

## **Study of hydraulic fracturing for gas drainage in a coalmine in Iran**

Davood Mahdavian, BEng, MSc, PhD candidate, University of Exeter, UK  
Akbar Javadi, BEng, MSc, PhD, Professor, University of Exeter, UK

### **Abstract**

Hydraulic fracturing (HF) is one of the methods to make coalmining operation safer and more economic. One of the hazards in underground coal mining operation is the sudden coal gas emission leading to coal explosion. To reduce the risk of gas emissions to ensure safer mining, it is necessary to pre-drain coal seams and surrounding layers. The most important parameters affecting the HF process of a coal seam are: dip, thickness, seam uniformity, roof and floor conditions, reserve of coal seam and coal strength. This paper presents the development and application of a fuzzy model to predict the efficiency of hydraulic fracturing, considering the above factors. In the developed model, the efficiency of hydraulic fracturing of coal seams is calculated as a dimensionless numerical index within the range 0-100. The suggested numerical scale categorizes the efficiency of HF of seams to very low, low, medium, high and very high, each one being specified by a numerical range as a subset of the above range (0-100). The model is used to study the potential of hydraulic fracturing in a coal bed in PARVADEH 4 coalmine in Iran, which will be undergoing stress variation due to future mining activities. The mine consists of 5 seams C1, C2, B1, B2 and D with different characteristics. The results show that the seams C1 and B2 with predicted 94.6% and 81.2% efficiency, have high potential for gas drainage, and considering dip, uniformity and thickness it is suitable to use HF technique. The B1 seam with 31.8 percent efficiency has low potential for gas drainage by HF. HF would not be appropriate for both of C2 and D seams with 7.5 percent efficiency.

Keywords: Hydraulic Fracturing; Gas Drainage; Fuzzy Logic; Coal bed methane; Energy

### **1- Introduction**

In underground coalmines, the gas content of coal seam increases with depth and mining intensity and is a primary factor in mining safety and efficiency. Coal is a complex porous medium that consists of primary pores and fissures that result from tectonic movement, therefore, it has a large amount of free space and multiple pore surfaces. Coal seam gas exists in adsorbed and free states. Only free gas can flow to a working face or be extracted. Coal bed methane is one of the major causes of underground coalmine explosions. Despite the negative financial and environmental impacts of coal bed gas, it is still considered as a fuel source (MacDonald, 1990).

Methane is present within the natural pores of coal and micro pores of coal matrix. Some of this methane is absorbed by coal molecules and bonded to them (Holditch, 1989). If underground coal seams are pressurized, coal molecules will be trapped within the seams. If there is a pressure drop (due to mining, construction of a front or gas drainage drilling), coal molecules will start to move towards the low pressure area. As coal has high potential for absorbing methane, coal seams will accumulate a considerable amount of gas (Sereshki, et al, 2003).

Although coal is of a porous nature with low permeability, its pore structure is far more

complex than ordinary layers of other rocks (Soeder, 1991). Natural fractures, coal permeability and hydraulic fractures create a route for gas and water to flow into coal seams from the cleats. Cleats in a coal seam are natural systematic fractures similar to those of sedimentary rocks (Kendall & Briggs, 1993). Cleat systems are among the features of gas reservoirs that influence the economic viability of gas drainage from coal seams. This affects the success or failure of such projects, and is influential in the progress of gas drainage operations (Dhir, 1991).

The fundamental task of mining engineers is to produce more coal and methane gas at a given level of labour input and material costs, optimum quality and maximum efficiency. To achieve these goals, it is necessary to automate and mechanize mining operations. HF can result in significant cost reductions and higher levels of profitability for coalmines. Therefore, mining engineers are continuously looking for different ways to mechanize mining procedures, especially gas drainage of coal mines that provides a large potential for reduced cost of ventilation, increased safety and improved profitability.

Methane drainage operation is carried out in underground coalmines to prevent sudden gas and coal outbursts and to enhance safety. Generally, coal beds possess low gas recovery. When the coalface is mined, a pressure difference is generated between the faces and somewhere deep inside the coal bed strata. This results in methane emission into the working face. Gas emission is further facilitated by horizontal and vertical fractures induced by the changing ground stress conditions.

In this study, the development of an incremental approach to evaluate the methane production for various given parameters is investigated. The factors affecting the development of coal bed methane extraction by the HF method are examined. As a case study, HF in the coal bed in PARVADEH 4 Tabas mine in Iran, which will be undergoing stress variation due to future mining activities, is investigated. Tabas Coal Mine is located about 60 km South West of Tabas City where the extraction is carried out by longwall mining. The average coal bed gas content is in the order of 15 m<sup>3</sup>/t.

## **2- Hydraulic Fracturing of Coal**

Hydraulic fracturing is the process of creating the fractures in rock and placing proppants into the fractures. Hydraulic fracturing is routinely applied for stimulation of oil, gas, and coalbed methane wells around the world. The stimulation effect is achieved in coal seams as in other reservoirs, by producing conductive fractures, connecting the well to the coal reservoir. The conductivity of the fracture is usually maintained by placing round and sieved sand proppant in the fracture channel. The proppant prevents the fracture faces from closing back completely on one another after the treatment (Jeffrey, 2012).

The coalbed methane (CBM) industry began after the realization that large methane contents of coals could often be produced profitably if the seams were dewatered and if a permeable path to the wellbore could be established for the gas.

Although hydraulic fracturing had been highly developed for conventional gas reservoirs of low-permeability, adjustments to the process were necessary for the coal because of the following phenomena (Jeffrey, 2012):

- The surface of the coal adsorbs chemicals of the fracturing fluid.

- The coal has an extensive natural network of primary, secondary, and tertiary fractures that open to accept fluid during hydraulic fracturing but close upon the fluid afterwards, introducing damage, fluid loss, fines, and treating pressures higher than expected.
- Fracturing fluid can leak deep into natural fractures of coal without forming a filter cake.
- Multiple, complex fractures develop during treatment.
- High pressures are often required to fracture coal.
- Young's modulus for coal is much lower than that for conventional rock.
- Induced fractures in some vertical CBM wells may be observed in subsequent mine troughs.
- Horizontal fractures occur in very shallow coals.
- Fines and rubble result from fracturing brittle coal.
- Coal seams to be fractured may be multiple and thin, perhaps only 0.3 or 0.6 meter thick, requiring a strict economical approach to the operations

To produce the water and gas from the coal seam, holes are introduced that penetrate the casing, cement and a short distance into the coal. These holes are typically created using perforating guns that consist of a string of shaped explosive charges that, when set off, shoot an explosively generated jet through the steel, cement and rock to a distance of 200 to 400 mm into the coal. Alternatively, a high pressure water and sand slurry can be directed at the casing to cut a hole or slot through the casing and into the coal. Hydraulic fracturing is then done by isolating the perforated section, typically by installing a plug inside the casing that presses against the casing to hold itself in place. Pumping fluid down the well then pressurises the section perforated. The fluid pressure increases until the in situ stress and strength of the rock are exceeded, resulting in formation of a fracture. This fracture is extended as a hydraulic fracture by continuing to pump the fracturing fluid into it as it grows in size into the reservoir. The rate of growth of the fracture depends on the fluid injection rate, its overall shape and a number of other rock properties and fluid characteristics.

