Comparison of multi-objective optimization methods applied to urban drainage adaptation problems

Qi Wang\textsuperscript{a} Ph.D., Qianqian Zhou\textsuperscript{*} Ph.D., Xiaohui Lei\textsuperscript{b} Ph.D., Dragan A. Savi\textsuperscript{c} Ph.D.

\textsuperscript{a}School of Civil and Transportation Engineering, Guangdong University of Technology, Panyu District, Guangzhou, China, 510006

\textsuperscript{b}China Institute of Water Resources and Hydropower Research, State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, Beijing China, 100038

\textsuperscript{c}Centre for Water Systems, College of Engineering, Mathematics and Physical Sciences, University of Exeter, North Park Road, Exeter, UK, EX4 4QF

Corresponding author: Dr. Qianqian Zhou, qiaz@foxmail.com

Abstract: This article compares three multi-objective evolutionary algorithms (MOEAs) with application to the urban drainage system (UDS) adaptation of a capital city in North China. Particularly, the well-known NSGA-II, the built-in solver in the MATLAB Global Optimization Toolbox (MLOT), and a newly-developed hybrid MOEA called GALAXY are considered. A variety of parameter combinations of each MOEA are systemically applied to examine their impacts on optimization efficiency. Results suggest that traditional MOEAs suffer from severe parameterization issues. For NSGA-II, the distribution indices of crossover and mutation operators were found to have dominant impacts, while the probabilities of the two operators played a secondary role. For MLOT, the two-point and scattered crossover accompanying the adaptive feasible mutation gained the best Pareto fronts, provided that the crossover fraction is set to lower values. In contrast, GALAXY was the most robust and easy-to-use tool among the
three MOEAs, owing to its mechanism for substantially alleviating the parameterization issues. This study contributes to the literature by showing how to improve solution robustness through better selection of operators and associated parameter settings for real-world UDS applications. **Keywords:** urban drainage system adaptation; multi-objective evolutionary algorithm; method selection; parameter setting

**Introduction**

Many regions across the world are suffering from growing losses of life and property due to urban floods, and the levels of risks are likely to increase due to impacts of climate change and urbanization (IPCC 2014; Kaspersen et al. 2017). The urban drainage system (UDS) has been a crucial infrastructure to manage floods in cities. The proper adaptation of existing UDSs to meet challenges of future non-stationary conditions has significant socioeconomic benefits (Ranger et al. 2011; Yazdanfar and Sharma 2015; Zhou et al. 2012). In research, links between UDS adaptation and optimization methods are quickly made, since optimization tools can provide much higher computational efficiency and stronger robustness in solving complex problems compared to traditional design methods. Over the past two decades, applications of multi-objective evolutionary algorithms (MOEAs) to UDSs have been greatly expanded (Nicklow et al. 2010; Siriwardene and Perera 2006). Among those, the optimal design of system configuration (e.g., layout and component design) considering the constraints of budget and system performance has been one of the most popular topics (Giacomoni and Joseph 2017; Haghighi 2013; Navin and Mathur 2016; Steele et al. 2016). Automated model calibration and rehabilitation of UDS, particularly using Genetic Algorithms (GAs) and hydrological models, have attracted the attention of a number of researchers (Barco et al. 2008; Barreto et al. 2010;
Meanwhile, the application of MOEAs to optimizing investment strategies (e.g., extent and timing) for UDS adaptation has also gained increasing attention given the recent development in flood modeling and damage evaluation capabilities (Barreto et al. 2010; Delelegn et al. 2011; Maharjan et al. 2009; Yazdi et al. 2017a).

Most comparative studies of MOEAs have been carried out in the field of water distribution systems (WDSs) (Choi et al. 2017; Perelman and Ostfeld 2007; Zheng et al. 2016) and groundwater management (Ketabchi and Ataie-Ashtiani 2015; Kollat and Reed 2006). It is shown that certain MOEAs perform better than others for selected problems and case studies (Gibbs et al. 2010; Maier et al. 2014; Nicklow et al. 2010; Siriwardene and Perera 2006; Zheng et al. 2017). In the field of urban drainage systems, however, understanding on the performances of different MOEAs in solving UDS problems is still very limited (Yazdi et al. 2017a). Despite a large number of applications of MOEAs to UDS design and management studies, there has been surprisingly little research on how to robustly identify optimal solutions through more powerful search operators and better settings of associated parameters. In a majority of the UDS literature reviewed, besides the common parameters (e.g., population size, generations), the settings of operators (e.g., crossover, mutation) are either not discussed (Barreto et al. 2010; Dong et al. 2012; Muschalla 2008) or simply based on subjective judgment or recommended values in literature (Delelegn et al. 2011; Udias et al. 2012). Even with limited studies exploring the sensitivity of optimization results (Chui et al. 2016; Yazdi et al. 2017b; Zare et al. 2012; Zhou et al. 2017), the major attention has been dedicated to examining the settings of decision variables and objective functions rather than understanding the working principles of different
Despite important conclusions made in previous comparative studies of MOEAs on other types of design problems (e.g., WDS design), they offer limited insight into how the particular algorithms perform in the UDS adaptation problems. It is not straightforward to directly transfer the conclusions to the drainage systems due to the complexity and difference in hydrological and hydraulic equations to be solved (Yazdi et al. 2017a). Gaining improved knowledge on the linkage between algorithmic behaviors and associated parameter settings of different MOEAs for UDS adaptation problems is important for practitioners who rely on the output of these tools for design, planning, and management. This paper contributes to the urban water community by exploring the suitability and efficiency of three representative MOEAs, thus offering guidance for users who lack the expertise of optimization algorithms to select appropriate MOEAs for UDS adaptation problems.
Methodology

