Emission of coherent spin waves from a magnetic layer excited by a uniform microwave magnetic field

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Abstract

We have developed an analytical theory of the Schlömann spin wave generation from a ferromagnetic layer sandwiched between two semi-infinite media of another ferromagnetic material and pumped by a uniform microwave magnetic field. Our calculations show that, under identical conditions, such a non-uniformity can boost more than twice the emitted spin wave amplitude relative to that emitted from an isolated magnetic interface. The theory provides further support in favour of the dominant role played in the process by the local difference of the microwave magnetic susceptibilities of the adjacent magnetic materials.

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I. Introduction

The use of wave excitations of the magnetic order (so called spin waves[1]) as data or signal carriers may enable data manipulation and storage within the same devices. The development of such devices forms one of the main promises of the magnonic technology and an active direction in the research of spin waves. The topic of spin wave generation is therefore central for both applied and fundamental investigations in magnonics.[2,3]

In our earlier report, we demonstrated how magnetic interfaces can mediate generation of coherent spin waves with wavelength much shorter than the length scales associated with the incident driving microwave magnetic field.[4] Our theory revealed the decisive role of the difference in dynamic magnetic properties of the two magnonic media separated by the interface in this process. This is different from the conventional interpretation of the Schlömann mechanism of spin wave excitation,[5,6] which emphasized the role of matching between the incident microwave frequency and the local resonant frequency.[7-9] The latter interpretation appears especially questionable in the case of spin wave generation from local decreases rather than increases of the local resonance frequency, as identified in Refs. 10,11 where the frequency of the incident microwave field did not match the local resonance frequency in any point of the sample. Here, we further explore the relationship between the two interpretations using a ferromagnetic layer sandwiched between two semi-infinite media of another ferromagnetic material as a model. Indeed, such a non-uniformity could be considered either as an increase / decrease of the local resonance frequency, or as a system of two interfaces. Also, our analysis reveals a close connection between the Schlömann spin wave emission efficiency and the spin wave scattering properties of a magnetic non-uniformity.

The paper is organized as follows. In the next Section (Section II), we describe our theoretical model and present analytical expressions for the complex amplitude of spin waves generated from the magnetic layer pumped by a uniform microwave magnetic field. These results are then used in Section III compare the two alternative interpretations of the emission mechanism. Finally, Section IV is devoted to conclusions and outlook for further development of the theory for more complex models.

II. Model and main results

Let us consider a layer of magnetic material A ($-d/2 < z < +d/2$) sandwiched between two semi-infinite media made of magnetic material B ($z < -d/2$ and $z > +d/2$), as shown in

Fig. 1. Here, d is the thickness of the layer. The layer is parallel to the x-y plane. We consider the case when the static magnetizations $\boldsymbol{M}_{\mathrm{A},0}$ and $\boldsymbol{M}_{\mathrm{B},0}$ are parallel everywhere in the system, i.e. $\boldsymbol{M}_{\mathrm{A},0} \parallel \boldsymbol{M}_{\mathrm{B},0}$. Here and in the following, we use indices A and B to denote quantities characterizing materials A and B, respectively. Materials A and B have uniaxial anisotropies with different strengths K_{A} and K_{B} but with the parallel axes of the easy magnetization, e.g. $\hat{\boldsymbol{z}}$, alone which the media are magnetized by a bias magnetic field $\boldsymbol{H}$. Hence, the system has axially symmetric properties.

The magnetization dynamics are described by the Landau-Lifshitz equation

$$\frac{\partial \boldsymbol{M}_{\mathrm{A(B)}}}{\partial t} = -\gamma_{\mathrm{A(B)}}\left[\boldsymbol{M}_{\mathrm{A(B)}} \times \boldsymbol{H}_{\mathrm{eff,A(B)}}\right] + \frac{\alpha_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}}\left[\boldsymbol{M}_{\mathrm{A(B)}} \times \frac{\partial \boldsymbol{M}_{\mathrm{A(B)}}}{\partial t}\right], \qquad (1)$$

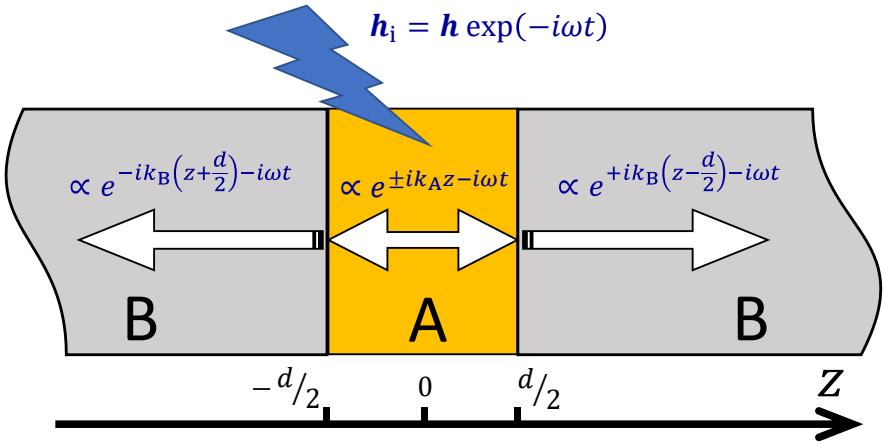

Fig. 1 The geometry of the problem is schematically shown. The system of a magnetic layer A sandwiched between semi-infinite media B is irradiated by a uniform microwave magnetic field $\boldsymbol{h}_{\mathrm{i}}$, which leads to emission of spin waves from layer A into the semi-infinite media B.

