z) \end{pmatrix} \begin{pmatrix} \frac{\alpha}{\phi+4} \\ \frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+10\phi+6} \\ \frac{\alpha}{\phi+4} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0

In this case $q_j^{(i)}(g_z)$ = $q_l^{(i)}(g_z)$ = $q_j^{(j)}(g_z)$ = $q_l^{(l)}(g_z)$ = 0. Therefore it holds that $q_i^{(i)}(g_z)$ = $q_i^{(j)}(g_z)$ = $q_i^{(l)}(g_z)$ = 0.2500; $q_k^{(k)}(g_z)$ = 0.5000; $CS_i(g_z)$ = $\frac{1}{2}(0.2500+0.2500+0.2500)^2$ = 0.2813; $CS_j(g_z)$ = $CS_l(g_z)$ = 0; $CS_k(g_z)$ = $\frac{1}{2}(0.5000)^2$ = 0.1250; $\pi_i^{(i)}(g_z)$ = $\pi_i^{(j)}(g_z)$ = $\pi_i^{(l)}(g_z)$ = $(0.2500)^2$ = 0.0625; $\pi_k^{(k)}(g_z)$ = $(0.5000)^2$ = 0.2500; and $PS_i(g_z)$ = $PS_j(g_z)$ = $PS_k(g_z)$ = $PS_l(g_z)$ = 0.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

In this case $q_j^{(i)}(g_z)$ = $q_l^{(i)}(g_z)$ = $q_j^{(j)}(g_z)$ = $q_l^{(l)}(g_z)$ = 0. Therefore it holds that $q_i^{(i)}(g_z)$ = $q_i^{(j)}(g_z)$ = $q_i^{(l)}(g_z)$ = 0.2222; $q_k^{(k)}(g_z)$ = 0.4000; $CS_i(g_z)$ = $\frac{1}{2}(0.2222+0.2222+0.2222)^2$ = 0.2222; $CS_j(g_z)$ = $CS_l(g_z)$ = 0; $CS_k(g_z)$ = $\frac{1}{2}(0.4000)^2$ = 0.0800; $\pi_i^{(i)}(g_z)$ = $\pi_i^{(j)}(g_z)$ = $\pi_i^{(l)}(g_z)$ = $\frac{(2.5)}{2}(0.2222)^2$ = 0.0617; $\pi_k^{(k)}(g_z)$ =

$\frac{(2.5)}{2}(0.4000)^2$ = 0.2000; $PS_i(g_z)$ = $PS_j(g_z)$ = $PS_l(g_z)$ = $\frac{0.5}{4}(0.2222)^2$ = 0.0062; and $PS_k(g_z)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

In this case $q_j^{(i)}(g_z)$ = $q_l^{(i)}(g_z)$= $q_j^{(j)}(g_z)$ = $q_l^{(l)}(g_z)$ = 0. Therefore it holds that $q_i^{(i)}(g_z)$ = $q_i^{(j)}(g_z)$ = $q_i^{(l)}(g_z)$ = 0.1818; $q_k^{(k)}(g_z)$ = 0.2857; $CS_i(g_z)$ = $\frac{1}{2}(0.1818+0.1818+0.1818)^2$ = 0.1487; $CS_j(g_z)$ = $CS_l(g_z)$ = 0; $CS_k(g_z)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $\pi_i^{(i)}(g_z)$ = $\pi_i^{(j)}(g_z)$ = $\pi_i^{(l)}(g_z)$ = $\frac{(3.5)}{2}(0.1818)^2$ = 0.0578; $\pi_k^{(k)}(g_z)$ = $\frac{(3.5)}{2}(0.2857)^2$ = 0.1428; $PS_i(g_z)$ = $PS_j(g_z)$ = $PS_l(g_z)$ = $\frac{1.5}{4}(0.1818)^2$ = 0.0124; and $PS_k(g_z)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306.

Simulation 14: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_z) \\ q_j^{(i)}(g_z) \\ q_i^{(j)}(g_z) \\ q_j^{(j)}(g_z) \end{pmatrix} \begin{pmatrix} 0.2500 \\ 0.1667 \\ 0.2500 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_z) = q_i^{(j)}(g_z) = q_i^{(l)}(g_z) = 0.2500$; $q_j^{(i)}(g_z) = q_l^{(i)}(g_z) = q_j^{(j)}(g_z) = q_l^{(l)}(g_z) = 0.1667$; $q_k^{(k)}(g_z) = 0.5000$; $CS_i(g_z) = \frac{1}{2}(0.2500 + 0.2500 + 0.2500)^2 = 0.2813$; $CS_j(g_z) = CS_l(g_z) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $CS_k(g_z) = \frac{1}{2}(0.5000)^2 = 0.1250$; $\pi_i^{(i)}(g_z) = \pi_i^{(j)}(g_z) = \pi_i^{(l)}(g_z) = (0.2500)^2 = 0.0625$; $\pi_j^{(i)}(g_z) = \pi_l^{(i)}(g_z) = \pi_j^{(j)}(g_z) = \pi_l^{(l)}(g_z) = (0.1667)^2 = 0.0278$; $\pi_k^{(k)}(g_z) = (0.5000)^2 = 0.2500$; and $PS_i(g_z) = PS_j(g_z) = PS_k(g_z) = PS_l(g_z) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2593 & 0 & -0.0741 \\ 0.1064 & 1 & -0.0426 & 0 \\ 0 & -0.0741 & 1 & 0.0926 \\ -0.0476 & -0.0476 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_z) \\ q_j^{(i)}(g_z) \\ q_i^{(j)}(g_z) \\ q_j^{(j)}(g_z) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.1277 \\ 0.2222 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_z) = 0.2020$; $q_j^{(i)}(g_z) = q_l^{(i)}(g_z) = 0.1155$; $q_i^{(j)}(g_z) = q_i^{(l)}(g_z) = 0.2185$; $q_j^{(j)}(g_z) = q_l^{(l)}(g_z) = 0.1320$; $q_k^{(k)}(g_z) = 0.4000$; $CS_i(g_z) = \frac{1}{2}(0.2020 + 0.2185 + 0.2185)^2 = 0.2042$; $CS_j(g_z) = CS_l(g_z) = \frac{1}{2}(0.1155 + 0.1320)^2 = 0.0306$; $CS_k(g_z) = \frac{1}{2}(0.4000)^2 = 0.0800$; $\pi_i^{(i)}(g_z) = \frac{(2.5)}{2}(0.2020)^2 = 0.0510$; $\pi_j^{(i)}(g_z) = \pi_l^{(i)}(g_z) = \frac{(2.5)}{2}(0.1155)^2 = 0.0167$; $\pi_i^{(j)}(g_z) = \pi_i^{(l)}(g_z) = \frac{(2.5)}{2}(0.2185)^2 = 0.0597$; $\pi_j^{(j)}(g_z) = \pi_l^{(l)}(g_z) = \frac{(2.5)}{2}(0.1320)^2 = 0.0218$; $\pi_k^{(k)}(g_z) = \frac{(2.5)}{2}(0.4000)^2 = 0.2000$;

$PS_i(g_z) = \frac{0.5}{4}(0.2020 + 0.1155 + 0.1155)^2 = 0.0234$; $PS_j(g_z) = PS_l(g_z) = \frac{0.5}{4}(0.2185 + 0.1320)^2 = 0.0154$; and $PS_k(g_z) = \frac{0.5}{4}(0.4000)^2 = 0.0200$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.4909 & 0 & -0.1091 \\ 0.1892 & 1 & -0.0541 & 0 \\ 0 & -0.1091 & 1 & 0.1909 \\ -0.0667 & -0.0667 & 0.2333 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_z) \\ q_j^{(i)}(g_z) \\ q_i^{(j)}(g_z) \\ q_j^{(j)}(g_z) \end{pmatrix} \begin{pmatrix} 0.1818 \\ 0.0901 \\ 0.1818 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_z) = 0.1569$; $q_j^{(i)}(g_z) = q_l^{(i)}(g_z) = 0.0698$; $q_i^{(j)}(g_z) = q_i^{(l)}(g_z) = 0.1730$; $q_j^{(j)}(g_z) = q_l^{(l)}(g_z) = 0.0859$; $q_k^{(k)}(g_z) = 0.2857$; $CS_i(g_z) = \frac{1}{2}(0.1569 + 0.1730 + 0.1730)^2 = 0.1265$; $CS_j(g_z) = CS_l(g_z) = \frac{1}{2}(0.0698 + 0.0859)^2 = 0.0121$; $CS_k(g_z) = \frac{1}{2}(0.2857)^2 = 0.0408$; $\pi_i^{(i)}(g_z) = \frac{(3.5)}{2}(0.1569)^2 = 0.0431$; $\pi_j^{(i)}(g_z) = \pi_l^{(i)}(g_z) = \frac{(3.5)}{2}(0.0698)^2 = 0.0085$; $\pi_i^{(j)}(g_z) = \pi_i^{(l)}(g_z) = \frac{(3.5)}{2}(0.1730)^2 = 0.0524$; $\pi_j^{(j)}(g_z) = \pi_l^{(l)}(g_z) = \frac{(3.5)}{2}(0.0859)^2 = 0.0129$; $\pi_k^{(k)}(g_z) = \frac{(3.5)}{2}(0.2857)^2 = 0.1428$; $PS_i(g_z) = \frac{1.5}{4}(0.1569 + 0.0698 + 0.0698)^2 = 0.0330$; $PS_j(g_z) = PS_l(g_z) = \frac{1.5}{4}(0.1730 + 0.0859)^2 = 0.0251$; and $PS_k(g_z) = \frac{1.5}{4}(0.2857)^2 = 0.0306$.

Network a'

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_{a'})$ = $q_k^{(k)}(g_{a'})$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_{a'})+\phi q_i^{(j)}(g_{a'})-\phi(\phi+2)q_j^{(i)}(g_{a'})}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_{a'})$ = $q_j^{(k)}(g_{a'})$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(j)}(g_{a'})-\phi(\phi+2)q_i^{(i)}(g_{a'})}{2(\phi+4)(\phi+1)}$; $q_i^{(j)}(g_{a'})$ = $q_k^{(j)}(g_{a'})$ = $\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_{a'})-\phi(\phi+2)q_j^{(j)}(g_{a'})}{3\phi^2+10\phi+6}$; $q_j^{(j)}(g_{a'})$ = $\frac{2\tilde{\alpha}(\phi+1)+2\phi q_i^{(i)}(g_{a'})-2\phi(\phi+3)q_i^{(j)}(g_{a'})}{2(\phi+4)(\phi+1)}$; $q_l^{(l)}(g_{a'})$ = $\frac{\tilde{\alpha}}{\phi+2}$; $CS_i(g_{a'})$ = $CS_k(g_{a'})$ = $\frac{1}{2}\left(q_i^{(i)}(g_{a'})+q_i^{(j)}(g_{a'})\right)^2$; $CS_j(g_{a'})$ = $\frac{1}{2}\left(q_j^{(i)}(g_{a'})+q_j^{(j)}(g_{a'})+q_j^{(k)}(g_{a'})\right)^2$; $CS_l(g_{a'})$ = $\frac{1}{2}\left(q_l^{(l)}(g_{a'})\right)^2$; $\pi_i^{(i)}(g_{a'})$ = $\pi_k^{(k)}(g_{a'})$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_{a'})\right)^2$; $\pi_j^{(i)}(g_{a'})$ = $\pi_j^{(k)}(g_{a'})$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_{a'})\right)^2$; $\pi_i^{(j)}(g_{a'})$ = $\pi_k^{(j)}(g_{a'})$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_{a'})\right)^2$; $\pi_j^{(j)}(g_{a'})$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_{a'})\right)^2$; $\pi_l^{(l)}(g_{a'})$ = $\frac{(2+\phi)}{2}\left(q_l^{(l)}(g_{a'})\right)^2$; $PS_i(g_{a'})$ = $PS_k(g_{a'})$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_{a'})+q_j^{(i)}(g_{a'})\right)^2$; $PS_j(g_{a'})$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_{a'})+q_j^{(j)}(g_{a'})+q_k^{(j)}(g_{a'})\right)^2$; and $PS_l(g_{a'})$ = $\frac{\phi}{4}\left(q_l^{(l)}(g_{a'})\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} & -\frac{\phi}{2(\phi+3)(\phi+1)} \\ \frac{\phi(\phi+2)}{2(\phi+4)(\phi+1)} & 1 & -\frac{\phi}{(\phi+4)(\phi+1)} & 0 \\ 0 & -\frac{\phi}{3\phi^2+10\phi+6} & 1 & \frac{\phi(\phi+2)}{3\phi^2+10\phi+6} \\ -\frac{\phi}{(\phi+4)(\phi+1)} & 0 & \frac{\phi(\phi+3)}{(\phi+4)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{a'}) \\ q_j^{(i)}(g_{a'}) \\ q_i^{(j)}(g_{a'}) \\ q_j^{(j)}(g_{a'}) \end{pmatrix} \begin{pmatrix} \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+4} \\ \frac{2\alpha(\phi+1)}{3\phi^2+10\phi+6} \\ \frac{\tilde{\alpha}}{\phi+4} \end{pmatrix}$$

Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$

In this case $q_j^{(i)}(g_{a'}) = q_j^{(j)}(g_{a'}) = q_j^{(k)}(g_{a'}) = q_l^{(l)}(g_{a'}) = 0$. Therefore it holds that $q_i^{(i)}(g_{a'}) = q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = q_k^{(k)}(g_{a'}) = 0.3333$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_{a'}) = CS_l(g_{a'}) = 0$; $\pi_i^{(i)}(g_{a'}) = \pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = (0.3333)^2 = 0.1111$; and $PS_i(g_{a'}) = PS_j(g_{a'}) = PS_k(g_{a'}) = PS_l(g_{a'}) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(i)}(g_{a'}) = q_j^{(j)}(g_{a'}) = q_j^{(k)}(g_{a'}) = q_l^{(l)}(g_{a'}) = 0$. Therefore it holds that $q_i^{(i)}(g_{a'}) = q_k^{(k)}(g_{a'}) = 0.2979$; $q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = 0.2553$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.2979+0.2553)^2 = 0.1530$; $CS_j(g_{a'}) = CS_l(g_{a'}) = 0$; $\pi_i^{(i)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = \frac{(2.5)}{2}(0.2979)^2 = 0.1109$; $\pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = \frac{(2.5)}{2}(0.2553)^2 = 0.0815$; $PS_i(g_{a'}) = PS_k(g_{a'}) = \frac{0.5}{4}(0.2979)^2 = 0.0111$; $PS_j(g_{a'}) = \frac{0.5}{4}(0.2553+0.2553)^2 = 0.0323$; and $PS_l(g_{a'}) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(i)}(g_{a'}) = q_j^{(j)}(g_{a'}) = q_j^{(k)}(g_{a'}) = q_l^{(l)}(g_{a'}) = 0$. Therefore it holds that $q_i^{(i)}(g_{a'}) = q_k^{(k)}(g_{a'}) = 0.2342$; $q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = 0.1802$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.2342 + 0.1802)^2 = 0.0859$; $CS_j(g_{a'}) = CS_l(g_{a'}) = 0$; $\pi_i^{(i)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = \frac{(3.5)}{2}(0.2342)^2 = 0.0960$; $\pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = \frac{(3.5)}{2}(0.1802)^2 = 0.0568$; $PS_i(g_{a'}) = PS_k(g_{a'}) = \frac{1.5}{4}(0.2342)^2 = 0.0206$; $PS_j(g_{a'}) = \frac{1.5}{4}(0.1802 + 0.1802)^2 = 0.0487$; and $PS_l(g_{a'}) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{a'}) \\ q_j^{(i)}(g_{a'}) \\ q_i^{(j)}(g_{a'}) \\ q_j^{(j)}(g_{a'}) \end{pmatrix} \begin{pmatrix} 0.3333 \\ 0.1250 \\ 0.3333 \\ 0.125 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{a'}) = q_k^{(k)}(g_{a'}) = q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = 0.3333$; $q_j^{(i)}(g_{a'}) = q_j^{(k)}(g_{a'}) = q_j^{(j)}(g_{a'}) = 0.1250$; $q_l^{(l)}(g_{a'}) = 0.2500$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_j(g_{a'}) = \frac{1}{2}(0.1250 + 0.1250 + 0.1250)^2 = 0.0703$; $CS_l(g_{a'}) = \frac{1}{2}(0.2500)^2 = 0.0313$; $\pi_i^{(i)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = \pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_{a'}) =$

$\pi_j^{(k)}(g_{a'}) = \pi_j^{(j)}(g_{a'}) = (0.1250)^2 = 0.0156$; $\pi_l^{(l)}(g_{a'}) = (0.2500)^2 = 0.0625$; and $PS_i(g_{a'}) = PS_j(g_{a'}) = PS_k(g_{a'}) = PS_l(g_{a'}) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1190 & -0.0476 & -0.0476 \\ 0.0926 & 1 & -0.0741 & 0 \\ 0 & -0.0426 & 1 & 0.1064 \\ -0.0741 & 0 & 0.2593 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{a'}) \\ q_j^{(i)}(g_{a'}) \\ q_i^{(j)}(g_{a'}) \\ q_j^{(j)}(g_{a'}) \end{pmatrix} \begin{pmatrix} 0.2857 \\ 0.1111 \\ 0.2553 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{a'}) = q_k^{(k)}(g_{a'}) = 0.2886$; $q_j^{(i)}(g_{a'}) = q_j^{(k)}(g_{a'}) = 0.1031$; $q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = 0.2526$; $q_j^{(j)}(g_{a'}) = 0.0670$; $q_l^{(l)}(g_{a'}) = 0.2000$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.2886+0.2526)^2 = 0.1464$; $CS_j(g_{a'}) = \frac{1}{2}(0.1031+0.0670+0.1031)^2 = 0.0373$; $CS_l(g_{a'}) = \frac{1}{2}(0.2000)^2 = 0.0200$; $\pi_i^{(i)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = \frac{(2.5)}{2}(0.2886)^2 = 0.1041$; $\pi_j^{(i)}(g_{a'}) = \pi_j^{(k)}(g_{a'}) = \frac{(2.5)}{2}(0.1031)^2 = 0.0133$; $\pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = \frac{(2.5)}{2}(0.2526)^2 = 0.0798$; $\pi_j^{(j)}(g_{a'}) = \frac{(2.5)}{2}(0.0670)^2 = 0.0056$; $\pi_l^{(l)}(g_{a'}) = \frac{(2.5)}{2}(0.2000)^2 = 0.0500$; $PS_i(g_{a'}) = PS_k(g_{a'}) = \frac{0.5}{4}(0.2886+0.1031)^2 = 0.0192$; $PS_j(g_{a'}) = \frac{0.5}{4}(0.2526+0.0670+0.2526)^2 = 0.0409$; and $PS_l(g_{a'}) = \frac{0.5}{4}(0.2000)^2 = 0.0050$.

Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2333 & -0.0667 & -0.0667 \\ 0.1909 & 1 & -0.0667 & 0 \\ 0 & -0.0541 & 1 & 0.1892 \\ -0.0667 & 0 & 0.4909 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{a'}) \\ q_j^{(i)}(g_{a'}) \\ q_i^{(j)}(g_{a'}) \\ q_j^{(j)}(g_{a'}) \end{pmatrix} \begin{pmatrix} 0.2222 \\ 0.0909 \\ 0.1802 \\ 0.0909 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{a'}) = q_k^{(k)}(g_{a'}) = 0.2212$; $q_j^{(i)}(g_{a'}) = q_j^{(k)}(g_{a'}) = 0.0607$; $q_i^{(j)}(g_{a'}) = q_k^{(j)}(g_{a'}) = 0.1802$; $q_j^{(j)}(g_{a'}) = 0.0172$; $q_l^{(l)}(g_{a'}) = 0.1429$; $CS_i(g_{a'}) = CS_k(g_{a'}) = \frac{1}{2}(0.2212+0.1802)^2 = 0.0806$; $CS_j(g_{a'}) = \frac{1}{2}(0.0607+0.0172+0.0607)^2 = 0.0096$; $CS_l(g_{a'}) = \frac{1}{2}(0.1429)^2 = 0.0102$; $\pi_i^{(i)}(g_{a'}) = \pi_k^{(k)}(g_{a'}) = \frac{(3.5)}{2}(0.2212)^2 = 0.0856$; $\pi_j^{(i)}(g_{a'}) = \pi_j^{(k)}(g_{a'}) = \frac{(3.5)}{2}(0.0607)^2 = 0.0064$; $\pi_i^{(j)}(g_{a'}) = \pi_k^{(j)}(g_{a'}) = \frac{(3.5)}{2}(0.1802)^2 = 0.0568$; $\pi_j^{(j)}(g_{a'}) = \frac{(3.5)}{2}(0.0172)^2 = 0.0005$; $\pi_l^{(l)}(g_{a'}) = \frac{(3.5)}{2}(0.1429)^2 = 0.0357$; $PS_i(g_{a'}) = PS_k(g_{a'}) = \frac{1.5}{4}(0.2212+0.0607)^2 = 0.0298$; $PS_j(g_{a'}) = \frac{1.5}{4}(0.1802+0.0172+0.1802)^2 = 0.0535$; and $PS_l(g_{a'}) = \frac{1.5}{4}(0.1429)^2 = 0.0077$.

Network b'

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_{b'}) = q_k^{(k)}(g_{b'}) = \dfrac{2\alpha(\phi+1)+\phi q_j^{(j)}(g_{b'})-\phi(\phi+2)q_j^{(i)}(g_{b'})}{2(\phi+3)(\phi+1)}$; $q_j^{(i)}(g_{b'}) = q_l^{(k)}(g_{b'}) =$

$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(j)}(g_{b'})-\phi(\phi+2)q_i^{(i)}(g_{b'})}{2(\phi+3)(\phi+1)}$; $q_i^{(j)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ =

$\frac{2\alpha(\phi+1)+\phi q_j^{(i)}(g_{b'})-\phi(\phi+2)q_j^{(j)}(g_{b'})}{2(\phi+3)(\phi+1)}$; $q_j^{(j)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ =

$\frac{2\tilde{\alpha}(\phi+1)+\phi q_i^{(i)}(g_{b'})-\phi(\phi+2)q_i^{(j)}(g_{b'})}{2(\phi+3)(\phi+1)}$; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ = $\frac{1}{2}\left(q_i^{(i)}(g_{b'})+q_i^{(j)}(g_{b'})\right)^2$;

$CS_j(g_{b'})$ = $CS_l(g_{b'})$ = $\frac{1}{2}\left(q_j^{(i)}(g_{b'})+q_j^{(j)}(g_{b'})\right)^2$; $\pi_i^{(i)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_{b'})\right)^2$;

$\pi_j^{(i)}(g_{b'})$ = $\pi_l^{(k)}(g_{b'})$ = $\frac{(2+\phi)}{2}\left(q_j^{(i)}(g_{b'})\right)^2$; $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\frac{(2+\phi)}{2}\left(q_i^{(j)}(g_{b'})\right)^2$;

$\pi_j^{(j)}(g_{b'})$ = $\pi_l^{(l)}(g_{b'})$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_{b'})\right)^2$; $PS_i(g_{b'})$ = $PS_k(g_{b'})$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_{b'})+q_j^{(i)}(g_{b'})\right)^2$;

and $PS_j(g_{b'})$ = $PS_l(g_{b'})$ = $\frac{\phi}{4}\left(q_i^{(j)}(g_{b'})+q_j^{(j)}(g_{b'})\right)^2$.

The outputs involved in this network can be obtained by solving the following output matrix:

$$\begin{pmatrix} 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} \\ \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 \\ 0 & -\frac{\phi}{2(\phi+3)(\phi+1)} & 1 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} \\ -\frac{\phi}{2(\phi+3)(\phi+1)} & 0 & \frac{\phi(\phi+2)}{2(\phi+3)(\phi+1)} & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{b'}) \\ q_j^{(i)}(g_{b'}) \\ q_i^{(j)}(g_{b'}) \\ q_j^{(j)}(g_{b'}) \end{pmatrix} \begin{pmatrix} \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+3} \\ \frac{\alpha}{\phi+3} \\ \frac{\tilde{\alpha}}{\phi+3} \end{pmatrix}$$

<u>*Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0*</u>

In this case $q_j^{(i)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0. Therefore it holds that $q_i^{(i)}(g_{b'})$ = $q_i^{(j)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = 0.3333; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ =

$\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_j(g_{b'}) = CS_l(g_{b'}) = 0$; $\pi_i^{(i)}(g_{b'})$ = $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_{b'}) = PS_j(g_{b'}) = PS_k(g_{b'}) = PS_l(g_{b'}) = 0$.

Simulation 12: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0.5

In this case $q_j^{(i)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0. Therefore it holds that $q_i^{(i)}(g_{b'})$ = $q_i^{(j)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = 0.2857; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ = $\frac{1}{2}(0.2857+0.2857)^2$ = 0.1632; $CS_j(g_{b'}) = CS_l(g_{b'}) = 0$; $\pi_i^{(i)}(g_{b'})$ = $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\frac{(2.5)}{2}(0.2857)^2$ = 0.1020; and $PS_i(g_{b'})$ = $PS_j(g_{b'})$ = $PS_k(g_{b'})$ = $PS_l(g_{b'})$ = $\frac{0.5}{4}(0.2857)^2$ = 0.0102.

Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5

In this case $q_j^{(i)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0. Therefore it holds that $q_i^{(i)}(g_{b'})$ = $q_i^{(j)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = 0.2222; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ = $\frac{1}{2}(0.2222+0.2222)^2$ = 0.0988; $CS_j(g_{b'}) = CS_l(g_{b'}) = 0$; $\pi_i^{(i)}(g_{b'})$ = $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\frac{(3.5)}{2}(0.2222)^2$ = 0.0864; and $PS_i(g_{b'})$ = $PS_j(g_{b'})$ = $PS_k(g_{b'})$ = $PS_l(g_{b'})$ = $\frac{1.5}{4}(0.2222)^2$ = 0.0185.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{b'}) \\ q_j^{(i)}(g_{b'}) \\ q_i^{(j)}(g_{b'}) \\ q_j^{(j)}(g_{b'}) \end{pmatrix} \begin{pmatrix} 0.3333 \\ 0.1667 \\ 0.3333 \\ 0.1667 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{b'}) = q_k^{(k)}(g_{b'}) = q_i^{(j)}(g_{b'}) = q_k^{(l)}(g_{b'}) = 0.3333$; $q_j^{(i)}(g_{b'}) = q_l^{(k)}(g_{b'}) = q_j^{(j)}(g_{b'}) = q_l^{(l)}(g_{b'}) = 0.1667$; $CS_i(g_{b'}) = CS_k(g_{b'}) = \frac{1}{2}(0.3333+0.3333)^2 = 0.2222$; $CS_j(g_{b'}) = CS_l(g_{b'}) = \frac{1}{2}(0.1667+0.1667)^2 = 0.0056$; $\pi_i^{(i)}(g_{b'}) = \pi_k^{(k)}(g_{b'}) = \pi_i^{(j)}(g_{b'}) = \pi_k^{(l)}(g_{b'}) = (0.3333)^2 = 0.1111$; $\pi_j^{(i)}(g_{b'}) = \pi_l^{(k)}(g_{b'}) = \pi_j^{(j)}(g_{b'}) = \pi_l^{(l)}(g_{b'}) = (0.1667)^2 = 0.0278$; and $PS_i(g_{b'}) = PS_j(g_{b'}) = PS_k(g_{b'}) = PS_l(g_{b'}) = 0$.

Simulation 15: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0.5$

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.1190 & 0 & -0.0476 \\ 0.1190 & 1 & -0.0476 & 0 \\ 0 & -0.0476 & 1 & 0.1190 \\ -0.0476 & 0 & 0.1190 & 1 \end{pmatrix} \begin{pmatrix} q_i^{(i)}(g_{b'}) \\ q_j^{(i)}(g_{b'}) \\ q_i^{(j)}(g_{b'}) \\ q_j^{(j)}(g_{b'}) \end{pmatrix} \begin{pmatrix} 0.2857 \\ 0.1429 \\ 0.2857 \\ 0.1429 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_i^{(j)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = 0.2769; $q_j^{(i)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0.1231; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ = $\frac{1}{2}(0.2769+0.2769)^2$ = 0.1533; $CS_j(g_{b'})$ = $CS_l(g_{b'})$ = $\frac{1}{2}(0.1231+0.1231)^2$ = 0.0303; $\pi_i^{(i)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\frac{(2.5)}{2}(0.2769)^2$ = 0.0958; $\pi_j^{(i)}(g_{b'})$ = $\pi_l^{(k)}(g_{b'})$ = $\pi_j^{(j)}(g_{b'})$ = $\pi_l^{(l)}(g_{b'})$ = $\frac{(2.5)}{2}(0.1231)^2$ = 0.0189; $PS_i(g_{b'})$ = $PS_j(g_{b'})$ = $PS_k(g_{b'})$ = $PS_l(g_{b'})$ = $\frac{0.5}{4}(0.2769+0.1231)^2$ = 0.0200.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

In this case the output matrix is given by:

$$\begin{pmatrix} 1 & 0.2333 & 0 & -0.0667 \\ 0.2333 & 1 & -0.0667 & 0 \\ 0 & -0.0667 & 1 & 0.2333 \\ -0.0667 & 0 & 0.2333 & 1 \end{pmatrix}\begin{pmatrix} q_i^{(i)}(g_{b'}) \\ q_j^{(i)}(g_{b'}) \\ q_i^{(j)}(g_{b'}) \\ q_j^{(j)}(g_{b'}) \end{pmatrix}\begin{pmatrix} 0.2222 \\ 0.1111 \\ 0.2222 \\ 0.1111 \end{pmatrix}$$

Therefore, $q_i^{(i)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_i^{(j)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = 0.2095; $q_j^{(i)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0.0762; $CS_i(g_{b'})$ = $CS_k(g_{b'})$ = $\frac{1}{2}(0.2095+0.2095)^2$ = 0.0878; $CS_j(g_{b'})$ = $CS_l(g_{b'})$ = $\frac{1}{2}(0.0762+0.0762)^2$ = 0.0116; $\pi_i^{(i)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_i^{(j)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\frac{(3.5)}{2}(0.2095)^2$ = 0.0768; $\pi_j^{(i)}(g_{b'})$ = $\pi_l^{(k)}(g_{b'})$ = $\pi_j^{(j)}(g_{b'})$ = $\pi_l^{(l)}(g_{b'})$ =

$\frac{(3.5)}{2}(0.0762)^2$ = 0.0102; and $PS_i(g_{b'})$ = $PS_j(g_{b'})$ = $PS_k(g_{b'})$ = $PS_l(g_{b'})$ = $\frac{1.5}{4}(0.2095+0.0762)^2$ = 0.0306.

Network c'

In considering the equations presented in Section 4.2.1.2 it holds that $q_i^{(i)}(g_{c'})$ = $q_k^{(i)}(g_{c'})$ = $q_i^{(k)}(g_{c'})$ = $q_k^{(k)}(g_{c'})$ = $\frac{2\alpha(\phi+1)}{3\phi^2+9\phi+6}$; $q_j^{(j)}(g_{c'})$ = $q_l^{(j)}(g_{c'})$ = $q_j^{(l)}(g_{c'})$ = $q_l^{(l)}(g_{c'})$ = $\frac{2\tilde{\alpha}(\phi+1)}{3\phi^2+9\phi+6}$; $CS_i(g_{c'}) = CS_k(g_{c'}) = \frac{1}{2}\left(q_i^{(i)}(g_{c'})+q_i^{(k)}(g_{c'})\right)^2$; $CS_j(g_{c'}) = CS_l(g_{c'})$ = $\frac{1}{2}\left(q_j^{(j)}(g_{c'})+q_j^{(l)}(g_{c'})\right)^2$; $\pi_i^{(i)}(g_{c'})$ = $\pi_k^{(i)}(g_{c'})$ = $\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = $\frac{(2+\phi)}{2}\left(q_i^{(i)}(g_{c'})\right)^2$; $\pi_j^{(j)}(g_{c'})$ = $\pi_l^{(j)}(g_{c'})$ = $\pi_j^{(l)}(g_{c'})$ = $\pi_l^{(l)}(g_{c'})$ = $\frac{(2+\phi)}{2}\left(q_j^{(j)}(g_{c'})\right)^2$; $PS_i(g_{c'})$ = $PS_k(g_{c'})$ = $\frac{\phi}{4}\left(q_i^{(i)}(g_{c'})+q_k^{(i)}(g_{c'})\right)^2$; and $PS_j(g_{c'})$ = $PS_l(g_{c'})$ = $\frac{\phi}{4}\left(q_j^{(j)}(g_{c'})+q_l^{(j)}(g_{c'})\right)^2$.

Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0

In this case $q_j^{(j)}(g_{c'})$ = $q_l^{(j)}(g_{c'})$ = $q_j^{(l)}(g_{c'})$ = $q_l^{(l)}(g_{c'})$ = 0. Therefore $q_i^{(i)}(g_{c'})$ = $q_k^{(i)}(g_{c'})$ = $q_i^{(k)}(g_{c'})$ = $q_k^{(k)}(g_{c'})$ = 0.3333; $CS_i(g_{c'})$ = $CS_k(g_{c'})$ = $\frac{1}{2}(0.3333+0.3333)^2$ = 0.2222; $CS_j(g_{c'}) = CS_l(g_{c'}) = 0$; $\pi_i^{(i)}(g_{c'})$ = $\pi_k^{(i)}(g_{c'})$ = $\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = $(0.3333)^2$ = 0.1111; and $PS_i(g_{c'}) = PS_j(g_{c'}) = PS_k(g_{c'}) = PS_l(g_{c'}) = 0$.

Simulation 12: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0.5$

In this case $q_j^{(j)}(g_{c'}) = q_l^{(j)}(g_{c'}) = q_j^{(l)}(g_{c'}) = q_l^{(l)}(g_{c'}) = 0$. Therefore $q_i^{(i)}(g_{c'}) = q_k^{(i)}(g_{c'}) = q_i^{(k)}(g_{c'}) = q_k^{(k)}(g_{c'}) = 0.2667$; $CS_i(g_{c'}) = CS_k(g_{c'}) = \frac{1}{2}(0.2667 + 0.2667)^2 = 0.1423$; $CS_j(g_{c'}) = CS_l(g_{c'}) = 0$; $\pi_i^{(i)}(g_{c'}) = \pi_k^{(i)}(g_{c'}) = \pi_i^{(k)}(g_{c'}) = \pi_k^{(k)}(g_{c'}) = \frac{(2.5)}{2}(0.2667)^2 = 0.0889$; $PS_i(g_{c'}) = PS_k(g_{c'}) = \frac{0.5}{4}(0.2667 + 0.2667)^2 = 0.0356$; and $PS_j(g_{c'}) = PS_l(g_{c'}) = 0$.

Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$

In this case $q_j^{(j)}(g_{c'}) = q_l^{(j)}(g_{c'}) = q_j^{(l)}(g_{c'}) = q_l^{(l)}(g_{c'}) = 0$. Therefore $q_i^{(i)}(g_{c'}) = q_k^{(i)}(g_{c'}) = q_i^{(k)}(g_{c'}) = q_k^{(k)}(g_{c'}) = 0.1905$; $CS_i(g_{c'}) = CS_k(g_{c'}) = \frac{1}{2}(0.1905 + 0.1905)^2 = 0.0726$; $CS_j(g_{c'}) = CS_l(g_{c'}) = 0$; $\pi_i^{(i)}(g_{c'}) = \pi_k^{(i)}(g_{c'}) = \pi_i^{(k)}(g_{c'}) = \pi_k^{(k)}(g_{c'}) = \frac{(3.5)}{2}(0.1905)^2 = 0.0635$; $PS_i(g_{c'}) = PS_k(g_{c'}) = \frac{1.5}{4}(0.1905 + 0.1905)^2 = 0.0544$; and $PS_j(g_{c'}) = PS_l(g_{c'}) = 0$.

Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$

In this case it holds that $q_i^{(i)}(g_{c'}) = q_k^{(i)}(g_{c'}) = q_i^{(k)}(g_{c'}) = q_k^{(k)}(g_{c'}) = 0.3333$; $q_j^{(j)}(g_{c'}) = q_l^{(j)}(g_{c'}) = q_j^{(l)}(g_{c'}) = q_l^{(l)}(g_{c'}) = 0.1667$; $CS_i(g_{c'}) = CS_k(g_{c'}) = \frac{1}{2}(0.3333 + 0.3333)^2 = 0.2222$; $CS_j(g_{c'}) = CS_l(g_{c'}) = \frac{1}{2}(0.1667 + 0.1667)^2 = 0.0556$; $\pi_i^{(i)}(g_{c'}) = \pi_k^{(i)}(g_{c'}) =$

$\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = $(0.3333)^2$ = 0.1111; $\pi_j^{(j)}(g_{c'})$ = $\pi_l^{(j)}(g_{c'})$ = $\pi_j^{(l)}(g_{c'})$ = $\pi_l^{(l)}(g_{c'})$ = $(0.1667)^2$ = 0.0278; $PS_i(g_{c'})$ = $PS_j(g_{c'})$ = $PS_k(g_{c'})$ = $PS_l(g_{c'})$ = 0.

Simulation 15: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 0.5

In this case it holds that $q_i^{(i)}(g_{c'})$ = $q_k^{(i)}(g_{c'})$ = $q_i^{(k)}(g_{c'})$ = $q_k^{(k)}(g_{c'})$ = 0.2667; $q_j^{(j)}(g_{c'})$ = $q_l^{(j)}(g_{c'})$ = $q_j^{(l)}(g_{c'})$ = $q_l^{(l)}(g_{c'})$ = 0.1333; $CS_i(g_{c'})$ = $CS_k(g_{c'})$ = $\frac{1}{2}(0.2667+0.2667)^2$ = 0.1423; $CS_j(g_{c'})$ = $CS_l(g_{c'})$ = $\frac{1}{2}(0.1333+0.1333)^2$ = 0.0355; $\pi_i^{(i)}(g_{c'})$ = $\pi_k^{(i)}(g_{c'})$ = $\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = $\frac{(2.5)}{2}(0.2667)^2$ = 0.0889; $\pi_j^{(j)}(g_{c'})$ = $\pi_l^{(j)}(g_{c'})$ = $\pi_j^{(l)}(g_{c'})$ = $\pi_l^{(l)}(g_{c'})$ = $\frac{(2.5)}{2}(0.1333)^2$ = 0.0222; $PS_i(g_{c'})$ = $PS_k(g_{c'})$ = $\frac{0.5}{4}(0.2667+0.2667)^2$ = 0.0356; and $PS_j(g_{c'})$ = $PS_l(g_{c'})$ = $\frac{0.5}{4}(0.1333+0.1333)^2$ = 0.0089.

