Supplementary Information

Reliability, risk and resilience relationships in an integrated urban wastewater system under the Safe & SuRe framework

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RISK AND RESILIENCE FORMULATIONS

All possible formulations of risk and resilience in the Safe & SuRe framework (Butler et al. 2016) are given in Table S1 and Table S2. Where multiple risk descriptions are given for a single formulation (for example S2), each risk value will be different since different probabilities are used; however no additional information will be gained from calculation of multiple risk values under one formulation as all are based on the same cause of failure. Different resilience descriptions under a single formulation are all equivalent since calculation of resilience is not dependent on the probability of the event considered to cause the failure.

Table S1: Risk and resilience formulations in the Safe & SuRe framework with specified (known) cause of failure; grey shading represents incalculable formulations.

Formulation	Failure / non-failure state measurement	Causal event	Risk		Resilience	
			Description(s) and equation(s)	**Unknowns**	**Description(s) and equation(s)**	**Unknowns**
S1	All system components	Specified (known) threat	• Risk to system from specified threat, $Risk_{S1} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{S_K} \vert T_{K,w})$	None	• Resilience of system to specified threat, $Res_{S1} = g\left(\mathbf{mag}(\boldsymbol{S_K} \vert T_{K,w}), \mathbf{dur}(\boldsymbol{S_K} \vert T_{K,w})\right)$	None
S2	All impacts	Specified (known) threat	• Risk to level of service from specified threat, $Risk_{S2a} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{I_K} \vert T_{K,w})$ • Risk to level of service from any system failure, given a specified threat, $Risk_{S2b} = \mathbf{P}(\boldsymbol{S_K} \vert T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{I_K} \vert T_{K,w})$	None	• Resilience of level of service provision to specified threat, • Resilience of level of service provision to any system failure which results from specified threat, $Res_{S2} = g\left(\mathbf{mag}(\boldsymbol{I_K} \vert T_{K,w}), \mathbf{dur}(\boldsymbol{I_K} \vert T_{K,w})\right)$	None
S3	All known consequences	Specified (known) threat	• Risk to SEE from specified threat, $Risk_{S3a} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{C_K} \vert T_{K,w}, \boldsymbol{C_U} \vert T_{K,w})$ • Risk to SEE from any system failure, given a specified threat, $Risk_{S3b} = \mathbf{P}(\boldsymbol{S_K} \vert T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{C_K} \vert T_{K,w}, \boldsymbol{C_U} \vert T_{K,w})$ • Risk to SEE from any level of service failure, given a specified threat, $Risk_{S3c} = \mathbf{P}(\boldsymbol{I_K} \vert T_{K,w}) \cdot \mathbf{mag}(\boldsymbol{C_K} \vert T_{K,w}, \boldsymbol{C_U} \vert T_{K,w})$	$\mathbf{mag}(\boldsymbol{C_U} \vert T_{K,w})$	• Resilience of SEE to specified threat, • Resilience of SEE to any system failure which results from specified threat, • Resilience of SEE to any level of service failure which results from specified threat $Res_{S3} = g\left(\mathbf{mag}(\boldsymbol{C_K} \vert T_{K,w}), \mathbf{dur}(\boldsymbol{C_K} \vert T_{K,w}), \mathbf{mag}(\boldsymbol{C_U} \vert T_{K,w}), \mathbf{dur}(\boldsymbol{C_U} \vert T_{K,w})\right)$	$\mathbf{mag}(\boldsymbol{C_U} \vert T_{K,w})$, $\mathbf{dur}(\boldsymbol{C_U} \vert T_{K,w})$

