Design and Validation of a Distributed Observer-Based Estimation Scheme for Power Grids

Gianmario Rinaldi, Prathyush P. Menon, Christopher Edwards and Antonella Ferrara

Abstract—**This paper presents a novel estimation scheme for power grids based on distributed observers. Assuming only the generator voltage phase angles are measured and the electrical load active power demands are specified, we design an observer for each bus of the power grid, exploiting only knowledge of local information about the power system. In particular, we propose a super-twisting-like sliding mode observer to estimate the frequency deviation for each generator bus, and a so-called algebraic observer to estimate the load voltage phase angle for each load bus based on distributed iterative algorithms. The observer-based estimation scheme is validated by considering the IEEE 39 bus SimPowerSystems model.**

Index Terms—**Large-scale systems, Observers, State estimation, Power systems, Variable structure systems.**

I. Introduction

THE increasing demand for electrical energy in the twenty-first century, the limited fossil fuel reserves, and environmental issues requiring the reduction of greenhouse gases, call for fast worldwide developments in the area of renewable energy sources. According to the recent Directive of the European Parliament [1], it is predicted that significantly more renewable energy generation sources will be connected so that 20% of the overall electricity consumption will be supplied from such sources by 2020. A big effort is being made to reach this goal and it is worth mentioning two changes occurring in power systems [2]: i) the increase of so-called renewable non-programmable power plants (such as photovoltaic power plants and wind power plants) with their intermittent power generation profiles; ii) the development of geographically distributed generation. These changes increase the uncertainty in the operation of power systems and call for the use of advanced monitor and control strategies [3] [4] [5].

In order to deal with more advanced and smarter monitoring schemes for power networks, so-called Phasor Measurement Units (PMUs) have been recently proposed for large-scale implementation in wide-area measurement systems [6]. PMUs provide synchronized measurements of real-time voltage and current phasors in each branch of a power network. The synchronization is achieved in practice by means of timing signals from Global Positioning System Satellites (GPS) [6]. However, it is worth noting that additional problems arise concerning possible corruptions or losses of measurements during monitoring and control (also in the case of PMUs-based measurement network).

To this end, in recent years, worldwide interest has been shown both in the literature and in the real world to develop suitable estimation schemes for large-scale dynamical systems, with application to power grids specifically to tackle the aforementioned issues. In particular, a survey of the state of art about power grid estimation schemes has been summarized in [7]. Observer-based estimation schemes have been successfully proposed in a wide range of works. For example, in [8], a distributed network of sliding mode observers for large-scale systems has been designed. Specifically, each node has been modeled using ordinary nonlinear differential equations, while the interconnections amongst the nodes are governed by linear time-varying dynamics. A small number of measurements at certain key-nodes within the network have been used to reduce the number of required sensors, and yet estimate the entire state of the network. Following a similar approach, in [9], a distributed adaptive sliding mode observer for a network of dynamical systems has been presented. Each node has been modeled by ordinary differential equations with a known linear part and an unknown bounded nonlinearity, whilst the coupling between each node has been assumed to be governed by linear algebraic equations. In this approach, the observers are capable of adapting their gains with respect to the magnitude of the disturbance affecting the dynamics of the system. In [10], a distributed state estimator for multi-area power systems has been proposed: each area performs its own state estimation, using local measurements, and exchanges border information to its neighboring areas. In [11], continuous-time distributed observers with discrete communication have been designed for large-scale, linear, and continuous-time systems. Moreover, in [11], the system has been decomposed into weakly-coupled areas and state estimation of a linearized model of a power grid have been discussed, in which the algebraic equations for the loads have not been considered. In [12], a third order sliding mode observer-based approach has been presented for optimal load frequency control in power networks divided into control areas. Each area has been modeled using nonlinear ordinary differential equations and the estimation scheme has been revealed to be completely decentralized. In [13], a second-order multi-variable sliding mode observer has been designed to detect and reconstruct a certain class of load faults in power networks. In order to perform fault reconstruction, a centralized estimation scheme has been adopted making use of the Kron-reduction of the power network.

Main Contribution: In this paper we propose a novel estimation scheme based on distributed observers with ap-

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plication to power grid monitoring. By considering a power grid as a graph, we design an observer for each generator bus and each load bus. The architecture relies on only an exchange of information between an observer and other ones in its neighborhood. Specifically, we introduce a distinction between the generator bus dynamics, governed by differential equations, and the load bus, governed by algebraic equations. This is a key-novelty with respect to the work in [8] - [13]. Furthermore, we propose a novel super-twisting-like sliding mode observer for the local estimation of the electrical angular speed of each generator. This kind of observer is known to be very robust to a class of matched uncertainties and disturbances and is even applicable to a wide range of nonlinear systems. Moreover, this observer makes use of periodically sampled signals from its neighboring observers. Discrete-time distributed iterative algorithms are introduced to estimate the state of each load bus. In particular, we compare the Jacobi method, originally applied for this purpose in [14], with the Successive Over Relaxation (SOR) method [15]. The comparison shows a faster speed of convergence of the SOR method. The combination of sliding mode observers for the differential part of the dynamics and the so-called algebraic observers for the algebraic part represents the main contribution of the paper. Taken as a whole, the proposed estimation scheme constitutes a distributed observer for power grids. In this paper we validate in detail the designed scheme by considering the IEEE 39 bus benchmark simulation. With respect to [14], which can be considered as a starting point for the present paper, we significantly improve the algebraic estimator, ensuring better performance in terms of speed of convergence, by employing the SOR method. Again, in contrast to [14], in which a linearized model of a the IEEE 14 bus benchmark has been considered for the simulations, in the present work, the IEEE 39 bus benchmark implemented in the Matlab-SimPowerSystems environment is considered. The nonlinearities in the power grid dynamics are treated as disturbances and therefore their impact on the performance of the distributed observers is expected to be minimal using of the underlying design technique.

Structure of the Paper: The structure of the paper is as follows: in Section II, we derive a viable mathematical model of a power grid commonly adopted in the literature for the purpose of designing observer-based estimation schemes. Furthermore, ideas from graph theory are recalled and the central role of the Laplacian matrix is highlighted. In Section III, the design of the novel distributed observer scheme is discussed in detail. In Section IV, we show simulations based on the IEEE 39 bus benchmark to prove the validity of the proposed approach. Section V concludes the paper.

