

Power Spectral Density Model for Pedestrian Walking Load

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Abstract: Intensive vibrations may occur in slender structures like footbridges and long-span floors due to movement of pedestrians. Problems are usually treated in time domain as Fourier series models of the forcing function, but most methods have disadvantages of neglecting the stochastic character of human walking, being computationally inefficient for random vibration analysis and overestimating responses in the case of resonance. Meanwhile, frequency domain models of other types of structural loading are efficient while being a more acceptable approach widely adopted for dealing with stochastic response problems. Hence, an experiment-based power spectral density (PSD) model normalized to walking frequency and order of harmonic is proposed. To construct this model, 1528 individual walking load time histories were collected from an experiment on a rigid floor. These records were then linked to obtain a smaller number of longer samples for a good frequency resolution in spectral analysis. Using the linked samples and for frequency normalized to mean walking frequency, PSD models for the range 1 ± 0.05 for the harmonic and the sub-harmonic are suggested as Gaussian mixture with eight model parameters. Via the stationary and non-stationary stochastic vibration theory, the proposed model is used to predict the structural response in terms of root-mean-square and peak of acceleration. The framework is finally tested via field measurements demonstrating applicability in practical design work.

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23 **Keywords:** Walking load; Power spectral density; Stochastic vibrations; Vibration serviceability.

24 **Introduction**

25 Due to the significant advance and wide application of light-weight and high-strength construction
26 materials, structures like long-span floors and footbridges are even more distinctly characterized with
27 lightness and slenderness. Accounting for the low natural frequency and damping ratio of this type of
28 slender structure, when human activities like walking and jumping take place, excessive unpleasant
29 vibration might appear. In history, many buildings and bridges have experienced this human-induced
30 vibration problem, with occasional but rare tragic consequences. The most infamous case is the
31 Millennium Bridge in London which was caused to vibrate dramatically due to crowd loading (Dallard et
32 al. 2001). In 2013, a pedestrian bridge in Jiangxi Province, China collapsed when a group of tourists
33 crowded into the bridge, causing some people to fall into the water. To avoid such occurrences, a better
34 understanding of the human-induced load on such structures is in great demand.

35 Numerous researchers have studied this field: Blanchard et al. (1977), Bachmann and Ammann (1987),
36 Rainer et al. (1988), Allen and Murray (1993), Petersen (1996), Kerr (1998), Yoneda (2002) and Chen et al.
37 (2014) have put forward deterministic walking load models in Fourier series. These previous researches
38 mainly focused on dynamic load factors (DLF), coefficients for the deterministic models of walking load
39 in time domain. However, such time-domain models for walking are based on an assumption that both
40 human feet produce exactly the same force and that the force is periodic. In fact, human walking load is a
41 kind of stochastic load by nature because people cannot maintain identical amplitude and duration in each
42 walking step. The proposed models neglect the inter-subject variability (e.g. walking frequency, velocity,
43 force amplitude, and body weight differing between humans) and the intra-subject variability (e.g. the
44 inability of humans to repeat the same force in each step) in the walking process (Živanović et al. 2007).
45 Moreover, though simple in concept and application, deterministic models can overestimate structural

46 response in resonance, especially when resonant with higher orders of harmonics of walking load, as
47 pointed out by Brownjohn et al. (2004) for walking load, and by Racic and Pavic (2010) for jumping load.
48 This is owing to the energy leakage around each harmonic center frequency in real walking, instead of the
49 perfect concentration of energy in deterministic models. Furthermore, the higher the harmonic, the greater
50 the spread of energy. This phenomenon is a result of the stochastic nature of human walking as mentioned.

51 Probabilistic models, which take account of the randomness, have then been proposed. In time domain,
52 such models have been suggested by Živanović et al. (2007, 2015) and Racic and Brownjohn (2010). The
53 probabilistic models in time domain are quite complicated for stochastic problem analysis, requiring time
54 history calculation and Monte Carlo simulation, which is time-consuming particularly for complex
55 structures (Živanović et al. 2010, Piccardo & Tubino 2012, Caprani 2014). With the help of random
56 vibration theory, load models in frequency domain are relatively easy to use in obtaining the structural
57 response and are more acceptable and widely adopted in dealing with stochastic problems in earthquake
58 and wind engineering (Haselton et al. 2011, Huang et al. 2015). The theories of stationary and
59 non-stationary random processes for application to vibration serviceability appeared earliest in the 1980s
60 (Ohlsson 1982) and later, Brownjohn et al. (2004) first proposed a spectral model based on experimental
61 data from three test subjects when determining the model parameters. The model was presented in the
62 form of Fourier amplitude spectra, which was not scaled to the walking frequency and order of harmonic.
63 Thus, parameters of the first six harmonics in the spectra were identified separately by curve fitting.
64 Drawbacks of this model are that merely three test subjects were involved in the experiment which is not
65 enough for describing inter-subject variability; the spectrum neglects the sub-harmonics which is distinct
66 particularly for the fundamental; the spectrum is only suggested around a certain frequency range of each
67 harmonic and finally energy for higher harmonics presents noticeable overlap in the spectrum. Thereafter,
68 Caprani (2014) took advantage of the previous model (Brownjohn et al. 2004) by digitizing their

69 experimental data to provide a new DLF spectral model, which solves the overlap problem but still has the
70 similar deficiency of limited records. Piccardo and Tubino (2012) and Ferrarotti and Tubino (2016)
71 suggested models for crowd load with non-dimensional parameters derived from the Fourier series
72 walking load function, and the final model considers only the first harmonic. Li et al. (2010) studied the
73 complex crowd-footbridge resonant vibration mechanism with the random vibration approach. Krenk
74 (2012) developed a spectral model with respect to bandwidth of pedestrian load and a compact explicit
75 formula to estimate the structural response. Looking through all these models, large volumes of testing
76 data is prerequisite to represent the randomness of walking load and thus to develop a more accurate
77 experimental spectral model. In addition, applying parallels to the approach for modelling random wind
78 gust loading, the spectral model is better presented in a non-dimensional form, as employed in this
79 research. Thus, a unified function can be adopted to model all the main and sub harmonics.

80 In this paper, an experiment to collect walking record was conducted, serving as the data source for
81 power spectral density (PSD) modelling. The PSD model is proposed with a special linking method to
82 obtain samples with longer duration, and the model parameters are determined accordingly. To predict
83 structural response, including root-mean-square of acceleration for continuous vibrations as well as the
84 peak acceleration, a methodology making use of stochastic vibration theory is introduced. Field
85 verifications of this methodology on a floor model in the lab and an as-built floor in a railway station were
86 carried out via a comparison of calculated and experimental result. Some limitations and further
87 development of the proposed model are also discussed in the end.

88 **Experiments for collecting walking load samples**

89 A large number of individual walking load data is necessary to develop a PSD model for a stochastic
90 dynamic excitation, and force plates are commonly adopted in walking load measurement. The force plate
91 can only record a single footfall trace (SFT) due to its relatively small size and fixed installation position.

92 The SFT records are too short, typically less than one second, for spectral property investigation.
93 Although an instrumented treadmill can be used to measure continuous walking load forces (Brownjohn et
94 al. 2004), there are reservations about potential influence of the predefined treadmill belt speed on the test
95 subject's gait (thus variability of walking force). To tackle this problem, a novel wireless insole pressure
96 system (Pedar-x, Germany) was used to measure the continuous walking load time history. While a person
97 was walking, steps were recorded at sampling frequency of 100 Hz with instrumented insoles in the
98 subject's shoes. The Pedar system recorded the pressure distribution under the foot sole using one hundred
99 micro pressure sensors and transmitted the records to data acquisition center (a laptop) wirelessly via
100 Bluetooth technology (Fig. 1). On the laptop, the analysis software Novel helped to collect and present
101 the pressure data, and provided the vertical force and the center of pressure of each foot estimated by
102 integrating all the pressures. The Pedar system has been widely used in gait and biomechanical studies and
103 its measurement accuracy is well acknowledged (Lee and Hong 2005, Forner Cordero et al. 2004).
104 Nevertheless, the Pedar system used in this study was calibrated by comparing its measurements with
105 those from force plate. Moreover, for each test case the standard test protocol was strictly followed to
106 reduce pressure sensor's zeroing effect. As an example, Fig. 2a show time histories of the measured
107 walking load of the left, right foot and their summation, the Fourier amplitude spectrum of the total force
108 is shown in Fig. 2b.