S4	All impacts	Specified system failure	• Risk to level of service from specified system failure, $Risk_{S4} = \mathbf{P}(S_{K,x}) \cdot \mathbf{mag}(\boldsymbol{I_K}\vert S_{K,x})$	$\mathbf{P}(S_{K,x})$	• Resilience of level of service provision to specified system failure, $Res_{S4} = g\left(\mathbf{mag}(\boldsymbol{I_K}\vert S_{K,x}), \mathbf{dur}(\boldsymbol{I_K}\vert S_{K,x})\right)$	None
S5	All known consequences	Specified system failure	• Risk to SEE from specified system failure, $Risk_{S5a} = \mathbf{P}(S_{K,x}) \cdot \mathbf{mag}(\boldsymbol{C_K}\vert S_{K,x}, \boldsymbol{C_U}\vert S_{K,x})$ • Risk to SEE from any level of service failure, given specified system failure, $Risk_{S5b} = \mathbf{P}(\boldsymbol{I_K}\vert S_{K,x}) \cdot \mathbf{mag}(\boldsymbol{C_K}\vert S_{K,x}, \boldsymbol{C_U}\vert S_{K,x})$	$\mathbf{P}(S_{K,x})$ or $\mathbf{P}(\boldsymbol{I_K}\vert S_{K,x})$, $\mathbf{mag}(\boldsymbol{C_U}\vert S_{K,x})$	• Resilience of SEE to specified system failure, $Res_{S5} = g\left(\mathbf{mag}(\boldsymbol{C_K}\vert S_{K,x}), \mathbf{dur}(\boldsymbol{C_K}\vert S_{K,x}), \mathbf{mag}(\boldsymbol{C_U}\vert S_{K,x}), \mathbf{dur}(\boldsymbol{C_U}\vert S_{K,x})\right)$	$\mathbf{mag}(\boldsymbol{C_U}\vert S_{K,x})$, $\mathbf{dur}(\boldsymbol{C_U}\vert S_{K,x})$
S6	All known consequences	Specified level of service failure	• Risk to SEE from specified level of service failure, $Risk_{S6} = \mathbf{P}(I_{K,y}) \cdot \mathbf{mag}(\boldsymbol{C_K}\vert I_{K,y}, \boldsymbol{C_U}\vert I_{K,y})$	$\mathbf{P}(I_{K,y})$, $\mathbf{mag}(\boldsymbol{C_U}\vert I_{K,y})$	• Resilience of SEE to specified level of service failure, $Res_{S6} = g\left(\mathbf{mag}(\boldsymbol{C_K}\vert I_{K,y}), \mathbf{dur}(\boldsymbol{C_K}\vert I_{K,y}), \mathbf{mag}(\boldsymbol{C_U}\vert I_{K,y}), \mathbf{dur}(\boldsymbol{C_U}\vert I_{K,y})\right)$	$\mathbf{mag}(\boldsymbol{C_U}\vert I_{K,y})$, $\mathbf{dur}(\boldsymbol{C_U}\vert I_{K,y})$
S7	Specified system component(s)	Specified (known) threat	• Risk to specified system from specified threat, $Risk_{S7} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(S_{K,x}\vert T_{K,w})$	None	• Resilience of specified system to specified threat, $Res_{S1} = g\left(\mathbf{mag}(S_{K,x}\vert T_{K,w}), \mathbf{dur}(S_{K,x}\vert T_{K,w})\right)$	None
S8	Specified impact(s)	Specified (known) threat	• Risk to specified level of service from specified threat, $Risk_{S8a} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(I_{K,y}\vert T_{K,w})$ • Risk to specified level of service from any system failure, given a specified threat, $Risk_{S8b} = \mathbf{P}(\boldsymbol{S_K}\vert T_{K,w}) \cdot \mathbf{mag}(I_{K,y}\vert T_{K,w})$	None	• Resilience of specified level of service provision to specified threat, • Resilience of specified level of service provision to any system failure which results from specified threat, $Res_{S8} = g\left(\mathbf{mag}(I_{K,y}\vert T_{K,w}), \mathbf{dur}(I_{K,y}\vert T_{K,w})\right)$	None
S9	Specified (known) consequence(s)	Specified (known) threat	• Risk to specified SEE component from specified threat, $Risk_{S9a} = \mathbf{P}(T_{K,w}) \cdot \mathbf{mag}(C_{K,x}\vert T_{K,w})$ • Risk to specified SEE component from any system failure, given a specified	None	• Resilience of specified SEE component to specified threat, • Resilience of specified SEE component to any system failure which results from specified threat,	None