Notation: We adopt the following (standard) notation throughout the paper: For a matrix Y, Y_{ij} denotes its $(i,j)^{th}$ element, Y^{xz} denotes a submatrix of Y, while Y^T denotes its transpose. The symbol I_n denotes the Identity Matrix of size n. For a continuous-time signal $x(t)$, $x[k]$ denotes its value sampled at $k\tau$ seconds (where τ is the sampling time) while x^{ZOH} denotes the Zero Order Hold (ZOH) approximation of $x(t)$.

II. Preliminaries and System Description

Even though power grids have complex and nonlinear dynamics (see e.g. [16]), the following (standard) assumptions have been used in the literature to obtain viable models for the purpose of designing estimation algorithms (see, for example, [13], [14], and [17]):

Assumption 1 *It assumed that:*

- *The power grid is a lossless system. This implies that the resistance of each power transmission line is negligible.*
- *The voltage profile is flat, which implies that the magnitude of the voltage at each bus is constant and equal to 1 p.u. (where 1 p.u. is the expression of the actual value with respect to the base value, assuming the units of both are the same).*
- *The difference between the i^{th} and j^{th} voltage phase angle is sufficiently small such that the small angle approximations hold.*
- *The electrical reactive power flow through the power transmission lines is not considered.*

These commonly adopted assumptions allow us to derive a linear model for the power grid dynamics comprising linear differential equations for the generators and linear algebraic equations for the loads. However, the specific observers that we design using the linear dynamics perform acceptably due to the extreme robustness of the adopted method, specifically with respect to the matched uncertainties/nonlinearities present in the system.

Remark 1 *The work [18] has demonstrated that Assumption 1 is accurate if, for each power transmission line, the ratio* inductance/resistance *is greater than 4 [p.u.], and the standard deviation of the voltage magnitudes is smaller than 0.1 [p.u.]. This holds for the IEEE 39 bus benchmark, according to the data [19].*

A. Graph Theory Preliminaries

A power grid can be considered as a graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, defined in terms of a set of nodes $\mathcal{N} = \{1,\ldots,N\}$ and a set of edges $\mathcal{E}$. Here in the case of power grids, the nodes (called equivalently the buses) are the generators and the loads, whereas the edges linking the buses are the power transmission lines and the power transformers. Each edge is considered an entity linking an unordered pair of distinct nodes (i,j) and is characterized by its weight. Under Assumption 1, the weight of an edge is the reciprocal of corresponding power transmission line or power transformer reactance, denoted as b'_{ij}. Therefore, each edge can be written as: $\left\{(i,j),\ b'_{ij}\right\} \in \mathcal{E}$.

For the reader's convenience, it is worth recalling the concepts of adjacent buses and the neighborhood set.

Definition 1 (Adjacent Buses and Neighborhood Set) *The i^{th} bus and the j^{th} bus of a power grid are said to be directly adjacent if they are linked by an edge, or, in other words:*

$$\exists\left\{(i,j),\ b'_{ij}\right\} \in \mathcal{E}. \tag{1}$$

The neighborhood of the i^{th} bus of a power grid is the set (denoted as $\mathcal{N}_i$) of the buses directly adjacent to the i^{th} bus. Formally:

$$\mathcal{N}_i := \left\{ k \mid \left\{ (i,k),\ b'_{ik} \right\} \in \mathcal{E} \right\}. \tag{2}$$

The set $\mathcal{N}_i$ is split into the neighboring generator bus set $\mathcal{M}_i$ and the neighboring load bus set $\mathcal{O}_i$ such that $\mathcal{N}_i = \mathcal{M}_i \cup \mathcal{O}_i$.

B. Laplacian Matrix Description

For a given power grid interpreted as a graph, suppose that n_g represents the number of generators and n_a represents the number of loads. Define $N = n_g + n_a$, then the power grid is said to be an N-bus power grid.

Assumption 2 *The enumeration of the power grid buses is chosen in such a way that the first n_g buses refer to the generators.*

The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$, capturing the interconnection topology of the power network has a central role, and its elements are defined as follows (see e.g. [16] and [20]):

$$\mathcal{L} \quad = \quad \begin{cases} \mathcal{L}_{ii} & = \sum_{k\in\mathcal{N}_i} b'_{ik}, \\ \mathcal{L}_{ij} & = -b'_{ij}. \end{cases} \tag{3}$$

In (3), $\mathcal{N}_i$ is the neighborhood set of the i^{th} bus defined according to (2). Exploiting equation (3), the diagonal elements $\mathcal{L}_{ii}$ are the sums of the reciprocals of the reactances b'_{ik} of all the edges connecting the i^{th} bus to its neighboring buses. The off-diagonal element $\mathcal{L}_{ij}$ is the negative reciprocal of the reactance b'_{ij} of the edge $\left\{(i,j),\ b'_{ij}\right\}$ directly connecting the i^{th} and j^{th} buses. If there is no connection between the i^{th} and the j^{th} buses, $\mathcal{L}_{ij} = 0$.

By using Assumption 2, the $\mathcal{L}$ matrix can be partitioned into four sub-matrices as follows:

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}^{gg} & \mathcal{L}^{gl} \\ \mathcal{L}^{lg} & \mathcal{L}^{ll} \end{bmatrix}, \tag{4}$$

where the sub-matrices $\mathcal{L}^{gg} \in \mathbb{R}^{n_g\times n_g}$, $\mathcal{L}^{gl} \in \mathbb{R}^{n_g\times n_a}$, $\mathcal{L}^{lg} \in \mathbb{R}^{n_a\times n_g}$, $\mathcal{L}^{ll} \in \mathbb{R}^{n_a\times n_a}$. Furthermore, in (4), the matrix $\mathcal{L}^{ll}$ is invertible [17], [21].