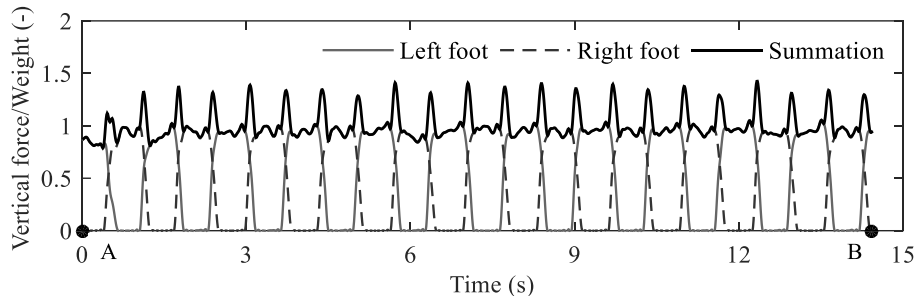
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Fig.1. Test subject with Pedar insole system in the experiment

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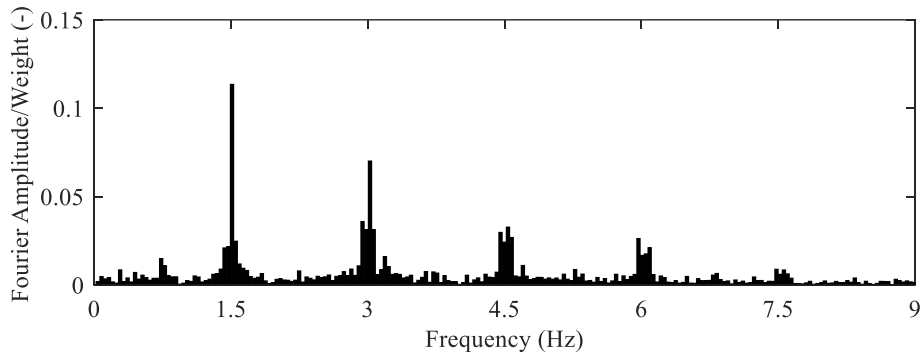
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(a) Normalized vertical force of left, right foot and their summation

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(b) Fourier amplitude spectrum of total force

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Fig. 2. Typical testing samples at $f_p = 1.50$ Hz

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Fifty-six test subjects participated in the experiment. Statistical features of all the test subjects are shown in Table 1. Each test subject performed 11 test cases, including eight cases with fixed walking frequencies f_p (1.50, 1.65, 1.75, 1.80, 1.95, 2.00, 2.10, and 2.25 Hz) and three free-walking cases (self-controlled slow, normal, and fast rates). The eight fixed walking frequencies cover the range for a test subject to maintain normal gait. Each frequency was instructed by a metronome, e.g. 90 beats per minute for 1.5 Hz. For each case, each test subject walked along a 40 m long path on a rigid floor and repeated three times. The test protocol satisfied the requirements by Tongji Medical Ethics Committee. In total, 1528 continuous walking loads were collected from the experiment.

Table 1. Statistics of age, body mass and height of the test subjects

Gender	No.	Age (Year)			Body Mass (kg)			Height (cm)		
		Mean	Std.	Range	Mean	Std.	Range	Mean	Std.	Range
Male	39	24.0	2.52	20-29	68.3	15.00	49.8-95.1	174.6	47.58	156-197
Female	17	23.7	6.37		60.3	4.98		162.6	21.28	

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129 **Power spectral density model**

130 This section describes the development of a PSD model for individual walking load that is treated as a
 131 narrowband stationary stochastic process in time domain. The same assumption was adopted in the work
 132 of Brownjohn et al. (2004) and Eriksson (1994). A mathematical expression for PSD is first proposed
 133 whose parameters are identified from the experiment records.

134 *Basic equations*

135 The PSD of a stationary stochastic process $x(t)$ is defined as (Wirsching and Paez 2006):

$$136 \quad S_x(f) = \lim_{T \rightarrow \infty} \frac{1}{T} E[|X(f, T)|^2] \quad -\infty < f < \infty \quad (1)$$

137 where $E[\square]$ is the ensemble average, $|\square|$ means the absolute value, f is frequency in Hertz, T and
 138 $X(f, T)$ are duration and Fourier transform of $x(t)$

$$139 \quad X(f, T) = \int_{-\infty}^{+\infty} x(t) e^{-i2\pi ft} dt \quad (2)$$

140 To avoid use of negative frequency, single-sided PSD is defined as

$$141 \quad G_x(f) = \begin{cases} 2S_x(f) & f > 0 \\ S_x(f) & f = 0 \\ 0 & f < 0 \end{cases} \quad (3)$$

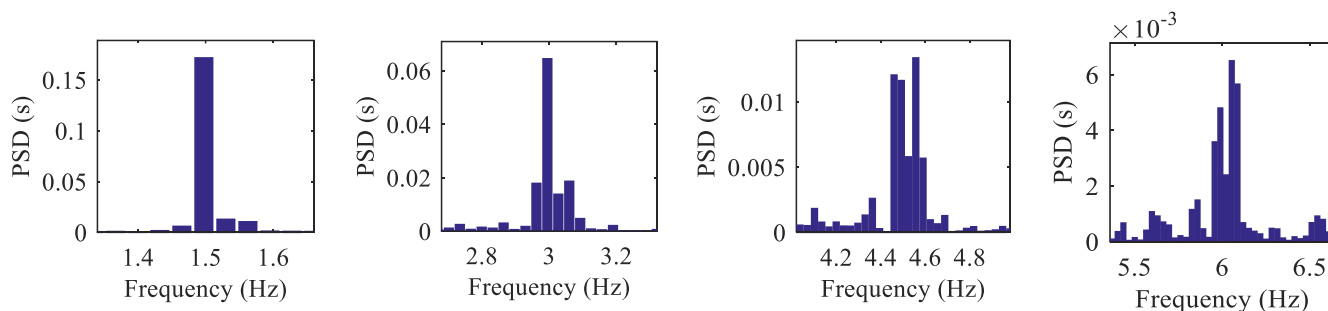
142 *Construction of new samples*

143 Generally, in practical work, with limited number of samples of limited durations, PSD is usually

144 calculated through the periodogram method: PSD of each measured record, i.e., sample realizations of the
 145 stochastic process, can be obtained as sample PSDs and their average is taken as the PSD of the stochastic
 146 dynamic excitation. Equation (2) indicates that in theory a sample with infinite duration is required for
 147 PSD calculation. Moreover, because the walking load is a near-periodic random process, its PSD curve is
 148 expected to have a sharp change at each peak's adjacent region (Brownjohn et al. 2004). A fine frequency
 149 resolution is therefore imperative for describing local features around each PSD's peak. The duration of
 150 the original recorded sample is typically 20-36 s leading to a frequency resolution up to 0.05 Hz, which is
 151 very poor for a near-periodic process. As an example, Fig. 3 shows the PSD, around the first four
 152 harmonics, of a 1.5 Hz walking test sample whose actual frequency resolution is $df = 0.0325$ Hz.

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(a) The 1st harmonic

(b) The 2nd harmonic

(c) The 3rd harmonic

(d) The 4th harmonic

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Fig. 3. The PSD of a walking load sample normalized to body weight at $f_p = 1.50$ Hz ($df = 0.0325$ Hz)

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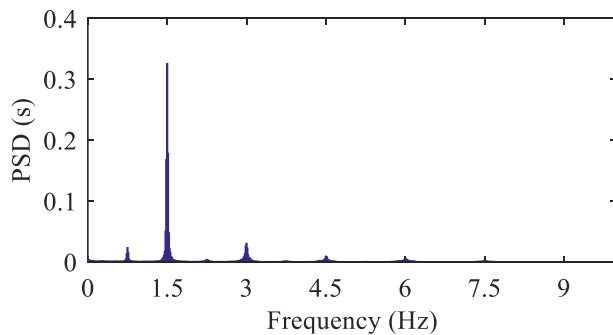
To tackle the resolution problem, new samples of longer duration are constructed using original records by the following steps. Each original record, summation of records of both feet, has been firstly normalized by the test subject's static body weight. The normalized record was then truncated from one foot's initial contact with the ground of the first footfall (i.e., point A in Fig. 2a) to the same foot's final contact with the ground of the last footfall (i.e., point B in Fig. 2a) in the time history. The dominant frequency of each truncated record of around 20 s was identified from its Fourier amplitude spectrum as shown in Fig. 2b. Those records whose dominant frequencies are not in the range $f_p \pm 0.025$ Hz were taken

165 as unqualified ones and excluded from further analysis, where f_p is the specified walking frequency in the
 166 test. All the qualified records from the same test case (i.e. the same prompted frequency f_p) can be assumed
 167 as samples of the same random stationary process. They were then connected end-to-end to form a new
 168 sample. The durations of the constructed new samples are 3200, 2600, 4000, 2200, 2000, 3000, 1800, and
 169 1800 seconds for walking frequencies 1.50, 1.65, 1.75, 1.80, 1.95, 2.00, 2.10, and 2.25 Hz, respectively.
 170 Run tests (Bendat and Piersol 2000) were conducted on all the new samples, and they all passed the test
 171 with a significance level of 0.05 and a test segment number of 50, indicating that the stationarity
 172 assumption is tenable for all the constructed new samples.