Simulation 16: α = 1, $\tilde{\alpha}$ = 0.5 and ϕ = 1.5

In this case it holds that $q_i^{(i)}(g_{c'})$ = $q_k^{(i)}(g_{c'})$ = $q_i^{(k)}(g_{c'})$ = $q_k^{(k)}(g_{c'})$ = 0.1905; $q_j^{(j)}(g_{c'})$ = $q_l^{(j)}(g_{c'})$ = $q_j^{(l)}(g_{c'})$ = $q_l^{(l)}(g_{c'})$ = 0.0952; $CS_i(g_{c'})$ = $CS_k(g_{c'})$ = $\frac{1}{2}(0.1905+0.1905)^2$ = 0.0726; $CS_j(g_{c'})$ = $CS_l(g_{c'})$ = $\frac{1}{2}(0.0952+0.0952)^2$ = 0.0181; $\pi_i^{(i)}(g_{c'})$ = $\pi_k^{(i)}(g_{c'})$ = $\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = $\frac{(3.5)}{2}(0.1905)^2$ = 0.0635; $\pi_j^{(j)}(g_{c'})$ = $\pi_l^{(j)}(g_{c'})$ = $\pi_j^{(l)}(g_{c'})$ =

$\pi_l^{(l)}(g_{c'}) = \frac{(3.5)}{2}(0.0952)^2 = 0.0159$; $PS_i(g_{c'}) = PS_k(g_{c'}) = \frac{1.5}{4}(0.1905 + 0.1905)^2 =$ 0.0544; and $PS_j(g_{c'}) = PS_l(g_{c'}) = \frac{1.5}{4}(0.0952 + 0.0952)^2 = 0.0136$.

APPENDIX D

Simulations for the case of asymmetric countries in terms of farmers' productivity

Network a

In considering the equations presented in Section 4.2.1.3, the following results are obtained: $q_i^{(i)}(g_a) = q_k^{(k)}(g_a) = 0.2857$; and $q_j^{(j)}(g_a) = q_l^{(l)}(g_a) = 0.4000$. Therefore $CS_i(g_a) = CS_k(g_a)$ = $\frac{1}{2}(0.2857)^2$ = 0.0408; $CS_j(g_a) = CS_l(g_a)$ = 0.0800; $\pi_i(g_a)$ = $\pi_k(g_a)$ = $\frac{(4.5)}{2}(0.4000)^2$ = 0.1837; $\pi_j(g_a)$ = $\pi_l(g_a)$ = $\frac{(4.5)}{2}(0.4000)^2$ = 0.2000; $PS_i(g_a) = PS_k(g_a)$ = $\frac{1.5}{4}(0.2857)^2$ = 0.0306; $PS_j(g_a) = PS_l(g_a)$ = $\frac{0.5}{4}(0.4000)^2$ = 0.0200; $W_i(g_a)$ = $W_k(g_a)$ = 0.2551; $W_j(g_a)$ = $W_l(g_a)$ = 0.3000.

Network b

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_b) = 0.1935 - 0.2419 q_j^{(i)}(g_b) + 0.0323 q_j^{(j)}(g_b)$$

$$q_j^{(i)}(g_b) = 0.1935 - 0.2419 q_i^{(i)}(g_b) + 0.0323 q_i^{(j)}(g_b)$$

$$q_i^{(j)}(g_b) = 0.3226 + 0.0968 q_j^{(i)}(g_b) - 0.1129 q_j^{(j)}(g_b)$$

$q_j^{(j)}(g_a) = 0.3226 + 0.0968 q_i^{(i)}(g_a) - 0.1129 q_i^{(j)}(g_a)$

$q_k^{(k)}(g_a) = 0.4000$

$q_l^{(l)}(g_a) = 0.1600$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_b)$ = $q_j^{(i)}(g_b)$ = 0.1637; $q_i^{(j)}(g_b)$ = $q_j^{(j)}(g_b)$ = 0.3041; $q_k^{(k)}(g_b)$ = 0.2857; $q_l^{(l)}(g_b)$ = 0.4000; $CS_i(g_b)$ = $CS_j(g_b)$ = 0.1094; $CS_k(g_b)$ = 0.0408; $CS_l(g_b)$ = 0.0800; $\pi_i^{(i)}(g_b)$ = $\pi_j^{(i)}(g_b)$ = 0.0603; $\pi_i^{(j)}(g_b)$ = $\pi_j^{(j)}(g_b)$ = 0.1156; $\pi_k^{(k)}(g_b)$ = 0.1837; $\pi_l^{(l)}(g_b)$ = 0.2000; $PS_i(g_b)$ = 0.0402; $PS_j(g_b)$ = 0.0462; $PS_k(g_b)$ = 0.0306; $PS_l(g_b)$ = 0.0200; $W_i(g_b)$ = 0.2703; $W_j(g_b)$ = 0.3869; $W_k(g_b)$ = 0.2551; and $W_l(g_b)$ = 0.3000.

Network c

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$q_i^{(i)}(g_c) = 0.2222 - 0.2333 q_k^{(i)}(g_c) + 0.0667 q_k^{(k)}(g_c)$

$q_k^{(i)}(g_c) = 0.2222 - 0.2333 q_i^{(i)}(g_c) + 0.0667 q_i^{(k)}(g_c)$

$q_j^{(j)}(g_c) = 0.4000$

$q_i^{(k)}(g_c) = 0.2222 + 0.0667 q_k^{(i)}(g_c) - 0.2333 q_k^{(k)}(g_c)$

$q_k^{(k)}(g_c) = 0.2222 + 0.0667 q_i^{(i)}(g_c) - 0.2333 q_i^{(k)}(g_c)$

$q_l^{(l)}(g_c) = 0.4000$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_c) = q_k^{(i)}(g_c) = q_i^{(k)}(g_c) = q_k^{(k)}(g_c) = 0.1905$; $q_j^{(j)}(g_c) = q_l^{(l)}(g_c) = 0.4000$; $CS_i(g_c) = CS_k(g_c) = 0.0726$; $CS_j(g_c) = CS_l(g_c) = 0.0800$; $\pi_i^{(i)}(g_c) = \pi_k^{(i)}(g_c) = \pi_i^{(k)}(g_c) = \pi_k^{(k)}(g_c) = 0.0816$; $\pi_j^{(j)}(g_c) = \pi_l^{(l)}(g_c) = 0.4000$; $PS_i(g_c) = PS_k(g_c) = 0.0544$; $PS_j(g_c) = PS_l(g_c) = 0.0200$; $W_i(g_c) = W_k(g_c) = 0.2902$; and $W_j(g_c) = W_l(g_c) = 0.3000$.

Network d

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_d) = 0.2857$$

$$q_j^{(j)}(g_d) = 0.2857 - 0.1190 q_l^{(j)}(g_d) + 0.0476 q_l^{(l)}(g_d)$$

$$q_l^{(j)}(g_d) = 0.2857 - 0.1190 q_j^{(j)}(g_d) + 0.0476 q_j^{(l)}(g_d)$$

$$q_k^{(k)}(g_d) = 0.2857$$

$$q_j^{(l)}(g_d) = 0.2857 + 0.0476 q_l^{(j)}(g_d) - 0.1190 q_l^{(l)}(g_d)$$

$$q_l^{(l)}(g_d) = 0.2857 + 0.0476 q_j^{(j)}(g_d) - 0.1190 q_j^{(l)}(g_d)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_d)$ = $q_k^{(k)}(g_d)$ = 0.2857; $q_j^{(j)}(g_d)$ = $q_l^{(j)}(g_d)$ = $q_j^{(l)}(g_d)$ = $q_l^{(l)}(g_d)$ = 0.2667; $CS_i(g_d)$ = $CS_k(g_d)$ = 0.0408; $CS_j(g_d)$ = $CS_l(g_d)$ = 0.1422; $\pi_i^{(i)}(g_d)$ = $\pi_k^{(k)}(g_d)$ = 0.1837; $\pi_j^{(j)}(g_d)$ = $\pi_l^{(j)}(g_d)$ = $\pi_j^{(l)}(g_d)$ = $\pi_l^{(l)}(g_d)$ = 0.0889; $PS_i(g_d)$ = $PS_k(g_d)$ = 0.0306; $PS_j(g_d)$ = $PS_l(g_d)$ = 0.0356; $W_i(g_d)$ = $W_k(g_d)$ = 0.2551; and $W_j(g_d)$ = $W_l(g_d)$ = 0.3556.

Network e

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_e) = 0.1622 - 0.2514q_j^{(i)}(g_e) - 0.2514q_k^{(i)}(g_e) + 0.0270q_j^{(j)}(g_e) + 0.0486q_k^{(k)}(g_e)$$

$$q_j^{(i)}(g_e) = 0.1935 - 0.2419q_i^{(i)}(g_e) - 0.2419q_k^{(i)}(g_e) + 0.0323q_i^{(j)}(g_e)$$

$$q_k^{(i)}(g_e) = 0.2222 - 0.2333q_i^{(i)}(g_e) - 0.2333q_j^{(i)}(g_e) + 0.0667q_i^{(k)}(g_e)$$

$$q_i^{(j)}(g_e) = 0.2703 + 0.0811q_j^{(i)}(g_e) + 0.0811q_k^{(i)}(g_e) - 0.1216q_j^{(j)}(g_e) + 0.0811q_k^{(k)}(g_e)$$

$$q_j^{(j)}(g_e) = 0.3226 + 0.0968q_i^{(i)}(g_e) + 0.0968q_k^{(i)}(g_e) - 0.1129q_i^{(j)}(g_e)$$

$$q_i^{(k)}(g_e) = 0.1622 + 0.0486q_j^{(i)}(g_e) + 0.0486q_k^{(i)}(g_e) + 0.0270q_j^{(j)}(g_e) - 0.2514q_k^{(k)}(g_e)$$

$$q_k^{(k)}(g_e) = 0.1935 + 0.0581q_i^{(i)}(g_e) + 0.0581q_j^{(i)}(g_e) - 0.2419q_i^{(k)}(g_e)$$

$$q_l^{(l)}(g_e) = 0.4000$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_e)$ = 0.1008; $q_j^{(i)}(g_e)$ = 0.1352; $q_k^{(i)}(g_e)$ = 0.1767; $q_i^{(j)}(g_e)$ = 0.2708; $q_j^{(j)}(g_e)$ = 0.3189; $q_i^{(k)}(g_e)$ = 0.1425; $q_k^{(k)}(g_e)$ = 0.1728; $q_l^{(l)}(g_e)$ = 0.2000; $CS_i(g_e)$ = 0.1322; $CS_j(g_e)$ = 0.1031; $CS_k(g_e)$ = 0.0611; $CS_l(g_e)$ = 0.0800; $\pi_i^{(i)}(g_e)$ = 0.0229; $\pi_j^{(i)}(g_e)$ = 0.0411; $\pi_k^{(i)}(g_e)$ = 0.0702; $\pi_i^{(j)}(g_e)$ = 0.0917; $\pi_j^{(j)}(g_e)$ = 0.1271; $\pi_i^{(k)}(g_e)$ = 0.0457; $\pi_k^{(k)}(g_e)$ = 0.0672; $\pi_l^{(l)}(g_e)$ = 0.2000; $PS_i(g_e)$ = 0.0638; $PS_j(g_e)$ = 0.0435; $PS_k(g_e)$ = 0.0373; $PS_l(g_e)$ = 0.0200; $W_i(g_e)$ = 0.3302; $W_j(g_e)$ = 0.3653; $W_k(g_e)$ = 0.2112; and $W_l(g_e)$ = 0.3000.

Network f

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_f) = 0.1935 - 0.2419 q_j^{(i)}(g_f) + 0.0323 q_j^{(j)}(g_f) + 0.0323 q_l^{(j)}(g_f)$$

$$q_j^{(i)}(g_f) = 0.1463 - 0.2561 q_i^{(i)}(g_f) + 0.0244 q_i^{(j)}(g_f) + 0.0244 q_l^{(j)}(g_f) + 0.0244 q_l^{(l)}(g_f)$$

$$q_i^{(j)}(g_f) = 0.3226 + 0.0968 q_j^{(i)}(g_f) - 0.1129 q_j^{(j)}(g_f) - 0.1129 q_l^{(j)}(g_f)$$

$$q_j^{(j)}(g_f) = 0.2439 + 0.0732 q_i^{(i)}(g_f) - 0.1260 q_i^{(j)}(g_f) - 0.1260 q_l^{(j)}(g_f) + 0.0407 q_l^{(l)}(g_f)$$

$$q_l^{(j)}(g_f) = 0.2857 - 0.1190 q_i^{(j)}(g_f) - 0.1190 q_j^{(j)}(g_f) + 0.0476 q_j^{(l)}(g_f)$$

$$q_k^{(k)}(g_f) = 0.2857$$

$$q_j^{(l)}(g_f) = 0.2439 + 0.0732 q_i^{(i)}(g_f) + 0.0407 q_i^{(j)}(g_f) + 0.0407 q_j^{(j)}(g_f) - 0.1260 q_l^{(l)}(g_f)$$

$$q_l^{(l)}(g_f) = 0.2857 + 0.0476 q_i^{(j)}(g_f) + 0.0476 q_j^{(j)}(g_f) - 0.1190 q_j^{(l)}(g_f)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_f)$ = 0.1787; $q_j^{(i)}(g_f)$ = 0.1202; $q_i^{(j)}(g_f)$ = 0.2843; $q_j^{(j)}(g_f)$ = 0.2024; $q_l^{(j)}(g_f)$ = 0.2393; $q_k^{(k)}(g_f)$ = 0.2857; $q_j^{(l)}(g_f)$ = 0.2415; $q_l^{(l)}(g_f)$ = 0.2801; $CS_i(g_f)$ = 0.1072; $CS_j(g_f)$ = 0.1591; $CS_k(g_f)$ = 0.0408; $CS_l(g_f)$ = 0.1349; $\pi_i^{(i)}(g_f)$ = 0.0719; $\pi_j^{(i)}(g_f)$ = 0.0325; $\pi_i^{(j)}(g_f)$ = 0.1011; $\pi_j^{(j)}(g_f)$ = 0.0512; $\pi_l^{(j)}(g_f)$ = 0.0716; $\pi_k^{(k)}(g_f)$ = 0.1837; $\pi_j^{(l)}(g_f)$ = 0.0729; $\pi_l^{(l)}(g_f)$ = 0.0981; $PS_i(g_f)$ = 0.0335; $PS_j(g_f)$ = 0.0659; $PS_k(g_f)$ = 0.0306; $PS_l(g_f)$ = 0.0340; $W_i(g_f)$ = 0.2451; $W_j(g_f)$ = 0.4488;$W_k(g_f)$ = 0.2551; and $W_l(g_f)$ = 0.3399.

Network g

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_g) = 0.1277 - 0.2617q_j^{(i)}(g_g) - 0.2617q_k^{(i)}(g_g) - 0.2617q_l^{(i)}(g_g) + 0.0213q_j^{(j)}(g_g) + 0.0383q_k^{(k)}(g_g) + 0.0213q_l^{(l)}(g_g)$$

$$q_j^{(i)}(g_g) = 0.1935 - 0.2419q_i^{(i)}(g_g) - 0.2419q_k^{(i)}(g_g) - 0.2419q_l^{(i)}(g_g) + 0.0323q_i^{(j)}(g_g)$$

$$q_k^{(i)}(g_g) = 0.2222 - 0.2333q_i^{(i)}(g_g) - 0.2333q_j^{(i)}(g_g) - 0.2333q_l^{(i)}(g_g) + 0.0667q_i^{(k)}(g_g)$$

$$q_l^{(i)}(g_g) = 0.1935 - 0.2419q_i^{(i)}(g_g) - 0.2419q_j^{(i)}(g_g) - 0.2419q_k^{(i)}(g_g) + 0.0323q_i^{(l)}(g_g)$$

$$q_i^{(j)}(g_g) = 0.2128 + 0.0638 q_j^{(i)}(g_g) + 0.0638 q_k^{(i)}(g_g) + 0.0638 q_l^{(i)}(g_g) - 0.1312 q_j^{(j)}(g_g) + 0.0638 q_k^{(k)}(g_g) + 0.0355 q_l^{(l)}(g_g)$$

$$q_j^{(j)}(g_g) = 0.3226 + 0.0968 q_i^{(i)}(g_g) + 0.0968 q_k^{(i)}(g_g) + 0.0968 q_l^{(i)}(g_g) - 0.1129 q_i^{(j)}(g_g)$$

$$q_i^{(k)}(g_g) = 0.1277 + 0.0383 q_j^{(i)}(g_g) + 0.0383 q_k^{(i)}(g_g) + 0.0383 q_l^{(i)}(g_g) + 0.0213 q_j^{(j)}(g_g) - 0.2617 q_k^{(k)}(g_g) + 0.0213 q_l^{(l)}(g_g)$$

$$q_k^{(k)}(g_g) = 0.2222 + 0.0667 q_i^{(i)}(g_g) + 0.0667 q_j^{(i)}(g_g) + 0.0667 q_l^{(i)}(g_g) - 0.23333 q_i^{(k)}(g_g)$$

$$q_i^{(l)}(g_g) = 0.2128 + 0.0638 q_j^{(i)}(g_g) + 0.0638 q_k^{(i)}(g_g) + 0.0638 q_l^{(i)}(g_g) + 0.0355 q_j^{(j)}(g_g) + 0.0638 q_k^{(k)}(g_g) - 0.1312 q_l^{(l)}(g_g)$$

$$q_l^{(l)}(g_g) = 0.3226 + 0.0968 q_i^{(i)}(g_g) + 0.0968 q_j^{(i)}(g_g) + 0.0968 q_k^{(i)}(g_g) - 0.1129 q_i^{(l)}(g_g)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_g)$ = 0.0441; $q_j^{(i)}(g_g)$ = 0.1215; $q_k^{(i)}(g_g)$ = 0.1619; $q_l^{(i)}(g_g)$ = 0.1215; $q_i^{(j)}(g_g)$ = 0.2210; $q_j^{(j)}(g_g)$ = 0.3293; $q_i^{(k)}(g_g)$ = 0.1001; $q_k^{(k)}(g_g)$ = 0.2180; $q_i^{(l)}(g_g)$ = 0.2210; $q_l^{(l)}(g_g)$ = 0.3293; $CS_i(g_g)$ = 0.1718; $CS_j(g_g)$ = 0.1016; $CS_k(g_g)$ = 0.0722; $CS_l(g_g)$ = 0.1016; $\pi_i^{(i)}(g_g)$ = 0.0044; $\pi_j^{(i)}(g_g)$ = 0.0332; $\pi_k^{(i)}(g_g)$ = 0.0590; $\pi_l^{(i)}(g_g)$ = 0.0332; $\pi_i^{(j)}(g_g)$ = 0.0610; $\pi_j^{(j)}(g_g)$ = 0.1356; $\pi_i^{(k)}(g_g)$ = 0.0226; $\pi_k^{(k)}(g_g)$ = 0.1069; $\pi_i^{(l)}(g_g)$ = 0.0610; $\pi_l^{(l)}(g_g)$ = 0.1356; $PS_i(g_g)$ = 0.0756; $PS_j(g_g)$ = 0.0379; $PS_k(g_g)$ = 0.0380; $PS_l(g_g)$ = 0.0379; $W_i(g_g)$ = 0.3771; $W_j(g_g)$ =0.3361; $W_k(g_g)$ = 0.2396; and $W_l(g_g)$ = 0.3361.

Network h

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_h) = 0.1622 - 0.2514 q_j^{(i)}(g_h) - 0.2514 q_k^{(i)}(g_h) + 0.0270 q_j^{(j)}(g_h) + 0.0486 q_k^{(k)}(g_h) + 0.0486 q_l^{(k)}(g_h)$$

$$q_j^{(i)}(g_h) = 0.1935 - 0.2419 q_i^{(i)}(g_h) - 0.2419 q_k^{(i)}(g_h) + 0.0323 q_i^{(j)}(g_h)$$

$$q_k^{(i)}(g_h) = 0.1622 - 0.2514 q_i^{(i)}(g_h) - 0.2514 q_j^{(i)}(g_h) + 0.0486 q_i^{(k)}(g_h) + 0.0486 q_l^{(k)}(g_h) + 0.0270 q_l^{(l)}(g_h)$$

$$q_i^{(j)}(g_h) = 0.2703 + 0.0811 q_j^{(i)}(g_h) + 0.0811 q_k^{(i)}(g_h) - 0.1216 q_j^{(j)}(g_h) + 0.0811 q_k^{(k)}(g_h) + 0.0811 q_l^{(k)}(g_h)$$

$$q_j^{(j)}(g_h) = 0.3226 + 0.0968 q_i^{(i)}(g_h) + 0.0968 q_k^{(i)}(g_h) - 0.1129 q_i^{(j)}(g_h)$$

$$q_i^{(k)}(g_h) = 0.1622 + 0.0486 q_j^{(i)}(g_h) + 0.0486 q_k^{(i)}(g_h) + 0.0270 q_j^{(j)}(g_h) - 0.2514 q_k^{(k)}(g_h) - 0.2514 q_l^{(k)}(g_h)$$

$$q_k^{(k)}(g_h) = 0.1622 + 0.0486 q_i^{(i)}(g_h) + 0.0486 q_j^{(i)}(g_h) - 0.2514 q_i^{(k)}(g_h) - 0.2514 q_l^{(k)}(g_h) + 0.0270 q_l^{(l)}(g_h)$$

$$q_l^{(k)}(g_h) = 0.1935 - 0.2419 q_i^{(k)}(g_h) - 0.2419 q_k^{(k)}(g_h) + 0.0323 q_k^{(l)}(g_h)$$

$$q_k^{(l)}(g_h) = 0.2703 + 0.0811 q_i^{(i)}(g_h) + 0.0811 q_j^{(i)}(g_h) + 0.0811 q_i^{(k)}(g_h) + 0.0811 q_l^{(k)}(g_h) - 0.1216 q_l^{(l)}(g_h)$$

$$q_l^{(l)}(g_h) = 0.3226 + 0.0968 q_i^{(k)}(g_h) + 0.0968 q_k^{(k)}(g_h) - 0.1129 q_k^{(l)}(g_h)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_h)$ = $q_k^{(i)}(g_h)$ = $q_i^{(k)}(g_h)$ = $q_k^{(k)}(g_h)$ = 0.1174; $q_j^{(i)}(g_h)$ = $q_l^{(k)}(g_h)$ = 0.1456; $q_i^{(j)}(g_h)$ = $q_k^{(l)}(g_h)$ = 0.2747; $q_j^{(j)}(g_h)$ = $q_l^{(l)}(g_h)$ = 0.3143; $CS_i(g_h)$ = $CS_k(g_h)$ = 0.1297; $CS_j(g_h)$ = $CS_l(g_h)$ = 0.1058; $\pi_i^{(i)}(g_h)$ = $\pi_k^{(i)}(g_h)$ = $\pi_i^{(k)}(g_h)$ = $\pi_k^{(k)}(g_h)$ = 0.0310; $\pi_j^{(i)}(g_h)$ = $\pi_l^{(k)}(g_h)$ = 0.0477; $\pi_i^{(j)}(g_h)$ = $\pi_k^{(l)}(g_h)$ = 0.0943; $\pi_j^{(j)}(g_h)$ = $\pi_l^{(l)}(g_h)$ = 0.1235; $PS_i(g_h)$ = $PS_k(g_h)$ = 0.0542; $PS_j(g_h)$ = $PS_l(g_h)$ = 0.0434; $W_i(g_h)$ = $W_k(g_h)$ = 0.2937; and $W_j(g_h)$ = $W_l(g_h)$ = 0.3669.

Network i

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_i) = 0.1622 - 0.2514q_j^{(i)}(g_i) - 0.2514q_k^{(i)}(g_i) + 0.0270q_j^{(j)}(g_i) + 0.0270q_l^{(j)}(g_i) + 0.0486q_k^{(k)}(g_i)$$

$$q_j^{(i)}(g_i) = 0.1463 - 0.2561q_i^{(i)}(g_i) - 0.2561q_k^{(i)}(g_i) + 0.0244q_i^{(j)}(g_i) + 0.0244q_l^{(j)}(g_i) + 0.0244q_l^{(l)}(g_i)$$

$$q_k^{(i)}(g_i) = 0.2222 - 0.2333q_i^{(i)}(g_i) - 0.2333q_j^{(i)}(g_i) + 0.0667q_i^{(k)}(g_i)$$

$$q_i^{(j)}(g_i) = 0.2703 + 0.0811q_j^{(i)}(g_i) + 0.0811q_k^{(i)}(g_i) - 0.1216q_j^{(j)}(g_i) - 0.1216q_l^{(j)}(g_i) + 0.0811q_k^{(k)}(g_i)$$

$$q_j^{(j)}(g_i) = 0.2439 + 0.0732q_i^{(i)}(g_i) + 0.0732q_k^{(i)}(g_i) - 0.1260q_i^{(j)}(g_i) - 0.1260q_l^{(j)}(g_i) + 0.0407q_l^{(l)}(g_i)$$

$$q_l^{(j)}(g_i) = 0.2857 - 0.1190q_i^{(j)}(g_i) - 0.1190q_j^{(j)}(g_i) + 0.0476q_j^{(l)}(g_i)$$

$q_i^{(k)}(g_i) = 0.1818 + 0.0545 q_j^{(i)}(g_i) + 0.0545 q_k^{(i)}(g_i) + 0.0303 q_j^{(j)}(g_i)$
$+ 0.0303 q_l^{(j)}(g_i) - 0.2818 q_k^{(k)}(g_i)$

$q_k^{(k)}(g_i) = 0.2222 + 0.0667 q_i^{(i)}(g_i) + 0.0667 q_j^{(i)}(g_i) - 0.2333 q_i^{(k)}(g_i)$

$q_j^{(l)}(g_i) = 0.2439 + 0.0732 q_i^{(i)}(g_i) + 0.0732 q_k^{(i)}(g_i) + 0.0407 q_i^{(j)}(g_i)$
$+ 0.0407 q_l^{(j)}(g_i) - 0.1260 q_l^{(l)}(g_i)$

$q_l^{(l)}(g_i) = 0.2857 + 0.0476 q_i^{(j)}(g_i) + 0.0476 q_j^{(j)}(g_i) - 0.1190 q_j^{(l)}(g_i)$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_i)$ = 0.1156; $q_j^{(i)}(g_i)$ = 0.0882; $q_k^{(i)}(g_i)$ = 0.1850; $q_i^{(j)}(g_i)$ = 0.2531; $q_j^{(j)}(g_i)$ = 0.2148; $q_l^{(j)}(g_i)$ = 0.2420; $q_i^{(k)}(g_i)$ = 0.1542; $q_k^{(k)}(g_i)$ = 0.1998; $q_j^{(l)}(g_i)$ = 0.2510; $q_l^{(l)}(g_i)$ = 0.2781; $CS_i(g_i)$ = 0.1367; $CS_j(g_i)$ = 0.1535; $CS_k(g_i)$ = 0.0740; $CS_l(g_i)$ = 0.1352; $\pi_i^{(i)}(g_i)$ = 0.0300; $\pi_j^{(i)}(g_i)$ = 0.0175; $\pi_k^{(i)}(g_i)$ = 0.0770; $\pi_i^{(j)}(g_i)$ = 0.0801; $\pi_j^{(j)}(g_i)$ = 0.0577; $\pi_l^{(j)}(g_i)$ = 0.0732; $\pi_i^{(k)}(g_i)$ = 0.0535; $\pi_k^{(k)}(g_i)$ = 0.0898; $\pi_j^{(l)}(g_i)$ = 0.0787; $\pi_l^{(l)}(g_i)$ = 0.0967; $PS_i(g_i)$ = 0.0567; $PS_j(g_i)$ = 0.0630; $PS_k(g_i)$ = 0.0470; $PS_l(g_i)$ = 0.0350; $W_i(g_i)$ = 0.3179; $W_j(g_i)$ = 0.4274; $W_k(g_i)$ = 0.2644; and $W_l(g_i)$ = 0.3457.

Network j

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_j) = 0.1622 - 0.2514 q_j^{(i)}(g_j) - 0.2514 q_k^{(i)}(g_j) + 0.0270 q_j^{(j)}(g_j) + 0.0270 q_k^{(j)}(g_j) + 0.0486 q_j^{(k)}(g_j) + 0.0486 q_k^{(k)}(g_j)$$

$$q_j^{(i)}(g_j) = 0.1622 - 0.2514 q_i^{(i)}(g_j) - 0.2514 q_k^{(i)}(g_j) + 0.0270 q_i^{(j)}(g_j) + 0.0270 q_k^{(j)}(g_j) + 0.0486 q_i^{(k)}(g_j) + 0.0486 q_k^{(k)}(g_j)$$

$$q_k^{(i)}(g_j) = 0.1622 - 0.2514 q_i^{(i)}(g_j) - 0.2514 q_j^{(i)}(g_j) + 0.0270 q_i^{(j)}(g_j) + 0.0270 q_j^{(j)}(g_j) + 0.0486 q_i^{(k)}(g_j) + 0.0486 q_j^{(k)}(g_j)$$

$$q_i^{(j)}(g_j) = 0.2703 + 0.0811 q_j^{(i)}(g_j) + 0.0811 q_k^{(i)}(g_j) - 0.1216 q_j^{(j)}(g_j) - 0.1216 q_k^{(j)}(g_j) + 0.0811 q_j^{(k)}(g_j) + 0.0811 q_k^{(k)}(g_j)$$

$$q_j^{(j)}(g_j) = 0.2703 + 0.0811 q_i^{(i)}(g_j) + 0.0811 q_k^{(i)}(g_j) - 0.1216 q_i^{(j)}(g_j) - 0.1216 q_k^{(j)}(g_j) + 0.0811 q_i^{(k)}(g_j) + 0.0811 q_k^{(k)}(g_j)$$

$$q_k^{(j)}(g_j) = 0.2703 + 0.0811 q_i^{(i)}(g_j) + 0.0811 q_j^{(i)}(g_j) - 0.1216 q_i^{(j)}(g_j) - 0.1216 q_j^{(j)}(g_j) + 0.0811 q_i^{(k)}(g_j) + 0.0811 q_j^{(k)}(g_j)$$

$$q_i^{(k)}(g_j) = 0.1622 + 0.0486 q_j^{(i)}(g_j) + 0.0486 q_k^{(i)}(g_j) + 0.0270 q_j^{(j)}(g_j) + 0.0270 q_k^{(j)}(g_j) - 0.2514 q_j^{(k)}(g_j) - 0.2514 q_k^{(k)}(g_j)$$

$$q_j^{(k)}(g_j) = 0.1622 + 0.0486 q_i^{(i)}(g_j) + 0.0486 q_k^{(i)}(g_j) + 0.0270 q_i^{(j)}(g_j) + 0.0270 q_k^{(j)}(g_j) - 0.2514 q_i^{(k)}(g_j) - 0.2514 q_k^{(k)}(g_j)$$

$$q_k^{(k)}(g_j) = 0.1622 + 0.0486 q_i^{(i)}(g_j) + 0.0486 q_j^{(i)}(g_j) + 0.0270 q_i^{(j)}(g_j) + 0.0270 q_j^{(j)}(g_j) - 0.2514 q_i^{(k)}(g_j) - 0.2514 q_j^{(k)}(g_j)$$

$$q_l^{(l)}(g_j) = 0.4000$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_j) = q_j^{(i)}(g_j) = q_k^{(i)}(g_j) = q_i^{(k)}(g_j) = q_j^{(k)}(g_j) = q_k^{(k)}(g_j) = 0.1250$; $q_i^{(j)}(g_j) = q_j^{(j)}(g_j) = q_k^{(j)}(g_j) = 0.2500$; $q_l^{(l)}(g_j) = 0.4000$; $CS_i(g_j) = CS_j(g_j) = CS_k(g_j) = 0.1250$;

$CS_l(g_j)$ = 0.0800; $\pi_i^{(i)}(g_j)$ = $\pi_j^{(i)}(g_j)$ = $\pi_k^{(i)}(g_j)$ = $\pi_i^{(k)}(g_j)$ = $\pi_j^{(k)}(g_j)$ = $\pi_k^{(k)}(g_j)$ = 0.0352; $\pi_i^{(j)}(g_j)$ = $\pi_j^{(j)}(g_j)$ = $\pi_k^{(j)}(g_j)$ = 0.0781; $\pi_l^{(l)}(g_j)$ = 0.0200; $PS_i(g_j)$ = $PS_k(g_j)$ = 0.0527; $PS_j(g_j)$ = 0.0703; $PS_l(g_j)$ = 0.0200; $W_i(g_j)$ = $W_k(g_j)$ = 0.2832; $W_j(g_j)$ = 0.4297; and $W_l(g_j)$ = 0.3000.

Network k

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_k) = 0.1935 - 0.2419q_j^{(i)}(g_k) + 0.0323q_j^{(j)}(g_k) + 0.0323q_l^{(j)}(g_k)$$

$$q_j^{(i)}(g_k) = 0.1463 - 0.2561q_i^{(i)}(g_k) + 0.0244q_i^{(j)}(g_k) + 0.0244q_l^{(j)}(g_k) + 0.0244q_k^{(l)}(g_k) + 0.0244q_l^{(l)}(g_k)$$

$$q_i^{(j)}(g_k) = 0.3226 + 0.0968q_j^{(i)}(g_k) - 0.1129q_j^{(j)}(g_k) - 0.1129q_l^{(j)}(g_k)$$

$$q_j^{(j)}(g_k) = 0.2439 + 0.0732q_i^{(i)}(g_k) - 0.1260q_i^{(j)}(g_k) - 0.1260q_l^{(j)}(g_k) + 0.0407q_k^{(l)}(g_k) + 0.0407q_l^{(l)}(g_k)$$

$$q_l^{(j)}(g_k) = 0.2439 - 0.1260q_i^{(j)}(g_k) - 0.1260q_j^{(j)}(g_k) + 0.0732q_k^{(k)}(g_k) + 0.0407q_j^{(l)}(g_k) + 0.0407q_k^{(l)}(g_k)$$

$$q_k^{(k)}(g_k) = 0.1935 - 0.2419q_l^{(k)}(g_k) + 0.0323q_j^{(l)}(g_k) + 0.0323q_l^{(l)}(g_k)$$

$$q_l^{(k)}(g_k) = 0.1463 + 0.0244q_i^{(j)}(g_k) + 0.0244q_j^{(j)}(g_k) - 0.2561q_k^{(k)}(g_k) + 0.0244q_j^{(l)}(g_k) + 0.0244q_k^{(l)}(g_k)$$

$$q_j^{(l)}(g_k) = 0.2439 + 0.0732q_i^{(i)}(g_k) + 0.0407q_i^{(j)}(g_k) + 0.0407q_l^{(j)}(g_k) - 0.1260q_k^{(l)}(g_k) - 0.1260q_l^{(l)}(g_k)$$

$$q_k^{(l)}(g_k) = 0.3226 + 0.0968q_l^{(k)}(g_k) - 0.1129q_j^{(l)}(g_k) - 0.1129q_l^{(l)}(g_k)$$

$$q_l^{(l)}(g_k) = 0.2439 + 0.0407q_i^{(j)}(g_k) + 0.0407q_j^{(j)}(g_k) + 0.0732q_k^{(k)}(g_k)$$
$$- 0.1260q_j^{(l)}(g_k) - 0.1260q_k^{(l)}(g_k)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_k)$ = $q_k^{(k)}(g_k)$ = 0.1770; $q_j^{(i)}(g_k)$ = $q_l^{(k)}(g_k)$ = 0.1254; $q_i^{(j)}(g_k)$ = $q_k^{(l)}(g_k)$ = 0.2864; $q_j^{(j)}(g_k)$ = $q_l^{(j)}(g_k)$ = $q_j^{(l)}(g_k)$ = $q_l^{(l)}(g_k)$ = 0.2141; $CS_i(g_k)$ = $CS_k(g_k)$ = 0.1074; $CS_j(g_k)$ = $CS_l(g_k)$ = 0.1533; $\pi_i^{(i)}(g_k)$ = $\pi_k^{(k)}(g_k)$ = 0.0705; $\pi_j^{(i)}(g_k)$ = $\pi_l^{(k)}(g_k)$ = 0.0354; $\pi_i^{(j)}(g_k)$ = $\pi_k^{(l)}(g_k)$ = 0.1025; $\pi_j^{(j)}(g_k)$ = $\pi_l^{(j)}(g_k)$ = $\pi_j^{(l)}(g_k)$ = $\pi_l^{(l)}(g_k)$ = 0.0573; $PS_i(g_k)$ = $PS_k(g_k)$ = 0.0343; $PS_j(g_k)$ = $PS_l(g_k)$ = 0.0638; $W_i(g_k)$ = $W_k(g_k)$ = 0.2476; and $W_j(g_k)$ = $W_l(g_k)$ = 0.4343.