C. Generator Dynamics

The linear swing equations governing the i^{th} generator bus comprise two linear differential equations [16] and are given by:

$$\dot{\delta}_i(t) = \Delta\omega_i(t), \tag{5}$$

$$\frac{2H_i}{\omega_0}\Delta\dot{\omega}_i(t) = -D_{g_i}\Delta\omega_i(t) + P_{G_i}(t) - P_{T_i}(t), \tag{6}$$

$$y_i(t) = \delta_i(t), \tag{7}$$

where $\delta_i(t)$ is the voltage phase angle of the generator, measured in $[rad]$; $\Delta\omega_i(t)$ is the electrical angular speed deviation from the rated value ω_0[1], measured in $[rad/s]$; D_{g_i} is the damping coefficient, measured in $[p.u.\cdot s/rad]$; H_i is the inertia constant, measured in $[s]$; $P_{G_i}(t)$ is the mechanical input power at the i^{th} generator (which can be considered as the control input signal), in $[p.u.]$; $P_{T_i}(t)$ is the total electrical active power injected into the grid by the i^{th} generator, measured in $[p.u.]$. (An expanded expression for $P_{T_i}(t)$, depending on the topology of the power grid, is provided in sequel.)

In this paper, we locally measure only $y_i(t)$, which is the voltage phase angle $\delta_i(t)$. This is reasonable in practice, since each synchronous generator is generally equipped with an encoder to measure the angular position of the rotor, which is nothing but the generator angle $\delta_i(t)$ [22]. On the other hand, the measurement of $\Delta\omega_i(t)$ may present some drawbacks. In particular, as illustrated in [23], the lack of a speed sensor can enhance the reliability of the generator, and reduces the system cost.

D. Load Dynamics

In this paper the so-called static model of the loads is adopted [16]. In such an approach, the load electrical active power demand $P_{L_i}(t)$ (measured in $[p.u.]$) is specified and treated as a known input [18]. According to Assumption 1, the following algebraic constraint holds:

$$0 = P_{L_i}(t) - \sum_{j\in\mathcal{N}_i} P_{T_{L_j}}(t), \tag{8}$$

where $\sum_{j\in\mathcal{N}_i} P_{T_{L_j}}(t)$ is the total electrical active power transmitted from the i^{th} load bus to its neighboring buses, also measured in $[p.u.]$. Equation (8) can be expanded considering the so-called DC power flow, which is a linear mathematical method expressing the value of the electrical active power flow through each power transmission line (see [18] for further details). Specifically, consider a power transmission line linking the i^{th} and the j^{th} bus and let b'_{ij} be the reciprocal of its reactance. The DC power flow method expresses the power flow (denoted as $P_{ij}(t)$) through the transmission line as follows:

$$P_{ij}(t) = b'_{ij}\left(\vartheta_i(t) - \vartheta_j(t)\right), \tag{9}$$

where $\vartheta_i(t)$ and $\vartheta_j(t)$ are the i^{th} and the j^{th} bus voltage phase angles (and represent algebraic state variables). Using the DC power flow method together with the Laplacian matrix introduced earlier, equation (8) can be rewritten as follows:

$$P_{L_i}(t) = \overbrace{\sum_{j\in\mathcal{M}_i} \mathcal{L}^{lg}_{ij}\delta_j(t) + \mathcal{L}^{ll}_{ii}\vartheta_i(t) + \sum_{k\in\mathcal{O}_i} \mathcal{L}^{ll}_{ik}\vartheta_k(t)}^{\sum_{j\in\mathcal{N}_i} P_{T_{L_j}}(t)}, \tag{10}$$

where $\delta_j(t)$ is the j^{th} generator bus voltage phase angle, $\vartheta_i(t)$ is the i^{th} load bus voltage phase angle, and $\vartheta_k(t)$ is the k^{th} load bus voltage phase angle.

By considering all the n_a equations in the form of (10), it is possible to write the following algebraic relation

$$P_L(t) = \mathcal{L}^{lg}\delta(t) + \mathcal{L}^{ll}\vartheta(t), \tag{11}$$

where $\mathcal{L}^{ll}$ and $\mathcal{L}^{lg}$ are the submatrices in (4), the vector $P_L(t) := [P_{L_1}(t), \ldots, P_{L_{n_a}}(t)]^T \in \mathbb{R}^{n_a}$, the vector $\delta(t) :=$

[1] ω_0 is associated with the electrical frequency of the power system f_0 by the relation $\omega_0 = 2\pi f_0$.

$[\delta_1(t),\ldots,\delta_{n_g}(t)]^T \in \mathbb{R}^{n_g}$, and $\vartheta(t) := [\vartheta_1(t),\ldots,\vartheta_{n_a}(t)]^T \in \mathbb{R}^{n_a}$. In accordance with the partition in (4), and using the DC power flow method again, the expression for $P_{T_i}(t)$ in (6) can be rewritten as:

$$P_{T_i}(t) = \mathcal{L}^{gg}_{ii}\delta_i(t) + \sum_{j\in\mathcal{N}_i}\mathcal{L}^{gl}_{ij}\vartheta_j(t), \tag{12}$$

and the differential equation (6) can be rewritten:

$$\frac{2H_i}{\omega_0}\Delta\dot{\omega}_i(t) = -D_{g_i}\Delta\omega_i(t) + P_{G_i}(t) - \mathcal{L}^{gg}_{ii}\delta_i(t) - \sum_{j\in\mathcal{N}_i}\mathcal{L}^{gl}_{ij}\vartheta_j(t). \tag{13}$$

III. Distributed Observers Design

A. Preliminaries

A distributed observer formulation typically relies on the interconnection of a number of observers able to estimate the state variables of a large-scale system using available local information at node level, together with information gathered from its neighboring nodes. In this section, we design a novel distributed observer scheme which comprises a combination of n_a "algebraic observer schemes" for each load bus, and n_g super-twisting-like sliding mode observers for each generator bus.

B. Algebraic Observers Design

Suppose the voltage phase angle of each generator is measured and the electrical active power demand is specified in each load bus using a sample period of τ [s], where reasonable values for τ can be selected from an understanding of the system and according to [24]. The time at which the measurements are acquired is given by $k\tau$, where $k = 0,1,2,\ldots$ Considering (11), at the k^{th} sampling instance the following algebraic equation is satisfied

$$0 = -\mathcal{L}^{ll}\vartheta[k] - \mathcal{L}^{lg}\delta[k] + P_L[k], \tag{14}$$

where the $\mathcal{L}^{ll}$ and $\mathcal{L}^{lg}$ are the submatrices in (4), $\vartheta[k] \in \mathbb{R}^{n_a}$, $\delta[k] \in \mathbb{R}^{n_g}$ and $P_L[k] \in \mathbb{R}^{n_a}$ are the sampled versions of $\vartheta(t)$, $\delta(t)$ and $P_L(t)$. Equation (14) can be compactly rewritten as:

$$\mathcal{L}^{ll}\vartheta[k] = b[k]. \tag{15}$$

where $b[k] := -\mathcal{L}^{lg}\delta[k] + P_L[k]$. The signal $b[k]$ is fixed within the time interval $[k\tau,\ k\tau+\tau]$ and the idea is to estimate $\vartheta[k]$ by solving, in a distributed fashion, the linear system of n_a equations

$$\mathcal{L}^{ll}\hat{\vartheta}[k] = b[k], \tag{16}$$

in order to obtain an estimate $\hat{\vartheta}[k]$. This will be achieved by using an iterative scheme operating within each sampling period. In this paper two methods are discussed for this purpose: the Jacobi Method and the Successive Over Relaxation (SOR) Method [15].