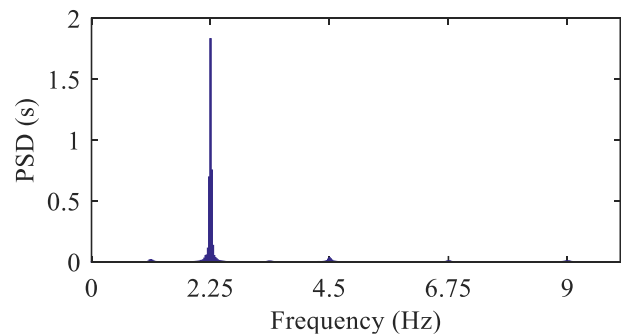
173 ***Power spectral density of new sample***

174 To determine the PSD of the new samples, Welch method (Welch 1967) with Hanning window
 175 (Grandke 1983) was used: each new sample was divided into several segments (around 40-70 segments)
 176 with fifty percent overlap. Duration of each segment was so selected that it was long enough for a good
 177 frequency resolution should be N (integer) times over the test pacing period $T_p = 1/f_p$. The second criterion
 178 was adopted to avoid numerical leakage in Fourier analysis and for walking frequency normalization later.
 179 Figure 4 shows the so-obtained PSD $G_{X_w}(f)$ (where X_w denotes the walking load) for 1.50 and 2.25 Hz
 180 test walking frequency cases, the frequency resolution in both cases is $df = f_p/100$, i.e. the duration of each
 181 segment is 100 times T_p .

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183 (a) $f_p = 1.50$ Hz



184 (b) $f_p = 2.25$ Hz,

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Fig. 4. The PSD $G_{x_w}(f)$ of the new samples normalized to body weight with $df = f_p/100$

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Note from Fig. 4 that the walking load energy mainly distributes around the first several harmonics (i.e.,

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frequencies at 1, 2, 3, and 4 times of f_p) and sub-harmonics (i.e., frequencies at 0.5, 1.5, 2.5, and 3.5 times

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of f_p). The distribution range becomes more pronounced for higher harmonics. This phenomenon justifies

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once again the narrowband stochastic process assumption for walking load. Based on the above

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observation, spectral modelling is focused mainly on a range of each harmonic or sub-harmonic of the

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PSD curve, which is $[0.95nf_p, 1.05nf_p]$ where n is the order of harmonic or sub-harmonic. The energy, in

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other words, the enclosed area of PSD curve within the distribution frequency range, for each harmonic

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(sub-harmonic) can be calculated as

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$$S_n = \int_{0.95nf_p}^{1.05nf_p} G_{x_w}(f) df \quad (4)$$

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The calculation results for the first four harmonics, i.e., $n = 1, 2, 3,$ and $4,$ and the first four

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sub-harmonics, i.e., $n = 0.5, 1.5, 2.5,$ and $3.5,$ are shown in Table 2. To account for the load energy between

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each harmonic later, total energy of the PSD curve in the whole frequency range (0-50 Hz) was also

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calculated and given in Table 2 as S_0 .

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$$S_0 = \int_0^{50} G_{x_w}(f) df \quad (5)$$

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Table 2. PSD energy normalized to body weight of every harmonic S_n (10^{-3})

f_p (Hz)	Harmonics				Sub-harmonics				Total	$\Sigma S_{\text{sub}} / \Sigma S_n^*$ (-)	$\Sigma S_n / S_0$ (-)
	S_1	S_2	S_3	S_4	$S_{0.5}$	$S_{1.5}$	$S_{2.5}$	$S_{3.5}$	S_0		
1.50	12.72	2.48	1.02	0.86	0.87	0.35	0.21	0.15	22.61	8.47%	83%
1.65	21.05	1.91	0.85	0.92	0.89	0.44	0.20	0.13	30.73	6.29%	86%
1.75	28.76	1.48	0.90	1.03	1.12	0.51	0.21	0.18	40.03	5.91%	85%
1.80	33.52	1.34	0.85	1.05	0.94	0.49	0.16	0.09	43.38	4.37%	89%
1.95	52.63	2.33	0.76	1.19	1.47	0.75	0.22	0.13	63.47	4.32%	94%
2.00	56.25	2.30	0.83	1.29	1.21	0.70	0.22	0.13	66.92	3.59%	94%
2.10	62.73	2.84	0.80	1.22	1.17	0.67	0.20	0.12	74.17	3.10%	94%
2.25	84.76	3.33	1.05	1.46	1.20	0.75	0.23	0.14	99.09	2.50%	94%

* $\Sigma S_{\text{sub}} = S_{0.5} + S_{1.5} + S_{2.5} + S_{3.5}$, $\Sigma S_n = \Sigma S_{\text{sub}} + S_1 + S_2 + S_3 + S_4$.

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Non-dimensional experimental PSD model

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Note from Table 2 that the first four harmonics and sub-harmonics contain most of the walking load

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energy, so rationally, they are considered in the following modelling process. First, the PSD values within

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$[0.95nf_p, 1.05nf_p]$ of each curve are divided by its corresponding S_n , i.e., the energy for each harmonic as

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shown in Table 2, thus making the enclosed area of each curve to be unity. Second, the frequency axis was

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divided by walking frequency f_p and the amplitude axis multiplied f_p , leading to the eight

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frequency-normalized PSD curves (four harmonics and four sub-harmonics) as shown in Fig. 5 together

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with an average curve. Third, all the eight average PSD curves in Fig. 5 were normalized to their

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corresponding order of harmonic (or sub-harmonic). This was done by normalizing the frequency axis by

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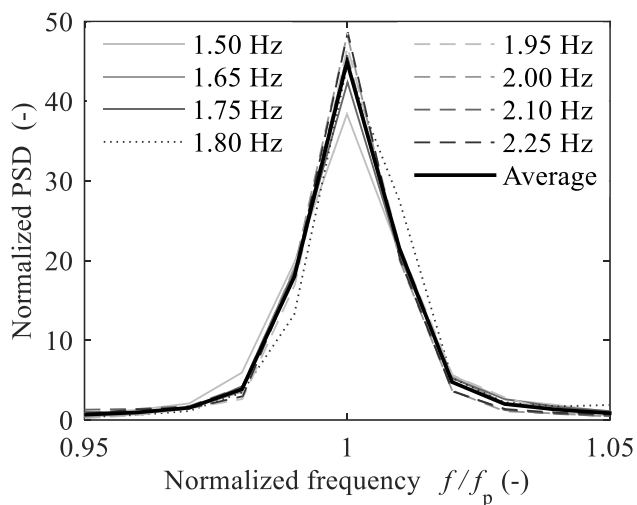
n and multiplying the amplitude axis by n . The normalized curves are shown in Fig. 6a for harmonic case

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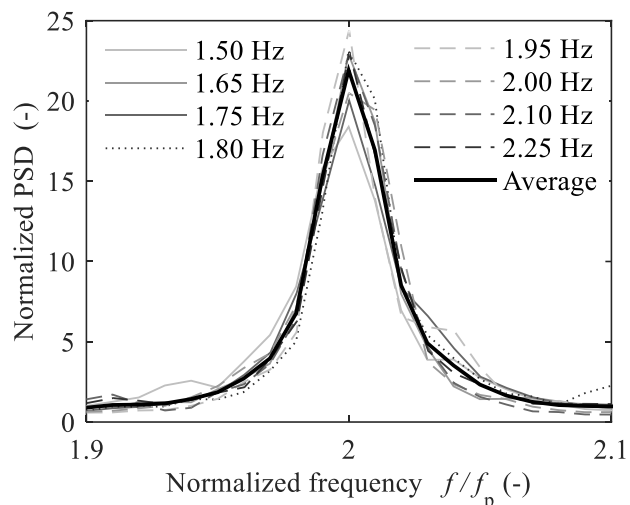
and Fig. 6b for sub-harmonic case.

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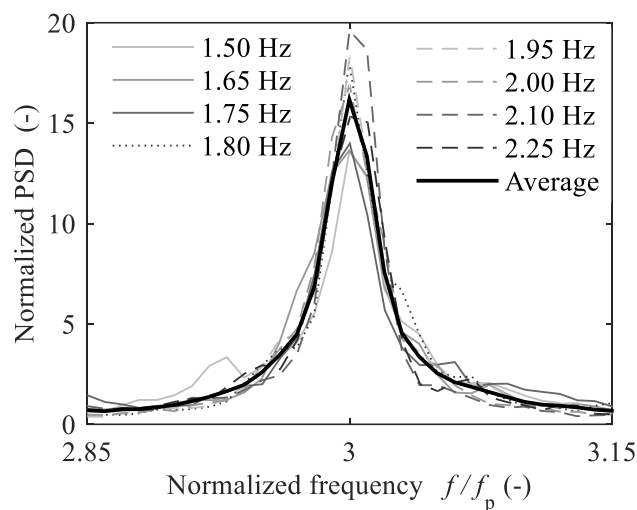