where $\boldsymbol{M}_{\mathrm{A(B)}}$ is the magnetization, $\gamma_{\mathrm{A(B)}}$ is the gyromagnetic ratio, $\alpha_{\mathrm{A(B)}}$ is the dimensionless (Gilbert) damping constant and t is the time. The effective magnetic field $\boldsymbol{H}_{\mathrm{eff,A(B)}}$ is then

$$\boldsymbol{H}_{\mathrm{eff,A(B)}} = \frac{\partial}{\partial z}\left(\frac{2A_{\mathrm{A(B)}}}{M^2_{\mathrm{A(B)}}}\frac{\partial \boldsymbol{M}_{\mathrm{A(B)}}}{\partial z}\right) + \frac{2K_{\mathrm{A(B)}}}{M^2_{\mathrm{A(B)}}}\left(\boldsymbol{M}_{\mathrm{A(B)}}\hat{\boldsymbol{z}}\right)\hat{\boldsymbol{z}} + \boldsymbol{H} + \boldsymbol{h}_{\mathrm{i}}\,, \qquad (2)$$

where $A_{\mathrm{A(B)}}$ is the exchange constant and the microwave magnetic field $\boldsymbol{h}_{\mathrm{i}} = \boldsymbol{h}\exp(-i\omega t)$ is assumed to be effectively uniform on the length scales of the problem and incident circularly

polarized in the *x*-*y* plane. We note that our expression for the effective field does not contain any demagnetizing field. This is because, in our geometry, the dynamic demagnetizing fields vanish, while the effect of the static demagnetizing field can be reduced to simple additive corrections to the anisotropy constants.

We represent the magnetization and effective field as sums of their static values and small dynamic perturbations

$$\begin{aligned} \boldsymbol{M}_{\mathrm{A(B)}} &= \boldsymbol{M}_{\mathrm{A(B),0}} + \boldsymbol{m}_{\mathrm{A(B)}}(t),\ m_{\mathrm{A(B)}} \ll M_{\mathrm{A(B)}}, \\ \boldsymbol{H}_{\mathrm{eff,A(B)}} &= \boldsymbol{H}_{\mathrm{eff,A(B),0}} + \boldsymbol{h}_{\mathrm{eff,A(B)}}(t),\ \ h_{\mathrm{eff,A(B)}} \ll H_{\mathrm{eff,A(B),0}}\ , \end{aligned} \tag{3}$$

and then linearize the Landau-Lifshitz equation in $\boldsymbol{m}$ and $\boldsymbol{h}_{\mathrm{eff}}$. As a result, we obtain

$$\begin{aligned} \frac{\partial \boldsymbol{m}_{\mathrm{A(B)}}}{\partial t} = -\gamma_{\mathrm{A(B)}}\left[\boldsymbol{m}_{\mathrm{A(B)}} \times \boldsymbol{H}_{\mathrm{eff,A(B),0}}\right] - \gamma_{\mathrm{A(B)}}\left[\boldsymbol{M}_{\mathrm{A(B),0}} \times \boldsymbol{h}_{\mathrm{eff,A(B)}}\right] + \\ \frac{\alpha_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}}\left[\boldsymbol{M}_{\mathrm{A(B),0}} \times \frac{\partial \boldsymbol{m}_{\mathrm{A(B)}}}{\partial t}\right]. \end{aligned} \tag{4}$$

Considering the full coupling limit and long wavelength spin waves, we require that, at each interface, solutions of the linearized Landau-Lifshitz equation satisfy the following pair of boundary conditions[12]

$$\begin{aligned} \left[\boldsymbol{M}_{\mathrm{B,0}} \times \boldsymbol{m}_{\mathrm{A}}\right] &= \left[\boldsymbol{M}_{\mathrm{A,0}} \times \boldsymbol{m}_{\mathrm{B}}\right], \\ \frac{A_{\mathrm{B}}}{M_{\mathrm{B}}^2}\left[\boldsymbol{M}_{\mathrm{B,0}} \times \frac{\partial \boldsymbol{m}_{\mathrm{B}}}{\partial z}\right] &= \frac{A_{\mathrm{A}}}{M_{\mathrm{A}}^2}\left[\boldsymbol{M}_{\mathrm{A,0}} \times \frac{\partial \boldsymbol{m}_{\mathrm{A}}}{\partial z}\right]. \end{aligned} \tag{5}$$

Following the method from Ref. 4, we seek solutions in the form

$$\boldsymbol{m}_{\mathrm{A(B)}}(t) = \boldsymbol{m}_{\mathrm{U,A(B)}} \exp(-i\omega t) + \boldsymbol{\mu}_{\mathrm{A(B)}} \exp(-i\omega t), \tag{6}$$

where $\boldsymbol{\mu}_{\mathrm{A(B)}}$ are the new unknown functions. We require that functions $\boldsymbol{m}_{\mathrm{U,A(B)}}$ describe the linear response of the system to the incident uniform microwave magnetic field in the absence of the interfacial coupling. Hence, $\boldsymbol{m}_{\mathrm{U,A(B)}}$ satisfy the following *inhomogeneous* linear differential equations