1) Jacobi Method:
The Jacobi Method can be used to solve the linear system (16), and is governed by the iteration scheme:

$$\hat{\vartheta}[k,h+1] = \left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)\hat{\vartheta}[k,h] + \mathcal{D}^{-1}b[k], \tag{17}$$

where $\hat{\vartheta}[k,h]$ represents the h^{th} estimate of $\vartheta[k]$, during the iteration cycle occurring in the time interval $[k\tau,\ k\tau+\tau]$, and the matrix $\mathcal{D}$ is defined according to:

$$\mathcal{D}_{ij} = \begin{cases} \mathcal{L}^{ll}_{ij} & \text{if } i = j, \\ 0. \end{cases} \tag{18}$$

The iteration scheme in (17) can be implemented in a distributed way since

$$\hat{\vartheta}_i[k,h+1] = -\frac{1}{\mathcal{L}^{ll}_{ii}}\left(\sum_{j\in\mathcal{N}_i}\mathcal{L}^{ll}_{ij}\hat{\vartheta}_j[k,h] - b_i[k]\right), \tag{19}$$

where $\hat{\vartheta}_i[k,h+1]$ is the $(h+1)^{th}$ update of the i^{th} load voltage phase angle estimate, only depends on the other estimates of neighboring load voltage phase angles the h^{th} step. The exchange of information about the estimates of the load voltage phase angles constitutes the distributed architecture in our approach. The convergence of (17), which has to be guaranteed, depends on the eigenvalues of $\left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)$.

Remark 2 *Since $\mathcal{L}^{ll}$ is (weakly) diagonally dominant according to the Laplacian matrix definition, using Gershgorin's Theorem [15], each eigenvalue λ of $\left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)$ satisfies $|\lambda| \leq 1$. Furthermore, if all the eigenvalues of $\left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)$ lie (strictly) inside the unit disk in the complex plane, it follows* (17) *converges and $\hat{\vartheta}[k,h+1] \to \hat{\vartheta}[k,h]$ as $h \to \infty$.*

The steady state solution, denoted as $\hat{\vartheta}[k,\cdot]$, satisfies:

$$\hat{\vartheta}[k,\cdot] = \left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)\hat{\vartheta}[k,\cdot] + \mathcal{D}^{-1}b[k], \tag{20}$$

which implies

$$\mathcal{L}^{ll}\hat{\vartheta}[k,\cdot] = b[k]. \tag{21}$$

In practice, during the time interval $[k\tau,\ k\tau+\tau]$, a sufficient number of iteration can be executed to obtain a close to equilibrium solution of (17), satisfying the required accuracy.

2) Successive Over Relaxation Method:
By using the Jacobi Method, it has been possible to guarantee that each eigenvalue of the matrix $\left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)$ satisfies $\lambda \leq 1$. However, if there are eigenvalues of absolute value close to 1, the Jacobi Method may converge too slowly or not converge at all. The so-called Successive Over Relaxation (SOR) Method [15] can be used to ensure faster convergence, and is governed by the following iterative scheme:

$$\begin{aligned}\hat{\vartheta}[k,h+1] = &\overbrace{\left(I_{n_a} - \kappa\left(\mathcal{D} - \kappa\mathcal{E}\right)^{-1}\mathcal{L}^{ll}\right)}^{M_\kappa}\hat{\vartheta}[k,h] \\ &+ \kappa\left(\mathcal{D} - \kappa\mathcal{E}\right)^{-1}b[k],\end{aligned} \tag{22}$$

where the matrix $M_\kappa := \left(I_{n_a} - \kappa(\mathcal{D}-\kappa\mathcal{E})^{-1}\mathcal{L}^{ll}\right)$ is a function of the design weight $\kappa \in \mathbb{R}^+$, and $\mathcal{E}$ is defined according to:

$$\mathcal{E}_{ij} \begin{cases} -\mathcal{L}^{ll}_{ij} & \text{if } i<j, \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

In (22), the weight κ has to be selected according to the convergence criteria detailed in [25] in order to ensure that the eigenvalues of M_κ lie inside the unit disk. In particular, κ has to satisfy the condition [25]:

$$\kappa\left(\mathcal{D}+k\mathcal{E}\mathcal{L}^{llT}\right)+\kappa\mathcal{L}^{ll}\left(\mathcal{D}+\kappa\mathcal{L}^{llT}\right)-\kappa^2\mathcal{L}^{ll}\mathcal{L}^{llT}>0, \quad (24)$$

where $\mathcal{L}^{llT}$ denotes the transpose of the matrix $\mathcal{L}^{ll}$. Furthermore, the optimal value κ^* of κ satisfying equation (24), is given by:

$$\kappa^* = \arg\min_{\kappa}\left(\rho(M_\kappa)\right), \quad (25)$$

where $\rho(M_\kappa)$ denotes the spectral radius of the matrix M_κ. The steady-state solution of (22) satisfies:

$$\begin{aligned} \hat{\vartheta}[k,\cdot] &= \left(I_{n_a}-\kappa(\mathcal{D}-\kappa\mathcal{E})^{-1}\mathcal{L}^{ll}\right)\hat{\vartheta}[k,\cdot] \\ &\quad + \kappa(\mathcal{D}-\kappa\mathcal{E})^{-1}b[k], \end{aligned} \quad (26)$$

which again implies equation (21) is satisfied.

Additional practical details regarding the choice of the weight κ and further discussions about the faster convergence of the SOR method are given in Section IV.