(a) The 1st harmonic

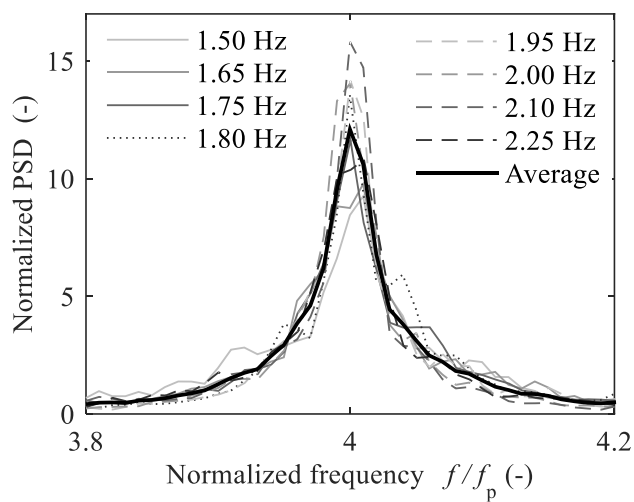


(b) The 2nd harmonic

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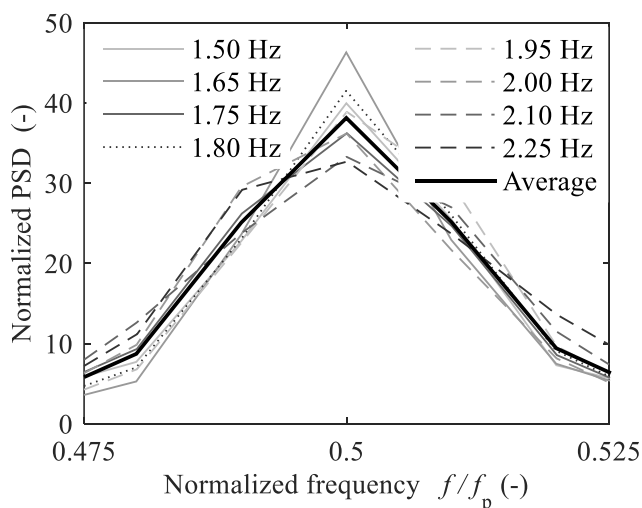


(c) The 3rd harmonic

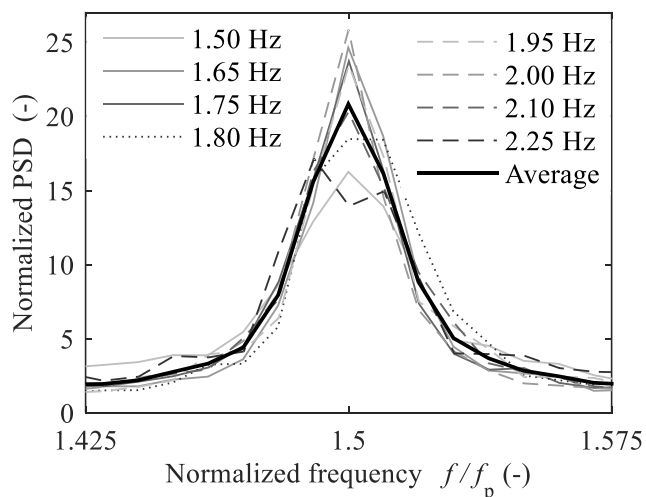


(d) The 4th harmonic

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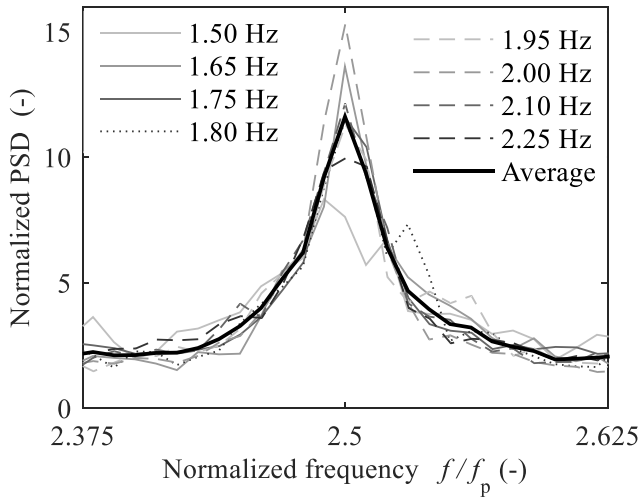
(e) The 1st sub-harmonic



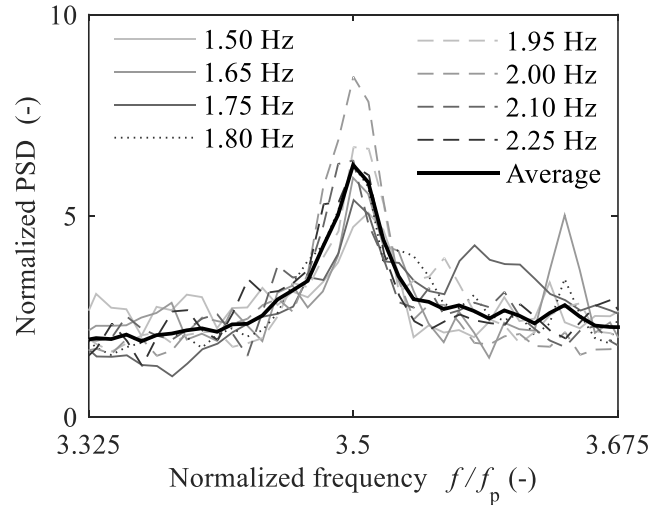
(f) The 2nd sub-harmonic

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(g) The 3rd sub-harmonic



(h) The 4th sub-harmonic

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Fig. 5. Superimposed PSD of the new samples and their average

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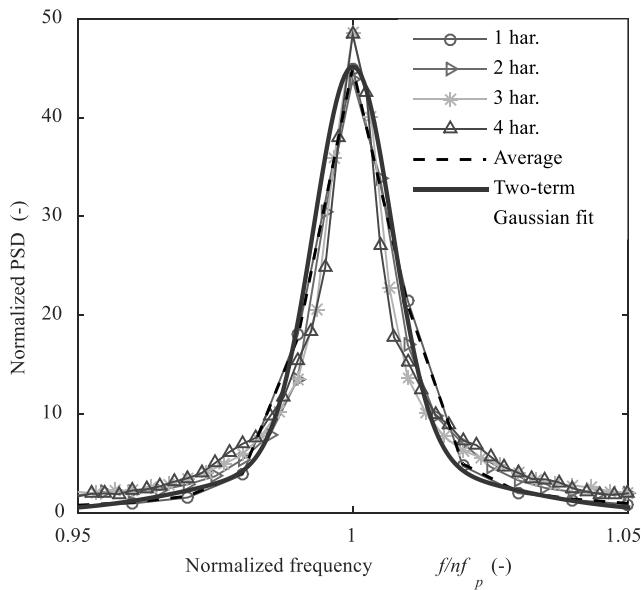
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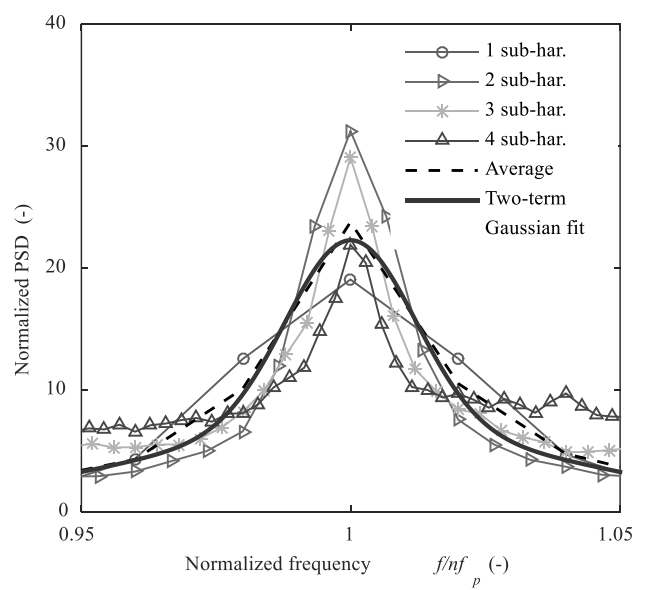
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After order-normalization, for different harmonics, the frequency spacing (i.e., df/n) is not the same, as demonstrated by the symbol of each curve in Fig. 6. Thus, all the curves were resampled to the same length and their average, which was weighted with $E(S_n)$, i.e., mean of S_n of the eight walking frequencies corresponding to each harmonic, was calculated and plotted in Fig. 6.

231



(a) Harmonic



(b) Sub-harmonic

232

233 **Fig. 6.** Normalized PSDs of four harmonics (sub-harmonics), the average and the two-term Gaussian fit
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235 ***Mathematical expression for PSD of walking load***

236 After all the above operations, i.e. normalization of area, frequency, order and taking average, a unified
 237 non-dimensional PSD model can then be developed based on results in Fig. 6. Notice the symmetrical
 238 bell-shape of the average curve in Fig. 6 and by trial-and-error procedure, a Gaussian mixture for the PSD
 239 of walking load at any given walking frequency f_p within [1.5, 2.25] Hz were given as :

$$240 \left\{ \begin{array}{l} G_n(f) = \frac{\beta S_n}{nf_p} \left\{ A_1 \exp \left[-\left(\frac{\bar{f}-1}{\sigma_1} \right)^2 \right] + A_2 \exp \left[-\left(\frac{\bar{f}-1}{\sigma_2} \right)^2 \right] \right\}, \quad f \in [0.95nf_p, 1.05nf_p] \\ G_n(f) \equiv 0, \quad \text{otherwise} \end{array} \right. \quad (6)$$

241 where $G_n(f)$ (unit in second) is the PSD of each harmonic for given walking frequency f_p , normalized
 242 frequency $\bar{f} = f/(nf_p)$ ($n = 0.5, 1, 1.5, 2, 2.5, 3, 3.5,$ and 4), f (unit in Hz) is frequency variable, β
 243 (dimensionless) is an energy compensation factor, S_n (unit in second) is the energy of each considered
 244 harmonic. Parameters in the model, i.e., A_1 , σ_1 , A_2 , and σ_2 (all dimensionless), were determined by fitting
 245 Eq. (6) to the two average curves in Figs. 6a and 6b, and the results are shown in Table 3. Considering the
 246 numerical error of curve-fitting, the fitted parameters A_1 and A_2 were scaled by a factor, which is around 1
 247 ± 0.02 , to make sure the area enclosed by the normalized PSD function (i.e., right side of Eq.(6)) is exactly
 248 unity.