$$\begin{aligned} i\omega \boldsymbol{m}_{\mathrm{U,A(B)}} = \gamma_{\mathrm{A(B)}}\left[\boldsymbol{m}_{\mathrm{U,A(B)}} \times \left(\frac{2K_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}}\hat{\boldsymbol{z}} + \boldsymbol{H}\right)\right] + \gamma_{\mathrm{A(B)}}\left[\boldsymbol{M}_{\mathrm{A(B),0}} \times \right. \\ \left.\left(\frac{2A_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}^2}\frac{\partial^2 \boldsymbol{m}_{\mathrm{U,A(B)}}}{\partial z^2} + \boldsymbol{h}\right)\right] + \frac{i\alpha_{\mathrm{A(B)}}\omega}{M_{\mathrm{A(B)}}}\left[\boldsymbol{M}_{\mathrm{A(B),0}} \times \boldsymbol{m}_{\mathrm{U,A(B)}}\right], \end{aligned} \tag{7}$$

with the *homogeneous* boundary conditions

$$\left[\boldsymbol{M}_{\mathrm{A(B)},0} \times \frac{\partial \boldsymbol{m}_{\mathrm{U,A(B)}}}{\partial z}\right] = 0\,. \tag{8}$$

The solution can be written in a general form as

$$\boldsymbol{m}_{\mathrm{A(B)},\mathrm{U}} = \hat{\chi}_{\mathrm{A(B)}}\boldsymbol{h}\,, \tag{9}$$

where $\hat{\chi}_{\mathrm{A(B)}}$ are the uniform susceptibility tensors of the materials A and B. Due to the axial symmetry of our problem, the tensors can be replaced by scalar susceptibilities

$$\chi_{\mathrm{A(B)}} = -\frac{\gamma_{\mathrm{A(B)}}M_{\mathrm{A(B)}}}{\omega-\omega_{0,\mathrm{A(B)}}+i\alpha_{\mathrm{A(B)}}\omega} = -\frac{M_{\mathrm{A(B)}}^2}{2A_{\mathrm{A(B)}}k_{\mathrm{A(B)}}^2}\,. \tag{10}$$

Here, $\omega_{0,\mathrm{A(B)}} = \gamma_{\mathrm{A(B)}}\left(H + \frac{2K_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}}\right)$ are the frequencies of the uniform ferromagnetic resonance (FMR), and $k_{\mathrm{A(B)}} = \sqrt{\frac{M_{\mathrm{A(B)}}}{2A_{\mathrm{A(B)}}\gamma_{\mathrm{A(B)}}}\left(\omega - \omega_{0,\mathrm{A(B)}} + i\alpha_{\mathrm{A(B)}}\omega\right)}$ are the complex wave numbers of spin waves in the materials A and B, for the case of propagation along the $\hat{\boldsymbol{z}}$ axis.

The spin waves emitted from each interface described by functions $\boldsymbol{\mu}_{\mathrm{A(B)}}$, which are found from the *homogeneous* linear differential equation

$$\begin{aligned} i\omega\boldsymbol{\mu}_{\mathrm{A(B)}} = \gamma_{\mathrm{A(B)}}\left[\boldsymbol{\mu}_{\mathrm{A(B)}} \times \left(\frac{2K_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}}\hat{\boldsymbol{z}} + \boldsymbol{H}\right)\right] + \gamma_{\mathrm{A(B)}}\left[\boldsymbol{M}_{\mathrm{A(B)},0} \times \frac{2A_{\mathrm{A(B)}}}{M_{\mathrm{A(B)}}^2}\frac{\partial^2\boldsymbol{\mu}_{\mathrm{A(B)}}}{\partial z^2}\right] + \\ \frac{i\alpha_{\mathrm{A(B)}}\omega}{M_{\mathrm{A(B)}}}\left[\boldsymbol{M}_{\mathrm{A(B)},0} \times \boldsymbol{\mu}_{\mathrm{A(B)}}\right] \end{aligned} \tag{11}$$

with the *inhomogeneous* boundary conditions

$$\begin{aligned} \left[\boldsymbol{\mu}_{\mathrm{A}} \times \boldsymbol{M}_{\mathrm{B},0}\right] + \left[\boldsymbol{M}_{\mathrm{A},0} \times \boldsymbol{\mu}_{\mathrm{B}}\right] = \left[\boldsymbol{M}_{\mathrm{B},0} \times \chi_{\mathrm{A(B)}}\boldsymbol{h}\right] + \left[\chi_{\mathrm{A(B)}}\boldsymbol{h} \times \boldsymbol{M}_{\mathrm{A},0}\right], \\ \frac{A_{\mathrm{B}}}{M_{\mathrm{B}}^2}\left[\boldsymbol{M}_{\mathrm{B},0} \times \frac{\partial\boldsymbol{\mu}_{\mathrm{B}}}{\partial z}\right] = \frac{A_{\mathrm{A}}}{M_{\mathrm{A}}^2}\left[\boldsymbol{M}_{\mathrm{A},0} \times \frac{\partial\boldsymbol{\mu}_{\mathrm{A}}}{\partial z}\right]. \end{aligned} \tag{12}$$

In our model, the emitted spin waves are circularly polarized and can be described by scalar functions

$$\mu_{\mathrm{A(B)}} \propto \exp\left(\pm ik_{\mathrm{A(B)}}z\right). \tag{13}$$

The boundary conditions (12) for the emitted spin waves are then simplified to the following scalar form

$$\frac{\mu_{\mathrm{B}}}{M_{\mathrm{B}}} - \frac{\mu_{\mathrm{A}}}{M_{\mathrm{A}}} = \left(\frac{\chi_{\mathrm{A}}}{M_{\mathrm{A}}} - \frac{\chi_{\mathrm{B}}}{M_{\mathrm{B}}}\right)h\,, \tag{14}$$

$$\frac{A_\mathrm{B}}{M_\mathrm{B}}\frac{\partial \mu_\mathrm{B}}{\partial z}=\frac{A_\mathrm{A}}{M_\mathrm{A}}\frac{\partial \mu_\mathrm{A}}{\partial z},$$

where *h* is now the amplitude of the incident circularly polarized microwave magnetic field.