Initial Conditions:
For both the Jacobi and the SOR method, the following initial conditions are selected:

$$\hat{\vartheta}[k,h=0] = \begin{cases} \hat{\vartheta}[0] & \text{if } k=1, \\ \hat{\vartheta}[k-1,\cdot] & \text{if } k>1, \end{cases} \quad (27)$$

where $\hat{\vartheta}[0]$ is an arbitrarily chosen initial condition, while $\hat{\vartheta}[k-1,\cdot]$ is the steady-state estimate related to the $(k-1)^{th}$ sample period. From equation (27), it is assumed that for $k>1$ the steady state estimates $\hat{\vartheta}[k,\cdot]$ are close to the previous one $\hat{\vartheta}[k-1,\cdot]$.

RMSE Error:
The Root-Mean-Square Error (RMSE) statistic is introduced in this paper to compute the global performance of the algebraic observer scheme, defined as:

$$\text{RMSE}[k,h] = \sqrt{\frac{\sum_{i=1}^{n_a}\left(\vartheta_i[k]-\hat{\vartheta}_i[k,h]\right)^2}{n_a}}, \quad \forall k. \quad (28)$$

According to the development in this section, the RMSE Error is expected to converge to zero within each time interval $[k\tau, k\tau+\tau]$.

Remark 3 *It is worth highlighting that the number of iterations in the algebraic observers affects the accuracy of the estimations. Specifically, the greater the number of iterations, the greater the estimation accuracy at the expense of a prolonged sampling time. However, another aspect which influences the accuracy of the estimation is the position of the eigenvalues of the matrix M_κ in the complex plane. More precisely, if these eigenvalues are placed close to the origin (by selecting the optimal value of the weight κ according to* (25)*), it is possible to obtain better accuracy by using the same number of iterations. In practice, the optimisation as in* (25) *can be performed off-line and only once, given a set of values for the weight $\kappa \in (0,2)$ and the matrix $M_\kappa := \left(I_{n_a}-\kappa(\mathcal{D}-\kappa\mathcal{E})^{-1}\mathcal{L}^{ll}\right)$ is computed accordingly [25].*

C. Super-Twisting-Like Sliding Mode Observers Design

To estimate $\Delta\omega_i$, consider at each generator node an original super-twisting-like sliding mode observer of the form[2]:

$$\dot{\hat{z}}_1 = \hat{z}_2 - a_1e_1 - \alpha|e_1|^{1/2}\text{sign}(e_1), \quad (29)$$

$$\begin{aligned} \dot{\hat{z}}_2 &= -\frac{\omega_0}{2H_i}\mathcal{L}^{gg}_{ii}\delta_i + a_1\hat{z}_2 - a_1^2e_1 - \alpha a_1|e_1|^{1/2}\text{sign}(e_1) \\ &\quad - \beta\,\text{sign}(e_1) + \frac{\omega_0}{2H_i}P_{G_i} + \varphi^{ZOH}. \end{aligned} \quad (30)$$

In (29) and (30), $\hat{z}_1$ represents the estimate of δ_i; $\hat{z}_2$ represents the estimate of $\Delta\omega_i$; $a_1 \equiv -\omega_0 D_{g_i}/2H_i$; $e_1 := \hat{z}_1 - \delta_i$, sign$(\cdot)$ denotes the signum function, and α, β are positive scalar design constants to be tuned.

Remark 4 *In equation* (29)*, the additional term added with respect to the super-twisting sliding mode observer in [26] is $-a_1e_1$, while in equation* (30)*, the additional terms are $-a_1^2e_1$, and $-\alpha a_1|e_1|^{1/2}\text{sign}(e_1)$. The usefulness of these choices are detailed in sequel.*

In (30), the i^{th} generator super-twisting-like sliding mode observer receives estimates from its neighboring algebraic observers. In particular:

$$\varphi^{ZOH} := -\frac{\omega_0}{2H_i}\left(\sum_{j\in\mathcal{N}_i}\mathcal{L}^{gl}_{ij}\hat{\vartheta}^{ZOH}_j\right), \quad (31)$$

where $\hat{\vartheta}^{ZOH}_j$ is the Zero Order Hold (ZOH) version (piecewise constant) [15] of the j^{th} load bus voltage phase angle estimate communicated by the j^{th} neighboring algebraic observer.

Remark 5 *The ZOH implementation is a basic requirement, since the algebraic observers have discrete-time dynamics, while the super-twisting-like ones have continuous-time dynamics.*

Subtracting (5) from (29) and (13) (divided by $\omega_0/2H_i$) from (30), we obtain the error system dynamics:

$$\dot{e}_1 = e_2 - a_1e_1 - \alpha|e_1|^{1/2}\text{sign}(e_1), \quad (32)$$

$$\begin{aligned} \dot{e}_2 &= a_1e_2 - a_1^2e_1 - \alpha a_1|e_1|^{1/2}\text{sign}(e_1) \\ &\quad - \beta\,\text{sign}(e_1) + \Phi, \end{aligned} \quad (33)$$

where $e_2 := \hat{z}_2 - \Delta\omega_i$ and

$$\Phi := \varphi^{ZOH} + \frac{\omega_0}{2H_i}\left(\sum_{j\in\mathcal{N}_i}\mathcal{L}^{gl}_{ij}\vartheta_j\right). \quad (34)$$

[2]The time dependence on the variables is omitted here for the sake of simplicity.

Note that Φ is a matched uncertain term in the error dynamics (32)-(33). By virtue of the structure of the power grid dynamics, it is possible to nicely decouple the discrete-time part of the estimation schemes with respect to the continuous-time one. Moreover, it is also possible to lump in Φ bounded nonlinearities related to unmodeled dynamics of the system, as detailed in sequel.

