Wiener-Hopf analysis of the scattering by a two dimensional periodic semi-infinite array of dipoles

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***Abstract*—We present a rigorous solution of the scattering of plane waves by a truncated planar array of dipoles which is infinite and periodic in one direction and semi-infinite in the orthogonal direction, thus presenting an edge truncation. By applying the Wiener-Hopf technique to the Z-transformed system of equations derived from the electric field integral equation, the contributions to the current on the dipoles due to the scattering by the edge of the array and the excitation of surface waves are obtained rigorously.**

I. Introduction

The scattering by finite arrays has been subject of great interest to the engineering community since such geometry includes the coupling between free space radiation and surface waves due to the edge diffraction. This coupling cannot be studied using the usual local periodicity assumption, widely applied in the design of adiabatically-modified periodic arrangements. The presence of these surface waves has been shown to have great importance near the onset of diffraction orders, affecting elements far from the edge truncation and leading to standing-wave like patterns over the array [1]. Although the analysis of finite arrays has been approached via a truncated Floquet-wave version of the Method of Moments [2], the rigorous study of a single truncation in the present paper provides us with further insight in the excitation of those surface waves.

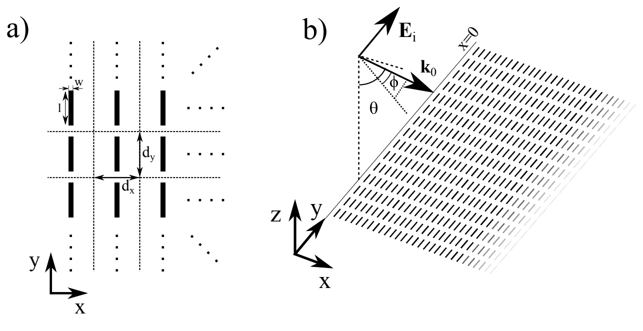

Fig. 1. (a) Top view of the semi-infinite array of flat dipoles, each of length l and width w, placed in a rectangular lattice with spacings d_x and d_y along the x and y directions respectively. (b) Perspective view of the array truncation with the incident plane wave. The array is infinitely long along y.

II. Integral Equation Formulation

Let us consider the problem of an arbitrary polarized wave impinging on the semi-infinite planar array of dipoles whose geometry is shown in Fig. 1. The incident electric field on the plane of the array is written as $\mathbf{E}^{\mathrm{i}} = \mathbf{E}_0^{\mathrm{i}} \mathrm{e}^{-\mathrm{j}(k_{\mathrm{x}0}x + k_{\mathrm{y}0}y)}$ where $k_{\mathrm{x}0} = k_0 \sin\theta\cos\phi$ and $k_{\mathrm{y}0} = k_0 \sin\theta \sin\phi$ following the notation in [3]. The dipoles are assumed to be perfectly conducting and flat with negligible thickness. In this case, the tangential electric field on their surface must vanish, i.e.,

$$\hat{\mathbf{z}} \times \left(\mathbf{E}^{\mathrm{i}}(\mathbf{r}) + \mathbf{E}^{\mathrm{sc}}(\mathbf{r})\right) = \mathbf{0}. \tag{1}$$

The integral equation is obtained by representing the tangential component of the electric field scattered by the array in terms of the dyadic Green's function as

$$\mathbf{E}_{\mathrm{t}}^{\mathrm{sc}}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{G}_{\mathrm{J}}(x-x', y-y') \cdot \mathbf{j}^{\mathrm{sc}}(x',y') \mathrm{d}x' \mathrm{d}y' \tag{2}$$

By using the periodicity of the array along the direction of the truncation (i.e.,y) the problem is reduced to the study of a semi-infinite chain by introducing the periodic Green's function of a linear array of dipoles. Then, by expanding the unknown current flow on the surface on the dipoles in terms of singular basis functions [4] ($\mathbf{b}(\mathbf{r})$ such that $\mathbf{j}^{\mathrm{sc}}(\mathbf{r}) \approx i_n \mathbf{b}(\mathbf{r} - \mathbf{r}_{\mathrm{c}n})$ at the n-th unit cell with its center at $\mathbf{r}_{\mathrm{c}n}$) one can derive a linear system of equations for the unknown current weights given by

$$\sum_{n=0}^{\infty} k_{n-m} i_n = v_m \tag{3}$$

where $m, n = 0, 1, ..., \infty$ represent the dipole on the m-th and n-th unit cell of the linear array respectively and where

$$k_{n-m} = \int_{\eta_m}\int_{\eta_n} \mathbf{b}_m^*(\mathbf{r}) \mathbf{G}_{\mathrm{J}}^{\mathrm{per}}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{b}_n(\mathbf{r}') \,\mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}' \tag{4}$$

with η_m representing the surface of the dipole on the m-th unit cell along the linear array off dipoles and where

$$v_m = -\int_{\eta_m} \mathbf{b}_m^*(\mathbf{r}) \cdot \mathbf{E}_{\mathrm{i}} \,\mathrm{d}\mathbf{r} = V e^{-\mathrm{j}k_{x0} m d} \tag{5}$$