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Table 3. Parameters fitting Eq.(6) of normalized Gaussian fitting PSD

Parameter	A_1	σ_1	A_2	σ_2
Harmonic	40.094	0.0100	5.063	0.034
Sub-harmonic	15.771	0.017	6.515	0.060

251

252 To calculate PSD for walking frequency other than that used in the experiment, S_n in Eq. (6) can be
 253 obtained through Eq. (7) which is fitted by the experiment results of S_n in Table 2, and the fitting results are
 254 summarized in Table 4.

$$255 \quad S_n = af_p^3 + bf_p^2 + cf_p + d, \quad f_p \in [1.5, 2.25] \text{ Hz} \quad (7)$$

256 where $n = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5,$ and 4 .

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Table 4. Parameters for the PSD energy

Para.	Harmonic				Sub-harmonic			Total	
	S_1	S_2	S_3	S_4	$S_{0.5}$	$S_{1.5}$	$S_{2.5}$	$S_{3.5}$	S_0
a	-0.1383	-0.0082	0.0029	-0.0016	0.0011	-0.0017	0.0009	0.0015	-0.0821
b	0.7937	0.0557	-0.0144	0.0089	-0.0052	0.0089	-0.0047	-0.0080	0.4952
c	-1.4124	-0.1193	0.0229	-0.0158	0.0086	-0.0153	0.0081	0.0144	-0.8875
d	0.8122	0.0838	-0.0107	0.0099	-0.0039	0.0088	-0.0044	-0.0083	0.5169

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260 The total energy enclosed by PSD curve is crucial for determining response of a structure subjected to
 261 the stochastic excitation. Since only a certain range of the first four harmonics and the first four
 262 sub-harmonics are considered in the proposed PSD model, their summation is less than the total energy of
 263 the PSD curve (as demonstrated in the last column of Table 2). The factor β is therefore introduced to
 264 balance the total energy of the proposed PSD model and that of the real sample, which is defined as

$$265 \quad \beta = \frac{S_0}{\sum_{n=0.5}^4 S_n} \quad (8)$$

266 where S_0 is calculated by Eq. (7), which is obtained with the same method as S_n in Eq. (7).

267 Finally, the whole PSD $G(f)$ can be obtained from the superposition of $G_n(f)$ of all the

268 harmonics

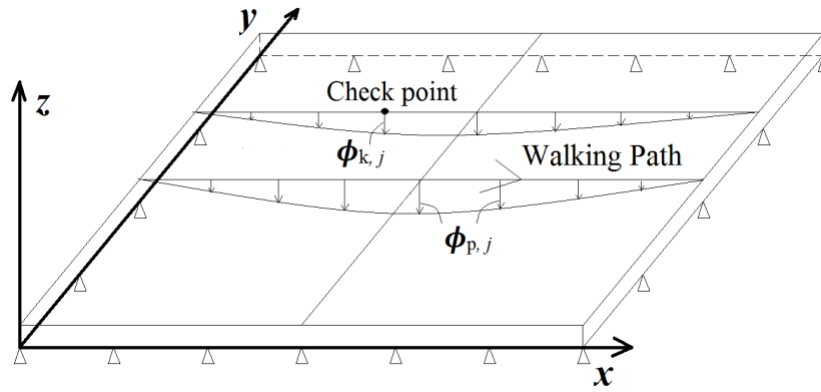
$$269 \quad G(f) = \sum_{n=0.5}^4 G_n(f) \quad (9)$$

270 **Structure response prediction using the power spectral density model**

271 *Root-mean-square response*

272 With the proposed PSD models $G(f)$ of walking load, the root-mean-square (RMS) and the peak
273 acceleration of a floor or footbridge under single pedestrian walking load can be evaluated using
274 stochastic vibration theory (Wirsching and Paez 2006).

275



276

277 **Fig. 7.** Walking path on a structure

278

279 Suppose that a pedestrian walks across a floor along a path as illustrated in Fig. 7, the induced
280 acceleration at a specified location (hereafter termed as check point) is to be calculated. By the
281 mode-superposition method (Chopra 2005), the linear equation of motion of the j th mode of the floor can
282 be expressed as

$$283 \quad \frac{1}{W} (M_j \ddot{q}_j + 2\xi_j \omega_j M_j \dot{q}_j + \omega_j^2 q_j M_j) = \phi_{p,j}(t) x_w(t), \quad (j = 1, 2, \dots, m) \quad (10)$$

284 where q_j , \dot{q}_j , \ddot{q}_j are, respectively, the modal displacement, velocity and acceleration, M_j is the modal

285 mass, ξ_j is the damping ratio, ω_j denotes the natural circular frequency of vibration of the j th mode, W
 286 is the pedestrian's body weight, $x_w(t)$ is the walking load normalized with the body weight W , m is the
 287 total number of vibration modes considered for the floor, $\phi_{p,j}(t)$ is the mode shape value at the step point
 288 of the pedestrian on the floor at time instant t .

289 Unlike a jumping load that usually remains on a fixed location, the walking load changes in a
 290 temporal-spatial manner. The right side of Eq.(10), i.e., the modal force, is non-stationary, though the
 291 walking load $x_w(t)$ is stationary, because $\phi_{p,j}(t)$ changes with time. To deal with this problem, the
 292 spectral representation method with amplitude and frequency modulation proposed by Deodatis (1996) is
 293 adopted which has been commonly used for seismic response simulation.

294 Modal force of the j th mode can be written as

$$295 \quad z_j(t) = \phi_{p,j}(t)x_w(t) \quad (11)$$

296 where $x_w(t)$ is the stationary stochastic process and the amplitude modulating function $\phi_{p,j}(t)$ is not
 297 related to frequency, but only to time.

298 To start with, the evolution cross PSD $G_{z,j,l}(f)$ of the j th and the l th modal forces, $z_j(t)$ and $z_l(t)$, can be
 299 derived from the PSD of $x_w(t)$ (Deodatis 1996), giving

$$300 \quad G_{z,j,l}(t, f) = \phi_{p,j}(t)\phi_{p,l}(t)G(f) \quad (12)$$

301 Moreover, when the energy distribution is relaxed along with time, e.g. taking its average over time, the
 302 evolution PSD of the non-stationary process can be reduced to PSD of a stationary process as an average
 303 of time integral (Liang et al. 2007)

$$304 \quad G_{z,j,l}(f) = \frac{1}{T_z} \int_{-\infty}^{\infty} G_{z,j,l}(t, f) dt = G(f) \frac{1}{T_z} \int_{-\infty}^{\infty} \phi_{p,j}(t)\phi_{p,l}(t) dt \quad (13)$$

305 where T_z is the effective duration of the load $z(t)$.

306 When there are totally N_p steps on the structure, Eq. (13) can be rewritten in a discrete form as:

$$307 \quad G_{Z,j,l}(f) = \left(\sum_{p=1}^{N_p} \phi_{p,j} \phi_{p,l} / N_p \right) G(f) \quad (14)$$

308 where $\phi_{p,j}$ is the j th mode shape value at p th step point.

309 According to the mode superposition method, the single-sided PSD $G_R(f)$ of the acceleration at the
 310 check point will be:

$$311 \quad G_R(f) = \sum_{j=1}^m \sum_{l=1}^m \phi_{k,j} \phi_{k,l} W^2 H_j^*(f) H_l(f) G_{Z,j,l}(f) \quad (15)$$

312 where $\phi_{k,j}$ is the j th mode shape value at the check point (denoted by subscript k), $H_j(f)$ is known as
 313 the complex frequency-response function of acceleration and the asterisk denotes a complex conjugate

$$314 \quad H_j(f) = \frac{-f^2}{M_j(-f^2 + f_j^2 + 2i\xi_j f f_j)} \quad (16)$$

315 where f_j is the j th mode's cyclic frequency of the structure, i is the unit imaginary quantity.