Assumption 3 *The modulus of Φ in* (33) *is bounded as:*

$$|\Phi| \leq \Delta_M. \tag{35}$$

Assumption 3 is reasonable, since Φ depends on the sum of the neighboring algebraic observer estimation errors. In particular, expanding the expression for Φ from (34), one gets:

$$\Phi = -\frac{\omega_0}{2H_i}\left(\sum_{j\in\mathcal{N}_i}\mathcal{L}_{ij}^{gl}\left(\hat{\vartheta}_j^{ZOH}-\vartheta_j\right)\right). \tag{36}$$

In (36), $\left(\hat{\vartheta}_j^{ZOH}-\vartheta_j\right)$ remains bounded $\forall j\in\mathcal{N}_i$ and asymptotically converges to zero, according to the convergence arguments of the algebraic observer scheme in (17) and in (22). Therefore, (36) remains bounded also. Define:

$$\tilde{e}_2 := e_2 - a_1 e_1, \tag{37}$$

then, the derivative of (37) yields:

$$\dot{\tilde{e}}_2 = \dot{e}_2 - a_1\dot{e}_1. \tag{38}$$

It is possible to rewrite the error dynamics considering: i) equation (32) rewritten substituting for $\tilde{e}_2$ from (37); ii) equation (38) substituting for $\dot{e}_1$ from (32) and $\dot{e}_2$ from (33) respectively. Thanks to the additional terms highlighted in Remark 4, after some algebraic manipulations, one gets a simpler and more viable form for the error dynamics as follows:

$$\dot{e}_1 = \tilde{e}_2 - \alpha\,|e_1|^{1/2}\operatorname{sign}(e_1), \tag{39}$$
$$\dot{\tilde{e}}_2 = -\beta\operatorname{sign}(e_1) + \Phi. \tag{40}$$

Equations (39)-(40) are in the form of the standard Super Twisting Algorithm [27] and $e_1=\dot{e}_1=\tilde{e}_2=e_2=0$ is achieved in a finite time by properly tuning the gains α, β. For example, following the procedure in [28], a suitable choice of the gains is:

$$\alpha > 0, \tag{41}$$
$$\beta > \Delta_M\left(3+\frac{2\Delta_M}{\alpha^2}\right), \tag{42}$$

where Δ_M is the bound on the uncertainty from Assumption 3. It follows $\hat{z}_1=\delta_i$ and $\hat{z}_2=\Delta\omega_i$ in a finite time, which means that the estimates are equal to the generator states in finite time.

Remark 6 *In the case when constant a_1 in the super-twisting-like sliding mode observer is uncertain or not known, defining the signal*

$$\tilde{\Phi} = \Phi + \frac{\omega_0}{2H_i}D_{g_i}\Delta\omega_i, \tag{43}$$

and assuming that $|\tilde{\Phi}|\leq\Delta_M$, it is possible to introduce the following super-twisting sliding mode observer to estimate $\Delta\omega_i$

$$\dot{\hat{z}}_1 = \hat{z}_2 \tag{44}$$
$$\dot{\hat{z}}_2 = -\beta\operatorname{sign}(e_1) + \varphi^{ZOH}, \tag{45}$$

where the meanings of the variables are as described above. The convergence of the observer in the form of (44)-(45) *can be easily proven as above, since the error dynamics are in the form of the standard Super Twisting Algorithm.*

Figure 1 shows on the left a schematic of the proposed estimation scheme to illustrate the observer design presented in this section.

IV. Simulation Examples and Validation

The robustness of the distributed estimation scheme proposed in this paper is investigated, by considering a nonlinear power grid model. In particular, the nonlinearities in the power grid dynamics are treated as disturbances and therefore their impact on the performance of the distributed observer is expected to be minimal by exploiting the robustness of the underlying design technique. Specifically, certain nonlinearities affecting the super-twisting-like sliding mode observers can be lumped in the matched disturbance Φ in (34).

Remark 7 *Assumption 1, introduced to obtain a viable model for design, has to be relaxed in this situation. Detailed expressions for the simulated nonlinear dynamics can be found in [16] and [20] (for example).*

The IEEE 39 bus (also called the 10-machine New-England) power grid model is considered and has been implemented in a Matlab-SimPowerSystems environment. The basic data for this power system is available in the literature in [19]. Figure 1 shows on the right the single-line diagram of this power network comprising 10 synchronous generators, 39 buses, and 46 power transmission lines. Figure 2 shows on the right and in the center the SimPowerSystems implementation of the IEEE 39 bus benchmark. An enlarged view is also provided in the center of Figure 2, in which it is possible to distinguish the generators, the power transformers, the loads and the power transmission lines. If t is the simulation time, for $0\leq t<5$ $[s]$ the entire power system is at steady state. A sudden variation in the electrical active power demand at the 16^{th} load bus takes place as shown in Figure 1. This disturbance makes the electrical frequency of each generator deviate from its rated value as shown in Figure 4. Furthermore, when the load alteration ends (for $t=10$ $[s]$), the frequency of each generator tends to reach again the rated value of 60 Hz.

1) Algebraic Observers:

The algebraic observer scheme introduced in Section III relies on the Jacobi method or on the SOR method. In this section we show the results using the two methods and the faster convergence of the SOR is discussed. The sampling time τ is set equal to 0.1 seconds and within each sampling interval a maximum of 1000 iterations can be executed. Considering the Jacobi method first, Figure 3 shows that the matrix $\left(I_{n_a}-\mathcal{D}^{-1}\mathcal{L}^{ll}\right)$ in the iterative scheme (17) has a spectral

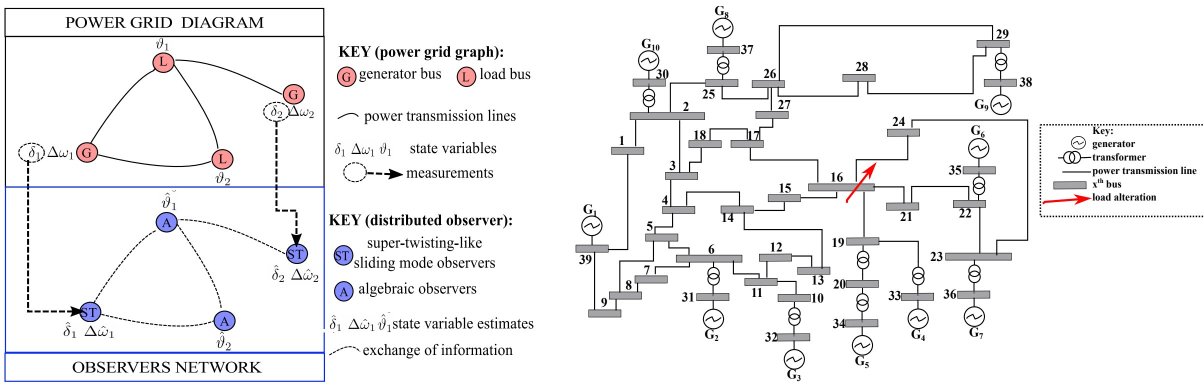

Figure 1. A schematic of the novel distributed observers scheme proposed in this paper (left). The single-line diagram of the IEEE 39 bus benchmark with the highlighted load alteration acting on the 16^{th} bus (right).