Let us now calculate the spin wave amplitude emitted from the layer A into the semi-infinite media B. We use the boundary conditions (14) to match solutions $\mu_{\mathrm{A(B)}}$ in materials A and B, obtaining for the left interface

$$\begin{aligned}&\frac{C_\mathrm{A,R}}{M_\mathrm{A}}\exp\left(-i\frac{k_\mathrm{A}d}{2}\right)+\frac{C_\mathrm{A,L}}{M_\mathrm{A}}\exp\left(i\frac{k_\mathrm{A}d}{2}\right)-\frac{C_\mathrm{B,L}}{M_\mathrm{B}}\exp\left(i\frac{k_\mathrm{B}d}{2}\right)=-\left(\frac{\chi_\mathrm{A}}{M_\mathrm{A}}-\frac{\chi_\mathrm{B}}{M_\mathrm{B}}\right)h,\\&\frac{C_\mathrm{A,R}}{M_\mathrm{A}}k_\mathrm{A}A_\mathrm{A}\exp\left(-i\frac{k_\mathrm{A}d}{2}\right)-\frac{C_\mathrm{A,L}}{M_\mathrm{A}}k_\mathrm{A}A_\mathrm{A}\exp\left(i\frac{k_\mathrm{A}d}{2}\right)+\frac{C_\mathrm{B,L}}{M_\mathrm{B}}k_\mathrm{B}A_\mathrm{B}\exp\left(i\frac{k_\mathrm{B}d}{2}\right)=0,\end{aligned}\tag{15}$$

and for the right interface

$$\begin{aligned}&\frac{C_\mathrm{A,R}}{M_\mathrm{A}}\exp\left(i\frac{k_\mathrm{A}d}{2}\right)+\frac{C_\mathrm{A,L}}{M_\mathrm{A}}\exp\left(-i\frac{k_\mathrm{A}d}{2}\right)-\frac{C_\mathrm{B,R}}{M_\mathrm{B}}\exp\left(i\frac{k_\mathrm{B}d}{2}\right)=-\left(\frac{\chi_\mathrm{A}}{M_\mathrm{A}}-\frac{\chi_\mathrm{B}}{M_\mathrm{B}}\right)h,\\&\frac{C_\mathrm{A,R}}{M_\mathrm{A}}k_\mathrm{A}A_\mathrm{A}\exp\left(i\frac{k_\mathrm{A}d}{2}\right)-\frac{C_\mathrm{A,L}}{M_\mathrm{A}}k_\mathrm{A}A_\mathrm{A}\exp\left(-i\frac{k_\mathrm{A}d}{2}\right)-\frac{C_\mathrm{B,R}}{M_\mathrm{B}}k_\mathrm{B}A_\mathrm{B}\exp\left(i\frac{k_\mathrm{B}d}{2}\right)=0,\end{aligned}\tag{16}$$

where $C_\mathrm{A,L}$ ($C_\mathrm{B,L}$) and $C_\mathrm{A,R}$ ($C_\mathrm{B,R}$) are the complex amplitudes of spin waves propagating in material A (B) to the left and right, respectively.

The solutions of the system of equations (15-16) are

$$\begin{aligned}&\frac{C_\mathrm{A,R}}{M_\mathrm{A}}=\frac{C_\mathrm{A,L}}{M_\mathrm{A}}=-\frac{1}{2}\frac{k_\mathrm{B}A_\mathrm{B}}{k_\mathrm{B}A_\mathrm{B}\cos\left(\frac{k_\mathrm{A}d}{2}\right)-ik_\mathrm{A}A_\mathrm{A}\sin\left(\frac{k_\mathrm{A}d}{2}\right)}\left(\frac{\chi_\mathrm{A}}{M_\mathrm{A}}-\frac{\chi_\mathrm{B}}{M_\mathrm{B}}\right)h,\\&\frac{C_\mathrm{B,L}}{M_\mathrm{B}}=\frac{C_\mathrm{B,R}}{M_\mathrm{B}}=\frac{k_\mathrm{A}A_\mathrm{A}\sin\left(\frac{k_\mathrm{A}d}{2}\right)\exp\left(-i\frac{k_\mathrm{B}d}{2}\right)}{k_\mathrm{A}A_\mathrm{A}\sin\left(\frac{k_\mathrm{A}d}{2}\right)+ik_\mathrm{B}A_\mathrm{B}\cos\left(\frac{k_\mathrm{A}d}{2}\right)}\left(\frac{\chi_\mathrm{A}}{M_\mathrm{A}}-\frac{\chi_\mathrm{B}}{M_\mathrm{B}}\right)h.\end{aligned}\tag{17}$$