316 Putting Eqs.(10)-(16) together gives $G_R(f)$, and RMS of acceleration α_{rms} is

$$317 \quad \alpha_{\text{rms}}^2 = \int_0^{+\infty} G_R(f) df \quad (17)$$

318 **Peak response**

319 Peak acceleration is deduced by a peak factor η

$$320 \quad \alpha_{\text{peak}} = \eta \alpha_{\text{rms}} \quad (18)$$

321 The peak factor suggested by Vanmarcke (1975) is recommended, which can be used for narrowband
 322 random process. The cumulative probability distribution $P(\eta)$ of peak factor η is defined as

323
$$P(\eta) = \left[1 - \exp\left(-\frac{\eta^2}{2}\right) \right] \exp\left[-v\bar{T} \frac{1 - \exp\left(-\sqrt{\frac{\pi}{2}}\delta^{1.2}\eta\right)}{\exp\left(\frac{\eta^2}{2}\right) - 1} \right] \quad (19)$$

324 where v is a parameter defined as

325
$$v = 2 \left(\frac{m_2}{m_0} \right)^{\frac{1}{2}} \quad (20)$$

326 δ is the bandwidth coefficient defined as

327
$$\delta = \sqrt{\frac{m_0 m_2 - m_1^2}{m_0 m_2}} \quad (21)$$

328
$$m_h = \int_0^{+\infty} f^h G_R(f) df \quad (22)$$

329 where \bar{T} is the duration of steady response of structure, and m_h is the h th spectrum moment of the
 330 stationary stochastic process.

331 For a given guarantee probability q , the corresponding peak factor η can be obtained by

332
$$P(\eta) = q \quad (23)$$

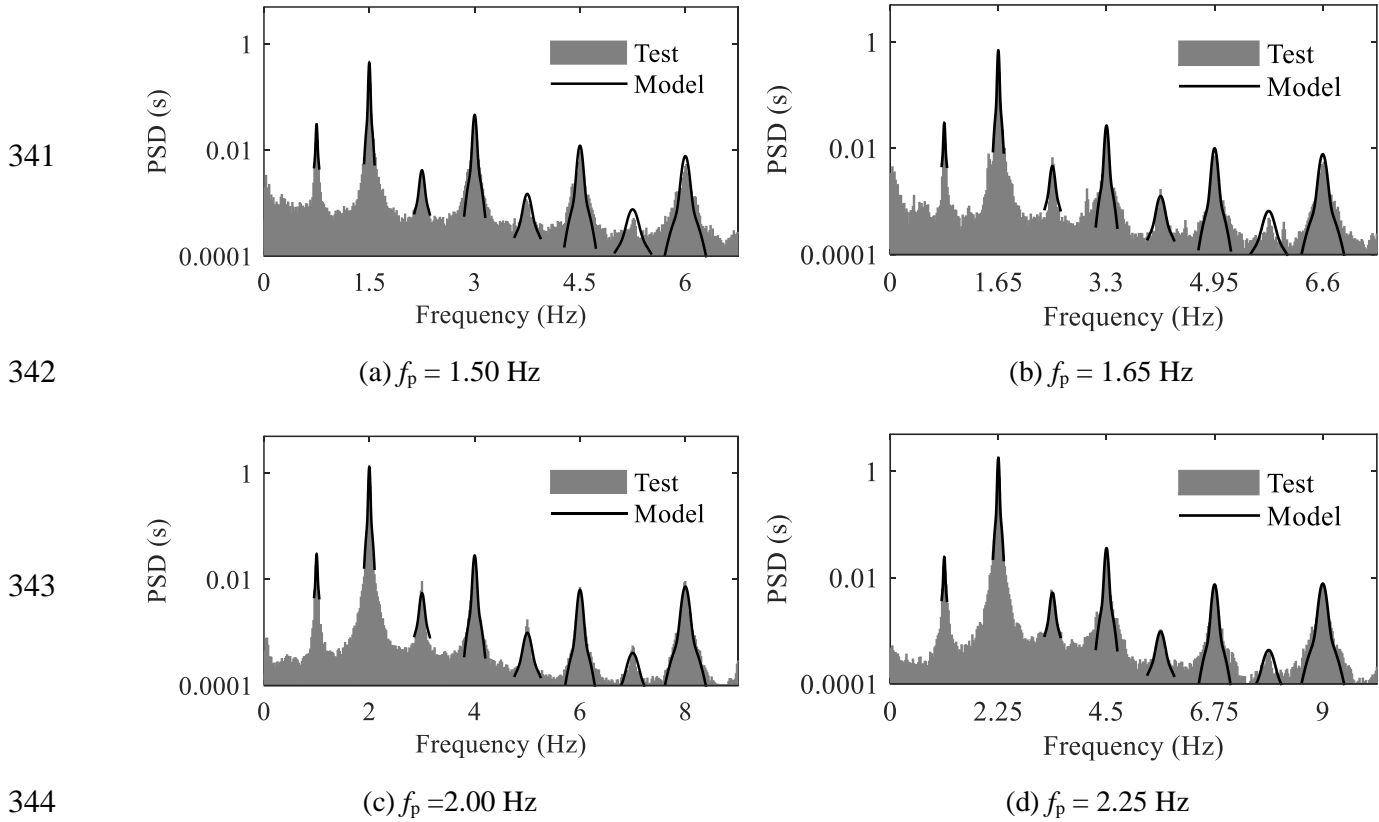
333 Two values as $q = 75\%$ and $q = 50\%$ were used in this study to determine η .

334 **Verification of the proposed PSD model**

335 *Guided walking case*

336 Figure 8 compares the proposed PSDs of $f_p = 1.5, 1.65, 2.0,$ and 2.25 Hz calculated from Eq. (6)-(9)
 337 with those from experiments. It is demonstrated that the proposed PSD model fits well with the
 338 experimental samples. The observation is the same for other test cases with prompted walking
 339 frequencies.

340



344 **Fig. 8.** Comparison of calculated and experimental PSD curves (y axis in logarithmic scale, normalized to weight)

345

346

347 ***Free-walking case***

348 Experimental records in the free-walking tests were not utilized in developing the PSD model and

349 they were therefore adopted to validate the proposed PSD model. For each record, its experimental PSD

350 was compared with the corresponding theoretical PSD, which was obtained through Eqs. (4)-(7)

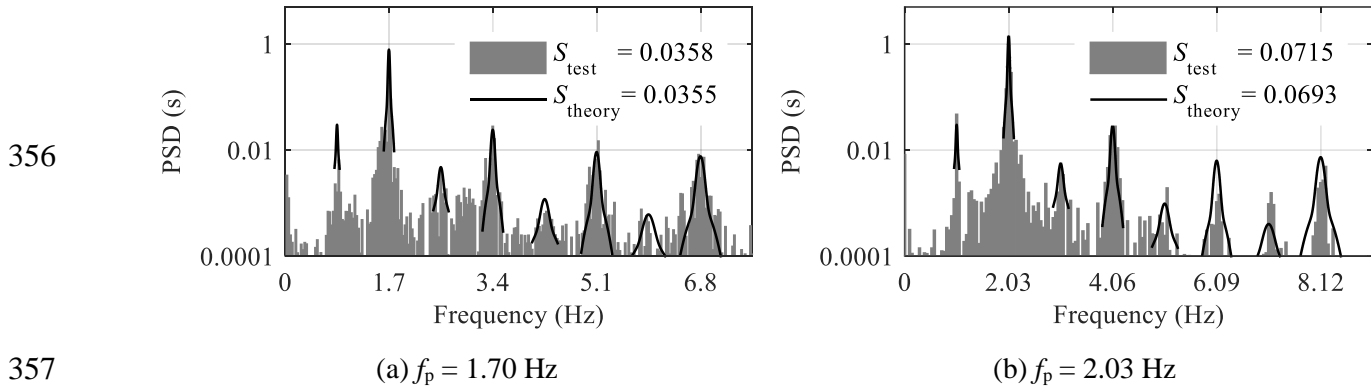
351 assuming f_p equals the dominant frequency of the record. Figure 9 shows results of two examples

352 ($f_p=1.70$ and 2.03 Hz respectively). For both cases, the total spectral energy calculated by the proposed

353 PSD model is in good agreement with the experimental value; the difference is only 0.7% and 0.3% for

354 $f_p=1.70$ and 2.03 Hz in terms of total energy, respectively. The results for other test cases are similar.

355



358 **Fig. 9.** Theoretical and experimental PSD of free-walking cases (y axis in logarithmic scale, normalized to weight)

359

360 **Experiments on floor model**

361 This section compares theoretical response prediction with experimental measurements. To this end, a

362 10 m × 6 m rectangular concrete floor model was constructed in a lab. The model floor had cast-in-place

363 concrete of grade C40 (characteristic value of compressive strength $f_{ck} = 26.8$ N/mm² and modulus of

364 elasticity $E_c = 3.25 \times 10^4$ N/mm²) (National Standard of the People's Republic of China 2010) and a

365 thickness of 110 mm. It was simply supported at two ends of the long span (see Fig. 10a).

366



369 **Fig. 10.** Experimental setup for the floor model

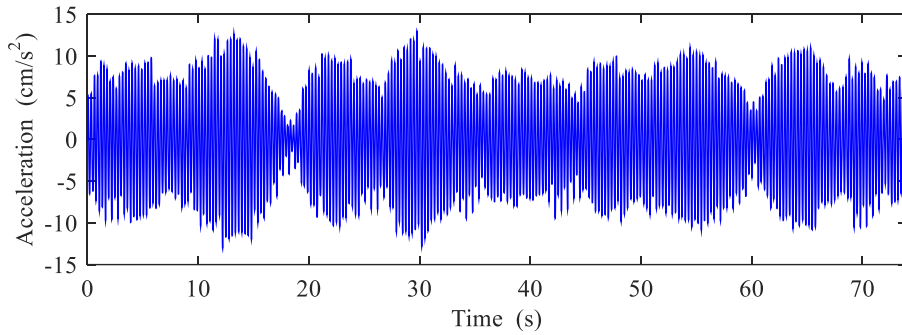
370

371 Fifteen accelerometers (type: Lance LC0132T) were installed beneath the floor to record its responses

372 at various locations including the floor center whose sampling frequency is 200 Hz. The modal properties

373 of the floor were obtained by hammer test. The natural frequencies of the first four vertical vibration
 374 modes are 3.48, 6.14, 6.74, and 14.12 Hz, the corresponding modal masses are 8583, 2587, 9625, and
 375 2423 kg, and the damping ratios are 0.02 for all modes.

