Three Essays on the Interaction Between Monetary and Fiscal Policy in Commodity-Rich Economies

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Submitted in October 2018
for the degree of
Doctor of Philosophy in Economics

March 2019

I would like to dedicate this thesis to my family and loved ones . . .

Declaration

Thesis Title: Three Essays on the Interaction Between Monetary and Fiscal Policy in Commodity-Rich Economies

Submitted by Haytem Troug to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Economics, August 2018

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This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

(Signature) .....

Haytem Troug
March 2019

Acknowledgements

First and foremost, I would like to express my most profound gratitude to my supervisors. Namely, Hrishikesh “Rish” Singhania who has supported me from the time I was doing my master, by providing unlimited support throughout my study at the university; to Sebastian Kripfganz, who was always there for me whenever I needed help; to Dudley Cooke, who provided support to me whenever I knocked on his door.

I would also like to thank the faculty of the economics department for their continues and unlimited support. Namely, Rajiv Sarin, James Davidson, Brit Grosskopf, Dieter Balkenborg, Carlos Cortinhas, Miguel Fonseca, Ramon Cobo Reyes, Jack Rogers, Jan Auerbach, Simone Meraglia, Julian Neira, and Elisa Keller. Furthermore, I would like to thank Exeter University for their financial support throughout my PhD. A special thanks to Kate Gannon, Rajesh Tharyan, and Natalie Bartram.

Furthermore, I am thankful to my colleagues and dear friends at the business school: Rami Chehab, Keqing Liu, Lorenz Adams, Lorenz Hartmann, Zuoxiang Zhao, Qizhi Wang, Bing Chao, Pei Sun, Jinlin Li, Mengyu Wang, Yaming Han, Lutfi Rahimi, Aya Ghalayini, Laura Colombo, Elif Soytoprak, and Ernil Sabaj.

A special thanks to my dearest friends: Michael Melidis, Florence Heath, Manni Eteeh, Matt Murray, Marwan Elmajin, Tarik Salel, Osama Jelani, and Osama Zawam.

Abstract

Assuming prior separability in preferences between private consumption and government consumption can produce stagnant estimates of the response of macroeconomic variables to government consumption shocks. These estimates conflict with some of the empirical findings in the literature. To overcome this discrepancy, and make DSGE models compatible with empirical models, we extend an otherwise standard New Keynesian model to allow for the presence of non-separable government consumption in the utility function, which is financed by means of lump-sum taxes. This introduction will affect the entire structure of New Keynesian models, and will make it deviate from the standard case.

In the first chapter, we conduct our analysis in a closed economy framework. We demonstrate how the introduction of government consumption in a non-separable form affects the transmission of monetary policy. We find that when government consumption has a crowding-in effect on private consumption, it will have a crowding-out effect on monetary policy. However, when government consumption has a crowding out effect on private consumption, it will have a crowding-in effect on monetary policy. This is a result of the effect of introducing non-separable government consumption on the slope of the IS curve. In this regard, when government consumption is introduced as a complement to private consumption in the model, macroeconomic variables become less responsive to changes in interest rates and vice versa. We also show how monetary policy should optimally respond to demand and supply shocks when government consumption is a complement to private consumption in one scenario, and once it is a substitute to private consumption in the other. If monetary policy fails to keep track of developments in government consumption, this will cause inflationary (deflationary) pressure when government consumption is a complement (substitute) to private consumption.

In the second chapter, we extend our analysis to a small open economy framework. Extending the model to a small open economy case complicates the problem for monetary policy to the extent that the authorities must additionally take into account how the exchange rate affects other macroeconomic variables. The other extension that we add to the canonical small open economy model is that we also model the rest of the world economy in our

framework. This will allow us to trace the spillover effects of supply and demand shocks in the foreign economy on the domestic economy.

In the open-economy case, the degree of openness will minimise the deviation of the slope of the IS curve from the standard case both in the complementarity case and the substitutability case. Moreover, the degree of openness minimises the crowding-out (-in) effect of government consumption towards monetary policy when the former is a complement (substitute) to private consumption. We also find that the fiscal multiplier is also minimised by the degree of openness of the economy, in comparison to the closed-economy version of the model. We additionally find that the size of the fiscal multiplier is adversely affected by the response of monetary policy and the flexibility of the exchange rate, which is in line with the findings of the existing literature. Moreover, we show that, in the case of the spillover effect of external shocks, the amount of exchange rate volatility will determine how much the domestic economy will be affected by external shocks. This will result in different effects of domestic and foreign government consumption on domestic private consumption, both in the substitutability and complementarity case, which contradicts the findings of some of the existing literature.

In the third chapter, we adopt a small open economy model for a commodity-rich country to quantitatively study the triggers of business cycles in different commodity-rich economies, and to highlight the existence of heterogeneity among commodity-rich economies. We extend the model used in the second chapter by adding some features to our model to make it more relevant for a commodity-rich economy. Our model allows for a quadruple role for commodities. First, the domestic government collects the windfalls of selling commodities to the rest of the world. Second, commodities are consumed by households both in the domestic economy and the foreign economy. Third, firms both in the domestic economy and the foreign economy use commodities as an input in their production. Lastly, the domestic economy is affected by the second-round effect of an increase in commodity prices in the form of high foreign inflation and low world demand. Moreover, the prices of commodities are endogenously determined in the model, and are affected by developments in the rest of the world economy.

The primary behavioural parameters that the chapter focuses on are the elasticity of substitution between government consumption and private consumption and the response of government consumption to fluctuations in commodity prices. The former parameter is an indicator of the efficiency of government consumption and its effect on private consumption (crowding-in versus crowding-out). The other parameter captures the behaviour and the stance of fiscal policy during booms and busts of commodity prices, along with the size of the commodity windfalls in the government's revenue. We feed the model with a variety of

shocks that were previously proposed in the literature. The estimations of the model show that oil-rich economies are more vulnerable to external shocks than their commodity-rich counterparts. This is mainly the result of the size of commodity windfalls in the economy, as the shares of oil revenues are significantly higher than the revenues of other commodities, as a ratio of output. The results also show that there exists a policy crowding out effect of fiscal policy to monetary policy in oil-rich economies.

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Chapter 1

Monetary Policy with Non-Separable Government Spending

1.1 Introduction

Government expenditure plays a significant role in stabilising and/or stimulating economic activities both in developed and developing countries. Conventional monetary policy, when not constrained at the zero-lower-bound, reacts to changes in its targeted variables of interest which might, in return, be affected by changes in fiscal policy. The significant role of government consumption in affecting economic conditions raises the necessity for monetary policy to take into account the behaviour of fiscal policy and to also take into account how the presence of the fiscal sector might affect the transmission mechanism of monetary policy in the economy. These dynamics raise the question of how government consumption affects the economy. Moreover, how the dynamics of the economy change under this effect?

Government consumption was usually introduced to DSGE models under the standard hypothesis that it is either complete waste (Obstfeld & Rogoff 1995, 1996) or included to preferences in a separable form. While the former approach became obsolete in the literature, the inclusion of government consumption to preferences in a separable form was adopted both in RBC models (Baxter & King (1993)) and in New Keynesian models (Smets & Wouters (2007) and Gali & Monacelli (2008)). The inclusion of government consumption in the utility function, despite being understudied as highlighted by Cantore et al. (2014), seems appealing since agents gain utility from government consumption, making the introduction of government consumption meaningful in the model. The inclusion is also supported by the fact that the primary purpose of government consumption in any economy is to provide goods services for the agents of that economy. Nevertheless, assuming prior separability in

preferences between private consumption and government consumption can produce stagnant estimates of the response of private consumption, labour supply, and, hence, of output to a government consumption shock, as recently highlighted by Ercolani & e Azevedo (2014).

Under the standard hypothesis of DSGE models, any increase in government consumption will be financed by current and future lump-sum taxes. This tax increase would lower the present value of the after-tax income of the Ricardian consumers. Thus, the adverse wealth effect of government consumption is the primary mechanism of a government consumption shock. The adverse wealth effect will cause private consumption to decline, labour supply to increase, and in equilibrium this will cause lower real wages, higher employment, and higher output. Gali et al. (2007) is one of the most influential papers that challenged the adverse effect of government consumption on private consumption. Where the paper highlighted the discrepancy between the results generated by standard DSGE models and the ones displayed by some empirical models, which show a positive effect of government consumption on private consumption. To overcome this discrepancy, the paper includes rule-of-thumb consumers who do not have access to the financial markets. The fraction of this type of consumers was adjusted in an *ad-hoc* manner until the prediction of the model replicated empirical findings in U.S. data. Nevertheless, the findings of the paper were not empirically robust for other countries as found by Coenen & Straub (2005).

Moreover, the literature is yet to reach consensus on the effect of government consumption on private consumption. Ganelli (2003) introduced government consumption as a substitute for private consumption. The elasticity of substitution between government consumption and private consumption, in his model, governs how much private consumption needs to decline in response to an increase in government consumption to keep the utility on the same indifference curve. Ercolani & e Azevedo (2014) also show results that indicate that government consumption is a substitute for private consumption in a New Keynesian framework. Bouakez & Rebei (2007) and Pieschacon (2012), among others, support the complementarity assumption between private consumption and government consumption. The two papers employ an RBC model and use the complementarity assumption to analyse the effect of a government consumption shock on the main macro variables of the model. The disparity in the literature goes beyond the theoretical models, where even empirical models show different estimates depending on the modelled time frame and the adopted method[1].

The above complications raise a crucial question about the reaction of monetary policy to different shocks once government consumption is included in the utility function in a non-separable form. Introducing non-separable government consumption to a standard New

[1] Karras (1994), Fiorito & Kollintzas (2004), and Coenen et al. (2013) find complementarity between the two variables while Aschauer (1985) and Ahmed (1986) find substitutability between the two.

Keynesian model would affect the labour supply condition and the consumption smoothing condition. This, in return, will have an effect on the structure of the whole model and would affect the transmission mechanism of monetary policy. To the best of our knowledge, this issue has not yet been addressed by the existing literature, and we aim to fill in this gap in this paper. The mechanism of this model applies to both the complementarity case and the substitutability case, and we will focus on the changes in the reaction and the transmission mechanism of monetary policy, after the government sector is incorporated to a standard New Keynesian model in a non-separable form.

We use the term "government consumption" in this paper for non-fixed capital formation government expenditure. For instance, it represents government provision of goods and services, excluding compensations of state employees. In practice, government consumption could either be a substitute or a complement to private consumption. For instance, government consumption on health and education could crowd-out private consumption on those two items, while government spending on security encourages private consumption in general. Also, the quality of government consumption is different from one country to another, and this might produce different effects of government consumption among those countries.

We extend an otherwise standard New Keynesian model to allow for meaningful government consumption to the utility function in a non-separable form. Means of lump-sum taxes finance this government consumption. The inclusion of government consumption will affect the slope of the IS curve, and will make it flatter in the complementarity case than in the standard case. As a result, the response of output and consumption to changes in interest rates will weaken, showing an indication of a crowding-out effect of fiscal policy on monetary policy. In the substitutability case, however, the response of the macro variables to changes in interest rates will be higher, as the IS curve becomes steeper than in the standard case. This addition of the fiscal sector makes the model comprehensive to all degrees of substitutability between private and government consumption and would overcome the limitations of the *ad-hoc* approach of including rule-of-thumb consumers.

In addition, our results on the fiscal multiplier conflict with the findings of Woodford (2011) and Koh (2017) as unlike these papers, we find that the size of the fiscal multiplier is not sensitive to the reaction of monetary policy. We also find, similar to Ercolani & e Azevedo (2015), that the fiscal multiplier is substantially lower in the substitutability case.

The remainder of the paper is organised as follows: We first demonstrate the structure of our model in the second section. In the third section, we show the parametrisation of the model. The equilibrium dynamics of the model will be discussed in the fourth section and the analysis of the impulse response functions is presented in the fifth section. In the sixth

section, we show the results of the welfare loss calculations and the second moments of the main variables of the economy. Lastly, the seventh section provides the concluding remarks.

1.2 The Model

1.2.1 Households

Our economy is populated by a representative household that derives utility from aggregate consumption and leisure. The household is assumed to live infinitely, and in each period the household is endowed with one unit of time, which is divided between work and leisure: $N_t + L_t = 1$. The representative consumer seeks to maximise the following discounted lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{C}_t, N_t) \tag{1.1}$$

The utility function is assumed to be continuous and twice differentiable. N_t is the number of hours worked; β is the discount factor; $\bar{C}_t$ is the aggregate consumption bundle. $\bar{C}_t$ is a constant elasticity of substitution aggregate consisting of private consumption C_t and government consumption G_t[2]:

$$\bar{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{1.2}$$

Where δ is the share of private consumption in the aggregate consumption bundle, and χ is the inverse elasticity of substitution between private consumption and government consumption. In this setup, for government to play a role in the utility function, δ has to be strictly less than 1. Moreover, the value of χ has to deviate from 1 in order for government consumption to influence the rest of the dynamics in the model. C_t is the private consumption of goods produced in the economy and it is represented by the unit interval: $C_t \equiv \left(\int_0^1 C_t^{\frac{\varepsilon-1}{\varepsilon}}(j)dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ for all $j \in [0,1]$. From equations (1.1) and (1.2) we can notice that the utility function is non-decreasing in government consumption G_t. The above utility function is subject to the following budget constraint:

$$\int_0^1 P_t(j)C_t(j)dj + E_t Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t \tag{1.3}$$

[2]As noted above, government consumption in this framework can be thought of as a public good that households consume at a free cost. It can also be thought of as government expenditure on security and defence which stimulates private consumption and increases the utility of households.

Where D_t is the nominal payoff at period $t+1$ of bonds held at the end of period t including shares in firms, government bonds and deposits. W_t is wages and T_t is lump-sum transfers to the households net of lump-sum taxes.

The utility function that we use assumes two separabilities. The first one is the separation between consumption and the number of hours worked, and the second one is time separability. The household's problem is also analysed in two stages in this paper. We first deal with the expenditure minimisation problem faced by the representative household to derive the demand functions for goods. In the second stage, households choose the level of C_t and N_t, given the optimally chosen combination of goods.

Now as a first step, the households must minimise their expenditure by optimally choosing the share of each good in the aggregate consumption bundle. Doing so will yield the following demand functions:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t \tag{1.4}$$

Where $P_t \equiv (\int_0^1 P_t(j)^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index. ε is the elasticity of substitution between goods in the economy, and it shows how much the demand for good (j) will decline if the relative price of that good increased by 1 unit. A lower elasticity of substitution indicates higher consumption of the good of interest. This assumption shows that goods in the consumption bundle are not perfect substitutes.

Now we turn our attention to the per-period utility function in the following form[3]:

$$U(C_t, N_t; G_t) = \frac{\bar{C}_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{1.5}$$

In the above equation, σ is the inverse elasticity of intertemporal substitution. Setting σ equal to 1 implies that the household has log-utility in consumption. φ is the inverse Frisch labour supply coefficient. Parameter $\varphi > 0$ also measures the curvature of the marginal disutility of labour. The above equation is subject to the aggregate budget constraint, which we get by plugging the above demand bundles and price indices in equation (1.3):

$$P_t C_t + E_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \tag{1.6}$$

Where E_t is the conditional expectations operator. The household's aggregate expenditure basket is equal to: $P_t C_t = \int_0^1 P_t(j) C_t(j) dj$. From Equation (1.5) and (1.6) we can write the

[3] We replaced private consumption in the utility function with the aggregate consumption bundle. As noted above, this is one of the deviations that we make from Gali (2008).

standard optimality condition for the households as follows:

$$\frac{W_t}{P_t} = N_t^{\varphi} \bar{C}_t^{\sigma} \left(\frac{C_t}{\bar{C}_t}\right)^{\chi} \delta^{-\chi} \tag{1.7}$$

The intertemporal optimality condition is:

$$\beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{C_t}{C_{t+1}}\right)^{\chi} = Q_{t,t+1} \tag{1.8}$$

Taking the conditional expectation of equation (1.8) and rearranging the terms we get:

$$\beta R_t E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{C_t}{C_{t+1}}\right)^{\chi} \right] = 1 \tag{1.9}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is the one-period return from a riskless bond and $Q_{t,t+1}$ is the expected price of that bond. The form of equations (1.7) and (1.9) deviate from the standard literature. This deviation from the standard case is due to the fact that we have included government consumption in an aggregate CES basket with private consumption in a non-separable form. The first equation depicts the labour supply dynamics; it shows labour supply as a function of real wages, given the aggregate consumption bundle and private consumption, and it also shows how the effect of the aggregate consumption bundle on labour supply depends on the value of χ. Noting that in the Cobb-Douglas case, when $\chi = \sigma$, the labour supply equation collapses to its standard form as government consumption would not affect private consumption. When $\chi > \sigma$, government consumption will have a negative effect on real wages given its positive effect on labour supply. When $\chi < \sigma$, on the other hand, government consumption will have a positive effect on real wages given its negative effect on labour supply. The second equation is the Euler equation that characterises consumption smoothing. The Euler equation in this model deviates from the standard form that is found in the literature. In our case, the smoothing of the aggregate consumption bundle $\bar{C}$ is a component of the above Euler equation. In the Cobb-Douglas case, when $\chi = \sigma$, the above equation would collapse to the canonical version of the Euler equation. When $\chi > \sigma$, any changes in the current value of the aggregate consumption bundle will have a positive effect on private consumption. Conversely the former will have an adverse effect on private consumption when $\chi < \sigma$.

