Evolution of a generic, dynamic and multicomponent tumbling mill model structure incorporating a wide-range 4D appearance function

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ABSTRACT

This work aims to build a generic dynamic model structure, which can accommodate interchangeable sub-models of each sub-process, making it amenable to continuous upgrade without the need for redevelopment, for multicomponent tumbling grinding mills. The Generic Tumbling Mill Model Structure (GTMMS) is based on a population mass balance framework which incorporates breakage characteristics, transport, classification along the mill, a discharge function, and energy consumption incorporated in a dynamic mill model structure. Version III builds on two earlier versions by incorporating energy distributions derived from discrete element modelling, an updated version of the 4D breakage appearance function which applies to a broader size range, and addresses multi-component ore breakage via the probability distribution of energy split based on material stiffness. The model has been tested against multi-component plant survey data.

GTMMS III suggests a mechanistic insight into mixture prediction through component analysis and is a step forward towards the unified comminution model (UCM) with its mechanistic, generic, and dynamic prediction capability.

Keywords: generic mill model, dynamic, 4D appearance function, multicomponent ore, energy splitting, DEM

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1. Introduction

Comminution, including crushing and grinding, consumes a considerable amount of energy in the mining industry. Ballantyne and Powell [3] estimated that about 0.2% of global, and 1.3% of Australia's electricity is consumed by comminution of gold and copper ores alone. Ball mills and SAG/AG mills are the most commonly used ore grinding equipment in the mining industry, responsible for the majority of the comminution energy consumption. The goal of dynamic tumbling mill modelling is to improve design and mill sizing for varying feeds within the dynamics of a milling circuit, apply the model to the design and application of control systems, improve energy utilisation, reduce operating costs and increase productivity.

There are many kinds of classification schemes for comminution models, but these can be classified broadly into two categories: fundamental and black box models [4]. Fundamental models aim to describe interactions of ore particles in the mills largely based on Newtonian mechanics coupled with particle fracture mechanics and fluid transport models, requiring considerable computing power. Black box models are phenomenological and predict products from feeds, breakage characterisation and back-fitted response relationships.

A second classification can categorise mill models as non-mechanistic or mechanistic[1]. The non-mechanistic models are empirical, and they do not consider the fundamental physical mechanism in the comminution process. Mechanistic models focus on the comminution process, and fundamental physical laws are applied to analyse the comminution progress. Because of the mechanistic features, their parameters are related to physical phenomena, and the models tend to be more robust in extrapolation than non-mechanistic models. Austin, Menacho and Pearcy [5] developed a general model for SAG/AG mills based on the assumption of mass balance and mean residence time. Three breakage modes were proposed: normal breakage (caused by nipping of particles between media), abnormal (caused by media without nipping), and self-breakage (caused by chipping fracture and abrasion of the tumbling action of rock lumps) [5]. Powell [6] proposed a conceptually new model - the unified comminution model (UCM) to overcome the limitations of the current comminution models. It is proposed in the UCM that comminution is a generic process of ore breakage independent of the device and that the mechanical breakage environment can be modelled fundamentally. Govender, Tupper and Mainza [7] developed a mechanistic cell model based on combining space and time-averaged Navier - Stokes equations to simulate the slurry transport in dynamic beds. Thanks to the rapid growth of computational power,

Discrete Element Method (DEM) used in mechanistic modelling has gone through remarkable development. DEM is recognised as one of the 'High-Fidelity Simulation (HFS) tools' for comminution simulation and optimisation based on the analysis of basic physics theories [8]. DEM can give a wide range of information such as the mill mechanical and energy environment [9].

The third classification method is according to the classes of matrix model, i.e., kinetic model and perfect mixing model [10]. Schuhmann [11] developed a matrix model with a selection and breakage function for ball mills and rod mills. Kinetic models assume that breakage rate conforms to the first order law and the mill charge is fully mixed. The breakage function can be measured by conducting a monosized particle grinding test over a short grinding time. The following Eq. (1) can describe the kinetic models [12]:

$$\frac{dm_i(t)}{dt} = -s_i\, m_i(t) + \sum_{j=1,i>1}^{i} b_{ij}\, s_j m_j(t) \quad j \geq i \geq 1 \qquad (1)$$

where

$m_i(t)$: mass fraction of the ith size fraction in the mill

t: grinding time

s_i : breakage rate for the ith size fraction

b_{ij}: appearance function which describes the fraction of material broken into size class i due to breakage of the size class j

A perfect mixing model is based on the population balance with the assumption of perfect mixed mill contents [13, 14]. The basic equation for this is:

$$\frac{dm_i(t)}{dt} = f_i(t) - p_i(t) + \sum_{j=1}^{i} b_{ij}\, s_j m_j(t)$$

$$p_i(t) = d_i m_i(t) \qquad (2)$$

Where f_i(t) is the mass flow rate of feed in size class i; p_i(t) is the mass flow rate of product in size class i; d_i is the discharge rate of size class i; all the other symbols are the same as in eq.(1).

The above modelling approaches are developed by assuming that the milling component is homogeneous. Given that so many mine deposits are heterogeneous, modelling of multi-component grinding becomes important and has attracted some attention in recent years. Compared with single component ore grinding,

multi-component (heterogeneous) grinding modelling is less studied. The above theories though have laid a firm foundation for multi-component modelling.

de Paiva Bueno [10, 15] conducted a comprehensive survey of the LKAB iron ore operations (the LKAB Kiirunavaara Mine, Kiruna, Northern Sweden) and collected a large data set to build a preliminary multi-component model. de Paiva Bueno [10, 15] proposed a multi-component AG/SAG mill model structure based on the JKSimMet version of Leung's model (Figure 1).