			threat, $Risk_{S9b} = \mathbf{P}(\boldsymbol{S_K} \vert T_{K,w}) \cdot \mathbf{mag}(C_{K,z} \vert T_{K,w})$ • Risk to specified SEE component from any level of service failure, given a specified threat, $Risk_{S9} = \mathbf{P}(\boldsymbol{I_K} \vert T_{K,w}) \cdot \mathbf{mag}(C_{K,z} \vert T_{K,w})$		• Resilience of specified SEE component to any level of service failure which results from specified threat • $Res_{S9} = g\left(\mathbf{mag}(C_{K,z} \vert T_{K,w}), \mathbf{dur}(C_{K,z} \vert T_{K,w})\right)$	
S10	Specified impact(s)	Specified system failure	• Risk to specified level of service from specified system failure, $Risk_{S10} = \mathbf{P}(S_{K,x}) \cdot \mathbf{mag}(I_{K,y} \vert S_{K,x})$	$\mathbf{P}(S_{K,x})$	• Resilience of level of service provision to specified system failure, • $Res_{S10} = g\left(\mathbf{mag}(I_{K,y} \vert S_{K,x}), \mathbf{dur}(I_{K,y} \vert S_{K,x})\right)$	None
S11	Specified (known) consequence(s)	Specified system failure	• Risk to specified SEE component from specified system failure, $Risk_{S11a} = \mathbf{P}(S_{K,x}) \cdot \mathbf{mag}(C_{K,z} \vert S_{K,x})$ • Risk to specified SEE component from any level of service failure, given specified system failure, $Risk_{S11b} = \mathbf{P}(\boldsymbol{I_K} \vert S_{K,x}) \cdot \mathbf{mag}(C_{K,z} \vert S_{K,x})$	$\mathbf{P}(S_{K,x})$ or $\mathbf{P}(\boldsymbol{I_K} \vert S_{K,x})$	• Resilience of specified SEE component to specified system failure, • Resilience of specified SEE component to any level of service failure which results from specified system failure, • $Res_{S11} = g\left(\mathbf{mag}(C_{K,z} \vert S_{K,x}), \mathbf{dur}(C_{K,z} \vert S_{K,x})\right)$	None
S12	Specified (known) consequence(s)	Specified level of service failure	• Risk to specified SEE component from specified level of service failure, $Risk_{S6} = \mathbf{P}(I_{K,y}) \cdot \mathbf{mag}(C_{K,z} \vert I_{K,y})$	$\mathbf{P}(I_{K,y})$	• Resilience of specified SEE component to specified level of service failure, • $Res_{S6} = g\left(\mathbf{mag}(C_{K,z} \vert I_{K,y}), \mathbf{dur}(C_{K,z} \vert I_{K,y})\right)$	None

Table S2: Risk and resilience formulations in the Safe & SuRe framework with any cause of failure; grey shading represents incalculable formulations.