1.2.2 The Effect of Government Consumption on Private Consumption

In this section, we demonstrate how the effect of government consumption on private consumption is subject to the value of the elasticity of substitution between the two. We show this under three different values for the inverse elasticity of substitution between government consumption and private consumption χ: a) The Cobb-Douglas scenario when the elasticity between government consumption and private consumption is equal to the inverse elasticity of intertemporal substitution $\chi = \sigma$. b) The complementarity case when $\chi > \sigma$. And c) $\chi < \sigma$ when the two variables are substitutes. Maximising the utility function (1.5) with respect to the budget constraint (1.6) yields:

$$\ell = \frac{\left[\left(\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right)^{\frac{1}{1-\chi}}\right]^{1-\sigma}}{1-\sigma} - 1 - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t (D_t + W_t N_t + P_t T_t - P_t C_t - E_t[Q_{t,t+1} D_{t+1}]) \tag{1.10}$$

We then take the first-order condition with respect to private consumption to show how the marginal utility of consumption reacts to changes in government consumption under different values of elasticity of substitution:

$$\frac{\partial \ell}{\partial C_t} = \delta^{\chi} \bar{C}_t^{-\sigma} \left(\frac{C_t}{\bar{C}_t}\right)^{-\chi} - P_t \lambda_t = 0 \tag{1.11}$$

Now we check the response of the marginal utility of consumption to changes in G_t:

$$\frac{\partial \lambda_t}{\partial G_t} = \chi - \sigma \bar{C}_t^{\chi-\sigma-1} \left(\frac{C_t G_t}{\bar{C}_t}\right)^{-\chi} \left(\frac{\delta^{\chi}(1-\delta)^{\chi}}{P_t}\right) \tag{1.12}$$

The above equation shows that the reaction of the marginal utility of consumption for a given level of consumption will depend on the value of $\chi - \sigma$:

a) $\chi = \sigma$: in the Cobb-Douglas case when $\chi = 1$, the above ratio will collapse to 0, regardless of the size of δ in the utility function.

b) $\chi > \sigma$: in this case, the effect of government consumption will be positive, and as $\chi \to \infty$ the two variables will be perfect complements.

c) $\chi < \sigma$, in this case, the sign of the term above will be negative, and any changes in government consumption will have an adverse effect on private consumption. Also, as $\chi \to 0$, the two variables will be perfect substitutes.

It is easy to see from the above analysis that once we change the size of χ, the dynamics of the whole model will follow, as we will show below. In the separable case when $\chi = \sigma$, the

entire model collapses to the standard version of the model since the government consumes different goods than the ones consumed by the representative consumer.

1.2.3 Firms

Price Setting Behaviour

The firms in this model set their prices in a staggered way following Calvo (1983)[4]. Under Calvo contracts, we have a random fraction $1-\theta$ of the firms that can reset their prices at period t, while the remaining firms, of size θ, keep their prices fixed at the previous period's price levels. Therefore, θ^k is the probability that a price set at period t will still be valid at period $t+k$. Also, the likelihood of the firm re-optimising its prices will be independent of the time passed since it last re-optimised its prices, and the average duration of prices not to change is $\frac{1}{1-\theta}$. Given the above information, the aggregate price level will take the following form:

$$P_t = \left[\theta(P_{t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_t^{1-\varepsilon})\right]^{\frac{1}{1-\varepsilon}} \tag{1.13}$$

Where $\bar{P}_t$ is the new price set by the optimising firms. From the derivations shown in Appendix A.2, we get the following form for inflation at period t:

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\bar{P}_t}{P_{t-1}}\right)^{1-\varepsilon} \tag{1.14}$$

The above equation shows that the inflation rate at any given period is solely determined by the fraction of firms that reset their prices at that period. In addition, when a given firm in the economy sets its prices, it seeks to maximise the expected discounted value of its stream of profits, conditional that the price it sets remains effective:

$$max_{\bar{P}_t} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}[c_{jt+k|t}(\bar{P}_t - \Psi_{t+k})] \right\} \tag{1.15}$$

The above equation is subject to a sequence of demand constraints: $c_{jt+k} = \left(\frac{\bar{P}_t}{P_{t+k}}\right)^{-\varepsilon} C_t$. Solving this problem (also shown in Appendix A.2) yields the following optimal decision rule:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} C_{t+k} \left[\frac{\bar{P}_t}{P_{t-1}} - \mathscr{M} MC_{t+k|t} \Pi^H_{t-1,t+k} \right] \right\} = 0 \tag{1.16}$$

[4]The Calvo model makes aggregation easier because it gets rid of the heterogeneity in the economy. The alternative pricing scheme is the quadratic cost of price adjustment by Rotemberg (1982). The two dynamics are equivalent up to a first-order approximation.

In the above equation, $\mathscr{M}$ is the firm's markup at the steady state, and MC_t is the real marginal cost. As shown in equation (1.16), in the sticky price scheme, producers, given their forward-looking behaviour, adjust their prices at a random period to maximise the expected discounted value of their profits at that period and in the future. Thus, firms in this model will set their prices equal to a markup plus the present value of the future expected stream of their marginal costs. The price-setting behaviour takes this form because firms know that the price they set at period t will remain valid for a random period of time in the future. We also assume that all firms in the economy face the same marginal cost, given the constant returns to scale assumption imposed in the model and the subsidy that the government pays to firms, as we will see in the following section. The firms also use the same discount factor β as the one used by households because the households are the shareholders of these firms. All the firms that optimise their prices in any given period will choose the same price, which is also a consequence of the firms facing the same marginal cost. Equation (1.16) also shows that the inflation rate is proportional to the discounted sum of the future real marginal costs additional to a markup resulting from the monopolistic power of the firms.

Production

A certain firm in the domestic economy produces a differentiated good following a linear production function:

$$Y_t(j) = A_t N_t(j) \tag{1.17}$$

Where $Y_t(j)$ is the output of final good (j) in the home economy. A_t is the level of technology in the production function. Technology is assumed to be common across all firms in the economy, and it evolves exogenously. $N_t(j)$ is the labour force employed by firm (j). The log form of total factor productivity $a_t = log(A_t)$ is assumed to follow an AR(1) process: $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$. Where ρ_a is the autocorrelation of technology and the innovation to technology $\varepsilon_{a,t}$ has a zero mean and a finite variance σ_a. Capital was excluded from production in this model for the sake of tractability. Aggregate output and aggregate employment in the domestic economy are defined by the Dixit & Stiglitz (1977) aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}; N_t = \left(\int_0^1 N_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.18}$$

Given the common technology across all firms of the economy, the total cost function for firm (j) is defined as follows:

$$TC_t(j) = \frac{(1-\tau)W_t Y_t(j)}{A_t} \tag{1.19}$$

Where τ is the subsidy that the government gives to firms in order to eliminate the markup distortion, which is created by the firms' monopolistic power. Taking the first-order condition of the above equation yields the following marginal cost equation:

$$MC_t(j) = \frac{(1-\tau)W_t}{A_t} \tag{1.20}$$

From the above equation, it is clear that the subsidy and the constant return to scale assumption make the marginal cost independent of the firm's production level. This will make marginal cost common across all firms: $MC_t(j) = MC_t$, and the common real marginal cost will look like:

$$MC_t^r = \frac{(1-\tau)}{A_t}\frac{W_t}{P_t} \tag{1.21}$$

The marginal cost equation is expressed in terms of the aggregate price level P_t, wages W_t, total factor productivity A_t, and the subsidy that the government gives to firms (τ). The latter, as explained above, is paid to eliminate the markup distortion created by the firms' monopolistic power.

Lastly, after the aggregation of output and employment, we get the following aggregate production function:

$$Y_t = A_t N_t \tag{1.22}$$

1.2.4 The Supply Side of the Economy

We now turn our attention to the supply side of the economy. From the firm's section, the log-linearised version of the marginal cost equation of the firms in the economy takes the following form:

$$\begin{aligned} mc_t &= -v + w_t - p_t - a_t \\ &= -v + \varphi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t - a_t \\ &= -v + \varphi y_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t - (1+\varphi) a_t \\ &= -v + (\varphi + \sigma_\delta) y_t + (\sigma - \sigma_\delta) g_t - (1+\varphi) a_t \end{aligned} \tag{1.23}$$

Where $\sigma_\delta = \delta\sigma + (1-\delta)\chi$ is a weighted sum of the intertemporal elasticity of substitution and the elasticity of substitution between government consumption and private consumption. In the above equation, we made use of the log forms of the labour supply equation (eq. 1.7), the production function (eq. 1.22), and the market clearing condition

($c_t = y_t$). The above equation shows the positive effect of the increase in demand on the marginal cost. It also shows that technology has a negative effect on the marginal cost. The effect of government consumption on the marginal cost depends on the value of σ_δ. In the Cobb-Douglas case, government consumption will have no effect on the marginal cost, as $\sigma_\delta = \sigma$, and the above equation will collapse to its standard version. If the inverse elasticity of substitution between government consumption and household's consumption is greater than the inverse elasticity of intertemporal substitution ($\chi > \sigma$), the effect of government consumption on the marginal cost is negative, and this is a result of the negative effect of government consumption on real wages in this case. This effect is inherited from the labour supply equation (eq. 1.7) where we show that government consumption has a negative effect on real wages, given the increase in labour supply. On the other hand, when $\chi < \sigma$ then government consumption will have a positive effect on marginal cost, as it also has a positive effect on real wages through its negative effect on labour supply. To calculate the natural level of output, we equate the marginal cost to $(-\mu)$, as this is the state of marginal cost under flexible prices[5]. Getting rid of the constant terms, the natural rate of output equation will be:

$$\bar{y}_t = -\left(\frac{\sigma - \sigma_\delta}{\varphi + \sigma_\delta}\right) g_t + \left(\frac{1+\varphi}{\varphi + \sigma_\delta}\right) a_t \tag{1.24}$$

The above equation shows the positive effect of government consumption and productivity on the natural rate of output when government consumption is a complement to private consumption. Also, when $\chi > \sigma$, the effect of technology on the natural rate of output is less than its effect in the standard case. When government consumption is a substitute to private consumption, however, the effect of government consumption on the natural rate of output will be negative, and the effect of technology will be greater than its effect in the standard case. To get the relationship between the output gap and the marginal cost, we subtract the above equation from equation (1.23):

$$\hat{mc}_t = (\varphi + \sigma_\delta) x_t \tag{1.25}$$

Where $x_t = y_t - \bar{y}_t$ is the output gap. Plugging the value of the marginal cost in the above equation into the derived Phillips curve equation in Appendix A.2 yields inflation as a function of the output gap and inflation expectations one-period ahead:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(\varphi + \sigma_\delta) x_t \tag{1.26}$$

[5] $\mu = 0$ in the perfect competition case.

1.2.5 The Demand Side of the Economy

Moving to the demand side of the economy, we add the loglinearised form of the Euler equation (eq. 1.9) to the market clearing equation ($c_t = y_t$) to get:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\delta}[r_t - E_t\{\pi_{t+1}\}] + \frac{\sigma - \sigma_\delta}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \tag{1.27}$$

In the special case of a Cobb-Douglas utility function, the above IS curve converges back to the canonical representation of the IS curve with the slope being equal to 1. Government consumption when $\chi = \sigma$ has no effect on output as $\sigma = \sigma_\delta$. In the case of complementarity between government consumption and private consumption $\chi > \sigma$, the slope of the IS curve is flatter than the standard case (Figure 1). The fact that $\sigma_\delta > 1$ when $\chi > \sigma$ dampens the response of output to changes in interest rates. In the case of substitutability between government consumption and private consumption $\chi < \sigma$, the slope of the IS curve is steeper, which strengthens the response of output to changes in interest rates. Also, adding government consumption to the model will shift the IS curve on the right of the standard IS curve. Nevertheless, the new curve will not be parallel to the old curve once we introduce government expenditure to the utility function in a non-separable form. Solving the above IS curve for the output gap yields:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\delta}[r_t - E_t\{\pi_{t+1}\} - \bar{r}r_t] \tag{1.28}$$

Where:

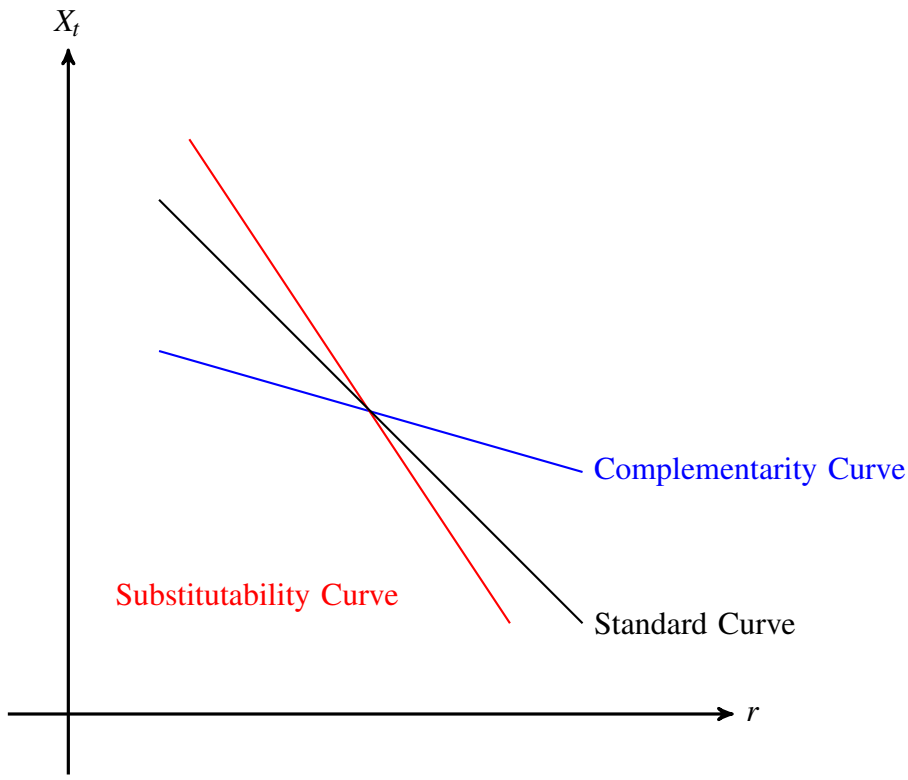
$$\begin{aligned} \bar{r}r_t &= \sigma_\delta \Delta y_{t+1} + (\sigma - \sigma_\delta)\Delta g_{t+1} \\ &= -\frac{\sigma_\delta(1-\rho_a)(1+\varphi)}{\varphi + \sigma_\delta} a_t - \frac{(\sigma - \sigma_\delta)(1-\rho_g)\varphi}{\varphi + \sigma_\delta} g_t \end{aligned} \tag{1.29}$$

The above natural rate of interest still, consistent with the canonical case, negatively reacts to changes in productivity. Nevertheless, the introduction of government consumption in a non-separable form changes the magnitude of the response of the natural rate of interest to a technology shock through the changes in the value of the weighted elasticity of substitution σ_δ. In the complementarity case when $\chi > \sigma$, the response of the natural rate of interest is dampened, given that the slope of the IS curve is flatter in this case. In the substitutability case when $\chi < \sigma$, the response of the natural rate of interest to a technology shock is magnified, given that the IS curve is steeper in this case.

Additionally, when government consumption is included in a non-separable form ($\sigma \neq \sigma_\delta$), the natural rate of interest reacts to changes in government consumption. In the

complementarity case when $\chi > \sigma$, the natural rate of interest positively reacts to changes in government consumption, given the inflationary pressures that the latter causes. In the substitutability case when $\chi < \sigma$, on the other hand, the natural rate of interest negatively reacts to changes in government consumption, given the latter's negative effect on output.

Figure 1.1 The IS curve



1.2.6 Fiscal and monetary policy

Fiscal Policy

The fiscal sector of this model has the following budget constraint:

$$P_t G_t + (1 + R_{t-1}) B_{t-1} = B_t + P_t T_t \tag{1.30}$$

Where B_t is the quantity of a riskless one-period bond maturing in the current period, and it pays one unit. R_t denotes the gross nominal return on bonds purchased in period (t). The government levies a non-distortionary lump-sum tax T_t to finance its expenditure. Following

the existing literature[6], G_t is government expenditure, and it evolves exogenously according to the following first order autoregressive process:

$$\frac{G_t}{G} = \left\{\frac{G_{t-1}}{G}\right\}^{\rho_g} exp(\zeta_{G,t}) \tag{1.31}$$

Where $0 < \rho_g < 1$ is the autocorrelation of government consumption, and $\zeta_{G,t}$ represents an i.i.d government consumption shock with constant variance σ^2. Another important feature that we add to this model is to consider that the government consumes from a different market than the one occupied by the private agents. Moreover, following the above mentioned literature, government consumption is produced costlessly[7].

Monetary Policy

The central bank in this model uses a short-term interest rate as its policy tool. In our case, we have a cashless economy where the money supply is implicitly determined to achieve the interest rate target. We also assume that the central bank will meet all money demand under the policy rate it sets.

We first demonstrate a number of possible policy tools that might be employed by the monetary authority in the economy. The first rule in the model will be the optimal rule:

$$\frac{R_t}{R} = \left\{\frac{RR_t}{RR}\right\}\left\{\frac{\Pi_t}{\Pi}\right\}^{\phi_\pi}\left\{\frac{Y_t}{Y}\right\}^{\phi_x} \tag{1.32}$$

The optimal rule illustrates how by setting domestic CPI inflation and the output gap to zero, the policy rate will be equal to the natural rate of interest, and it will be able to accommodate developments in the natural rate of output. The optimal policy reproduces the flexible price equilibrium output, given that the government will pay a subsidy τ to offset the monopolistic distortion in the economy. The above policy rule will provide a useful benchmark to evaluate the performance of different monetary policy rates.