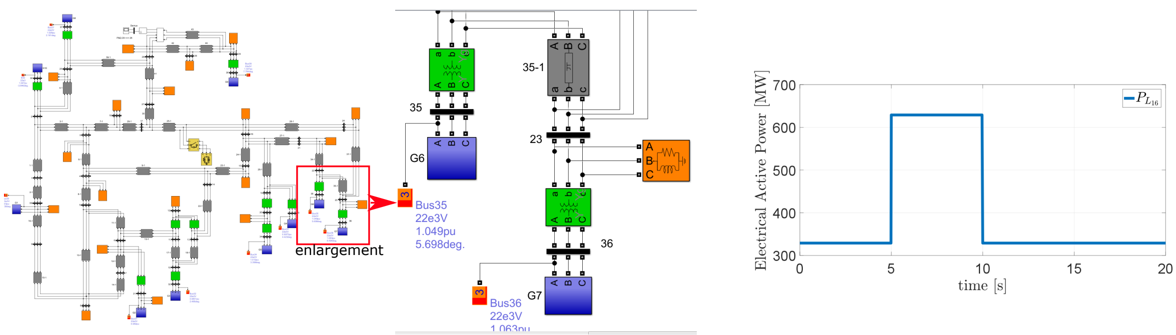

Figure 2. The IEEE 39 bus benchmark model in SimPowerSystems nonlinear environment (left). An enlarged view (center): it is worth noting the generators, the power transformers, the loads and the power transmission lines. The load alteration acting on the 16^{th} bus of the power grid (right).

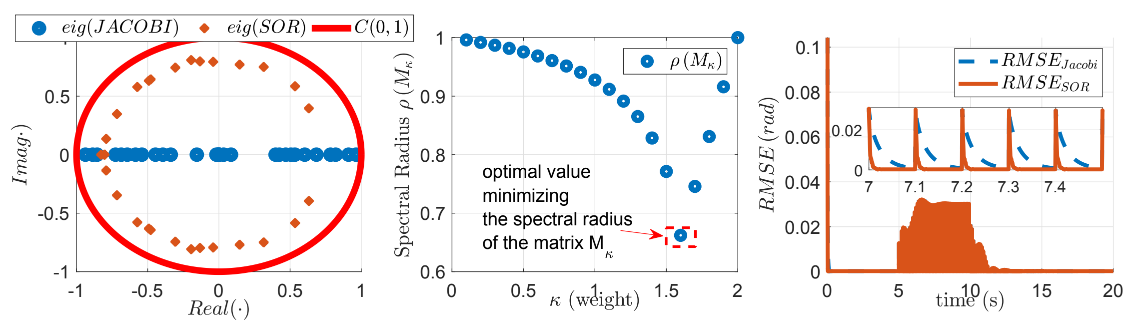

Figure 3. The eigenvalues on the complex plane for the matrices in the Jacobi and SOR methods (left). The spectral radius of the matrix M_κ as function of the weight κ and the optimal choice for the weight κ^* (center). Load bus voltage phase angles RMSE error for the Jacobi and SOR method and an enlarged view during the time interval $[7, 7.5]$ $[s]$ (right).

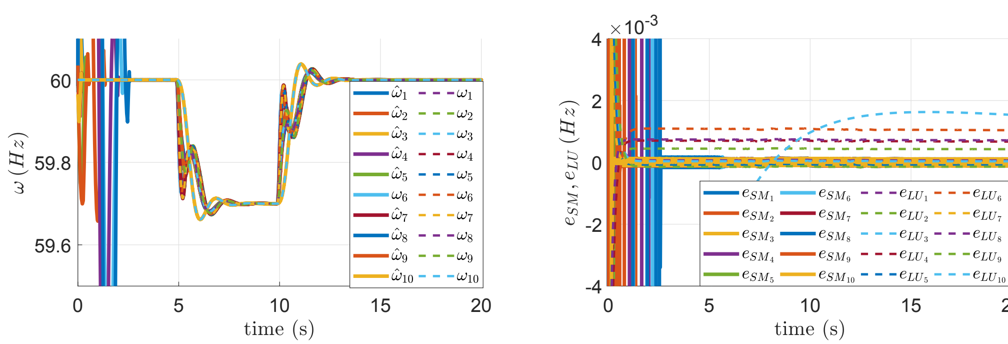

Figure 4. The frequency of each generator and its estimate via super-twisting-like sliding mode observer (left). Frequency estimation error for each super-twisting-like sliding mode observer e_{SM_i} and for UI Luenberger observer e_{LU_i} (right).

radius too close to one and the convergence speed is expected to be slow. Therefore, it is useful to introduce the SOR method, selecting the optimal value for the weight κ according to (25). Figure 3 summarizes the optimal choice of the weight which was made. The resulting eigenvalues of the matrix M_{κ^*} are drawn in the complex plane together with the eigenvalues of $\left(I_{n_a} - \mathcal{D}^{-1}\mathcal{L}^{ll}\right)$ for the Jacobi method, and the unit disk. From Figure 3, the convergence of the SOR algorithm is expected to be faster than the Jacobi one. The RMSE Error defined in (28) accounts for the global performance of the algebraic observer scheme. For $t > 5[s]$, the transient behaviors of the algebraic observers are shown in the enlargement in Figure 3. This transient behavior is due to the update of the term $b[k]$ and the following iterations within each time interval $[k\tau; k\tau + \tau]$ according to equation (17) and (22). The proposed SOR method displays a faster speed of convergence when compared to the Jacobi method, originally adopted in [14].

2) Super-Twisting-Like Sliding Mode Observers: The design constants for the super-twisting-like sliding mode observer are set as follows: $\Delta_M = 10$, $\alpha = 5$, and $\beta = 50$. Since these observers have continuous-time dynamics, the solver *Ode1* (Euler) is select in Matlab, with a fixed integration step size equal to 0.1 milliseconds. In order to demonstrate the superiority of the proposed observer, we compare their performances with the well-enstablished Unknown Input (UI) Luenberger observers [29]. From Figure 4, one can conclude that a correct estimation of the frequency in each generator is achieved in finite time (a couple seconds). Moreover, the accuracy of the sliding mode observers is clearly higher than the UI Luenberger ones, as shown in Figure 4.