The three MOEAs applied to the UDS adaptation in this study include: (1) the classic non-dominated sorting genetic algorithm II (NSGA-II); (2) the built-in multi-objective solver located in the MATLAB Global Optimization toolbox (MLOT); and (3) a newly developed hybrid MOEA called GALAXY. The three MOEAs are chosen to cover a range of user groups. First, NSGA-II is widely used to solve various multi-objective optimization problems in water resources due to its reliability and optimization efficiency (Nicklow et al. 2010). Nevertheless, the parameterization of this algorithm has not received sufficient attention in UDS applications. Second, MLOT is a popular optimization tool for practitioners who prefer a user-friendly interface. It reduces the burden of the coding work, thus often being regarded as an easy-to-use tool by users who are proficient in using MOEAs. Third, GALAXY is reported to perform competitively well with Borg (Hadka and Reed 2013) and other representative MOEAs for various design problems of water distribution systems (Wang et al. 2017), which have similar features (e.g., discrete variables, non-linear, and highly combinatorial) as the UDS. Therefore, GALAXY is included in this study to examine its performance in UDS adaptation problems.

In the next section, a brief introduction to the three MOEAs is provided, with a special emphasis on the associated operators and parameterization. Note that this study focuses on the comparisons of features and performances of the three MOEAs under different parameter settings, rather than elaborating on their basic structures and functionalities. Readers are referred to the following references for further details on each method (Deb 2001; Deb et al. 2002; Wang et al. 2017).
NSGA-II

NSGA-II (Deb et al. 2002) is one of the most popular MOEAs in the community of water research (Maier et al. 2014; Reed et al. 2013). It is acknowledged as an “industry standard” algorithm and has been successfully applied to a variety of water resource optimization problems (Carlucci et al. 2015; Li et al. 2015). NSGA-II adopts a fast non-dominated sorting approach to rank solutions through an implicit elitist selection method based on the Pareto dominance concept and a secondary selection method based on the crowding distance. The use of the two selection methods can significantly improve its performance in solving complex multi-objective problems. Moreover, this MOEA provides a constraint-handling technique to efficiently deal with constrained problems and supports both binary and real coding representations. The standard NSGA-II applies the Simulated Binary Crossover (SBX) and the Polynomial Mutation (PM) to reproduce children from parental generations with designated probabilities. The functionalities of SBX and PM are further controlled by their distribution indices, respectively. A large value of the distribution index means a small variation in the distance from children to their parents during the reproduction process, and vice versa (Deb et al. 2002). For UDS adaptation problems, the default/recommended settings in the literature may not be suitable because the optimal settings of the SBX and PM operators can vary from case to case. Guidance on the selection of appropriate operators and associated parameters should be given individually according to the characteristics of the given problems.

MLOT

MATLAB is a well-known commercial software in scientific and technical computing. It
provides a broad range of functions and toolboxes in a number of application fields. The multi-objective solver in MLOT is a variant of NSGA-II with controlled elitism, which favors individuals with better fitness values in the Pareto sense (i.e., convergence), as well as those helping to increase the population diversity (Matlab R2017b Documentation, 2017). The balance between population convergence and diversity plays a key role in steering the search towards the true optimal region, in particular for non-linear, multi-modal and non-convex problems.

MLOT provides six crossover operators (i.e., heuristic, intermediate, scattered, single-point, two-point, and arithmetic) and three mutation operators (i.e., adaptive-feasible, Gaussian and uniform). The island model is also supported, in which the whole population is divided into a series of subpopulations (i.e., islands). At first, the subpopulation evolves by itself in each island. At a given interval, the best individuals from one island will replace the worst individuals in another one (known as migration). Many papers show that using the island model helps improve the quality of solutions (Alba and Tomassini 2002; Skolicki and De Jong 2005).

However, there is no intuitive rule-of-thumb for setting appropriate island models for UDS problems, such as the direction, fraction, and interval of migration. Note that in comparison to NSGA-II, MLOT provides more options for users to solve optimization problems. On the other hand, it may require a large amount of extra work on the selection of appropriate operators and parameters before using the tool. This is especially challenging for users who lack expertise in MOEAs and usually, they are inclined to adopt the default or recommend settings in literature with or without minor adjustments. To guarantee the efficiency of MLOT, a systematical tuning
approach is generally requested to deal with the parameterization issue.