The first rule is a Taylor rule:

$$\frac{R_t}{R} = \left\{\frac{\Pi_t}{\Pi}\right\}^{\phi_\pi}\left\{\frac{Y_t}{Y}\right\}^{\phi_x} \tag{1.33}$$

[6]See, for example, Bouakez & Rebei (2007) and Sims & Wolff (2018).

[7]Despite the fact that the assumption of costless government consumption is counterintuitive, it suits the purpose of this paper to focus on how the non-fiscal sectors react to different shocks once government consumption is introduced in a non-separable format.

The other policy rule is a CPI-targeting Taylor rule:

$$\frac{R_t}{R} = \left\{ \frac{\Pi_t}{\Pi} \right\}^{\phi_\pi} \tag{1.34}$$

The parameters of the above policy rules describe the strength of the response of the policy rate to deviations in the variables on the right-hand side. These parameters are also assumed to be non-negative. The last rule is often referred to as a naive interest rule, and that is because it only makes use of observable variables. Also, the inflation response parameter ϕ_π in the above policy rates must be higher than 1 in order for the solution of the model to be unique, as shown by Bullard & Mitra (2002) and depicted in Appendix A.4.

1.3 Parametrisation

β	Discount factor	0.99
σ	inverse elasticity of intertemporal substitution	1
φ	inverse Frisch labour supply elasticity	3
χ	inverse elasticity of substitution between C_t&G_t	20/0.01
ε	elasticity of substitution	6
δ	share of private consumption in the aggregate consumption bundle	0.95
θ	Calvo probability	0.75
ρ_g	AR(1) coefficient of government expenditure	0.9
ϕ_π	inflation elasticity of the nominal interest rate	1.5
ϕ_x	output gap elasticity of the nominal interest rate	0.5
ρ_a	AR(1) coefficient of productivity shock	0.9

The values of the parameters of the model are listed in the above table. We Set θ equal to 0.75, which implies that firms only change their prices once a year. Our discount factor β is equal to 0.99. This parameter value implies that, given that $\beta = 1/r$ at the steady state, annual return is approximately equal to 4 %. We set φ equal to 3, under the assumption that the labour supply elasticity is $\frac{1}{3}$. We set ϕ_π & ϕ_x equal to 1.5 and 0.5 following Taylor (1993). We also set the inverse elasticity of substitution between government consumption and private consumption χ is set equal to 20 following Bouakez & Rebei (2007) and Pieschacon (2012), and we use 0.01 to illustrate the dynamics of the model in the substitutability case. The size of household's consumption in the aggregate consumption bundle δ equal to 0.95. In this regard, and as mentioned above, the weight of δ hast to be strictly less than 1 for government consumption to influence the dynamics of the model, and different values of δ between 0 and

1 only affect the model quantitatively. Moreover, change in the value of χ do not qualitatively affect the behaviour of the model.

The inverse elasticity of intertemporal substitution of consumption is set equal to 1 which implies a log utility form. The elasticity of substitution between the domestically produced goods ε equals 6 which corresponds to a steady state markup of 1.2. Also, we adopt the persistence parameter of government consumption ρ_g from Gali et al. (2007). As for the standard deviations of the two shock processes, we use the standard deviation of the TFP shock in Gali & Monacelli (2005) $\sigma_a = 0.0071$, and for the government consumption shock we use the one in Coenen & Straub (2005) $\sigma_g = 0.323$.

1.4 Equilibrium Dynamics

The key equations that we use to analyse the model's equilibrium implications are the non-policy block equations (IS demand curve, NKPC) and the interest rate rule adopted by the monetary authority:

$$\begin{aligned} x_t &= E_t\{x_{t+1}\} - \frac{1}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t) \\ \pi_t &= \beta E_t\{\pi_{t+1}\} + \kappa_g x_t \\ r_t &= \overline{rr}_t + \phi_\pi \pi_t + \phi_x x_t \end{aligned} \tag{1.35}$$

Where $\kappa_g = \kappa(\varphi + \sigma_\delta)$. Combining the first equation with the third equation allows us to simplify the above system of equations to only two equations. Solving for the output gap and domestic CPI inflation as a function of their respective expectations yields the following system of equations:

$$\begin{aligned} x_t &= \frac{\sigma_\delta}{\sigma_\delta + \phi_x + \kappa_g \phi_\pi} E_t\{x_{t+1}\} + \frac{(1 - \beta\phi_\pi)}{\sigma_\delta + \phi_x + \kappa_g \phi_\pi} E_t\{\pi_{H,t+1}\} \\ \pi_t &= \frac{\kappa_g \sigma_\delta}{\sigma_\delta + \phi_x + \kappa_g \phi_\pi} E_t\{x_{t+1}\} + \frac{\beta(\sigma_\delta + \phi_x) + \kappa_g}{\sigma_\delta + \phi_x + \kappa_g \phi_\pi} E_t\{\pi_{H,t+1}\} \end{aligned} \tag{1.36}$$

The above system of equation could easily be presented in a matrix form:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix}$$

Where $A = \Omega \begin{bmatrix} \sigma_\delta & (1-\beta\phi_\pi) \\ \kappa_g\sigma_\delta & \beta(\sigma_\delta+\phi_x)+\kappa_g \end{bmatrix}$ and $\Omega = \frac{1}{\sigma_\delta+\phi_x+\kappa_g\phi_\pi}$.

Under the assumed values of the policy parameters and the non-policy parameters of the model shown in the calibration section, we find that both eigenvalues of matrix A lie inside the unit circle making the solution of the model unique. The results also apply to the CPI targeting rule and the Taylor rule as well. The equilibrium in each model is unique, and it satisfies the following condition[8]:

$$\kappa_g(\phi_\pi - 1) + (1-\beta)\phi_x > 0. \tag{1.37}$$

From the above equation, it is clear that the inflation parameter has to be strictly greater than one ($\phi_\pi > 1$) for this rule to be determined, along with a trivial condition which requires $0 < \beta < 1$.

1.5 IRF

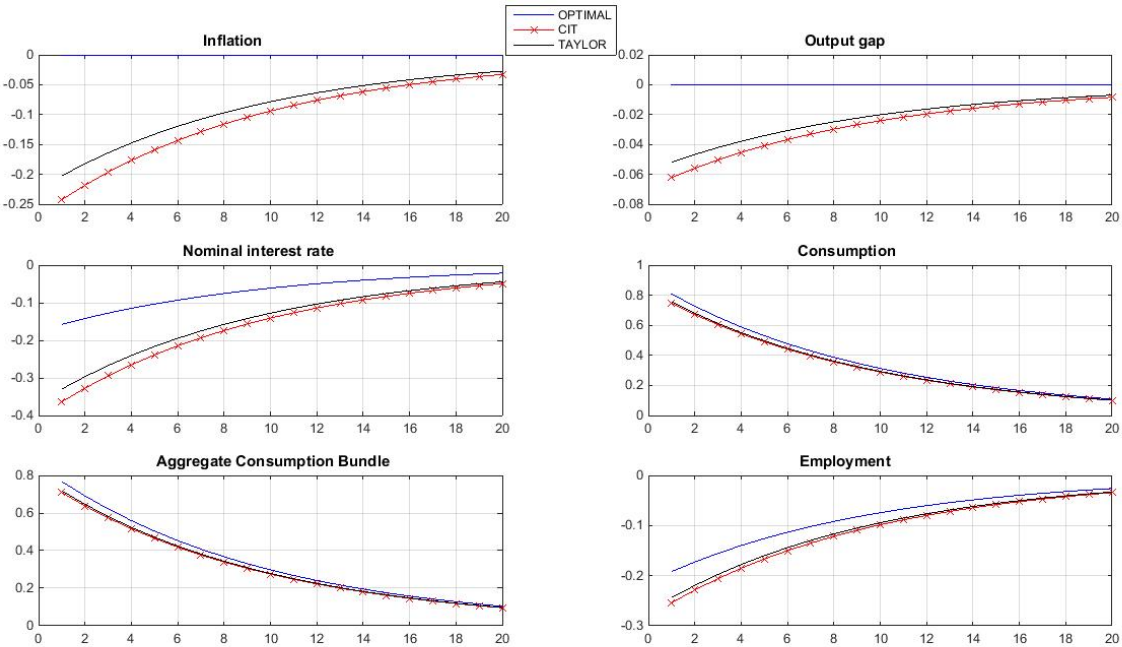
1.5.1 The Complementarity Case

Technology Shock:

The effect of a technology shock on the output gap and inflation under the optimal policy is zero, by construction. Once the changes in the output gap and inflation are set to zero, the optimal policy rule will follow the path of the natural rate of interest (eq. 1.29). Given that the natural rate of interest takes into account development in the natural rate of output, it will have a neutral monetary stance which is neither expansionary nor contractionary. The above graph shows a persistent reduction in the interest rates, following the technology shock, to support the transitory expansion in output/consumption, and this is consistent with the flexible prices case. The interest rates will only affect the economy in this model via the traditional intertemporal channel in the IS curve. Also, different from the standard case, the natural rate of output and the actual rate of output grow at a rate less than the size of the technology shock. This, in return, causes a decline in employment under the optimal policy rule. If the optimal policy tries to stimulate output to alleviate the decline in employment, it will cause inflationary pressures.

[8] See Appendix A.4.

Figure 1.2 Response to a TFP Shock (Complementarity)



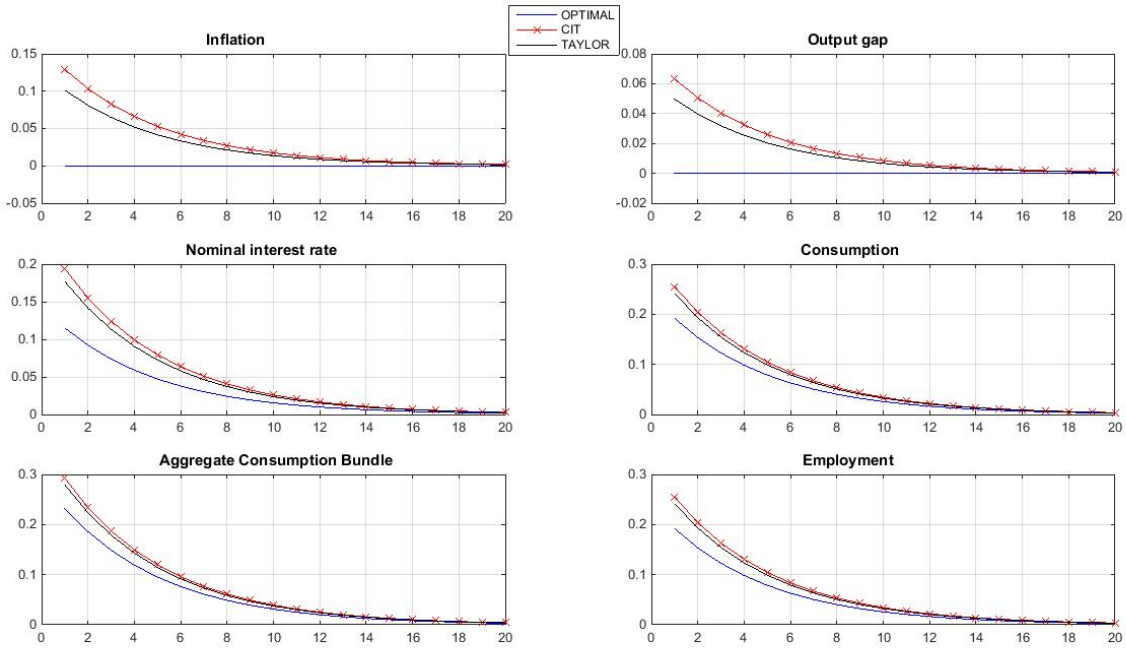
The other two policy rules in the model take an expansionary stance in response to a technology shock, given that both of them operate below the optimal policy rule. Despite this reaction, both policies fail to stimulate the actual rate of output to reach its natural level, and this what causes a negative output gap and negative inflation levels (deflation). This is a consequence of the inability of the two policies to maintain inflation expectations at the zero level and given the forward-looking behaviour of the agents of the economy. As a result, the expansionary stance of both policies will not be fully reflected in the actual rate of output. Employment under these two policies falls at a higher rate than the case under the optimal policy rule, given the relatively higher difference between the growth of output and the size of the technology shock.

Moreover, the effect of the shock on private consumption is identical to its effect on the actual rate of output, given the market clearing condition of this model. Also, from the above IRF response, it is clear that the Taylor rule outperforms the CPI-targeting Taylor rule. This is attributed to the fact that the earlier keeps track of more variables in the model, and it closely resembles the behaviour of the optimal policy.

Government Consumption Shock:

A shock to government consumption will immediately cause an increase in private consumption, given the assumption that the two are complements in this scenario. The increase in consumption will be mirrored by output, following the market clearing condition. The effect of the government shock will be positive both on the natural rate of output and the natural rate of interest. We also notice that the size of the fiscal multiplier is not sensitive to the reaction of monetary policy. This can be seen in the below figure, as consumption and output are not sensitive to the strength of the interest rates.

Figure 1.3 Response to a Government Consumption Shock (Complementarity)



The role of monetary policy under this shock is to control the inflationary increase in demand. Similar to the above analysis, the output gap and inflation are, by construction, set to zero under the optimal policy rule. The policy rule increases in response to a government consumption shock, and this increase aims to reduce the inflationary pressure caused by the shock. Employment under this scenario, since technology is muted, will mirror the behaviour of output.

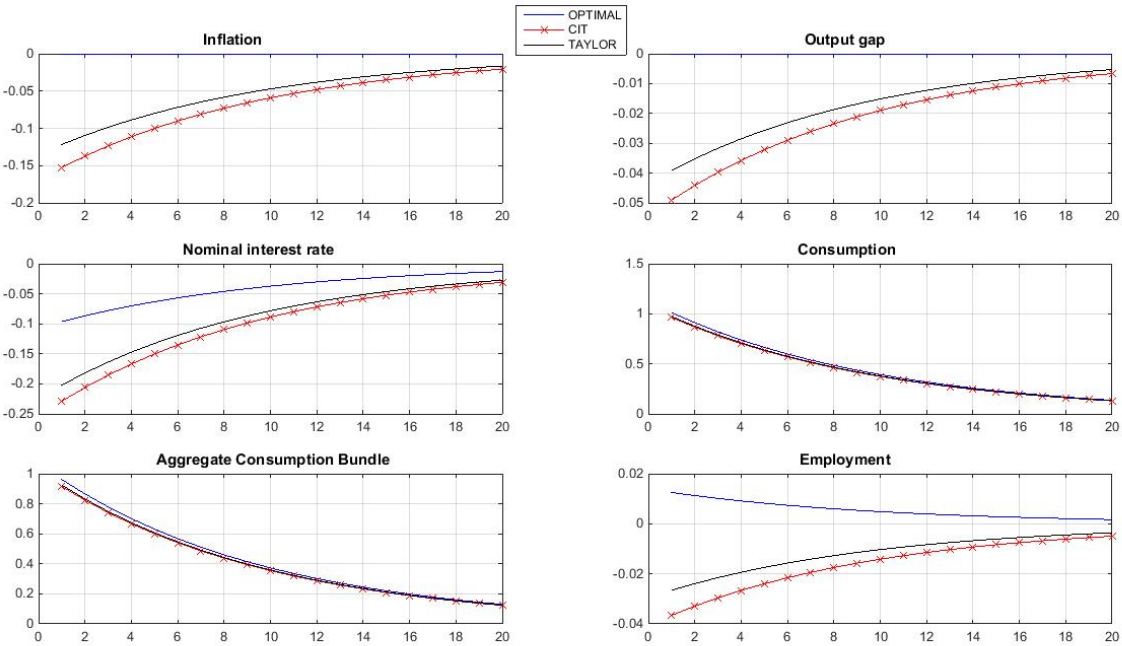
Despite the contractionary stance of the Taylor rule and the CPI-targeting rule, their inability to guide inflation expectations to zero levels will weaken the effect of the policy rates on output. As a result, these two policies will fail to mitigate the inflationary pressure of a government consumption shock and actual output will grow above its natural level,

which, in return, will cause an increase in inflation. The Taylor rule still outperforms the CPI-targeting rule under the government consumption shock.