Separate perfect mixing model equations, independent breakage distribution functions (A, b and t_a), independent breakage rates and discharge rates were applied to each component. de Paiva Bueno [10] proposed an energy sharing method. The average energy level E_1 was considered to be the same for all the components.

$$E_1 = \frac{4\pi(S_{20})^3}{3}\rho_{20}gD \quad (1)$$

where s_{20} is the coarsest 20% size in the mill load; ρ_{20} is the bulk specific gravity of s_{20} ; g is gravitational acceleration; D is the mill diameter (m).

The specific energy E_{cs} (kWh/t) for components is:

$$E_{cs}(m,s) = \frac{E_1}{\frac{4\pi(X_{m,s})^3}{3}\rho_{m,s}} = \frac{(S_{20})^3\rho_{20}gD}{(X_{m,s})^3\rho_{m,s}} \quad (2)$$

where subscripts "m" and "s" denote magnetite and silicate. $\rho_{m,s}$ is the specific gravity of magnetite or silicate. $X_{m,s}$ is the top size of magnetite or silicate. It can be seen from Eq. (1) and Eq. (2) that the energy split is related to the mass ratio of components.

However, there are some limitations of de Paiva Bueno's multi-component model, such as the incomplete revelation of the interactions between components and the energy split method not considering the hardness of components.

It is challenging to apply DEM for modelling tumbling mill performance because the existing computational power limits the lower limit of particle sizes that can be modelled and several different modes of breakage are happening at the same time. However, recently, there are many researchers (Delaney et al. [16]; Xu et al. [17]; Govender et al. [18]) making progress on applying DEM for SAG mill modelling including breakage, mill

charge and wear in the mill. The increasing knowledge from DEM simulation will definitely help in the further development of the generic tumbling mill model structure.

The need for treating hard ore and soft ore differently has been raised by de Paiva Bueno [10, 15]; Pourghahramani [19]; Dündar and Benzer [20]. Since each component in the multicomponent system exhibits different breakage characteristics with different grindabilities, the traditional grinding mill models, which ignore the grindability differences, lead to less accurate model predictions.

In the previous papers, an integrated, mechanistic mill model structure for tumbling mills has been developed. It was based on the population mass balance framework combining the JK appearance function, transport function, and other sub-models. That model structure has been named the Generic Tumbling Mill Model Structure Version I (GTMMS I) [1]. GTMMS I is based on the population balance framework for tracking the production of progeny; incorporates a full breakage function based on detailed ore characterisation; has a size-dependent transport function along the mill, an independent discharge function, and a slurry hold-up function. The transport function is included as a principle sub-model in GTMMS I structure. GTMMS I was later upgraded to GTMMS II [2], incorporating a 4D (four-dimensional) appearance function sub-model based on JK RBT experimental results that was developed to describe the breakage characteristics. In addition, the Discrete Element Method (DEM) energy distribution was integrated and the model structure made dynamic by applying a time-stepping technique for non-steady-state simulation [2].

In this paper, GTMMS III is proposed. The 4D appearance function in GTMMS II is upgraded to a wide-size-range 4D appearance function [21], which was developed specifically to better cover the range of particle sizes, down to fine size below 300 µm, required for accurate mill modelling. A multi-component model structure based on a probability-based energy split scheme is incorporated and simulated by treating the components separately and as a whole. The simulations are also compared and verified with de Paiva Bueno's LKAB KA2 plant survey data [10].

2. Generic Tumbling Mill Model Structure Version III (GTMMS III)

GTMMS III (Figure 2) uses the Population Balance Method (PBM), which was successfully applied in the earlier versions of the model framework.

The sub-models are presented in GTMMS I [1] with some of them interchanged with upgraded sub-models. GTMMS II replaces the JK A-b appearance function in GTMMS I with a 4D appearance function and

supersedes the selection function in GTMMS I with a Discrete Element Method (DEM) energy distribution [2]. There are three upgrades in GTMMS III compared with GTMMS II [2]. A wide-size-range 4D appearance function developed from the data of JK Mini DWT (Drop Weight Tester) and JK standard DWT replaces the previous 4D model based on JK RBT (Rotary Breakage Tester) data [21]. A DEM energy distribution model derived for each component in the mixture is used in place of the DEM energy model in GTMMS II. At this stage of the model development, only impact breakage is considered and shear is not accounted for in the breakage rate or appearance function. Lastly, multi-component modelling, utilising the energy split between components is incorporated and tested against plant survey data.

2.1 Energy split model

The energy split between the individual components is the foundation of multi-component modelling. In many past applications, the comminution energy for individual components in a mixture is shared according to volumetric ratios. However, due to complicated interactions between components in a mixture, the energy split is not only related to the volumetric ratio but also related to the ore hardness, density, feed fineness, mill type and the grinding environment [22]. It was reported that the softer component consumes a greater proportion of the grinding energy when the soft and hard components are ball milled together [23]. Tavares and King [24] proposed an energy split method based on stiffness to calculate the energy actually absorbed by the particle during the UFLC test. de Carvalho [25] applied the approach of Tavares and King [24] to estimate the energy sharing relationship between two particles in a collision based on stiffness in his mechanistic mill model:

$$e_1 = \frac{k_1}{k_1+k_2} \tag{3}$$

$$e_2 = \frac{k_2}{k_1+k_2} \tag{4}$$

Where subscripts 1 and 2 denote particle 1 and particle 2 in the collision, e_i is the fraction of collision energy assigned to particle i and k is stiffness.

The above energy fraction relationship is combined with the collision probability, which is related to the volumetric ratio, to develop an energy split fraction model for magnetite-silicate two-component grinding. All collisions among the magnetite-silicate mixture ore can be divided into three groups: magnetite vs. magnetite, silicate vs. silicate, and magnetite vs. silicate. It is reasonable to assume that the energy shared between the same components is unrelated to stiffness but dependent on the collision probability. On the other hand, the

energy shared between the different components is not only related to the collision probability but also related to stiffness ratios.