Combining equations (10), (13) and (17), we obtain

$$\begin{aligned}&\frac{\mu_\mathrm{A}}{M_\mathrm{A}}=\frac{C_\mathrm{A}h}{M_\mathrm{A}}\cos(k_\mathrm{A}z),\\&\frac{\mu_\mathrm{B}}{M_\mathrm{B}}=\frac{C_\mathrm{B}h}{M_\mathrm{B}}\exp\left(\pm ik_\mathrm{B}\left(z\mp\frac{d}{2}\right)\right).\end{aligned}\tag{18}$$

where

$$C_\mathrm{A}=\frac{1}{2}\frac{M_\mathrm{A}}{\cos\left(\frac{k_\mathrm{A}d}{2}\right)-i\frac{k_\mathrm{A}A_\mathrm{A}}{k_\mathrm{B}A_\mathrm{B}}\sin\left(\frac{k_\mathrm{A}d}{2}\right)}\left(\frac{M_\mathrm{A}}{A_\mathrm{A}k_\mathrm{A}^2}-\frac{M_\mathrm{B}}{A_\mathrm{B}k_\mathrm{B}^2}\right),\tag{19}$$

$$C_{\mathrm{B}} = \frac{1}{2}\frac{M_{\mathrm{B}}}{1+i\frac{k_{\mathrm{B}}A_{\mathrm{B}}}{k_{\mathrm{A}}A_{\mathrm{A}}}\cot\left(\frac{k_{\mathrm{A}}d}{2}\right)}\left(\frac{M_{\mathrm{B}}}{A_{\mathrm{B}}k_{\mathrm{B}}^2} - \frac{M_{\mathrm{A}}}{A_{\mathrm{A}}k_{\mathrm{A}}^2}\right).$$

III. Discussion

For the sake of gaining an analytical insight, we begin by considering the case of zero dissipation in the system. Furthermore, since we are interested in the case of propagating spin waves emitted into media B, it is useful to express k_{A} in terms of $k_{\mathrm{B}} > 0$, i.e.

$$k_{\mathrm{A}} = \sqrt{\frac{M_{\mathrm{A}}}{2\gamma_{\mathrm{A}}A_{\mathrm{A}}}\left(\omega_{0,\mathrm{B}} - \omega_{0,\mathrm{A}} + \frac{2\gamma_{\mathrm{B}}A_{\mathrm{B}}}{M_{\mathrm{B}}}k_{\mathrm{B}}^2\right)}. \tag{20}$$

We can see that two different situations can be realized depending on the difference in the FMR frequencies $\omega_{0,\mathrm{B}} - \omega_{0,\mathrm{A}}$. If this difference is positive, i.e. $\omega_{0,\mathrm{B}} > \omega_{0,\mathrm{A}}$, k_{A} is real for any frequency value. This corresponds to two counter-propagating spin waves emitted into medium A from the two interfaces. The two waves undergo multiple scattering and interference in layer A, which leads in Equations (19) to the factors containing trigonometric functions of $\frac{k_{\mathrm{A}}d}{2}$. If, however, $\omega_{0,\mathrm{B}} - \omega_{0,\mathrm{A}}$ is negative, i.e. $\omega_{0,\mathrm{B}} < \omega_{0,\mathrm{A}}$, k_{A} is real only for $k_{\mathrm{B}} > \sqrt{\frac{M_{\mathrm{B}}}{2A_{\mathrm{B}}\gamma_{\mathrm{B}}}\left(\omega_{0,\mathrm{A}} - \omega_{0,\mathrm{B}}\right)}$, i.e. for $\omega > \omega_{0,\mathrm{A}}$, but is imaginary otherwise, i.e. $k_{\mathrm{A}} = i\kappa_{\mathrm{A}}$, where κ_{A} is real.

The presence of factor $\cot\left(\frac{k_{\mathrm{A}}d}{2}\right)$ in the denominator of Equation (19) for C_{B} should lead to a beating of C_{B} as a function of k_{A} and therefore also of the frequency of the incident microwave field. We expect zeros of C_{B} at poles of $\cot\left(\frac{k_{\mathrm{A}}d}{2}\right)$, i.e. when $k_{\mathrm{A}} = \frac{2\pi n}{d}$, where n is an integer number. Furthermore, maxima of C_{B} are expected when $\cot\left(\frac{k_{\mathrm{A}}d}{2}\right) = 0$, i.e. when $k_{\mathrm{A}} = \frac{\pi}{d} + \frac{2\pi n}{d}$. The spin wave amplitude C_{A} does not become equal zero at any of the k_{A} values above. Instead, the emission efficiency is determined by the values of the total wave field in layer A $\mu_{\mathrm{A}} = C_{\mathrm{A}}h\cos(k_{\mathrm{A}}z)$ at the interfaces. It is easy to check using Equations (15-16) that, for $k_{\mathrm{A}} = \frac{2\pi n}{d}$, we obtain $\cos\left(\pm\frac{k_{\mathrm{A}}d}{2}\right) = -1$, $\sin\left(\pm\frac{k_{\mathrm{A}}d}{2}\right) = 0$, and so, $C_{\mathrm{B}} = 0$, while for $k_{\mathrm{A}} = \frac{\pi}{d} + \frac{2\pi n}{d}$, we obtain $\cos\left(\pm\frac{k_{\mathrm{A}}d}{2}\right) = 0$, and so, $C_{\mathrm{B}} = M_{\mathrm{B}}\left(\frac{\chi_{\mathrm{A}}}{M_{\mathrm{A}}} - \frac{\chi_{\mathrm{B}}}{M_{\mathrm{B}}}\right)$.