**GALAXY**

GALAXY stands for the Genetically Adaptive Leaping Algorithm for approximatio\textsuperscript{ation and diversity}, which is a brand-new hybrid MOEA proposed for solving discrete and combinatorial multi-objective design problems (Wang et al. 2017). It is distinguished from traditional MOEAs by employing six search operators simultaneously and adaptively. Specifically, the SBX and PM are adapted to Simulated Binary Crossover for Integer (SBXI) and Gaussian Mutation (GM) to fit in the integer coding scheme. Another four operators include Turbulence Factor (TF), Differential Evolution (DE), Uniform Mutation (UM) and Dither Creeping (DC). GALAXY relies on global searching operators (i.e., TF, DE and SBXI) to drive the population towards the near-optimal region and then employs local searching operators (i.e., UM, GM and DC) to improve the Pareto front through fine-grained tuning. In addition, this MOEA implements several strategies (e.g., hybrid replacement, global information sharing, and duplicates handling) to guarantee the quality of Pareto fronts obtained (Wang et al. 2017). Despite the multiple operators employed, GALAXY eliminates the setting requirements in the reproduction process, which to a great extent alleviates the parameterization issue and the high computational overhead in optimization. The control parameters include only the population size and the number of function evaluations which are common ones in MOEAs, thus making GALAXY an easy-to-use optimization tool.
Applications

Case study

A portion of the drainage network in the city of Hohhot is selected for the case study to investigate the performance of the three MOEAs for UDS adaptation applications (Figure 1a).

Hohhot, the capital of the Inner Mongolia Autonomous Region located in the northern China, has experienced an accelerated demographic and spatial growth in the past 20 years (Ding and Zhang 2012). The city area is about 1,398 hectares and mainly covered by high-density residential districts, with an overall mean imperviousness of 71%. The case study is located within the watershed adjacent to the Xiaohei River. The main land-use is categorized into residential, commercial, green space and others (see Figure 1b). A few key public facilities, such as institutes, hospitals, municipal administrative buildings, sport and recreational sites, are scattered over the area.

The stormwater drainage system (Figure 1c) is composed of 53 manholes, pipelines, subcatchments, and 3 outlet structures, with a total pipe length of approximately 37 km. The stormwater flows are conveyed from north to south and discharged to the recipient river in the southeast part of the city. Due to past promotion of industrial and high-density residential constructions in the area and limited system upgrades since the early 1970s, flooding has occurred more frequently in the catchment (Zhou et al. 2016). The current drainage system cannot cope with even 1-yr rainfall event. With increasing flood losses in the study site, the local authorities are under intense pressure to implement effective adaptation plans in the near future to improve the system performance up to a 3-yr service level (Zhou et al. 2016). Pipe
enlargement is applied as the adaptation measures to enhance the hydraulic capacity of existing pipelines (i.e., using larger pipes). Costs of pipe enhancement take into account primary construction and maintenance costs, including the installation/relocation of new pipes, costs of earthwork, and evacuation and clearing. The costs can vary depending on a number of factors, such as pipe materials, soil conditions and types of roads. In this study, costs are calculated using average costs derived from regional projects and construction budget manuals, which are a function of pipe diameter, pipe length and buried depth of the pipeline.

**Drainage model**

SWMM (Rossman and Huber 2016) was used to simulate the hydrological and hydraulic response of the drainage system. The model was built based on data such as rainfall hyetograph, subcatchment properties (area, width, imperviousness and slope), network dimensions (manholes and pipes) and related spatial locations and elevations. The kinematic-wave method (Guo and Urbonas 2009; Xiong and Melching 2005) and the Horton equation were used in the flood routing and infiltration calculations, respectively. The input rainfall series corresponded to a 3-yr event with 45.6 mm rain depth over 4-hour duration, with a temporal resolution of 10 minutes. Evaluation of system performance after incorporating different pipe enhancement measurements was conducted for each simulation during the optimization. Note that SWMM is not capable of simulating 2D surface inundation conditions and overflow from overloaded manholes is expressed as the value of total flood volume (TFV). As the adaptation goal in this study is to upgrade the system to the 3-yr service level (i.e., no system overloading occurs under 3-yr event), it is reasonable to use TFV to reflect the UDS performance. Surface inundation
models, e.g. (Vojinovic and Tutulic 2009), are applicable if overland flow characteristics and related damage assessment are desirable for system performance.

**Multi-objective optimization model**

The optimization problem concerned two objective functions: i.e., minimization of the expected TFV provided by adapted pipe capacities and the related pipe enhancement costs. These two objectives are conflicting, meaning that more investment reduces TFV and vice versa. There were in total 53 decision variables (i.e., 53 pipelines) for the identification of optimal locations and capacities of pipe enhancement. The optimization goal was to identify a set of optimal solutions (i.e., Pareto front) which reflect the trade-off between system overloading and required costs.

**Experimental setup**

The parameterization of MOEAs was investigated from two aspects: common parameters and specific ones. The former refers to the parameters required by all MOEAs, including the population size (PS) and the number of function evaluations (NFEs). The latter is related to the individual MOEA applied to the optimization and may differ from case to case. Also note that in some cases similar parameter terms may have very different meanings and functionalities (e.g., the probability of SBX in NSGA-II versus the crossover fraction in MLOT). For a given combination of parameter settings, each MOEA was run 10 times independently using different random seeds. The main parameters considered for each MOEA are shown in Table 1.
The settings of common parameters were determined by preliminary sensitivity analyses to ensure a satisfactory level of convergence of all MOEAs (see Figures S1-S3 and Tables S1-S3 in Supplemental Materials for more details). It was found that NSGA-II and MLOT with a PS of 200 and 20,000 NFEs delivered the best results among all combinations. In contrast, GALAXY with a PS of 20 (denoted as GALAXY\(_{20}\)) and 20,000 NFEs reported the best Pareto fronts. To compare the performance of GALAXY with NSGA-II and MLOT, both GALAXY\(_{20}\) and GALAXY\(_{200}\) were tested in this study.