The rate of fracture growth decreases with time and, typically after 15 to 20 minutes, growth has slowed to a few metres per minute. Hydraulic fracture treatments in coal would typically create fractures extending to between 100 and 300 m, but smaller and larger fractures can be formed depending on the injection rates, seam thickness, fracturing fluid type and volume, and other details of the coal, surrounding rock, in situ stress and treatment execution. Volumes used per fracture treatment range from a few hundred litres for test fractures up to approximately one million litres. Average treatments might be approximately 250,000 litres in volume. Injection pressures depend on the depth of the interval being fractured and typically range from 10 MPa to 40 MPa. Average pressures might be 25 MPa. Both the volume injected and the pressure responses observed are dependent on details of the site and the stimulation design (Jeffrey, 2012).

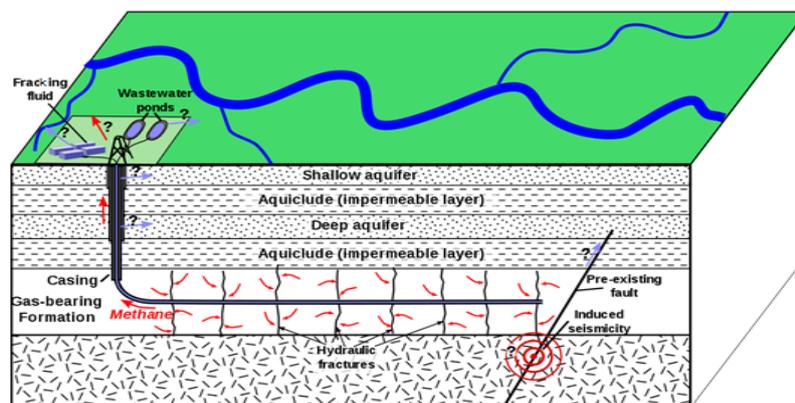


Figure 1: Hydraulic fractures in coal bed methane (en.wikipedia.org)

### **3- Injection of fluids and proppants in hydraulic fracturing**

The next step is the injection of fluids and proppant into the well to initiate fracturing in the coal seam and to keep the fractures open so that gas and water can flow to the well. Injection takes from tens of minutes to a few hours (Taleghani 2009).

The fracture faces expose a large area of the seam to the lower producing pressure, allowing the water and gas to drain directly into the propped fractures at an accelerated rate. Hydraulic fracture treatments are designed to place a propped conductive fracture in the coal seam that will efficiently stimulate production from the seam. The stimulation effect achieved depends both on the conductivity and size in length and height of the fracture and on the permeability and thickness of the coal seam. Effective stimulation of a low-permeability seam requires longer moderate conductivity hydraulic fractures, while stimulation of a high-permeability seam requires shorter high-conductivity fractures.

It is usually intended that fluids and particles be only injected into the target coal seam and not the units above and below. This is achieved through accurate subsurface characterisation so that perforation and subsequent injection only occurs at the target coal seam. However, some fracture treatments are designed to produce a fracture that grows vertically through several adjacent thin seams because stimulating each seam individually would not be cost effective. Water makes up the majority of the fracturing fluid, with the next largest component being the proppant, which is transported into the fractures to prevent them from closing once the high fluid pressure is removed. Proppant is typically sand but can also be nut shells, ceramics or bauxite (Beckwith 2010).

Some hydraulic fracturing fluids also contain either a gel mixed in with the water to increase viscosity or a friction-reducing additive. Viscosity is a measure of a fluid's resistance to flow. The main difference between fracturing with water or 'slickwater', which is water with a friction reducing additive, or a water-gel mixture, is that the increase in viscosity from the addition of gel allows more proppant to be carried into the fractures. Fracturing with gel may require a volume of up to 1.2 percent of additives, compared to water fracturing which typically contains a 0.1 per cent volume of additives (APLNG 2013b). Most operators in Australia use water-gel mixtures (APLNG 2013b; Golder Associates 2010b).

A range of other chemicals are used including acid, biocides, stabilisers, pH buffers and breakers. A summary of the fracturing fluids and proppants used is provided in Table 1. The fluid composition and volume changes during injection and is tailored to suit the site-specific condition at each well. The general order of operations involves the following considerations:

If there is significant calcium carbonate present in the coal, then a dilute mix of acid and corrosion inhibitors is injected to dissolve it. Acid is also used to stabilise pH and to clean the perforation tunnels. Injection of high pressure water to initiate fracturing using corrosion inhibitors, clay stabilisers, biocides and optionally gelling agents continues until a drop in pressure is recorded that signifies initiation of fracturing. If a gelling agent is used then 'breaker' chemicals are progressively added to the slurry to breakdown the gel and reduce the viscosity close to that of water to make it easier to extract the injected fluid back. A small volume of water or uncrosslinked gel is injected at the end of the treatment to flush the last slurry to the perforations so that no proppant is left in the well. The most common gelling agents are natural polymers such as guar gum derived from the pods of the guar bean (Economides & Martin 2007).

**Table1: Summary of the fluids and particles used in hydraulic fracturing fluid in Australia ( Economides & Martin 2007; Golder Associates 2010b; APLNG 2011; AGL 2011; Santos 2011; QGC 2011; Arrow Energy 2012b).**

Injected substance	Purpose and notes	Used materials
Water	Fractures the coal when injected under high pressure. Volume of water required is ~0.2 to 1.3 ML per well.	Bore water, farm pond water or groundwater previously extracted from coal seams is often used
Proppant	Keeps the fractures open once the high pressure fluid is removed. The latest technology advances in proppants include high strength ceramics and sintered bauxite	Sand, Resin-coated sand, Ceramics, Bauxite
Acid	Dissolves calcite in the coal prior to fracturing. Not all wells require this treatment because coal seams do not always contain calcite	Hydrochloric acid, Muriatic acid, Acetic acid
Gelling agent or Clay stabilisers	Increases the viscosity of the fluid, to allow more proppant to be carried into fractures. Not all hydraulic fracturing uses a gel; gel-free fracturing is termed 'slickwater'	Guar gum, Starches, Cellulose derivatives  Polydimethyldiallylammonium chloride (Claytrol)  Tetramethylammonium chloride (Claytreat 3C)
Crosslinker	Increase the viscosity of gelling agents.  There are different crosslinkers for different gelling agents	Borate salt , Ethyl glycol, Isopropanol  Disodium octaborate tetrahydrate  Boric acid, Boric oxide
Biocide	Limits or prevents growth of bacteria that could damage the gelling agent.  The natural polymer gelling agents are good food for bacteria so they encourage bacterial growth - biocides kill these bacteria	Glutaraldehyde, Boric acid, Caustic soda  2,2-Dibromo-2-cyanoacetamide, bronopol  Tetrakis(hydroxymethyl)phosphonium sulfate  Sodium hypochlorite, Sodium thiosulfate
pH buffer	Keeps the pH of the fluid in a specified range. Required for the stability of crosslinked polymers	Acetic acid, Sodium hydroxide  Potassium carbonate, Sodium carbonate,
Breaker	Chemically break the bonds of the gel in order to reduce the viscosity back to that of water. Only required if a gel is used	Hydrogen peroxides, Sodium persulfate  Diammonium peroxidisulphate
Friction reducers	Reduce fluid surface tension	Oxyalkylated alcohol