Network l

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_l) = 0.1935 - 0.2419q_j^{(i)}(g_l) + 0.0323q_j^{(j)}(g_l) + 0.0323q_k^{(j)}(g_l) + 0.0323q_l^{(j)}(g_l)$$

$$q_j^{(i)}(g_l) = 0.1277 - 0.2617q_i^{(i)}(g_l) + 0.0213q_i^{(j)}(g_l) + 0.0213q_k^{(j)}(g_l) + 0.0213q_l^{(j)}(g_l)$$
$$+ 0.0383q_k^{(k)}(g_l) + 0.0213q_l^{(l)}(g_l)$$

$$q_i^{(j)}(g_l) = 0.3226 + 0.0968q_j^{(i)}(g_l) - 0.1129q_j^{(j)}(g_l) - 0.1129q_k^{(j)}(g_l) - 0.1129q_l^{(j)}(g_l)$$

$$q_j^{(j)}(g_l) = 0.2128 + 0.0638q_i^{(i)}(g_l) - 0.1312q_i^{(j)}(g_l) - 0.1312q_k^{(j)}(g_l) - 0.1312q_l^{(j)}(g_l)$$
$$+ 0.0638q_k^{(k)}(g_l) + 0.0355q_l^{(l)}(g_l)$$

$$q_k^{(j)}(g_l) = 0.3226 - 0.1129q_i^{(j)}(g_l) - 0.1129q_j^{(j)}(g_l) - 0.1129q_l^{(j)}(g_l) + 0.0968q_j^{(k)}(g_l)$$

$$q_l^{(j)}(g_l) = 0.2857 - 0.1190q_i^{(j)}(g_l) - 0.1190q_j^{(j)}(g_l) - 0.1190q_k^{(j)}(g_l) + 0.0476q_j^{(l)}(g_l)$$

$$q_j^{(k)}(g_l) = 0.1277 + 0.0383q_i^{(i)}(g_l) + 0.0213q_i^{(j)}(g_l) + 0.0213q_k^{(j)}(g_l) + 0.0213q_l^{(j)}(g_l)$$
$$- 0.2617q_k^{(k)}(g_l) + 0.0213q_l^{(l)}(g_l)$$

$$q_k^{(k)}(g_l) = 0.1935 + 0.0323q_i^{(j)}(g_l) + 0.0323q_j^{(j)}(g_l) + 0.0323q_l^{(j)}(g_l) - 0.02419q_j^{(k)}(g_l)$$

$$q_j^{(l)}(g_l) = 0.2128 + 0.0638q_i^{(i)}(g_l) + 0.0355q_i^{(j)}(g_l) + 0.0355q_k^{(j)}(g_l) + 0.0355q_l^{(j)}(g_l)$$
$$+ 0.0638q_k^{(k)}(g_l) - 0.1312q_l^{(l)}(g_l)$$

$$q_l^{(l)}(g_l) = 0.2857 + 0.0476q_i^{(j)}(g_l) + 0.0476q_j^{(j)}(g_l) + 0.0476q_k^{(j)}(g_l) - 0.1190q_j^{(l)}(g_l)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_l)$ = $q_k^{(k)}(g_l)$ = 0.1878; $q_j^{(i)}(g_l)$ = $q_j^{(k)}(g_l)$ = 0.1077; $q_i^{(j)}(g_l)$ = $q_k^{(j)}(g_l)$ = 0.2621; $q_j^{(j)}(g_l)$ = 0.1499; $q_l^{(j)}(g_l)$ = 0.2162; $q_j^{(l)}(g_l)$ = 0.2248; $q_l^{(l)}(g_l)$ = 0.2911; $CS_i(g_l)$ = $CS_k(g_l)$ = 0.1012; $CS_j(g_l)$ = 0.1741; $CS_l(g_l)$ = 0.1286; $\pi_i^{(i)}(g_l)$ = $\pi_k^{(k)}(g_l)$ = 0.0793; $\pi_j^{(i)}(g_l)$ = $\pi_j^{(k)}(g_l)$ = 0.0261; $\pi_i^{(j)}(g_l)$ = $\pi_k^{(j)}(g_l)$ = 0.0859; $\pi_j^{(j)}(g_l)$ = 0.0281; $\pi_l^{(j)}(g_l)$ = 0.0584; $\pi_j^{(l)}(g_l)$ = 0.0632; $\pi_l^{(l)}(g_l)$ = 0.1059; $PS_i(g_l)$ = $PS_k(g_l)$ = 0.0327; $PS_j(g_l)$ = 0.0991; $PS_l(g_l)$ = 0.0333; $W_i(g_l)$ = $W_k(g_l)$ = 0.2393; $W_j(g_l)$ = 0.5314; and $W_l(g_l)$ = 0.3310.

Network m

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_m) = 0.1463 - 0.2561q_j^{(i)}(g_m) - 0.2561q_l^{(i)}(g_m) + 0.0244q_j^{(j)}(g_m) + 0.0244q_l^{(j)}(g_m)$$
$$+ 0.0244q_j^{(l)}(g_m) + 0.0244q_l^{(l)}(g_m)$$

$$q_j^{(i)}(g_m) = 0.1463 - 0.2561q_i^{(i)}(g_m) - 0.2561q_l^{(i)}(g_m) + 0.0244q_i^{(j)}(g_m) + 0.0244q_l^{(j)}(g_m)$$
$$+ 0.0244q_i^{(l)}(g_m) + 0.0244q_l^{(l)}(g_m)$$

$$q_l^{(i)}(g_a) = 0.1463 - 0.2561q_i^{(i)}(g_a) - 0.2561q_j^{(i)}(g_a) + 0.0244q_i^{(j)}(g_a) + 0.0244q_j^{(j)}(g_a)$$
$$+ 0.0244q_i^{(l)}(g_a) + 0.0244q_j^{(l)}(g_a)$$

$$q_i^{(j)}(g_m) = 0.2439 + 0.0732q_j^{(i)}(g_m) + 0.02732q_l^{(i)}(g_m) - 0.1260q_j^{(j)}(g_m) - 0.1260q_l^{(j)}(g_m)$$
$$+ 0.0407q_j^{(l)}(g_m) + 0.0407q_l^{(l)}(g_m)$$

$$q_j^{(j)}(g_m) = 0.2439 + 0.0732q_i^{(i)}(g_m) + 0.0732q_l^{(i)}(g_m) - 0.1260q_i^{(j)}(g_m) - 0.1260q_l^{(j)}(g_m)$$
$$+ 0.0407q_i^{(l)}(g_m) + 0.0407q_l^{(l)}(g_m)$$

$$q_l^{(j)}(g_m) = 0.2439 + 0.0732q_i^{(i)}(g_m) + 0.0732q_j^{(i)}(g_m) - 0.1260q_i^{(j)}(g_m) - 0.1260q_j^{(j)}(g_m)$$
$$+ 0.0407q_i^{(l)}(g_m) + 0.0407q_j^{(l)}(g_m)$$

$$q_k^{(k)}(g_m) = 0.2857$$

$$q_i^{(l)}(g_m) = 0.2439 + 0.0732q_j^{(i)}(g_m) + 0.0732q_l^{(i)}(g_m) + 0.0407q_j^{(j)}(g_m) + 0.0407q_l^{(j)}(g_m)$$
$$- 0.1260q_j^{(l)}(g_m) - 0.1260q_l^{(l)}(g_m)$$

$$q_j^{(l)}(g_m) = 0.2439 + 0.0732q_i^{(i)}(g_m) + 0.0732q_l^{(i)}(g_m) + 0.0407q_i^{(j)}(g_m) + 0.0407q_l^{(j)}(g_m)$$
$$- 0.1260q_i^{(l)}(g_m) - 0.1260q_l^{(l)}(g_m)$$

$$q_l^{(l)}(g_m) = 0.2439 + 0.0732q_i^{(i)}(g_m) + 0.0732q_j^{(i)}(g_m) + 0.0407q_i^{(j)}(g_m) + 0.0407q_j^{(j)}(g_m)$$
$$- 0.1260q_i^{(l)}(g_m) - 0.1260q_j^{(l)}(g_m)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_m) = q_j^{(i)}(g_m) = q_l^{(i)}(g_m) = 0.1111$; $q_i^{(j)}(g_m) = q_j^{(j)}(g_m) = q_l^{(j)}(g_m) = q_i^{(l)}(g_m) = q_j^{(l)}(g_m) = q_l^{(l)}(g_m) = 0.2222$; $q_k^{(k)}(g_m) = 0.2857$; $CS_i(g_m) = CS_j(g_m) = CS_l(g_m) = 0.1543$; $CS_k(g_m) = 0.0408$; $\pi_i^{(i)}(g_m) = \pi_j^{(i)}(g_m) = \pi_l^{(i)}(g_m) = 0.0278$; $\pi_i^{(j)}(g_m) = \pi_j^{(j)}(g_m) = \pi_l^{(j)}(g_m) = \pi_i^{(l)}(g_m) = \pi_j^{(l)}(g_m) = \pi_l^{(l)}(g_m) = 0.0617$; $\pi_k^{(k)}(g_m) = 0.1837$; $PS_i(g_m) = 0.0417$; $PS_j(g_m) = PS_l(g_m) = 0.0556$; $PS_k(g_m) = 0.0306$; $W_i(g_m) = 0.2793$; $W_j(g_m) = W_l(g_m) = 0.3951$; and $W_k(g_m) = 0.2551$.

Network n

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_n) = 0.1277 - 0.2617q_j^{(i)}(g_n) - 0.2617q_k^{(i)}(g_n) - 0.2617q_l^{(i)}(g_n) + 0.0213q_j^{(j)}(g_n) + 0.0383q_k^{(k)}(g_n) + 0.0383q_l^{(k)}(g_n) + 0.0213q_k^{(l)}(g_n) + 0.0213q_l^{(l)}(g_n)$$

$$q_j^{(i)}(g_n) = 0.1935 - 0.2419q_i^{(i)}(g_n) - 0.2419q_k^{(i)}(g_n) - 0.2419q_l^{(i)}(g_n) + 0.0323q_i^{(j)}(g_n)$$

$$q_k^{(i)}(g_n) = 0.1622 - 0.2514q_i^{(i)}(g_n) - 0.2514q_j^{(i)}(g_n) - 0.2514q_l^{(i)}(g_n) + 0.0486q_i^{(k)}(g_n) + 0.0486q_l^{(k)}(g_n) + 0.0270q_i^{(l)}(g_n) + 0.0270q_l^{(l)}(g_n)$$

$$q_l^{(i)}(g_n) = 0.1622 - 0.2514q_i^{(i)}(g_n) - 0.2514q_j^{(i)}(g_n) - 0.2514q_k^{(i)}(g_n) + 0.0486q_i^{(k)}(g_n) + 0.0486q_k^{(k)}(g_n) + 0.0270q_i^{(l)}(g_n) + 0.0270q_k^{(l)}(g_n)$$

$$q_i^{(j)}(g_n) = 0.2128 + 0.0638q_j^{(i)}(g_n) + 0.0638q_k^{(i)}(g_n) + 0.0638q_l^{(i)}(g_n) - 0.1312q_j^{(j)}(g_n) + 0.0638q_k^{(k)}(g_n) + 0.0638q_l^{(k)}(g_n) + 0.0638q_k^{(l)}(g_n) + 0.0638q_l^{(l)}(g_n)$$

$$q_j^{(j)}(g_a) = 0.3226 + 0.0968q_i^{(i)}(g_a) + 0.0968q_k^{(i)}(g_a) + 0.0968q_l^{(i)}(g_a) - 0.1129q_i^{(j)}(g_a)$$

$$q_i^{(k)}(g_a) = 0.1277 + 0.0383 q_j^{(i)}(g_a) + 0.0383 q_k^{(i)}(g_a) + 0.0383 q_l^{(i)}(g_a) + 0.0213 q_j^{(j)}(g_a)$$
$$- 0.2617 q_k^{(k)}(g_a) - 0.2617 q_l^{(k)}(g_a) + 0.0213 q_k^{(l)}(g_a) + 0.0213 q_l^{(l)}(g_a)$$

$$q_k^{(k)}(g_n) = 0.1622 + 0.0486 q_i^{(i)}(g_n) + 0.0486 q_j^{(i)}(g_n) + 0.0486 q_l^{(i)}(g_n) - 0.2514 q_i^{(k)}(g_n)$$
$$- 0.2514 q_l^{(k)}(g_n) + 0.0270 q_i^{(l)}(g_n) + 0.0270 q_l^{(l)}(g_n)$$

$$q_l^{(k)}(g_n) = 0.1622 + 0.0486 q_i^{(i)}(g_n) + 0.0486 q_j^{(i)}(g_n) + 0.0486 q_k^{(i)}(g_n) - 0.2514 q_i^{(k)}(g_n)$$
$$- 0.2514 q_k^{(k)}(g_n) + 0.0270 q_i^{(l)}(g_n) + 0.0270 q_k^{(l)}(g_n)$$

$$q_i^{(l)}(g_n) = 0.2128 + 0.0638 q_j^{(i)}(g_n) + 0.0638 q_k^{(i)}(g_n) + 0.0638 q_l^{(i)}(g_n) + 0.0638 q_k^{(k)}(g_n)$$
$$+ 0.0638 q_l^{(k)}(g_n) - 0.1312 q_k^{(l)}(g_n) - 0.1312 q_l^{(l)}(g_n)$$

$$q_k^{(l)}(g_n) = 0.2703 + 0.0811 q_i^{(i)}(g_n) + 0.0811 q_j^{(i)}(g_n) + 0.0811 q_l^{(i)}(g_n) + 0.0811 q_i^{(k)}(g_n)$$
$$+ 0.0811 q_l^{(k)}(g_n) - 0.1216 q_i^{(l)}(g_n) - 0.1216 q_l^{(l)}(g_n)$$

$$q_l^{(l)}(g_n) = 0.2703 + 0.0811 q_i^{(i)}(g_n) + 0.0811 q_j^{(i)}(g_n) + 0.0811 q_k^{(i)}(g_n) + 0.0811 q_i^{(k)}(g_n)$$
$$+ 0.0811 q_k^{(k)}(g_n) - 0.1216 q_i^{(l)}(g_n) - 0.1216 q_k^{(l)}(g_n)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_n)$ = 0.0642; $q_j^{(i)}(g_n)$ = 0.1335; $q_k^{(i)}(g_n)$ = $q_l^{(i)}(g_n)$ = 0.1081; $q_i^{(j)}(g_n)$ = 0.2429; $q_j^{(j)}(g_n)$ = 0.3223; $q_i^{(k)}(g_n)$ = 0.0893; $q_k^{(k)}(g_n)$ = $q_l^{(k)}(g_n)$ = 0.1331; $q_i^{(l)}(g_n)$ = 0.1841; $q_k^{(l)}(g_n)$ = $q_l^{(l)}(g_n)$ = 0.2592; $CS_i(g_n)$ = 0.1685; $CS_j(g_n)$ = 0.1039; $CS_k(g_n)$ = $CS_l(g_n)$ = 0.1252; $\pi_i^{(i)}(g_n)$ = 0.0093; $\pi_j^{(i)}(g_n)$ = 0.0401; $\pi_k^{(i)}(g_n)$ = $\pi_l^{(i)}(g_n)$ = 0.0263; $\pi_i^{(j)}(g_n)$ = 0.0737; $\pi_j^{(j)}(g_n)$ = 0.1298; $\pi_i^{(k)}(g_n)$ = 0.0179; $\pi_k^{(k)}(g_n)$ = $\pi_l^{(k)}(g_n)$ = 0.0399; $\pi_i^{(l)}(g_n)$ = 0.0424; $\pi_k^{(l)}(g_n)$ = $\pi_l^{(l)}(g_n)$ = 0.0840; $PS_i(g_n)$ = 0.0643; $PS_j(g_n)$ = 0.0399; $PS_k(g_n)$ = 0.0474; $PS_l(g_n)$ = 0.0617; $W_i(g_n)$ = 0.3347; $W_j(g_n)$ = 0.3474; $W_k(g_n)$ = 0.2703; and $W_l(g_n)$ = 0.3972.

Network o

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_o) = 0.1277 - 0.2617q_j^{(i)}(g_o) - 0.2617q_k^{(i)}(g_o) - 0.2617q_l^{(i)}(g_o) + 0.0213q_j^{(j)}(g_o)$$
$$+ 0.0213q_l^{(j)}(g_o) + 0.0383q_k^{(k)}(g_o) + 0.0213q_j^{(l)}(g_o) + 0.0213q_l^{(l)}(g_o)$$

$$q_j^{(i)}(g_o) = 0.1463 - 0.2561q_i^{(i)}(g_o) - 0.2561q_k^{(i)}(g_o) - 0.2561q_l^{(i)}(g_o) + 0.0244q_i^{(j)}(g_o)$$
$$+ 0.0244q_l^{(j)}(g_o) + 0.0244q_i^{(l)}(g_o) + 0.0244q_l^{(l)}(g_o)$$

$$q_k^{(i)}(g_o) = 0.2222 - 0.2333q_i^{(i)}(g_o) - 0.2333q_j^{(i)}(g_o) - 0.2333q_l^{(i)}(g_o) + 0.0667q_i^{(k)}(g_o)$$

$$q_l^{(i)}(g_o) = 0.1463 - 0.2561q_i^{(i)}(g_o) - 0.2561q_j^{(i)}(g_o) - 0.2561q_k^{(i)}(g_o) + 0.0244q_i^{(j)}(g_o)$$
$$+ 0.0244q_j^{(j)}(g_o) + 0.0244q_i^{(l)}(g_o) + 0.0244q_j^{(l)}(g_o)$$

$$q_i^{(j)}(g_o) = 0.2439 + 0.0732q_j^{(i)}(g_o) + 0.0732q_k^{(i)}(g_o) + 0.0732q_l^{(i)}(g_o) - 0.1260q_j^{(j)}(g_o)$$
$$- 0.1260q_l^{(j)}(g_o) + 0.0732q_k^{(k)}(g_o) + 0.0407q_j^{(l)}(g_o) + 0.0407q_l^{(l)}(g_o)$$

$$q_j^{(j)}(g_o) = 0.2439 + 0.0732q_i^{(i)}(g_o) + 0.0732q_k^{(i)}(g_o) + 0.0732q_l^{(i)}(g_o) - 0.1260q_i^{(j)}(g_o)$$
$$- 0.1260q_l^{(j)}(g_o) + 0.0407q_i^{(l)}(g_o) + 0.0407q_l^{(l)}(g_o)$$

$$q_l^{(j)}(g_o) = 0.2439 + 0.0732q_i^{(i)}(g_o) + 0.0732q_j^{(i)}(g_o) + 0.0732q_k^{(i)}(g_o) - 0.1260q_i^{(j)}(g_o)$$
$$- 0.1260q_j^{(j)}(g_o) + 0.0407q_i^{(l)}(g_o) + 0.0407q_j^{(l)}(g_o)$$

$$q_i^{(k)}(g_o) = 0.1277 + 0.0383q_j^{(i)}(g_o) + 0.0383q_k^{(i)}(g_o) + 0.0383q_l^{(i)}(g_o) + 0.0213q_j^{(j)}(g_o)$$
$$+ 0.0213q_l^{(j)}(g_o) - 0.2617q_k^{(k)}(g_o) + 0.0213q_j^{(l)}(g_o) + 0.0213q_l^{(l)}(g_o)$$

$$q_k^{(k)}(g_o) = 0.2222 + 0.0667q_i^{(i)}(g_o) + 0.0667q_j^{(i)}(g_o) + 0.0667q_l^{(i)}(g_o) - 0.2333q_i^{(k)}(g_o)$$

$$q_i^{(l)}(g_o) = 0.2439 + 0.0732q_j^{(i)}(g_o) + 0.0732q_k^{(i)}(g_o) + 0.0732q_l^{(i)}(g_o) + 0.0407q_j^{(j)}(g_o)$$
$$+ 0.0407q_l^{(j)}(g_o) + 0.0703q_k^{(k)}(g_o) - 0.1260q_j^{(l)}(g_o) - 0.1260q_l^{(l)}(g_o)$$

$$q_j^{(l)}(g_o) = 0.2439 + 0.0732 q_i^{(i)}(g_o) + 0.0732 q_k^{(i)}(g_o) + 0.0732 q_l^{(i)}(g_o) + 0.0407 q_i^{(j)}(g_o) + 0.0407 q_l^{(j)}(g_o) - 0.1260 q_i^{(l)}(g_o) - 0.1260 q_l^{(l)}(g_o)$$

$$q_l^{(l)}(g_o) = 0.2439 + 0.0732 q_i^{(i)}(g_o) + 0.0732 q_j^{(i)}(g_o) + 0.0732 q_k^{(i)}(g_o) + 0.0407 q_i^{(j)}(g_o) + 0.0407 q_j^{(j)}(g_o) - 0.1260 q_i^{(l)}(g_o) - 0.1260 q_j^{(l)}(g_o)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_o)$ = 0.0645; $q_j^{(i)}(g_o)$ = $q_l^{(i)}(g_o)$ = 0.0863; $q_k^{(i)}(g_o)$ = 0.1739; $q_i^{(j)}(g_o)$ = $q_i^{(l)}(g_o)$ = 0.2461; $q_j^{(j)}(g_o)$ = $q_l^{(j)}(g_o)$ = $q_j^{(l)}(g_o)$ = $q_l^{(l)}(g_o)$ = 0.2273; $q_i^{(k)}(g_o)$ = 0.1043; $q_k^{(k)}(g_o)$ = 0.2137; $CS_i(g_o)$ = 0.2185; $CS_j(g_o)$ = $CS_l(g_o)$ = 0.1462; $CS_k(g_o)$ = 0.0751; $\pi_i^{(i)}(g_o)$ = 0.0094; $\pi_j^{(i)}(g_o)$ = $\pi_l^{(i)}(g_o)$ = 0.0168; $\pi_k^{(i)}(g_o)$ = 0.0680; $\pi_i^{(j)}(g_o)$ = $\pi_i^{(l)}(g_o)$ = 0.0757; $\pi_j^{(j)}(g_o)$ = $\pi_l^{(j)}(g_o)$ = $\pi_j^{(l)}(g_o)$ = $\pi_l^{(l)}(g_o)$ = 0.0646; $\pi_i^{(k)}(g_o)$ = 0.0245; $\pi_k^{(k)}(g_o)$ = 0.1027; $PS_i(g_o)$ = 0.0633; $PS_j(g_o)$ = $PS_l(g_o)$ = 0.0614; $PS_k(g_o)$ = 0.0379; $W_i(g_o)$ = 0.3927; $W_j(g_o)$ = $W_l(g_o)$ = 0.4124; and $W_k(g_o)$ = 0.2403.

Network p

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_p) = 0.1622 - 0.2514 q_j^{(i)}(g_p) - 0.2514 q_k^{(i)}(g_p) + 0.0270 q_j^{(j)}(g_p) + 0.0270 q_l^{(j)}(g_p) + 0.0486 q_k^{(k)}(g_p) + 0.0486 q_l^{(k)}(g_p)$$

$$q_j^{(i)}(g_p) = 0.1463 - 0.2561 q_i^{(i)}(g_p) - 0.2561 q_k^{(i)}(g_p) + 0.0244 q_i^{(j)}(g_p) + 0.0244 q_l^{(j)}(g_p) + 0.0244 q_k^{(l)}(g_p) + 0.0244 q_l^{(l)}(g_p)$$

$$q_k^{(i)}(g_p) = 0.1622 - 0.2514 q_i^{(i)}(g_p) - 0.2514 q_j^{(i)}(g_p) + 0.0486 q_i^{(k)}(g_p) + 0.0486 q_l^{(k)}(g_p)$$
$$+ 0.0270 q_j^{(l)}(g_p) + 0.0270 q_l^{(l)}(g_p)$$

$$q_i^{(j)}(g_p) = 0.2703 + 0.0811 q_j^{(i)}(g_p) + 0.0811 q_k^{(i)}(g_p) - 0.1216 q_j^{(j)}(g_p) - 0.1216 q_l^{(j)}(g_p)$$
$$+ 0.0911 q_k^{(k)}(g_p) + 0.0811 q_l^{(k)}(g_p)$$

$$q_j^{(j)}(g_p) = 0.2439 + 0.0732 q_i^{(i)}(g_p) + 0.0732 q_k^{(i)}(g_p) - 0.1260 q_i^{(j)}(g_p) - 0.1260 q_l^{(j)}(g_p)$$
$$+ 0.0407 q_k^{(l)}(g_p) + 0.0407 q_l^{(l)}(g_p)$$

$$q_l^{(j)}(g_p) = 0.2439 - 0.1260 q_i^{(j)}(g_p) - 0.1260 q_j^{(j)}(g_p) + 0.0732 q_i^{(k)}(g_p) + 0.0732 q_k^{(k)}(g_p)$$
$$+ 0.0407 q_j^{(l)}(g_p) + 0.0407 q_k^{(l)}(g_p)$$

$$q_i^{(k)}(g_p) = 0.1622 + 0.0486 q_j^{(i)}(g_p) + 0.0486 q_k^{(i)}(g_p) + 0.0270 q_j^{(j)}(g_p) + 0.0270 q_l^{(j)}(g_p)$$
$$- 0.2514 q_k^{(k)}(g_p) - 0.2514 q_l^{(k)}(g_p)$$

$$q_k^{(k)}(g_p) = 0.1622 + 0.0486 q_i^{(i)}(g_p) + 0.0486 q_j^{(i)}(g_p) - 0.2514 q_i^{(k)}(g_p) - 0.2514 q_l^{(k)}(g_p)$$
$$+ 0.0270 q_j^{(l)}(g_p) + 0.0270 q_l^{(l)}(g_p)$$

$$q_l^{(k)}(g_p) = 0.1463 + 0.0244 q_i^{(j)}(g_p) + 0.0244 q_j^{(j)}(g_p) - 0.2561 q_i^{(k)}(g_p)$$
$$- 0.2561 q_k^{(k)}(g_p) + 0.0244 q_j^{(l)}(g_p) + 0.0244 q_k^{(l)}(g_p)$$

$$q_j^{(l)}(g_p) = 0.2439 + 0.0732 q_i^{(i)}(g_p) + 0.0732 q_k^{(i)}(g_p) + 0.0407 q_i^{(j)}(g_p) + 0.0407 q_l^{(j)}(g_p)$$
$$- 0.1260 q_k^{(l)}(g_p) - 0.1260 q_l^{(l)}(g_p)$$

$$q_k^{(l)}(g_p) = 0.2703 + 0.0811 q_i^{(i)}(g_p) + 0.0811 q_j^{(i)}(g_p) + 0.0811 q_i^{(k)}(g_p) + 0.0811 q_l^{(k)}(g_p)$$
$$- 0.1216 q_j^{(l)}(g_p) - 0.1216 q_l^{(l)}(g_p)$$

$$q_l^{(l)}(g_p) = 0.2439 + 0.0407 q_i^{(j)}(g_p) + 0.0407 q_j^{(j)}(g_p) + 0.0732 q_i^{(k)}(g_p) + 0.0732 q_k^{(k)}(g_p)$$
$$- 0.1260 q_j^{(l)}(g_p) - 0.1260 q_k^{(l)}(g_p)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_p)$ = $q_k^{(i)}(g_p)$ = $q_i^{(k)}(g_p)$ = $q_k^{(k)}(g_p)$ = 0.1722; $q_j^{(i)}(g_p)$ = $q_l^{(k)}(g_p)$ = 0.1044; $q_i^{(j)}(g_p)$ = $q_k^{(l)}(g_p)$ = 0.2539; $q_j^{(j)}(g_p)$ = $q_l^{(j)}(g_p)$ = $q_j^{(l)}(g_p)$ = $q_l^{(l)}(g_p)$ = 0.2219; $CS_i(g_p)$ = $CS_k(g_p)$ = 0.1292; $CS_j(g_p)$ = $CS_l(g_p)$ = 0.1503; $\pi_i^{(i)}(g_p)$ = $\pi_k^{(i)}(g_p)$ = $\pi_i^{(k)}(g_p)$ = $\pi_k^{(k)}(g_p)$ = 0.0364; $\pi_j^{(i)}(g_p)$ = $\pi_l^{(k)}(g_p)$ = 0.0245; $\pi_i^{(j)}(g_p)$ = $\pi_k^{(l)}(g_p)$ = 0.0806; $\pi_j^{(j)}(g_p)$ = $\pi_l^{(j)}(g_p)$ = $\pi_j^{(l)}(g_p)$ = $\pi_l^{(l)}(g_p)$ = 0.0616; $PS_i(g_p)$ = $PS_k(g_p)$ = 0.0483; $PS_j(g_p)$ = $PS_l(g_p)$ = 0.0608; $W_i(g_p)$ =$W_k(g_p)$ = 0.2748 and $W_j(g_p)$ = $W_l(g_p)$ = 0.4148.

Network q

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_q) = 0.1622 - 0.2514 q_j^{(i)}(g_q) - 0.26514 q_k^{(i)}(g_q) + 0.0270 q_j^{(j)}(g_q) + 0.0270 q_k^{(j)}(g_q) + 0.0270 q_l^{(j)}(g_q) + 0.0486 q_j^{(k)}(g_q) + 0.0486 q_k^{(k)}(g_q)$$

$$q_j^{(i)}(g_q) = 0.1277 - 0.2617 q_i^{(i)}(g_q) - 0.2617 q_k^{(i)}(g_q) + 0.0213 q_i^{(j)}(g_q) + 0.0213 q_k^{(j)}(g_q) + 0.0213 q_l^{(j)}(g_q) + 0.0383 q_i^{(k)}(g_q) + 0.0383 q_k^{(k)}(g_q) + 0.0213 q_l^{(l)}(g_q)$$

$$q_k^{(i)}(g_q) = 0.1622 - 0.2514 q_i^{(i)}(g_q) - 0.2514 q_j^{(i)}(g_q) + 0.0270 q_i^{(j)}(g_q) + 0.0270 q_j^{(j)}(g_q) + 0.0270 q_l^{(j)}(g_q) + 0.0486 q_i^{(k)}(g_q) + 0.0486 q_j^{(k)}(g_q)$$

$$q_i^{(j)}(g_q) = 0.2703 + 0.0811 q_j^{(i)}(g_q) + 0.0811 q_k^{(i)}(g_q) - 0.1216 q_j^{(j)}(g_q) - 0.1216 q_k^{(j)}(g_q) - 0.1216 q_l^{(j)}(g_q) + 0.0811 q_j^{(k)}(g_q) + 0.0811 q_k^{(k)}(g_q)$$

$$q_j^{(j)}(g_q) = 0.2128 + 0.0638 q_i^{(i)}(g_q) + 0.0638 q_k^{(i)}(g_q) - 0.1312 q_i^{(j)}(g_q) - 0.1312 q_k^{(j)}(g_q) - 0.1312 q_l^{(j)}(g_q) + 0.0638 q_i^{(k)}(g_q) + 0.0638 q_k^{(k)}(g_q) + 0.0355 q_l^{(l)}(g_q)$$

$$q_k^{(j)}(g_q) = 0.2703 + 0.0811 q_i^{(i)}(g_q) + 0.0811 q_j^{(i)}(g_q) - 0.1216 q_i^{(j)}(g_q) - 0.1216 q_j^{(j)}(g_q)$$
$$- 0.1216 q_l^{(j)}(g_q) + 0.0811 q_i^{(k)}(g_q) + 0.0811 q_j^{(k)}(g_q)$$

$$q_l^{(j)}(g_q) = 0.2857 - 0.1190 q_i^{(j)}(g_q) - 0.1190 q_j^{(j)}(g_q) - 0.1190 q_k^{(j)}(g_q) + 0.0476 q_j^{(l)}(g_q)$$

$$q_i^{(k)}(g_q) = 0.1622 + 0.0486 q_j^{(i)}(g_q) + 0.0486 q_k^{(i)}(g_q) + 0.0270 q_j^{(j)}(g_q) + 0.0270 q_k^{(j)}(g_q)$$
$$+ 0.0270 q_l^{(j)}(g_q) - 0.2514 q_j^{(k)}(g_q) - 0.2514 q_k^{(k)}(g_q)$$

$$q_j^{(k)}(g_q) = 0.1277 + 0.0383 q_i^{(i)}(g_q) + 0.0383 q_k^{(i)}(g_q) + 0.0213 q_i^{(j)}(g_q) + 0.0213 q_k^{(j)}(g_q)$$
$$+ 0.0213 q_l^{(j)}(g_q) - 0.2617 q_i^{(k)}(g_q) - 0.2617 q_k^{(k)}(g_q) + 0.0213 q_l^{(l)}(g_q)$$

$$q_k^{(k)}(g_q) = 0.1622 + 0.0486 q_i^{(i)}(g_q) + 0.0486 q_j^{(i)}(g_q) + 0.0270 q_i^{(j)}(g_q) + 0.0270 q_j^{(j)}(g_q)$$
$$+ 0.0270 q_l^{(j)}(g_q) - 0.2514 q_i^{(k)}(g_q) - 0.2514 q_j^{(k)}(g_q)$$

$$q_j^{(l)}(g_q) = 0.2128 + 0.0638 q_i^{(i)}(g_q) + 0.0638 q_k^{(i)}(g_q) + 0.0355 q_i^{(j)}(g_q) + 0.0355 q_k^{(j)}(g_q)$$
$$+ 0.0355 q_l^{(j)}(g_q) + 0.0638 q_i^{(k)}(g_q) + 0.0638 q_k^{(k)}(g_q) - 0.1312 q_l^{(l)}(g_q)$$

$$q_l^{(l)}(g_q) = 0.2857 + 0.0476 q_i^{(j)}(g_q) + 0.0476 q_j^{(j)}(g_q) + 0.0476 q_k^{(j)}(g_q) - 0.1190 q_j^{(l)}(g_q)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_q)$ = $q_k^{(i)}(g_q)$ = $q_i^{(k)}(g_q)$ = $q_k^{(k)}(g_q)$ = 0.1338; $q_j^{(i)}(g_q)$ = $q_j^{(k)}(g_q)$ = 0.0885; $q_i^{(j)}(g_q)$ = $q_k^{(j)}(g_q)$ = 0.2309; $q_j^{(j)}(g_q)$ = 0.1674; $q_l^{(j)}(g_q)$ = 0.2219; $q_j^{(l)}(g_q)$ = 0.2334; $q_l^{(l)}(g_q)$ = 0.2879; $CS_i(g_q)$ = $CS_k(g_q)$ = 0.1243; $CS_j(g_q)$ = 0.1670; $CS_l(g_q)$ = 0.1300; $\pi_i^{(i)}(g_q)$ = $\pi_k^{(i)}(g_q)$ = $\pi_i^{(k)}(g_q)$ = $\pi_k^{(k)}(g_q)$ = 0.0403; $\pi_j^{(i)}(g_q)$ = $\pi_j^{(k)}(g_q)$ = 0.0176; $\pi_i^{(j)}(g_q)$ = $\pi_k^{(j)}(g_q)$ = 0.0666; $\pi_j^{(j)}(g_q)$ = 0.0350; $\pi_l^{(j)}(g_q)$ = 0.0616; $\pi_j^{(l)}(g_q)$ = 0.0681; $\pi_l^{(l)}(g_q)$ = 0.1036; $PS_i(g_q)$ = $PS_k(g_q)$ = 0.0476; $PS_j(g_q)$ = 0.0906; $PS_l(g_q)$ = 0.0340; $W_i(g_q)$ = $W_k(g_q)$ = 0.2701; $W_j(g_q)$ = 0.4874; and $W_l(g_q)$ = 0.3356.