For NSGA-II, four specific parameters were selected, including the probabilities of SBX and PM (denoted as \(P_c\) and \(P_m\) respectively) and the associated distribution indices (denoted as \(D_{c}\) and \(D_{m}\) respectively). The \(P_c\) was varied from 0.6 to 0.9 with an increment of 0.1, which was believed to cover the most effective range of SBX (Zheng et al. 2017). The recommended setting of the \(P_m\) in the literature is the inverse of the number of decision variables (Wang et al. 2015). In this case, this rate is roughly equal to 0.02 (i.e., 1/53). Besides, three additional values of the \(P_m\) (i.e., 0.002, 0.2 and 0.05) were examined to represent the much lower, much higher and a comparable level of mutation rates, respectively. The minimum and maximum distribution indices of SBX and PM (i.e., \(D_{c}\) and \(D_{m}\)) were bounded to 1 and 20, respectively, based upon preliminary tests with 100,000 random samples from the decision variable space to investigate their impacts on the distribution of children from parents (see Figures S4-S5 in Supplemental Materials for details). As a result, there were in total 64 groups of parameter combinations for NSGA-II.
The parameterization of MLOT was arranged in two stages. At the first stage, the focus was given to examining the impact of operator settings (types of operators and related functional options) on optimization. More specifically, the six crossover operators, the three mutation operators and four types of island models (i.e., 1, 2, 4, and 8 subpopulations implemented on a single processing core) were considered. The first stage only evaluated the performance of the combination of the aforementioned operators and functional options with the associated parameters set to their default values. As a result, there were a total of 72 groups of parameter combinations for MLOT at this stage. At the second stage, only the combinations of operators with high efficiency found at the first stage were considered. The focus was then shifted to investigate the influence of the associated parameters within their corresponding effective ranges.

**Performance Metrics**

Four kinds of performance criteria were used to evaluate the quality of solutions, including the hypervolume (HV, Zitzler and Thiele 1999) and the generational distance (GD, Veldhuizen 1999) metrics to assess the diversity and convergence of Pareto fronts obtained by different MOEAs, respectively. Both metrics range from 0 to 1, and a larger HV and a smaller GD suggest better performance in terms of diversity and convergence, and vice versa. On the other hand, since the solutions which can eliminate TFV (i.e., located on the x-axis) are of special interest to local decision makers, the averaged costs of such solutions from multiple independent runs (denoted as Cost_{TFV=0}) and the associated frequency to identify them are used as complementary indicators to measure the convergence and reliability of MOEAs.
simultaneously.

Note that the optimization efficiency in terms of CPU running time was not taken into consideration in this study, because all the optimization tools were implemented within the MATLAB environment, and the most time-intensive part during optimization came from the hydraulic simulations via the SWMM toolkit. Therefore, the differences among three tools can be neglected as long as the same NFEs are permitted.

Results and discussion

Pareto fronts from tested MOEAs

The best Pareto fronts (BPFs) obtained by each MOEA are shown as colored solid dots in Figure 2a, with the colored hollow circles demonstrating the corresponding Pareto fronts derived by all parameter combinations. Both NSGA-II and MLOT suffered from severe parameterization issues since their solutions disperse widely in the objective space. The Pareto fronts achieved by NSGA-II are mainly within a range of costs less than $4.5 million and a TFV lower than 60,000 m$^3$. Impacts of parameterization on MLOT are more pronounced as the obtained solutions in Figure 2a disperse over a wider area than those by NSGA-II. In contrast, GALAXY converged to a satisfactory level compared with the two traditional MOEAs despite the use of two different population sizes. Main differences in MOEA performances are found in the middle part of the BPFs, in which traditional MOEAs converged better. However, it should be emphasized that the $\text{BPF}_{\text{NSGA-II}}$ and $\text{BPF}_{\text{MLOT}}$ were not achieved by any single combination of their parameters, but actually composed of Pareto fronts from several efficient parameter
combinations. In addition, in the region near the maximum TFV, NSGA-II converged worse than GALAXY and MLOT, and missed the boundary point which represents the solution without any pipe enlargement. In the region near the x-axis (Figure 2b), the differences between GALAXY and traditional MOEAs become smaller. By comparing the solutions with zero TFV (i.e., no system overloading) on each BPF (Figure 2c), results show that GALAXY with a PS of 20 (i.e., the green solid dot) found the cheapest solution with an investment cost of approximately $4.4 million. This cost saves 0.3% and 16.2% of the ones found by NSGA-II and MLOT, respectively.