#### **4- Vertical and horizontal wells for gas drainage in HF**

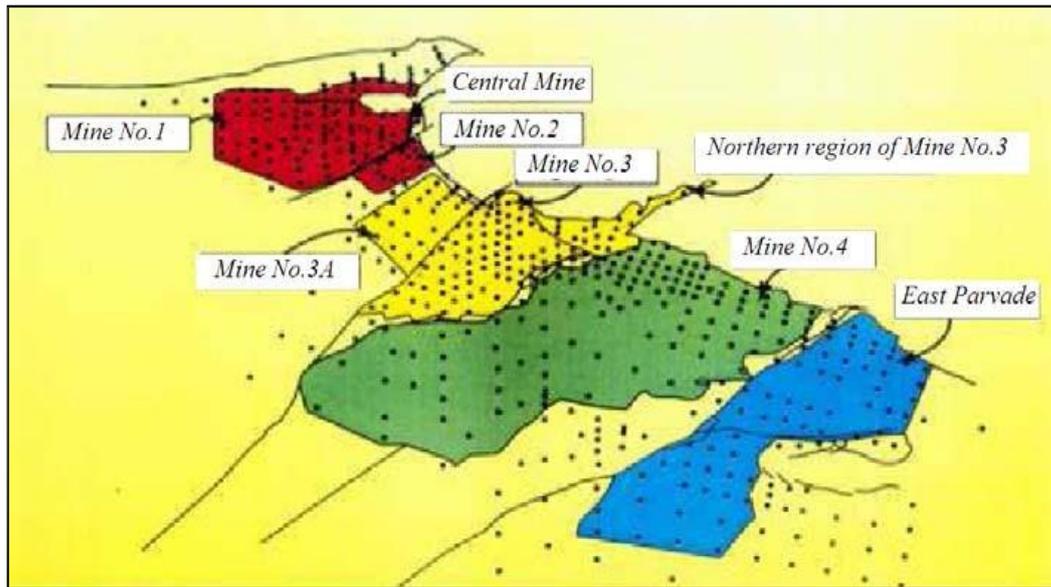
Vertical wells drilled in advance of mining to drain seam gas require stimulation to accelerate the drainage process and to allow fewer wells to effectively drain the area targeted. A typical distance between wells might be 200 to 400 m. Hydraulically fractured wells at this spacing might require five years or more to drain 50 percent of the gas in place. Closer spaced wells drain the gas more quickly, but the total costs of drilling, completion and operating rapidly increase. Therefore, using vertical wells to drain gas before mining requires significant lead-time and upfront investment. There is good scope for mines to partner with a coal seam methane producer to reduce the cost to the mine significantly. Hydraulic fracturing is routinely used to stimulate coal seam methane wells (Jeffrey et al., 1997, Jeffrey et al., 1998, Diamond and Oyler, 1987).

Horizontal wells are drilled and hydraulically fractured in oil and gas reservoirs. The fracture treatments are undertaken to stimulate production and connect the horizontal well into layered reservoir formation. The horizontal layering in the reservoir invariably imparts a permeability anisotropy to the rock. The vertical permeability is typically significantly lower than the horizontal permeability. In addition, hydraulic fractures bypass the near wellbore damage zone, which can be a significant factor in reducing the productivity of any horizontal well or drainage borehole. Hydraulic fractures can be placed in horizontal drain holes by running inflatable straddle packers on an injection string. Fluid bypass or even fracturing of the coal under the packers may occur (Jeffrey, 1999).

Several trials of placing hydraulic fractures in coal seams have been carried out (Croft, 1980, Kravits, 1993, Jeffrey, 1999) with some success reported by Kravits. Special pumps and blenders are needed if sand is included in the treatment, but some stimulation effect can be achieved using only water. Fracturing horizontal wells have been developed in the petroleum industry and might be adapted to fracturing horizontal drain wells in coal seams.

#### **5- Tabas Coal Mine**

Tabas coal region is one of the most comprehensive coal resources in Iran. Tabas coalmine is located in central part of Iran near the city of Tabas in Yazd province and situated 75 km far from southern Tabas. The mine area is a part of Tabas-Kerman coal field. The coalfield is divided into 3 parts in which PARVADEH region, with the extent of 1200 Km<sup>2</sup> and 1.1 billion tones of estimated coal reserve, is the largest and main part for excavation and exploitation for future years. The coal seam has eastern-western expansion with reducing trend in thickness toward east. Its thickness ranges from 0.5 to 2.2 m but in the most places it has a consistent 1.8 m thickness. The large volume of coal reserve and appropriate geometry of coal seams in Tabas have created the required condition for application of HF. The most important coal seam in the Tabas region is C1 with the average thickness 1.8 m.



**Figure 2: PARVADEH 4 Coal mine in PARVADEH Region (IMPASCO 2005)**

In this region, the longwall mining method has been applied for a section of the C1 seam in the mine No 1. The development and opening of the orebody have been carried out through inclined openings. Table 2 shows the average geomechanical parameters of the coal and the overburden rocks of PARVADEH 4 Tabas coal seam (IMPASCO 2005).

**Table 2: PARVADEH 4 Tabas coal seams data (IMPASCO 2005)**

Seam	B1	B2	C1	C2	D
Thickness (m)	0.4-0.9	0.5-1	0.8-1.1	0.4-0.8	0.4-0.7
Dip (degree)	9	8	7	9	11
Uniformity	Semi-uniform	Uniform	Semi Uniform-Uniform	Un-uniform	Un-uniform
Roof Type	Claystone	Siltstone	Siltstone	Siltstone	Siltstone
Floor Type	Siltstone	Claystone	Sandstone	Siltstone	Siltstone
Strength (MPa)	4.7	4.5	4.4	4.4	4.4
Reserve (ton)	29785000	72877000	64936000	30862000	7753000

## 6- Objectives of HF and the parameters affecting the HF

The most important objectives of HF in a coal mine are to achieve: reduced costs; faster development; faster mining; safer mining; concentrating production at fewer locations; achieving higher production rates per shift; mining with smaller underground crews; smaller capital expenditure per extracted ton of coal; working under protectively supported roofs and more productive crews.

To take advantage of HF in a coal seam, a number of factors should be considered. Seam dip, seam thickness, seam uniformity, seam floor condition, seam roof condition and gas concentration are some of the most important factors that affect the potential of coal seam gas to be extracted by HF (Robert 2002). Due to high quantity of methane gas in PARVADEH Tabas coal mine the factor of gas concentration is not considered in this study.