From the point of view of collision types between particles, the energy share in the magnetite-silicate mixture ore can be represented by the following three parts:

$$S_{mm} + S_{ss} + S_{ms} = 1 \quad (5)$$

Where S_{mm}, S_{ss}, and S_{ms} are the energy share ratios for the collisions of magnetite vs. magnetite, silicate vs. silicate, and magnetite vs. silicate. It is assumed that the energy share ratio equals the collision probability of each collision type (i.e. magnetite vs. magnetite, silicate vs. silicate, and magnetite vs. silicate) under an ideal condition. Furthermore, the ideal probability of selecting a magnetite particle from the mixture for a collision is equal to the volumetric ratio of magnetite ϕ_m.Strictly, the probability of selecting the second particle is a little bit larger than the first one because the total number of particles is the original number minus one. However, because there are a huge number of particles, such variation can be ignored. Thus, the probability of selecting another magnetite particle from the mixture is also equal to the volumetric ratio of magnetite ϕ_m. The collision probability (which equals energy share ratio) between these two magnetite particles can be expressed as:

Magnetite vs. magnetite:

$$S_{mm} = \phi_m \cdot \phi_m = \phi_m^2 \quad (6)$$

Similarly, the collision probability between two silicate particles can be obtained based on the volumetric ratio of silicate ϕ_s:

Silicate vs. silicate:

$$S_{ss} = \phi_s \cdot \phi_s = \phi_s^2 \quad (7)$$

Substituting Eq.(6) and Eq.(7) into Eq.(5), the balance of the energy share ratio for the collisions between magnetite and silicate particles is:

Magnetite vs. silicate:

$$S_{ms} = 1 - S_{mm} - S_{ss} = 1 - \phi_m^2 - \phi_s^2 \quad (8)$$

On the other hand, from the point view of components, the total energy is split into two components: magnetite and silicate:

$$S_m + S_s = 1 \tag{9}$$

where, S_m and S_s are the energy ratio assigned to magnetite and silicate. The total energy ratio assigned to magnetite S_m should have two parts: one is the energy of the magnetite-magnetite collision (where all the collision energy is absorbed by magnetite), S_{mm}, and the other is the energy portion for magnetite in the magnetite-silicate collision (where the collision energy is split between magnetite and silicate), S_{ms_m}. So the energy ratio assigned to magnetite S_m is:

$$S_m = S_{mm} + S_{ms\_m} \tag{10}$$

Similarly, the total energy ratio assigned to silicate S_s is:

$$S_s = S_{ss} + S_{ms\_s} \tag{11}$$

where S_{ms_s} is the energy portion for silicate in the magnetite-silicate collision.

The energy for magnetite-silicate collision is shared by magnetite and silicate:

$$S_{ms} = S_{ms\_m} + S_{ms\_s} \tag{12}$$

The hypothesis for the energy split of a collision according to Eq. (6) and Eq.(7), provides the basis for the energy of a magnetite-silicate collision to be shared according to stiffness ratio [25]:

$$S_{ms\_m} = S_{ms} \cdot \frac{k_s}{k_s + k_m} = S_{ms} \cdot \psi_m \tag{13}$$

$$S_{ms\_s} = S_{ms} \cdot \frac{k_m}{k_s + k_m} = S_{ms} \cdot \psi_s \tag{14}$$

where subscripts *s* and *m* denote silicate and magnetite; ψ is relative stiffness ratio:

$$\psi_m = \frac{k_s}{k_s + k_m} \tag{15}$$

$$\psi_s = \frac{k_m}{k_s + k_m} \tag{16}$$

Substituting Eq.(6), Eq.(13) and Eq.(8) into Eq.(10):

$$S_m = \phi_m^2 + (1 - \phi_m^2 - \phi_s^2) \cdot \psi_m \tag{17}$$

Substituting Eq.(7), Eq.(14) and Eq.(8) into Eq.(11)

$$S_s = \phi_s^2 + (1 - \phi_m^2 - \phi_s^2) \cdot \psi_s \tag{18}$$

As a discussion, if the subscript "*m*" and "*s*" denote the same component, i.e. the stiffness $S_s = k_m$, then, $\psi_m = \psi_s = 0.5$, substituting this into Eq. (17) and Eq. (18), considering the volumetric ratio relationship $\phi_m + \phi_s = 1$, thus, it is clear that: $S_m = \phi_m$; $S_s = S_m$. That means for the same components (or the different components but with the same stiffness), the energy share is merely according to the volumetric ratio. However, if the components are different, the energy is split not only by volumetric ratio but also by stiffness ratio.

Interestingly, if $\phi_m = \phi_s = 0.5$,

$$S_m = 0.25 + 0.5 \cdot \psi_m \tag{19}$$

$$S_s = 0.25 + 0.5 \cdot \psi_s \tag{20}$$

That means if two components have the same volumetric ratio, the energy split ratio is not equal to the stiffness ratio because the magnetite-magnetite collisions and silicate-silicate collisions are also considered besides the magnetite-silicate collisions.

Furthermore, the softer component will receive more energy than if split only according to the volumetric ratio. Of course, the harder component will receive less energy. For example, in this case, if the magnetite is a softer component, the energy adjustment factor A is:

$$A_m = \frac{S_m}{\phi_m} = \frac{\phi_m^2 + (1 - \phi_m^2 - \phi_s^2) \cdot \psi_m}{\phi_m} \tag{21}$$

Considering $\phi_s = 1 - \phi_m$ and $\psi_m = k_s/(k_s + k_m)$,

$$A_m = (1 - 2\psi_m)\phi_m + 2\psi_m = \frac{k_m - k_s}{k_s + k_m}\phi_m + \frac{2k_s}{k_s + k_m} \tag{22}$$

In the same way, the energy split adjustment factor for the harder component silicate is:

$$A_s = \frac{S_s}{\phi_s} = (1 - 2\psi_s)\phi_s + 2\psi_s = \frac{k_s - k_m}{k_s + k_m}\phi_s + \frac{2k_m}{k_s + k_m} \tag{23}$$

Therefore if $k_s = k_m$, then $A_m = A_s = 1$. That means if there is no difference in stiffness, there is no adjustment based on stiffness and the energy will be split solely according to volumetric ratio.