It is interesting to note that both the maxima and zeros of the spin wave emission from layer A correspond to the condition of zero reflection of spin waves incident upon layer A from

medium B. Indeed, after a standard calculation, the reflection coefficient R can be written in our notations as

$$R = \frac{\frac{i}{2}\left(\frac{k_A A_A}{k_B A_B} - \frac{k_B A_B}{k_A A_A}\right)\sin(k_A d)}{\cos(k_A d) - \frac{i}{2}\left(\frac{k_A A_A}{k_B A_B} + \frac{k_B A_B}{k_A A_A}\right)\sin(k_A d)}. \quad (21)$$

The reflection vanishes when $k_A = \frac{\pi n}{d}$, which includes both $k_A = \frac{2\pi n}{d}$ and $k_A = \frac{\pi}{d} + \frac{2\pi n}{d}$ values, corresponding to emission zeros and peaks, respectively. This is consistent with the findings from Ref. 11, where both zeros and maxima of the spin wave emission from a Pöschl-Teller-like anisotropy profile were also observed to correspond to the condition of its being reflectionless.[13] This connection between the spin wave emission efficiency and scattering properties of a magnetic non-uniformity is likely to be general.[14]

From the analysis above, it follows that the process of spin wave emission is mainly influenced by the difference in the FMR frequencies of the two media and the thickness of layer A. Hence, for the sake of simplicity, we may assume that the gyromagnetic ratio, exchange stiffness and magnetization in materials A and B have the same values γ, A and M, respectively. In addition, we introduce the exchange length $\sigma = \sqrt{\frac{2A}{M^2}}$ and the dimensionless thickness of layer A as $\delta = d/\sigma$. Then, Equations (19) read

$$C_A = \frac{\frac{1}{k_A^2\sigma^2} - \frac{1}{k_B^2\sigma^2}}{\cos\left(\frac{k_A\sigma\delta}{2}\right) - i\frac{k_A}{k_B}\sin\left(\frac{k_A\sigma\delta}{2}\right)}, \quad C_B = \frac{\frac{1}{k_B^2\sigma^2} - \frac{1}{k_A^2\sigma^2}}{1 + i\frac{k_B}{k_A}\cot\left(\frac{k_A\sigma\delta}{2}\right)}, \quad (22)$$

where $k_{A(B)}$ are functions of $k = \frac{1}{\sigma}\sqrt{\frac{\omega}{\gamma M}} > k_{B,0}$:

$$k_A = \sqrt{k^2(1 + i\alpha_A) - k_{A,0}^2}, \quad k_B = \sqrt{k^2(1 + i\alpha_B) - k_{B,0}^2}, \quad (23)$$

with $k_{A,0}^2 = \frac{1}{\sigma^2}\frac{\omega_{0,A}}{\gamma M}$ and $k_{B,0}^2 = \frac{1}{\sigma^2}\frac{\omega_{0,B}}{\gamma M}$.

In the following, we set $\sigma = 1$, using the exchange length as the length unit. Then, for realistic magnetic materials, the dimensionless parameters and variables in Equations (22-23) could have values in the following ranges: $k_{A(B),0} \in (0, 1)$, $k \in (0,3)$, $\delta \in (0.5,10)$ and $\alpha_{A(B)} \in (0.001, 0.05)$.

The spin wave emission efficiency of our system, which is characterised by the presence of two interfaces, is compared to that for a single magnetic interface as studied in Ref.

4. Under the assumption of a full exchange coupling and using our notations introduced above, the complex amplitude $C_{\mathrm{B},1}$ of spin waves emitted into medium B by an isolated interface reads

$$C_{\mathrm{B},1} = \left(\frac{1}{k_{\mathrm{B}}^2} - \frac{1}{k_{\mathrm{A}}^2}\right) \Big/ \left(1 + \frac{k_{\mathrm{B}}}{k_{\mathrm{A}}}\right). \tag{24}$$

Fig. 2 shows the dependence of $C_{\mathrm{B},1}$ on k. As expected, this dependence is characterized by a strong decrease of $C_{\mathrm{B},1}$ at high values of k. When $\omega_{0,\mathrm{B}} < \omega_{0,\mathrm{A}}$, the spin wave efficiency gets a boost for k values around $k_{\mathrm{A},0}$, i.e. near the FMR in layer A. We also observe that an increase in the damping coefficient in material A leads to a slower decay of $C_{\mathrm{B},1}$ at high k values.