### 6.1- Seam dip

Most coal gas drainage activities occur in flat or nearly flat seams. In general, seams with low slope are more amenable for HF. Seams with slope of over 35 degrees have low potential for hydraulic fracturing. With increased seam slope, the application of HF becomes more difficult. The best operational conditions are level seams.

### 6.2- Seam thickness

The thickness of the seam and its regularity are important parameters in coal seam hydraulic fracturing and great irregularities cannot be accommodated. The thickness that can be worked, at present, ranges from 0.5m and 5m. Where coal seams are limited in thickness and the individual seams within the coal measures are typically less than 0.5 m thick, hydraulic fracturing may be used to connect separate seams over a target horizon of 2 to 5 m.

### 6.3- Seam uniformity

The effect of faulting on the geomechanics of HF is one of the most difficult issues to predict. In some cases, the presence of faults or jointing can have a dominant effect on the geomechanics of a retreating mining operation. If there are complex geological conditions such as faults and seam pinch-outs, the applicability of HF will be reduced. The amount of coal seam displacement and the number of faults present over the length of a seam are very important factors that affect the condition of the working face and the decision to mechanize the operation of the seam. In our study, we have defined the displacement index ( $I_m$ ) as a parameter, to quantify geological disturbances as follows:

$$I_m = \frac{m}{t} \quad (1)$$

where  $m$  is the displacement of a seam by faults and  $t$  is the thickness of the seam.

Table 3 shows the level of seam uniformity with respect to the displacement index. In this classification, seam uniformity ranges from 0 to 1, where seams with an index  $I_m = 0$  are completely uniform and seams with a displacement index of more than 3, are considered to be non-uniform (Unrug and Szwilski, 1982).

**Table 3: Seam uniformity classification (Unrug and Szwilski, 1982)**

Seam uniformity Condition	Seam Uniformity Score	Seam Displacement index
Uniform	1-0.6	0-0.05
Semi-uniform	0.6-0.35	0.5-1
	0.35-0.2	1-1.5
Non-uniform	0.2-0.13	1.5-2
	0.13-0.08	2-2.5
	0.08-0.04	2.5-3
	0	3

#### 6.4- Roof conditions

During hydraulic fracturing process the fractures should be kept open in order to extract the methane gas but if the roof of coal seam is not strong enough, it will push the fractures and close them. Also after hydraulic fracturing in coal seam due to cutting coal seam by shearer machines, strong roof is unavoidable. Both operational experience and research results have demonstrated that roof stability is relative (Unrug and Szwilski, 1982).

For an unstable roof, certain techniques are required to control and change the factors contributing to the unstable conditions and to upgrade its stability after HF.

Quantitative methods are available to evaluate the propensity of roofs to cave in. These methods employ various factors such as lithological sequences, amount of roof convergence at the gob edge, lack of support over a certain time period before caving, seismic wave velocity, drill core strength, average frequency of bedding plane and rock strength and bed separation resistance. The empirical formula for determining the roof strength index is given as:

$$Q_r = 0.016 \times \sigma_c \times K_1 \times K_2 \times K_3 \times \frac{m}{K-1} \quad (2)$$

where  $Q_r$  is the roof strength index,  $\sigma_c$  is the average uniaxial compressive strength of the core (kg/cm<sup>2</sup>),  $K_1$  a factor to account for decrease in strength from the laboratory to a field specimen,  $K_2$  a factor to account for decrease in strength with creep loading,  $K_3$  a factor to account for decrease in strength with an increase in humidity,  $m$  the thickness of the immediate roof (cm) and  $K$  a swelling coefficient with a value between 1.3–1.5. The various design parameters are based on a roof classification system represented by the roof strength index (see Table 4). Table 5 shows the values of different factors for various types of roofs (Unrug and Szwilski, 1982).

**Table 4: Roof strength and time exposure classification (Unrug and Szwilski, 1982)**

Roof type	Roof strength index	Description
Unstable	$0 \leq Q_r \leq 18$	After exposure, roof caves in immediately or after a short delay
Low stable	$18 \leq Q_r \leq 35$	Roof very difficult to control. Full of cavities, fractures and fissures, caves in easily
Medium stable	$35 \leq Q_r \leq 60$	Easily to be caved. From fractured roof with local falls to fairly good roof
Stable	$60 \leq Q_r \leq 130$	Good roof with excellent caving properties to hardly any caving
Very stable	$Q_r \geq 130$	Very strong and very stable. Artificial caving is necessary

**Table 5: K value for different rocks (Unrug and Szwilski, 1982)**

Rock type	Sandstone	Mudstone	Siltstone
$K_1$	0.33	0.42	0.5
$K_2$	0.7	0.6	0.6
$K_3$	0.6	0.4	0.4

## 6.5- Floor conditions

The floor should be strong enough to resist intrusions. Intrusion of soft floors is troublesome for advancing and also makes the roof conditions difficult to control owing to high convergence. During mining operations, some coal may be left if the coal is hard. The reaction of floors to any kind of support, installed along or behind mining faces, significantly affects stability of strata. If the design of the support is to be based on an acceptable rate of closure or deformation along a mining face and its ends, then, in order to ensure support balance and stability, the stratum pressure within the face region should be controlled. This requires: (a) uniform pressure and deformation distribution along the face; (b) a floor bearing capacity in excess of the effective stratum pressure exerted upon it through the supports. Where footwall rocks are weak, support systems may fail by punching into the peripheral rock of ore bodies. The failure mode is analogous to bearing capacity failure of a foundation and may be analysed as such. The floor rock bearing capacity is directly related with the uniaxial compressive strength of rocks. In general, a higher strength implies a greater bearing capacity and a greater potential for HF of the coal seam (Hartman, 1987).

## 6.6- Seam reserve

The coal reserve should be large enough to use hydraulic fracturing. It takes 15–20 days to drill the wellbore and install the equipment in order to begin the hydraulic fracturing. If an individual coal seam is thin and does not contain enough gas to be a viable target for production by itself the HF will not be economical. A large coal reserve would result in lower installation cost per cubic meter of extracted coal bed methane (Ataee, 2005).

## 6.7- Coal Strength

Coal strength is one of important factors that can affect in initiation and propagation of cracks in HF. As seen in the Table 6, by increasing uniaxial compressive strength of coal, more shear force is required to overcome the coal strength (Peng and Chiang, 1984).

**Table 6: Shear stress and uniaxial compressive strength of coals by considering stiffness (Peng and Chiang, 1984)**

Coal Type	Uniaxial Compressive Strength (MPa)	Shear Stress (MP)
Soft	9.81	14.7
Medium	9.81-19.61	14.7-29.4
Hard	19.61-29.42	2.94-44.1

## 7- Fuzzy logic in hydraulic fracturing of coal seam

Over the past decades, the Fuzzy Set Theory (FST) has been used in geotechnical engineering problems to cope with uncertain data due to lack of precision, incompleteness, vagueness and randomness of the information as well as incorporating subjective judgment from experts into problem analyses. Introduced by Zadeh in 1965, FST provides the means for representing epistemic uncertainty using set theory and describes the concept of gradualness and bipolarity (Dubois and Prade, 2010).