As shown in [5], this system of an infinite number of equations can be solved without truncating it through the use of the Z-transform of the convolution shown in (3), which is given by

$$K(z)I^+(z) = V\frac{O^-(z)}{O^-(z_\gamma)}\frac{z}{z-z_\gamma} \tag{6}$$

where $K(z)$ is obtained by using the conformal mapping $z = \mathrm{e}^{-\mathrm{j}k'_x d_x}$ and the Poisson formula $\sum_{n=-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}(k_x-k'_x)nd_x} = \frac{2\pi}{d_x}\sum_{p=-\infty}^{\infty}\delta(k_x - k'_x - \frac{2\pi p}{dx})$, in terms of the Fourier transforms of the basis functions $B(k_x, k_y)$, as

$$K(z) = -\frac{\zeta}{2d_x k_0}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} B\left(k'_x + \frac{2\pi p}{d}, k_{yq}\right) \tag{7}$$

$$B^*\left(k'^*_x + \frac{2\pi p}{d}, k_{yq}\right) \cdot \left.\frac{k_0^2 - k_{yq}^2}{\sqrt{k_0^2 - \left(k'_x + \frac{2\pi p}{d}\right)^2 - k_{yq}^2}}\right|_{k'_x = \frac{\mathrm{j}}{d_x}\ln z}.$$

If one is able to factorize $K(z) = K^+(z)K^-(z)$ such that $K^+(z)$ ($K^-(z)$) is analytic for $|z| > 1$ ($|z| \leq 1$), then by Liouville's theorem, the unknown Z-transform of the discrete current magnitudes is evaluated as

$$I^+(z) = I(z) = \frac{V}{K^+(z)K^-(z_\gamma)}\frac{z}{z-z_\gamma}. \tag{8}$$

The value of i_n is then obtained through the inverse Z-transform as

$$i_n = \frac{V}{2\pi \mathrm{j}}\frac{1}{K^-(z_\gamma)}\oint_C \frac{1}{K^+(z)}\frac{z^n}{z-z_\gamma}\mathrm{d}z, \tag{9}$$

where C corresponds to an integration path along the unit circle. From Figure 2 and equation (9), one can see that the current on each dipole is represented as the sum of the three contributions arising from the different poles and branch cuts found inside the unit circle. The additional branch cuts with respect to the case shown in [5] are found close to the origin for frequencies below the first diffraction onset and their contribution is negligible. We also find an additional pole z_{sw} due to the zero of (7), which can be readily located using an iterative complex-zero search algorithm.

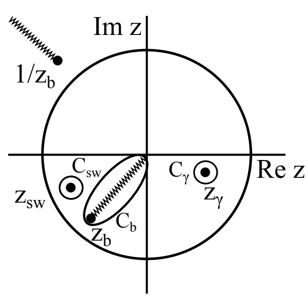

Fig. 2. Diagram of the position of the poles and branch cuts inside the integration path of (9).

III. Results

The different contributions arising from the branch cut and pole introduced inside the unit circle by the presence of $1/K^+(z)$ in addition to the pole introduced by the impressed field (depicted in Fig. 2) have been integrated numerically for the case of a TE-polarized plane wave with unit electric field amplitude with $\theta = 20^o$, $\phi = 0$, $d_x = 0.4\lambda$, $d_y = 0.5\lambda$, $l = 0.4\lambda$ and $w = 0.01\lambda$ leading to the results in Fig. 3. In contrast to the case shown in [5], the presence of the surface wave excited by the truncation introduces long-range effects on the behavior of the array which do not decrease with the distance from the array truncation, when losses can be neglected.

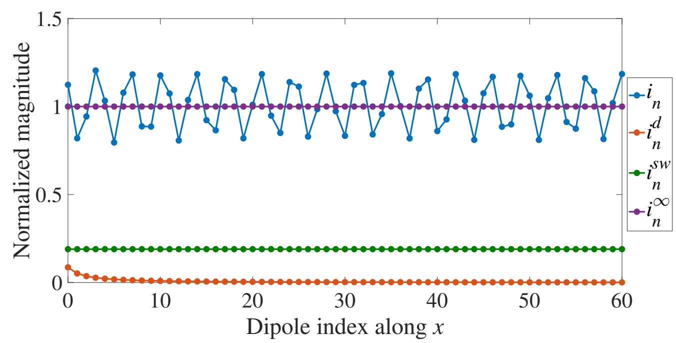

Fig. 3. Amplitude of the different contributions to the total current i_n arising from the contour integrals shown in Fig.2 comprising the edge-diffracted field (i_n^d), excited surface wave (i_n^{sw}) and the solution for the non-truncated (i.e., infinitely extended in both directions) array (i_n^∞), which the various components are normalized to

IV. Conclusion

A rigorous solution of the scattering by a two-dimensional periodic semi-infinite array of dipoles has been presented, using the Wiener-Hopf approach applied to the Z-transformed array currents and voltages pertaining to the infinite system of equations. This lead to the exact decomposition of the induced currents on the array in terms of (i) currents of the infinitely extended planar array plus (ii) edge diffracted currents, decaying algebraically away from the edge, and (iii) edge-excited surface waves, if supported by the array.

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