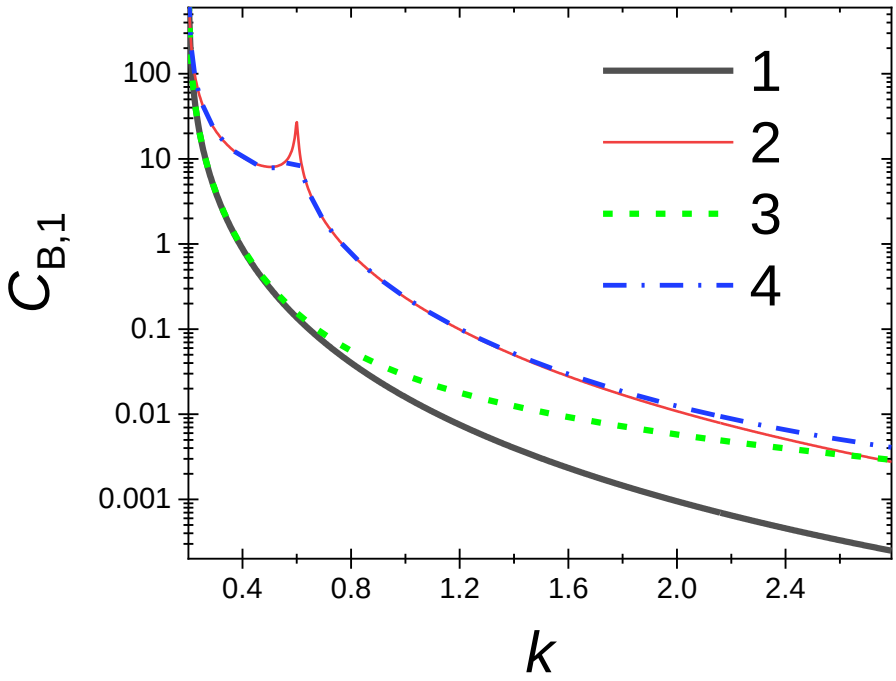

Fig. 2 The k dependence of the spin wave emission amplitude $C_{\mathrm{B},1}$ from an isolated A/B interface into medium B is shown for 1. $k_{\mathrm{A},0} = 0.1$, $\alpha_{\mathrm{A}} = \alpha_{\mathrm{B}} = 0.01$; 2. $k_{\mathrm{A},0} = 0.6$, $\alpha_{\mathrm{A}} = \alpha_{\mathrm{B}} = 0.01$; 3. $k_{\mathrm{A},0} = 0.1$, $\alpha_{\mathrm{A}} = 0.05$, $\alpha_{\mathrm{B}} = 0.005$; 4. $k_{\mathrm{A},0} = 0.6$, $\alpha_{\mathrm{A}} = 0.05$, $\alpha_{\mathrm{B}} = 0.005$. In each case, $k_{\mathrm{B},0} = 0.2$.

To analyse the effect of spin wave confinement in layer A, Fig. 3 shows the dependence of the ratio $C_{\mathrm{B}}/C_{\mathrm{B},1}$ on k for different thicknesses of layer A. The top panel, which corresponds to the case of $\omega_{0,\mathrm{B}} > \omega_{0,\mathrm{A}}$, shows a periodic beating of the ratio on k, with the period of this beating decreasing as the thickness of layer A increases. This is due to the variation of the phase of the total wave field in layer A at interfaces, as discussed earlier. In contrast, the bottom panel of Fig. 3, which corresponds to $\omega_{0,\mathrm{B}} < \omega_{0,\mathrm{A}}$, shows this beating only

for $k > k_{0,\mathrm{A}} = 0.6$. At lower k values, the two spin waves emitted into layer A are not of propagating character. Instead, they decay exponentially into layer A. Let us recall that (i) the spin waves emitted in opposite directions from an isolated interface have opposite phases,[4] and (ii) that the exponential decay (i.e. tunnelling) does not lead to a phase change. Combining facts (i) and (ii), we expect that the waves emitted by the two interfaces of layer A into the same direction should interfere destructively in medium B. This is indeed observed in the bottom panel of Fig. 3 as a sharp dip of $C_{\mathrm{B}}/C_{\mathrm{B},1}$ at k values just below $k_{0,\mathrm{A}}$ for $\delta = 10$, and as a general suppression of the spin wave emission at small k values for small thicknesses of layer A (e.g. $\delta = 0.5$ or 1). For $\delta = 10$, we observe that, at k values significantly lower than $k_{0,\mathrm{A}}$, this decay is very steep, so that the ratio $C_{\mathrm{B}}/C_{\mathrm{B},1}$ becomes equal to unity, i.e. the spin wave emission amplitudes from an isolated interface and from a layer comprising two such interfaces become equal.