Network r

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_r) = 0.1463 - 0.2561 q_j^{(i)}(g_r) - 0.2561 q_l^{(i)}(g_r) + 0.0244 q_j^{(j)}(g_r) + 0.0244 q_l^{(j)}(g_r)$$
$$+ 0.0244 q_j^{(l)}(g_r) + 0.0244 q_k^{(l)}(g_r) + 0.0244 q_l^{(l)}(g_r)$$

$$q_j^{(i)}(g_r) = 0.1453 - 0.2561 q_i^{(i)}(g_r) - 0.2561 q_l^{(i)}(g_r) + 0.0244 q_i^{(j)}(g_r) + 0.0244 q_l^{(j)}(g_r)$$
$$+ 0.0244 q_i^{(l)}(g_r) + 0.0244 q_k^{(l)}(g_r) + 0.0244 q_l^{(l)}(g_r)$$

$$q_l^{(i)}(g_r) = 0.1277 - 0.2617 q_i^{(i)}(g_r) - 0.2617 q_j^{(i)}(g_r) + 0.0213 q_i^{(j)}(g_r) + 0.0213 q_j^{(j)}(g_r)$$
$$+ 0.0383 q_k^{(k)}(g_r) + 0.0213 q_i^{(l)}(g_r) + 0.0213 q_j^{(l)}(g_r) + 0.0213 q_k^{(l)}(g_r)$$

$$q_i^{(j)}(g_r) = 0.2439 + 0.0732 q_j^{(i)}(g_r) + 0.0732 q_l^{(i)}(g_r) - 0.1260 q_j^{(j)}(g_r) - 0.1260 q_l^{(j)}(g_r)$$
$$+ 0.0407 q_j^{(l)}(g_r) + 0.0497 q_k^{(l)}(g_r) + 0.0407 q_l^{(l)}(g_r)$$

$$q_j^{(j)}(g_r) = 0.2439 + 0.0732 q_i^{(i)}(g_r) + 0.0732 q_l^{(i)}(g_r) - 0.1260 q_i^{(j)}(g_r) - 0.1260 q_l^{(j)}(g_r)$$
$$+ 0.0407 q_i^{(l)}(g_r) + 0.0407 q_k^{(l)}(g_r) + 0.0407 q_l^{(l)}(g_r)$$

$$q_l^{(j)}(g_r) = 0.2128 + 0.0638 q_i^{(i)}(g_r) + 0.0638 q_j^{(i)}(g_r) - 0.1312 q_i^{(j)}(g_r) - 0.1312 q_j^{(j)}(g_r)$$
$$+ 0.0638 q_k^{(k)}(g_r) + 0.0355 q_i^{(l)}(g_r) + 0.0355 q_j^{(l)}(g_r) + 0.0355 q_k^{(l)}(g_r)$$

$$q_k^{(k)}(g_r) = 0.1935 - 0.2419 q_l^{(k)}(g_r) + 0.0323 q_i^{(l)}(g_r) + 0.0323 q_j^{(l)}(g_r) + 0.0323 q_l^{(l)}(g_r)$$

$$q_l^{(k)}(g_r) = 0.1277 + 0.0383 q_i^{(i)}(g_r) + 0.0383 q_j^{(i)}(g_r) + 0.0213 q_i^{(j)}(g_r) + 0.0213 q_j^{(j)}(g_r)$$
$$- 0.2617 q_k^{(k)}(g_r) + 0.0213 q_i^{(l)}(g_r) + 0.0213 q_j^{(l)}(g_r) + 0.0213 q_k^{(l)}(g_r)$$

$$q_i^{(l)}(g_r) = 0.2439 + 0.0732 q_j^{(i)}(g_r) + 0.0732 q_l^{(i)}(g_r) + 0.0407 q_j^{(j)}(g_r) + 0.0407 q_l^{(j)}(g_r)$$
$$- 0.1260 q_j^{(l)}(g_r) - 0.1260 q_k^{(l)}(g_r) - 0.1260 q_l^{(l)}(g_r)$$

$$q_j^{(l)}(g_r) = 0.2439 + 0.0732 q_i^{(i)}(g_r) + 0.0732 q_l^{(i)}(g_r) + 0.0407 q_i^{(j)}(g_r) + 0.0407 q_l^{(j)}(g_r)$$
$$- 0.1260 q_i^{(l)}(g_r) - 0.1260 q_k^{(l)}(g_r) - 0.1260 q_l^{(l)}(g_r)$$

$$q_k^{(l)}(g_r) = 0.3226 + 0.0968 q_l^{(k)}(g_r) - 0.1667 q_i^{(l)}(g_r) - 0.1667 q_j^{(l)}(g_r) - 0.1667 q_l^{(l)}(g_r)$$

$$q_l^{(l)}(g_r) = 0.2128 + 0.0638q_i^{(i)}(g_r) + 0.0638q_j^{(i)}(g_r) + 0.0355q_i^{(j)}(g_r) + 0.0355q_j^{(j)}(g_r)$$
$$+ 0.0638q_k^{(k)}(g_r) - 0.1312q_i^{(l)}(g_r) - 0.1312q_j^{(l)}(g_r) - 0.1312q_k^{(l)}(g_r)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_r)$ = $q_j^{(i)}(g_r)$ = 0.1170; $q_l^{(i)}(g_r)$ = 0.0969; $q_i^{(j)}(g_r)$ = $q_j^{(j)}(g_r)$ = 0.2299; $q_l^{(j)}(g_r)$ = 0.2018; $q_k^{(k)}(g_r)$ = 0.1850; $q_l^{(k)}(g_r)$ = 0.1116; $q_i^{(l)}(g_r)$ = $q_j^{(l)}(g_r)$ = 0.2002; $q_k^{(l)}(g_r)$ = 0.2380; $q_l^{(l)}(g_r)$ = 0.1721; $CS_i(g_r)$ = $CS_j(g_r)$ = 0.1497; $CS_k(g_r)$ = 0.0895; $CS_l(g_r)$ = 0.1695; $\pi_i^{(i)}(g_r)$ = $\pi_j^{(i)}(g_r)$ = 0.0308; $\pi_l^{(i)}(g_r)$ = 0.0211; $\pi_i^{(j)}(g_r)$ = $\pi_j^{(j)}(g_r)$ = 0.0661; $\pi_l^{(j)}(g_r)$ = 0.0509; $\pi_k^{(k)}(g_r)$ = 0.0770; $\pi_l^{(k)}(g_r)$ = 0.0280; $\pi_i^{(l)}(g_r)$ = $\pi_j^{(l)}(g_r)$ = 0.0501; $\pi_k^{(l)}(g_r)$ = 0.0708; $\pi_l^{(l)}(g_r)$ = 0.0370; $PS_i(g_r)$ = 0.0410; $PS_j(g_r)$ = 0.0547; $PS_k(g_r)$ = 0.0330; $PS_l(g_r)$ = 0.0821; $W_i(g_r)$ = 0.2734; $W_j(g_r)$ = 0.3875; $W_k(g_r)$ = 0.2275; and $W_l(g_r)$ = 0.4596.

Network s

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_s) = 0.1277 - 0.2617q_j^{(i)}(g_s) - 0.2627q_k^{(i)}(g_s) - 0.2617q_l^{(i)}(g_s) + 0.0213q_j^{(j)}(g_s)$$
$$+ 0.0213q_l^{(j)}(g_s) + 0.0383q_k^{(k)}(g_s) + 0.0383q_l^{(k)}(g_s) + 0.0213q_j^{(l)}(g_s) + 0.0213q_k^{(l)}(g_s)$$
$$+ 0.0213q_l^{(l)}(g_s)$$

$$q_j^{(i)}(g_s) = 0.1463 - 0.2561q_i^{(i)}(g_s) - 0.2561q_k^{(i)}(g_s) - 0.2561q_l^{(i)}(g_s) + 0.0244q_i^{(j)}(g_s)$$
$$+ 0.0244q_l^{(j)}(g_s) + 0.0244q_i^{(l)}(g_s) + 0.0244q_k^{(l)}(g_s) + 0.0244q_l^{(l)}(g_s)$$

$$q_k^{(i)}(g_s) = 0.1622 - 0.2514q_i^{(i)}(g_s) - 0.2514q_j^{(i)}(g_s) - 0.2514q_l^{(i)}(g_s) + 0.0486q_i^{(k)}(g_s) + 0.0486q_l^{(k)}(g_s) + 0.0270q_i^{(l)}(g_s) + 0.0270q_j^{(l)}(g_s) + 0.0270q_l^{(l)}(g_s)$$

$$q_l^{(i)}(g_s) = 0.1277 - 0.2617q_i^{(i)}(g_s) - 0.2617q_j^{(i)}(g_s) - 0.2617q_k^{(i)}(g_s) + 0.0213q_i^{(j)}(g_s) + 0.0213q_j^{(j)}(g_s) + 0.0383q_i^{(k)}(g_s) + 0.0383q_k^{(k)}(g_s) + 0.0213q_i^{(l)}(g_s) + 0.0213q_j^{(l)}(g_s) + 0.0213q_k^{(l)}(g_s)$$

$$q_i^{(j)}(g_s) = 0.2128 + 0.0638q_j^{(i)}(g_s) + 0.0638q_k^{(i)}(g_s) + 0.0638q_l^{(i)}(g_s) - 0.1312q_j^{(j)}(g_s) - 0.1312q_l^{(j)}(g_s) + 0.0638q_k^{(k)}(g_s) + 0.0638q_l^{(k)}(g_s) + 0.0355q_j^{(l)}(g_s) + 0.0355q_k^{(l)}(g_s) + 0.0355q_l^{(l)}(g_s)$$

$$q_j^{(j)}(g_s) = 0.2439 + 0.0732q_i^{(i)}(g_s) + 0.0732q_k^{(i)}(g_s) + 0.0732q_l^{(i)}(g_s) - 0.1260q_i^{(j)}(g_s) - 0.1260q_l^{(j)}(g_s) + 0.0407q_i^{(l)}(g_s) + 0.0407q_k^{(l)}(g_s) + 0.0407q_l^{(l)}(g_s)$$

$$q_l^{(j)}(g_s) = 0.2128 + 0.0638q_i^{(i)}(g_s) + 0.0638q_j^{(i)}(g_s) + 0.0638q_k^{(i)}(g_s) - 0.1312q_i^{(j)}(g_s) - 0.1312q_j^{(j)}(g_s) + 0.0638q_i^{(k)}(g_s) + 0.0638q_k^{(k)}(g_s) + 0.0355q_i^{(l)}(g_s) + 0.0355q_j^{(l)}(g_s) + 0.0355q_k^{(l)}(g_s)$$

$$q_i^{(k)}(g_s) = 0.1277 + 0.0383q_j^{(i)}(g_s) + 0.0383q_k^{(i)}(g_s) + 0.0383q_l^{(i)}(g_s) + 0.0213q_j^{(j)}(g_s) + 0.0213q_l^{(j)}(g_s) - 0.2617q_k^{(k)}(g_s) - 0.2617q_l^{(k)}(g_s) + 0.0213q_j^{(l)}(g_s) + 0.0213q_k^{(l)}(g_s) + 0.0213q_l^{(l)}(g_s)$$

$$q_k^{(k)}(g_s) = 0.1622 + 0.0486q_i^{(i)}(g_s) + 0.0486q_j^{(i)}(g_s) + 0.0486q_l^{(i)}(g_s) - 0.2514q_i^{(k)}(g_s) - 0.2514q_l^{(k)}(g_s) + 0.0270q_i^{(l)}(g_s) + 0.0270q_j^{(l)}(g_s) + 0.0270q_l^{(l)}(g_s)$$

$$q_l^{(k)}(g_s) = 0.1277 + 0.0383q_i^{(i)}(g_s) + 0.0383q_j^{(i)}(g_s) + 0.0383q_k^{(i)}(g_s) + 0.0213q_i^{(j)}(g_s) + 0.0213q_j^{(j)}(g_s) - 0.2617q_i^{(k)}(g_s) - 0.2617q_k^{(k)}(g_s) + 0.0213q_i^{(l)}(g_s) + 0.0213q_j^{(l)}(g_s) + 0.0213q_k^{(l)}(g_s)$$

$$q_i^{(l)}(g_s) = 0.2128 + 0.0638q_j^{(i)}(g_s) + 0.0638q_k^{(i)}(g_s) + 0.0638q_l^{(i)}(g_s) + 0.0355q_j^{(j)}(g_s) + 0.0355q_l^{(j)}(g_s) + 0.0638q_k^{(k)}(g_s) + 0.0638q_l^{(k)}(g_s) - 0.1312q_j^{(l)}(g_s) - 0.1312q_k^{(l)}(g_s) - 0.1312q_l^{(l)}(g_s)$$

$$q_j^{(l)}(g_s) = 0.2439 + 0.0732q_i^{(i)}(g_s) + 0.0732q_k^{(i)}(g_s) + 0.0732q_l^{(i)}(g_s) + 0.0407q_i^{(j)}(g_s) + 0.0407q_l^{(j)}(g_s) - 0.1260q_i^{(l)}(g_s) - 0.1260q_k^{(l)}(g_s) + 0.01260q_l^{(l)}(g_s)$$

$$q_k^{(l)}(g_s) = 0.2703 + 0.0811q_i^{(i)}(g_s) + 0.0811q_j^{(i)}(g_s) + 0.0811q_l^{(i)}(g_s) + 0.0811q_i^{(k)}(g_s) + 0.0811q_l^{(k)}(g_s) - 0.1216q_i^{(l)}(g_s) - 0.1216q_j^{(l)}(g_s) - 0.1216q_l^{(l)}(g_s)$$

$$q_l^{(l)}(g_s) = 0.2128 + 0.0638q_i^{(i)}(g_s) + 0.0638q_j^{(i)}(g_s) + 0.0638q_k^{(i)}(g_s) + 0.0355q_i^{(j)}(g_s) + 0.0355q_j^{(j)}(g_s) + 0.0638q_i^{(k)}(g_s) + 0.0638q_k^{(k)}(g_s) - 0.1312q_i^{(l)}(g_s) - 0.1312q_j^{(l)}(g_s) - 0.1312q_k^{(l)}(g_s)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_s)$ = $q_l^{(i)}(g_s)$ = 0.0808; $q_j^{(i)}(g_s)$ = 0.0987; $q_k^{(i)}(g_s)$ = 0.1218; $q_i^{(j)}(g_s)$ = $q_l^{(j)}(g_s)$ = 0.2109; $q_j^{(j)}(g_s)$ = 0.2359; $q_i^{(k)}(g_s)$ = $q_l^{(k)}(g_s)$ = 0.0993; $q_k^{(k)}(g_s)$ = 0.1403; $q_i^{(l)}(g_s)$ = $q_l^{(l)}(g_s)$ = 0.1811; $q_j^{(l)}(g_s)$ = 0.2061; $q_k^{(l)}(g_s)$ = 0.2384; $CS_i(g_s)$ = $CS_l(g_s)$ = 0.1636; $CS_j(g_s)$ = 0.1462; $CS_k(g_s)$ = 0.1252; $\pi_i^{(i)}(g_s)$ = $\pi_l^{(i)}(g_s)$ = 0.0147; $\pi_j^{(i)}(g_s)$ = 0.0219; $\pi_k^{(i)}(g_s)$ = 0.0334; $\pi_i^{(j)}(g_s)$ = $\pi_l^{(j)}(g_s)$ = 0.0556; $\pi_j^{(j)}(g_s)$ = 0.0696; $\pi_i^{(k)}(g_s)$ = $\pi_l^{(k)}(g_s)$ = 0.0222; $\pi_k^{(k)}(g_s)$ = 0.0443; $\pi_i^{(l)}(g_s)$ = $\pi_l^{(l)}(g_s)$ = 0.0410; $\pi_j^{(l)}(g_s)$ = 0.0531; $\pi_k^{(l)}(g_s)$ = 0.0710; $PS_i(g_s)$ = 0.0547; $PS_j(g_s)$ = 0.0541; $PS_k(g_s)$ = 0.0431; $PS_l(g_s)$ = 0.0813; $W_i(g_s)$ = 0.3030; $W_j(g_s)$ = 0.3810; $W_k(g_s)$ = 0.2569; and $W_l(g_s)$ = 0.4510.

Network t

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_t) = 0.1277 - 0.2617q_j^{(i)}(g_t) - 0.2617q_k^{(i)}(g_t) - 0.2617q_l^{(i)}(g_t) + 0.0213q_j^{(j)}(g_t)$$
$$+ 0.0213q_k^{(j)}(g_t) + 0.0383q_j^{(k)}(g_t) + 0.0383q_k^{(k)}(g_t) + 0.0383q_l^{(k)}(g_t) + 0.0213q_k^{(l)}(g_t)$$
$$+ 0.0213q_l^{(l)}(g_t)$$

$$q_j^{(i)}(g_t) = 0.1622 - 0.2514q_i^{(i)}(g_t) - 0.2514q_k^{(i)}(g_t) - 0.2514q_l^{(i)}(g_t) + 0.0270q_i^{(j)}(g_t)$$
$$+ 0.0270q_k^{(j)}(g_t) + 0.0486q_i^{(k)}(g_t) + 0.0486q_k^{(k)}(g_t) + 0.0486q_l^{(k)}(g_t)$$

$$q_k^{(i)}(g_t) = 0.1277 - 0.2617q_i^{(i)}(g_t) - 0.2617q_j^{(i)}(g_t) - 0.2617q_l^{(i)}(g_t) + 0.0213q_i^{(j)}(g_t)$$
$$+ 0.0213q_j^{(j)}(g_t) + 0.0383q_i^{(k)}(g_t) + 0.0383q_j^{(k)}(g_t) + 0.0383q_l^{(k)}(g_t) + 0.0213q_i^{(l)}(g_t)$$
$$+ 0.0213q_l^{(l)}(g_t)$$

$$q_l^{(i)}(g_t) = 0.1622 - 0.2514q_i^{(i)}(g_t) - 0.2514q_j^{(i)}(g_t) - 0.2514q_k^{(i)}(g_t) + 0.0486q_i^{(k)}(g_t)$$
$$+ 0.0486q_j^{(k)}(g_t) + 0.0486q_k^{(k)}(g_t) + 0.0270q_i^{(l)}(g_t) + 0.0270q_k^{(l)}(g_t)$$

$$q_i^{(j)}(g_t) = 0.2128 + 0.0638q_j^{(i)}(g_t) + 0.0638q_k^{(i)}(g_t) + 0.0638q_l^{(i)}(g_t) - 0.131q_j^{(j)}(g_t)$$
$$- 0.1312q_k^{(j)}(g_t) + 0.0638q_j^{(k)}(g_t) + 0.0638q_k^{(k)}(g_t) + 0.0638q_l^{(k)}(g_t) + 0.0355q_k^{(l)}(g_t)$$
$$+ 0.0355q_l^{(l)}(g_t)$$

$$q_j^{(j)}(g_t) = 0.2703 + 0.0811q_i^{(i)}(g_t) + 0.0811q_k^{(i)}(g_t) + 0.0811q_l^{(i)}(g_t) - 0.1216q_i^{(j)}(g_t)$$
$$- 0.1216q_k^{(j)}(g_t) + 0.0811q_i^{(k)}(g_t) + 0.0811q_k^{(k)}(g_t) + 0.0811q_l^{(k)}(g_t)$$

$$q_k^{(j)}(g_t) = 0.2128 + 0.0638q_i^{(i)}(g_t) + 0.0638q_j^{(i)}(g_t) + 0.0638q_l^{(i)}(g_t) - 0.1312q_i^{(j)}(g_t)$$
$$- 0.1312q_j^{(j)}(g_t) + 0.0638q_i^{(k)}(g_t) + 0.0638q_j^{(k)}(g_t) + 0.0638q_l^{(k)}(g_t) + 0.0355q_i^{(l)}(g_t)$$
$$+ 0.0355q_l^{(l)}(g_t)$$

$$q_i^{(k)}(g_t) = 0.1277 + 0.0383q_j^{(i)}(g_t) + 0.0383q_k^{(i)}(g_t) + 0.0383q_l^{(i)}(g_t) + 0.0213q_j^{(j)}(g_t)$$
$$+ 0.0213q_k^{(j)}(g_t) - 0.2617q_j^{(k)}(g_t) - 0.2617q_k^{(k)}(g_t) - 0.2617q_l^{(k)}(g_t) + 0.0213q_k^{(l)}(g_t)$$
$$+ 0.0213q_l^{(l)}(g_t)$$

$$q_j^{(k)}(g_t) = 0.1622 + 0.0486q_i^{(i)}(g_t) + 0.0846q_k^{(i)}(g_t) + 0.0486q_l^{(i)}(g_t) + 0.0270q_i^{(j)}(g_t)$$
$$+ 0.0270q_k^{(j)}(g_t) - 0.2514q_i^{(k)}(g_t) - 0.2514q_k^{(k)}(g_t) - 0.2514q_l^{(k)}(g_t)$$

$$q_k^{(k)}(g_t) = 0.1277 + 0.0383q_i^{(i)}(g_t) + 0.0383q_j^{(i)}(g_t) + 0.0383q_l^{(i)}(g_t) + 0.0213q_i^{(j)}(g_t)$$
$$+ 0.0213q_j^{(j)}(g_t) - 0.2617q_i^{(k)}(g_t) - 0.2617q_j^{(k)}(g_t) - 0.2617q_l^{(k)}(g_t) + 0.0213q_i^{(l)}(g_t)$$
$$+ 0.0213q_l^{(l)}(g_t)$$

$$q_l^{(k)}(g_t) = 0.1622 + 0.0486q_i^{(i)}(g_t) + 0.0486q_j^{(i)}(g_t) + 0.0486q_k^{(i)}(g_t) - 0.2514q_i^{(k)}(g_t)$$
$$- 0.2514q_j^{(k)}(g_t) - 0.2514q_k^{(k)}(g_t) + 0.0270q_i^{(l)}(g_t) + 0.0270q_k^{(l)}(g_t)$$

$$q_i^{(l)}(g_t) = 0.2128 + 0.0638q_j^{(i)}(g_t) + 0.0638q_k^{(i)}(g_t) + 0.0638q_l^{(i)}(g_t) + 0.0355q_j^{(j)}(g_t)$$
$$+ 0.0355q_k^{(j)}(g_t) + 0.0638q_j^{(k)}(g_t) + 0.0638q_k^{(k)}(g_t) + 0.0638q_l^{(k)}(g_t) - 0.1312q_k^{(l)}(g_t)$$
$$- 0.1312q_l^{(l)}(g_t)$$

$$q_k^{(l)}(g_t) = 0.2128 + 0.0638q_i^{(i)}(g_t) + 0.0638q_j^{(i)}(g_t) + 0.0638q_l^{(i)}(g_t) + 0.0355q_i^{(j)}(g_t)$$
$$+ 0.0355q_j^{(j)}(g_t) + 0.0638q_i^{(k)}(g_t) + 0.0638q_j^{(k)}(g_t) + 0.0638q_l^{(k)}(g_t) - 0.1312q_i^{(l)}(g_t)$$
$$- 0.1312q_l^{(l)}(g_t)$$

$$q_l^{(l)}(g_t) = 0.2703 + 0.0811q_i^{(i)}(g_t) + 0.0811q_j^{(i)}(g_t) + 0.0811q_k^{(i)}(g_t) + 0.0811q_i^{(k)}(g_t)$$
$$+ 0.0811q_j^{(k)}(g_t) + 0.0811q_k^{(k)}(g_t) - 0.1216q_i^{(l)}(g_t) - 0.1216q_k^{(l)}(g_t)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_t)$ = $q_k^{(i)}(g_t)$ = $q_i^{(k)}(g_t)$ = $q_k^{(k)}(g_t)$ = 0.0777; $q_j^{(i)}(g_t)$ = $q_l^{(i)}(g_t)$ = $q_j^{(k)}(g_t)$ = $q_l^{(k)}(g_t)$ = 0.1180; $q_i^{(j)}(g_t)$ = $q_k^{(j)}(g_t)$ = $q_i^{(l)}(g_t)$ = $q_k^{(l)}(g_t)$ = 0.2076; $q_j^{(j)}(g_t)$ = $q_l^{(l)}(g_t)$ = 0.2641; $CS_i(g_t)$ = $CS_k(g_t)$ = 0.1628; $CS_j(g_t)$ = $CS_l(g_t)$ = 0.1250; $\pi_i^{(i)}(g_t)$ = $\pi_k^{(i)}(g_t)$ = $\pi_i^{(k)}(g_t)$ = $\pi_k^{(k)}(g_t)$ = 0.0136; $\pi_j^{(i)}(g_t)$ = $\pi_l^{(i)}(g_t)$ = $\pi_j^{(k)}(g_t)$ = $\pi_l^{(k)}(g_t)$ = 0.0313; $\pi_i^{(j)}(g_t)$ = $\pi_k^{(j)}(g_t)$ = $\pi_i^{(l)}(g_t)$ = $\pi_k^{(l)}(g_t)$ = 0.0539; $\pi_j^{(j)}(g_t)$ = $\pi_l^{(l)}(g_t)$ = 0.0872; $PS_i(g_t)$

= $PS_k(g_t)$ = 0.0574; $PS_j(g_t)$ = $PS_l(g_t)$ = 0.0577; $W_i(g_t)$ =$W_k(g_t)$ = 0.3100; and $W_j(g_t)$ = $W_l(g_t)$ = 0.3777.

Network u

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_u) = 0.1277 - 0.2617q_j^{(i)}(g_u) - 0.2617q_k^{(i)}(g_u) - 0.2617q_l^{(i)}(g_u) + 0.0213q_j^{(j)}(g_u) + 0.0213q_k^{(j)}(g_u) + 0.0213q_l^{(j)}(g_u) + 0.0383q_j^{(k)}(g_u) + 0.0383q_k^{(k)}(g_u) + 0.0213q_j^{(l)}(g_u) + 0.0213q_l^{(l)}(g_u)$$

$$q_j^{(i)}(g_u) = 0.1277 - 0.2617q_i^{(i)}(g_u) - 0.2617q_k^{(i)}(g_u) - 0.2617q_l^{(i)}(g_u) + 0.0213q_i^{(j)}(g_u) + 0.0213q_k^{(j)}(g_u) + 0.0213q_l^{(j)}(g_u) + 0.0383q_i^{(k)}(g_u) + 0.0383q_k^{(k)}(g_u) + 0.0213q_i^{(l)}(g_u) + 0.0213q_l^{(l)}(g_u)$$

$$q_k^{(i)}(g_u) = 0.1622 - 0.2514q_i^{(i)}(g_u) - 0.2514q_j^{(i)}(g_u) - 0.2514q_l^{(i)}(g_u) + 0.0270q_i^{(j)}(g_u) + 0.0270q_j^{(j)}(g_u) + 0.0270q_l^{(j)}(g_u) + 0.0486q_i^{(k)}(g_u) + 0.0486q_j^{(k)}(g_u)$$

$$q_l^{(i)}(g_u) = 0.1463 - 0.2561q_i^{(i)}(g_u) - 0.2561q_j^{(i)}(g_u) - 0.2561q_k^{(i)}(g_u) + 0.0244q_i^{(j)}(g_u) + 0.0244q_j^{(j)}(g_u) + 0.0244q_k^{(j)}(g_u) + 0.0244q_i^{(l)}(g_u) + 0.0244q_j^{(l)}(g_u)$$

$$q_i^{(j)}(g_u) = 0.2128 + 0.0638q_j^{(i)}(g_u) + 0.0638q_k^{(i)}(g_u) + 0.0638q_l^{(i)}(g_u) - 0.1312q_j^{(j)}(g_u) - 0.1312q_k^{(j)}(g_u) - 0.1312q_l^{(j)}(g_u) + 0.0638q_j^{(k)}(g_u) + 0.0638q_k^{(k)}(g_u) + 0.0355q_j^{(l)}(g_u) + 0.0355q_l^{(l)}(g_u)$$

$$q_j^{(j)}(g_u) = 0.2128 + 0.0638q_i^{(i)}(g_u) + 0.0638q_k^{(i)}(g_u) + 0.0638q_l^{(i)}(g_u) - 0.1312q_i^{(j)}(g_u) - 0.1312q_k^{(j)}(g_u) - 0.1312q_l^{(j)}(g_u) + 0.0638q_i^{(k)}(g_u) + 0.0638q_k^{(k)}(g_u) + 0.0355q_i^{(l)}(g_u) + 0.0366q_l^{(l)}(g_u)$$

$$q_k^{(j)}(g_u) = 0.2703 + 0.0811q_i^{(i)}(g_u) + 0.0811q_j^{(i)}(g_u) + 0.0811q_l^{(i)}(g_u) - 0.1216q_i^{(j)}(g_u) - 0.1216q_j^{(j)}(g_u) - 0.1216q_l^{(j)}(g_u) + 0.0811q_i^{(k)}(g_u) + 0.0811q_j^{(k)}(g_u)$$

$$q_l^{(j)}(g_u) = 0.2439 + 0.0732q_i^{(i)}(g_u) + 0.0732q_j^{(i)}(g_u) + 0.0732q_k^{(i)}(g_u) - 0.1260q_i^{(j)}(g_u)$$
$$- 0.1260q_j^{(j)}(g_u) - 0.1260q_k^{(j)}(g_u) + 0.0407q_i^{(l)}(g_u) + 0.0407q_j^{(l)}(g_u)$$

$$q_i^{(k)}(g_u) = 0.1277 + 0.0383q_j^{(i)}(g_u) + 0.0383q_k^{(i)}(g_u) + 0.0383q_l^{(i)}(g_u) + 0.0213q_j^{(j)}(g_u)$$
$$+ 0.0213q_k^{(j)}(g_u) + 0.0213q_l^{(j)}(g_u) - 0.2617q_j^{(k)}(g_u) - 0.2617q_k^{(k)}(g_u) + 0.0213q_j^{(l)}(g_u)$$
$$+ 0.0213q_l^{(l)}(g_u)$$

$$q_j^{(k)}(g_u) = 0.1277 + 0.0383q_i^{(i)}(g_u) + 0.0279q_k^{(i)}(g_u) + 0.0383q_l^{(i)}(g_u) + 0.0213q_i^{(j)}(g_u)$$
$$+ 0.0213q_k^{(j)}(g_u) + 0.0213q_l^{(j)}(g_u) - 0.2617q_i^{(k)}(g_u) - 0.2617q_k^{(k)}(g_u) + 0.0213q_i^{(l)}(g_u)$$
$$+ 0.0213q_l^{(l)}(g_u)$$

$$q_k^{(k)}(g_u) = 0.1622 + 0.0486q_i^{(i)}(g_u) + 0.0486q_j^{(i)}(g_u) + 0.0486q_l^{(i)}(g_u) + 0.0270q_i^{(j)}(g_u)$$
$$+ 0.0270q_j^{(j)}(g_u) + 0.0270q_l^{(j)}(g_u) - 0.2514q_i^{(k)}(g_u) - 0.2514q_j^{(k)}(g_u)$$

$$q_i^{(l)}(g_u) = 0.2128 + 0.0638q_j^{(i)}(g_u) + 0.0638q_k^{(i)}(g_u) + 0.0638q_l^{(i)}(g_u) + 0.0355q_j^{(j)}(g_u)$$
$$+ 0.0355q_k^{(j)}(g_u) + 0.0355q_l^{(j)}(g_u) + 0.0638q_j^{(k)}(g_u) + 0.0638q_k^{(k)}(g_u) - 0.1312q_j^{(l)}(g_u)$$
$$- 0.1312q_l^{(l)}(g_u)$$

$$q_j^{(l)}(g_u) = 0.2128 + 0.0638q_i^{(i)}(g_u) + 0.0638q_k^{(i)}(g_u) + 0.0638q_l^{(i)}(g_u) + 0.0355q_i^{(j)}(g_u)$$
$$+ 0.0355q_k^{(j)}(g_u) + 0.0355q_l^{(j)}(g_u) + 0.0638q_i^{(k)}(g_u) + 0.0638q_k^{(k)}(g_u) - 0.1312q_i^{(l)}(g_u)$$
$$- 0.1312q_l^{(l)}(g_u)$$

$$q_l^{(l)}(g_u) = 0.2439 + 0.0732q_i^{(i)}(g_u) + 0.0732q_j^{(i)}(g_u) + 0.0732q_k^{(i)}(g_u) + 0.0407q_i^{(j)}(g_u)$$
$$+ 0.0407q_j^{(j)}(g_u) + 0.0407q_k^{(j)}(g_u) - 0.1260q_i^{(l)}(g_u) - 0.1260q_j^{(l)}(g_u)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_u) = q_j^{(i)}(g_u) = 0.0808$; $q_k^{(i)}(g_u) = 0.1218$; $q_l^{(i)}(g_u) = 0.0987$; $q_i^{(j)}(g_u) = q_j^{(j)}(g_u) = 0.1811$; $q_k^{(j)}(g_u) = 0.2384$; $q_l^{(j)}(g_u) = 0.2061$; $q_i^{(k)}(g_u) = q_j^{(k)}(g_u) = 0.0993$; $q_k^{(k)}(g_u) = 0.1403$; $q_i^{(l)}(g_u) = q_j^{(l)}(g_u) = 0.2109$; $q_l^{(l)}(g_u) = 0.2359$; $CS_i(g_u) = CS_j(g_u) = 0.1636$; $CS_k(g_u) = 0.1252$; $CS_l(g_u) = 0.1462$; $\pi_i^{(i)}(g_u) = \pi_j^{(i)}(g_u) = 0.0147$; $\pi_k^{(i)}(g_u) = 0.0334$;

$\pi_l^{(i)}(g_u)$ = 0.0219; $\pi_i^{(j)}(g_u)$ = $\pi_j^{(j)}(g_u)$ = 0.0410; $\pi_k^{(j)}(g_u)$ = 0.0710; $\pi_l^{(j)}(g_u)$ = 0.0531;

$\pi_i^{(k)}(g_u)$ = $\pi_j^{(k)}(g_u)$ = 0.0222; $\pi_k^{(k)}(g_u)$ = 0.0443; $\pi_i^{(l)}(g_u)$ = $\pi_j^{(l)}(g_u)$ = 0.0556; $\pi_l^{(l)}(g_u)$ = 0.0696; $PS_i(g_u)$ = 0.0547; $PS_j(g_u)$ = 0.0813; $PS_k(g_u)$ = 0.0431; $PS_l(g_u)$ = 0.0541;

$W_i(g_u)$ = 0.3030; $W_j(g_u)$ = 0.4510; $W_k(g_u)$ = 0.2569; and $W_l(g_u)$ = 0.3810.

Network v

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_v) = 0.1463 - 0.2561q_j^{(i)}(g_v) - 0.2561q_l^{(i)}(g_v) + 0.0244q_j^{(j)}(g_v) + 0.0244q_k^{(j)}(g_v) + 0.0244q_l^{(j)}(g_v) + 0.0244q_j^{(l)}(g_v) + 0.0244q_k^{(l)}(g_v) + 0.0244q_l^{(l)}(g_v)$$

$$q_j^{(i)}(g_v) = 0.1277 - 0.2617q_i^{(i)}(g_v) - 0.2617q_l^{(i)}(g_v) + 0.0213q_i^{(j)}(g_v) + 0.0213q_k^{(j)}(g_v) + 0.0213q_l^{(j)}(g_v) + 0.0383q_k^{(k)}(g_v) + 0.0383q_l^{(k)}(g_v) + 0.0213q_i^{(l)}(g_v) + 0.0213q_k^{(l)}(g_v) + 0.0213q_l^{(l)}(g_v)$$

$$q_l^{(i)}(g_v) = 0.1277 - 0.2617q_i^{(i)}(g_v) - 0.2617q_j^{(i)}(g_v) + 0.0213q_i^{(j)}(g_v) + 0.0213q_j^{(j)}(g_v) + 0.0213q_k^{(j)}(g_v) + 0.0383q_j^{(k)}(g_v) + 0.0383q_k^{(k)}(g_v) + 0.0213q_i^{(l)}(g_v) + 0.0213q_j^{(l)}(g_v) + 0.0213q_k^{(l)}(g_v)$$

$$q_i^{(j)}(g_v) = 0.2439 + 0.0732q_j^{(i)}(g_v) + 0.0732q_l^{(i)}(g_v) - 0.1260q_j^{(j)}(g_v) - 0.1260q_k^{(j)}(g_v) - 0.1260q_l^{(j)}(g_v) + 0.0407q_j^{(l)}(g_v) + 0.0407q_k^{(l)}(g_v) + 0.0407q_l^{(l)}(g_v)$$

$$q_j^{(j)}(g_v) = 0.2128 + 0.0638q_i^{(i)}(g_v) + 0.0638q_l^{(i)}(g_v) - 0.1312q_i^{(j)}(g_v) - 0.1312q_k^{(j)}(g_v) - 0.1312q_l^{(j)}(g_v) + 0.0638q_k^{(k)}(g_v) + 0.0638q_l^{(k)}(g_v) + 0.0355q_i^{(l)}(g_v) + 0.0355q_k^{(l)}(g_v) + 0.0355q_l^{(l)}(g_v)$$

$$q_k^{(j)}(g_v) = 0.2439 - 0.1260q_i^{(j)}(g_v) - 0.1260q_j^{(j)}(g_v) - 0.1260q_l^{(j)}(g_v) + 0.0732q_j^{(k)}(g_v) + 0.0732q_l^{(k)}(g_v) + 0.0407q_i^{(l)}(g_v) + 0.0407q_j^{(l)}(g_v) + 0.0407q_l^{(l)}(g_v)$$

$$q_l^{(j)}(g_v) = 0.2128 + 0.0638 q_i^{(i)}(g_v) + 0.0638 q_j^{(i)}(g_v) - 0.1312 q_i^{(j)}(g_v) - 0.1312 q_j^{(j)}(g_v) - 0.1312 q_k^{(j)}(g_v) + 0.0638 q_j^{(k)}(g_v) + 0.0638 q_k^{(k)}(g_v) + 0.0355 q_i^{(l)}(g_v) + 0.0355 q_j^{(l)}(g_v) + 0.0355 q_k^{(l)}(g_v)$$

$$q_j^{(k)}(g_v) = 0.1277 + 0.0383 q_i^{(i)}(g_v) + 0.0383 q_l^{(i)}(g_v) + 0.0213 q_i^{(j)}(g_v) + 0.0213 q_k^{(j)}(g_v) + 0.0213 q_l^{(j)}(g_v) - 0.2617 q_k^{(k)}(g_v) - 0.2617 q_l^{(k)}(g_v) + 0.0213 q_i^{(l)}(g_v) + 0.0213 q_k^{(l)}(g_v) + 0.0213 q_l^{(l)}(g_v)$$

$$q_k^{(k)}(g_v) = 0.1463 + 0.0244 q_i^{(j)}(g_v) + 0.0244 q_j^{(j)}(g_v) + 0.0244 q_l^{(j)}(g_v) - 0.2561 q_j^{(k)}(g_v) - 0.2561 q_l^{(k)}(g_v) + 0.0244 q_i^{(l)}(g_v) + 0.0244 q_j^{(l)}(g_v) + 0.0244 q_l^{(l)}(g_v)$$

$$q_l^{(k)}(g_v) = 0.1277 + 0.0383 q_i^{(i)}(g_v) + 0.0383 q_j^{(i)}(g_v) + 0.0213 q_i^{(j)}(g_v) + 0.0213 q_j^{(j)}(g_v) + 0.0213 q_k^{(j)}(g_v) - 0.2617 q_j^{(k)}(g_v) - 0.2617 q_k^{(k)}(g_v) + 0.0213 q_i^{(l)}(g_v) + 0.0213 q_j^{(l)}(g_v) + 0.0213 q_k^{(l)}(g_v)$$

$$q_i^{(l)}(g_v) = 0.2439 + 0.0732 q_j^{(i)}(g_v) + 0.0732 q_l^{(i)}(g_v) + 0.0407 q_j^{(j)}(g_v) + 0.0407 q_k^{(j)}(g_v) + 0.0407 q_l^{(j)}(g_v) - 0.1260 q_j^{(l)}(g_v) - 0.1260 q_k^{(l)}(g_v) - 0.1260 q_l^{(l)}(g_v)$$

$$q_j^{(l)}(g_v) = 0.2128 + 0.0638 q_i^{(i)}(g_v) + 0.0638 q_l^{(i)}(g_v) + 0.0355 q_i^{(j)}(g_v) + 0.0355 q_k^{(j)}(g_v) + 0.0355 q_l^{(j)}(g_v) + 0.0638 q_k^{(k)}(g_v) + 0.0638 q_l^{(k)}(g_v) - 0.1312 q_i^{(l)}(g_v) - 0.1312 q_k^{(l)}(g_v) - 0.1312 q_l^{(l)}(g_v)$$

$$q_k^{(l)}(g_v) = 0.2439 + 0.0407 q_i^{(j)}(g_v) + 0.0407 q_j^{(j)}(g_v) + 0.0407 q_l^{(j)}(g_v) + 0.0732 q_j^{(k)}(g_v) + 0.0732 q_l^{(k)}(g_v) - 0.1260 q_i^{(l)}(g_v) - 0.1260 q_j^{(l)}(g_v) - 0.1260 q_l^{(l)}(g_v)$$

$$q_l^{(l)}(g_v) = 0.2128 + 0.0638 q_i^{(i)}(g_v) + 0.0638 q_j^{(i)}(g_v) + 0.0355 q_i^{(j)}(g_v) + 0.0355 q_j^{(j)}(g_v) + 0.0355 q_k^{(j)}(g_v) + 0.0638 q_j^{(k)}(g_v) + 0.0638 q_k^{(k)}(g_v) - 0.1312 q_i^{(l)}(g_v) - 0.1312 q_j^{(l)}(g_v) - 0.1312 q_k^{(l)}(g_v)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_v) = q_k^{(k)}(g_v) = 0.1217$; $q_j^{(i)}(g_v) = q_l^{(i)}(g_v) = q_j^{(k)}(g_v) = q_l^{(k)}(g_v) = 0.1031$; $q_i^{(j)}(g_v) = q_k^{(j)}(g_v) = q_i^{(l)}(g_v) = q_k^{(l)}(g_v) = 0.2097$; $q_j^{(j)}(g_v) = q_l^{(j)}(g_v) = q_j^{(l)}(g_v) = q_l^{(l)}(g_v) = 0.1837$;

$CS_i(g_v)$ = $CS_k(g_v)$ = 0.1464; $CS_j(g_v)$ = $CS_l(g_v)$ = 0.1645; $\pi_i^{(i)}(g_v)$ = $\pi_k^{(k)}(g_v)$ = 0.0333; $\pi_j^{(i)}(g_v)$ = $\pi_l^{(i)}(g_v)$ = $\pi_j^{(k)}(g_v)$ = $\pi_l^{(k)}(g_v)$ = 0.0239; $\pi_i^{(j)}(g_v)$ = $\pi_k^{(j)}(g_v)$ = $\pi_i^{(l)}(g_v)$ = $\pi_k^{(l)}(g_v)$ = 0.0550; $\pi_j^{(j)}(g_v)$ = $\pi_l^{(j)}(g_v)$ = $\pi_j^{(l)}(g_v)$ = $\pi_l^{(l)}(g_v)$ = 0.0422; $PS_i(g_v)$ = $PS_k(g_v)$ = 0.0403; $PS_j(g_v)$ = $PS_l(g_v)$ = 0.0774; $W_i(g_v)$ = $W_k(g_v)$ = 0.2679; and $W_j(g_v)$ = $W_l(g_v)$ = 0.4363.