1.5.2 The Substitutability Case

Technology Shock:

Figure 1.4 Response to a TFP Shock (Substitutability)



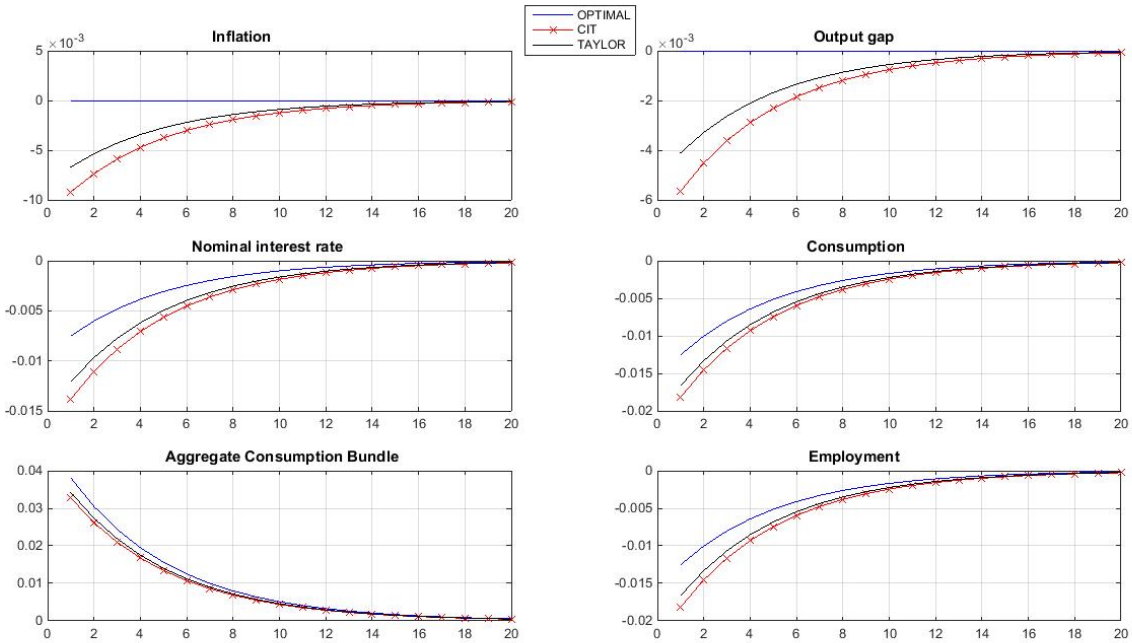
The main difference in the technology shock, in this case, is the response of the optimal policy interest rate. The economy is more sensitive to changes in interest rates, and this makes the required change in the policy rate less than the one under the complementarity case, which is shown in the response of the optimal policy rule in both cases. The effect of a TFP shock on the natural rate of output is greater than the size of the shock itself, as the parameter which governs the relationship between the two is greater than one in the substitutability case. Consequently, this will cause growth in employment above its steady-state level, in order for monetary policy to close the gap between the actual rate of output and its natural level.

The reaction of the Taylor rule and the CPI-targeting rule is not different in this case than the complementarity case. Both rules take an expansionary stance since they are both lower than the optimal rule. Nevertheless, their inability to manage expectations will cause a negative output gap, and in return, this will transmit to deflation and a drop in employment.

Government Consumption Shock:

A shock in government consumption will immediately cause a decline in private consumption, given that the two are substitutes. The drop in output and employment will be identical to the drop in consumption given the market clearing condition and the production function, respectively. Additionally, the effect of government consumption on the natural rate of output and the natural rate of interest will be adverse, under the substitutability assumption between government consumption and private consumption.

Figure 1.5 Response to a Government Consumption Shock (Substitutability)

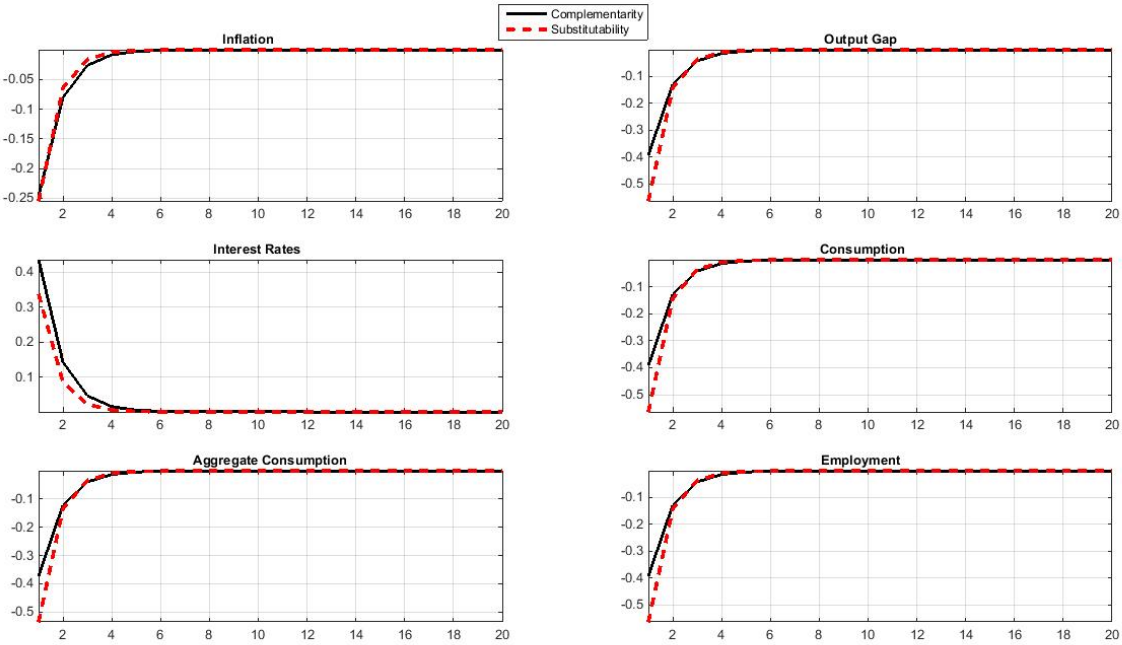


In this case, the role of monetary policy is to lessen the adverse effect of government consumption on the economy. Under the optimal policy rule, the output gap and inflation are set to zero, by construction. The decline in the policy rule will help in mitigating the drop in output and consumption. Similar to the complementarity case, the fiscal multiplier in this case will not be sensitive to the reaction of monetary policy.

The Taylor rule and the CPI-targeting rule take an expansionary monetary stance. Nevertheless, their inability to guide inflation expectations to zero levels will weaken the effect of the policy rates on output. This will cause the actual rate of output to decline and in return, this will cause a drop in employment. The Taylor rule still outperforms the CPI-targeting rule under this government consumption shock, similar to all the above simulations.

1.5.3 Monetary Shock

Figure 1.6 Response to a Monetary Shock



In the above figure, we show how the introduction of government consumption affects the transmission of monetary policy. We do this by comparing the effect of a monetary policy shock on the economy under the complementarity assumption and under the substitutability assumption. We use the Taylor rule (eq. 1.33) for this exercise by adding an exogenous component $\varepsilon_{r,t}$ to the rule. $\varepsilon_{r,t}$ represents an i.i.d monetary shock with constant variance σ_r^2. A one standard deviation of a monetary shock has a contractionary effect on the economy, as it will depress output and push prices downwards. Monetary policy will not have an effect on the natural rate of interest and the natural rate of output.

The above figure shows that an increase in the policy rate depresses output. The decline in output pushes prices downward (deflation) and the decline in output is mirrored by a drop in consumption and employment, following the market clearing condition and the production function, respectively. The above figure mainly shows that the effect of monetary policy is higher under the substitutability assumption than under the complementarity assumption. All of the central macroeconomic variables are more affected by the monetary shock when government consumption is a substitute to private consumption. The only exception is the behaviour of interest rates. This is attributed to the endogenous changes in the interest rates which are induced by the output gap and inflation. The results mainly support the paper's

claim that government consumption has a crowding-out effect on monetary policy when it has a crowding-in effect on private consumption and vice versa.

1.6 Second Moments and Welfare Losses

Table 1.1 Cyclical Properties of Alternative Policy Regimes

A. Under a TFP shock			
	Optimal	CIT	Taylor
Inflation	0.00	0.39	0.33
Output gap	0.00	0.10	0.09
Consumption	1.3	1.21	1.23
Nominal interest rate	0.26	0.59	0.53
Fiscal Gap	1.3	1.21	1.23
B. Under a government shock			
	Optimal	CIT	Taylor
Inflation	0.00	6.92	5.40
Output gap	0.00	3.48	2.71
Consumption	10.3	13.81	13.05
Nominal interest rate	6.20	10.38	9.46
Fiscal Gap	43.50	40.02	40.79

Note: the above figures are standard deviation of selected macroeconomic variables.

Table (1) reports results of business cycle properties which also confirm the visual findings of the impulse response functions found above [9]. The results of the second moments show how the Taylor rule outperforms the simple CPI-targeting rule, given that the former closely resembles the behaviour of the optimal policy rule and as it keeps track of more than one variable in the economy.

Moving to the welfare loss analysis, the goal of the monetary authority is to reduce the utility losses of the representative household. The welfare function will be used to assess the implications of different policies and rank them based on the loss each of these policies causes to the welfare loss function. This is done by taking a quadratic approximation of the utility function after imposing log utility of consumption. In Appendix A.3, we derive the following welfare loss function:

[9]The second moments of the substitutability case are similar to the ones introduced in the paper for the complementarity case and they are available upon request.

$$W = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_t^2 + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\right] \tag{1.38}$$

Assuming that β tends to 1 and we take the unconditional expectations, we get the average per-period loss function:

$$L = -\frac{1}{2}\left[\frac{\varepsilon}{\lambda}var.\pi_t + (1+\varphi)var.\hat{x}_t + (1-\delta)var.(\hat{g}_t - \hat{y}_t)\right] \tag{1.39}$$

The above equation shows that the increase in the size of government consumption in the utility function will give more weight to the fiscal gap $(\hat{g}_t - \hat{y}_t)$ in the welfare loss function, which is in line with the derivations of Gali & Monacelli (2008). The equilibrium under the optimal policy mixture satisfies zero levels of inflation, zero output gap and zero levels of the fiscal gap. In other words, the combined monetary-fiscal policy mixture ensures that all gaps remain at constant, zero values. Nevertheless, as this model assumes that government consumption evolves exogenously, the policy mixture is above the scope of this paper. The monetary authorities, in this case, should solely focus on stabilising prices and closing the output gap, resisting any temptations to accommodate expansionary/contractionary fiscal stances. Setting the inverse elasticity of substitution between government expenditure and private consumption to unity will make the above equation collapse to its standard version. Excluding government consumption from the model will make the equation further collapse to the standard New Keynesian model. The equation is also consistent with Gali & Monacelli (2005), where a deviation of the level of employment from its steady state level determines the inefficiency gap between the marginal rate of substitution and the marginal rate of transformation. This is the main reason why we see that the welfare loss function is increasing in φ.

The welfare loss results shown in Table (2) illustrate consistent results with the ones found above in the impulse responses. They are also consistent with the findings of Gali & Monacelli (2005) where the two authors highlight that this kind of welfare loss exercise in the literature typically generates low welfare losses for all policy regimes for the TFP shock. The welfare losses for the government expenditure shock are higher than those of the TFP shock, reflecting the fact that the size of the standard deviation of the government shock is much higher than the size of the standard deviation of the TFP shock.

Table 1.2 Contribution to Welfare Losses

A. Under a government shock		
	CIT	Taylor
Var(Domestic Infl.)	16.7300	10.2074
Var(Output Gap)	0.2416	0.1474
Var(Fiscal Gap)	0.4005	0.4159
Total	17.3721	10.7707
B. Under a TFP shock		
	CIT	Taylor
Var(Domestic Infl.)	0.0543	0.0376
Var(Output Gap)	0.0020	0.0015
Total	0.0563	0.0391

1.7 Conclusion

How does the inclusion of government consumption in the utility function affect the dynamics of a standard New Keynesian model? To address this question, we developed a standard New Keynesian model which incorporates meaningful government consumption in the utility function in a non-separable form. The inclusion of the fiscal sector to the model gives us more insightful analysis of the dynamics of the model. We find that the reaction of monetary policy does not affect the fiscal multiplier in the economy under the structure of this model.

The introduction of government consumption to the utility function in a non-separable form will affect the slope of the IS curve as it will make it flatter in the complementarity case between government consumption and private consumption. As a result, the response of output to changes in the interest rates will weaken, showing an indication of a crowding-out effect of fiscal policy towards monetary policy. When government consumption and private consumption are substitutes, the response of output to changes in the interest rates will be higher than both the traditional and the complementarity case. Failure to account for the presence of the fiscal sector will result in deflationary pressure in case of a technology (supply) shock and inflationary pressure in case of government consumption (demand) shock when government consumption is a complement to private consumption. In the substitutability case, the two shocks will cause deflationary pressure if the presence of government consumption is not taken into account.

The effect of the interest rate in this economy transmits through the traditional intertemporal channel in the IS curve. Different to the response to a TFP shock under the optimal policy

response in the canonical version of the model, output will not grow at an equivalent rate to the technology shock in this model. In the complementarity case, employment will drop, and if monetary policy tries to stimulate output to alleviate the decline in employment, it will cause inflationary pressure. In the substitutability case, a TFP shock will cause an increase in employment above its steady-state level. This results from the size of the parameter that governs the relationship between technology and the natural rate of output, which is larger than the one in the standard case.

Moreover, the effect of monetary policy is higher under the substitutability assumption than under the complementarity assumption, and all of the central macroeconomic variables are more sensitive to a monetary shock when government consumption is a substitute to private consumption. These results show that when government consumption has a crowding-out effect on monetary policy, it has a crowding-in effect on private consumption and vice versa.

We also derive the welfare loss function of the representative household to measure the costs of demand and supply shocks under different monetary policy rules. Among the two sub-optimal rules, the Taylor rule outperforms the CPI-targeting rule under all the shocks, given that the former keeps track of more variables than the latter. The welfare losses for the government consumption shock are higher than those of the TFP shock. This difference is due to the fact that the size of the standard deviation of the government shock is much higher than the size of the standard deviation of the TFP shock.

In this paper, government consumption was assumed to be exogenous, and the setting of the model was in a closed economy framework. Developing the current setting to an open economy model would provide more insight on the dynamics of the model regarding developments in the internal balances versus external balances framework. Also, the current framework could be further developed to a two-country framework, and the spillover effect could be studied with the introduction of non-separable government consumption. Another possible extension to this model is to add government investment to produce capital. This capital could be rented to firms as the only available capital in the economy so that it could be a complement to the other factors of production. On the other hand, public capital could be included as a substitute for private capital.

Chapter 2

Monetary Policy in a Small Open Economy with Non-Separable Government Spending

2.1 Introduction

The substantial role of government consumption in influencing economic activity raises the necessity for monetary policy to take into account the behaviour of fiscal policy and to also take into account how the presence of the fiscal sector affects the transmission mechanism of monetary policy. Despite being a flexible tool that can address several macroeconomic issues, DSGE models have been rarely used to analyse the interaction between monetary and fiscal policy until the post financial crisis. The recent literature (see, e.g., Christiano et al. (2011); Davig & Leeper (2011)) focused on the impact of fiscal policy only when monetary policy is constrained by the zero-lower-bound, paying no attention to the role of monetary policy in the presence of non-separable government consumption that would affect dynamics of the model beyond the traditional wealth channel[1]. That motivates this paper to analyse how government consumption affects the dynamics of a small open economy, once the former is included in a non-separable form in the utility function. To the best of our knowledge, this issue has not been addressed by the literature, and we aim to do so in this paper.

The standard hypothesis of DSGE models introduces government consumption as either complete waste (Obstfeld & Rogoff 1995, 1996) or included in preferences in a non-separable form. While the former became an obsolete assumption in the recent literature, the inclusion

[1]Following the work of the first chapter, we use the term "government consumption" in this paper for non-fixed capital formation government expenditure. For instance, it represents government provisions of goods and services, excluding compensations of state employees.

of government consumption to preferences in a separable form was adopted both in New Keynesian models (Smets & Wouters (2007) and Gali & Monacelli (2008)) and in RBC models (Baxter & King (1993)). Despite being understudied, as noted by Cantore et al. (2014), the inclusion of government consumption seems appealing, given that agents gain utility from government consumption and the purpose of the government delivering services to households supports this claim. However, assuming prior separability in preferences between private consumption and government consumption can produce biased estimates of the response of private consumption, labour supply, and, hence, of output to a government consumption shock, as recently highlighted by Ercolani & e Azevedo (2014). Under the assumption of prior separability, current and future lump-sum taxes finance any increase in government consumption which in return would lower the present value of the after-tax income. The adverse wealth effect of government consumption is the primary mechanism of a government consumption shock. As a result, private consumption declines, labour supply increases, and in equilibrium, this will lead to lower real wages, higher employment, and higher output.

Gali et al. (2007) challenged the adverse effect of government consumption on private consumption produced by these DSGE models. They highlighted the discrepancy between the estimates of these DSGE models and the ones produced by some empirical models which illustrate a positive effect of government consumption on private consumption. Bouakez & Rebei (2007) and Pieschacon (2012), among others, have also supported the complementarity assumption between private consumption and government consumption. The two papers employ an RBC model and use the complementarity assumption to analyse the effect of a government consumption shock on the economy. Nevertheless, Ercolani & e Azevedo (2014) showed results that indicate that government consumption is a substitute for private consumption in a New Keynesian framework. Ganelli (2003) also introduced government consumption as a substitute for private consumption. The elasticity of substitution between government consumption and private consumption in Ganelli's model governs how much private consumption needs to decline in response to an increase in government consumption in order to hold the utility on the same indifference curve. The discrepancy in the literature goes beyond the theoretical models, where even empirical models illustrate different estimates for the effect of government consumption, which depend on the modelled time frame and the adopted estimation method[2].

The above complications raise a crucial question regarding the reaction of monetary policy to different shocks once government consumption is included in the utility function

[2] For instance, while Aschauer (1985) and Ahmed (1986) find substitutability between government consumption and private consumption, Karras (1994), Fiorito & Kollintzas (2004), and Coenen et al. (2013) find complementarity between the two variables.

in a non-separable form. Once government consumption is included in a non-separable form, it will affect the marginal utility of consumption and consequently the labour supply condition and the consumption smoothing condition. As a result, this inclusion will affect the structure of the whole model, and the transmission mechanism of monetary policy. As noted above, the existing literature has not addressed this issue, and we aim to fill in this gap in this paper. The mechanism of this model applies to both the complementarity case and the substitutability case, and we will focus on changes in the reaction and the transmission mechanism of monetary policy once government consumption is incorporated to a standard New Keynesian model in a non-separable form.