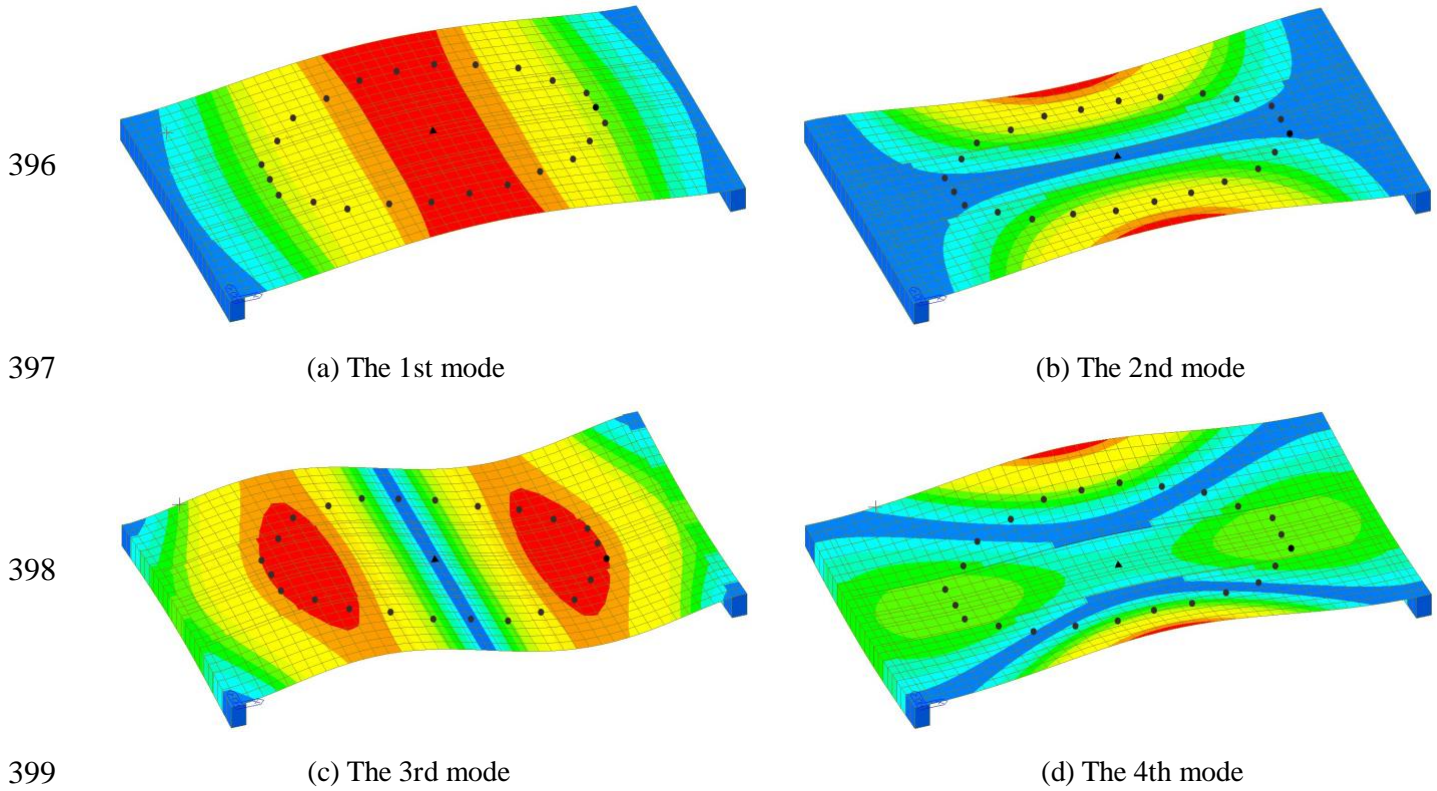
376 As demonstrated in Fig. 10b, a test subject weight 813 N was asked to walk along an oval path (see Fig.
 377 10c) at different walking frequencies guided by a metronome. The steady state acceleration measurement
 378 at the floor center for 1.75 Hz walking frequency is illustrated in Figure 11. The test results, including
 379 RMS for the whole period, peak acceleration and corresponding peak factor, at the floor center for the test
 380 cases at resonant frequency ($f_p = 1.75$ Hz) and three normal walking frequencies ($f_p = 1.5, 2$ and 2.25 Hz)
 381 are summarized in Table 5. The PSD for test response shown in Fig. 12 is obtained via Eq. (1)-(3).
 382



384 **Fig. 11.** Measured acceleration of steady state from the test at $f_p = 1.75$ Hz
 385

386 The theoretical prediction by spectral model was obtained by the following procedures. First, for a
 387 walking frequency f_p in the field test, the corresponding PSD $G(f)$ was obtained from Eqs. (6)-(8). Second,
 388 the evolution cross PSD $G_{z,j,l}(f)$ of modal forces was determined by Eq. (14) where the mode shape
 389 values were extracted from a finite element model (mode shapes are shown in Fig. 12). The first four
 390 vertical vibration frequencies of the FE model were 3.52, 6.16, 8.97, and 13.19 Hz showing a good match
 391 with the measured values. Third, via Eq. (15), the PSD $G_R(f)$ of acceleration at the check point (center
 392 of the floor, as shown in Fig. 10c and Fig. 12) was obtained. Finally, RMS was derived from Eq. (17) and

393 peak acceleration with $P(\eta) = 0.5$ and $P(\eta) = 0.75$ from Eqs. (18)-(22). The predicted results of RMS, peak
 394 acceleration and peak factor of the proposed model at $f_p = 1.5, 1.75, 2,$ and 2.2 Hz are summarized in Table
 395 5.



399 **Fig. 12.** Mode shapes of FE model of the floor with the step points (●) and the check point (▲)

400

401

402 Note from Table 5 that the predicted RMS response at resonance frequency (1.75Hz) is about 9% larger
 403 than the measured value. For other three walking frequencies, the predicted RMS responses are close to
 404 the measured ones with a maximum underestimation of about 20% at 2.2 Hz case. As for peak response,
 405 the predicted values for $P = 0.5$ and $P = 0.75$ are all larger than the experimental values with only one
 406 exception at 2.2 Hz case. The comparison also shows that $P = 0.75$ will overestimate the structure's peak
 407 response especially for resonance situation. Since the floor vibration serviceability problem is not a safety
 408 issue in most cases, a guarantee probability $P = 0.5$ is recommended to predict floor response using the
 409 proposed PSD model. Figure 13 further compares the predicted PSD $G_R(f)$ of response around the

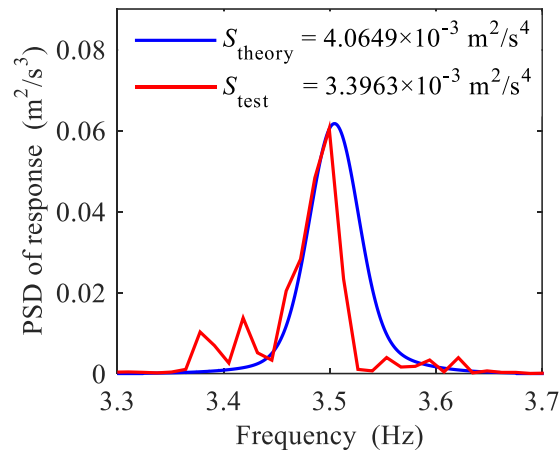
410 resonant frequency at $f_p = 1.75$ Hz with the experimental counterpart, illustrating that they are similar in
 411 shape, and the total energies of the two PSDs are close.