A further quantitative comparison among three MOEAs is presented in Table 2. The BPF<sub>MLOT</sub> achieved the best convergence and diversity according to HV and GD indicators. Recall that the BPF of each MOEA was generated by filtering all the non-dominated solutions via multiple runs using the fast non-dominated sorting procedure (Deb et al. 2002); consequently, the HV and GD values of each BPF (Rows 2-5) are superior to those of the averaged performance of each MOEA through independent runs (Rows 6-9). Despite the BPF<sub>MLOT</sub> exhibiting the best diversity and convergence, it was actually achieved at the price of the highest computational overhead (i.e., 810 runs). In contrast, GALAXY of both population sizes achieved a much higher level of diversity and similar convergence compared with NSGA-II and MLOT when comparing their averaged HV and GD values. Furthermore, there is a high reliability in GALAXY<sub>20</sub>’s performance to identify better (cheaper) boundary solutions on the x-axis (i.e., no TFV). In contrast, traditional MOEAs, especially the MLOT, suffered different levels of difficulties in locating such boundary solutions (i.e., more expensive with lower frequencies).
This implies that GALAXY can provide better solutions to UDS problems at a much lower computational burden.

Comparison of best solutions

Figure 3 shows the best pipe enlargement configurations (i.e., most economical solutions to eliminate system overloading under 3-yr event) optimized by GALAXY, NSGA-II, and MLOT, respectively. The extents of pipe increments are distinguished by the line thickness and associated colors as shown in the figure legend. Note that pipe diameters were not enlarged gradually, since different parts of current drainage networks are served with varying pipe capacities due to uncoordinated historical adaptations in the area. Generally speaking, all the three MOEAs found similar locations for pipe increment, but with different sizes. Pipelines in the upstream were augmented appropriately through the optimization. It is agreed that the pipes near the Outlet O1 have sufficient capacities to cope with the 3-yr event and are therefore kept unchanged (black lines). Nevertheless, the pipes near the Outlet O2 and O3 are found to have different extents of system overloading and were enlarged accordingly. In particular, GALAXY managed to identify the most cost-effective solution for pipe enlargement (i.e., about $4.4 million), with most of optimized pipe increments ranging between 0.2 m and 0.6 m. In contrast, NSGA-II and MLOT suggested 0.4-0.8 m for the same pipelines, which resulted in additional investment costs of $0.011 and $0.8 million, respectively.
Parameterization of MOEAs

NSGA-II

Figure 4 demonstrates the parameterization impacts of NSGA-II in the compass plot, in which the combinations of controlling parameters of NSGA-II (i.e., $D_{Ic}$, $D_{Im}$, $P_c$, and $P_m$) are shown in four colored rings and their contributions to the $BPF_{NSGA-II}$ are shown in the outermost grey ring. For each combination, the contribution is computed as the ratio of the number of solutions by the specific parameter settings found in the $BPF_{NSGA-II}$ to the total number of solutions in the $BPF_{NSGA-II}$. The whole plot is sorted by the contribution ratio in a descending order in the counter-clockwise direction. Results show that $D_{Ic}$ and $D_{Im}$ have significant impacts on the performance of NSGA-II. That is, the combinations with a larger $D_{Ic}$ and a smaller $D_{Im}$ made more than 5% contributions (i.e., the top seven slots) to the $BPF_{NSGA-II}$. The $D_{Ic}$ seems to dominate the NSGA-II performance in solving the UDS adaptation problem presented. With the $D_{Ic}$ set to 1, the majority of the combinations made minor or no contributions to the $BPF_{NSGA-II}$, regardless how the other three parameters were set. When the $D_{Ic}$ was set to 20, half of associated parameter combinations (i.e., 16 out of 32 groups) made identifiable contributions. In terms of delivering efficient solutions, the $D_{Im}$ with a value of 20 has a 31.25% chance of failure, which is 1.7 time the chance (i.e., 18.75%) of those with a value of 1. This finding is essential to guide the further use of NSGA-II as previous applications neglected the setting of these two parameters and default values were often adopted (Barreto et al. 2010; Jia et al. 2015; Yazdi et al. 2017a) without investigating their impacts.

The probabilities of SBX and PM played a secondary role on NSGA-II’s performance, when
the DI\textsubscript{c} and DI\textsubscript{m} were fixed at 20 and 1, respectively. The combination with P\textsubscript{c} and P\textsubscript{m} set to 0.9 and 0.05 obtained the best optimization results across all 64 groups. A larger P\textsubscript{m} generally yielded a higher contribution to the BPF\textsubscript{NSGA-II}, despite the variations in the P\textsubscript{c}. For example, the second best combination had a contribution rate of 16.4% and was achieved with the P\textsubscript{m} set to 0.2. When reducing the P\textsubscript{m} by an order of magnitude (i.e., from 0.2 to 0.02), a noticeable damping effect on the contribution rate (i.e., from 9.2% to 1.7%, highlighted by the black dashed line in Figure 4) was observed. In general, the combinations with larger P\textsubscript{c} and P\textsubscript{m} captured more solutions in the BPF\textsubscript{NSGA-II}. Nevertheless, both parameters, in particular the P\textsubscript{c}, showed no dominant impacts on the performance of NSGA-II. This is somewhat different to the previous literature that treated the probabilities of SBX and PM as the main driving parameters (Khu et al. 2006; Yazdi et al. 2017a). In summary, the results imply that for the case study the best parameter settings for NSGA-II should consider a larger DI\textsubscript{c} (i.e., a smaller search step) with a higher crossover probability, coupling with a smaller DI\textsubscript{m} (i.e., a larger search step) with a much higher mutation probability than the recommended value in the literature (i.e., 0.02).