In this paper, we employ a New Keynesian model to study the optimal response of monetary policy to supply and demand shocks in the presence of fiscal policy for a small open economy. To do this, we extend an otherwise standard New Keynesian model for a small open economy (Gali & Monacelli (2005)) and build from the model used in Chapter 1 to allow for meaningful non-separable government consumption, financed by means of lump-sum taxes, in the utility function. Extending the model to a small open economy case complicates the problem for monetary policy to the extent that the authorities must additionally take into account how the exchange rate affects other macroeconomic variables. Similar to the closed-economy case, we are able to derive an optimal monetary policy rule which takes into account developments in government consumption, in addition to the foreign economy's foreign government consumption and output. The other extension that we add to the canonical small open economy model is that we also model the rest of the world economy in our framework, as an aggregate of identical small open economies, and each of them has a size of zero, following the work of Unalmis et al. (2008). This will allow us to trace the spillover effects of supply and demand shocks in the foreign economy on the domestic economy. I choose this framework over the two-country model, adopted in (Obstfeld & Rogoff 1995, 1996) and Ganelli (2003), to prevent spillovers from the domestic economy to the rest of the world economy that might complicate the analysis.

In the open-economy case, the degree of openness will minimise the deviation of the slope of the IS curve from the standard case both in the complementarity and the substitutability case. Moreover, the degree of openness minimises the crowding-out (-in) effect of government consumption towards monetary policy when the former is a complement (substitute) to private consumption. We also find that the fiscal multiplier is also minimised by the degree of openness of the economy, in comparison to the closed-economy version of the model. We additionally find that the size of the fiscal multiplier is negatively affected by the response of monetary policy and the flexibility of the exchange rate, which is in line with the theoretical findings of Woodford (2011), and the empirical results of Koh (2017).

Moreover, we show that, in the case of the spillover effect of external shocks, the amount of exchange rate volatility will determine how much the domestic economy will be affected by external shocks. In this regard, we find that the main difference in the dynamics between our model and the one used in Ganelli (2003) is the exchange rate channel in the two models. In our model, the exchange rate is a product of the interest rates differentials, and the purchasing power of the domestic consumers will be affected by any changes in the exchange rate. On the other hand, in Ganelli's model, the exchange rate is a function of the money demand, which is, in return, a function of private consumption and government consumption. This difference in the dynamics between the two models produces a conflicting effect between domestic and foreign government consumption on domestic private consumption in our model, while the two have the same adverse effect on domestic consumption in Ganelli's model.

The remainder of the paper is organised as follows. We will first demonstrate the structure of our model in the second section. In the third section, we show the parametrisation of the model. The equilibrium dynamics of the model will be discussed in the fourth section. The analysis of the impulse response functions is presented in the fifth section. In the sixth section, we show the results of the welfare loss calculations and the second moments of the primary variables of the small open economy. Lastly, the concluding remarks will take place in the seventh section.

2.2 Small Open Economy Model[3]

2.2.1 Households in the Domestic Economy

Our economy is populated by a representative household that derives utility from aggregate consumption and leisure. The household is assumed to live infinitely and in each period is endowed with one unit of time that is divided between work and leisure: $N_t + L_t = 1$. The representative consumer seeks to maximise the following discounted lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{C}_t, N_t) \tag{2.1}$$

The utility function is assumed to be continuous and twice differentiable. Where N_t is the number of hours worked; β is the discount factor; $\bar{C}_t$ is the aggregate consumption bundle. The aggregate consumption bundle is a constant elasticity of substitution that consists of

[3] We will only display the model for the small open economy in the main body of this paper. We show the log-linearised version of the rest of the world economy in Appendix B.1, where we adopted the same model used in Chapter 1 for the closed economy.

private consumption C_t and government consumption G_t[4]:

$$\bar{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{2.2}$$

Where δ is the share of private consumption in the aggregate consumption bundle and χ is the inverse elasticity of substitution between private consumption and government consumption. Equations (2.1) and (2.2) show that the utility function is non-decreasing in government consumption G_t. The above utility function is subject to the following budget constraint:

$$\int_0^1 P_{H,t}(j) C_{H_t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i_t}(j) dj di + E_t Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t \tag{2.3}$$

Where D_t is the nominal payoff at period t+1 of bonds held at the end of period t including shares in firms, government bonds, and different types of deposits. $Q_{t,t+1}$ is a stochastic discount factor of nominal payoffs and it is equal to $\frac{1}{R_t}$; W_t is the wage; T_t is lump-sum transfers to the households net of lump-sum taxes. All units are expressed in terms of domestic currency.

The utility function that we use assumes two separabilities. The first one is the separation between consumption and the amount of hours worked, and the second one is time separability. The household's problem is also analysed in two stages here: we first deal with the expenditure minimisation problem faced by the representative household to derive the demand functions for domestic and foreign goods. In the second stage, the households choose the level of C_t and N_t, given the optimally chosen combination of goods. C_t is our basic private consumption bundle, and it is a CES composite of home and foreign goods defined as follows:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.4}$$

The above equation is the same household consumption bundle used by Gali & Monacelli (2005), which is the workhorse for small open economies. α here is the degree of openness in the economy which represents the share of imported goods $C_{F,t}$ in the household's consumption bundle. Conversely, the home bias parameter $(1-\alpha)$ produces the possibility of a different consumption bundle in each economy. This is a consequence of having different consumption baskets in each country, despite the law of one price holding for

[4]Government consumption in this framework can be thought of as a public good that households consume at a free cost. It can also be thought of as government expenditure on security and defence which stimulates private consumption and increases the utility of households. Basically, the government is assumed to consume non-tradable goods only.

each individual good. $\eta > 0$ is the elasticity of substitution between domestically produced goods and imported goods in the household's consumption bundle. Consumption goods that are produced either at home or in any foreign country are represented by the unit interval: $C_{j,t} = \int_0^1 C_{j,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di$ for $i \in [0,1]$ and $j = [H, F, i \in [0,1]]$. Now as a first step, the households must minimise their expenditure by optimally choosing the share of each good in the aggregate consumption bundle. Doing so will yield the following demand functions:

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}\,, C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}\,, C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{2.5}$$

Where $P_{i,t} \equiv (\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj)^{1-\varepsilon}$ is the aggregate price index for imported goods from country (i), $P_{H,t} \equiv (\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj)^{1-\varepsilon}$ is the aggregate price index for domestic goods, and $P_{F,t}$ is the aggregate price index for imported goods. The first two terms show domestic demand for good (j) in one of the foreign economies and in the home economy, respectively. The parameter ε represents how much the demand for good (j) will decline if the relative price of that good increased by 1 unit. A lower elasticity of substitution indicates higher consumption of the good of interest. This shows that the goods in the consumption bundle are not perfect substitutes. The third equality is domestic demand for goods produced in country (i) as a function of total domestic demand for foreign goods, and $\gamma > 0$ is the elasticity of substitution between goods from different origins. We finally show the demand functions of domestic and foreign goods from their expenditure minimisation given total consumption:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\,; \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2.6}$$

Now we turn our attention to the per-period utility function in the following form[5]:

$$U(C_t, N_t; G_t) = \frac{\bar{C}_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{2.7}$$

Where σ is the inverse elasticity of intertemporal substitution. Setting σ equal to 1 implies that the household has a log-utility in consumption; φ is the inverse Frisch labour supply coefficient, and $\varphi > 0$ also measures the curvature of the marginal disutility of labour. The above equation is subject to the aggregate budget constraint, which we get by plugging the above demand bundles and price indices in equation (2.3):

[5]We replaced private consumption in the utility function with the aggregate consumption bundle. As noted above, this is one of the deviations that we make from the standard New Keynesian models.

$$P_tC_t + E_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_tN_t + T_t \tag{2.8}$$

Where E_t is the conditional expectations operator. The household's total expenditure basket is equal to: $P_tC_t = \int_0^1 P_{H,t}(j)C_{H_t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i_t}(j)djdi$. P_t is the consumer price index (CPI) and it is equal to: $P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$. From Equation (2.7) and (2.8) we can write the standard optimality condition for households as follows:

$$\frac{W_t}{P_t} = N_t^{\varphi}\bar{C}_t^{\sigma}\left(\frac{C_t}{\bar{C}_t}\right)^{\chi}\delta^{-\chi} \tag{2.9}$$

The intertemporal optimality condition is:

$$\beta\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_t}{C_{t+1}}\right)^{\chi} = Q_{t,t+1} \tag{2.10}$$

Taking the conditional expectation of equation (2.10) and rearranging the terms we get:

$$\beta R_t E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_t}{C_{t+1}}\right)^{\chi}\right] = 1 \tag{2.11}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is the one-period return from a riskless bond and $Q_{t,t+1}$ is the price of that bond. Equations (2.9) and (2.11) deviate from the standard open economy literature. This deviation is due to the fact that we have included government consumption in the aggregate CES basket with private consumption in a non-separable form. The first equation depicts the labour supply dynamics. It shows labour supply as a function of the real wage given the aggregate consumption bundle and private consumption, and it shows how the effect of the aggregate consumption bundle on labour supply depends on the value of χ. Noting that in the Cobb-Douglas case, when $\chi = \sigma$, the labour supply equation collapses to its canonical form as government consumption would have no effect on private consumption. When $\chi > \sigma$, government consumption will have a negative effect on real wages given its positive effect on labour supply. On the other hand when $\chi < \sigma$, government consumption will have a positive effect on real wages, resulting from its negative effect on labour supply. The second equation is the Euler equation which characterises consumption smoothing. The Euler equation in this model also deviates from the standard form found in the literature. In this case, the smoothing of the aggregate consumption bundle $\bar{C}$ is included in the above Euler equation. In the Cobb-Douglas case, when $\chi = \sigma$, the above equation would also collapse to the canonical version of the Euler equation. When $\chi > \sigma$, changes in the current value of the aggregate consumption bundle will have a positive effect on private consumption. However,

the aggregate consumption bundle will have an adverse effect on private consumption when $\chi < \sigma$.

2.2.2 Firms

Price Setting Behaviour

The firms in this model set their prices in a staggered way following Calvo (1983)[6]. Under Calvo contracts, we have a random fraction $1-\theta$ of firms that can reset their prices at period t, while the remaining firms of size θ keep their prices fixed at the previous period's price level. Therefore, we can say that θ^k is the probability that a price set at period t will still be valid at period $t+k$. Thus, the probability of a firm re-optimising its prices will be independent of the time elapsed since it last re-optimised its prices, and the average duration for prices not to change is $\frac{1}{1-\theta}$. Given the above information, the aggregate domestic price level will have the following form:

$$P_{H,t} = \left[\theta(P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t}^{1-\varepsilon})\right]^{\frac{1}{1-\varepsilon}} \tag{2.12}$$

Where $\bar{P}_{H,t}$ is the new price set by the optimising firms. From the derivations shown in Appendix B.2, we get the following form for inflation:

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{t-1}}\right)^{1-\varepsilon} \tag{2.13}$$

The above equation shows that the inflation rate at any given period will be solely determined by the fraction of firms that reset their prices at that period. When a given firm in the economy sets its prices, it seeks to maximise the expected discounted value of its stream of profits, conditional that the price it sets remains valid:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k}[c_{jt+k|t}(\bar{P}_{H,t} - \Psi_{t+k})]\right\} \tag{2.14}$$

The above equation is subject to a sequence of demand constraints: $c_{jt+k} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_t$. Solving this problem (also shown in Appendix B.2) yields the following optimal decision rule:

$$\sum_{k=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k}C_{t+k}\left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathscr{M}MC_{t+k|t}\Pi_{t-1,t+k}^{H}\right]\right\} = 0 \tag{2.15}$$

[6]The Calvo model makes aggregation easier because it gets rid of the heterogeneity in the economy. The alternative pricing scheme is the quadratic cost of price adjustment by Rotemberg (1982). The two dynamics are equivalent up to a first-order approximation.

Where $\mathscr{M}$ is the firm's price markup at the steady state and MC_t is the real marginal cost that firms face in the domestic economy. As we can see from equation (2.15), in the sticky price scheme producers, given their forward-looking behaviour, adjust their prices at a random period to maximise the expected discounted value of their profits at that period and in the future. Thus, firms in this model will set their prices to equal a price markup plus the present value of the future expected stream of their marginal costs. This is done because firms know that the price they set at period t will remain valid for a random period of time in the future. We also assume that all firms in the economy face the same marginal cost, given the constant return to scale assumption imposed on the model and the subsidy that the government pays to firms, as we will see in the following section. The firms also use the same discount factor β as the one used by households, and this is attributed to the fact that the households are the shareholders of these firms. Also, all the firms that optimise their prices in any given period will choose the same price, and this is also a consequence of the firms facing the same marginal cost. Equation (2.15) also shows that the inflation rate is proportional to the discounted sum of the future real marginal costs additional to a markup resulting from the monopolistic power of the firms.

Production

Firm (j) in the domestic economy produces a differentiated good following a linear production function:

$$Y_t(j) = A_t N_t(j) \tag{2.16}$$

Where $Y_t(j)$ is the output of final good (j) in the domestic economy. A_t is the level of technology in the production function, and it is assumed to be common across all firms in the economy and exogenously evolves. $N_t(j)$ is the labour force employed by firm (j). The log form of total factor productivity $a_t = log(A_t)$ is assumed to follows an AR(1) process: $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$. $0 < \rho_a > 1$ is the autocorrelation of the shock, and the innovation to technology $\varepsilon_{a,t}$ has a zero mean and a finite variance σ_a. We exclude capital from production in this model for the sake of tractability. Aggregate output and aggregate employment in the domestic economy are defined by the Dixit & Stiglitz (1977) aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}; N_t = \left(\int_0^1 N_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.17}$$

Given the common technology assumption across all firms of the economy, the total cost function for firm (j) is defined as follows:

$$TC_t(j) = \frac{(1-\tau)W_t Y_t(j)}{A_t} \tag{2.18}$$

In the above equation, we left W_t without any firm specification, which is attributed to the fact that we have a competitive labour market. Also, τ is the subsidy that the government gives to firms in order to eliminate the markup distortion created by the firms' monopolistic power. Taking the first order condition of equation (2.18), with respect to production, yields the following marginal cost equation:

$$MC_t(j) = \frac{(1-\tau)W_t}{A_t} \tag{2.19}$$

From the above equation, it is clear that the constant return to scale assumption makes the marginal cost independent of the firm's production level and this will make marginal cost common across all firms: $MC_t(j) = MC_t$. Now the common real marginal cost will look like:

$$MC_t^r = \frac{(1-\tau)}{A_t}\frac{W_t}{P_{H,t}} \tag{2.20}$$

The marginal cost equation is expressed in terms of the domestic prices level $P_{H,t}$, wages W_t, total factor productivity A_t and τ, which is the subsidy that the government gives to firms, in order to eliminate the markup distortion created by the firms' monopolistic power.

Lastly, after the aggregation of output and employment, we get the following aggregate production function:

$$Y_t = A_t N_t \tag{2.21}$$

2.2.3 International Linkages

We first start by the defining the terms of trade as the ratio of imported prices to domestic prices. The bilateral terms of trade between the domestic economy and another small economy (country i) is defined as: $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$. Thus, the aggregate terms of trade is defined as: $S_t = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$. Defining $P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ allows us to define the aggregate effective terms of trade as:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{2.22}$$

If we plug in the log-linearised representation of the imported prices index from the above equation ($p_{F,t} = s_t + p_{H,t}$) in the log-linearised form of the CPI price index equation, we will be able to derive the CPI index as a function of the domestic prices index and the terms of trade:

$$p_t = p_{H,t} + \alpha s_t \tag{2.23}$$

The above function shows that a gap exists between the CPI index and the domestic price index which is filled with the terms of trade. The gap is parametrised by the degree of openness of the domestic economy. Before progressing to further derivations, we first define the bilateral exchange rate $\mathscr{E}_{i,t}$ as the value of country i's currency in terms of the domestic currency. Assuming that the law of one price holds, the price of any good in country (i) will be equal to:

$$P_{i,t}(j) = \mathscr{E}_{i,t} P^i_{i,t}(j) \tag{2.24}$$

Integrating the above equation yields the price index for country (i). Solving this integrate for the imported prices index in the domestic economy yields:

$$P_{F,t} = \mathscr{E}_t P^*_t \tag{2.25}$$

The nominal effective exchange rate is equal to $\mathscr{E}_t \equiv \int_0^1 \mathscr{E}_{i,t} di$, and the world price index is defined as $P^*_t \equiv \int_0^1 P_{i,t} di$. Plugging the value of the imported prices index from the above equation in the definition of the terms of trade yields:

$$S_t = \frac{\mathscr{E}_t P^*_t}{P_{H,t}} \tag{2.26}$$

We now define the bilateral real exchange rate as the ratio of the price index in country (i) to the CPI index in the domestic economy: $REER_{i,t} = \frac{\mathscr{E}_{i,t} P^i_t}{P_t}$. Integrating the bilateral real exchange rate equation yields the real effective exchange rate equation for the domestic economy: $REER_t = \frac{\mathscr{E}_t P^*_t}{P_t}$. From the definitions of the terms of trade and the real effective exchange rate, we can define the equation that links the two variables in a log-linearised form as follows:

$$q_t = (1-\alpha) s_t \tag{2.27}$$

Under the assumption of complete international financial markets, the price of a one-period riskless bond from country (i) dominated in the domestic economy's currency is equal to: $\mathscr{E}_{i,t} Q^i_t = E[\mathscr{E}_{i,t+1} Q_{t,t+1}]$. If we add this equation to the domestic bond's price equation ($Q_t = E[Q_{t,t+1}]$), we get the uncovered interest rate parity condition:

$$\frac{Q^i_t}{Q_t} = E_t\left(\frac{\mathscr{E}_{i,t+1}}{\mathscr{E}_{i,t}}\right) \tag{2.28}$$

The uncovered interest parity condition is crucial for the no-arbitrage condition to hold in the international bonds market. Under the uncovered interest parity we assume that foreign

bonds are perfect substitutes for domestic bonds once both of them are expressed in the same currency. The uncovered interest parity equation also implies that higher foreign interest rates or depreciation in the exchange rate will put upward pressure on domestic interest rates.