Fuzzy logic is a powerful tool for the analysis of systems that work with vague parameters and which receive qualitative inputs, uncertain and simple analytical information of the conditional IF-THEN type in an algorithm in the shortest time and with suitable results. A fuzzy set is an extension of the concept of a crisp set. While a crisp set only allows full membership or no membership to every element of a universe of discourse, a fuzzy set allows for partial membership. The fuzzy set theory includes fuzzy variables or fuzzy functions, fuzzy logic, fuzzy inference system, fuzzy probability, and hybrid fuzzy set. Fuzzy inference system models define relationships between input and output variables of a system by using linguistic labels in a collection of IF-THEN rules, Mandani and Takagi-Sugeno systems being the most commonly used. Ample details on the FST can be found in e.g., Zimmermann, (1991) and Celikyilmaz and Turksen, (2009).

Since early 80's when the first applications of FST in geotechnical engineering appeared, it has been developing intensively and currently it is employed in wide variety of problems for instance, slope stability, rock engineering, tunneling, project management, and even constitutive modelling of geomaterials. In this study, the potential of HF in coal seams is studied using the fuzzy logic. The effect of each parameter on potential of HF should be defined. This definition is subjective. When the result are evaluated with respect to geotechnical parameters, qualitative terms are usually used as excellent, favourable, poor, etc. These terms are ambiguous and vague.

### 7.1- Fuzzy logic base

Fuzzy logic refers to the study of methods and principles of human reasoning. The classical logic, as common practice, deals with propositions (e.g., conclusions or decisions) that are either true or false. Each proposition has an opposite. This classical logic, therefore, deals with combinations of variables that represent propositions. As each variable stands for a hypothetical proposition, any combination of them eventually assumes a true value. The main content of classical logic is the study of rules that allow new logical variables to be produced as functions of certain existing variables (Chen and Pham, 2000).

A fuzzy set can be simply defined as a set with fuzzy boundaries. Let  $X$  be the universe of discourse and its elements be denoted as  $x$ . In classical set theory, crisp set  $A$  of  $X$  is defined as function  $f_A(x)$  called the characteristic function of  $A$

$$f_A(x): x \rightarrow [0,1] \quad (3)$$

where

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

This set maps universe  $X$  to a set of two elements. For any element  $x$  of universe  $X$ , characteristic function  $f_A(x)$  is equal to 1 if  $x$  is an element of set  $A$ , and is equal to 0 if  $x$  is not an element of  $A$ . In the fuzzy theory, fuzzy set  $A$  of universe  $X$  is defined by function  $\mu_A(x)$  called the membership function of set  $A$

$$\mu_A(x): x \rightarrow [0,1] \quad (4)$$

where

$$\mu_A(x) = 1 \text{ if } x \text{ is totally in } A$$

$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A$$

$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A$$

This set allows a continuum of possible choices. For any element  $x$  of universe  $X$ , membership function  $\mu_A(x)$  equals the degree to which  $x$  is an element of set  $A$ . This degree, a value between 0 and 1, represents the degree of membership, also called membership value, of element  $x$  in set  $A$  (Zadeh, 1992). The evaluations of the fuzzy rules and the combination of the results of the individual rules are performed using fuzzy set operations. The operations on fuzzy sets are different than the operations on non-fuzzy sets. Let  $\mu_A$  and  $\mu_B$  be the membership functions for fuzzy sets  $A$  and  $B$ . Table 7 contains possible fuzzy operations for OR and AND operators on these sets, comparatively. The mostly-used operations for OR and AND operators are max and min, respectively. For complement (NOT) operation, Eq.5 is used for fuzzy sets.

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad (5)$$

**Table 7: Fuzzy set operations**

OR (Union)		AND (intersection)	
MAX	$\text{Max}\{\mu_A(x), \mu_B(x)\}$	MIN	$\text{Min}\{\mu_A(x), \mu_B(x)\}$
ASUM	$\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$	PROD	$\mu_A(x)\mu_B(x)$
BSUM	$\text{Min}\{1, \mu_A(x) + \mu_B(x)\}$	BDIF	$\text{Max}\{0, \mu_A(x) + \mu_B(x) - 1\}$

After evaluating the result of each rule, these results should be combined to obtain a final result. This process is called inference. The results of individual rules can be combined in different ways. Table 8 contains possible accumulation methods that are used to combine the results of individual rules. The maximum algorithm is generally used for accumulation (Mendel, 1995).

**Table 8: Accumulation methods**

Operation	Formula
Maximum	$\text{Max}\{\mu_A(x), \mu_B(x)\}$
Bounded sum	$\text{Min}\{1, \mu_A(x) + \mu_B(x)\}$
Normalized sum	$\mu_A(x) + \mu_B(x)$ $\text{Max}\{1, \text{Max}\{\mu_A(x'), \mu_B(x')\}\}$

## 7.2- Membership degrees of effective parameters in hydraulic fracturing

A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The membership degree quantifies the grade of membership of each element in the fuzzy set.

Seam dip is one of the major parameters that determine the HF potential of coal seams. A fuzzy membership grade of seam dip has been developed, as shown in Figure 3. It should be noted that “very low” is allocated a membership grade of 1.0 at a seam dip  $\leq 8$  degrees after which it gradually declines to 0. On the other hand, “very high” means a membership grade of 0 for a seam dip  $\leq 45$  degree and gradually increases to 1.0 at a seam dip  $\geq 50$  degree. Other qualitative legends (low, medium and high) are defined as shown in Figure 3.

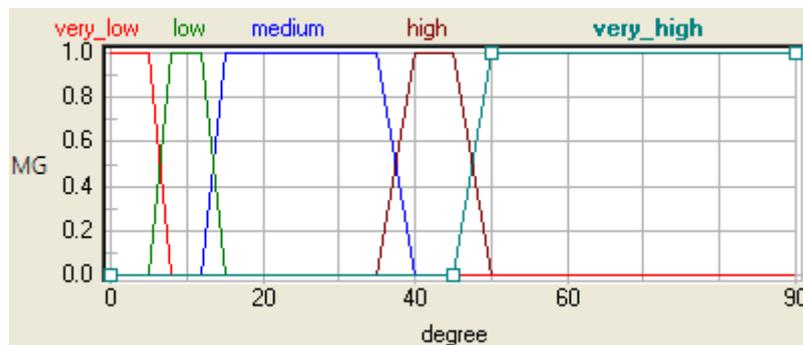


Figure 3: Membership function diagram for seam dip

Seam thickness is defined between 0 and 6 m. Figure 4 describes five qualitative legends “very low”, “low”, “medium”, “high” and “very high”. “Low” is allocated a membership grade of 1.0 when the thickness of the seam is between 0.8 and 1 m and 0.0 when the seam thickness is  $\geq 1.4$  m or  $\leq 0.4$  m. Other qualitative legends are defined as shown in Figure 4.