412

413 **Table 5.** Comparison between acceleration response at floor center between field test and theoretical prediction

f_p (Hz)	Dur. (s)	Experimental			Theoretical				
		α_{rms} (cm/s ²)	α_{peak} (cm/s ²)	η (-)	α_{rms} (cm/s ²)	$P = 50\%$		$P = 75\%$	
						α_{peak} (cm/s ²)	η (-)	α_{peak} (cm/s ²)	η (-)
1.50	82	1.11	4.97	4.46	1.61	5.59	3.48	6.01	3.74
1.75	84	5.83	13.00	2.23	6.38	19.00	2.98	20.98	3.29
2.00	45	1.95	5.26	2.70	1.84	6.26	3.40	6.74	3.66
2.20	34	2.98	7.95	2.67	2.34	7.89	3.37	8.50	3.63

414



415

416 **Fig. 13.** Theoretical and experimental PSD of response at $f_p = 1.75$ Hz

417

418 ***Field measurements of an as-built long-span floor***

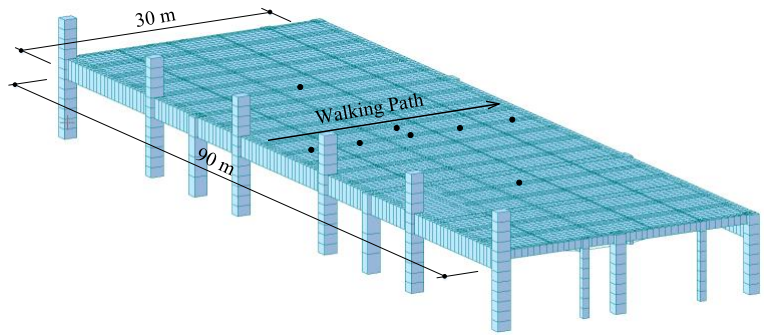
419 Field measurements of a long-span concrete floor in a railway station in China is used to validate the
 420 suggested PSD model. The floor serves as a waiting hall and it has a very long span of 30 m (Fig. 14a). The
 421 floor has a special structural form composed of a concrete plate and external prestressing tendons

422 underneath the plate (Figs 14b, 14c). The fundamental frequency of the floor was tested as 2.2 Hz and the
 423 damping ratio 0.03. Individual walking tests were conducted on this floor and the floor's responses were
 424 recorded by accelerometers installed on eight test points (Fig. 14b). The sampling frequency was set as
 425 200 Hz. Figure 15 shows measured accelerations at the floor center from two test cases where the same
 426 test subject weight 813 N was asked to walk across the floor along the path shown in Fig. 14b at a
 427 frequency of 2.3 Hz twice. More details about the floor and field measurement can be found in Chen et al.
 428 (2016).

429



430

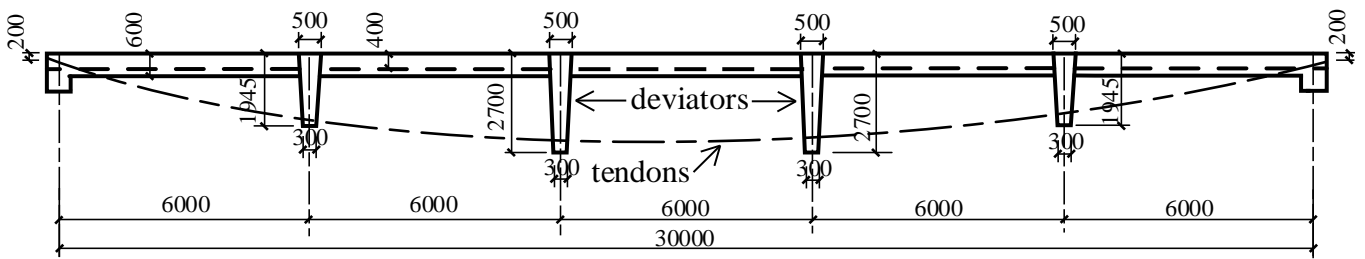


431

(a) The floor in use

(b) Finite element model with walking path and test points (●)

432



433

(c) Side view of the floor system (unit: mm)

434

Fig. 14. The long-span floor in a railway station in China

435

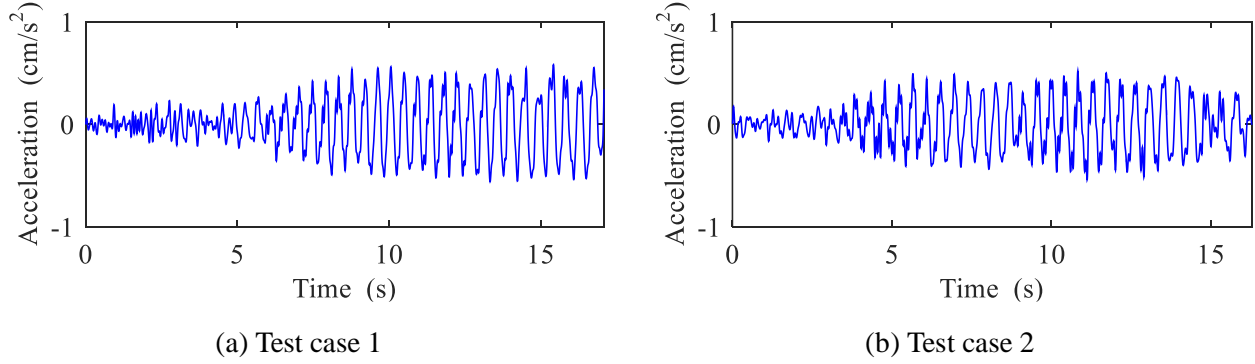


Fig. 15. Measured acceleration at floor center at $f_p = 2.3$ Hz

For verification purpose, RMS acceleration response at the center of the floor has been predicted by the proposed PSD model with computational parameters given in Table 6. The predicted RMS value calculated from the procedure proposed in this paper is 0.3305 cm/s^2 whilst the RMS values of the two test cases (Fig. 15) are 0.3323 cm/s^2 and 0.2808 cm/s^2 , respectively. The comparison demonstrates that the proposed PSD model can be used to predict floor's response to individual walking load.

Table 6. Natural frequencies and modal masses of the floor in the station

Mode	1	2	3	4	5	6	7	8
f_n (Hz)	2.26	2.67	3.01	3.80	4.52	4.62	4.79	4.80
M_n ($\times 10^5$ kg)	7.425	9.069	7.429	8.115	5.325	5.649	4.504	5.514

Discussion

Since records of the individual walking load on rigid floor were utilized to develop the PSD model, the corresponding structural response prediction framework has one limitation that the human-structure interaction (HSI) effect is not considered and this might be one cause for overrating the structure's response (Van Nimmen et al. 2017, Wang et al. 2017). Yet, in most researches relevant to HSI, load models put forward by experiments on a rigid floor are used as portion of contact forces (Van Nimmen et al. 2017), indicating that when the HSI is studied in frequency domain, the proposed PSD load model

454 can be adopted.

455 In practical design, most codes provide comfort criteria in terms of maximum acceleration or RMS
 456 limits with regard to resonant structural response induced by a single pedestrian (ISO 2007, BSI 2003).
 457 With the assistance of this proposed PSD model, it is not difficult to estimate the response. A further
 458 development of the present work is to model crowd load. To obtain the structural response due to crowd
 459 walking, a similar method in this paper can be used, but only with the prerequisite of knowing PSD for
 460 crowd load $\mathbf{G}(\mathbf{f})$ presented in matrix as

$$461 \quad \mathbf{G}(\mathbf{f}) = \begin{bmatrix} W_1 W_1 G_{11}(f) & \cdots & W_1 W_u G_{1u}(f) & \cdots & W_1 W_{N_h} G_{1N_h}(f) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_s W_1 G_{s1}(f) & \cdots & W_s W_u G_{su}(f) & \cdots & W_s W_{N_h} G_{sN_h}(f) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{N_h} W_1 G_{N_h1}(f) & \cdots & W_{N_h} W_u G_{N_hu}(f) & \cdots & W_{N_h} W_{N_h} G_{N_hN_h}(f) \end{bmatrix} \quad (24)$$

462 where $G_{su}(f)$ is the cross power spectral density between the s th and u th pedestrians. In the matrix, the
 463 diagonal elements are just the auto-PSD for each pedestrian proposed in this paper, while the
 464 off-diagonal elements $G_{su}(f)$ ($s \neq u$) are undetermined and serve as the kernel of crowd loading problem.

465 To cope with this, a synchronization factor γ is introduced (Wirsching and Paez 2006)

$$466 \quad G_{su}(f) = \gamma \sqrt{G_s(f) G_u(f)}. \quad (25)$$

467 For two perfectly correlated loads, the synchronization factor $\gamma = 1$, while for completely
 468 uncorrelated ones, the synchronization factor $\gamma = 0$. However, a more realistic situation is that the
 469 pedestrian loads present a limited correlation (i.e., $0 < \gamma < 1$), so evaluating the synchronization factor
 470 would be the primary task in subsequent research.

471 **Conclusions**

472 This study proposes a power spectral density model for individual walking load based on statistical
 473 analysis of a great number of experimental records, employing a larger database compared with previous

474 walking models in frequency domain. Accounting for the fact that people cannot maintain identical load at
475 each walking step, the new model treats walking load as a random process rather than a perfectly periodic
476 process typically assumed in the majority of existing models. The suggested model describes the spectral
477 amplitudes around a certain range of the first four harmonic and first four sub-harmonic by summation of
478 two Gaussian functions, one accounts for the energy concentration degree and the other accounts for the
479 energy distribution degree of each harmonic or sub-harmonic. The model is expressed in a
480 non-dimensional form similar to that for wind gust load. Model parameters are identified from the
481 experimental data. The proposed spectral density model and its application for predicting structure's
482 acceleration responses by stochastic vibration theory are finally verified by comparing its predictions with
483 measured values from an experimental floor model and an as-built floor.

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488 project making possible the data collection.

489 **Supplemental materials**

490 Experimental dataset of walking load samples in Matlab format is available online in the ASCE Library
491 (ascelibrary.org).

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