The adjustment factor for soft component (magnetite in this case) has a maximum value:

$$A_m|_{max} = \frac{2k_s}{k_s+k_m} \quad (24)$$

The adjustment factor for hard component (silicate in this case) has a minimum value:

$$A_s|_{min} = \frac{2k_m}{k_s+k_m} \quad (25)$$

If the energy is shared by volumetric ratio only, for magnetite,

$$\frac{E_{m,v}}{E} = \phi_m \quad (26)$$

where $E_{m,v}$ is the energy share of magnetite with only the volumetric ratio considered, E is the total energy.

If the energy is shared not only by the volumetric ratio but also by the stiffness ratio,

$$\frac{E_{m,v,stiff}}{E} = S_m \quad (27)$$

where $E_{m,v,stiff}$ is the energy share of magnetite with both the volumetric ratio and the stiffness ratio considered.

Eq.(27) is divided by Eq.(26),

$$\frac{E_{m,v,stiff}}{E_{m,v}} = \frac{S_m}{\phi_m} = A_m \quad (28)$$

In the same way, for silicate,

$$\frac{E_{s,v,stiff}}{E_{s,v}} = \frac{S_s}{\phi_s} = A_s \quad (29)$$

If the energy share based solely on the volumetric ratio is known, the energy share based on volumetric ratios and stiffness ratios can be obtained via the adjustment factor (Eq.(28) and Eq.(22), Eq.(29) and Eq.(23)). However, it should be acknowledged that the energy sharing relationship based on stiffness only (Eqs.(3) and (7)) is not a tested relationship as the data is not easily sourced. Therefore, the maximum and minimum values of the adjustment factor and the exact energy split between the two components cannot be determined precisely. Nevertheless, it illustrates the principal effect of stiffness on energy sharing and is a pathway towards quantitatively taking stiffness ratios into account.

2.2 4D appearance function

The appearance function is an important sub-model in a generic model structure for tumbling mills [1] and some AG/SAG mill models [26]. The 4D appearance function (Eq. (30), Eq.(31) and Eq.(32)) have been shown [2, 21] to be more accurate than the existing JK appearance function:

$$B(x) = 1 - e^{\ln(0.2)\left(\frac{x}{P_{80}}\right)^{m}} \quad (30)$$

Where B(x) is the cumulative appearance function in continuous form, x is the product progeny size class, and P_{80} and m are model parameters related to feed size and specific energy respectively [21]. The P_{80} parameter is;

$$P_{80} = e^{[(10X_{nd}^{3}+\alpha X_{nd}^{2}+\beta X_{nd}+1)e^{\gamma E_{cs_{nd}}^{\delta}}]} - 1 \quad (31)$$

where, α, β, γ, and δ are fitting parameters. α and β account for the effect of dimensionless feed ore particle size on P_{80}, while γ and δ provide for the effect of dimensionless specific energy on P_{80}.

X_{nd}: dimensionless feed ore size $X_{nd} = X/X_{top}$

$E_{cs_{nd}}$: dimensionless specific energy $E_{cs_{nd}} = E_{cs}/E_{cs_{top}}$

X: original size of feed ore (mm);

X_{top}: top feed ore size (mm)

E_{cs}: specific energy (kWh/t);

$E_{cs_{top}}$: top specific energy (kWh/t), the largest E_{cs} value among all the E_{cs} values at different X

m is a parameter that determines the size distribution shape:

$$m = e^{[\frac{\lambda X_{nd}^{\eta} E_{cs_{nd}}^{\kappa}}{1+\phi X_{nd}^{0.02}+\psi E_{cs_{nd}}^{0.1}} - 5]} \quad (32)$$

where λ, η, ϕ, ψ, κ are fitting parameters. λ, η, and ϕ provide for the effect of dimensionless feed ore particle size on m, while ψ and κ parameters apply the influence of dimensionless specific energy on m.

From the cumulative appearance function Eq.(33), one can calculate the discrete breakage function b_{ij} in Eq.(2) below:

$$b_{ij} = B(x_j) - B(x_i) \quad (36)$$

From previous results with other ore types [21], it is expected that the appearance function form is also suitable for silicate and magnetite. de Paiva Bueno [10] conducted JKDWT breakage tests over the standard sizes from 11 mm upwards for silicates and magnetite. Using the above appearance functions (Eqs. (33)-(36)), the model parameters for both silicate and magnetite were obtained (Table 1):

Table 1: The 4D appearance function model parameters for silicate and magnetite

Parameters	P_{80} Group Parameters				m Group Parameters				
	α	β	γ	δ	λ	η	κ	ϕ	ψ
Magnetite	-21.63	17.16	-1.34	0.27	1.17	-0.04	-0.11	-0.55	-0.20
Silicate	-21.80	16.47	-1.05	0.32	0.74	-0.08	-0.20	-0.55	-0.30

3. Simulation

Two simulations are presented in this paper. Simulation 1 treats the multi-component mixture ore as a blend, and Simulation 2 deals with independent ore components individually after splitting the energy according to the proposed energy sharing scheme and then combining the simulation results of individual components to obtain the prediction of the ore blend. The results of the two simulations are compared and analysed.