Formulation	Failure / non-failure state measurement	Causal event	Risk		Resilience	
			Description(s) and equation(s)	Unknowns	Description(s) and equation(s)	Unknowns
G1	All system components	Any threat	• Risk to system from any threat, $Risk_{G1} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}(\boldsymbol{S_K})$	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$, $\mathbf{mag}(\boldsymbol{S_K})$	• Resilience of system, • Resilience of system to any threat, • $Res_{G1} = g(\mathbf{mag}(\boldsymbol{S_K}\vert\boldsymbol{T_K}, \boldsymbol{S_K}\vert\boldsymbol{T_U}), \mathbf{dur}(\boldsymbol{S_K}\vert\boldsymbol{T_K}, \boldsymbol{S_K}\vert\boldsymbol{T_U}))$	$\mathbf{mag}(\boldsymbol{S_K}\vert\boldsymbol{T_U})$, $\mathbf{dur}(\boldsymbol{S_K}\vert\boldsymbol{T_U})$
G2	All impacts	Any threat or system failure	• Risk to level of service from any threat, $Risk_{G2a} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}(\boldsymbol{I_K})$ • Risk to level of service from any infrastructure failure, $Risk_{G2b} = \mathbf{P}(\boldsymbol{S_K}) \cdot \mathbf{mag}(\boldsymbol{I_K})$	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$ or $\mathbf{P}(\boldsymbol{S_K})$, $\mathbf{mag}(\boldsymbol{I_K})$	• Resilience of level of service, • Resilience of level of service to any threat, • Resilience of level of service to any system failure, • $Res_{G2} = g(\mathbf{mag}(\boldsymbol{I_K}\vert\boldsymbol{S_K}), \mathbf{dur}(\boldsymbol{I_K}\vert\boldsymbol{S_K}))$	
G3	All known consequences	Any threat, system failure or level of service failure	• Risk to SEE from any threat, $Risk_{G3a} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}(\boldsymbol{C_K}, \boldsymbol{C_U})$ • Risk to SEE from any system failure, $Risk_{G3b} = \mathbf{P}(\boldsymbol{S_K}) \cdot \mathbf{mag}(\boldsymbol{C_K}, \boldsymbol{C_U})$ • Risk to SEE from any level of service failure, $Risk_{G3c} = \mathbf{P}(\boldsymbol{I_K}) \cdot \mathbf{mag}(\boldsymbol{C_K}, \boldsymbol{C_U})$	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$, $\mathbf{P}(\boldsymbol{S_K})$ or $\mathbf{P}(\boldsymbol{I_K})$, $\mathbf{mag}(\boldsymbol{C_K}, \boldsymbol{C_U})$	• Resilience of SEE, • Resilience of SEE to any threat, • Resilience of SEE to any system failure, • Resilience of SEE to any level of service failure, • $Res_{G3} = g\big(\mathbf{mag}(\boldsymbol{C_K}\vert\boldsymbol{I_K}, \boldsymbol{C_U}\vert\boldsymbol{I_K}), \mathbf{dur}(\boldsymbol{C_K}\vert\boldsymbol{I_K}, \boldsymbol{C_U}\vert\boldsymbol{I_K})\big)$	$\mathbf{mag}(\boldsymbol{C_U}\vert\boldsymbol{I_K})$, $\mathbf{dur}(\boldsymbol{C_U}\vert\boldsymbol{I_K})$
G4	Specified system component(s)	Any threat	• Risk to specified system component from any threat, $Risk_{G4} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}(S_{K,x})$	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$, $\mathbf{mag}(S_{K,x})$	• Resilience of specified system component, • Resilience of specified system component to any threat, • $Res_{G4} = g\big(\mathbf{mag}(S_{K,x}\vert\boldsymbol{T_K}, S_{K,x}\vert\boldsymbol{T_U}), \mathbf{dur}(S_{K,x}\vert\boldsymbol{T_K}, S_{K,x}\vert\boldsymbol{T_U})\big)$	$\mathbf{mag}(\boldsymbol{S_{K,x}}\vert\boldsymbol{T_U})$, $\mathbf{dur}(\boldsymbol{S_{K,x}}\vert\boldsymbol{T_U})$
G5	Specified impact(s)	Any threat or system	• Risk to specified level of service from any threat,	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$ or $\mathbf{P}(\boldsymbol{S_K})$,	• Resilience of specified level of service provision,	