Network w

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_w) = 0.1463 - 0.2561q_j^{(i)}(g_w) - 0.2561q_l^{(i)}(g_w) + 0.0244q_j^{(j)}(g_w) + 0.0244q_k^{(j)}(g_w) + 0.0244q_k^{(l)}(g_w) + 0.0244q_l^{(l)}(g_w)$$

$$q_j^{(i)}(g_w) = 0.1622 - 0.2514q_i^{(i)}(g_w) - 0.2514q_l^{(i)}(g_w) + 0.0270q_i^{(j)}(g_w) + 0.0270q_k^{(j)}(g_w) + 0.0486q_k^{(k)}(g_w) + 0.0486q_l^{(k)}(g_w)$$

$$q_l^{(i)}(g_w) = 0.1622 - 0.2514q_i^{(i)}(g_w) - 0.2514q_j^{(i)}(g_w) + 0.0486q_j^{(k)}(g_w) + 0.0486q_k^{(k)}(g_w) + 0.0270q_i^{(l)}(g_w) + 0.0270q_k^{(l)}(g_w)$$

$$q_i^{(j)}(g_w) = 0.2439 + 0.0732q_j^{(i)}(g_w) + 0.0732q_l^{(i)}(g_w) - 0.1260q_j^{(j)}(g_w) - 0.1260q_k^{(j)}(g_w) + 0.0407q_k^{(l)}(g_w) + 0.0407q_l^{(l)}(g_w)$$

$$q_j^{(j)}(g_w) = 0.2703 + 0.0811q_i^{(i)}(g_w) + 0.0811q_l^{(i)}(g_w) - 0.1216q_i^{(j)}(g_w) - 0.1216q_k^{(j)}(g_w) + 0.0811q_k^{(k)}(g_w) + 0.0811q_l^{(k)}(g_w)$$

$$q_k^{(j)}(g_w) = 0.2439 - 0.1260q_i^{(j)}(g_w) - 0.1260q_j^{(j)}(g_w) + 0.0732q_j^{(k)}(g_w) + 0.0732q_l^{(k)}(g_w) + 0.0407q_i^{(l)}(g_w) + 0.0407q_l^{(l)}(g_w)$$

$$q_j^{(k)}(g_w) = 0.1622 + 0.0486 q_i^{(i)}(g_w) + 0.0486 q_l^{(i)}(g_w) + 0.0270 q_i^{(j)}(g_w) + 0.0270 q_k^{(j)}(g_w)$$
$$- 0.2514 q_k^{(k)}(g_w) - 0.2514 q_l^{(k)}(g_w)$$

$$q_k^{(k)}(g_w) = 0.1463 + 0.0244 q_i^{(j)}(g_w) + 0.0244 q_j^{(j)}(g_w) - 0.2561 q_j^{(k)}(g_w) - 0.2561 q_l^{(k)}(g_w)$$
$$+ 0.0244 q_i^{(l)}(g_w) + 0.0244 q_l^{(l)}(g_w)$$

$$q_l^{(k)}(g_w) = 0.1622 + 0.0486 q_i^{(i)}(g_w) + 0.0486 q_j^{(i)}(g_w) - 0.2514 q_j^{(k)}(g_w) - 0.2514 q_k^{(k)}(g_w)$$
$$+ 0.0270 q_i^{(l)}(g_w) + 0.0270 q_k^{(l)}(g_w)$$

$$q_i^{(l)}(g_w) = 0.2439 + 0.0732 q_j^{(i)}(g_w) + 0.0732 q_l^{(i)}(g_w) + 0.0407 q_j^{(j)}(g_w) + 0.0407 q_k^{(j)}(g_w)$$
$$- 0.1260 q_k^{(l)}(g_w) - 0.1260 q_l^{(l)}(g_w)$$

$$q_k^{(l)}(g_w) = 0.2439 + 0.0407 q_i^{(j)}(g_w) + 0.0407 q_j^{(j)}(g_w) + 0.0732 q_j^{(k)}(g_w) + 0.0732 q_l^{(k)}(g_w)$$
$$- 0.1260 q_i^{(l)}(g_w) - 0.1260 q_l^{(l)}(g_w)$$

$$q_l^{(l)}(g_w) = 0.2703 + 0.0811 q_i^{(i)}(g_w) + 0.0811 q_j^{(i)}(g_w) + 0.0811 q_j^{(k)}(g_w) + 0.0811 q_k^{(k)}(g_w)$$
$$- 0.1216 q_i^{(l)}(g_w) - 0.1216 q_k^{(l)}(g_w)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_w) = q_k^{(k)}(g_w) = 0.1044$; $q_j^{(i)}(g_w) = q_l^{(i)}(g_w) = q_j^{(k)}(g_w) = q_l^{(k)}(g_w) = 0.1272$; $q_i^{(j)}(g_w) = q_k^{(j)}(g_w) = q_i^{(l)}(g_w) = q_k^{(l)}(g_w) = 0.2219$; $q_j^{(j)}(g_w) = q_l^{(l)}(g_w) = 0.2539$; $CS_i(g_w) = CS_k(g_w) = 0.1503$; $CS_j(g_w) = CS_l(g_w) = 0.1292$; $\pi_i^{(i)}(g_w) = \pi_k^{(k)}(g_w) = 0.0245$; $\pi_j^{(i)}(g_w) = \pi_l^{(i)}(g_w) = \pi_j^{(k)}(g_w) = \pi_l^{(k)}(g_w) = 0.0364$; $\pi_i^{(j)}(g_w) = \pi_k^{(j)}(g_w) = \pi_i^{(l)}(g_w) = \pi_k^{(l)}(g_w) = 0.0616$; $\pi_j^{(j)}(g_w) = \pi_l^{(l)}(g_w) = 0.0806$; $PS_i(g_w) = PS_k(g_w) = 0.0483$; $PS_j(g_w) = PS_l(g_w) = 0.0608$; $W_i(g_w) = W_k(g_w) = 0.2959$; and $W_j(g_w) = W_l(g_w) = 0.3937$.

Network x

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_x) = 0.1277 - 0.2617q_j^{(i)}(g_x) - 0.2617q_k^{(i)}(g_x) - 0.2617q_l^{(i)}(g_x) + 0.0213q_j^{(j)}(g_x) + 0.0213q_k^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) + 0.0383q_j^{(k)}(g_x) + 0.0383q_k^{(k)}(g_x) + 0.0383q_l^{(k)}(g_x) + 0.0213q_j^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_j^{(i)}(g_x) = 0.1277 - 0.2617q_i^{(i)}(g_x) - 0.2617q_k^{(i)}(g_x) - 0.2617q_l^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x) + 0.0213q_k^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) + 0.0383q_i^{(k)}(g_x) + 0.0383q_k^{(k)}(g_x) + 0.0383q_l^{(k)}(g_x) + 0.0213q_i^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_k^{(i)}(g_x) = 0.1277 - 0.2617q_i^{(i)}(g_x) - 0.2617q_j^{(i)}(g_x) - 0.2617q_l^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x) + 0.0213q_j^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) + 0.0383q_i^{(k)}(g_x) + 0.0383q_j^{(k)}(g_x) + 0.0383q_l^{(k)}(g_x) + 0.0213q_i^{(l)}(g_x) + 0.0213q_j^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_l^{(i)}(g_x) = 0.1277 - 0.2617q_i^{(i)}(g_x) - 0.2617q_j^{(i)}(g_x) - 0.2617q_k^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x) + 0.0213q_j^{(j)}(g_x) + 0.0213q_k^{(j)}(g_x) + 0.0383q_i^{(k)}(g_x) + 0.0383q_j^{(k)}(g_x) + 0.0383q_k^{(k)}(g_x) + 0.0213q_i^{(l)}(g_x) + 0.0213q_j^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x)$$

$$q_i^{(j)}(g_x) = 0.2128 + 0.0638q_j^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) - 0.1312q_j^{(j)}(g_x) - 0.1312q_k^{(j)}(g_x) - 0.1312q_l^{(j)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x) + 0.0355q_j^{(l)}(g_x) + 0.0355q_k^{(l)}(g_x) + 0.0355q_l^{(l)}(g_x)$$

$$q_j^{(j)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) - 0.1312q_i^{(j)}(g_x) - 0.1312q_k^{(j)}(g_x) - 0.1312q_l^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x) + 0.0355q_i^{(l)}(g_x) + 0.0355q_k^{(l)}(g_x) + 0.0355q_l^{(l)}(g_x)$$

$$q_k^{(j)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_j^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) - 0.1312q_i^{(j)}(g_x) - 0.1312q_j^{(j)}(g_x) - 0.1312q_l^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x) + 0.0355q_i^{(l)}(g_x) + 0.0355q_j^{(l)}(g_x) + 0.0355q_l^{(l)}(g_x)$$

$$q_l^{(j)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_j^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) - 0.1312q_i^{(j)}(g_x)$$
$$- 0.1312q_j^{(j)}(g_x) - 0.1312q_k^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x)$$
$$+ 0.0355q_i^{(l)}(g_x) + 0.0355q_j^{(l)}(g_x) + 0.0355q_k^{(l)}(g_x)$$

$$q_i^{(k)}(g_x) = 0.1277 + 0.0383q_j^{(i)}(g_x) + 0.0383q_k^{(i)}(g_x) + 0.0383q_l^{(i)}(g_x) + 0.0213q_j^{(j)}(g_x)$$
$$+ 0.0213q_k^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) - 0.2617q_j^{(k)}(g_x) - 0.2617q_k^{(k)}(g_x) - 0.2617q_l^{(k)}(g_x)$$
$$+ 0.0213q_j^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_j^{(k)}(g_x) = 0.1277 + 0.0383q_i^{(i)}(g_x) + 0.0383q_k^{(i)}(g_x) + 0.0383q_l^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x)$$
$$+ 0.0213q_k^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) - 0.2617q_i^{(k)}(g_x) - 0.2617q_k^{(k)}(g_x) - 0.2617q_l^{(k)}(g_x)$$
$$+ 0.0213q_i^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_k^{(k)}(g_x) = 0.1277 + 0.0383q_i^{(i)}(g_x) + 0.0383q_j^{(i)}(g_x) + 0.0383q_l^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x)$$
$$+ 0.0213q_j^{(j)}(g_x) + 0.0213q_l^{(j)}(g_x) - 0.2617q_i^{(k)}(g_x) - 0.2617q_j^{(k)}(g_x) - 0.2617q_l^{(k)}(g_x)$$
$$+ 0.0213q_i^{(l)}(g_x) + 0.0213q_j^{(l)}(g_x) + 0.0213q_l^{(l)}(g_x)$$

$$q_l^{(k)}(g_x) = 0.1277 + 0.0383q_i^{(i)}(g_x) + 0.0383q_j^{(i)}(g_x) + 0.0383q_k^{(i)}(g_x) + 0.0213q_i^{(j)}(g_x)$$
$$+ 0.0213q_j^{(j)}(g_x) + 0.0213q_k^{(j)}(g_x) - 0.2617q_i^{(k)}(g_x) - 0.2617q_j^{(k)}(g_x) - 0.2617q_k^{(k)}(g_x)$$
$$+ 0.0213q_i^{(l)}(g_x) + 0.0213q_j^{(l)}(g_x) + 0.0213q_k^{(l)}(g_x)$$

$$q_i^{(l)}(g_x) = 0.2128 + 0.0638q_j^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) + 0.0355q_j^{(j)}(g_x)$$
$$+ 0.0355q_k^{(j)}(g_x) + 0.0355q_l^{(j)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x)$$
$$- 0.1312q_j^{(l)}(g_x) - 0.1312q_k^{(l)}(g_x) - 0.1312q_l^{(l)}(g_x)$$

$$q_j^{(l)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) + 0.0355q_i^{(j)}(g_x)$$
$$+ 0.0355q_k^{(j)}(g_x) + 0.0355q_l^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x)$$
$$- 0.1312q_i^{(l)}(g_x) - 0.1312q_k^{(l)}(g_x) - 0.1312q_l^{(l)}(g_x)$$

$$q_k^{(l)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_j^{(i)}(g_x) + 0.0638q_l^{(i)}(g_x) + 0.0355q_i^{(j)}(g_x)$$
$$+ 0.0355q_j^{(j)}(g_x) + 0.0355q_l^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_l^{(k)}(g_x)$$
$$- 0.1312q_i^{(l)}(g_x) - 0.1312q_j^{(l)}(g_x) - 0.1312q_l^{(l)}(g_x)$$

$$q_l^{(l)}(g_x) = 0.2128 + 0.0638q_i^{(i)}(g_x) + 0.0638q_j^{(i)}(g_x) + 0.0638q_k^{(i)}(g_x) + 0.0355q_i^{(j)}(g_x)$$
$$+ 0.0355q_j^{(j)}(g_x) + 0.0355q_k^{(j)}(g_x) + 0.0638q_i^{(k)}(g_x) + 0.0638q_j^{(k)}(g_x) + 0.0638q_k^{(k)}(g_x)$$
$$- 0.1312q_i^{(l)}(g_x) - 0.1312q_j^{(l)}(g_x) - 0.1312q_k^{(l)}(g_x)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_x)$ = $q_j^{(i)}(g_x)$ = $q_k^{(i)}(g_x)$ = $q_l^{(i)}(g_x)$ = $q_i^{(k)}(g_x)$ = $q_j^{(k)}(g_x)$ = $q_k^{(k)}(g_x)$ = $q_l^{(k)}(g_x)$ = 0.0911; $q_i^{(j)}(g_x)$ = $q_j^{(j)}(g_x)$ = $q_k^{(j)}(g_x)$ = $q_l^{(j)}(g_x)$ = $q_i^{(l)}(g_x)$ = $q_j^{(l)}(g_x)$ = $q_k^{(l)}(g_x)$ = $q_l^{(l)}(g_x)$ = 0.1924; $CS_i(g_x)$ = $CS_j(g_x)$ = $CS_k(g_x)$ = $CS_l(g_x)$ = 0.1608; $\pi_i^{(i)}(g_x)$ = $\pi_j^{(i)}(g_x)$ = $\pi_k^{(i)}(g_x)$ = $\pi_l^{(i)}(g_x)$ = $\pi_i^{(k)}(g_x)$ = $\pi_j^{(k)}(g_x)$ = $\pi_k^{(k)}(g_x)$ = $\pi_l^{(k)}(g_x)$ = 0.0187; $\pi_i^{(j)}(g_x)$ = $\pi_j^{(j)}(g_x)$ = $\pi_k^{(j)}(g_x)$ = $\pi_l^{(j)}(g_x)$ = $\pi_i^{(l)}(g_x)$ = $\pi_j^{(l)}(g_x)$ = $\pi_k^{(l)}(g_x)$ = $\pi_l^{(l)}(g_x)$ = 0.0463; $PS_i(g_x)$ = $PS_k(g_x)$ = 0.0498; $PS_j(g_x)$ = $PS_l(g_x)$ = 0.0740; $W_i(g_x)$ = $W_k(g_x)$ = 0.2854; and $W_j(g_x)$ = $W_l(g_x)$ = 0.4199.

Network y

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_y) = 0.1935 - 0.2419q_j^{(i)}(g_y) + 0.0323q_j^{(j)}(g_y) + 0.0323q_k^{(j)}(g_y)$$

$$q_j^{(i)}(g_y) = 0.1622 - 0.2514q_i^{(i)}(g_y) + 0.0270q_i^{(j)}(g_y) + 0.0270q_k^{(j)}(g_y) + 0.0486q_k^{(k)}(g_y) + 0.0486q_l^{(k)}(g_y)$$

$$q_i^{(j)}(g_y) = 0.3226 + 0.0968q_j^{(i)}(g_y) - 0.1129q_j^{(j)}(g_y) - 0.1129q_k^{(j)}(g_y)$$

$$q_j^{(j)}(g_y) = 0.2703 + 0.0811q_i^{(i)}(g_y) - 0.1216q_i^{(j)}(g_y) - 0.1216q_k^{(j)}(g_y) + 0.0811q_k^{(k)}(g_y) + 0.0811q_l^{(k)}(g_y)$$

$$q_k^{(j)}(g_y) = 0.2439 - 0.1260q_i^{(j)}(g_y) - 0.1260q_j^{(j)}(g_y) + 0.0732q_j^{(k)}(g_y) + 0.0732q_l^{(k)}(g_y) + 0.0407q_l^{(l)}(g_y)$$

$$q_j^{(k)}(g_y) = 0.1622 + 0.0486q_i^{(i)}(g_y) + 0.0270q_i^{(j)}(g_y) + 0.0270q_k^{(j)}(g_y) - 0.2514q_k^{(k)}(g_y) - 0.2514q_l^{(k)}(g_y)$$

$$q_k^{(k)}(g_y) = 0.1463 + 0.0244q_i^{(j)}(g_y) + 0.0244q_j^{(j)}(g_y) - 0.2561q_j^{(k)}(g_y) - 0.2561q_l^{(k)}(g_y) + 0.0244q_l^{(l)}(g_y)$$

$$q_l^{(k)}(g_y) = 0.1935 - 0.2419q_j^{(k)}(g_y) - 0.2419q_k^{(k)}(g_y) + 0.0323q_k^{(l)}(g_y)$$

$$q_k^{(l)}(g_y) = 0.2439 + 0.0407q_i^{(j)}(g_y) + 0.0407q_j^{(j)}(g_y) + 0.0732q_j^{(k)}(g_y) + 0.0732q_l^{(k)}(g_y) - 0.1260q_l^{(l)}(g_y)$$

$$q_l^{(l)}(g_y) = 0.3226 + 0.0968q_j^{(k)}(g_y) + 0.0968q_k^{(k)}(g_y) - 0.1129q_k^{(l)}(g_y)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_y)$ = 0.1734; $q_j^{(i)}(g_y)$ = 0.1439; $q_i^{(j)}(g_y)$ = 0.2853; $q_j^{(j)}(g_y)$ = 0.2441; $q_k^{(j)}(g_y)$ = 0.2098; $q_j^{(k)}(g_y)$ = 0.1221; $q_k^{(k)}(g_y)$ = 0.0977; $q_l^{(k)}(g_y)$ = 0.1483; $q_k^{(l)}(g_y)$ = 0.2454; $q_l^{(l)}(g_y)$ = 0.3162; $CS_i(g_y)$ = 0.1052; $CS_j(g_y)$ = 0.1301; $CS_k(g_y)$ = 0.1529; $CS_l(g_y)$ = 0.1078; $\pi_i^{(i)}(g_y)$ = 0.0676; $\pi_j^{(i)}(g_y)$ = 0.0466; $\pi_i^{(j)}(g_y)$ = 0.1017; $\pi_j^{(j)}(g_y)$ = 0.0745; $\pi_k^{(j)}(g_y)$ = 0.0550; $\pi_j^{(k)}(g_y)$ = 0.0336; $\pi_k^{(k)}(g_y)$ = 0.0215; $\pi_l^{(k)}(g_y)$ = 0.0495; $\pi_k^{(l)}(g_y)$ = 0.0753; $\pi_l^{(l)}(g_y)$ = 0.1249; $PS_i(g_y)$ = 0.0378; $PS_j(g_y)$ = 0.0683; $PS_k(g_y)$ = 0.0508; $PS_l(g_y)$ = 0.0394; $W_i(g_y)$ = 0.2572; $W_j(g_y)$ = 0.4296; $W_k(g_y)$ = 0.3082; and $W_l(g_y)$ = 0.3475.

Network z

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_z) = 0.1463 - 0.2561 q_j^{(i)}(g_z) - 0.2561 q_l^{(i)}(g_z) + 0.0244 q_j^{(j)}(g_z) + 0.0244 q_l^{(l)}(g_z)$$

$$q_j^{(i)}(g_z) = 0.1935 - 0.2419 q_i^{(i)}(g_z) - 0.2419 q_l^{(i)}(g_z) + 0.0323 q_i^{(j)}(g_z)$$

$$q_l^{(i)}(g_z) = 0.1935 - 0.2419 q_i^{(i)}(g_z) - 0.2419 q_j^{(i)}(g_z) + 0.0323 q_i^{(l)}(g_z)$$

$$q_i^{(j)}(g_z) = 0.2439 + 0.0732 q_j^{(i)}(g_z) + 0.0732 q_l^{(i)}(g_z) - 0.1260 q_j^{(j)}(g_z) + 0.0407 q_l^{(l)}(g_z)$$

$$q_j^{(j)}(g_z) = 0.3226 + 0.0968 q_i^{(i)}(g_z) + 0.0968 q_l^{(i)}(g_z) - 0.1129 q_i^{(j)}(g_z)$$

$$q_k^{(k)}(g_z) = 0.2857$$

$$q_i^{(l)}(g_z) = 0.2439 + 0.0732 q_j^{(i)}(g_z) + 0.0732 q_l^{(i)}(g_z) + 0.0407 q_j^{(j)}(g_z) - 0.1260 q_l^{(l)}(g_z)$$

$$q_l^{(l)}(g_z) = 0.3226 + 0.0968 q_i^{(i)}(g_z) + 0.0968 q_j^{(i)}(g_z) - 0.1129 q_i^{(l)}(g_z)$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_z)$ = 0.0876; $q_j^{(i)}(g_z)$ = $q_l^{(i)}(g_z)$ = 0.1450; $q_i^{(j)}(g_z)$ = $q_i^{(l)}(g_z)$ = 0.2380; $q_j^{(j)}(g_z)$ = $q_l^{(l)}(g_z)$ = 0.3182; $q_k^{(k)}(g_z)$ = 0.2857; $CS_i(g_z)$ = 0.1588; $CS_j(g_z)$ = $CS_l(g_z)$ = 0.1073; $CS_k(g_z)$ = 0.0408; $\pi_i^{(i)}(g_z)$ = 0.0173; $\pi_j^{(i)}(g_z)$ = $\pi_l^{(i)}(g_z)$ = 0.0473; $\pi_i^{(j)}(g_z)$ = $\pi_i^{(l)}(g_z)$ = 0.0708; $\pi_j^{(j)}(g_z)$ = $\pi_l^{(l)}(g_z)$ = 0.1266; $\pi_k^{(k)}(g_z)$ = 0.1837; $PS_i(g_z)$ = 0.0534; $PS_j(g_z)$ = $PS_l(g_z)$ = 0.0387; $PS_k(g_z)$ = 0.0306; $W_i(g_z)$ = 0.3241; $W_j(g_z)$ = $W_l(g_z)$ = 0.3433; and $W_k(g_z)$ = 0.2551.

Network a’

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_{a'}) = 0.1935 - 0.2419 q_j^{(i)}(g_{a'}) + 0.0323 q_j^{(j)}(g_{a'}) + 0.0323 q_k^{(j)}(g_{a'})$$

$$q_j^{(i)}(g_{a'}) = 0.1622 - 0.2514 q_i^{(i)}(g_{a'}) + 0.0270 q_i^{(j)}(g_{a'}) + 0.0270 q_k^{(j)}(g_{a'}) + 0.0486 q_k^{(k)}(g_{a'})$$

$$q_i^{(j)}(g_{a'}) = 0.3226 + 0.0968 q_j^{(i)}(g_{a'}) - 0.1129 q_j^{(j)}(g_{a'}) - 0.1129 q_k^{(j)}(g_{a'})$$

$$q_j^{(j)}(g_{a'}) = 0.2703 + 0.0811 q_i^{(i)}(g_{a'}) - 0.1216 q_i^{(j)}(g_{a'}) - 0.1216 q_k^{(j)}(g_{a'}) + 0.0811 q_k^{(k)}(g_{a'})$$

$$q_k^{(j)}(g_{a'}) = 0.3226 - 0.1129 q_i^{(j)}(g_{a'}) - 0.1129 q_j^{(j)}(g_{a'}) + 0.0968 q_j^{(k)}(g_{a'})$$

$$q_j^{(k)}(g_{a'}) = 0.1622 + 0.0486 q_i^{(i)}(g_{a'}) + 0.0270 q_i^{(j)}(g_{a'}) + 0.0270 q_k^{(j)}(g_{a'}) - 0.2514 q_k^{(k)}(g_{a'})$$

$$q_k^{(k)}(g_{a'}) = 0.1935 + 0.0323 q_i^{(j)}(g_{a'}) + 0.0323 q_j^{(j)}(g_{a'}) - 0.2419 q_j^{(k)}(g_{a'})$$

$$q_l^{(l)}(g_{a'}) = 0.4000$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_{a'})$ = $q_k^{(k)}(g_{a'})$ = 0.1757; $q_j^{(i)}(g_{a'})$ = $q_j^{(k)}(g_{a'})$ = 0.1416; $q_i^{(j)}(g_{a'})$ = $q_k^{(j)}(g_{a'})$ = 0.2787; $q_j^{(j)}(g_{a'})$ = 0.2310; $q_l^{(l)}(g_{a'})$ = 0.4000; $CS_i(g_{a'})$ = $CS_k(g_{a'})$ = 0.1033; $CS_j(g_{a'})$ = 0.1322; $CS_l(g_{a'})$ = 0.0800; $\pi_i^{(i)}(g_{a'})$ = $\pi_k^{(k)}(g_{a'})$ = 0.0695; $\pi_j^{(i)}(g_{a'})$ = $\pi_j^{(k)}(g_{a'})$ = 0.0451; $\pi_i^{(j)}(g_{a'})$ = $\pi_k^{(j)}(g_{a'})$ = 0.0971; $\pi_j^{(j)}(g_{a'})$ = 0.0667; $\pi_l^{(l)}(g_{a'})$ = 0.2000; $PS_i(g_{a'})$ = $PS_k(g_{a'})$ = 0.0378; $PS_j(g_{a'})$ = 0.0777; $PS_l(g_{a'})$ = 0.0200; $W_i(g_{a'})$ = $W_k(g_{a'})$ = 0.2556; $W_j(g_{a'})$ = 0.4708; and $W_l(g_{a'})$ = 0.3000.

Network b’

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_{b'}) = 0.1935 - 0.2419 q_j^{(i)}(g_{b'}) + 0.0323 q_j^{(j)}(g_{b'})$$

$$q_j^{(i)}(g_{b'}) = 0.1935 - 0.2419 q_i^{(i)}(g_{b'}) + 0.0323 q_i^{(j)}(g_{b'})$$

$$q_i^{(j)}(g_{b'}) = 0.3226 + 0.0968 q_j^{(i)}(g_{b'}) - 0.1129 q_j^{(j)}(g_{b'})$$

$$q_j^{(j)}(g_{b'}) = 0.3226 + 0.0968 q_i^{(i)}(g_{b'}) - 0.1129 q_i^{(j)}(g_{b'})$$

$$q_k^{(k)}(g_{b'}) = 0.1935 - 0.2419 q_l^{(k)}(g_{b'}) + 0.0323 q_l^{(l)}(g_{b'})$$

$$q_l^{(k)}(g_{b'}) = 0.1935 - 0.2419 q_k^{(k)}(g_{b'}) + 0.0323 q_k^{(l)}(g_{b'})$$

$$q_k^{(l)}(g_{b'}) = 0.3226 + 0.0968 q_l^{(k)}(g_{b'}) - 0.1129 q_l^{(l)}(g_{b'})$$

$$q_l^{(l)}(g_{b'}) = 0.3226 + 0.0968 q_k^{(k)}(g_{b'}) - 0.1129 q_l^{(k)}(g_{b'})$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_{b'})$ = $q_j^{(i)}(g_{b'})$ = $q_k^{(k)}(g_{b'})$ = $q_l^{(k)}(g_{b'})$ = 0.1637; $q_i^{(j)}(g_{b'})$ = $q_j^{(j)}(g_{b'})$ = $q_k^{(l)}(g_{b'})$ = $q_l^{(l)}(g_{b'})$ = 0.3041; $CS_i(g_{b'})$ = $CS_j(g_{b'})$ = $CS_k(g_{b'})$ = $CS_l(g_{b'})$ = 0.1094; $\pi_i^{(i)}(g_{b'})$ = $\pi_j^{(i)}(g_{b'})$ = $\pi_k^{(k)}(g_{b'})$ = $\pi_l^{(k)}(g_{b'})$ = 0.0603; $\pi_i^{(j)}(g_{b'})$ = $\pi_j^{(j)}(g_{b'})$ = $\pi_k^{(l)}(g_{b'})$ = $\pi_l^{(l)}(g_{b'})$ = 0.1156; $PS_i(g_{b'})$ = $PS_k(g_{b'})$ = 0.0402; $PS_j(g_{b'})$ = $PS_l(g_{b'})$ = 0.0462; $W_i(g_{b'})$ = $W_k(g_{b'})$ = 0.2703; and $W_j(g_{b'})$ = $W_l(g_{b'})$ = 0.3869.

Network c'

In considering the equations presented in Section 4.2.1.3, the following results are obtained:

$$q_i^{(i)}(g_{c'}) = 0.2222 - 0.2333 q_k^{(i)}(g_{c'}) + 0.0667 q_k^{(k)}(g_{c'})$$

$$q_k^{(i)}(g_{c'}) = 0.2222 - 0.2333 q_i^{(i)}(g_{c'}) + 0.0667 q_i^{(k)}(g_{c'})$$

$$q_j^{(j)}(g_{c'}) = 0.2857 - 0.1190 q_l^{(j)}(g_{c'}) + 0.0476 q_l^{(l)}(g_{c'})$$

$$q_l^{(j)}(g_{c'}) = 0.2857 - 0.1190 q_j^{(j)}(g_{c'}) + 0.0476 q_j^{(l)}(g_{c'})$$

$$q_i^{(k)}(g_{c'}) = 0.2222 + 0.0667 q_k^{(i)}(g_{c'}) - 0.2333 q_k^{(k)}(g_{c'})$$

$$q_k^{(k)}(g_{c'}) = 0.2222 + 0.0667 q_i^{(i)}(g_{c'}) - 0.2333 q_i^{(k)}(g_{c'})$$

$$q_j^{(l)}(g_{c'}) = 0.2857 + 0.0476 q_l^{(j)}(g_{c'}) - 0.1190 q_l^{(l)}(g_{c'})$$

$$q_l^{(l)}(g_{c'}) = 0.2857 + 0.0476 q_j^{(j)}(g_{c'}) - 0.1190 q_j^{(l)}(g_{c'})$$

Solving by substitution, the following expressions are obtained:

$q_i^{(i)}(g_{c'})$ = $q_k^{(i)}(g_{c'})$ = $q_i^{(k)}(g_{c'})$ = $q_k^{(k)}(g_{c'})$ = 0.1905; $q_j^{(j)}(g_{c'})$ = $q_l^{(j)}(g_{c'})$ = $q_j^{(l)}(g_{c'})$ = $q_l^{(l)}(g_{c'})$ = 0.2667; $CS_i(g_{c'})$ = $CS_k(g_{c'})$ = 0.0726; $CS_j(g_{c'})$ = $CS_l(g_{c'})$ = 0.1422; $\pi_i^{(i)}(g_{c'})$ = $\pi_k^{(i)}(g_{c'})$ = $\pi_i^{(k)}(g_{c'})$ = $\pi_k^{(k)}(g_{c'})$ = 0.0816; $\pi_j^{(j)}(g_{c'})$ = $\pi_l^{(j)}(g_{c'})$ = $\pi_j^{(l)}(g_{c'})$ = $\pi_l^{(l)}(g_{c'})$ = 0.0889; $PS_i(g_{c'})$ = $PS_k(g_{c'})$ = 0.0544; $PS_j(g_{c'})$ = $PS_l(g_{c'})$ = 0.0356; $W_i(g_{c'})$ = $W_k(g_{c'})$ = 0.2902; and $W_j(g_{c'})$ = $W_l(g_{c'})$ = 0.3556.

Appendix E

Numerical results of the simulations developed in the current research

This appendix shows the numerical results that were obtained from the simulations computed in the previous appendix. These results were tabulated as follows.

Tables for Simulation 1: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

Table E.1. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1250	0.1250	0.1250	0.1250
b	0.2222	0.1250	0.2222	0.1250
c	0.2222	0.2222	0.2222	0.2222
d	0.2813	0.2813	0.2222	0.2222
e	0.2813	0.2813	0.2813	0.2813
f	0.2813	0.2222	0.2222	0.1250
g	0.2813	0.2813	0.2813	0.1250
h	0.3200	0.2222	0.2222	0.2222
i	0.2813	0.2813	0.3200	0.2222
j	0.2813	0.3200	0.3200	0.2813
k	0.3200	0.3200	0.3200	0.3200

Table E.2. Profits made by the intermediary

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2500	0.2500	0.2500	0.2500
b	0.2222	0.2500	0.2222	0.2500
c	0.2222	0.2222	0.2222	0.2222
d	0.2361	0.2361	0.1736	0.1736
e	0.1875	0.1875	0.1875	0.1875
f	0.2847	0.1736	0.1736	0.2500
g	0.1875	0.1875	0.1875	0.2500
h	0.3733	0.1511	0.1511	0.1511
i	0.1650	0.1650	0.2761	0.1511
j	0.1425	0.2050	0.2050	0.1425
k	0.1600	0.1600	0.1600	0.1600

Table E.3. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3750	0.3750	0.3750	0.3750
b	0.4444	0.3750	0.4444	0.3750
c	0.4444	0.4444	0.4444	0.4444
d	0.5174	0.5174	0.3958	0.3958
e	0.4688	0.4688	0.4688	0.4688
f	0.5660	0.3958	0.3958	0.3750
g	0.4688	0.4688	0.4688	0.3750
h	0.6933	0.3733	0.3733	0.3733
i	0.4463	0.4463	0.5961	0.3733
j	0.4238	0.5250	0.5250	0.4238
k	0.4800	0.4800	0.4800	0.4800

Tables for Simulation 2: ϕ_i = 0.5 and α_i = 1 for all i $\in$ N.

Table E.4. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0800	0.0800	0.0800	0.0800
b	0.1422	0.0800	0.1422	0.0800
c	0.1422	0.1422	0.1422	0.1422
d	0.1813	0.1813	0.1396	0.1396
e	0.1800	0.1800	0.1800	0.1800
f	0.1850	0.1382	0.1382	0.0800
g	0.1800	0.1800	0.1800	0.0800
h	0.2175	0.1232	0.1232	0.1232
i	0.1769	0.1769	0.2107	0.1368
j	0.1759	0.2075	0.2075	0.1759
k	0.2048	0.2048	0.2048	0.2048

Table E.5. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.2000	0.2000	0.2000
b	0.1778	0.2000	0.1778	0.2000
c	0.1778	0.1778	0.1778	0.1778
d	0.1702	0.1702	0.1564	0.1564
e	0.1500	0.1500	0.1500	0.1500
f	0.1908	0.1558	0.1558	0.2000
g	0.1500	0.1500	0.1500	0.2000
h	0.2093	0.1321	0.1321	0.1321
i	0.1419	0.1419	0.1721	0.1462
j	0.1318	0.1456	0.1456	0.1318
k	0.1280	0.1280	0.1280	0.1280

Table E.6. Producer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0200	0.0200	0.0200	0.0200
b	0.0356	0.0200	0.0356	0.0200
c	0.0356	0.0356	0.0356	0.0356
d	0.0502	0.0502	0.0309	0.0309
e	0.0450	0.0450	0.0450	0.0450
f	0.0561	0.0306	0.0306	0.0200
g	0.0450	0.0450	0.0450	0.0200
h	0.0788	0.0255	0.0255	0.0255
i	0.0421	0.0421	0.0662	0.0279
j	0.0391	0.0575	0.0575	0.0391
k	0.0512	0.0512	0.0512	0.0512

Table E.7. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3000	0.3000	0.3000	0.3000
b	0.3556	0.3000	0.3556	0.3000
c	0.3556	0.3556	0.3556	0.3556
d	0.4017	0.4017	0.3269	0.3269
e	0.3750	0.3750	0.3750	0.3750
f	0.4319	0.3246	0.3246	0.3000
g	0.3750	0.3750	0.3750	0.3000
h	0.5056	0.2808	0.2808	0.2808
i	0.3609	0.3609	0.4490	0.3109
j	0.3468	0.4106	0.4106	0.3468
k	0.3840	0.3840	0.3840	0.3840

Tables for Simulation 3: ϕ_i = 1.5 and α_i = 1 for all i $\in$ N.