3.1 Mill and ore information

The LKAB mine uses fully autogenous grinding lines to process high-grade magnetite ore. The feed is a mixture of waste silicates (which are introduced by the sublevel caving mining method) and magnetite. The silicates are considerably more competent than magnetite. A*b values of silicates and magnetite from JKDWT tests are 45.3 and 114 respectively [10]. The lower A*b means a lower t_{10} at the same specific energy input, and thus indicates a harder ore [4]. The equipment dimensions of the AG mill and the trommel are given in Table 2:

Table 2 AG mill and trommel parameters [10]

AG Mill		AG Mill (cont.)	
Inside shell diameter, m	6.5	New lining backing plate thickness, mm	110
Inside liner diameter, m	6.3	Number of lifter rows	24
Belly length, m	5.3	New lifter height, mm	165

Inlet trunnion diameter, mm	1800	New lifter angle, ^{0} (from face)	27
cone angle, deg	0	Average lifter height, mm	120
Installed motor power, kW	4300	Average lifter angle, ^{0} (from face)	45
Speed, RPM	12.7	Speed, % Critical	75.1%
Grate		**Trommel**	
Grate aperture size, mm	30 × 90	Diameter, mm	2000
Grate % open area	3.80%	Length, mm	2860
Relative radial position	0.76	Aperture size, mm	6 × 15

A sampling survey at the Kiruna KA2 mill concentrator was carried out and the entire AG mill contents were emptied and assayed by size to quantify the build-up of competent silica in the mills. The breakage characteristics measured with the JKRBT for the blended feed and for the two components from Survey 2, whose data were used for validating the proposed models, are presented in Table 3 [27]. The blended survey sample was tested in the JKRBT, which gives slightly different breakage figures, which can then be adjusted to align with the DWT figures.

Table 3 Ore and waste characterization tests results (After [10])

Parameter	Survey 2	Magnetite	Silicate
SG (Specific Gravity)	4.1	4.9	2.6
A	55.9 (RBT)	68	69
b	1.76 (RBT)	1.67	0.66
A*b	98 (RBT)	114	45.3
Abrasion, ta	0.38	0.59	0.13
BWI@75 µm	13.4	13.2	15.8
XRF %Magnetite (X-ray fluorescence)	87.3	95.7	4.3
		Soft	Hard

3.2 DEM simulation of the mixture energy distribution

The DEM technique can simulate the tumbling mill mechanical environment and provide the collision energy spectrum of the charge (AG/SAG mill) and collision energy spectrum of balls or rocks. DEM approximates the contact between colliding particles with a simplified model such as the non-linear Hertz-Mindlin no-slip model and calculates the motion of each particle, which is governed by the linear momentum and angular momentum conservation law, through solving Newton's second law of motion [28]. DEM provides a mechanistically based insight into the rate and energy of contacts between particles in a mill.

For each size class, DEM can provide the number of collisions per particle per second by energy level. The collision energy can be expressed as the specific energy (Ecs, kWh/t) in the normal and tangential collision directions which has been related to body breakage and surface breakage respectively [28]. Figure 3 is a typical energy spectrum from DEM. With this information, the DEM energy distribution can be obtained as outlined above. The detailed derivation of the energy distribution has been fully illustrated in GTMMS II [2].

The derived energy distribution across each size class is shown in Figure 4. The smallest size class which the DEM could accommodate is around 4 mm for this top-size of 130 mm. The distribution is extended to below 4 mm based on the assumption that the size classes 2 - 5 mm have the highest specific energy. Napier-Munn, Morrell, Morrison and Kojovic [4] pointed out that the breakage rate peaks in the range of 2 - 5 mm, with all larger particles contributing to their breakage. The DEM research also proves that the particles at smaller size classes experience higher specific collision energy than the larger ones [28]. However, particles less than 2 mm tend to be in suspension in the slurry, with a far lower probability of being captured in collision events. Thus, the breakage probability decreases with the decreasing size below 2 mm. Based on this hypothesis, and the assumption of a turning point at 2 mm, the DEM specific energy distribution map was extended to fine sizes for the case in this study (Figure 4). The assumed energy distribution curve for fine particles below 4 mm is an adjustable fitting parameter in the model structure.

3.3 Energy split between the components

In this paper, the energy split is linked to volumetric and stiffness ratios so the energy distribution derived from DEM must be adjusted before applying the energy split sub-model proposed in Eq. (17) and Eq. (18).

The stiffness must be known in order to calculate the energy split. Here the stiffness is estimated by the relationship [24]:

$$k = \frac{Y}{1-\mu} \tag{37}$$

where Y is Young's modulus and μ is Poisson's ratio. The Young's Modulus for a crystal of magnetite is between 175 and 230 GPa, and the Poisson ratio is assumed to be between 0.26 and 0.33 [29]. The magnetite stiffness in this paper was estimated at $k_m = 230GPa$. Silicates have a range of stiffness depending on their structures, grains, densities, and elements. The stiffness of silicate gangue here was estimated at $k_s = 350GPa$. Thus Eq.(15) and Eq.(16) give $\psi_m = 0.6$, ψ_s =0.4. If the volumetric ratio of magnetite changes from 0 to 1, the collision probabilities can be calculated with Eq. (6), Eq. (7) and Eq. (8). In order to observe the trends of

the split factor under different stiffness ratios, two cases were calculated ($k_m = 230GPa, k_s = 350GPa$ and $k_m = 100GPa, k_s = 350GPa$). The energy split adjustment factors can be calculated with Eq. (22) and Eq. (23). The resulting energy split share is presented in Table 4, Figure 5, and Figure 6.