The last thing that we need do in this section is to derive the international risk condition. Under the assumptions of complete international markets and the identical preferences assumption, the foreign consumer's Euler equation can be transformed to:

$$\beta\left(\frac{\bar{C}^*_{t+1}}{\bar{C}^*_t}\right)^{\chi-\sigma}\left(\frac{P^*_t}{P^*_{t+1}}\right)\left(\frac{C^*_t}{C^*_{t+1}}\right)^{\chi}\left(\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right) = Q_{t,t+1} \tag{2.29}$$

We then divide the domestic inter-temporal optimality condition (eq. 2.10) by the foreign economy's inter-temporal optimality condition (eq. 2.29) to get:

$$1 = E_t\left(\frac{\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_t}{C_{t+1}}\right)^{\chi}}{\left(\frac{\bar{C}^*_{t+1}}{\bar{C}^*_t}\right)^{\chi-\sigma}\left(\frac{P^*_t}{P^*_{t+1}}\right)\left(\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right)\left(\frac{C^*_t}{C^*_{t+1}}\right)^{\chi}}\right) \tag{2.30}$$

Plugging the definition of the real effective exchange rate in the above equation yields:

$$C_t = \mathscr{V}_t C^*_t (REER_t)^{\frac{1}{\chi}}\left(\frac{\bar{C}_t}{\bar{C}^*_t}\right)^{\frac{\chi-\sigma}{\chi}} \tag{2.31}$$

Where $\mathscr{V}_t = \frac{C_{t+1}\bar{C}^{*\frac{\chi-\sigma}{\chi}}_t}{C^*_{t+1}\bar{C}^{\frac{\chi-\sigma}{\chi}}_{t+1} REER^{\frac{1}{\chi}}_{t+1}}$ is a constant which depends on the initial relative wealth position. We assume that we have a symmetric initial condition and set $\mathscr{V}_t = 1$, meaning that the net position of foreign assets is equal to zero. Thus, the international risk sharing condition simplifies to:

$$C_t = C^*_t (REER_t)^{\frac{1}{\chi}}\left(\frac{\bar{C}_t}{\bar{C}^*_t}\right)^{\frac{\chi-\sigma}{\chi}} \tag{2.32}$$

Complete security markets ensure that risk-averse consumers can trade away the risks and the shocks they encounter. Under this setting, consumers can purchase contingent claims for realisations of all idiosyncratic shocks, and this will enable them to diversify all idiosyncratic risk through the capital markets. Also, the above international risk sharing condition depicts how a depreciation in the real effective exchange rate would boost domestic consumption relative to the foreign economy's consumption. The log-linearised form of the above international risk sharing condition is:

$$c_t = c^*_t + \frac{(\sigma-\sigma_\delta)}{\sigma_\delta}(g^*_t - g_t) + \frac{1}{\sigma_\delta}q_t. \tag{2.33}$$

Where $\sigma_\delta = \delta\sigma + (1-\delta)\chi$ is a weighted average of the intertemporal elasticity of substitution σ and the inverse elasticity of substitution between government consumption and private consumption χ. The above equation illustrates how the effect of domestic and foreign government consumption is governed by χ. In the Cobb-Douglas case when $\chi = \sigma$, the above international risk sharing condition collapses back to its standard representation as $\sigma_\delta = \sigma$. In this case, government consumption both in the domestic and foreign economy will not affect private domestic consumption. When $\chi > \sigma$, domestic government consumption has a positive effect on private consumption, while foreign government consumption has a negative effect on domestic private consumption. On the other hand, when $\chi > \sigma$ domestic government consumption will have a negative effect on private consumption and foreign government consumption will have a positive effect on private consumption. This last point makes a clear distinction between our model and the one used in Ganelli (2003). Where in the latter's both domestic and foreign government consumption have the same negative effect on private consumption. In this setting, as will be made clear in the simulations below, monetary policy will react to changes in government consumption, conditional on how government consumption affects the economy. The exchange rate will react to the movement in the interest rates differential affecting the purchasing power of domestic private consumers.

2.2.4 Market Clearing Conditions

We start by identifying the market clearing condition for the domestically produced products in the small open economy. Where domestic output of good (j) is absorbed both by domestic demand and foreign demand:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di \tag{2.34}$$

In the above equation, $C_{H,t}(j)$ is domestic demand for good (j) and $C^i_{H,t}$ is country (i)'s demand for good (j) in the domestic economy. We plug the domestic demand function for good (j) (eq.2.5). As for foreign demand for domestic good (j), we use the assumption of symmetric preferences across all the countries of the world economy to get:

$$C^i_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\mathscr{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} \tag{2.35}$$

Plugging in the respective demand bundles transforms the market clearing condition for domestic production of good (j) to:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left((1-\alpha)\left(\frac{P_{H,t}}{P_t^i}\right)^{-\eta}C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\mathscr{E}_{i,t}P_{F,t}^i}\right)^{-\gamma}\left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta}C_t^i(j)di\right) \quad (2.36)$$

Using the Dixit-Stiglitz aggregater of domestic output, we can write the above equation in aggregate terms:

$$Y_t = \left(\frac{P_{H,t}}{P_t^i}\right)^{-\eta}\left((1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathscr{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}Q_{i,t}^{\eta}C_t^i di\right) \quad (2.37)$$

In the above equation, we took $\left(\frac{P_{H,t}}{P_t^i}\right)^{-\eta}$ as common factor. We have also used the definition of the bilateral real exchange rate. If we divide and multiply the term $\left(\frac{\mathscr{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}$ by $P_{i,t}$ we get: $\left(\frac{P_{i,t}}{P_{H,t}}\frac{\mathscr{E}_{i,t}P_{F,t}^i}{P_{i,t}}\right)^{\gamma-\eta}$. The two terms that we get are essentially the effective terms of trade for country (i) and the bilateral terms of trade between the domestic economy and country (i), and equation (2.37) simplifies to:

$$Y_t = \left(\frac{P_{H,t}}{P_t^i}\right)^{-\eta}\left((1-\alpha)C_t + \alpha\int_0^1\left(S_t^i Si,t\right)^{\gamma-\eta}Q_{i,t}^{\eta}C_t^i di\right) \quad (2.38)$$

Taking the first order log-linearisation of the above equation around a symmetric steady state yields:

$$y_t = (1-\alpha)c_t + \alpha c_t^* + \alpha[\gamma + \eta(1-\alpha)]s_t \quad (2.39)$$

Adding the international risk sharing condition to the above equation yields:

$$y_t = y_t^* + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}(g_t^* - g_t) + \frac{\omega_\alpha}{\sigma_\delta}s_t \quad (2.40)$$

where $\omega = \sigma_\delta\gamma + (1-\alpha)(\eta\sigma_\delta - 1)$ and $\omega_\alpha = (1-\alpha) + \alpha\omega$. The above equation links actual output to foreign and domestic government consumption, the rest of the world's output, and the terms of trade. From the above equation, we notice that the terms of trade variable is the only channel through which monetary policy could have an effect on the actual rate of output. In this regard, the monetary policy rule will be adjusted to achieve the required movement in the terms of trade to close the output gap.

The Supply Side of the Economy

The equation of the natural rate of output (derived in Appendix B.1) takes the following form:

$$\bar{y}_t = \left(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi+\sigma_\delta}\right)a_t + \left(\frac{\sigma_\delta(\omega_\alpha-1)}{\omega_\alpha\varphi+\sigma_\delta}\right)y_t^* - \left(\frac{\alpha\omega(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\right)g_t^* - \left(\frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\right)g_t \tag{2.41}$$

In the above equation, the effect of technology on the natural rate of output is positive and this positive effect is robust against different values of χ. Nevertheless, changes in the value of χ will determine the magnitude of the effect of technology on the natural rate of output. In the Cobb-Douglas case, when $\chi = 1$, the reaction of the natural rate of output to a TFP shock is identical to its reaction in the canonical version of the model. In the complementarity case, the reaction of the natural rate of output will be less than its reaction in the canonical version of the model. In the substitutability case, the reaction of the natural rate of output is magnified, and it is higher than its reaction in the canonical version of the mode. These findings are consistent with the findings of the closed-economy version of the model found in Chapter 1. However, the degree openness in the economy will minimise the deviation of the response of the natural rate of output to a TFP shock–under both the complementarity and the substitutability assumptions–from the reaction of the natural rate of output in the canonical version of the model.

Moreover, once we deviate from the Cobb-Douglas case, the foreign economy's output will have an effect on the natural rate of output. In the complementarity case, the foreign economy's output will have a positive effect on the natural rate of output. While in the substitutability case, the foreign economy's output will have an adverse effect on the natural rate of output. Also, both domestic and foreign government consumption will have a positive effect on the natural rate of output in the complementarity case, and an adverse effect in the substitutability case. To construct a relationship between the marginal cost variable and the output gap in the domestic economy, we subtract the above equation from equation (B.2) in Appendix B.1 to get:

$$\hat{mc}_t = \frac{\varphi\omega_\alpha+\sigma_\delta}{\omega_\alpha}x_t \tag{2.42}$$

Adding the above equation to the derived Phillips curve derived in Appendix B.2 enables us to write domestic inflation as a function of the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa\frac{\varphi\omega_\alpha+\sigma_\delta}{\omega_\alpha}x_t \tag{2.43}$$

In the above Phillips curve equation, the effect of the output gap on domestic inflation will be higher in the complementarity case than in the standard case. While in the substitutability case, it will be relatively lower.

The Demand Side of the Economy

In the open economy version of the model, adding the domestic economy's market clearing condition (eq. 2.39) to the log form of the Euler equation (eq. 2.10) yields:

$$\begin{aligned}
y_t =& E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\}) - \alpha[\gamma + \eta(1-\alpha)]\Delta E_t\{s_{t+1}\} - \alpha\Delta E_t\{y^*_{t+1}\} \\
&+ \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \frac{\alpha\omega}{\sigma_\delta}\Delta E_t\{s_{t+1}\} - \alpha\Delta E_t\{y^*_{t+1}\} \\
&+ \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \alpha(\omega-1)\Delta E_t\{y^*_{t+1}\} + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&+ \frac{\alpha(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g^*_{t+1}\}
\end{aligned} \tag{2.44}$$

In the above system of equations, we made use of the CPI index equation in the domestic economy (eq. 2.23) and replaced the value of the terms of trade in equation (2.40). Moreover, we show that the effects of the domestic variables (government consumption and domestic real interest rate) on output are parametrised by the home-bias parameter $(1-\alpha)$, while the effects of the external variables are parametrised by the degree of openness in the economy α. This is inherited from the market clearing condition of the domestic economy.

The slope of the IS curve, similar to the closed-economy version, changes with the degree of substitutability between government consumption and private consumption. Nevertheless, the degree of openness in the economy minimises the effect of introducing non-separable government consumption on the slope of the IS curve. Therefore, as the degree of openness in the economy tends to zero, the slope of the IS curve starts to converge to the slope of the closed-economy case both in the substitutability and the complementarity case[7]. Solving the

[7] In the closed-economy case, the slope of the IS curve in the complementarity case becomes flatter, making output less responsive to changes in interest rates. In the substitutability case, on the other hand, the IS curve becomes steeper making output more responsive to changes in interest rates.

above IS curve for the output gap yields:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t) \tag{2.45}$$

Where:

$$\begin{aligned}\bar{rr}_t = &-\frac{\sigma_\delta(1+\varphi)(1-\rho_a)}{\varphi\omega_\alpha}a_t + \frac{\alpha(\omega-1)\varphi\sigma_\alpha}{\varphi\omega_\alpha}\Delta E_t\{y^*_{t+1}\} - \frac{\alpha\omega(\sigma-\sigma_\delta)\varphi(1-\rho_{g^*})}{\varphi\omega_\alpha}g^*_t \\ &- \frac{(1-\alpha)\omega(\sigma-\sigma_\delta)\varphi(1-\rho_g)}{\varphi\omega_\alpha}g_t\end{aligned} \tag{2.46}$$

The reaction of the natural rate of interest to a percentage change in productivity includes σ_δ to account for the presence of government consumption, as in the closed-economy version of this model. Nevertheless, unlike the closed-economy version of the model, the open economy's natural rate of interest takes into account the degree of openness in the economy, represented by ω_α in the denominator.

In addition, the responses of the natural rate of interest to changes in the foreign economy's variables are governed by the degree of openness, and also take into account the inclusion of government consumption in the model. Also, the reaction to domestic government consumption, on the other hand, is governed by the home-bias parameter, and is also affected by the degree of openness in the domestic economy. Moreover, the effect of the foreign economy's output is positive in the complementarity case and negative in the substitutability case.

2.2.5 Fiscal and Monetary Policy

Fiscal Policy

The government budget constraint in the economy is:

$$P_tG_t + (1+R_{t-1})B_{t-1} = B_t + P_tT_t \tag{2.47}$$

B_t is the quantity of a riskless one-period bond maturing in the current period, paying one unit of the domestic currency. R_t denotes the gross nominal return on bonds purchased in period t. The government levies a non-distortionary lump-sum tax T_t to finance its expenditure. Following the logic of the first chapter, G_t is government consumption, and is

assumed to evolve exogenously according to a first order autoregressive process:

$$\frac{G_t}{G} = \left\{\frac{G_{t-1}}{G}\right\}^{\rho_g} exp(\zeta_{G,t}) \tag{2.48}$$

Where $0 < \rho_g < 1$ is the autocorrelation parameter of government consumption, and $\zeta_{G,t}$ represents an i.i.d government consumption shock with constant variance σ^2. Another important feature that we add to this model is assuming that the government consumes from a different market than the one occupied by the private agents[8]. For this instance, we assume that while the private agents only consume tradable goods, the government consumes non-tradable goods. Also, government consumption is assumed to be produced costlessly.

Monetary Policy

The monetary authorities in this model use short-term interest rates as their policy tool. As the model uses a cashless economy, money supply is implicitly determined to achieve the interest rate target. It is also assumed that the central bank will meet all the money demanded under the policy rate it sets.

The first rule in the model will be the optimal rule:

$$\frac{R_t}{R} = \left\{\frac{RR_t}{RR}\right\}\left\{\frac{\Pi_{H,t}}{\Pi_H}\right\}^{\phi_\pi}\left\{\frac{Y_t}{Y}\right\}^{\phi_x} \tag{2.49}$$

The optimal rule tells us that by setting domestic inflation and the output gap to zero, the policy rate will equal the natural rate of interest, and it will be able to follow the developments in the natural rate of output. Thus, the optimal policy reproduces the flexible price equilibrium output, given that the government will pay a subsidy τ to offset the monopolistic distortion in the economy. The above policy rule will provide a useful benchmark to evaluate the performance of different monetary policy rates on the basis of a welfare-loss function. The first policy rule we set to evaluate in the small open economy is a naive CPI-targeting Taylor rule equation:

$$\frac{R_t}{R} = \left\{\frac{\Pi_t}{\Pi}\right\}^{\phi_\pi} \tag{2.50}$$

The second rule is a Taylor rule domestic-inflation targeting equation:

$$\frac{R_t}{R} = \left\{\frac{\Pi_{H,t}}{\Pi_H}\right\}^{\phi_\pi} \tag{2.51}$$

[8] In Appendix B.3 we show how the effect of government consumption on private consumption changes under different values of the elasticity of substitution between the two.

The third rule is the exchange rate peg equation. Where the domestic economy will follow the policy rule implemented by the foreign economy:

$$\mathscr{E}_t = 0 \tag{2.52}$$

The parameters of the above equations (ϕ_π, ϕ_x) describe the strength of the response of the policy rate to deviations in the variables on the right-hand side. These parameters are also assumed to be non-negative. Also, the last three rules are referred to as naive interest rules, resulting from the fact that they only make use of observable variables. Finally, the inflation response parameter ϕ_π in the above policy rules must be strictly higher than one in order for the solution of the model to be unique, as shown by Bullard & Mitra (2002) and further explained in Appendix B.6.

2.3 Parametrisation

β	Discount factor	0.99
σ	inverse elasticity of intertemporal substitution	1
φ	inverse Frisch labour supply elasticity	3
α	share of foreign goods in core consumption	0.4
χ	inverse elasticity of substitution between C_t&G_t	20 & 0.01
η	elasticity of substitution between domestic and foreign goods	1
ε	elasticity of substitution	6
δ	share of private consumption in the aggregate consumption bundle	0.95
θ	Calvo probability	0.75
ρ_g	AR(1) coefficient of domestic government expenditure	0.9
ρ_{g^*}	AR(1) coefficient of foreign government expenditure	0.9
ϕ_π	inflation elasticity of the nominal interest rate	1.5
ϕ_x	output gap elasticity of the nominal interest rate	0.5
ρ_a	AR(1) coefficient of domestic productivity shock	0.9
ρ_{a^*}	AR(1) coefficient of foreign productivity shock	0.9
γ	elasticity of substitution between goods in the world economy	1

The values of the parameters of the model are listed in the above table. We Set θ equal to 0.75, implying that firms only change their prices once a year. Our discount factor β is equal to 0.99, which implies–given that $\beta = 1/r$ at the steady state–that annual return is approximately equal to 4 %. We set φ equal to 3, under the assumption that the labour supply elasticity is $\frac{1}{3}$. We set ϕ_π & ϕ_x equal to 1.5 and 0.5 following Taylor (1993). We also set the

inverse elasticity of substitution between government expenditure and private consumption χ equal to 20 following Bouakez & Rebei (2007) and Pieschacon (2012), and we use 0.01 for the substitutability case. The size of private consumption in the aggregate consumption bundle δ equals to 0.95. In this regard, and as mentioned in the first chapter, the weight of δ hast to be strictly less than 1 for government consumption to influence the dynamics of the model, and different values of δ between 0 and 1 only affect the model quantitatively. Moreover, change in the value of χ do not qualitatively affect the behaviour of the model.