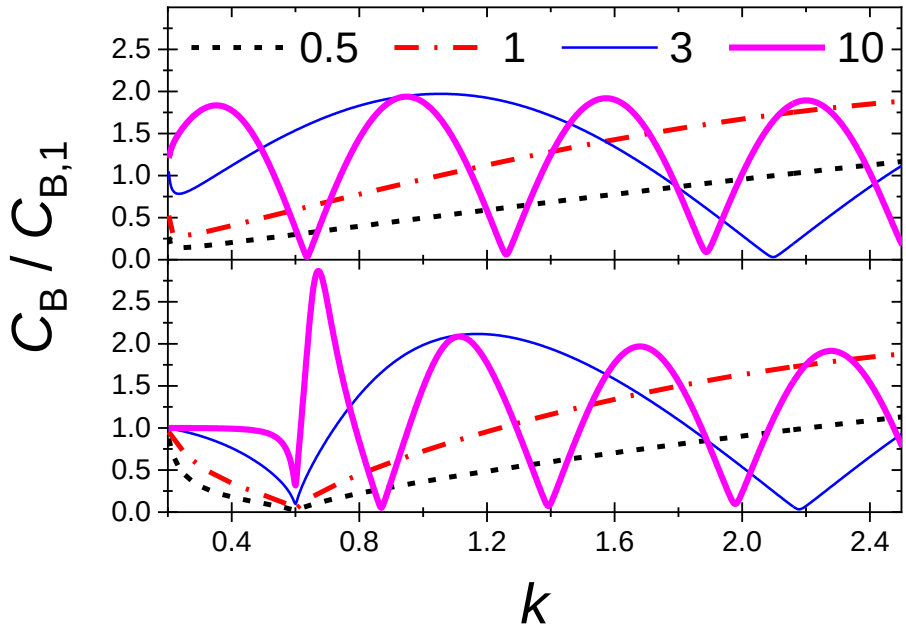

Fig. 3 The k dependence of the ratio $C_{\mathrm{B}}/C_{\mathrm{B},1}$ of spin wave emission amplitudes for layer A (C_{B}) and an isolated A/B interface ($C_{\mathrm{B},1}$) is shown for the indicated different values of $\delta = 0.5, 1, 3, 10$ - the dimensionless thickness of layer A. The top and bottom panels correspond to $k_{\mathrm{A},0} = 0.1$ and $k_{\mathrm{A},0} = 0.6$, respectively. In all cases, $k_{\mathrm{B},0} = 0.2$, $\alpha_{\mathrm{A}} = \alpha_{\mathrm{B}} = 0.01$.

In Fig. 4, we explore the effect of the damping constant in layer A upon the spin wave emission efficiency. We see that the effect is actually quite small, with only moderate suppression of the peaks and zeros of emission observed at higher k values. At even higher k

values (not shown), the ratio $C_{\mathrm{B}}/C_{\mathrm{B},1}$ slowly approaches unity, i.e. layer A behaves increasingly like an isolated interface.

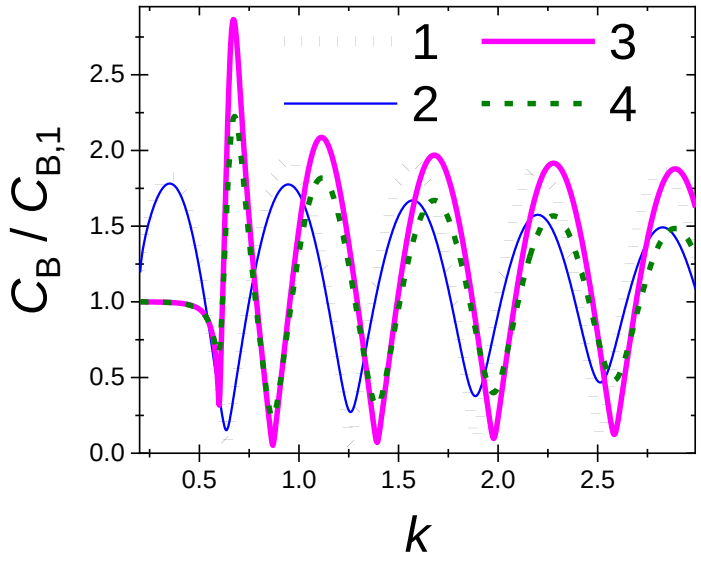

Fig. 4 The k dependence of the ratio $C_{\mathrm{B}}/C_{\mathrm{B},1}$ of spin wave emission amplitudes for layer A (C_{B}) and an isolated A/B interface ($C_{\mathrm{B},1}$) is shown for 1. $k_{\mathrm{A},0} = 0.1$, $\alpha_{\mathrm{A}} = 0.01$; 2. $k_{\mathrm{A},0} = 0.1$, $\alpha_{\mathrm{A}} = 0.05$; 3. $k_{\mathrm{A},0} = 0.6$, $\alpha_{\mathrm{A}} = 0.01$; 4. $k_{\mathrm{A},0} = 0.6$, $\alpha_{\mathrm{A}} = 0.05$. In all cases, $\delta = 10$, $k_{\mathrm{B},0} = 0.2$, $\alpha_{\mathrm{B}} = 0.01$.

IV. Conclusions and outlook

In conclusion, we have developed an analytical theory of spin wave emission induced by a uniform microwave magnetic field incident upon a ferromagnetic layer sandwiched between two semi-infinite media of another ferromagnetic material. Such a layer may be considered as a combination of two magnetic interfaces, studied in Ref.4. In the case of the incident frequency ω being greater than the FMR frequency of the layer, the efficiency of spin wave emission from the layer is determined by the character of the total spin wave field near the layer's interfaces with the outside medium. The efficiency has a periodic dependence on $k = \sqrt{\frac{\omega}{\gamma M}}$, characterised by the presence of both minima (which become zeroth in the dissipationless limit) and maxima. In the latter case, the layer can be a more than twice more efficient spin wave source than an isolated magnetic interface. At incident frequency values smaller than the FMR frequency of the layer, its spin wave emission efficiency is limited by that observed for an isolated magnetic interface. Overall, our results confirm the local

difference of the dynamic magnetic susceptibilities as the primary physical mechanism of the Schlömann spin wave emission. At the same time, the proximity of the incident microwave frequency to the FMR frequency in either material contributes to the spin wave emission by boosting the difference via a selective increase of the susceptibility of the resonating material.[4,15] Finally, our results reveal an intricate connection between the spin wave scattering properties and the Schlömann spin wave emission from magnetic non-uniformities, the details of which we address to future work. The future developments of the theory of spin wave excitation by a uniform microwave should also include magnetic multilayers[15,16] and planar thin film samples with a account of the different types of surface boundary conditions.[17]

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