Table 4 Probability-based energy split model

Volumetric ratio		Collision Probability			Case 1 k_m=100Gpa; k_s=350GPa				Case 2 k_m=230GPa; k_s=350GPa			
					Energy split ratio		Adjustment factors		Energy split ratio		Adjustment factors	
Mag.	Sil.	S-S	M-M	S-M	Mag.	Sil.	Mag.	Sil.	Mag.	Sil.	Mag.	Sil.
0.00	1.00	1.00	0.00	0.00	0.00	1.00	1.56	1.00	0.00	1.00	1.21	1.00
0.10	0.90	0.81	0.01	0.18	0.15	0.85	1.50	0.94	0.12	0.88	1.19	0.98
0.20	0.80	0.64	0.04	0.32	0.29	0.71	1.44	0.89	0.23	0.77	1.17	0.96
0.30	0.70	0.49	0.09	0.42	0.42	0.58	1.39	0.83	0.34	0.66	1.14	0.94
0.40	0.60	0.36	0.16	0.48	0.53	0.47	1.33	0.78	0.45	0.55	1.12	0.92
0.50	0.50	0.25	0.25	0.50	0.64	0.36	1.28	0.72	0.55	0.45	1.10	0.90
0.60	0.40	0.16	0.36	0.48	0.73	0.27	1.22	0.67	0.65	0.35	1.08	0.88
0.70	0.30	0.09	0.49	0.42	0.82	0.18	1.17	0.61	0.74	0.26	1.06	0.86
0.80	0.20	0.04	0.64	0.32	0.89	0.11	1.11	0.56	0.83	0.17	1.04	0.83
0.90	0.10	0.01	0.81	0.18	0.95	0.05	1.06	0.50	0.92	0.08	1.02	0.81
1.00	0.00	0.00	1.00	0.00	1.00	0.00	1.00	0.44	1.00	0.00	1.00	0.79

If the components have the same stiffness, the energy is shared solely according to the volumetric ratio (refer to the straight lines in Figure 5) and the adjustment factor equals to 1. For the soft component magnetite, the energy split ratio has been amplified by the stiffness ratio relative to the energy sharing solely by the volumetric ratio (refer to the curves above the diagonal line in Figure 5). This is because the hard components serve as grinding media and can make a contribution to the breakage of soft components, especially in AG mills where there is no steel grinding media. The softer the soft component, the more energy share it will receive and the less energy the hard component will absorb (Figure 5). The greater the stiffness difference, the greater the adjustment factor. If the components have the same stiffness, the adjustment factor is 1 (Figure 6). For example, when the stiffness of the soft component k_m decreases from 230GPa to 100GPa at the magnetite volumetric ratio of 0.8, the adjustment factor for Magnetite increases from 1.04 to 1.11 and that for Silicate decreases

from 0.83 to 0.56. With the soft component magnetite's volumetric ratio increasing, the adjustment factor of magnetite decreases. With the hard component silicate's volumetric ratio increasing, the adjustment factor of silicate increases (Figure 6).

From Eq. (22) and Eq. (23), the slope of Magnetite and Silicate in Figure 6 is (the volumetric ratio of Magnetite is the x-axis):

$$Slope = \frac{k_m - k_s}{k_m + k_s} \tag{38}$$

In this case, given that the volumetric ratio of magnetite and silicate are 0.66 and 0.34 respectively and the stiffness is 230GPa and 350GPa respectively, the adjustment factor for magnetite and silicate are 1.07 and 0.86 respectively ((Eq.(22) and Eq.(23)). The total energy was shared between magnetite and silicate via Eq.(17) and Eq.(18), where both volumetric ratios and stiffness ratios were considered. Then, the split energy was distributed proportionally according to the adjusted DEM energy distribution results.

The original energy distribution information provided by DEM simulation was based on volumetric ratios. It was adjusted with stiffness ratios via Eq.(22), Eq.(23), Eq.(28) and Eq.(29). As mentioned in section 2.1, it is not fully quantitatively known how the stiffness affects energy sharing. More test data and mechanism analysis are needed for the improvement of the energy split model. The multi-component model based on the above energy split theory is illustrated in Figure 7.

As one part of GTMMS III, this multi-component structure is based on a probability-based energy split scheme and the DEM energy distribution and considers the interactions between the components. The process of Simulation 2 can be divided into four steps: the first step is to calculate the energy split share of each component; the second step is to determine the appearance function parameters of each component; the third step is to calculate each component separately; the last step is to combine the outcomes of each component and derive the results for the mixture ore.

4. Results and discussion

The LKAB KA2 plant survey data used to validate GTMMS III was conducted in steady state and although GTMMS III is a dynamic model, the steady-state preditions can be evaluated against the survey data. The AG mill model is divided into six virtual segments along the flow direction to integrate the transport function. It is assumed that the mixture ore feed is added to the empty mill from zero to 453 t/h by step change.

Figure 8 shows the external and internal classification (The former is related to discharge function, and the latter is related to transport function [1]) used in Simulation 1. The internal classification function curve is a trilinear polyline in a semi-logarithmical coordinate system. It has a bypass above 1 mm to allow the coarser contents to flow through the mill. This internal classification function can be back-fitted, but the form has not been validated, being an area for on-going development.

Figure 9 shows the product size distribution evolution along the segments. By using a segmented mill with an internal classification function, the content particle size is getting finer gradually from the inlet to the outlet. The final overall product size distribution prediction is matched to the measured plant data with model parameters back-fitted as noted later.