For seam uniformity three qualitative legends (low, medium and high) are defined as shown in Figure 5. For roof and floor conditions, qualitative legends are defined as shown in Figures 6 and 7 respectively. There are three qualitative legends (low, medium and high) for seam strength also, as shown in Figure 8. For seam reservoir, qualitative legends are defined as shown in Figure 9. Seam membership grades of hydraulic fracturing potential are defined by five qualitative legends (very low, low, medium, high, and very high) are shown in Figure 10.

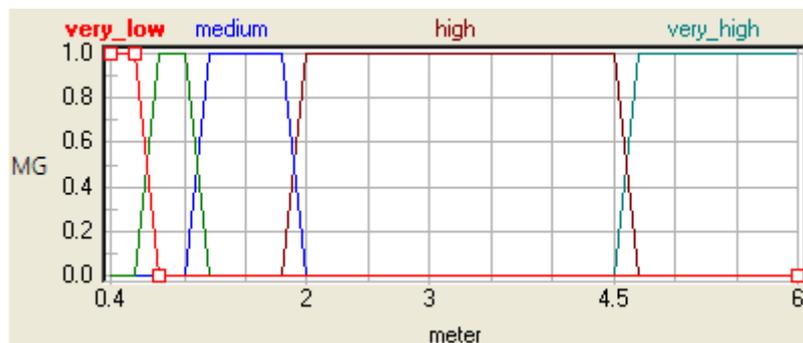


Figure 4: Membership function diagram for seam thickness

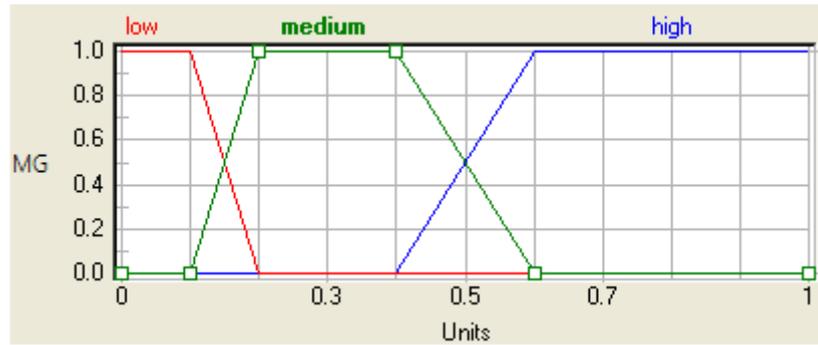


Figure 5: Membership function diagram for seam uniformity

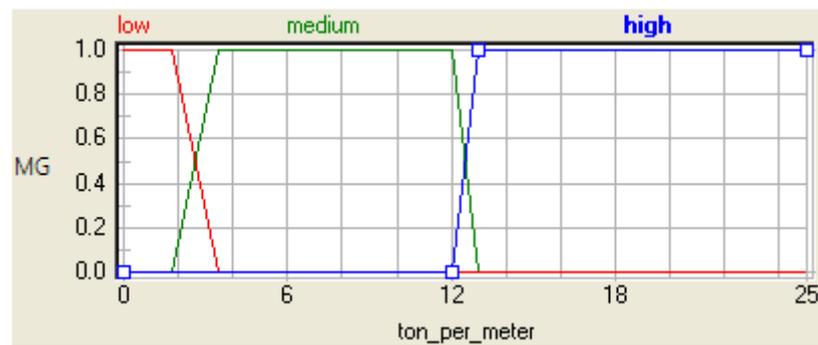


Figure 6: Membership function diagram for seam roof

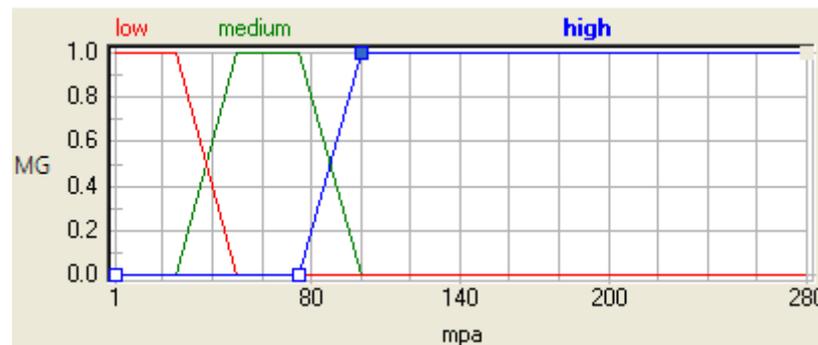


Figure 7: Membership function diagram for seam floor

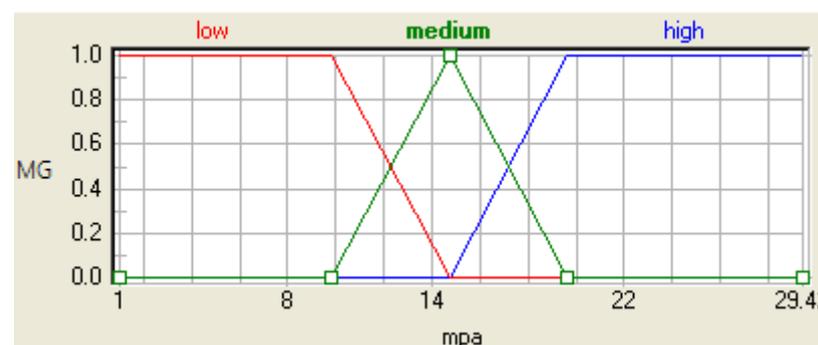
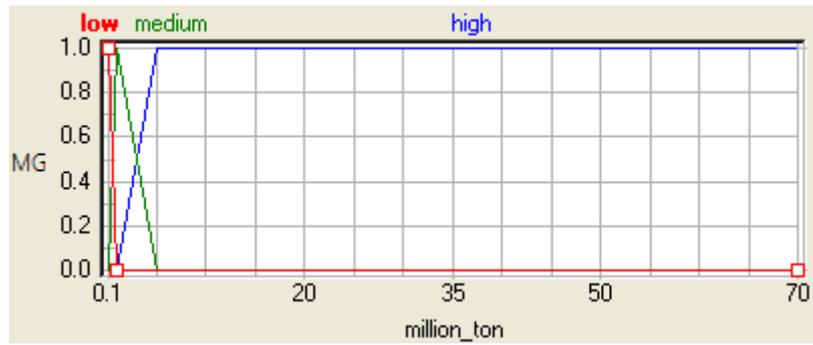
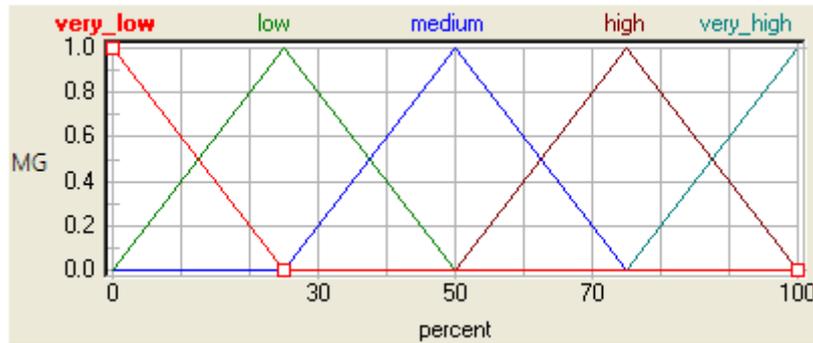