Table E.8. Consumer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0408	0.0408	0.0408	0.0408
b	0.0726	0.0408	0.0726	0.0408
c	0.0726	0.0726	0.0726	0.0726
d	0.0945	0.0945	0.0692	0.0692
e	0.0918	0.0918	0.0918	0.0918
f	0.0989	0.0683	0.0683	0.0408
g	0.0918	0.0918	0.0918	0.0408
h	0.1262	0.0531	0.0531	0.0531
i	0.0890	0.0890	0.1132	0.0664
j	0.0873	0.1082	0.1082	0.0873
k	0.1045	0.1045	0.1045	0.1045

Table E.9. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1428	0.1428	0.1428	0.1428
b	0.1270	0.1428	0.1270	0.1428
c	0.1270	0.1270	0.1270	0.1270
d	0.1142	0.1142	0.1191	0.1191
e	0.1071	0.1071	0.1071	0.1071
f	0.1223	0.1182	0.1182	0.1428
g	0.1071	0.1071	0.1071	0.1428
h	0.1219	0.0927	0.0927	0.0927
i	0.1039	0.1039	0.1075	0.1149
j	0.1002	0.0980	0.0980	0.1002
k	0.0916	0.0916	0.0916	0.0916

Table E.10. Producer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0306	0.0306	0.0306	0.0306
b	0.0544	0.0306	0.0544	0.0306
c	0.0544	0.0544	0.0544	0.0544
d	0.0725	0.0725	0.0505	0.0505
e	0.0689	0.0689	0.0689	0.0689
f	0.0773	0.0499	0.0499	0.0306
g	0.0689	0.0689	0.0689	0.0306
h	0.0971	0.0393	0.0393	0.0393
i	0.0662	0.0662	0.0891	0.0477
j	0.0639	0.0829	0.0829	0.0639
k	0.0784	0.0784	0.0784	0.0784

Table E.11. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2142	0.2142	0.2142	0.2142
b	0.2540	0.2142	0.2540	0.2142
c	0.2540	0.2540	0.2540	0.2540
d	0.2812	0.2812	0.2388	0.2388
e	0.2678	0.2678	0.2678	0.2678
f	0.2985	0.2364	0.2364	0.2142
g	0.2678	0.2678	0.2678	0.2142
h	0.3452	0.1851	0.1851	0.1851
i	0.2591	0.2591	0.3098	0.2290
j	0.2514	0.2891	0.2891	0.2514
k	0.2745	0.2745	0.2745	0.2745

Tables for Simulation 4: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

Table E.12. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2025	0.2025	0.2025	0.2025
b	0.2813	0.2025	0.2813	0.2025
c	0.2813	0.2813	0.2813	0.2813
d	0.3072	0.3072	0.2813	0.2813
e	0.3072	0.3072	0.3072	0.3072
f	0.3072	0.2813	0.2813	0.2025
g	0.3072	0.3072	0.3072	0.2025
h	0.3200	0.2813	0.2813	0.2813
i	0.3072	0.3072	0.3200	0.2813
j	0.3072	0.3200	0.3200	0.3072
k	0.3200	0.3200	0.3200	0.3200

Table E.13. Profits made by the intermediary

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1570	0.1570	0.1570	0.1570
b	0.1415	0.1717	0.1415	0.1717
c	0.1563	0.1563	0.1563	0.1563
d	0.1716	0.1716	0.1431	0.1431
e	0.1585	0.1585	0.1585	0.1585
f	0.1800	0.1331	0.1331	0.1817
g	0.1485	0.1485	0.1485	0.1870
h	0.2275	0.1338	0.1338	0.1338
i	0.1491	0.1491	0.1960	0.1390
j	0.1450	0.1735	0.1735	0.1450
k	0.1600	0.1600	0.1600	0.1600

Table E.14. Tariff revenue

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0744	0.0744	0.0744	0.0744
b	0.0313	0.0744	0.0313	0.0744
c	0.0313	0.0313	0.0313	0.0313
d	0.0110	0.0110	0.0313	0.0313
e	0.0110	0.0110	0.0110	0.0110
f	0.0110	0.0313	0.0313	0.0744
g	0.0110	0.0110	0.0110	0.0744
h	0.0000	0.0313	0.0313	0.0313
i	0.0110	0.0110	0.0000	0.0313
j	0.0110	0.0000	0.0000	0.0110
k	0.0000	0.0000	0.0000	0.0000

Table E.15. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.4339	0.4339	0.4339	0.4339
b	0.4541	0.4486	0.4541	0.4486
c	0.4689	0.4689	0.4689	0.4689
d	0.4898	0.4898	0.4557	0.4557
e	0.4767	0.4767	0.4767	0.4767
f	0.4982	0.4457	0.4457	0.4586
g	0.4667	0.4667	0.4667	0.4639
h	0.5475	0.4464	0.4464	0.4464
i	0.4673	0.4673	0.5160	0.4516
j	0.4632	0.4935	0.4935	0.4632
k	0.4800	0.4800	0.4800	0.4800

Tables for Simulation 5: ϕ_i = 0.5 and α_i = 1 for all i $\in$ N.

Table E.16. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1383	0.1383	0.1383	0.1383
b	0.1860	0.1363	0.1860	0.1363
c	0.1824	0.1824	0.1824	0.1824
d	0.1986	0.1986	0.1807	0.1807
e	0.1973	0.1973	0.1973	0.1973
f	0.2019	0.1825	0.1825	0.1351
g	0.2001	0.2001	0.2001	0.1345
h	0.2084	0.1792	0.1792	0.1792
i	0.1971	0.1971	0.2067	0.1791
j	0.1961	0.2056	0.2056	0.1961
k	0.2048	0.2048	0.2048	0.2048

Table E.17. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1203	0.1203	0.1203	0.1203
b	0.1166	0.1276	0.1166	0.1276
c	0.1241	0.1241	0.1241	0.1241
d	0.1308	0.1308	0.1196	0.1196
e	0.1266	0.1266	0.1266	0.1266
f	0.1336	0.1140	0.1140	0.1323
g	0.1214	0.1214	0.1214	0.1354
h	0.1512	0.1151	0.1151	0.1151
i	0.1228	0.1228	0.1403	0.1183
j	0.1216	0.1327	0.1327	0.1216
k	0.1280	0.1280	0.1280	0.1280

Table E.18. Producer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0346	0.0346	0.0346	0.0346
b	0.0401	0.0400	0.0401	0.0400
c	0.0456	0.0456	0.0456	0.0456
d	0.0500	0.0500	0.0448	0.0448
e	0.0493	0.0493	0.0493	0.0493
f	0.0491	0.0412	0.0412	0.0435
g	0.0456	0.0456	0.0456	0.0456
h	0.0599	0.0425	0.0425	0.0425
i	0.0474	0.0474	0.0558	0.0446
j	0.0475	0.0530	0.0530	0.0475
k	0.0512	0.0512	0.0512	0.0512

Table E.19. Tariff revenue

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0599	0.0599	0.0599	0.0599
b	0.0246	0.0584	0.0246	0.0584
c	0.0239	0.0239	0.0239	0.0239
d	0.0087	0.0087	0.0239	0.0239
e	0.0083	0.0083	0.0083	0.0083
f	0.0084	0.0250	0.0250	0.0575
g	0.0085	0.0085	0.0085	0.0571
h	0.0000	0.0261	0.0261	0.0261
i	0.0091	0.0091	0.0000	0.0244
j	0.0088	0.0000	0.0000	0.0088
k	0.0000	0.0000	0.0000	0.0000

Table E.20. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3531	0.3531	0.3531	0.3531
b	0.3673	0.3623	0.3673	0.3623
c	0.3760	0.3760	0.3760	0.3760
d	0.3882	0.3882	0.3691	0.3691
e	0.3816	0.3816	0.3816	0.3816
f	0.3930	0.3627	0.3627	0.3683
g	0.3756	0.3756	0.3756	0.3725
h	0.4196	0.3629	0.3629	0.3629
i	0.3764	0.3764	0.4027	0.3664
j	0.3741	0.3912	0.3912	0.3741
k	0.3840	0.3840	0.3840	0.3840

Tables for Simulation 6: ϕ_i = 1.5 and α_i = 1 for all i $\in$ N.

Table E.21. Consumer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0764	0.0764	0.0764	0.0764
b	0.0974	0.0707	0.0974	0.0707
c	0.0932	0.0932	0.0932	0.0932
d	0.1024	0.1024	0.0913	0.0913
e	0.1006	0.1006	0.1006	0.1006
f	0.1061	0.0936	0.0936	0.0689
g	0.1037	0.1037	0.1037	0.0678
h	0.1096	0.0903	0.0903	0.0903
i	0.1006	0.1006	0.1072	0.0895
j	0.0992	0.1056	0.1056	0.0992
k	0.1045	0.1045	0.1045	0.1045

Table E.22. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0828	0.0828	0.0828	0.0828
b	0.0853	0.0875	0.0853	0.0875
c	0.0886	0.0886	0.0886	0.0886
d	0.0920	0.0920	0.0869	0.0869
e	0.0904	0.0904	0.0904	0.0904
f	0.0931	0.0842	0.0842	0.0899
g	0.0881	0.0881	0.0881	0.0914
h	0.0995	0.0847	0.0847	0.0847
i	0.0888	0.0888	0.0958	0.0864
j	0.0885	0.0931	0.0931	0.0885
k	0.0914	0.0914	0.0914	0.0914

Table E.23. Producer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0573	0.0573	0.0573	0.0573
b	0.0637	0.0616	0.0637	0.0616
c	0.0699	0.0699	0.0699	0.0699
d	0.0753	0.0753	0.0698	0.0698
e	0.0755	0.0755	0.0755	0.0755
f	0.0737	0.0655	0.0655	0.0655
g	0.0713	0.0713	0.0713	0.0680
h	0.0846	0.0670	0.0670	0.0670
i	0.0734	0.0734	0.0817	0.0698
j	0.0739	0.0797	0.0797	0.0739
k	0.0784	0.0784	0.0784	0.0784

Table E.24. Tariff revenue

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0406	0.0406	0.0406	0.0406
b	0.0184	0.0394	0.0184	0.0394
c	0.0169	0.0169	0.0169	0.0169
d	0.0065	0.0065	0.0167	0.0167
e	0.0060	0.0060	0.0060	0.0060
f	0.0066	0.0184	0.0184	0.0377
g	0.0067	0.0067	0.0067	0.0368
h	0.0000	0.0190	0.0190	0.0190
i	0.0069	0.0069	0.0000	0.0171
j	0.0065	0.0000	0.0000	0.0065
k	0.0000	0.0000	0.0000	0.0000

Table E.25. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2572	0.2572	0.2572	0.2572
b	0.2648	0.2592	0.2648	0.2592
c	0.2687	0.2687	0.2687	0.2687
d	0.2762	0.2762	0.2647	0.2647
e	0.2725	0.2725	0.2725	0.2725
f	0.2795	0.2616	0.2616	0.2620
g	0.2698	0.2698	0.2698	0.2641
h	0.2938	0.2611	0.2611	0.2611
i	0.2697	0.2697	0.2847	0.2628
j	0.2682	0.2784	0.2784	0.2682
k	0.2743	0.2743	0.2743	0.2743

Tables for Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ ***and*** $\phi = 0$ ***for all*** $i \in N$.

Table E.26. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1250	0.0000	0.1250	0.0000
b	0.2222	0.0000	0.1250	0.0000
c	0.2222	0.0000	0.2222	0.0000
d	0.1250	0.0000	0.1250	0.0000
e	0.2813	0.0000	0.2222	0.0000
f	0.2222	0.0000	0.1250	0.0000
g	0.3200	0.0000	0.2222	0.0000
h	0.2813	0.0000	0.2813	0.0000
i	0.2813	0.0000	0.2222	0.0000
j	0.2813	0.0000	0.2813	0.0000
k	0.2222	0.0000	0.2222	0.0000
l	0.2222	0.0000	0.2222	0.0000
m	0.2813	0.0000	0.1250	0.0000
n	0.3200	0.0000	0.2813	0.0000
o	0.3200	0.0000	0.2222	0.0000
p	0.2813	0.0000	0.2813	0.0000
q	0.2813	0.0000	0.2813	0.0000
r	0.2813	0.0000	0.2222	0.0000
s	0.3200	0.0000	0.2813	0.0000
t	0.3200	0.0000	0.3200	0.0000
u	0.3200	0.0000	0.2813	0.0000
v	0.2813	0.0000	0.2813	0.0000
w	0.2818	0.0000	0.2818	0.0000
x	0.3200	0.0000	0.3200	0.0000
y	0.2222	0.0000	0.2813	0.0000
z	0.2813	0.0000	0.1250	0.0000
a'	0.2222	0.0000	0.2222	0.0000
b'	0.2222	0.0000	0.2222	0.0000
c'	0.2222	0.0000	0.2222	0.0000

Table E.27. Profits made by the intermediary

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2500	0.0000	0.2500	0.0000
b	0.1111	0.1111	0.2500	0.0000
c	0.2222	0.0000	0.2222	0.0000
d	0.2500	0.0000	0.2500	0.0000
e	0.1736	0.0625	0.1736	0.0000
f	0.1111	0.1111	0.2500	0.0000
g	0.1511	0.0400	0.1511	0.0400
h	0.1250	0.0625	0.1250	0.0625
i	0.1736	0.0625	0.1736	0.0000
j	0.1250	0.1250	0.1250	0.0000
k	0.1111	0.1111	0.1111	0.1111
l	0.1111	0.2222	0.1111	0.0000
m	0.0625	0.0625	0.2500	0.0625
n	0.1025	0.0400	0.1025	0.1025
o	0.1511	0.0400	0.1511	0.0400
p	0.1250	0.0625	0.1250	0.0625
q	0.1250	0.1250	0.1250	0.0000
r	0.0625	0.0625	0.1111	0.1736
s	0.1025	0.0400	0.1025	0.1025
t	0.0800	0.0800	0.0800	0.0800
u	0.1025	0.1025	0.1025	0.0400
v	0.0625	0.1250	0.0625	0.1250
w	0.0625	0.1250	0.0625	0.1250
x	0.0800	0.0800	0.0800	0.0800
y	0.1111	0.1736	0.0625	0.0625
z	0.0625	0.0625	0.2500	0.0625
a’	0.1111	0.2222	0.1111	0.0000
b’	0.1111	0.1111	0.1111	0.1111
c’	0.2222	0.0000	0.2222	0.0000

Table E.28. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3750	0.0000	0.3750	0.0000
b	0.3333	0.1111	0.3750	0.0000
c	0.4444	0.0000	0.4444	0.0000
d	0.3750	0.0000	0.3750	0.0000
e	0.4549	0.0625	0.3958	0.0000
f	0.3333	0.1111	0.3750	0.0000
g	0.4711	0.0400	0.3733	0.0400
h	0.4063	0.0625	0.4063	0.0625
i	0.4549	0.0625	0.3958	0.0000
j	0.4063	0.1250	0.4063	0.0000
k	0.3333	0.1111	0.3333	0.1111
l	0.3333	0.2222	0.3333	0.0000
m	0.3438	0.0625	0.3750	0.0625
n	0.4225	0.0400	0.3838	0.1025
o	0.4711	0.0400	0.3733	0.0400
p	0.4063	0.0625	0.4063	0.0625
q	0.4063	0.1250	0.4063	0.0000
r	0.3438	0.0625	0.3333	0.1736
s	0.4225	0.0400	0.3838	0.1025
t	0.4000	0.0800	0.4000	0.0800
u	0.4225	0.1025	0.3838	0.0400
v	0.3438	0.1250	0.3438	0.1250
w	0.3443	0.1250	0.3443	0.1250
x	0.4000	0.0800	0.4000	0.0800
y	0.3333	0.1736	0.3438	0.0625
z	0.3438	0.0625	0.3750	0.0625
a'	0.3333	0.2222	0.3333	0.0000
b'	0.3333	0.1111	0.3333	0.1111
c'	0.4444	0.0000	0.4444	0.0000

Tables for Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ ***and*** $\phi = 0.5$ ***for all*** $i \in N$.

Table E.29. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0800	0.0000	0.0800	0.0000
b	0.1632	0.0000	0.0800	0.0000
c	0.1423	0.0000	0.1423	0.0000
d	0.0800	0.0000	0.0800	0.0000
e	0.2024	0.0000	0.1475	0.0000
f	0.1632	0.0000	0.0800	0.0000
g	0.2465	0.0000	0.1510	0.0000
h	0.2074	0.0000	0.2074	0.0000
i	0.2024	0.0000	0.1475	0.0000
j	0.1994	0.0000	0.1994	0.0000
k	0.1632	0.0000	0.1632	0.0000
l	0.1530	0.0000	0.1530	0.0000
m	0.2222	0.0000	0.0800	0.0000
n	0.2438	0.0000	0.2042	0.0000
o	0.2465	0.0000	0.1510	0.0000
p	0.2074	0.0000	0.2074	0.0000
q	0.1994	0.0000	0.1994	0.0000
r	0.2125	0.0000	0.1557	0.0000
s	0.2438	0.0000	0.2042	0.0000
t	0.2419	0.0000	0.2419	0.0000
u	0.2472	0.0000	0.2040	0.0000
v	0.2074	0.0000	0.2074	0.0000
w	0.2074	0.0000	0.2074	0.0000
x	0.2419	0.0000	0.2419	0.0000
y	0.1557	0.0000	0.2125	0.0000
z	0.2222	0.0000	0.0800	0.0000
a'	0.1530	0.0000	0.1530	0.0000
b'	0.1632	0.0000	0.1632	0.0000
c'	0.1423	0.0000	0.1423	0.0000

Table E.30. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.0000	0.2000	0.0000
b	0.1020	0.1020	0.2000	0.0000
c	0.1778	0.0000	0.1778	0.0000
d	0.2000	0.0000	0.2000	0.0000
e	0.1407	0.0734	0.1407	0.0000
f	0.1020	0.1020	0.2000	0.0000
g	0.1235	0.0493	0.1235	0.0493
h	0.1034	0.0704	0.1034	0.0704
i	0.1407	0.0734	0.1407	0.0000
j	0.1108	0.1108	0.1108	0.0000
k	0.1020	0.1020	0.1020	0.1020
l	0.1109	0.1630	0.1109	0.0000
m	0.0617	0.0617	0.2000	0.0617
n	0.0910	0.0506	0.0910	0.0910
o	0.1235	0.0493	0.1235	0.0493
p	0.1034	0.0704	0.1034	0.0704
q	0.1108	0.1108	0.1108	0.0000
r	0.0672	0.0672	0.1086	0.1309
s	0.0910	0.0506	0.0910	0.0910
t	0.0756	0.0756	0.0756	0.0756
u	0.0911	0.0911	0.0928	0.0506
v	0.0704	0.1034	0.0704	0.1034
w	0.0704	0.1034	0.0704	0.1034
x	0.0756	0.0756	0.0756	0.0756
y	0.1086	0.1309	0.0672	0.0672
z	0.0617	0.0617	0.2000	0.0617
a’	0.1109	0.1630	0.1109	0.0000
b’	0.1020	0.1020	0.1020	0.1020
c’	0.1778	0.0000	0.1778	0.0000

Table E.31. Producer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0200	0.0000	0.0200	0.0000
b	0.0102	0.0102	0.0200	0.0000
c	0.0356	0.0000	0.0356	0.0000
d	0.0200	0.0000	0.0200	0.0000
e	0.0274	0.0073	0.0274	0.0000
f	0.0102	0.0102	0.0200	0.0000
g	0.0228	0.0049	0.0228	0.0049
h	0.0207	0.0071	0.0207	0.0071
i	0.0274	0.0073	0.0274	0.0000
j	0.0222	0.0222	0.0222	0.0000
k	0.0102	0.0102	0.0102	0.0102
l	0.0111	0.0326	0.0111	0.0000
m	0.0062	0.0062	0.0200	0.0062
n	0.0179	0.0051	0.0179	0.0179
o	0.0228	0.0049	0.0228	0.0049
p	0.0207	0.0070	0.0207	0.0070
q	0.0222	0.0222	0.0222	0.0000
r	0.0067	0.0067	0.0109	0.0255
s	0.0179	0.0051	0.0179	0.0179
t	0.0151	0.0151	0.0151	0.0151
u	0.0179	0.0179	0.0183	0.0051
v	0.0070	0.0207	0.0070	0.0207
w	0.0070	0.0207	0.0070	0.0207
x	0.0151	0.0151	0.0151	0.0151
y	0.0109	0.0255	0.0067	0.0067
z	0.0062	0.0062	0.0200	0.0062
a'	0.0111	0.0323	0.0111	0.0000
b'	0.0102	0.0102	0.0102	0.0102
c'	0.0356	0.0000	0.0356	0.0000

Table E.32. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3000	0.0000	0.3000	0.0000
b	0.2754	0.1122	0.3000	0.0000
c	0.3557	0.0000	0.3557	0.0000
d	0.3000	0.0000	0.3000	0.0000
e	0.3705	0.0807	0.3156	0.0000
f	0.2754	0.1122	0.3000	0.0000
g	0.3928	0.0542	0.2973	0.0542
h	0.3315	0.0774	0.3315	0.0774
i	0.3705	0.0807	0.3156	0.0000
j	0.3324	0.1330	0.3324	0.0000
k	0.2754	0.1122	0.2754	0.1122
l	0.2750	0.1956	0.2750	0.0000
m	0.2901	0.0679	0.3000	0.0679
n	0.3527	0.0557	0.3131	0.1089
o	0.3928	0.0542	0.2973	0.0542
p	0.3315	0.0774	0.3315	0.0774
q	0.3324	0.1330	0.3324	0.0000
r	0.2864	0.0739	0.2752	0.1564
s	0.3527	0.0557	0.3131	0.1089
t	0.3326	0.0907	0.3326	0.0907
u	0.3562	0.1090	0.3151	0.0557
v	0.2848	0.1241	0.2848	0.1241
w	0.2848	0.1241	0.2848	0.1241
x	0.3326	0.0907	0.3326	0.0907
y	0.2752	0.1564	0.2864	0.0739
z	0.2901	0.0679	0.3000	0.0679
a’	0.2750	0.1953	0.2750	0.0000
b’	0.2754	0.1122	0.2754	0.1122
c’	0.3557	0.0000	0.3557	0.0000

Tables for Simulation 13: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 1.5 for all i $\in$ N.

Table E.33. Consumer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0408	0.0000	0.0408	0.0000
b	0.0987	0.0000	0.0408	0.0000
c	0.0726	0.0000	0.0726	0.0000
d	0.0408	0.0000	0.0408	0.0000
e	0.1206	0.0000	0.0786	0.0000
f	0.0988	0.0000	0.0408	0.0000
g	0.1614	0.0000	0.0829	0.0000
h	0.1270	0.0000	0.1270	0.0000
i	0.1206	0.0000	0.0786	0.0000
j	0.1152	0.0000	0.1152	0.0000
k	0.0988	0.0000	0.0988	0.0000
l	0.0859	0.0000	0.0859	0.0000
m	0.1487	0.0000	0.0408	0.0000
n	0.1560	0.0000	0.1216	0.0000
o	0.1614	0.0000	0.0829	0.0000
p	0.1270	0.0000	0.1270	0.0000
q	0.1152	0.0000	0.1152	0.0000
r	0.1349	0.0000	0.0890	0.0000
s	0.1560	0.0000	0.1216	0.0000
t	0.1521	0.0000	0.1521	0.0000
u	0.1593	0.0000	0.1213	0.0000
v	0.1270	0.0000	0.1270	0.0000
w	0.1270	0.0000	0.1270	0.0000
x	0.1521	0.0000	0.1521	0.0000
y	0.0890	0.0000	0.1349	0.0000
z	0.1487	0.0000	0.0408	0.0000
a'	0.0859	0.0000	0.0859	0.0000
b'	0.0988	0.0000	0.0988	0.0000
c'	0.0726	0.0000	0.0726	0.0000

Table E.34. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1428	0.0000	0.1428	0.0000
b	0.0864	0.0864	0.1428	0.0000
c	0.1270	0.0000	0.1270	0.0000
d	0.1428	0.0000	0.1428	0.0000
e	0.1049	0.0724	0.1049	0.0000
f	0.0864	0.0864	0.1428	0.0000
g	0.0943	0.0521	0.0943	0.0521
h	0.0816	0.0690	0.0816	0.0690
i	0.1049	0.0724	0.1049	0.0000
j	0.0896	0.0896	0.0896	0.0000
k	0.0864	0.0864	0.0864	0.0864
l	0.0960	0.1136	0.0960	0.0000
m	0.0578	0.0578	0.1428	0.0578
n	0.0757	0.0546	0.0757	0.0757
o	0.0943	0.0521	0.0943	0.0521
p	0.0816	0.0690	0.0816	0.0690
q	0.0896	0.0896	0.0896	0.0000
r	0.0646	0.0646	0.0935	0.0955
s	0.0757	0.0546	0.0757	0.0757
t	0.0666	0.0666	0.0666	0.0666
u	0.0758	0.0758	0.0776	0.0546
v	0.0690	0.0816	0.0690	0.0816
w	0.0690	0.0816	0.0690	0.0816
x	0.0666	0.0666	0.0666	0.0666
y	0.0935	0.0955	0.0646	0.0646
z	0.0578	0.0578	0.1428	0.0578
a’	0.0960	0.1136	0.0960	0.0000
b’	0.0864	0.0864	0.0864	0.0864
c’	0.1270	0.0000	0.1270	0.0000

Table E.35. Producer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0306	0.0000	0.0306	0.0000
b	0.0185	0.0185	0.0306	0.0000
c	0.0544	0.0000	0.0544	0.0000
d	0.0306	0.0000	0.0306	0.0000
e	0.0439	0.0155	0.0439	0.0000
f	0.0185	0.0185	0.0306	0.0000
g	0.0372	0.0112	0.0372	0.0112
h	0.0350	0.0148	0.0350	0.0148
i	0.0439	0.0155	0.0439	0.0000
j	0.0384	0.0384	0.0384	0.0000
k	0.0185	0.0185	0.0185	0.0185
l	0.0206	0.0487	0.0206	0.0000
m	0.0124	0.0124	0.0306	0.0124
n	0.0319	0.0117	0.0319	0.0319
o	0.0372	0.0112	0.0372	0.0112
p	0.0350	0.0148	0.0350	0.0148
q	0.0384	0.0384	0.0384	0.0000
r	0.0139	0.0139	0.0200	0.0398
s	0.0319	0.0117	0.0319	0.0319
t	0.0285	0.0285	0.0285	0.0285
u	0.0320	0.0320	0.0330	0.0117
v	0.0148	0.0350	0.0148	0.0350
w	0.0148	0.0350	0.0148	0.0350
x	0.0285	0.0285	0.0285	0.0285
y	0.0200	0.0398	0.0139	0.0139
z	0.0124	0.0124	0.0306	0.0124
a’	0.0206	0.0487	0.0206	0.0000
b’	0.0185	0.0185	0.0185	0.0185
c’	0.0544	0.0000	0.0544	0.0000

Table E.36. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2142	0.0000	0.2142	0.0000
b	0.2036	0.1049	0.2142	0.0000
c	0.2540	0.0000	0.2540	0.0000
d	0.2142	0.0000	0.2142	0.0000
e	0.2694	0.0879	0.2274	0.0000
f	0.2037	0.1049	0.2142	0.0000
g	0.2929	0.0633	0.2144	0.0633
h	0.2436	0.0838	0.2436	0.0838
i	0.2694	0.0879	0.2274	0.0000
j	0.2432	0.1280	0.2432	0.0000
k	0.2037	0.1049	0.2037	0.1049
l	0.2025	0.1623	0.2025	0.1136
m	0.2189	0.0702	0.2142	0.0702
n	0.2636	0.0663	0.2292	0.1076
o	0.2929	0.0633	0.2144	0.0633
p	0.2436	0.0838	0.2436	0.0838
q	0.2432	0.1280	0.2432	0.0000
r	0.2134	0.0785	0.2025	0.1353
s	0.2636	0.0663	0.2292	0.1076
t	0.2472	0.0951	0.2472	0.0951
u	0.2671	0.1078	0.2319	0.0663
v	0.2108	0.1166	0.2108	0.1166
w	0.2108	0.1166	0.2108	0.1166
x	0.2472	0.0951	0.2472	0.0951
y	0.2025	0.1353	0.2134	0.0785
z	0.2189	0.0702	0.2142	0.0702
a’	0.2025	0.1623	0.2025	0.0000
b’	0.2037	0.1049	0.2037	0.1049
c’	0.2540	0.0000	0.2540	0.0000

Tables for Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$ for all i $\in$ N.

Table E.37. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1250	0.0313	0.1250	0.0313
b	0.2222	0.0556	0.1250	0.0313
c	0.2222	0.0313	0.2222	0.0313
d	0.1250	0.0556	0.1250	0.0556
e	0.2813	0.0556	0.2222	0.0313
f	0.2222	0.0703	0.1250	0.0556
g	0.3200	0.0556	0.2222	0.0556
h	0.2813	0.0556	0.2813	0.0556
i	0.2813	0.0703	0.2222	0.0556
j	0.2813	0.0703	0.2813	0.0313
k	0.2222	0.0703	0.2222	0.0703
l	0.2222	0.0800	0.2222	0.0556
m	0.2813	0.0703	0.1250	0.0703
n	0.3200	0.0556	0.2813	0.0703
o	0.3200	0.0703	0.2222	0.0703
p	0.2813	0.0703	0.2813	0.0703
q	0.2813	0.0800	0.2813	0.0556
r	0.2813	0.0703	0.2222	0.0800
s	0.3200	0.0703	0.2813	0.0800
t	0.3200	0.0703	0.3200	0.0703
u	0.3200	0.0800	0.2813	0.0703
v	0.2813	0.0800	0.2813	0.0800
w	0.2813	0.0703	0.2813	0.0703
x	0.3200	0.0800	0.3200	0.0800
y	0.2222	0.0703	0.2813	0.0556
z	0.2813	0.0556	0.1250	0.0556
a’	0.2222	0.0703	0.2222	0.0313
b’	0.2222	0.0056	0.2222	0.0056
c’	0.2222	0.0556	0.2222	0.0556

Table E.38. Profits made by the intermediary

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2500	0.0625	0.2500	0.0625
b	0.1389	0.1389	0.2500	0.0625
c	0.2222	0.0625	0.2222	0.0625
d	0.2500	0.0556	0.2500	0.0556
e	0.2014	0.0903	0.1736	0.0625
f	0.1267	0.1545	0.2500	0.0434
g	0.2067	0.0678	0.1511	0.0678
h	0.1528	0.0903	0.1528	0.0903
i	0.1892	0.1059	0.1736	0.0434
j	0.1406	0.1406	0.1406	0.0625
k	0.1267	0.1423	0.1267	0.1423
l	0.1211	0.2600	0.1211	0.0378
m	0.0937	0.0937	0.2500	0.0937
n	0.1459	0.0678	0.1181	0.1181
o	0.1823	0.0712	0.1511	0.0712
p	0.1406	0.0937	0.1406	0.0937
q	0.1350	0.1628	0.1350	0.0378
r	0.0881	0.0881	0.1211	0.1992
s	0.1281	0.0656	0.1125	0.1281
t	0.1112	0.0956	0.1112	0.0956
u	0.1281	0.1281	0.1125	0.0656
v	0.0825	0.1450	0.0825	0.1450
w	0.0937	0.1406	0.0937	0.1406
x	0.1000	0.1000	0.1000	0.1000
y	0.1267	0.1892	0.1059	0.0903
z	0.1181	0.0903	0.2500	0.0903
a’	0.1267	0.2378	0.1267	0.0625
b’	0.1389	0.1389	0.1389	0.1389
c’	0.2222	0.0556	0.2222	0.0556

Table E.39. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3750	0.0938	0.3750	0.0938
b	0.3611	0.1945	0.3750	0.0938
c	0.4444	0.0938	0.4444	0.0938
d	0.3750	0.1112	0.3750	0.1112
e	0.4827	0.1459	0.3958	0.0938
f	0.3489	0.2248	0.3750	0.0990
g	0.5267	0.1234	0.3733	0.1234
h	0.4341	0.1459	0.4341	0.1459
i	0.4705	0.1762	0.3958	0.0990
j	0.4219	0.2109	0.4219	0.0938
k	0.3489	0.2126	0.3489	0.2126
l	0.3433	0.3400	0.3433	0.0934
m	0.3750	0.1640	0.3750	0.1640
n	0.4659	0.1234	0.3994	0.1884
o	0.5023	0.1415	0.3733	0.1415
p	0.4219	0.1640	0.4219	0.1640
q	0.4163	0.2428	0.4163	0.0934
r	0.3694	0.1584	0.3433	0.2792
s	0.4481	0.1359	0.3938	0.2081
t	0.4312	0.1659	0.4312	0.1659
u	0.4481	0.2081	0.3938	0.1359
v	0.3638	0.2250	0.3638	0.2250
w	0.3750	0.2109	0.3750	0.2109
x	0.4200	0.1800	0.4200	0.1800
y	0.3489	0.2595	0.3872	0.1459
z	0.3994	0.1459	0.3750	0.1459
a’	0.3489	0.3081	0.3489	0.0938
b’	0.3611	0.1445	0.3611	0.1445
c’	0.4444	0.1112	0.4444	0.1112

Tables for Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

Table E.40. Consumer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0800	0.0200	0.0800	0.0200
b	0.1533	0.0303	0.0800	0.0200
c	0.1422	0.0200	0.1422	0.0200
d	0.0800	0.0355	0.0800	0.0355
e	0.1944	0.0282	0.1436	0.0200
f	0.1520	0.0414	0.0800	0.0370
g	0.2320	0.0282	0.1438	0.0282
h	0.1956	0.0291	0.1956	0.0291
i	0.1932	0.0396	0.1446	0.0325
j	0.1908	0.0350	0.1908	0.0200
k	0.1579	0.0383	0.1579	0.0383
l	0.1455	0.0408	0.1455	0.0293
m	0.2017	0.0399	0.0800	0.0399
n	0.2284	0.0291	0.1917	0.0355
o	0.2296	0.0379	0.1452	0.0379
p	0.1948	0.0380	0.1948	0.0380
q	0.1903	0.0439	0.1903	0.0302
r	0.1957	0.0372	0.1476	0.0444
s	0.2268	0.0366	0.1917	0.0427
t	0.2257	0.0377	0.2257	0.0377
u	0.2268	0.0427	0.1917	0.0366
v	0.1931	0.0432	0.1931	0.0432
w	0.1948	0.0380	0.1948	0.0380
x	0.2247	0.0419	0.2247	0.0419
y	0.1485	0.0375	0.1978	0.0307
z	0.2042	0.0306	0.0800	0.0306
a'	0.1464	0.0373	0.1464	0.0200
b'	0.1533	0.0303	0.1533	0.0303
c'	0.1422	0.0355	0.1422	0.0355

Table E.41. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2000	0.0500	0.2000	0.0500
b	0.1147	0.1147	0.2000	0.0500
c	0.1778	0.0500	0.1778	0.0500
d	0.2000	0.0444	0.2000	0.0444
e	0.1431	0.0885	0.1471	0.0500
f	0.1114	0.1119	0.2000	0.0503
g	0.1271	0.0702	0.1333	0.0702
h	0.1141	0.0873	0.1141	0.0873
i	0.1414	0.0856	0.1470	0.0441
j	0.1157	0.1157	0.1157	0.0500
k	0.1123	0.1173	0.1123	0.1173
l	0.1159	0.1618	0.1159	0.0405
m	0.0782	0.0782	0.2000	0.0782
n	0.0989	0.0710	0.1004	0.1004
o	0.1240	0.0672	0.1330	0.0672
p	0.1114	0.0831	0.1114	0.0831
q	0.1152	0.1127	0.1152	0.0438
r	0.0811	0.0811	0.1144	0.1333
s	0.0968	0.0670	0.0997	0.0968
t	0.0850	0.0879	0.0850	0.0879
u	0.0968	0.0968	0.0997	0.0670
v	0.0810	0.1084	0.0810	0.1084
w	0.0831	0.1114	0.0831	0.1114
x	0.0834	0.0834	0.0834	0.0834
y	0.1156	0.1367	0.0846	0.0850
z	0.0844	0.0815	0.2000	0.0815
a’	0.1174	0.1652	0.1174	0.0500
b’	0.1147	0.1147	0.1147	0.1147
c’	0.1778	0.0444	0.1778	0.0444

Table E.42. Producer surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0200	0.0050	0.0200	0.0050
b	0.0200	0.0200	0.0200	0.0050
c	0.0356	0.0050	0.0356	0.0050
d	0.0200	0.0089	0.0200	0.0089
e	0.0380	0.0167	0.0288	0.0050
f	0.0178	0.0258	0.0200	0.0097
g	0.0422	0.0138	0.0250	0.0138
h	0.0318	0.0164	0.0318	0.0164
i	0.0350	0.0221	0.0288	0.0088
j	0.0313	0.0313	0.0313	0.0050
k	0.0184	0.0259	0.0184	0.0259
l	0.0176	0.0467	0.0176	0.0075
m	0.0200	0.0200	0.0200	0.0200
n	0.0354	0.0139	0.0275	0.0275
o	0.0375	0.0186	0.0250	0.0186
p	0.0297	0.0212	0.0297	0.0212
q	0.0294	0.0370	0.0294	0.0086
r	0.0202	0.0202	0.0177	0.0392
s	0.0323	0.0182	0.0261	0.0323
t	0.0310	0.0252	0.0310	0.0252
u	0.0323	0.0323	0.0261	0.0182
v	0.0196	0.0338	0.0196	0.0338
w	0.0212	0.0297	0.0212	0.0297
x	0.0288	0.0288	0.0288	0.0288
y	0.0189	0.0346	0.0225	0.0159
z	0.0234	0.0154	0.0200	0.0154
a’	0.0192	0.0409	0.0192	0.0050
b’	0.0200	0.0200	0.0200	0.0200
c’	0.0356	0.0089	0.0356	0.0089

Table E.43. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.3000	0.0750	0.3000	0.0750
b	0.2880	0.1650	0.3000	0.0750
c	0.3556	0.0750	0.3556	0.0750
d	0.3000	0.0888	0.3000	0.0888
e	0.3755	0.1334	0.3195	0.0750
f	0.2812	0.1791	0.3000	0.0970
g	0.4013	0.1122	0.3021	0.1122
h	0.3415	0.1328	0.3415	0.1328
i	0.3696	0.1473	0.3204	0.0854
j	0.3378	0.1820	0.3378	0.0750
k	0.2886	0.1815	0.2886	0.1815
l	0.2790	0.2493	0.2790	0.0773
m	0.2999	0.1381	0.3000	0.1381
n	0.3627	0.1140	0.3196	0.1634
o	0.3911	0.1237	0.3032	0.1237
p	0.3359	0.1423	0.3359	0.1423
q	0.3349	0.1936	0.3349	0.0826
r	0.2970	0.1385	0.2797	0.2169
s	0.3559	0.1218	0.3175	0.1718
t	0.3417	0.1508	0.3417	0.1508
u	0.3559	0.1718	0.3175	0.1218
v	0.2937	0.1854	0.2937	0.1854
w	0.2991	0.1791	0.2991	0.1791
x	0.3369	0.1541	0.3369	0.1541
y	0.2830	0.2088	0.3049	0.1316
z	0.3120	0.1275	0.3000	0.1275
a’	0.2830	0.2434	0.2830	0.0750
b’	0.2880	0.1650	0.2880	0.1650
c’	0.3556	0.0888	0.3556	0.0888

Tables for Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ ***and*** $\phi = 1.5$ ***for all i*** $\in N$.