V. Conclusions

In this paper, a distributed observer formulation involving the interconnection of super-twisting-like sliding mode observers for each generator, and distributed algebraic observers schemes, is proposed for state estimation and monitoring in power grids. The scheme exploits the underlying topology of the power grid and requires only local information available at the node level of the graph. The numerical simulations based on the IEEE 39 bus benchmark validate the effectiveness of the proposed method. In particular, it has been possible to ensure strong robustness for the super-twisting-like sliding mode observer with respect to matched disturbances arising from modeling the nonlinear dynamics of the networks. On the other hand, the algebraic observer estimation scheme, relying on the linearized DC power flow method, is affected by small errors in the estimation due to the disturbances caused by the nonlinear dynamics of the grid.

References

[1] “Directive 2012/27/EU on energy efficiency,” *Official Journal*, vol. 315.

[2] G. M. Masters, *Renewable and efficient electric power systems*. John Wiley & Sons, 2013.

[3] S. Trip, M. Bürger, and C. De Persis, “An internal model approach to (optimal) frequency regulation in power grids with time-varying voltages,” *Automatica*, vol. 64, pp. 240–253, 2016.

[4] S. Trip, M. Cucuzzella, A. Ferrara, and C. De Persis, “An energy function based design of second order sliding modes for automatic generation control,” in *Proc. 20th IFAC World Congr.*, Toulouse, France, July 2017, pp. 11 613–11 618.

[5] M. Cucuzzella, S. Trip, C. De Persis, and A. Ferrara, “Distributed second order sliding modes for optimal load frequency control,” in *Proc. American Control Conf.*, Seattle, WA, USA, May 2017, pp. 3451–3456.

[6] P. W. Sauer, M. A. Pai, and J. H. Chow, *Power System Dynamics and Stability: With Synchrophasor Measurement and Power System Toolbox*. John Wiley & Sons, 2017.

[7] Y.-F. Huang, S. Werner, J. Huang, N. Kashyap, and V. Gupta, “State estimation in electric power grids: Meeting new challenges presented by the requirements of the future grid,” *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 33–43, 2012.

[8] C. Edwards and P. P. Menon, “On distributed pinning observers for a network of dynamical systems,” *IEEE Transactions on Automatic Control*, vol. 61, no. 12, pp. 4081–4087, 2016.

[9] P. P. Menon and C. Edwards, “Distributed adaptive sliding mode observers for a network of dynamical systems,” in *Proc. American Control Conference*, Washington, DC, USA, 2013, pp. 1537–1542.

[10] G. N. Korres, “A distributed multiarea state estimation,” *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 73–84, 2011.

[11] F. Dorfler, F. Pasqualetti, and F. Bullo, “Continuous-time distributed observers with discrete communication,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 7, no. 2, pp. 296–304, 2013.

[12] G. Rinaldi, M. Cucuzzella, and A. Ferrara, “Third order sliding mode observer-based approach for distributed optimal load frequency control,” *IEEE Control Systems Letters*, vol. 1, no. 2, pp. 215–220, 2017.

[13] C. Mellucci, P. P. Menon, C. Edwards, and A. Ferrara, “Second-order sliding mode observers for fault reconstruction in power networks,” *IET Control Theory & Applications*, vol. 11, no. 16, pp. 2772–2782, 2017.

[14] G. Rinaldi, P. P. Menon, C. Edwards, and A. Ferrara, “Distributed observers for state estimation in power grids,” in *Proc. American Control Conf.*, Seattle, WA, USA, May 2017, pp. 5824–5829.

[15] Y. Saad, *Iterative methods for sparse linear systems*. SIAM, 2003.

[16] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*. McGraw-hill New York, 1994, vol. 7.

[17] F. Pasqualetti, A. Bicchi, and F. Bullo, “A graph-theoretical characterization of power network vulnerabilities,” in *Proc. American Control Conference*, San Francisco, California, USA, 2011, pp. 3918–3923.

[18] K. Purchala, L. Meeus, D. Van Dommelen, and R. Belmans, “Usefulness of DC power flow for active power flow analysis,” in *Proc. Power Engineering Society General Meeting*, San Francisco, California, USA, 2005, pp. 454–459.

[19] I. Hiskens, “IEEE PES task force on benchmark systems for stability controls,” *Technical Report*, 2013.

[20] D. P. Kothari and I. Nagrath, *Modern power system analysis*. Tata McGraw-Hill Education, 2003.

[21] F. Dorfler and F. Bullo, “Kron reduction of graphs with applications to electrical networks,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 1, pp. 150–163, 2013.

[22] M. M. Chowdhury, M. E. Haque, D. Das, A. Gargoom, and M. Negnevitsky, “Modeling, parameter measurement and sensorless speed estimation of IPM synchronous generator for direct drive variable speed wind turbine application,” *International Transactions on Electrical Energy Systems*, vol. 25, no. 9, pp. 1814–1830, 2015.

[23] M. F. Rahman, L. Zhong, M. E. Haque, and M. Rahman, “A direct torque-controlled interior permanent-magnet synchronous motor drive without a speed sensor,” *IEEE Transactions on Energy Conversion*, vol. 18, no. 1, pp. 17–22, 2003.

[24] V. C. Gungor, D. Sahin, T. Kocak, S. Ergut, C. Buccella, C. Cecati, and G. P. Hancke, “A survey on smart grid potential applications and communication requirements,” *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 28–42, 2013.

[25] K. R. James and W. Riha, “Convergence criteria for successive overrelaxation,” *SIAM Journal on Numerical Analysis*, vol. 12, pp. 137–143, 1975.

[26] J. Davila, L. Fridman, and A. Levant, “Second-order sliding-mode observer for mechanical systems,” *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1785–1789, 2005.

[27] A. Levant, “Sliding order and sliding accuracy in sliding mode control,” *International journal of control*, vol. 58, no. 6, pp. 1247–1263, 1993.

[28] J. A. Moreno and M. Osorio, “Strict Lyapunov functions for the super-twisting algorithm,” *IEEE Transactions on Automatic Control*, vol. 57, no. 4, pp. 1035–1040, 2012.

[29] C. Edwards and C. P. Tan, “A comparison of sliding mode and unknown input observers for fault reconstruction,” *European Journal of control*, vol. 12, no. 3, pp. 245–260, 2006.