		failure	$Risk_{G5a} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}\left(I_{K,y}\right)$ • Risk to specified level of service from any system failure, $Risk_{G5b} = \mathbf{P}(\boldsymbol{S_K}) \cdot \mathbf{mag}\left(I_{K,y}\right)$	$\mathbf{mag}\left(I_{K,y}\right)$	• Resilience of specified level of service provision to any threat , • Resilience of specified level of service provision to any system failure, • $Res_{G5} = g\left(\mathbf{mag}(I_{K,y}\|\boldsymbol{S_K}), \mathbf{dur}(I_{K,y}\|\boldsymbol{S_K})\right)$
G6	Specified consequence(s)	Any threat, system failure or level of service failure	• Risk to specified SEE component from any threat, $Risk_{G6a} = \mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U}) \cdot \mathbf{mag}\left(C_{K,z}\right)$ • Risk to specified SEE component from any system failure, $Risk_{G6a} = \mathbf{P}(\boldsymbol{S_K}) \cdot \mathbf{mag}\left(C_{K,z}\right)$ • Risk to specified SEE component from any level of service failure, $Risk_{G6a} = \mathbf{P}(\boldsymbol{I_K}) \cdot \mathbf{mag}\left(C_{K,z}\right)$	$\mathbf{P}(\boldsymbol{T_K}, \boldsymbol{T_U})$, $\mathbf{P}(\boldsymbol{S_K})$ or $\mathbf{P}(\boldsymbol{I_K})$, $\mathbf{mag}\left(C_{K,z}\right)$	• Resilience of specified SEE component, • Resilience of specified SEE component to any threat, • Resilience of specified SEE component to any system failure, • Resilience of specified SEE component to any level of service failure, • $Res_{G6} = g\left(\mathbf{mag}(C_{K,z}\|\boldsymbol{I_K}), \mathbf{dur}(C_{K,z}\|\boldsymbol{I_K})\right)$

RELIABILITY ASSESSMENT

Receiving water quality is considered to be in a state of non-compliance (failure) if the DO concentration is below 4 mg/l and/or the un-ionised ammonia concentration is greater than 0.068 mg/l. Reliability is calculated as the fraction of time during which the receiving water quality is not in a state of failure when simulated under design conditions, as given in Eq. S1.

$$Rel = 1 - \frac{T_{failure}}{T_{total}} \qquad \text{Eq. S1}$$

Where $T_{failure}$ is the number of time steps in which the modelled receiving water quality is in a state of non-compliance, and T_{total} is the total number of modelled time steps.

RISK ASSESSMENT

Risk is evaluated for population increases of 0 to 15% at 1.5% intervals, using Eq. S2.

$$Risk = P(event) \times magnitude(failure) \qquad \text{Eq. S2}$$

The probability function for population increase used in risk assessment (Figure S.1) is based on 95% prediction intervals reported by the United Nations (Raftery et al. 2012, United Nations 2012) for the UK population in 2035 and assumption of a normal distribution.

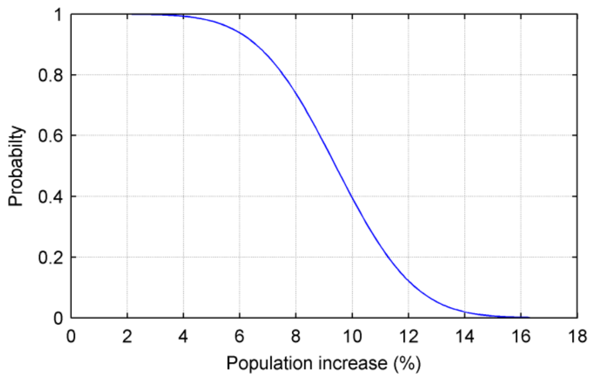

Figure S.1: Probability that a given population increase is equalled or exceeded by 2035

DO deficit and un-ionised ammonia exceedance are normalised with respect to their respective compliance limits, and the greater of these two values represents the failure magnitude.

This method yields 16 risk values for each IUWS option evaluated (risk from population increase of 0%, 1.5%, 3%,… 15%), from which the maximum is selected for use in further analysis.

RESILIENCE ASSESSMENT

Resilience measures

Resilience measures used are based on the system response curve (system performance as a function of disturbance magnitude) concept (Mens et al. 2011). In this study, the magnitude of population increase corresponds with the disturbance magnitude, and system performance is measured using multiple indicators related to the output DO and un-ionised ammonia concentrations.