Figure 8: Membership function diagram for seam strength



**Figure 9: Membership function diagram for seam reserve**



**Figure 10: Membership function diagram for HF**

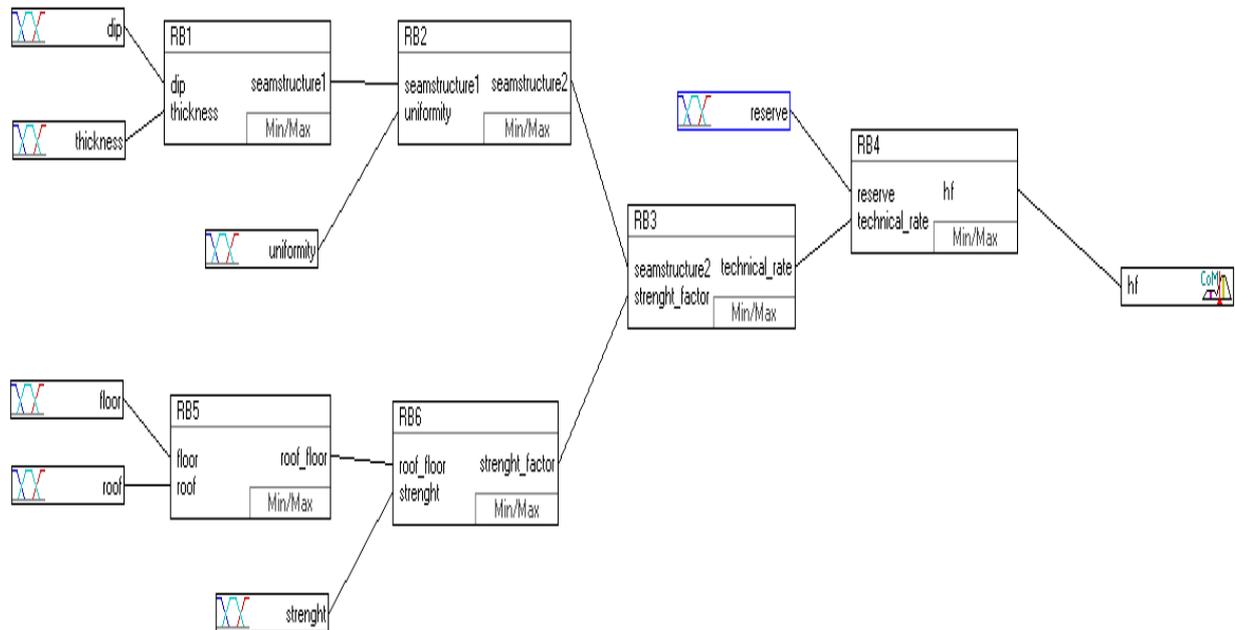
Seam membership grades of hydraulic fracturing potential are defined by five qualitative legends (very low, low, medium, high, and very high) are shown in Figure 10. Figure 10 shows the potential of hydraulic fracturing in the coal seam of Parvadeh 4 Tabas coal mine in percent and describes five qualitative legends “very low”, “low”, “medium”, “high” and “very high”. The horizontal axis shows the variation of hydraulic fracturing potential and the vertical axis shows the variation of membership grade.

### 7.3- Fuzzy rule-base

The main way to control a fuzzy logic system is to define the fuzzy rule-base in order to set up rules, which can combine different cases related to effective parameters in HF and the interaction of parameters with one another and eventually the overall effect on HF potential. At this stage, all input, intermediate and output variables and their interactions in a fuzzy logic system are determined. In order to predict the potential of hydraulic fracturing based on available inputs, a number of simple rules in the form of IF-THEN statements are needed to relate inputs to suitable results.

In this study 7 input variables, 5 intermediate variables, 6 rule bases, 104 fuzzy rules and 55 membership grades are used. Seam dip and uniformity variables which are the most effective parameters in the structure of coal seam in one side and roof, floor and coal quality which are strength parameters in the other side and the coal reserve which is an economical parameter, are classified separately (Figure 12). The intermediate variables are used in fuzzy logic system in order to simplify the analysis of fuzzy rules and eventually predict the impact of effective parameters on HF potential. The rule base of seam structure is divided into two parts. The first part is related to seam dip and thickness as seam structure1 and the second part by considering seam structure1 and seam uniformity as seam structure2 was investigated.

The rule base of strength parameters is also studied in two parts. The first part considers the quality of seam roof and floor as coal seam surrounding layers and second part considers the coal strength parameter and coal seam surrounding layers as strength factor.



**Figure 11: Fuzzy rule bases for determining the HF potential**

**Table 9: Fuzzy rule base of technical factors**

IF		THEN
Seam structure	Strength factor	Technical factor
Very low	Very low	Very low
Very low	Low	Very low
Very low	Medium	Very low
Very low	High	Very low
Very low	Very high	Very low
Low	Very low	Very low
Low	Low	Low
Low	Medium	Low
Low	High	Low
Low	Very high	Low
Medium	Very low	Very low
Medium	Low	Low
Medium	Medium	Medium
Medium	High	Medium
Medium	Very high	Medium
High	Very low	Very low
High	Low	Low
High	Medium	Medium
High	High	High
High	Very high	High
Very high	Very low	Very high
Very high	Low	Medium
Very high	Medium	Medium
Very high	High	High
Very high	Very high	Very high

**Table 10: Fuzzy rule base of hydraulic fracturing potential**

IF		THEN
Technical factor	Reserve	HF
Very low	Low	Very low
Very low	Medium	Very low
Very low	High	Very low
Low	Low	Low
Low	Medium	Low
Low	High	Low
Medium	Low	Very low
Medium	Medium	Medium
Medium	High	Medium
High	Low	Low
High	Medium	Medium
High	High	High
Very high	Low	Very low
Very high	Medium	Medium
Very high	High	Very high

Fuzzy rule-based systems are one of the most important areas of application of fuzzy sets and fuzzy logic. Constituting an extension of classical rule-based systems, these have been successfully applied to a wide range of problems in different domains for which uncertainty and vagueness emerge in multiple ways. A fuzzy rule is defined as a conditional statement in the form: IF x is A. THEN y is B. where x and y are linguistic variables; A and B are linguistic values determined by fuzzy sets on the universe of discourse X and Y, respectively. The rule base of seam structure is divided into two parts. The first part is related to seam dip and thickness as seam structure1 and the second part by considering seam structure1 and seam uniformity as seam structure2 was investigated. The rule base of strength parameters is also studied in two parts. The first part considers the quality of seam roof and floor as coal seam surrounding layers and second part considers the coal strength parameter and coal seam surrounding layers as strength factor.