The results of Simulation 2 are shown in Figs 10 and 11. With the DEM energy distribution information corrected by the proposed energy split scheme, the GTMMS III calculated the magnetite and silicate separately. It is supposed that both the Simulation 1 and Simulation 2 (i.e. the mixture, and the magnetite and the silicate) has the same external classification because the discharge grate is unchanged (Fig.8 and Fig. 10). The internal classification functions for the mixture, magnetite and silicate are different and were estimated to provide a flow resistance according to size between slices in the mill but with a constant ratio above a certain size, as shown in Figure 10. This transition size was back-fitted to be 1 mm for magnetite and over 10 mm for silicate because they could not be determined from the experimental data. Changing the form of this internal classification function will change the residence time of particles along the mill and represents an area for future refinement of this model structure. The silicate has a larger transition size (over 10 mm) and a higher bypass ratio (about 63%) in Figure 10. This is because the silicate is the harder component and it partly serves as grinding media for the soft component magnetite when moving towards the mill outlet. This back-fitted relationship is based on the premise that the more competent component (i.e. silicate in this case) survives longer and accumulates at the discharge end of the mill where it is slowly broken before discharging. This assumption is consistent with the experimental observation of Mwansa, Condori and Powell [30] who, for a mill that was grinding a blend of soft and hard components, sized the contents in four separate slices along the mill. Because the magnetite accounts for 86% (mass ratio) in the ore mixture, it has similar internal classification behaviour to the mixture (Figure 8 and Figure 10). Further experiments are needed to improve this simple transport model.

The separate simulation for silicate and magnetite were validated against the plant data of silicate and magnetite, and good agreement was achieved (Figure 11). For each size class, the cumulative percentage passing is obtained for silicate and magnetite through separate simulation. The total mixture mass in each size class is the sum of silicate mass and magnetite mass in that size class. This is plotted in Figure 12 together with the results from Simulation 1.

The P80 of product size of the magnetite, the silicate, and the mixture is around 0.3 mm, 3 mm and 0.4 mm respectively (Figure 11). Both predictions of Simulation 1 and Simulation 2 by GTMMS III can provide the product size distribution of the mixture and show good agreement with the plant survey data (Figure 12). Simulation 2 implies that the blend grinding can be predicted through understanding the individual components and their interactions in the mixture, whereas the simpler blend simulation 1 cannot be relied upon to predict the mill response to changing ratios of the blends.

In summary, there are many fitting parameters in the simulations of GTMMS III, such as internal classification curves and external classification curves and energy distribution curves for fine particles where the DEM cannot accommodate. Other minor fitting parameters may include the number of segments into which the mill is virtually divided, the relationship by which stiffness affects the energy sharing, etc. Furthermore, GTMMS III involves many empirical relationships such as the 4D appearance functions and the power draw model. The energy split model needs improvement and validation with more data.

Despite all this, GTMMS III offers a semi-mechanistic analytical method based on the combination of separate simulations of individual components with interactions considered through an energy sharing and energy split scheme and through including a transport relationship. The mechanistic solution by GTMMS III is potentially useful in understanding multicomponent grinding in the industry as the model responds to the relative competence and blend ratios of components, so is not dependent on being calibrated to a particular blend. This could dramatically reduce the cost associated with tests with multiple ore blends. Furthermore, the inter-grinding effect in multicomponent grinding (the hard component serves as the grinding media for the soft component) can be utilised in SAG mills to reduce the consumption of grinding balls. Operational optimisation

for multicomponent grinding can be achieved by simulation of GTMMS III. Given the lack of data for internal classification, further experiments and more data are needed to improve the model.

5. Conclusions

In this work, the GTMMS II model reported previously is upgraded to GTMMS III in which the new wide-size-range 4D appearance function and multi-component modelling are integrated. The key to multi-component modelling is energy distribution between the components. An energy sharing model based on the collision probability between the components followed by an energy split according to material stiffness is proposed and applied to modelling of LKAB's milling operations. The multi-component modelling shows the grinding progress of each component and their interaction in grinding. With the DEM energy distribution corrected by the probability-based energy split model, GTMMS III can predict each component and the mixture ore using a uniform external classification function. The different internal classification functions reveal the interactions between the components and indicate that transport plays an important role in multi-component grinding. It is concluded that the competent component survives longer in the mill. Two simulations, Simulation 1 (where the multicomponent ore is treated as a blend) and Simulation 2 (where the ore blend is predicted through each component with interactions considered), were conducted. The results of both simulations were validated against the plant survey data and good agreements were achieved.

Admittedly, GTMMS III involves many empirical relationships. Some sub-models, such as the energy split model and the DEM energy distribution, are not so mature. More plant survey data are needed for further improvement of the model structure. Despite this, GTMMS III provides a semi-mechanistic method to predict ore mixture in the mill through individual component analysis. It also offers a model structure that can accommodate future mechanistic, dynamic mill model evolution.

Due to the integration of the wide-range 4D appearance function, DEM energy distribution model, transport function, discharge function, power model, dynamic modelling and multi-component modelling, GTMMS III is a notable milestone towards the unified comminution model (UCM), the future of mechanistic grinding mill modelling.

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References

[1] P. Yu, W. Xie, L. Liu, M. Powell, Development of a dynamic mill model structure for tumbling mills. , in: J. Yianatos (Ed.) XXVII International Mineral Processing Congress - IMPC 2014 Conference Proceedings, Gecamin Digital Publications Santiago, Chile, 2014, pp. 41-51.

[2] P. Yu, W. Xie, L.X. Liu, N. Weerasekara, B. Bonfils, M. Powell, A generic dynamic model incorporating a 4D appearance function for tumbling mills, XXVIII International Mineral Processing Congress - IMPC 2016 Conference ProceedingsQuébec City, Canada, 2016.

[3] G.R. Ballantyne, M.S. Powell, Benchmarking comminution energy consumption for the processing of copper and gold ores, Minerals Engineering, 65 (2014) 109-114.

[4] T.J. Napier-Munn, S. Morrell, R.D. Morrison, T. Kojovic, Mineral comminution circuits: their operation and optimisation, Julius Kruttschnitt Mineral Research Centre, the University of Queensland, Indooroopilly, Brisbane, Australia, 1996.