\textbf{MLOT}

Impacts of the investigated operators on the performance of MLOT are shown in Figure 5a. There were only four combinations leading to noticeable contributions (i.e., with contribution rates larger than 5\%) to the BPF\textsubscript{MLOT}. Surprisingly, all these combinations corresponded to the use of a complete population rather than the island models. The scattered, two-point, and single-point crossover functions accompanying with the adaptive feasible mutation are found to be
more efficient in identifying optimal solutions. Although the Gaussian mutation was also competitive, it was not included at the second stage since it has two additional parameters and may yield infeasible solutions. The other three types of crossover operators (i.e., heuristic, intermediate and arithmetic) were generally inefficient no matter what types of mutation and/or island models were used.

To verify the effect of island models, additional sensitivity analyses were conducted to expand the testing ranges of the migration fraction (i.e., 0.5 and 0.8, the default value was 0.2) for island models with two subpopulations, using the top three combinations found in Figure 5a. No improvement was identified from the BPF_{MLOT} over the initial 720 runs (Figure 5b), which implies that the island models do not fit well to the optimization problem in this study. Consequently, at the second stage, only the three efficient crossover operators (i.e., scattered, two-point, and single-point), as well as the adaptive feasible mutation, were employed as the underlying settings, combined with varying crossover fractions (i.e., 0.6, 0.7, and 0.9, the default value was 0.8). It is worth noting that the crossover fraction controls both the crossover and mutation rates in MLOT, which is intrinsically different from the $P_c$ in NSGA-II. For instance, a crossover fraction of 0.7 means that 70% of the population will undergo crossover randomly and the remaining 30% will be mutated. However, in NSGA-II a child may be generated via both crossover and mutation. Figure 5c shows the Pareto fronts obtained by the additional 90 runs. It is clear that the original BPF_{MLOT} over the 720 runs was further improved by the combination of the two-point crossover, the adaptive feasible mutation, and the newly set crossover fraction with a value of 0.6. A subsequent statistical analysis revealed that the
two-point and scattered crossover functions with a relatively low crossover fraction (i.e., less than 0.8) delivered more efficient solutions, which gained more than 7.5% of the improved BPF$_{\text{MLOT}}$ over the 810 runs.

**GALAXY**

Figure 6 shows the dynamics of the six searching operators employed by GALAXY throughout the optimization process for solving the UDS adaptation problem. It is shown that the operators which are good at global search (i.e., TF, DE and SBXI) are found to dominate the behavior of GALAXY at very early generations (with clearer tendency in Figure 6b due to fewer generations). Afterwards, the operators which are good at local search (i.e., UM, GM and DC) gradually stepped in and steered the optimization from diversification to intensification. The patterns observed are in line with the original design concept of this algorithm, as well as the searching behavior when applied this MOEA to the WDS applications (Wang et al. 2017).

**Conclusions**

Three types of MOEAs, namely the NSGA-II, MLOT, and GALAXY, were compared in an application to the multi-objective adaptation of a district-wide urban drainage system in northern China. With a focus on the impacts of parameterization, the efficiency of each MOEA was evaluated to gain improved understanding of how different operators and associated parameter settings affect the performances of these MOEAs. Results indicate that both NSGA-II and MLOT suffered from severe parameterization issues due to the fact that they involve many controlling parameters for fine-tuning. It seems that the more parameters an MOEA
contains, the more significantly its performance would be affected. Among the three MOEAs, GALAXY turned out to be the most robust and easy-to-use tool for UDS users, especially for those who lack the expertise in evolutionary computation and are challenged by the parameter settings of MOEAs.

The distribution indices of SBX and PM dominate the optimization efficiency of NSGA-II, which have been generally ignored in previous applications. This implies that users need to pay special attention to fine-tuning of those two parameters before applying NSGA-II to given optimization problems. The reason lies in the fact that the variations of children from their parents mainly depend on the values of the distribution indices. The smaller the distribution indices are, the farther the children will be evolved from their parents. Proper settings of the two parameters will benefit the search by ensuring more balanced convergence and diversity in the population. The probabilities of SBX and PM play a secondary role in NSGA-II on the problem studied. A larger SBX rate (i.e., 0.9) with a relatively higher PM rate (i.e., 0.05) than the recommended literature value (i.e., the inverse of the number of decision variables) delivered the best performance. However, note that this finding is perhaps more suitable for problems involving only discrete decision variables. For those concerning continuous variables, the best combinations of SBX and PM rates should be fine-tuned via trial runs.

MLOT provides a good deal of flexibility, although the various options and parameters without appropriate/efficient guidance can also frustrate or even mislead users who are proficient in MOEAs. It was found that the two-point and scattered crossover functions accompanying the
adaptive feasible mutation gained the best Pareto fronts, provided that the crossover fraction was set to a lower value (e.g., 0.6 or 0.7). Importantly, this study showed that using the island GA models for multi-objective optimization seemed to be inefficient. The failure might be due to the loss of diversity by dividing the entire population into subgroups.