The inverse elasticity of intertemporal substitution of private consumption σ is set equal to 1, implying log utility in consumption. We set the elasticity of substitution between domestic and foreign produced goods η to 1. This elasticity describes the change in consumption of imported goods in response to changes in the prices of foreign goods relative to domestic prices. The value of the parameter implies that demand of imported goods increases precisely by 1 % when the relative price of foreign goods declines by 1 %. The share of foreign consumption goods in the private consumption basket is set to 40 %, while the elasticity of substitution between the domestically produced goods ε equals 6, corresponding to a steady state markup of 1.2. Also, we adopt the persistence parameter of government consumption in the two economies ($\rho_g \& \rho_{g^*}$) from Gali et al. (2007). As for the standard deviations of the two shock processes, we use the standard deviation of the TFP shock in Gali & Monacelli (2005) $\sigma_a = 0.0071$, and for the government consumption shock we use the one in Coenen & Straub (2005) $\sigma_g = 0.323$.

2.4 Equilibrium Dynamics

The key equations that we use to analyse the model's equilibrium implications are the non-policy block equations (IS demand curve, NKPC) and the interest rate rule conducted by the monetary authority:

$$\begin{aligned} x_t &= E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\} - \bar{rr}_t) \\ \pi_{H,t} &= \beta E_t\{\pi_{H,t+1}\} + \kappa_g x_t \\ r_t &= \overline{rr_t} + \phi_\pi \pi_{H,t} + \phi_x x_t \end{aligned} \tag{2.53}$$

Where $\kappa_g = \kappa \frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha}$. combining the first equation with the third equation allows us to simplify the above system of equations to only two equations. Solving for the output gap and domestic inflation as a function of their respective expectations yields:

$$
\begin{aligned}
x_t &= \frac{\sigma_\delta}{\sigma_\delta + \omega_\alpha \phi_x + \kappa_g \omega_\alpha \phi_\pi} E_t\{x_{t+1}\} + \frac{\omega_\alpha (1 - \beta \phi_\pi)}{\sigma_\delta + \omega_\alpha \phi_x + \kappa_g \omega_\alpha \phi_\pi} E_t\{\pi_{H,t+1}\} \\
\pi_{H,t} &= \frac{\kappa_g \sigma_\delta}{\sigma_\delta + \omega_\alpha \phi_x + \kappa_g \omega_\alpha \phi_\pi} E_t\{x_{t+1}\} + \frac{\beta(\sigma_\delta + \omega_\alpha \phi_x) + \kappa_g \omega_\alpha}{\sigma_\delta + \omega_\alpha \phi_x + \kappa_g \omega_\alpha \phi_\pi} E_t\{\pi_{H,t+1}\}
\end{aligned}
\tag{2.54}
$$

The above system of equations could be easily presented in a matrix form:

$$
\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} = A \begin{bmatrix} x_{t+1} \\ \pi_{H,t+1} \end{bmatrix}
$$

Where $A = \Omega \begin{bmatrix} \sigma_\delta & \omega_\alpha(1 - \beta\phi_\pi) \\ \kappa_g \sigma_\delta & \beta(\sigma_\delta + \omega_\alpha \phi_x) + \kappa_g \omega_\alpha \end{bmatrix}$ and $\Omega = \frac{1}{\sigma_\delta + \omega_\alpha \phi_x + \kappa_g \omega_\alpha \phi_\pi}$.

Under the assumed values of the policy parameters and the non-policy parameters of the model shown in the parametrisation section, we find that both eigenvalues of matrix A lie inside the unit circle, making the solution of the model unique. The results also apply to the domestic inflation targeting rule, the CPI targeting rule, and the exchange rate peg rule as well. Where the equilibrium in each model is unique, and it satisfies the following condition[9]:

$$
\kappa_g \omega_\alpha (\phi_\pi - 1) + (1 - \beta) \omega_\alpha \phi_x > 0 \tag{2.55}
$$

From the above equation it is clear that the inflation parameter has to be strictly greater than one for this rule to be determined, along with a trivial condition which requires $0 < \beta < 1$.

2.5 IRF

2.5.1 Open Economy

A Technology Shock in the Open Economy:

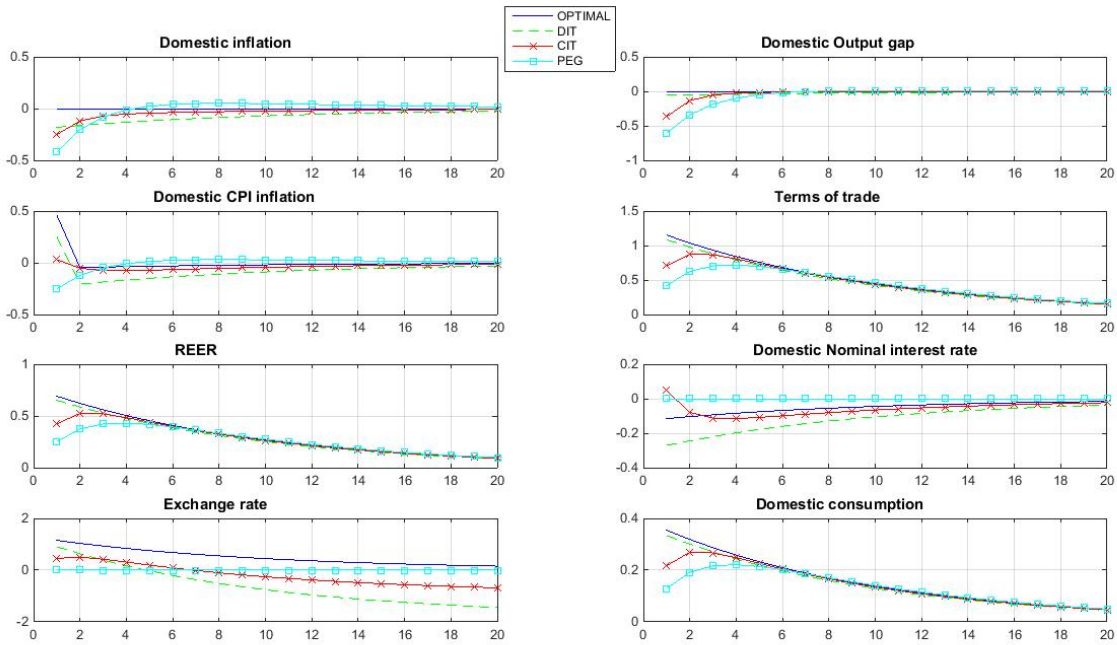
The analysis of this section focuses on the effect of a TFP shock in the small open economy. The shock is similar to the one found in Gali & Monacelli (2005), which gives us a good anchor, along with the closed-economy version of this model, on the effect of introducing government consumption to a New Keynesian model for a small open economy.

The response of the natural rate of interest in this version of the model is still consistent with the closed-economy version, where the natural rate of interest is reduced to accommodate

[9] See Appendix B.6.

the transitory expansion in the natural rate of output. The natural rate of interest still takes into account the introduction of government consumption to the model, similar to the closed-economy version case, and this makes the reduction of the interest rate higher than the standard case. Nevertheless, the degree of openness in the economy partly offsets the effect of introducing government consumption on the behaviour of the model. Conversely, the positive effect of the technology shock on the natural rate of output is less in this version of the model than in a standard New Keynesian one. However, the effect here is still higher than in the closed-economy version one, mainly due to the offsetting effect of the degree of openness in the economy on the introduction of government consumption to the model.

Figure 2.1 Response to a Domestic TFP shock



The responses of the output gap and domestic inflation are zero under the optimal policy rule. Once we set domestic inflation and the output gap to zero, the policy rate will be lowered to follow the path of the natural rate of interest. The exchange rate depreciates after the reduction of the domestic policy rule, given that the foreign economy's policy rate is constant in this case. This depreciation will fully reflect on the terms of trade since world inflation, and domestic inflation are both fixed at zero. The depreciation of the terms of trade will boost the growth of the actual rate of output until it equals its natural level.

Consumption is lower in the open economy version than in the closed economy version. This is attributed to the difference in the market clearing conditions between the

two economies; domestic output is not just absorbed by domestic consumption in the open economy, but by foreign consumption as well. Also, under a technology shock, the economy will still suffer from a decline in employment, similar to the closed-economy version of the model. Nevertheless, this decline in employment is minimised in the open economy case also due to the offsetting effect of the degree of openness of the economy.

The behaviour of the domestic inflation targeting rule closely resembles the behaviour of the optimal policy, and takes an apparent expansionary stance. Nonetheless, the policy's inability to guide inflation expectation to zero will lead the terms of trade to depreciate by less than the required amount to close the output gap to zero. The lack of depreciation in the terms of trade will lead to a negative output gap and negative inflation rates.

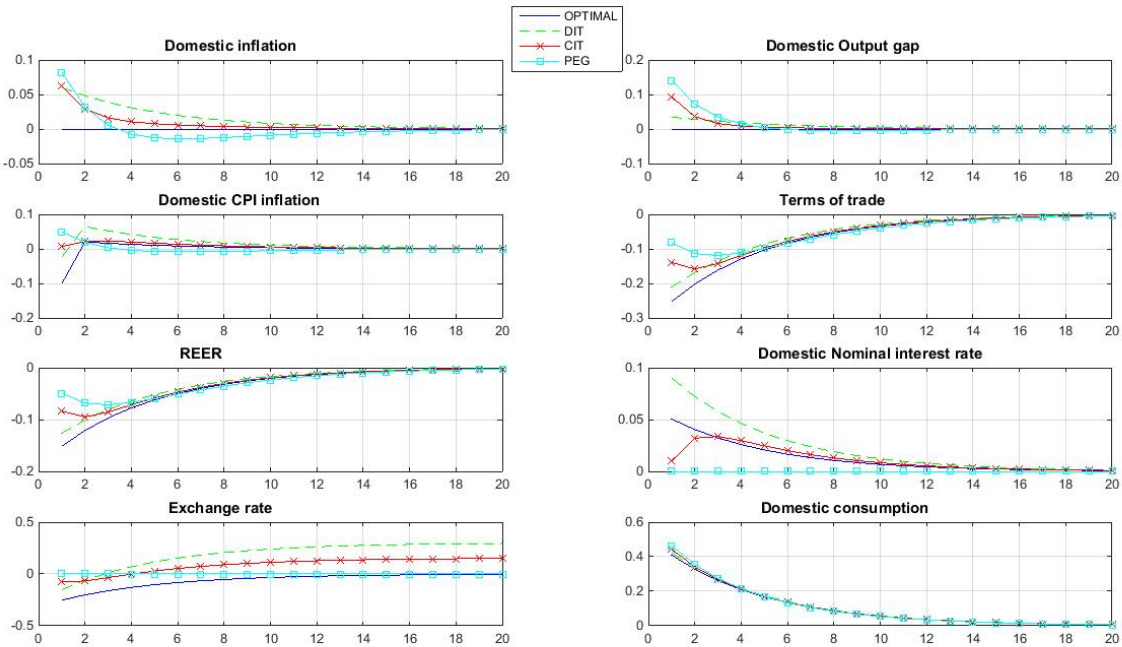
Under the exchange rate-peg regime, smoothing the terms of trade by keeping the exchange rate fixed will result in higher volatility in the domestic variables. On the other hand, the main difference between the CPI-targeting rule and the domestic inflation-targeting rule lies in the behaviour of the terms of trade. The CPI-rule targets both domestic inflation and the terms of trade. For this reason, the CPI-targeting rule will not allow the terms to depreciate enough to boost output until it reaches its potential level, and this explains the hump-shaped in the terms of trade and the exchange rate under the CPI-targeting rule. As a result, the output gap under the CPI-targeting rule will be higher than the one under the domestic inflation-targeting rule.

A Domestic Government Shock in the Open Economy:

The effect of a government consumption shock in the small open-economy follows the dynamics of the same shock in the closed-economy version of this model. An increase in government consumption will have a positive effect on the natural rate of output, and a positive response by the natural rate of interest to this shock, in order to limit the inflationary pressure of this demand shock. Nevertheless, the effect of this shock is minimised by the degree of openness in the economy when compared with the same shock in the closed-economy version of this model.

Given that domestic inflation and the output gap are set to zero under the optimal policy rate, the increase in the natural rate of interest causes an appreciation in the nominal exchange rate. This appreciation will sufficiently transmit to the terms of trade. The appreciation of the terms of trade will dampen the growth of domestic output in order to keep it at an equivalent value to its natural level.

Figure 2.2 Response to a Domestic Government Consumption Shock



The dynamics of the other policy rates follow their same behaviour under the technology shock, but in an opposite manner. We notice that under a domestic inflation-targeting rule, the policy rate increases above the neutral level of interest (contractionary monetary policy stance). Yet, this increase is not fully reflected in the exchange rate because of the inability of this policy rule to guide expectations of inflation to zero levels. The less than required appreciation in the exchange rate will cause the actual rate of output to grow above its natural level, causing a positive output gap. The overheating in domestic output will cause positive rates of domestic inflation, consequently.

Under the CPI-targeting rule, we notice a hump-shaped response in the policy rate. This, as explained above, is due to the fact that under this rule the terms of trade are also targeted by the monetary authorities, in addition to domestic inflation. As a result, the policy rate increases up to the point when the necessity of stabilising the terms of trade arises. While under the exchange rate-peg regime, smoothing the terms of trade by allowing them to only appreciate through domestic inflation causes more volatility in the output gap and domestic inflation.

Lastly, we notice that consumption in this open version of the model is at higher levels than consumption in the closed version of the model under the same government consumption shock. This is because the appreciation of the exchange rate boosts the purchasing power of domestic households and in return, that increases domestic consumption of imported goods

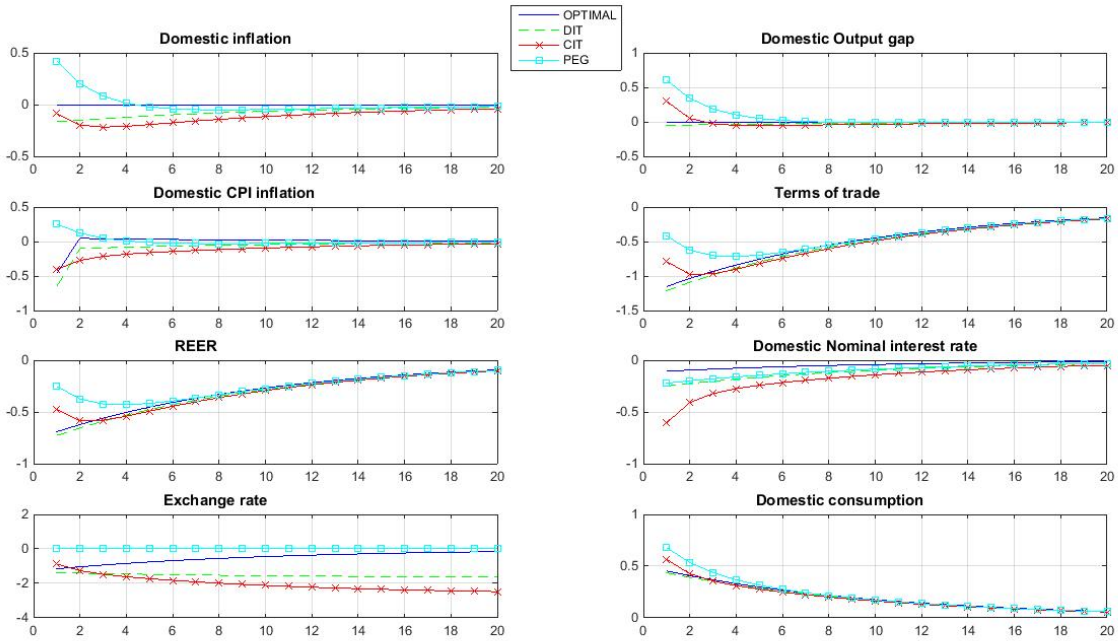
to the point that domestic consumption exceeds domestic output. We observe the opposite under a technology shock in the open economy.

2.5.2 Spillover Effect on the Domestic Economy

The structure of the model enables us to construct further analysis on the domestic small open economy. In this regard, by modelling the foreign economy, we can capture the effect of shocks in the foreign economy on the domestic variables via the three important channels. The first is changes in world demand for domestic goods, which is illustrated in the domestic market clearing condition. The second is the price differential in the two economies which affects the competitiveness between the two economies. The last, which is the vital one, is the interest rates differential between the two economies. We only limit this analysis to one policy rule in the foreign economy, and that rule is the optimal policy in the foreign (closed) economy.

The Effect of a Technology Shock in the Foreign Economy on the Domestic Economy:

Figure 2.3 Response to a TFP Shock in the Foreign Economy



A shock in the foreign economy's TFP causes growth in world demand for domestically produced goods. The main channel in these dynamics is, as noted above, the interest rates

differential between the two economies. In this regard, given that the monetary authority in the foreign economy will lower its policy rate to accommodate the expansion in the natural rate of output, the domestic authorities should try to manage how its exchange rate should behave in order to achieve internal stability in the domestic economy.