[5] L.G. Austin, J.M. Menacho, F. Pearcy, A general model for semi-autogenous and autogenous milling, APCOM 87. Proceedings of the Twentieth International Symposium on the Application of Computers and Mathematics in the Mineral Industries, SAIMM, Johannesburg, South Africa, 1987, pp. 107-126.

[6] M.S. Powell, The unified comminution model - a conceptually new model, IMPC 2006 Conference , Istanbul, Turkey, 2006, pp. 1783-1788.

[7] I. Govender, G.B. Tupper, A.N. Mainza, Towards a mechanistic model for slurry transport in tumbling mills, Minerals Engineering, 24 (2011) 230-235.

[8] J.A. Herbst, J.K. Lichter, Use of Multiphysics Models for the Optimization of Comminution Operations, in: S.K. Kawatra (Ed.) 3rd International Symposium, Society for Mining, Metallurgy and Exploration, Society for Mining, Metallurgy and Exploration (SME), 2006, pp. 447-460

[9] N.S. Weerasekara, M.S. Powell, S. Cole, J. Favier, Modeling breakage environment in tumbling mills using DEM and analyzing the outputs, in: A. Munjiza (Ed.) Discrete Element Methods - Simulation and discontinua: Theory and applications, DEM5: The Fifth International Conference on Discrete Element Methods Queen Mary, University of London 2010.

[10] M. de Paiva Bueno, E. Niva,, M. S. Powell, G. Adolfsson, T. Kojovic,, M. Henriksson, J. Worth, A. Partapuoli, P. Wikström, F. Shi, K. Tano and A. Fredriksson, The dominance of the competent. Proceedings International autogenous and semiautogenous grinding technology (2011), Sep. 25-28, Ed. Flintoff et al, Published CIM.

[11] J.R. Schuhmann, Energy input and size distribution in comminution, Mining Engineering, 12 (1960) 161-164.

[12] R.J. Charles, Energy-size reduction relationships in comminution, Mining Engineering, 9 (1957) 80-88.

[13] W.J. Whiten, A matrix theory of comminution machines, Chem. Eng. Sci., (1974) 585-599.

[14] D.W. Fuerstenau, D.A. Sullivan, Jr, Analysis of comminution of mixtures, Canadian Journal of Chemical Engineering, 40 (1962) 46-50.

[15] M.P. Bueno, T. Kojovic, M.S. Powell, F. Shi, Multi-component AG/SAG mill model, Minerals Engineering, 43-44 (2013) 12-21.

[16] G.W. Delaney, P.W. Cleary, R.D. Morrison, S. Cummins, B. Loveday, Predicting breakage and the evolution of rock size and shape distributions in Ag and SAG mills using DEM, Minerals Engineering, 50-51 (2013) 132-139.

[17] L. Xu, K. Luo, Y. Zhao, Numerical prediction of wear in SAG mills based on DEM simulations, Powder Technology 329 (2018) 353–363.

[18] N. Govender, R.K. Rajamani, S. Kok, D.N. Wilke, Discrete element simulation of mill charge in 3D using the BLAZE-DEM GPU framework, Minerals Engineering 79 (2015) 152–168.

[19] P. Pourghahramani, Effects of ore characteristics on product shape properties and breakage mechanisms in industrial SAG mills, Minerals Engineering 32 (2012) 30–37.

[20] H. Dündar, H. Benzer, Investigating multicomponent breakage in cement grinding, Minerals Engineering 77 (2015) 131–136.

[21] P. Yu, W. Xie, L.X. Liu, M.S. Powell, The development of the wide-range 4D appearance function for breakage characterisation in grinding mills, Minerals Engineering, 110 (2017) 1-11.

[22] K.S. Venkataraman, D.W. Fuerstenau, Application of the population balance model to the grinding of mixtures of minerals, Powder Technology, 39 (1984) 133-142.

[23] D.W. Fuerstenau, K.S. Venkataraman, The comminution of multicomponent feeds under batch and locked-cycle conditions: Kinetics, simulation and energy distribution, International Journal of Mineral Processing, 22 (1988) 105-118.

[24] L.M. Tavares, R.P. King, Single-particle fracture under impact loading, International Journal of Mineral Processing, 54 (1998) 1-28.

[25] R.M. de Carvalho, Mechanistic modelling of semi-autogenous grinding, Thesis submitted in fulfilment of PhD, Metallurgical and Materials Engineering, the Federal University of Rio de Janeiro, Rio de Janeiro, Brazil, 2013, pp. 204.

[26] K. Leung, An energy based, ore specific model for autogenous and semi-autogenous grinding mills, Thesis submitted in fulfilment of PhD, JKMRC, the University of Queensland, Brisbane, QLD, Australia, 1987.

[27] F.C. Bond, The Third Theory of Comminution, Mining Engineering 4(1952) 484-494.

[28] N. Weerasekara, M. Powell, Exploring the breakage environment in mills with discrete element methods, in: R.J. Holmes (Ed.) XXV International Mineral Processing Congress - IMPC 2010 'Smarter processing for the future', 6-10 September 2010 Australasian Institute of Mining and Metallurgy, Carlton, VIC, Australia Brisbane, QLD, Australia 2010, pp. 1357-1365.

[29] D. Chicot, J. Mendoza, A. Zaoui, G. Louis, V. Lepingle, F. Roudet, J. Lesage, Mechanical properties of magnetite (Fe3O4), hematite (α-Fe2O3) and goethite (α-FeO·OH) by instrumented indentation and molecular dynamics analysis, Materials Chemistry and Physics, 129 (2011) 862-870.

[30] S. Mwansa, P. Condori, M.S. Powell, Charge segregation and slurry transport in long SAG mills, Proceedings of the XXIII International Mineral Processing Congress (IMPC 2006) Promed Advertising Istanbul, Turkey 2006, pp. 81-86.