Table E.44. Consumer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0408	0.0102	0.0408	0.0102
b	0.0878	0.0116	0.0408	0.0102
c	0.0726	0.0102	0.0726	0.0102
d	0.0408	0.0181	0.0408	0.0181
e	0.1122	0.0094	0.0753	0.0102
f	0.0871	0.0186	0.0408	0.0138
g	0.1445	0.0099	0.0769	0.0099
h	0.1144	0.0101	0.1144	0.0101
i	0.1114	0.0162	0.0762	0.0140
j	0.1070	0.0107	0.1070	0.0102
k	0.0916	0.0146	0.0916	0.0146
l	0.0800	0.0157	0.0800	0.0115
m	0.1231	0.0163	0.0408	0.0163
n	0.1390	0.0103	0.1094	0.0116
o	0.1409	0.0142	0.0776	0.0142
p	0.1131	0.0140	0.1131	0.0140
q	0.1078	0.0167	0.1078	0.0116
r	0.1155	0.0136	0.0812	0.0175
s	0.1367	0.0127	0.1092	0.0153
t	0.1346	0.0128	0.1346	0.0128
u	0.1367	0.0153	0.1092	0.0127
v	0.1116	0.0158	0.1116	0.0158
w	0.1131	0.0140	0.1131	0.0140
x	0.1335	0.0143	0.1335	0.0143
y	0.0816	0.0138	0.1179	0.0119
z	0.1265	0.0121	0.0408	0.0121
a'	0.0806	0.0096	0.0806	0.0102
b'	0.0878	0.0116	0.0878	0.0116
c'	0.0726	0.0181	0.0726	0.0181

Table E.45. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1428	0.0357	0.1428	0.0357
b	0.0870	0.0870	0.1428	0.0357
c	0.1270	0.0357	0.1270	0.0357
d	0.1428	0.0318	0.1428	0.0318
e	0.1003	0.0731	0.1094	0.0357
f	0.0872	0.0810	0.1428	0.0342
g	0.0858	0.0589	0.1009	0.0589
h	0.0819	0.0717	0.0819	0.0717
i	0.1028	0.0664	0.1090	0.0353
j	0.0874	0.0874	0.0874	0.0357
k	0.0876	0.0901	0.0876	0.0901
l	0.0927	0.1103	0.0927	0.0324
m	0.0605	0.0605	0.1428	0.0605
n	0.0706	0.0608	0.0767	0.0767
o	0.0882	0.0539	0.1011	0.0539
p	0.0837	0.0657	0.0837	0.0657
q	0.0886	0.0824	0.0886	0.0368
r	0.0643	0.0643	0.0916	0.0935
s	0.0722	0.0556	0.0781	0.0722
t	0.0636	0.0694	0.0636	0.0694
u	0.0722	0.0722	0.0781	0.0556
v	0.0662	0.0800	0.0662	0.0800
w	0.0657	0.0837	0.0657	0.0837
x	0.0647	0.0647	0.0647	0.0647
y	0.0913	0.0966	0.0636	0.0694
z	0.0601	0.0653	0.1428	0.0653
a’	0.0920	0.1141	0.0920	0.0357
b’	0.0870	0.0870	0.0870	0.0870
c’	0.1270	0.0318	0.1270	0.0318

Table E.46. Producer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0306	0.0077	0.0306	0.0077
b	0.0306	0.0306	0.0306	0.0077
c	0.0544	0.0077	0.0544	0.0077
d	0.0306	0.0136	0.0306	0.0136
e	0.0528	0.0281	0.0460	0.0077
f	0.0281	0.0351	0.0306	0.0146
g	0.0532	0.0239	0.0407	0.0239
h	0.0460	0.0274	0.0460	0.0274
i	0.0498	0.0327	0.0459	0.0151
j	0.0479	0.0478	0.0479	0.0077
k	0.0296	0.0379	0.0296	0.0379
l	0.0289	0.0569	0.0289	0.0127
m	0.0306	0.0306	0.0306	0.0306
n	0.0485	0.0244	0.0428	0.0428
o	0.0502	0.0300	0.0410	0.0300
p	0.0448	0.0332	0.0448	0.0332
q	0.0452	0.0500	0.0452	0.0157
r	0.0320	0.0320	0.0294	0.0518
s	0.0466	0.0306	0.0417	0.0466
t	0.0455	0.0409	0.0455	0.0409
u	0.0466	0.0466	0.0417	0.0306
v	0.0321	0.0480	0.0321	0.0480
w	0.0332	0.0448	0.0332	0.0448
x	0.0441	0.0441	0.0441	0.0441
y	0.0307	0.0488	0.0334	0.0263
z	0.0330	0.0251	0.0306	0.0251
a'	0.0298	0.0535	0.0298	0.0077
b'	0.0306	0.0306	0.0306	0.0306
c'	0.0544	0.0136	0.0544	0.0136

Table E.47. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2142	0.0536	0.2142	0.0536
b	0.2054	0.1292	0.2142	0.0536
c	0.2540	0.0536	0.2540	0.0536
d	0.2142	0.0635	0.2142	0.0635
e	0.2653	0.1106	0.2307	0.0536
f	0.2024	0.1347	0.2142	0.0626
g	0.2835	0.0927	0.2185	0.0927
h	0.2423	0.1092	0.2423	0.1092
i	0.2640	0.1153	0.2311	0.0644
j	0.2423	0.1459	0.2423	0.0536
k	0.2088	0.1426	0.2088	0.1426
l	0.2016	0.1829	0.2016	0.0566
m	0.2142	0.1074	0.2142	0.1074
n	0.2581	0.0955	0.2289	0.1311
o	0.2793	0.0981	0.2197	0.0981
p	0.2416	0.1129	0.2416	0.1129
q	0.2416	0.1491	0.2416	0.0641
r	0.2118	0.1099	0.2022	0.1628
s	0.2555	0.0989	0.2290	0.1341
t	0.2437	0.1231	0.2437	0.1231
u	0.2555	0.1341	0.2290	0.0989
v	0.2099	0.1438	0.2099	0.1438
w	0.2120	0.1425	0.2120	0.1425
x	0.2423	0.1231	0.2423	0.1231
y	0.2036	0.1592	0.2149	0.1076
z	0.2196	0.1025	0.2142	0.1025
a’	0.2024	0.1772	0.2024	0.0536
b’	0.2054	0.1292	0.2054	0.1292
c’	0.2540	0.0635	0.2540	0.0635

Tables for Simulation 17: $\delta = 3$ for $\Omega = \{i, k\}$; $\delta = 1$ for $\Psi = \{j, l\}$; $\phi = 0.5$; and $\alpha = 1$.

Table E.48. Consumer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0408	0.0800	0.0408	0.0800
b	0.1094	0.1094	0.0408	0.0800
c	0.0726	0.0800	0.0726	0.0800
d	0.0408	0.1422	0.0408	0.1422
e	0.1322	0.1031	0.0611	0.0800
f	0.1072	0.1591	0.0408	0.1349
g	0.1718	0.1016	0.0722	0.1016
h	0.1297	0.1058	0.1297	0.1058
i	0.1367	0.1535	0.0740	0.1352
j	0.1250	0.1250	0.1250	0.0800
k	0.1074	0.1533	0.1074	0.1533
l	0.1012	0.1741	0.1012	0.1286
m	0.1543	0.1543	0.0408	0.1543
n	0.1685	0.1039	0.1252	0.1252
o	0.2185	0.1462	0.0751	0.1462
p	0.1292	0.1503	0.1292	0.1503
q	0.1243	0.1670	0.1243	0.1300
r	0.1497	0.1497	0.0895	0.1695
s	0.1636	0.1462	0.1252	0.1636
t	0.1628	0.1250	0.1628	0.1250
u	0.1636	0.1636	0.1252	0.1462
v	0.1464	0.1645	0.1464	0.1645
w	0.1503	0.1292	0.1503	0.1292
x	0.1608	0.1608	0.1608	0.1608
y	0.1052	0.1301	0.1529	0.1078
z	0.1588	0.1073	0.0408	0.1073
a'	0.1033	0.1322	0.1033	0.0800
b'	0.1094	0.1094	0.1094	0.1094
c'	0.0726	0.1422	0.0726	0.1422

Table E.49. Profits

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.1837	0.2000	0.1837	0.2000
b	0.1207	0.2312	0.1837	0.2000
c	0.1633	0.2000	0.1633	0.2000
d	0.1837	0.1778	0.1837	0.1778
e	0.1342	0.2187	0.1129	0.2000
f	0.1044	0.2238	0.1837	0.1710
g	0.1298	0.1966	0.1295	0.1966
h	0.1097	0.2178	0.1097	0.2178
i	0.1245	0.2109	0.1434	0.1754
j	0.1055	0.2344	0.1055	0.2000
k	0.1059	0.2171	0.1059	0.2171
l	0.1054	0.2582	0.1054	0.1691
m	0.0833	0.1852	0.1837	0.1852
n	0.1020	0.2036	0.0977	0.2103
o	0.1109	0.2048	0.1272	0.2048
p	0.0973	0.2037	0.0973	0.2037
q	0.0983	0.2299	0.0983	0.1717
r	0.0827	0.1831	0.1050	0.2080
s	0.0847	0.1807	0.0886	0.2061
t	0.0898	0.1950	0.0898	0.1950
u	0.0847	0.2061	0.0886	0.1807
v	0.0812	0.1943	0.0812	0.1943
w	0.0973	0.2037	0.0973	0.2037
x	0.0748	0.1851	0.0748	0.1851
y	0.1142	0.2312	0.1045	0.2002
z	0.1118	0.1974	0.1837	0.1974
a'	0.1146	0.2609	0.1146	0.2000
b'	0.1207	0.2312	0.1207	0.2312
c'	0.1633	0.1778	0.1633	0.1778

Table E.50. Producer Surplus

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.0306	0.0200	0.0306	0.0200
b	0.0402	0.0462	0.0306	0.0200
c	0.0544	0.0200	0.0544	0.0200
d	0.0306	0.0356	0.0306	0.0356
e	0.0638	0.0435	0.0373	0.0200
f	0.0335	0.0659	0.0306	0.0340
g	0.0756	0.0379	0.0380	0.0379
h	0.0542	0.0434	0.0542	0.0434
i	0.0567	0.0630	0.0470	0.0350
j	0.0527	0.0703	0.0527	0.0200
k	0.0343	0.0638	0.0343	0.0638
l	0.0327	0.0991	0.0327	0.0333
m	0.0417	0.0556	0.0306	0.0556
n	0.0643	0.0399	0.0474	0.0617
o	0.0633	0.0614	0.0379	0.0614
p	0.0483	0.0608	0.0483	0.0608
q	0.0476	0.0906	0.0476	0.0340
r	0.0410	0.0547	0.0330	0.0821
s	0.0547	0.0541	0.0431	0.0813
t	0.0574	0.0577	0.0574	0.0577
u	0.0547	0.0813	0.0431	0.0541
v	0.0403	0.0774	0.0403	0.0774
w	0.0483	0.0608	0.0483	0.0608
x	0.0498	0.0740	0.0498	0.0740
y	0.0378	0.0683	0.0508	0.0394
z	0.0534	0.0387	0.0306	0.0387
a’	0.0378	0.0777	0.0378	0.0200
b’	0.0402	0.0462	0.0402	0.0462
c’	0.0544	0.0356	0.0544	0.0356

Table E.51. Welfare

	Countries			
Networks	*i*	*j*	*k*	*l*
a	0.2551	0.3000	0.2551	0.3000
b	0.2703	0.3869	0.2551	0.3000
c	0.2902	0.3000	0.2902	0.3000
d	0.2551	0.3556	0.2551	0.3556
e	0.3302	0.3653	0.2112	0.3000
f	0.2451	0.4488	0.2551	0.3399
g	0.3771	0.3361	0.2396	0.3361
h	0.2937	0.3669	0.2937	0.3669
i	0.3179	0.4274	0.2644	0.3457
j	0.2832	0.4297	0.2832	0.3000
k	0.2476	0.4343	0.2476	0.4343
l	0.2393	0.5314	0.2393	0.3310
m	0.2793	0.3951	0.2551	0.3951
n	0.3347	0.3474	0.2703	0.3972
o	0.3927	0.4124	0.2403	0.4124
p	0.2748	0.4148	0.2748	0.4148
q	0.2701	0.4874	0.2701	0.3356
r	0.2734	0.3875	0.2275	0.4596
s	0.3030	0.3810	0.2569	0.4510
t	0.3100	0.3777	0.3100	0.3777
u	0.3030	0.4510	0.2569	0.3810
v	0.2679	0.4363	0.2679	0.4363
w	0.2959	0.3937	0.2959	0.3937
x	0.2854	0.4199	0.2854	0.4199
y	0.2572	0.4296	0.3082	0.3475
z	0.3241	0.3433	0.2551	0.3433
a’	0.2556	0.4708	0.2556	0.3000
b’	0.2703	0.3869	0.2703	0.3869
c’	0.2902	0.3556	0.2902	0.3556

Appendix F

Simulations for the case of governments biased in favour of consumers

This appendix shows and discusses the results obtained from the simulations carried out for the case of governments biased in favour of consumers. These results are presented as follows.

F.1 Simulations for bilateralism under symmetric countries

Two groups of simulations were developed for the case of symmetrical countries. One of them includes simulations under the assumption of exogenous tariffs and the other includes simulations under the assumption of endogenous tariffs.

F.1.1 Bilateralism under exogenous tariffs and symmetric countries

The simulations included in this group consider three different levels of monopsonistic power: (i) no monopsonistic power (i.e. $\phi_i = 0$ in Equation 4.1 which corresponds to the original model by Goyal and Joshi (2006)); (ii) moderate levels of monopsonistic power (i.e. $\phi_i = 0.5$ in Equation 4.1); and (iii) high levels of monopsonistic power (i.e. $\phi_i = 1.5$ in Equation 4.1). The results of these simulations for the case of governments biased in favour of consumers are presented as follows.

F.1.1.1 Simulation 1: ϕ_i = 0 and α_i = 1 for all i $\in$ N.

In considering Table 4.1 it is inferred that the set of link deletion and link addition proof networks are $D = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $A = \{k\}$, respectively. Consequently, the set of pairwise stable networks when governments are biased in favour of consumers is $P = D \cap A = \{k\}$, that is, global free trade.

The stability of this network is explained by the oligopolistic power exercised by the intermediaries. That is, when two countries sign an agreement, their domestic markets become more competitive because more intermediaries compete in these markets. Consumers are better off because consumer surplus increases when markets are more competitive. This explains why global free trade is the only stable network when governments are biased in favour of consumers: signing additional agreements always increases consumer surplus in this model reflecting the higher level of competition that is caused by free trade.

F.1.1.2 Simulation 2: ϕ_i = 0.5 and α_i = 1 for all i $\in$ N.

In considering Table E.4, it was inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of consumers are given by $D = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $A = \{k\}$. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A = \{k\}$. This is the same result obtained by Goyal and Joshi (2006). That is, when governments care about consumers, the pairwise stable network is global free trade and this is unique.

According to this result, the only stable network when governments are biased in favour of consumers is global free trade because this increases competition in domestic markets increasing consumer surplus. This is the same result as that found in Goyal and Joshi's world implying that their results are robust when there is a farming sector only when governments are biased in favour of consumers.

F.1.1.3 Simulation 3: $\phi_i = 1.5$ *and* $\alpha_i = 1$ *for all* $i \in N$.

In this simulation it is inferred from Table E.8 in Appendix E that the sets of link deletion and link addition proof networks are $D = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$, $A = \{k\}$, respectively. This implies that the set of pairwise stable networks in this case is $P = D \cap A = \{k\}$. This is the same result obtained in the previous simulation. This implies that this result is robust through different levels of monopsonistic power when governments are biased in favour of consumers.

F.1.2 Bilateralism under endogenous tariffs and symmetric countries

Simulations for the case of endogenous tariffs were developed only for politically unbiased governments as a consequence of the mathematical complexity of the model when assuming policy biases.

F.2 Simulations on bilateralism under asymmetric countries

This section extends the analysis by allowing asymmetry across countries. Two types of asymmetry are considered in this study: (1) asymmetry in market size: and (2) asymmetry in farmer's productivity.

F.2.1 Bilateralism under exogenous tariffs and asymmetry in market size

Asymmetry in market size is introduced by assuming that countries *i* and *k* have the same market size denoted by α, and countries *j* and *l* have the same market size denoted by $\tilde{\alpha} \neq \alpha$ (see Section 4.2.1.2). Using this assumption, six simulations were developed (i.e. simulations 11, 12, 13, 14, 15, and 16). The three first simulations considers the extreme case when $\tilde{\alpha}$ = 0. That is, they consider the case when countries *j* and *l* are extremely small in the sense that they don't have a domestic market. This assumption is relaxed in the next three simulations with the purpose of studying the incentive of large countries to trade with middle size countries. In these simulations it is assumed $\tilde{\alpha}$ = 0.5. That is, countries *j* and *l* are small but still have a significant domestic market.

F.2.1.1 Simulation 11: α = 1, $\tilde{\alpha}$ = 0 and ϕ = 0 for all i $\in$ N.

In this simulation it is assumed that there is no monopsonistic power (i.e. ϕ_i = 0). This implies that this simulation converges to the original model by Goyal and Joshi (2006) under asymmetry in market size.

According to the information presented in Table E.26, D = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and A = {*t*, *x*}. Consequently, the set of pairwise stable networks when governments are biased in favour of consumers is $P = D \cap A$ = {*t*, *x*}.

In contrast to the case of symmetric countries where global free trade is the only pairwise stable network, in the asymmetric case there are two stable networks: *t* and *x*. The stability of global free trade is explained by the fact large countries are unwilling to break an existing agreement because this causes a loss in consumer surplus as a result of the lower level of competition in their domestic markets. On the other hand, small countries are indifferent because they do not have relevant domestic markets. Consequently, breaking an existing agreement does not affect the level of consumer surplus in these countries because in any case consumer surplus is cero. On the other hand, network *t* is stable because the small countries are indifferent about breaking or signing a new agreement as this does not change the level of consumer surplus. Again, this is due to the fact that these countries do not have domestic markets and, therefore, consumer surplus is always cero.

F.1.1.2 Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$ for all $i \in N$.

This simulation introduces the farming sector into the analysis. This is done by assuming moderate level of monopsonistic power (i.e. $\phi = 0.5$ for all $i \in N$).

In considering Table E.29, it is inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of consumers are given by $D = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$ and $A = \{t, x\}$. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A = \{t, x\}$. This is the same result than the one obtained in the previous simulation.

This result revealed that when governments are biased in favour of consumers, the same pairwise stable networks are obtained with respect to the previous simulation. This implies that the results obtained from Goyal and Joshi's world are robust under relatively low levels of monopsonistic power.

F.2.1.3 Simulation 13: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 1.5$ for all $i \in N$.

The objective of this simulation is to determine whether the results obtained in the previous one are affected when intermediaries exercise larger levels of monopsonistic power (i.e. when $\phi = 1.5$).

In considering Table E.33, it is inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of consumers are $D = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$, $A = \{t, x\}$ and $P = D \cap A = \{t, x\}$. This is the same result that the one obtained in the previous simulation. It is concluded therefore that the network pairwise stability is not affected by different levels of monopsonistic power when countries are biased in favour of consumers.

F.2.1.4 Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$ for all i $\in$ N.

The next three simulation were introduced with the purpose of identifying the pairwise stable networks when there large and medium size countries (i.e. $\alpha = 1$ and $\tilde{\alpha} = 0.5$, respectively) under different degrees of monopsonistic power. The current simulation in particular considers the case when there is no monopsonistic power (i.e. Goyal and Joshi's world when there are large and medium size countries).

In considering Table E.37 it is inferred that when governments are politically biased in favour of consumers, *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and *A* = {*x*}. Consequently, the set of pairwise stable networks in this case is $P = D \cap A$ = {*x*}, that is, global free trade. This differs from the results obtained in Simulation 11. In that simulation network *t* is also stable and this is explained by the fact that the very small countries *j* and *l* in network *t* are indifferent about having an agreement with each other because they don't have domestic markets. In contrast, when countries *j* and *l* are medium size, they have relevant domestic markets that become more competitive after the agreement increasing in this way consumer surplus. This explains why network *t* is not stable in the current simulation.

F.2.1.5 Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

The objective of this simulation is to introduce the farming sector into the model when networks are formed of large and medium size countries. This is reflected by the assumption that intermediaries exercise a moderate level of monopsonistic power. (i.e. $\phi_i = 0.5$).

In considering Table E.40, it is inferred that the sets of link deletion proof and link addition proof networks when governments are biased in favour of consumers are *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and *A* = {*x*}, respectivelly. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A = \{x\}$. The same set of pairwise stable networks was obtained in the symmetrical case and the previous simulation. This implies therefore that the results obtained from Goyal and Joshi's when governments are biased in favour of consumers are robust through different moderate levels monopsonistic power and asymmetric countries in market size that do not include very small countries. However, a deviation was found in the set of link deletion proof networks with respect to the previous simulation. That is, network *l* is not link deletion proof when there is a farming sector. Likewise, a deviation was found in the set of pairwise stable networks in the simulation that considers moderate levels of monopsonistic power and very small countries (see Section 4.4.1.2). In that simulation network *t* is also pairwise stable. These deviations are explained as follows.

In the case of biased governments in favour of consumers the only pairwise stable network is global free trade and this differs from the case of large countries and very small countries with moderate level of monopsonistic power. In that case network *t* is also pairwise stable because the very small countries *j* and *k* are very small implying that they are indifferent about having an agreement with each other. That is, an agreement will not make any difference on consumer surplus because they have irrelevant domestic markets. In contrast when countries *j* and *k* are medium size, an agreement with each other will increase consumer surplus as their domestic markets become more competitive. It is concluded therefore that when governments are biased in favour of consumers and when the level of monopsonistic power exercised by intermediaries is moderate, deviations from global free trade are expected to be found in networks containing very small countries.

On the other hand, it was found in the previous simulation that network *l* is link deletion proof. However, this changes when there is a farming sector. This network is the star network with centre the medium size country *j*. In the previous simulation this network is link deletion proof because country *j* is unwilling to break any of the existing agreements. Breaking an agreement implies reducing the level of competition negatively affection consumer surplus in this country. In contrast, when there is a moderate level of monopsonistic power, this is not the case because the medium size country *j* has an incentive to break an existing agreement with a large country. The reason is because the agreement with a large country increases the export output to the latter country pushing the agricultural price up in the former. In order to deal with this higher cost, the intermediary of country *j* decreases the output that is sold in the domestic market in order to release pressure on the agricultural price and, in this way, improve the competitive position when exporting the food processed good to the large country. As a result, the agreement lowers the level of competition in the central medium size country *j* negatively affecting consumer surplus. This is why network *l* is not link deletion proof. This also proves the fact that free trade not always increases consumer surplus as is normally believed. Depending on the network, there are cases where consumer surplus decreases in medium size countries that occupy a central position in the network when there is a farming sector which is explained by the influence of this sector on the cost faced by the intermediaries.

F.2.1.6 Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ *and* $\phi = 1.5$ *for all* $i \in N$.

This last simulation in the section of asymmetry in market size introduced to study the effects of very high levels of monopsonistic power on the international trade system.

In considering Table E.44 it is concluded that when governments are biased in favour of consumers, the sets of link deletion proof, link addition proof and pairwise stable networks are *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *j*, *k*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *y*, *z*, *b*', *c*', *Eq*}, *A* = {*x*, *c*'} and $P = D \cap A$ = {*x*, *c*'}, respectively. This result is different from the one obtained in the previous simulation. In that simulation only global free trade is stable. This suggests therefore that high levels of monosponistic power can negatively affect the incentives of governments when they are biased in favour of consumers. The intuition behind this result is explained as follows.

In relation to the case of governments biased in favour of consumer surplus, the results revealed that when monopsonistic power if high, two networks become pairwise stable: global free trade; and network *c*'. The latter networks is composed of two blocks, one is formed of the large countries *i* and *k* and the other is formed of the medium size countries *j* and *l*. According to Table E.44, the medium size countries are unwilling to sign an agreement with a large size country (this can be seen when comparing consumer surplus in networks *c*' and *h* in Figure 4.5). The reason is because the agreement causes a net decrease in the total output that is sold in the domestic market of the medium size country reducing in this way the level of competition in this market. That is, the intermediary of this country has access to a large market after the agreement which implies a large quantity that is exported to the large country. This additional output pushes the price paid to farmers up as a consequence of the high monopsonistic power. In order cushion this increase in price, the intermediary decreases the output sold in the domestic market of the medium size country in order to take advantage of the large size of the external market. This decrease in output in the domestic market of the medium size country is not compensated by the additional output imported from the large country. As a result, the agreement causes a net decrease in the output that is traded in this country negatively affecting consumer surplus. It is concluded therefore that high levels of monopsonistic power negatively affect international trade when governments are biased in favour of consumers and when the network is formed of large and medium size countries. Under these conditions, regionalism of the south-north type arises. This is an interesting results because it is commonly believed that free trade always increase competition and consumer surplus by making cheaper the goods that are traded internationally. However, as shown in this simulation, this is not always the case. As proved in this section, the effect of free trade on consumer surplus depends on the architecture of the network, the level of monopsonistic power, and the existence of asymmetries across countries.

F.2.2 Bilateralism under exogenous tariffs and asymmetry in farmers' productivity

A key result obtained in the previous simulations is that monopsonistic power has an important effect on the architecture and stability of international networks of food processed goods. The objective of the simulation presented in this section is to assess how this type of imperfection affects the network architecture when farming sectors in different countries are asymmetric.

F.2.2.1 Simulation 17: $\delta = 3$ for $\Omega = \{i, k\}$; $\delta = 1$ for $\Psi = \{j, l\}$; $\phi = 0.5$; and $\alpha = 1$.

In considering Table E.48, it was found that the sets of link deletion proof and link addition proof networks when governments are biased in favour of consumers are given by *D* = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a*', *b*', *c*', *Eq*} and *A* = {*x*}. This implies that the set of pairwise stable networks in this case is given by $P = D \cap A = \{x\}$.

This result is the same as the ones obtained for the cases of symmetric countries with different levels of monosonistic power. This means that the same conclusions discussed in these simulations applies to the case of asymmetry in farmers' productivity when governments are politically biased in favour of consumers.

F.3 Simulations for global agreements under symmetric countries

This section explores the issue of global agreements when governments are biased in favour of consumers and when countries are symmetric. For this purpose, the global treaty stability developed in this dissertation is considered.

F.3.1 Global agreements under exogenous tariffs and symmetric countries

This part includes the simulations that assume symmetrical countries and exogenous tariffs. Each of these simulations corresponds to different levels of monopsonistic power associated with specific values of the parameter ϕ_i in equation 4.1: ϕ_i = 0; ϕ_i = 0.5; and ϕ_i = 1.5 for all $i \in N$. These simulations are explained as follows.

F.3.1.1 Simulation 1: $\phi_i = 0$ and $\alpha_i = 1$ for all $i \in N$.

The information presented in Table 4.1 revealed that when countries are biased in favour of consumers, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *Eq*} and Γ = {*h*, *i*, *j*, *k*, *Eq*}, respectively. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*h*, *i*, *j*, *k*, *Eq*}.

According to the results, the global stable networks in this case are networks *h*, *i*, *j* and *k*. The stability of the first three networks is explained by the fact that they contain at least one country that is indifferent about signing a global agreement in agriculture. The stability of network *k*, on the other hand, is explained by the fact that no country in this network is willing to break one or more links simultaneously. These networks are shown in Figure F.1. The countries that are indifferent about signing a global agreement are depicted as nodes highlighted with circles.

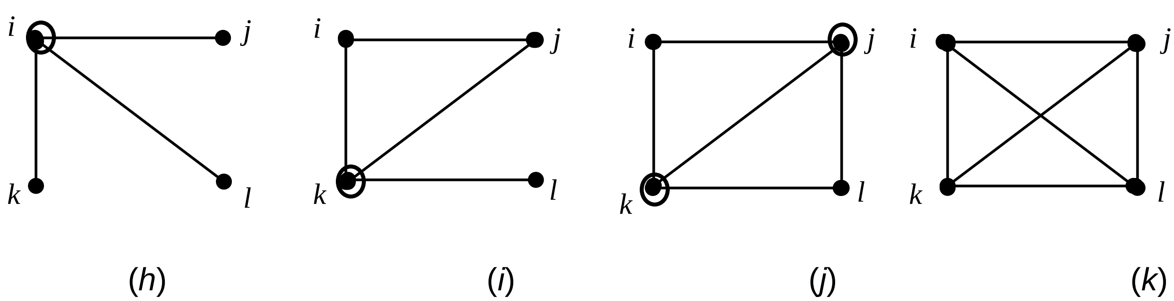


Figure F.1. Global treaty networks when governments are biased in favour of consumers.

To understand why networks *h*, *i* and *j* are global treaty stable, let us consider network *h* as an example. In this network, country *i* is connected with all countries of the world. As a result, the domestic market of this country has reached the highest possible level of competition as all players in the world are present in this market. This implies that country *i* enjoys the highest possible level of consumer surplus as a consequence of this high level of competition. Thus, if this country signed a global agreement in agriculture, the same countries would be playing in its domestic market because country *i* already have bilateral agreements with them. This means that this agreement would not affect the level of competition in the domestic market of country *i* and, therefore, the level of consumer surplus would remain the same. This is why this country is indifferent about signing a global agreement. The same explanation applies to networks *i* and *j*. It is concluded therefore that when countries are already connected to the other countries of the world, then biased governments in favour of consumers of the former are indifferent about signing a global agreement in agriculture because this would not affect the current level of consumer surplus.

In relation to network *k*, on the other hand, this network is global treaty stable because no country in this structure is willing to break one or more agreements simultaneously. If they did, then the resulting lower competition in the domestic market would negatively affect consumer surplus.

F.3.1.2 Simulation 2: $\phi_i = 0.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation introduces the farming sector in order to assess how moderate levels of monopsonistic power (i.e. $\phi_i = 0.5$) affects the global treaty stable networks identified in the previous case.

Using the information presented in Table E.4 it was found that when countries are biased in favour of consumers, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *Eq*} and Γ = {*h*, *i*, *j*, *k*, *Eq*}, respectively. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*h*, *i*, *j*, *k*, *Eq*}.

According to this result, the global treaty stability of the networks identified in the previous simulation is not affected either. However, while this sector does not affect this stability, it affects the incentives of the governments of central countries. To illustrate this effect, consider as an example the global treaty stable network *h*. In the previous section, this stability is explained by the fact that the central country *i* is indifferent about signing a global agreement because this does not affect consumer surplus. That is, because this country already has agreements with all countries in the world in network *h*, a global agreement does not increase the level of competition in this country. In contrast, when there is a farming sector, a global agreement causes an increase in the total output that is traded internationally. This pushes the price paid to the farming sector up implying that the intermediaries in the world face a higher marginal cost after the agreement. In order to adjust to this higher cost, these individuals reduce the total output that is traded globally negatively affecting the level of consumer surplus in the central country. This implies that when there is a farming sector, the governments of central countries are unwilling to sign a global agreement rather than being indifferent.

It is concluded therefore that the presence of the farming sector has a negative effect on the incentives of biased governments in favour of consumers in central countries because it reinforces the unwillingness to sign a global agreement in agriculture. It is also concluded that the claim arguing that global free trade always causes a gain in consumer surplus is not necessarily true. Rather, it depends on the existence of a farming sector and the position of countries in the network. This finding reinforces the advantage of studying agricultural trade liberalisation using a network approach as this framework can inform about the incentive of single countries in the network.

F.3.1.3 Simulation 3: $\phi_i = 1.5$ and $\alpha_i = 1$ for all $i \in N$.

This simulation was introduced with the purpose of investigating whether the results obtained under moderate levels of monopsonistic power remains robust when this power is high. The results are presented as follows.

It is inferred from Table E.8 in Appendix E that when countries are biased in favour of consumers, the sets of strong link deletion proof networks and global treaty proof networks are $D_S = \{a, b, c, d, e, f, g, h, i, j, k, Eq\}$ and $\Gamma = \{h, i, j, k, Eq\}$, respectively. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{h, i, j, k, Eq\}$.

This is the same result than the one obtained in the previous simulation. It is concluded therefore that the identified global treaty stable networks remain robust thtough different levels of monopsonistic power.

F.3.2 Global agreements under endogenous tariffs and symmetric countries

Given the complexity of the model when tariffs are placed endogenously, only the politically unbiased governments case was explored in the relevant simulations.

F.3.3 Global agreements under exogenous tariffs and asymmetry in market size

The objective of this section is to extend the analysis to determine whether asymmetry in market size can also affect the global treaty stability when countries are involved in global agreements.

F.3.3.1 Simulation 11: $\alpha = 1$, $\tilde{\alpha} = 0$ and $\phi = 0$ for all $i \in N$.

This simulation considers a world composed of large and very small countries without monopsonistic power. That is, it corresponds to Goyal and Joshis' world under this type of asymmetry.

According to information presented in Table E.26 in Appendix E the sets of strong link deletion proof networks and global treaty proof networks when governments are biased in favour of consumers are $D_S = \Gamma = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$.

According to the results, when governments are biased in favour of consumers, all the networks in Figure 4.5 are the global treaty stable. In contrast, in the symmetrical case only networks having central countries that are connected to all the countries of the world are global treaty stable. This difference is explained by the incentives of the very small countries. That is, because these countries have a very small domestic market, any gain in consumer surplus from a global agreement is irrelevant. This is why they are indifferent about signing an agreement in any possible network. This is also valid for large central countries that are connected to all countries of the world: because they are already connected to all countries of the world, a global agreement will not increase the level of competition in their domestic markets. It is concluded therefore that the only countries that are willing to sign a global agreement when governments are biased in favour of consumers are large countries that are not fully connected. This is because this agreement allows them to connect to all countries of the world and this, in turn, increases the level of competition positively affecting consumer surplus.

F.3.3.2 Simulation 12: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 0.5$ for all i $\in N$.

This simulation investigates how the results obtained in the previous simulation are affected when intermediaries exercise moderate levels of monopsonistic power (i.e. $\phi = 0.5$ in all i $\in N$).

According to the information presented in Table E.29 in Appendix E, the sets of strong link deletion proof networks and global treaty proof networks when countries are biased in favour of consumers are $D_S = \Gamma$ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n, o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c', Eq*}. Therefore, the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n, o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c', Eq*}.

According to the results, the same networks are global treaty stable when governments are biased in favour of either consumers or intermediaries. This implies that the results obtained from Goyal and Joshis' model are robust through moderate monopsonistic power when the world is composed of large and very small countries.

F.3.3.3 Simulation 13: $\alpha = 1, \tilde{\alpha} = 0$ and $\phi = 1.5$ for all i $\in N$.

This last simulation in the case of a world composed of large and very small countries is introduced to determine how the international trade structure of processed agricultural goods is affected when the level of monopsonistic power is high (i.e. when $\phi = 1.5$ for all $i \in N$).

According to the information presented in Table E.33 in Appendix E, the sets of strong link deletion proof networks and global treaty proof networks when countries are biased in favour of consumers are $D_S = \Gamma = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c', Eq\}$.

This is the same result that the one obtained in the previous simulations. It is concluded therefore that the identified global treaty stable networks remain robust through different levels of monoipsonistic power when governments are biased in favour of consumers.

F.3.3.4 Simulation 14: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 0$ for all $i \in N$.

The next three simulation are introduced to study global agreements when the world is composed of large and medium size countries (i.e. $\alpha = 1$ and $\tilde{\alpha} = 0.5$, respectively) under different degrees of monopsonistic power. The current simulation in particular considers the case when there is not monopsonistic power (i.e. Goyal and Joshi’s world).

Using Table E.39 in Appendix E, it is concluded that when countries are biased in favour of consumers, the sets of strong link deletion proof networks and global treaty proof networks are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*} and Γ = {*g*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *Eq*}, respectively. Therefore the set of global treaty stable networks is $GT = D_S \cap \Gamma$ = {*g*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *Eq*}.