#### **7.4- Fuzzy inference**

After constructing the rule bases, an inference engine is needed. A fuzzy inference system (FIS) is a system that uses the fuzzy set theory to map inputs to outputs. The most commonly used fuzzy inference technique is the so-called Mamdani method, which was proposed, by Mamdani and Assilian in 1957, as the very first attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Their work was inspired by an equally influential publication by Zadeh (Zadeh, 1973). Interest in fuzzy control has continued ever since, and the literature on the subject has grown rapidly. A survey of the field with fairly extensive references may be found in Lee (1990) or, more recently, in Sala et al., (2005). The rule of Mamdani inference is as follows:

$$\text{If } x \text{ is } \tilde{A} \text{ And } y \text{ is } \tilde{B} \text{ then } z \text{ is } \tilde{C}$$

$$\mu_{\tilde{C}}(c) = \text{SUP}_{z=F(x,y)} \{ \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)) \}$$

In fuzzy sets the SUP is upper limit of  $\mu_{\tilde{C}}(c)$ .

Mamdani's fuzzy inference system can be applied as a decision making model to classify geotechnical sites based on water, soil, support, infrastructure, input, and risk factor related information.

Many inferences can be used in fuzzy logic. For example methods of inference when the IF part is invoked, are GAMMA, MIN-AVG, MIN-MAX and methods of inference of the THEN part are MAX, BSUM. In this study, MAX-MIN operator is used. The reason of this choice is less membership degree in the IF part and maximum membership degree in the THEN part.

### 7.5- Defuzzification

After determination of fuzzy output, a certain level of defuzzification of the potential of hydraulic fracturing must be carried out. At this stage, a choice of different methods, such as the center of maximum (COM) and the mean of maximum (MOM) is available. The COM method was selected in this study.

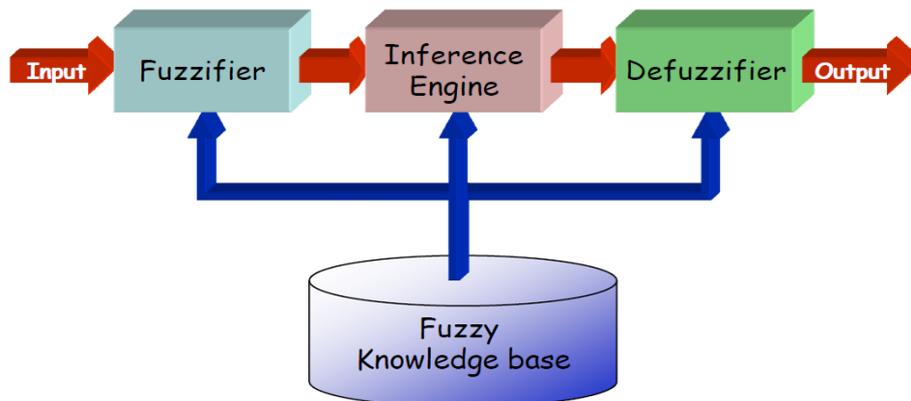


Figure 12: A Fuzzy Logic System

## 8- Results and Discussion

Due to productivity of HF method, this method is used as a method of gas drainage in this study in order to calculate the possibility of gas drainage in Tabas coal seams. The numerical value of HF potential is calculated by considering the technical restrictions. The main restrictions are, seam gradient, thickness, uniformity, roof and floor conditions, quantity of the reserve and coal strength. According to the fuzzy logic, the membership functions and then rule bases were established and eventually the numerical value of HF potential in coal seam of PARVADEH 4 Tabas coal seam was calculated.

Dominant natural conditions in coal seams are the most effective parameters in the determination of the potential of HF. Due to the verity of special conditions in each part of

the coal layer and variable seam specifications across the whole part of the seam, the fuzzy logic is used as a simple and strong tool for modelling and analysis of the systems which have various and vague parameters. In this study, a fuzzy logic system was designed using the Fuzzy Tech 5.54 software. Table 11 shows the result of the potential of HF in PARVADEH 4 Tabas coal seams.

**Table 11: Potential of hydraulic fracturing in PARVADEH 4 Tabas Coal Mine**

Seam	Dip (degree)	Thickness (m)	Uniformity	Roof (t/m)	Floor (MPa)	Strength (MPa)	Reserve (Mt)	Potential of HF (%)	
B1	9	0.65	0.3	10.2	83.4	4.7	29.875	31.83	Medium
B2	8	0.75	0.6	7.6	112.5	4.5	72.877	81.27	High
C1	7	0.95	0.58	11.2	273	4.4	64.936	94.6	High
C2	9	0.6	0.15	6.2	83.4	4.4	30.862	7.5	Very Low
D	11	0.55	0.15	7.08	83.4	4.4	7.753	7.5	Very Low

In order to construct a fuzzy system for calculating the potential of HF in coal seam, 7 input parameters, 5 intermediate parameters, 6 rule bases, 104 fuzzy roles and 55 membership functions were used. After constructing the fuzzy system, the potential of HF in PARVADEH 4 Tabas coal seam was calculated. The most significant parameter, which reduced the potential of using HF in this coalmine was low thickness of D and C2 seams and apart from thickness, seam uniformity is also very effective.

If an individual coal seam is thin and does not contain enough gas to be a viable target for production by itself, hydraulic fracturing can be used to fracture stimulate a number of seams with one treatment. Whether or not a number of seams can be successfully fractured is a function of the seam thickness, the thickness of the interburden rock between them and the stress acting in the seams and in the interburden rocks.

## 9- Conclusion

The PARVADEH 4 Tabas coalmine is one of the largest coal reserves in the Tabas coal basin. Hydraulic fracturing can be used to place a high conductivity channels in the coal seam. The conductive channel stimulates gas and water drainage rates by bypassing near borehole damage and forming a low pressure drain in the coal. As a result, gas drainage rates are increased. Coal has always been considered an important source of energy and despite short-term fluctuations, its long-term total demand in the world shows an upward trend. Reducing costs, achieving higher production rates per shift and increasing safety levels are the most important problems in Iranian coalmines.

Coal gas drainage plays an important role in exploitation of coal seam reserves. Coal seam gas drainage requires large amounts of investment and should therefore be studied carefully before final decision on the implementation of HF. In the present study, the most important parameters affecting the viability of HF of coal seams have been presented in terms of seam gradient and thickness, geological disturbances, seam floor conditions and seam roof conditions. These parameters are imprecise.

In this study by using the fuzzy logic, membership functions and fuzzy rule-bases were created and ultimately, the potential for HF was studied. 7 input variables, 5 intermediate variables, 6 rule bases, 104 fuzzy rules and 55 membership grades were used. Seam dip, thickness and uniformity variables, which are the most effective parameters in the structure of coal seam in one side and roof, floor and coal quality, which are strength parameters in the other side, are classified separately. The coal reserve is also classified separately as an economical parameter. The potential for HF of the PARVADEH 4 Tabas coalmine in Iran

was investigated as a case study. The results show a high potential for HF in B2 and C1 seams and low potential of HF for B1, C2 and D of the PARVADEH 4 Tabas coalmine.

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