GALAXY has distinct advantages in comparison to the two traditional MOEAs. First, except the common parameters (i.e., PS and NFEs), there is no need to set any accompanying parameters for GALAXY’s searching operators, which inherently ensures the robustness of its performance. Second, it deploys six searching operators based on their features in terms of the scale of variations in the objective space, and employs them adaptively and simultaneously rather than using them individually. This mechanism releases the capabilities of various operators in a synergetic way. Third, the performance of GALAXY is insensitive to the settings of population sizes as long as the NFEs are sufficient, which further simplifies the usage of this tool for real-world applications. In contrast, a sufficient population size is requested for traditional MOEAs to work properly. In summary, GALAXY can save substantial time and effort to cope with the parameterization issue of MOEAs. This is essential for users from a practical perspective as they only need to set up the appropriate objective functions and NFEs according to the scale/ characteristics of the optimization problem at hand.

In addition to the parameterization strategy as suggested in the GALAXY, there are other two ways to solve the parameterization issue: (1) the development of the self-adaptive strategy that is able to automatically adjust the parameters based on the searching performance, with a typical
example by Zheng et al. (2013) applied to WDS design problems; and (2) the development of
the hybrid methods to reduce the impacts of the parameterizations, as the starting positions of
MOEAs are the optimal solutions from some deterministic methods (e.g., Linear Programming
or Non-Linear Programming) rather than the randomly generated solutions. A number of
studies have been undertaken in the latter area, such as Ostfeld (2012) and Zheng et al. (2011).

For simplicity, this study only considered two-objective functions and one type of variables.
Nevertheless, the approach adopted in this study can be applied to other types of optimization
problems where different objectives and/or decision variables are concerned. However, it
should be born in mind that if more objectives (e.g., 3 or 4) are considered the performances of
tested MOEAs in identifying the near-optimal Pareto front may deteriorate dramatically due to
the “dominance resistance” encountered in the hyper Pareto space (Hadka and Reed 2013). In
that case, more efficient sorting procedures are required to maintain appropriate selection
pressure and prevent the population from premature. Moreover, given the complexity and
variability in applicable optimization methodologies (e.g., approach, configuration and
parameter), the role of this work is to propose appropriate optimization strategies that not only
enable an assessment of optimal measures in compliance with the predefined physical and/or
economic objectives, but more importantly provide insightful guidance on the selection and use
of efficient optimization approaches for UDS applications. In particular, a systematical analysis
on the impacts of parameter settings should be conducted before using any MOEAs, rather than
simply following the recommended settings in other applications. Furthermore, the fewer
parameters that require fine-tuning, the more robust an MOEA tends to be. On top of that, an
MOEA with multiple searching operators employed simultaneously is likely to provide a more balanced behavior between exploration and exploitation during optimization.

Although benchmark models are widely applied to WDS design problems to test the robustness of the performance of MOEAs (Choi et al. 2017; Zheng et al. 2016), due to a lack of well-acknowledged and publicly available benchmark drainage networks for UDS problems (Yazdi et al. 2017b), this work used only one case study for the comparative assessment of the MOEAs. Future work is planned on the applications of these MOEAs to multiple benchmark drainage networks of different scales and types. For instance, the current work only considers the option of drainage pipe enlargement. The potential of low impact development measures, such as green roofs and rain gardens, are not incorporated into the structure of decision variables. The increased complexity of decision variables and performance criteria will undoubtedly make the optimization even more challenging and is consequently worth investigating.

**Acknowledgements**

The study was funded by the Public welfare research and ability construction project of Guangdong Province (Grant No. 2017A020219003) and the Natural Science Foundation of Guangdong Province, China (Grant No. 2014A030310121).

**Supplemental Data**

Figs. S1–S5 are available online in the ASCE Library (ascelibrary.org).

Tables. S1–S5 are available online in the ASCE Library (ascelibrary.org).

**References**


Figure Captions

Fig. 1. Location (a), main land-use (b), and drainage system(c) of the case study in the city of Hohhot, North China

Fig. 2. Best Pareto fronts discovered by each MOEA via multiple independent runs

Fig. 3. Comparison of optimized pipe increments to upgrade the drainage to planned service level suggested by the three MOEAs

Fig. 4. Parameterization of NSGA-II contributing to its BPF over 640 runs (source data can be found in Table S4 in Supplemental Materials)

Fig. 5. Parameterization of MLOT contributing to the best Pareto front over 810 runs through two stages of test: (a) Parameterization over 720 runs at the 1st Stage (source data can be found in Table S5 in Supplemental Materials); (b) Pareto fronts considering the migration fraction; and (c) Pareto fronts obtained over 90 runs at the 2nd Stage