Chosen system performance indicators are based on resilience metrics used by Wang and Blackmore(2009). These are $P_{deficit}$ (based on the mean performance deficit during the period simulated) and $P_{duration,mean}$ (based on the mean failure duration during the period simulated), and are calculated as follows:

$$P_{deficit} = \frac{\sum_{i=1}^{N}(T_i - E_i)}{\sum_{i=1}^{N} T_i} \qquad \text{Eq. S3}$$

$$P_{duration,mean} = \frac{T_{total} - \frac{\sum_{k=1}^{N_e} F_k}{N_e}}{T_{total}} \qquad \text{Eq. S4}$$

where:

T_i	=	Threshold (maximum acceptable output) at time step i
E_i	=	Threshold exceedance at time step i
		= max(0, T_i – output_i)
N	=	Number of time steps
N_e	=	Number of times failure state is entered
k	=	Failure number
F_k	=	Duration of failure event k
T_{total}	=	Total duration of evaluation period

Both $P_{deficit}$ and $P_{duration,mean}$ take a value of one when no failures occur. $P_{duration,mean}$ has a minimum value of zero, which occurs when the failure duration equals the simulation duration. $P_{deficit}$ may be negative if the threshold is a ‘maximum’ acceptable output (as for un-ionised ammonia) and this is exceeded by more than 100%.

Knowledge of the performance measures resulting from a range of disturbance magnitudes yields response curves of the form shown in Figure S.2.

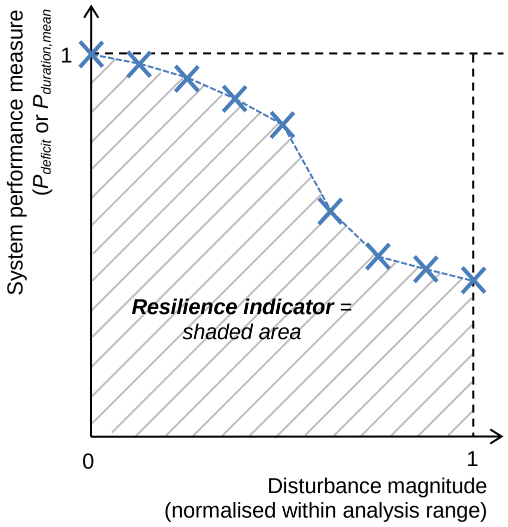

Figure S.2: Response curve used for calculation of resilience indicators

Resilience to a given threat is represented by the area under the response curve, as shown in Figure S.2. Two resilience indicators can be calculated for each combination of disturbance and system output: $R_{deficit}$ (based on the $P_{deficit}$ response) and $R_{duration,mean}$ (based on the $P_{duration,mean}$ response). A higher value indicates a more resilient system, with a resilience value of one indicating that no failures are expected under the specified disturbance type and maximum magnitude.

Assessment methodology

$P_{deficit}$ and $P_{duration,mean}$ are calculated for both DO and un-ionised ammonia outputs under 11 different magnitudes of population change, equally spaced in the range 0 to 150%. Resilience indicators are then calculated as follows:

$$R_{deficit} = 0.1 \sum_{k=1}^{10} \left(\min(P_{deficit,DO,k+1}, P_{deficit,AMM,k+1}) + \min\left(P_{deficit,DO,k}, P_{deficit,AMM,k}\right) \right) \quad \text{Eq. S5}$$

$$R_{duration,mean} = 0.1\sum_{k=1}^{10}\Big(\min(P_{duration,mean,DO,k+1}, P_{duration,mean,AMM,k+1}) + \min(P_{duration,mean,DO,k}, P_{duration,mean,AMM,k})\Big) \qquad \text{Eq. S6}$$

Where k represents the disturbance magnitude step number and subscripts DO and AMM denote performance measures based on output DO and un-ionised ammonia respectively.

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