In Section 5.4.3.1, it was found that when the world is composed of large and very small countries, all networks are global treaty stable. The reason is because the very small countries are indifferent about signing a global agreement because they have irrelevant domestic markets. Consequently, a global agreement does not help them to increase consumer surplus. In contrast, when the world is formed of large and medium size countries, the latter have domestic markets that are large enough to offer them gains in consumer surplus from a global agreement. However, there are a number of global treaty stable networks other than global free trade that is explained by centrality. That is networks *g*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u* and *v* in Figure 4.5 are all global treaty stable because they contain at least one country that is fully connected (i.e. they contain at least one central country). Now, because these countries are fully connected, they have already achieved the maximum possible level of competition in their domestic market and, therefore, the maximum possible level of consumer surplus. This is why these central countries are indifferent about signing a global agreement and why networks *g*, *l*, *n*, *o*, *q*, *r*, *s*, *t*, *u* and *v* are global treaty stable in the current simulation.

F.3.3.5 Simulation 15: $\alpha = 1, \tilde{\alpha} = 0.5$ and $\phi = 0.5$ for all $i \in N$.

In this simulation it is assumed a world composed of large and medium size countries with intermediaries that exercise moderate levels of monopsonistic power (i.e. $\phi_i = 0.5$).

Using the information presented in Table E.40 in Appendix E, it is concluded that when countries are biased in favour of consumers, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *a'*, *b'*, *c'*, *Eq*}, Γ = {*g*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*g*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *Eq*}, respectively.

The results revealed that when governments are biased in favour of consumers, network *l* is neither strong link deletion proof nor global treaty proof. This is explained by the incentives of the medium size country *j*. Note that this country is a central country in network *l*. Thus, because this country has already an agreement with all countries of the world, it is indifferent about signing a global agreement as this does not increase the level of competition and, therefore, consumer surplus. This is why network *l* is global treaty proof. However, when monopsonistic power is moderate, a global agreement increases the level of competition in non-central countries negatively affecting the output that is exported by the intermediary of the central country *j* to these countries. This net decrease in output pushed the price paid to the farming sector down. Thus, in response to this lower cost, the intermediary increases the level of output that is sold in the domestic market increasing competition and, therefore, consumer surplus. This is why in the current simulation country *j* has an incentive to sign a global agreement when there is monopsonistic power.

On the other hand, the central country *j* is not willing to break an existing agreement when there is not a farming sector because this lowers competition in the domestic market negatively affecting consumer surplus. However, when there is monopsonistic power this country is willing to break an existing agreement with a large country. This is because breaking this agreement causes a net decrease in the output that is traded by the intermediary of country *j* that pushes the price paid to the farming sector down. This lower price incentives the intermediary to increase the output that is sold in the domestic market positively affecting consumer surplus. This is why country *j* can increase consumer surplus by breaking the agreement with a large country, and this is another example that shows that free trade not always favour consumers.

F.3.3.6 Simulation 16: $\alpha = 1$, $\tilde{\alpha} = 0.5$ and $\phi = 1.5$ for all $i \in N$.

This last simulation in this section studies the global treaty stability of the international trade system when the world is composed of large countries and medium size countries and when monopsonistic power is high (i.e. $\phi = 1.5$).

Using the information presented in Table E.44 in Appendix E it is concluded that when countries are biased in favour of consumers, the sets of strong link deletion proof, global treaty proof, and global treaty networks are D_S = {*a*, *b*, *c*, *d*, *e*, *f*, *j*, *k*, *m*, *n*, *o*, *p*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y*, *z*, *b'*, *c'*, *Eq*}, Γ = {*d*, *f*, *g*, *i*, *k*, *l*, *m*, *n*, *o*, *q*, *r*, *s*, *t*, *u*, *v*, *x*, *c'*, *Eq*}, and $GT = D_S \cap \Gamma$ = {*d*, *f*, *k*, *m*, *n*, *o*, *r*, *s*, *t*, *u*, *v*, *x*, *c'*, *Eq*}, respectively.

In the case of governments biased in favour of consumers, the results revealed that the number of networks included in the set of strong link deletion proof networks decreases when monopsonistic power increases from $\phi = 0.5$ to $\phi = 1.5$ implying that countries have more incentives to break existing agreements. The networks that do not belong to this set when monopsonistic power is high are networks *g*, *h*, *i*, *q* and *a'* in Figure 4.5. The reason of why these networks are not strong link deletion proof is explained by the fact that medium size countries are unwilling to keep their agreements with large countries under high levels of monopsonistic power. Breaking and agreement with a large country reduces the total output that is traded by the intermediary of the medium size country pushing the price paid to the farming sector down. In response to this lower cost, the intermediary increases the level of output that is sold in the domestic market positively affecting consumer surplus. However, when monopsonistic power is moderate, this lower cost effect is not strong enough to offsets the negative effect of lower competition on consumer surplus that arises when the agreement is broken. This is why in the previous simulation networks *g*, *h*, *i*, *q* and *a'* are all strong link deletion proof.

The results also reveal that for the case of governments biased in favour of consumers, the number of networks in the set of global treaty proof increases when monopsonistic power increases from ϕ = 0.5 to ϕ = 1.5. This means that there are more network structures where countries are unwilling to sign a global agreement under high levels of monopsonistic power. These networks are *d*, *f*, *i*, *k*, *l*, *m* and *c'* in Figure 4.5. The reason why these networks become global treaty proof is because they contain medium size countries that are unwilling to sign a global agreement. For example, the medium size countries *j* and *l* are not connected to the large countries. By signing a global agreement, this connection would cause a large increase in export output that would strongly increase the price paid to the farming sector in the medium size countries. In response to this higher cost, the intermediaries of these countries would reduce the output that is sold in the domestic market negatively affecting consumer surplus. Another example is network *l*. In this case the medium size country *j* is a central country that is connected to all countries of the world. This country is unwilling to sign a global agreement because this agreement would increase the level of trade in the non-central countries pushing the price paid to the farming sector up. In response to this higher cost, the intermediaries of the non-central countries would decrease the output that is exported to the central country negatively affecting consumer surplus. This is why country *j* is against a global agreement.

Finally, it was found for the case of governments biased in favour of consumers that the set of global treaty stable networks includes new networks (i.e. *d*, *f*, *k*, *m* and *c'*), but it does not contains network *q*. This is a consequence of the changes discussed above and the main implications of this result are that high levels of monopsonistic power play against free trade, regionalism is a possible outcome, and centrality involving medium size central countries against a global agreement is also a possible outcome.

F.3.4 Global agreements under exogenous tariffs and asymmetry in farmers' productivity

The objective of this section is to extend the analysis to determine whether asymmetry in farmers' productivity (i.e. different levels of monopsonistic power across countries) affects countries' incentives to sign a global agreement. For this purpose, the same simulation developed in Section 4.4.2 is considered in this analysis.

F.3.4.1 Simulation 17: $\delta = 3$ for $\Omega = \{i, k\}$; $\delta = 1$ for $\Psi = \{j, l\}$; $\phi = 0.5$; and $\alpha = 1$.

In considering Table E.48, it was found that the sets of strong link deletion proof and global treaty proof networks when governments are biased in favour of consumers are $D_S = \{a, b, c, d, e, f, g, h, i, j, k, l\ m, n, o, p, q, r, s, t, u, v, w, x, y, z, a', b', c'\}$ and $\Gamma = \{g, l, n, o, q, r, s, t, u, v, x\}$, respectively. This implies that the set of global treaty stable networks in this case is given by $GT = D_S \cap \Gamma = \{g, l, n, o, q, r, s, t, u, v, x\}$.

According to the results obtained in the current simulation, there are no differences between the symmetric (see Sections 5.4.1.2 and 5.4.1.3) and the asymmetric cases when governments are biased in favour of consumers.

Appendix G

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International Trade Networks under Global Treaty Stability

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Abstract *The stability of international trade networks has been investigated using the pairwise stability concept. This concept is suitable to study the formation of bilateral agreements. However, it cannot be used to determine the stability of global trade agreements. This article proposes an alternative stability concept that can be adopted to determine the stability of global agreements such as the Doha agreement. This concept is named in this paper Global Treaty Stability.*

Key words: international trade networks; global treaty stability.

JEL classification: F12 F13

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1. Introduction

The international trade network model that extends the contribution of Jackson and Wolinsky (1996) was introduced independently by Goyal and Joshi (2006) and Furusawa and Konishi (2007). These researchers introduced this approach with the purpose of determining the possible stable trade networks when countries are involved in bilateral agreements. For this purpose, they adopted the pairwise stability concept proposed by Jackson and Wolinsky (2006).

Unfortunately this stability concept cannot be used to study the stability of global trade agreements (e.g. current Doha negotiations). The reason is because this concept assumes that countries can only sign or break one international agreement at time. Nonetheless, at least two conditions are needed to determine stability of a global agreement. Firstly, countries have to be able to sign several links simultaneously in order to form the multilateral agreement. Secondly, this stability demands that no country has an incentive to deviate unilaterally by breaking one or more agreements simultaneously.

In order to see why pairwise stability fails in capturing the outcome of a global agreement, note that according to the original model developed by Goyal and Joshi (2006), global free trade is always pairwise stable. This result implies that if all countries decided to sign an agreement simultaneously, then this agreement would be stable because this particular network is always pairwise stable. Nonetheless, current evidence shows that a global agreement of this nature has been unsuccessful. A good example of this failure is found in the agricultural sector. The first effort to achieve a global agreement in agriculture was the Uruguay Round Agreement in Agriculture (URRA) which concluded in 1993. The URRA is considered as an important achievement because it provided for the first time a foundation for establishing a rule-based world trading system that included both developed and developing countries (Athukorala and Kelegama, 1998; Anderson and Morris, 2000; and Anderson et al., 2001). However, this agreement has been considered unsuccessful because tariffs in agriculture remain high and also because agricultural trade liberalisation post URAA has been modest (Messerlin, 2003; and Gale, 1995). According to Josling (1998), tariffs on manufactured goods in the second half of the 1990s were of the order of 5-10%. In contrast, agricultural tariffs were on average 40% with tariffs picks of over 300% revealing that the URRA did little to liberalise trade in agriculture.

During the second half of the 1990s, the next step in promoting further integration of the agro-food sector into the multilateral trading system was carried out. This was triggered by three main factors: (i) lack of agricultural trade liberalisation post URRA; (ii) export subsidies and domestic support policies still being used by developed countries after this agreement; and (iii) the mandate in Article 20 of the URRA to hold new negotiations (Young et al., 1999; Coleman and Meilke, 2000; and Josling, 2000). These three factors led to new multilateral trade negotiations on agriculture with the purpose of strengthening the disciplines already established under the URRA (Devadoss, 2002). These negotiations were formally included in a round referred to as the Doha Round or the Doha Development Agenda (DDA). The DDA was launched at the World Trade Organisation (WTO)'s Fourth Ministerial Conference in Doha (Qatar) in November 2001, and was planned for conclusion in January 2005 (Matthews, 2001). After twelve years of talks, the Doha Round still has no framework (modalities) deal. In fact, the Geneva Ministerial Meeting in December 2009 ended without any substantial progress (Cho, 2010). The Doha's failure suggests that a global agreement in agriculture might never be attainable, a fact that has been recognised by some researchers (see, for example, Scott and Wilkinson, 2010).

This example illustrates the fact that the model developed by Goyal and Joshi (2006) is not able to capture the outcome of current global trade negotiations. As argued above, the main reason is because these researchers adopted pairwise stability which is not the most appropriate stability concept to analyse the issue of global agreements. The objective of the present article is to propose an alternative stability concept that is suitable to determine the stability of international trade networks when countries are involved in global trade agreements. This concept is called in this paper *Global treaty stability*.

In order to show how this stability concept works, an example based on the international network model of Goyal and Joshi (2006) is provided. According to these researchers, global free trade is stable because no country has an incentive to break a single agreement, a fact that is captured by pairwise stability. In contrast, global free trade is not always global treaty stable because biased countries have an incentive to simultaneously break several agreements. The reason of why this is the case is formally explained in Section 3.

2. Global Treaty Stability

Before defining global treaty stability, it is important to establish the international network model. The following version is based on Goyal and Joshi (2006). In this model, each node stands for a country with a single firm producing a home-based commodity. A network is a set of undirected binary links between these nodes/countries representing bilateral free trade treaties. Formally, an international agreement between countries i and j is described by a link, given by a binary variable $g_{ij} \in \{0,1\}$. If $g_{ij} = 0$, then no agreement exist between countries i and j. If $g_{ij} = 1$, then an agreement exists between them. A network $g \in \{(g_{ij})_{ij \in \mathrm{N}}\}$ is a description of the international agreements that exist between the countries in N, where N = {1, 2,..., N} is the set of identical countries, and N is the total number of countries. Network g^c is the complete network ($g_{ij} = 1 \ \forall \ i,j \in$ N) and corresponds to multilateral free trade (i.e. all countries have an agreement with each other), and Network g^e is the empty network ($g_{ij} = 0 \ \forall \ i,j \in$ N) and corresponds to the network in which all countries are in unattached. Let G denote the set of all possible networks of international agreements between countries. Let $N_i(g) = \{j \in \mathrm{N} : g_{ij} = 1\}$ be the set of countries with whom country i has an international trade agreement in network g. Assume that $i \in N_i(g)$. That is, $g_{ii} = 1$. The cardinality of $N_i(g)$ is denoted by $\eta_i(g)$. As described above, in this model $\eta_i(g)$ is also the number of active firms in country i and in network g because of the assumption that each country has a single firm producing a home-based commodity. Let $L_i(g) = \{(g_{ij})_{ij \in \mathrm{N}} : j \in N_i(g)\}$ be the set of links existing in country i in network g. Note that $g_{ii} \in L_i(g)$. Let $h_i \subset L_i(g) - \{g_{ii}\}$ be a link subset of the links existing in country i. Finally, let $W_i(g)$, $CS_i(g)$ and $\pi_i(g)$ be welfare, consumer surplus, and total profit, respectively, in country i and in network g. In this setting governments maximise the following welfare function:

$$W_i(g) = a_i CS_i(g) + b_i \pi_i(g) \qquad (1)$$

Where $a_i \geq 0$ and $b_i \geq 0$ represent weights that the government puts on consumer surplus and profits, respectively.

Let us now explain the global treaty stability concept. Gilles and Sarangi (2010), Gilles et al. (2012), and Chakrabarti and Gilles (2007) proposed a refinement of the pairwise stability concept introduced Jackson and Wolinsky (1996). This refinement is referred to as *strongly pairwise stability* and is based on the following concepts: (a) the marginal benefit of country i when deleting at the same time $h_i \subset L_i(g) - \{g_{ii}\}$ international agreements is: $D_i(g, h_i) = W_i(g) - W_i(g - h_i)$; (b) a network $g \in G$ is *strong link deletion proof* if for every player $i \in$ N and every $h_i \subset L_i(g) - \{g_{ii}\}$ it holds that $D_i(g, h_i) \geq 0$; and (c) a network $g \in G$ is *link addition proof* if for all $i,j \in$ N: $W_i(g + g_{ij}) >$

$W_i(g)$ implies that $W_j(g + g_{ij}) < W_j(g)$. A network $g \in G$ is *strongly pairwise stable* if g is strong link deletion proof as well as link addition proof.

The global treaty stability proposed in this article is a refinement of strongly pairwise stability that replaces the link addition proof condition by an alternative condition that has been named *global treaty proofness.* This is explained as follows. Let the marginal benefit of country i when forming a global agreement be $\Omega_i(g^c) = W_i(g^c) - W_i(g)$. A network $g \in G$ is *global treaty proofness* if for at least one country $i \in$ N it holds that $\Omega_i(g^c) \leq 0$. In words, a network $g \in G$ is *global treaty proofness* if at least one country $i \in$ N does not have an incentive to form a global agreement. Using this definition, a network $g \in G$ is said to be *global treaty stable* if g is strong link deletion proof as well as global treaty proofness. That is, network g is global treaty stable if: (i) no country has an incentive to break one or more international agreements; and (ii) at least one country is not willing to form a global trade agreement.

3. Examples

This section provides some examples showing that the use of global treaty stability in Goyal and Joshi's model generates results that are not the same than the ones identified by these researchers.

The first subsection considers examples assuming the case of exogenous tariffs (i.e. each country establishes a prohibitive tariff avoiding any trade between them. If two countries decide to sign an agreement, then each one offers the other a free market access). The reason for assuming exogenous tariffs is because most of the analysis developed by Goyal and Joshi (2006) was made under this assumption as the model becomes untractable in mathematic terms when adopting endogenous tariffs (this has formally pointed out by Goyal and Joshi, 2006, 768). Consequently, the main differences between global treaty stability and pairwise stability when considering the work of these researchers are better understood under the assumption of exogenous tariffs.

In recognising the relevance of endogenous tariffs and given the mathematical complexity of the model when adopting this assumption, the second subsection provides some examples based on simulations to identify differences between global treaty stability and pairwise stability under endogenous tariffs.

3.1. Examples under Exogenous Tariffs

Example 1

In order to show a concrete application of the global treaty stability concept, let us consider the following example based on the model of Goyal and Joshi (2006). In particular, this example is based on the case of exogenous tariffs and unbiased governments which means that governments put the same weight on the components of the welfare function, namely, consumer surplus and firms' profits. Using pairwise stability, Goyal and Joshi found that in this case two networks are pairwise stable: Global free trade; and a network composed of a complete component (i.e. a set of countries having an agreement with each other) and a singleton (i.e. an unattached country). The aim of this example is to show that among these two equilibriums, only global free trade is global treaty stable. As it will be shown, the reason is because the singleton is unwilling to sign a single agreement with a country of the complete component. This is because the gain it makes from accessing the new market is lower than the loss arising from the decrease in market power in the domestic market as a result of shearing this market with the new partner country. In contrast, the singleton is willing to sign a global agreement because in this case the gain from simultaneously accessing several foreign markets is larger than the loss associate with the lower market power in the domestic market after the global agreement.

Formally, assume that tariffs are determined exogenously and that governments are politically unbiased (i.e. a_i = b_i = 1 in Equation 1). According to Goyal and Joshi's results, two networks are pairwise stable in this case: (i) the complete network (i.e. g^c); and (ii) a network composed of one complete component of size $N - 1$ and a singleton (i.e. g^{N-1}). In order to show that the latter network is not global treaty stable, the following expressions were adopted (see Goyal and Joshi, 2006, 755). Note that it was assumed for simplicity and without losing generality that $\alpha - \gamma = 1$[1].

$$CS_i(g) = \frac{1}{2}\frac{\eta_i^2}{(\eta_i + 1)^2} \quad (2)$$

$$\pi_i(g) = \sum_{k \in N_i(g)} \frac{1}{(\eta_k + 1)^2} \quad (3)$$

Proposition 1: *The following statements hold in the case of unbiased governments and exogenous tariffs:(a) The complete network g^c is the unique network that is pairwise stable as well as global treaty stable; and (b) there exist global treaty stable networks that are not pairwise stable.*

Proof: Let us first prove statement (a). For this purpose, it will be proved that the following networks cannot be global treaty stable: (i) the empty network; (ii) a network formed of one or more complete components with one or more singletons; (iii) and a network formed of complete components without singletons. Firstly, note that the strong link deletion proof condition (i.e. $D_i(g, h_i) = W_i(g) - W_i(g - h_i) \geq 0$) is satisfied in all the countries that belong to these networks. In the case

[1] In Goyal and Joshi's model, the inverse demand for the homogeneous commodity in a determined country i is given by $P_i = \alpha - Q_i$, where P_i denotes the price of the commodity, α is a parameter representing market size, and Q_i is the total output demanded in that country. On the other hand, the parameter γ represents the marginal cost faced by the firms producing the commodity. The expression $\alpha - \gamma$ appears in the equations that result from the solution of the Cournot game adopted by Goyal and Joshi (2006).

of singleton countries, this is verified because for definition they cannot delete links as a consequence of being singletons (i.e. they do not have an incentive to delete inexistent links). Let us consider now the case of a country *i* that belongs to a complete component of size η. Using expressions 2 and 3, welfare in this country is given by $W_i(g)=\frac{1}{2}\frac{\eta^2}{(\eta+1)^2}+\frac{\eta}{(\eta+1)^2}$. Likewise, welfare in this country when the government deletes h_i links is given by $W_i(g-h_i)=\frac{1}{2}\frac{\zeta^2}{(\zeta+1)^2}+\frac{1}{(\zeta+1)^2}+\frac{\zeta-1}{(\eta+1)^2}$, where $\zeta < \eta$ is the number of agreements in country *i* in network $g - h_i$. Simple calculations shows that $W_i(g)>W_i(g-h_i)$ when $(\eta - \zeta)(2\zeta - 1) + (\eta - \zeta)(4\zeta - 2) > 0$ which is valid for all $\eta > \zeta$. This result implies, therefore, that no country in the networks described above has an incentive to break one or more existing agreements simultaneously. Secondly, the global treaty proofness condition (i.e. $\Omega_i(g^c) = W_i(g^c) - W_i(g) < 0$) is not satisfied in any of the countries that belong to these networks. To see why, note that welfare in an arbitrary country *i* in g^c is given by $W_i(g^c)=\frac{1}{2}\frac{N^2}{(N+1)^2}+\frac{N}{(N+1)^2}$; welfare in a singleton is given by $W_i(g')=\frac{3}{8}$; and welfare in a country that belongs to a complete component is given by $W_i(g'')=\frac{1}{2}\frac{\eta^2}{(\eta+1)^2}+\frac{\eta}{(\eta+1)^2}$. Because $W_i(g^c)-W_i(g')>0$ implies $N^2 + 2N - 3 > 0$ which is valid for all $N > 1$, and because $W_i(g^c)-W_i(g'')>0$ implies $N > \eta$ which is valid for definition, it is concluded that all countries in the networks described above have an incentive to form a global agreement. As a consequence, these networks are not global treaty stable. The main implication of this finding is that if the current network is consistent with any of the ones described above, then countries will sign a global agreement. Because the pairwise stable network g^{N-1} identified by Goyal and Joshi (2006) is actually one of these networks, it is concluded that this network is not global treaty stable. In contrast, the complete network is global treaty stable because this is a complete component. As shown in the proof, the strong link deletion proof condition is always satisfied when countries belong to complete components.

Let us now prove statement (b). We know from the results obtained above that networks composed of complete components cannot be global treaty stable. However, there are networks composed of incomplete components that can be. An example is a network composed of at least one incomplete star component (i.e. a component in which a country has a central position; it has a links with all the countries of the component; and the latter do not have an agreement with each other). Figure 1 shows an example of a network of this nature. This network is referred to as network *g*.

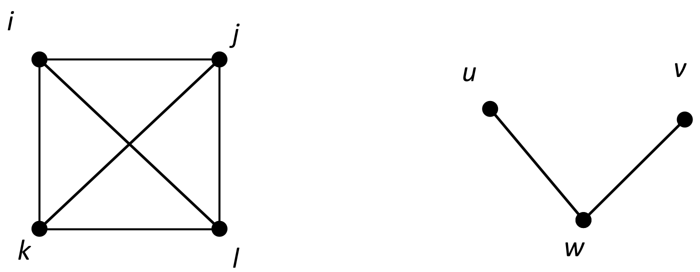

Figure 1: Network *g* composed of a complete component and an incomplete star component.

We know from the previous results that countries in the complete component do not have an incentive to break one or more existing agreements simultaneously. We also know that these countries are willing to sign a global agreement. Let us now analyse the motivation of the countries that belong to the incomplete component. For this purpose, consider the following information obtained from Expressions 2 and 3:

(i) $W_u(g) = W_v(g) = \frac{57}{144}$

(ii) $W_u(g - g_{uw}) = W_v(g - g_{vw}) = \frac{3}{8}$

(iii) $W_w(g) = \frac{163}{288}$

(iv) $W_w(g - g_{uw}) = W_w(g - g_{vw}) = \frac{4}{9}$

(v) $W_w(g - g_{uw} - g_{vw}) = \frac{3}{8}$

(vi) $W_u(g^c) = W_v(g^c) = W_w(g^c) = \frac{1}{2}\frac{N^2}{(N+1)^2} + \frac{N}{(N+1)^2}$

Because $W_u(g) = W_v(g) > W_u(g - g_{uw}) = W_v(g - g_{vw})$, it is concluded that countries *u* and *v* do not have an incentive to break their existing agreements with country *w*. Likewise, because $W_w(g) > W_w(g - g_{uw}) = W_w(g - g_{vw})$ and $W_w(g)$ > $W_w(g - g_{uw} - g_{vw})$, it is concluded that country *w* does not have an incentive to break one or more of their agreements simultaneously. It is concluded from this analysis that the strong link deletion proof condition is satisfied in this network. Let us now consider the global treaty proofness condition. According to this condition, a network *g* is global treaty stable if at least one country is not willing to form a global trade agreement. This is actually verified in country *w.* To see why, note that $W_w(g) > W_w(g^c)$ when $\frac{163}{288} > \frac{1}{2}\frac{N^2}{(N+1)^2} + \frac{N}{(N+1)^2}$. By rearranging terms, $19N^2 + 38N + 163 > 0$ which holds for all $N > 0$. □

This simple example illustrates the advantage of using the global treaty stability concept to study the stability of international networks when countries are involved in global agreements. That is, not all pairwise stable networks are global treaty stable networks; and not all global treaty stable networks are pairwise stable networks. As a consequence, pairwise stability fails in informing the possible outcomes of global international trade agreements.

Example 2

According to Proposition 5 in Goyal and Joshi (2006), the following networks are pairwise stable when governments are fully biased in favour of their domestic firms (i.e. $a_i = 0$ and $b_i = 1$ in Equation 1): the empty network; the complete network; networks composed of complete component if different size; and networks composed of complete components of different size with one or more singletons. The following proposition shows that among these networks, only the empty network is global treaty stable.

Proposition 2: *Among the pairwise stable networks identified by Goyal and Joshi (2006) in their Proposition 5, only the empty network is global treaty stable.*

Proof: The proof consists of showing that the strong link deletion proof condition does not hold when countries are organised in complete components. Using expression 3, welfare in an arbitrary country *i* that belongs to a complete component of size η is given by $W_i(g) = \eta/(\eta + 1)^2$. Let network g^* be the network in which country *i* is unattached. Because $W_i(g^*) = 1/4 > W_i(g) = \eta/(\eta + 1)^2$ for all $\eta > 1$, it is concluded that any country that belongs to a complete component (including the complete network) has an incentive to break all its existing agreements simultaneously. This means that the only pairwise stable network identified by Goyal and Joshi (2006) that is strong link deletion proof for the case of biased countries in favour of their domestic firms is the empty network. From the same analysis it is concluded that the global treaty proofness condition holds in this network. □

This result can better be understood by considering the trade effects arising when breaking one or more agreements simultaneously. When a country breaks one or more agreements, the profit that the domestic firm in this country makes in the domestic market increases because this market becomes less competitive (we call this change in profits the competition effect). However, this firm also loses the profits that it made in the ex-partner countries (we call this change in profits the expansion effect). Consequently, if the competition effect dominates the expansion effect, then the government has an incentive to break one or more agreements simultaneously. This is the key aspect that explains why pairwise stability differs from global treaty stability. In Goyal and Joshi's model global free trade is stable because countries are allowed to break only one agreement at time, and it is in this particular case when the competition effect is dominated by the expansion effect. However, if countries are allowed to break several links simultaneously, then the dominance of these effects is reversed. This is shown in the following figure:

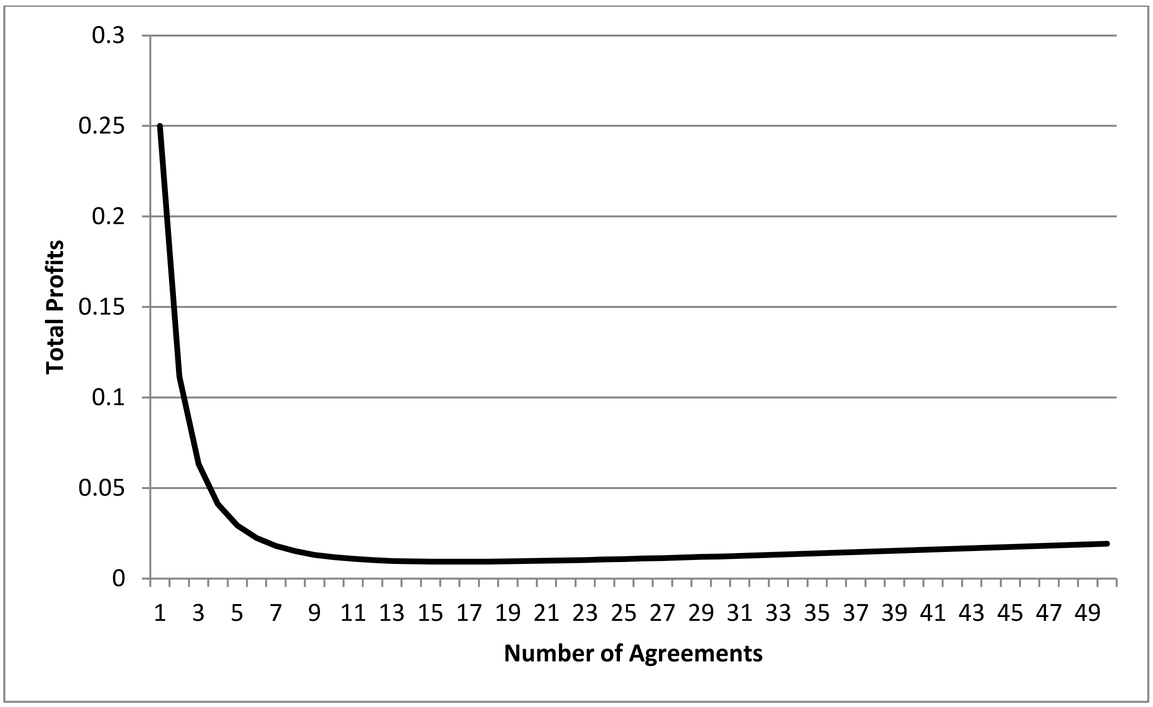

Figure 2: Total profit made by the domestic firm when the government deviates from global free trade.

This figure shows a simulation assuming the existence of 50 countries in the world. According to this figure, total profit made by the domestic firm of a determined country decreases as this country signs additional agreements. This trend is valid only until the country reaches about 15 agreements. After that, total profit increases as the country signs further agreements. As a result, if this country is in global free trade, then breaking a single link (i.e. passing from 50 to 49 agreements in the figure) decreases total profits making this change inappropriate if the country is only allowed to break one link. As explained above, this is why global free trade is pairwise stable. In contrast, if the country is allowed to break several agreements simultaneously, then the domestic firm can make higher profits in a less integrated network than in global free trade. This is why global free trade is not global treaty stable.

3.2. Examples under Endogenous Tariffs

As explained above, Goyal and Joshi's model becomes untractable in mathematic terms when assuming endogenous tariffs. It is for this reason that simulations assuming the existence of four countries in the world were adopted to overcome this problem to some extent. For a detailed explanation of the equations used in the simulations, please refer to Goyal and Joshi (2006, 765).

The simulations consider the set of possible networks that can be formed by countries *i*, *j*, *k* and *l*. These networks are shown in the following figure:

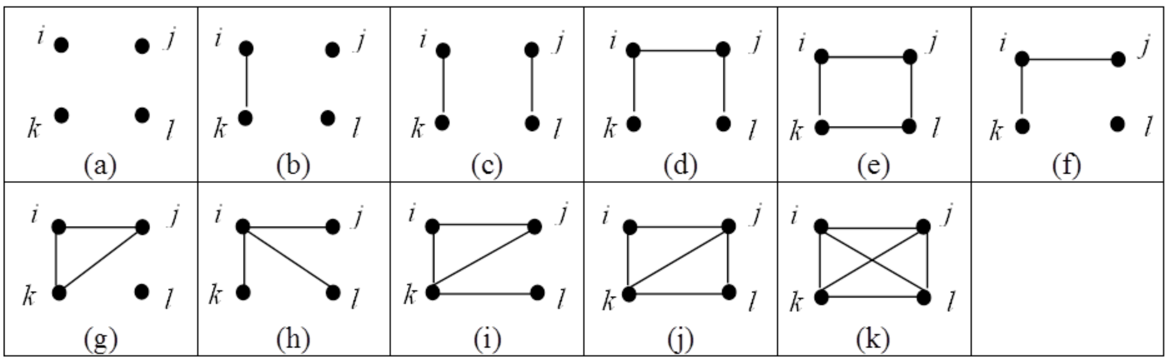

Figure 3: Possible network architectures formed by countries *i*, *j*, *k* and *l*.

Note that several possible network architectures were omitted. For example, country l in network *g* in this figure is a singleton. Similar network architectures could have been introduced in order to represent the cases when the other countries are singleton. However, information about these network topologies can be inferred from network *g* as a result of the assumption of symmetrical countries.

Using these network architectures, a simulation was developed assuming unbiased governments. The simulation is presented in Table 1.

Table 1. Welfare under the assumption of unbiased governments.

Network	**Country**				**Total**
	i	***j***	***k***	***l***	
a	0.4339	0.4339	0.4339	0.4339	1.7355
b	0.4540	0.4486	0.4540	0.4486	1.8053
c	0.4688	0.4688	0.4688	0.4688	1.8750
d	0.4897	0.4897	0.4556	0.4556	1.8908
e	0.4766	0.4766	0.4766	0.4766	1.9065
f	0.4981	0.4456	0.4456	0.4586	1.8480
g	0.4666	0.4666	0.4666	0.4639	1.8638
h	0.5475	0.4463	0.4463	0.4463	1.8863
i	0.4672	0.4672	0.5160	0.4515	1.9020
j	0.4631	0.4935	0.4935	0.4631	1.9133
k	0.4800	0.4800	0.4800	0.4800	1.9200

Using this simulation it is possible to infer that networks *a*, *b*, *c*, *e* and *g* are not global treaty stable because all countries in these networks have an incentive to form a global agreement (i.e. network *k*). Network *d* is not global treaty stable because country *l* has an incentive to break its agreement with country *j* (i.e. passing from network *d* to network *f*). Network *f* is not global treaty stable because country *j* has an incentive to break its agreement with country *i* (i.e. passing from network *f* to network *b*). Network *h* is not global treaty stable because country *l* has an incentive to break its agreement with country *i* (i.e. passing from network *h* to network *f*). Network *i* is not global treaty stable because country *l* has an incentive to break its agreement with country *k* (i.e. passing from network *i* to network *g*). Network *j* is not global treaty stable because country *l* has an incentive to break its agreements with countries *k* and *j* simultaneously (i.e. passing from network *j* to network *g*). In this simulation, the only global treaty stable network is network *k*, that is, global free trade.

It is interesting to note that two networks in this simulation are pairwise stable: networks *g* and *k*. However, only network *k* is also global treaty stable. This finding is consistent with the analysis conducted for the case of exogenous tariffs in that global treaty stability provides results that cannot been identified from pairwise stability. This confirms the fact that the analysis of global trade agreements cannot be conducted using pairwise stability.

The following table shows a second simulation that was developed with the purpose of proving that global free trade is not necessarily global treaty stable when governments are politically biased in favour of domestic firms. This simulation assumes that governments place the following weights on consumer surplus, profits and tariffs revenue, respectively, in Equation 27 of Goyal and Joshi (2006, 766): 0.75;1; and 1.

Table 2. Welfare under the assumption of biased governments.

Network	**Country**				**Total**
	i	***j***	***k***	***l***	
a	0.3739	0.3739	0.3739	0.3739	1.4957
b	0.3847	0.3866	0.3847	0.3866	1.5425
c	0.3973	0.3973	0.3973	0.3973	1.5893
d	0.4184	0.4184	0.3786	0.3786	1.5941
e	0.3997	0.3997	0.3997	0.3997	1.5988
f	0.4334	0.3697	0.3697	0.3955	1.5683
g	0.3908	0.3908	0.3908	0.4008	1.5731
h	0.4912	0.3669	0.3669	0.3669	1.5920
i	0.3881	0.3881	0.4485	0.3722	1.5968
j	0.3816	0.4181	0.4181	0.3816	1.5994
k	0.4000	0.4000	0.4000	0.4000	1.6000

It is not difficult to see in this simulation that the only global treaty stable network is network *g*. To see why, note that countries in networks *a*, *b*, *c* and *e* have an incentive to form a global agreement because welfare in network *k* is higher. This implies that networks *a*, *b*, *c* and *e* are not global treaty stable. On the other hand, network *k* is not global treaty stable because country *l* has an incentive to break all its agreements simultaneously (i.e. passing from network *k* to network *g*). The same holds for networks *j* and *i*: country *l* is better off in network *g*. Network *h* is not global treaty stable because country *l* has an incentive to break its agreement with country *i* (i.e. passing from network *h* to network *f*). Network *f* is not global treaty stable because country *j* has an incentive to break its agreement with country *i* (i.e. passing from network *f* to network *b*). Finally, network *d* is not global treaty stable because country *l* has an incentive to break its agreement with country *j* (i.e. passing from network *d* to network *f*).

The main implication of this simulation is that global free trade may not be reached when governments are involved in global agreements and when they are biased in favour of their domestic firms. This is consistent with the result obtained under the assumption of exogenous tariffs. This result, again, suggests that pairwise stability is not the best stability concept to study the issue of global agreements. To reinforce this argument, note that in this simulation global free trade is pairwise stable. This means that in the world of Goyal and Joshi, if countries were allowed to sign a global free agreement, then such an agreement would be feasible given the pairwise stability of this network. But as proved in this article, countries have an incentive to break several agreements simultaneously. As a consequence, results obtained under pairwise stability should be considered with caution.

5. Conclusions

The objective of this article is to propose a stability concept referred to as global treaty stability to study the stability of international networks when countries are involved in global agreements. Simple examples were used to show how this stability concept can be applied to identify stable networks when countries are involved in this type of agreements. Using these examples, two key results were found. Firstly, global free trade is not always global treaty stable. Secondly, pairwise stability is not able to identify some global treaty stable networks. This proves that fact that the proposed stability concept is more appropriate to study the formation of international network when countries are involved in global trade agreements.

It would be interesting to use this stability concept to determine the possible global treaty stable networks under different policy biases. Likewise, it would be interesting to use this concept to predict the outcome of the Doha agreement. These relevant extensions are left for future research.

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