Fig. 6. Dynamics of search operators within GALAXY over 10 runs
### Tables

<table>
<thead>
<tr>
<th>MOEAs</th>
<th>Main Parameters</th>
<th>Effective Range</th>
<th>Number of Combinations</th>
<th>Number of Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NSGA-II</strong></td>
<td><em>P&lt;sub&gt;c&lt;/sub&gt;</em></td>
<td>[0.6,0.7,0.8,0.9]</td>
<td>4x4x2x2=64</td>
<td>64x10=640</td>
</tr>
<tr>
<td></td>
<td><em>P&lt;sub&gt;m&lt;/sub&gt;</em></td>
<td>[0.002,0.02,0.05,0.2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>D&lt;sub&gt;Lc&lt;/sub&gt;</em></td>
<td>[1,20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>D&lt;sub&gt;Lm&lt;/sub&gt;</em></td>
<td>[1,20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MLOT&lt;sub&gt;1&lt;/sub&gt;</strong></td>
<td>crossover operators</td>
<td>[heu,sca,int,sin,two,ari]</td>
<td>6x3x4=72</td>
<td>72x10=720</td>
</tr>
<tr>
<td></td>
<td>mutation operators</td>
<td>[unif,adap,gaus]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>number of subpopulations</td>
<td>[1,2,4,8]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MLOT&lt;sub&gt;2&lt;/sub&gt;</strong></td>
<td>crossover operators</td>
<td>[sca,two,sin]</td>
<td>3x3=9</td>
<td>9x10=90</td>
</tr>
<tr>
<td></td>
<td>crossover fraction</td>
<td>[0.6,0.7,0.9]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GALAXY&lt;sub&gt;20&lt;/sub&gt;</strong></td>
<td>PS</td>
<td>20</td>
<td>1</td>
<td>1x10=10</td>
</tr>
<tr>
<td></td>
<td>NFEs</td>
<td>20,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GALAXY&lt;sub&gt;200&lt;/sub&gt;</strong></td>
<td>PS</td>
<td>200</td>
<td>1</td>
<td>1x10=10</td>
</tr>
<tr>
<td></td>
<td>NFEs</td>
<td>20,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The subscripts of MLOT refer to different stage of parameterization. The full spellings of the abbreviations of crossover and mutation operators in MLOT can be found in the corresponding subsection in Methodology. The subscripts of GALAXY denote the minimum and maximum population sizes used in this study.
Table 2. Comparison of Pareto fronts obtained by three MOEAs

<table>
<thead>
<tr>
<th>Row No.</th>
<th>Pareto Fronts</th>
<th>HV</th>
<th>HV&lt;sub&gt;relative&lt;/sub&gt;</th>
<th>GD</th>
<th>Avg(Cost&lt;sub&gt;TFV=0&lt;/sub&gt;)/Frequency million $/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Best-Known PF</td>
<td>0.8705</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>BPF&lt;sub&gt;NSGA-II&lt;/sub&gt;</td>
<td>0.8491</td>
<td>0.9754</td>
<td>0.0044</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>BPF&lt;sub&gt;MLOT&lt;/sub&gt;</td>
<td>0.8663</td>
<td>0.9951</td>
<td>0.0009</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>BPF&lt;sub&gt;GALAXY20&lt;/sub&gt;</td>
<td>0.8388</td>
<td>0.9636</td>
<td>0.0239</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>BPF&lt;sub&gt;GALAXY200&lt;/sub&gt;</td>
<td>0.8434</td>
<td>0.9689</td>
<td>0.0268</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>NSGA-II (640 runs)</td>
<td>0.7259</td>
<td>0.8339</td>
<td><strong>0.0223</strong></td>
<td>4.8774/99.2%</td>
</tr>
<tr>
<td>7</td>
<td>MLOT (810 runs)</td>
<td>0.5391</td>
<td>0.6193</td>
<td>0.0494</td>
<td>5.9189/1%</td>
</tr>
<tr>
<td>8</td>
<td>GALAXY&lt;sub&gt;20&lt;/sub&gt; (10 runs)</td>
<td>0.8166</td>
<td>0.9380</td>
<td>0.0319</td>
<td><strong>4.5502/100%</strong></td>
</tr>
<tr>
<td>9</td>
<td>GALAXY&lt;sub&gt;200&lt;/sub&gt; (10 runs)</td>
<td><strong>0.8244</strong></td>
<td><strong>0.9470</strong></td>
<td>0.0440</td>
<td>5.9941/100%</td>
</tr>
</tbody>
</table>

Notes: The maximum values of cost and TFV objectives were set to $8 million and 70,000 m<sup>3</sup>, respectively (i.e., the upper-right corner in Figure 2a). To avoid the impacts of different scales of the two objectives, the Pareto front obtained by each MOEA was first normalized using the maximum values of both objectives. The HV indicators of the four BPFs and the best-known PF were then calculated using the reference point (1,1). The HV<sub>relative</sub> indicator refers to the ratio of each HV indicator obtained by a specific MOEA to that of the best-known PF. The best value according to each criterion is shown in bold.
Figures

Fig. 1. Location (a), main land-use (b), and drainage system(c) of the case study in the city of Hohhot, North China.
Fig. 2. Best Pareto fronts discovered by each MOEA via multiple independent runs
Fig. 3. Comparison of optimized pipe increments to upgrade the drainage to planned service level suggested by the three MOEAs.
**Fig. 4.** Parameterization of NSGA-II contributing to $\text{BPF}_{\text{NSGA-II}}$ over 640 runs (Each colored ring represents one controlling parameter with different levels of darkness showing the considered values in the legend on the right-hand side. Therefore, every four-slot in the radial direction corresponds to a specific parameter combination of NSGA-II. The outermost grey ring indicates the contribution rates of different parameter combinations sorted in the descending order in the counter-clockwise direction. Source data can be found in Table S4 in Supplemental Materials)
Fig. 5. Parameterization of MLOT contributing to the best Pareto front over 810 runs through two stages of test: (a) Parameterization over 720 runs at the 1st Stage (source data can be found in Table S5 in Supplemental Materials); (b) Pareto fronts considering the migration fraction; and (c) Pareto fronts obtained over 90 runs at the 2nd Stage.
Fig. 6. Dynamics of search operators within GALAXY over 10 runs