Under the optimal policy rule, the domestic policy rule aims to achieve a positive interest rate differential against the foreign economy's interest rate. This positive gap in the interest rates will cause the terms of trade to appreciate to the extent that it keeps actual output at its natural level. Thus, under the optimal policy rule, inflationary external demand for domestic products is reduced by making domestic products more expensive for foreign households, and by increasing the purchasing power of domestic households to enable them to consume more from abroad.

Under the domestic inflation targeting rule however, the policy rate is lowered at a level equivalent to the foreign economy's policy rate. Nevertheless, its inability to manage expectation of zero-domestic inflation expectations makes the terms of trade appreciate more than required, causing a negative output gap. The double dimensional structure of the CPI-targeting rule will aim at lowering the policy rate to close the negative gap of domestic inflation (its first target), and this causes the actual output to grow above its natural level. Nevertheless, as the terms of trade start reaching undesirable negative values, the policy rule is reversed to achieve stabilisation of the terms of trade (its second target).

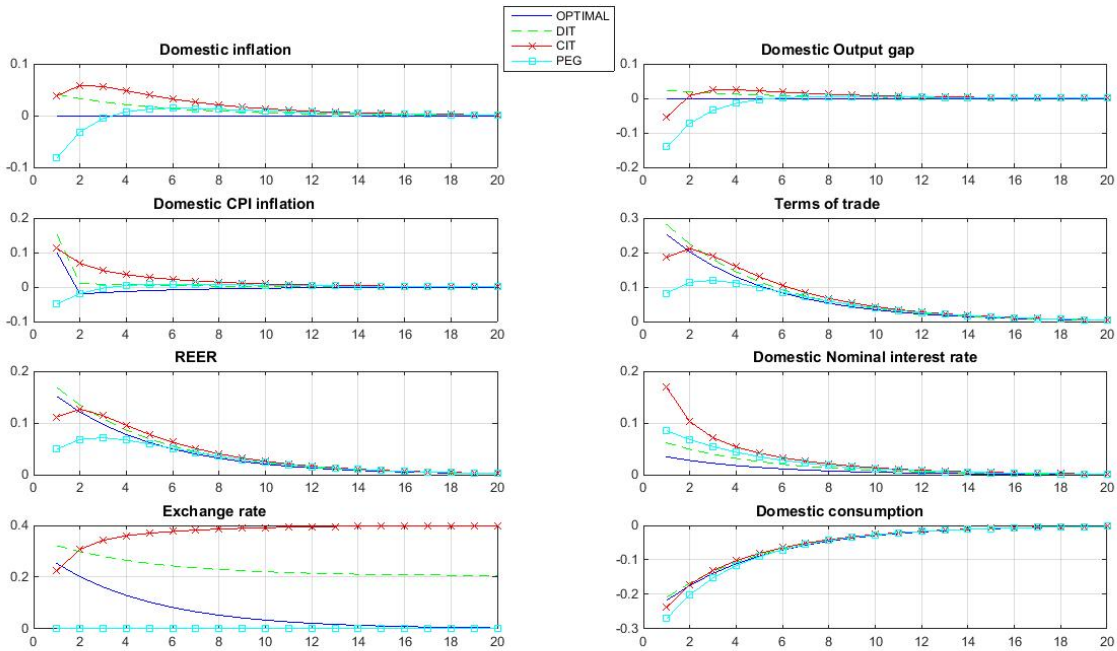
Under the exchange rate-peg regime, the domestic authorities follow the rule conducted by the foreign economy's authorities. As a result, the domestic policy rule will be lower than the neutral rate of interest. This expansionary stance will boost the actual rate of output to grow above its natural level, causing inflationary pressure. Consistent with the analysis of the shocks in the small open economy, pegging the exchange rate causes smooth behaviour in the terms of trade, given that they are only affected by the sticky prices of the two economies only. As a result, adopting this rule will cause more volatility in the domestic variables (domestic inflation, the output gap).

The Effect of a Government Shock in the Foreign Economy on the Domestic Economy:

Under a government consumption shock in the foreign economy, the foreign economy's policy rate will be increased to limit the inflationary pressure arising from the increase in demand. From the UIP condition, the relative increase in foreign interest rates will cause downward pressure on the value of the domestic economy's currency, and foreign demand for domestic goods will also increase in this case. Thus, leaving the domestic policy rule unchanged in this case will boost the actual rate of output to grow far beyond its natural level leading to high levels of inflation.

Under the optimal policy rule, the gap between the interest rates is reduced to limit the depreciation in the domestic currency. This depreciation will fully reflect on the terms of trade, leading the actual rate of output to grow at a rate equivalent to its natural level, and this will cause zero levels of domestic inflation. Limiting the depreciation of the domestic currency will also minimise the adverse effect of an increase in the foreign economy's policy rate on domestic consumption.

Figure 2.4 Response to a Government Consumption Shock in the Foreign Economy



We also notice that the domestic inflation-targeting rule fails, in this case as well, to guide expectation of inflation to the zero-levels, despite the fact that the policy stance, in this case, was contractionary (higher than the neutral rate of interest). Under this rule, the gap between the interest rates is minimised more than the optimal policy rule. Nevertheless, this interest rate differential will not fully reflect in the exchange rate, due to the presence of non-zero domestic inflation levels. The high level of depreciation in the domestic currency, in this case, will cause the actual rate of domestic output to grow above its natural level leading to positive rates of inflation in return.

Under the exchange rate-peg regime, the domestic authorities will follow the rate adopted by the foreign economy's monetary authorities in this case as well. This rate will cause the terms of trade to only grow via the relative sticky prices in the two economies. The actual rate of output will grow less than its potential level, causing a negative output gap and negative

inflation rates as a result. Thus, adopting the exchange rate-peg regime will lead the economy into recession because of the domestic authorities' inability to adjust the external balances (the exchange rate, the terms of trade), in order to achieve internal stability (the output gap, domestic inflation). Under the CPI-targeting rule, the policy rate is first increased to limit the high levels of domestic inflation causing a less than needed depreciation in the terms of trade. However, the policy rule reverts once domestic inflation starts going down. The terms of trade are never a concerning issue in this case since the depreciation in the exchange rate is offset by the positive levels of domestic inflation.

This section highlights the main differences between the dynamics of this model and the one used in Ganelli (2003). In our model, the interest rates differential will affect the purchasing power of the domestic consumers, due to the reaction of monetary policy to changes in aggregate demand. This will result in a contradictory effect between domestic government consumption and foreign government consumption on domestic private consumption, both in the complementarity and substitutability case.

2.5.3 The Substitutability Case

In this section we illustrate how the dynamics of the model change once we incorporate government consumption as a substitute to private consumption[10]:

A Technology Shock in the Open Economy:

The response of the natural rate of interest to a technology shock, in this case, is consistent with its response under the other cases. The natural rate of interest is reduced in this case as well to accommodate the expansion in the natural rate of output. Nevertheless, the reduction of the natural rate of interest, in this case, will be less than all of the other cases. This is attributed to the fact that output is more sensitive to changes in interest rates under the substitutability assumption, as noted earlier. Also, the positive effect of a technology shock on the natural rate of output is higher than all of the other cases.

Similar to all the other cases, under the exchange rate-peg rule and the CPI-targeting rule, the relative smoothing of the terms of trade will cause more volatility in domestic inflation and the output gap.

A Government Shock in the Open Economy:

Government consumption will have an adverse effect on output in this case. The role of monetary policy is to reduce interest rates to alleviate the adverse effect of this shock on the

[10]The impulse response function graphs are reported in Appendix B.7.

economy. The decline in the domestic rates will cause depreciation in the exchange rate, and this, in return, will direct private consumption towards domestically produced goods. The exchange rate-peg role is still the worst performing role among the evaluated rules.

The Effect of a Technology Shock in the Foreign Economy on the Domestic Economy:

The spillover effect of a technology shock in the foreign economy on the domestic economy in the substitutability case is similar to the effect in the complementarity case. Given that foreign interest rates are reduced to accommodate the expansion in output, the domestic interest rates must keep the appropriate positive interest rates differential to reduce foreign demand for domestically produced goods. This positive gap will make domestic goods more expensive than foreign goods, through the appreciation of the exchange rate. Failure to do so, similar to the CPI-targeting rule and the exchange rate-peg rule, will cause more welfare losses in the form of more fluctuations in the output gap and domestic inflation.

The Effect of a Government Shock in the Foreign Economy on the Domestic Economy:

The spillover effect of a government shock in the foreign economy will take place through two channels. The first is the direct positive effect of the foreign economy's government consumption on domestic private consumption in the substitutability case. Similar to the above technology shock, foreign interest rates will be reduced to limit the adverse effect of government consumption on the foreign economy, which creates the second channel (interest rates differential channel). The role of domestic monetary policy is to keep a positive gap between domestic interest rates and foreign interest rates to direct household consumption towards consumption of foreign goods. This, as a result, will reduce the inflationary effect of a shock in the foreign economy's government consumption.

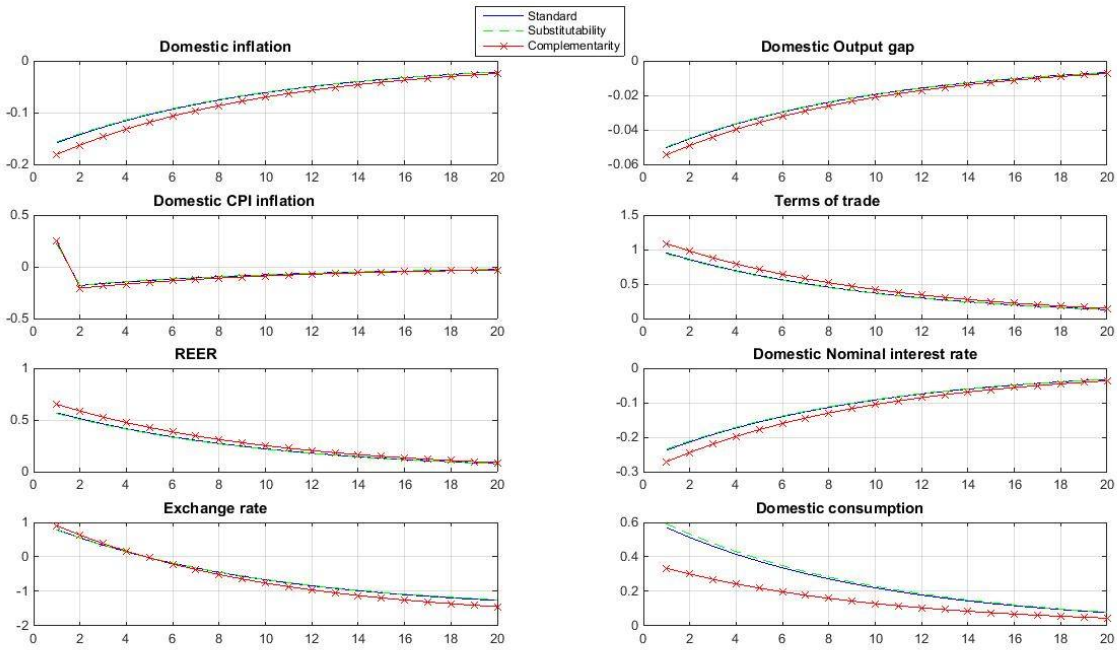
This section also illustrates the main difference between our model and the one used by Ganelli. We find that, even in the substitutability case, domestic government consumption and foreign government consumption have contradicting effects on domestic private consumption.

2.5.4 The Effect of Introducing Government Consumption on a Standard Model

In this section, I explicitly demonstrate how the behaviour of a standard New Keynesian model changes once government consumption is included in a non-separable form. In the below/above graph, I nest the effect of the same technology shock on the domestic economy under the standard case, the complementarity case and the substitutability case. I choose the

domestic inflation targeting rule as the monetary policy rule for all three scenarios to show how the output gap and domestic inflation are affected by the technology shock across all three scenarios.

Figure 2.5 Response to a Technology Shock



A TFP shock requires a reduction in interest rates. The highest reduction in interest rates occurs in the complementarity case, as domestic variables become less sensitive to changes in interest rates. The reduction under the substitutability assumption, on the other hand, resembles the standard case. The reduction of interest rates transmits to depreciation in the exchange rate and the terms of trade, which show the highest depreciation under the complementarity scenario. Private consumption in the complementarity case increases the least, despite the fact that interest rates are reduced the most in this case, reflecting the crowding out effect of government consumption to monetary policy under the complementarity assumption. Under the substitutability case, as domestic variables become more sensitive to changes in interest rates, private consumption increases the more than the other two scenarios, in spite of the fact that the reduction of interest rates in the complementarity case are equivalent to the reduction in the standard case. The simulations also show that the negative values of domestic inflation and the output gap are also greater in the complementarity case than the standard and substitutability case, also reflecting the effect of the introduction of government consumption in a non-separable form.

2.6 Second Moments and Welfare Losses

Table 2.1 Cyclical Properties of Alternative Policy Regimes in the Domestic Economy

A. Under a technology shock				
	Optimal	DIT	CIT	PEG
Domestic inflation	0.00	0.30	0.22	0.35
Output gap	0.00	0.09	0.28	0.52
CPI inflation	0.34	0.38	0.15	0.21
Nominal interest rate	0.19	0.44	0.22	0.00
Terms of trade	1.88	1.77	1.67	1.51
Nominal depr. rate	0.84	0.78	0.39	0.00
REER	1.13	1.06	1.00	0.91
Fiscal Gap	1.55	1.46	1.38	1.25
B. Under a government shock				
	Optimal	DIT	CIT	PEG
Domestic inflation	0.00	2.21	3.92	3.04
Output gap	0.00	1.27	2.42	5.27
CPI inflation	3.44	4.99	4.90	1.83
Nominal interest rate	1.85	3.31	7.35	4.57
Terms of trade	13.59	15.13	14.02	9.06
Nominal depr. rate	8.59	10.48	7.84	0.00
REER	8.15	9.08	8.41	5.43
Fiscal Gap	5.80	7.07	6.37	3.30

Note: the above figures are standard deviation of selected macroeconomic variables.

Table (1) reports results of business cycle properties which also confirm the visual findings in the impulse response functions found above. The results of the second moments show a clear negative relationship between the volatility of the external variables and the volatility of the internal variables. Additionally, we notice the poor performance of the CPI targeting regime and the exchange rate peg regime against the exogenous shocks. This is consistent with findings of Unalmis et al. (2008) and Levin et al. (2003).

Moving to the welfare loss analysis, the goal of the monetary authority is to reduce the utility losses of the representative household. The welfare function will be used to assess the implications of different policies and rank them based on the loss each of these policies cause to the welfare loss function. This is done by taking a quadratic approximation of the utility function after imposing log utility in consumption and by assuming that the elasticity of substitution between goods from different origins is unitary. In Appendix B.4, we derive the following welfare loss function:

$$W = -\frac{1-\alpha}{2\omega_\alpha}\sum_{t=0}^{\infty}\beta^t\left[\frac{\kappa}{\lambda}\pi_{H,t}^2 + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\right] \tag{2.56}$$

Assuming that β tends to 1 and we take the unconditional expectations, we get the average per-period loss function:

$$L = -\frac{1-\alpha}{2\omega_\alpha}\left[\frac{\kappa}{\lambda}var.\pi_{H,t} + (1+\varphi)var.\hat{x}_t + (1-\delta)var.(\hat{g}_t - \hat{y}_t)\right] \tag{2.57}$$

The above equation shows that the increase in the size of government expenditure in the utility function will give more weight to the fiscal gap $(\hat{g}_t - \hat{y}_t)$ in the welfare loss function. This finding is in line with the derivations of Gali & Monacelli (2008). The equilibrium under the optimal policy mixture satisfies zero levels of inflation, zero output gap and zero levels of the fiscal gap. In other words, the combined monetary-fiscal policy mixture ensures that all gaps remain at constant, zero values. Nevertheless, as this model assumes that government consumption evolves exogenously, the policy mixture is above the scope of this paper. The monetary authorities, in this case, should solely focus on stabilising prices and closing the output gap, resisting any temptations to accommodate expansionary/contractionary fiscal stances. Under the complementarity condition, ω_α reduces the utility losses of the targeted parameters in the utility loss function (domestic inflation and the output gap), and conversely in the substitutability case. Also, setting the inverse elasticity of substitution between government expenditure and private consumption to unity will make the above equation collapse to the same version in GM 2008. Excluding government expenditure from the model will make the equation further collapse to its counterpart equation in Gali & Monacelli (2005). Moreover, the equation is consistent with the latter, where a deviation of the level of employment from its steady state level determines the inefficiency gap between the marginal rate of substitution and the marginal rate of transformation. This explains why we see that the welfare loss function is increasing in φ[11].

The welfare loss results, shown in Table (2), illustrate consistent results with the ones found above in the impulse responses. They are also consistent with the findings of GM 2005. The two authors highlight that this kind of welfare-loss exercise in the literature typically generates low welfare losses for all policy regimes for the TFP shock. The welfare losses for the government consumption shock are higher than those of the TFP shock because the size of the standard deviation of the government shock is much higher than the size of the standard deviation of the TFP shock

[11] The results of the welfare losses and the second moments for the spillover effect are reported in Appendix B.5. Also, the results for the substitutability case resemble those reported for the complementarity case, and they were excluded from the analysis for the sake of brevity.

Table 2.2 Contribution to Welfare Losses in the Domestic Economy

A. Under a government shock			
	DIT	CIT	PEG
Var(Domestic Infl.)	1.3669	0.7138	1.2086
Var(Output Gap)	0.0259	0.0807	0.2072
Var(Fiscal Gap)	0.2093	0.2019	0.1887
Total	1.6021	0.9964	1.6045
B. Under a TFP shock			
	DIT	CIT	PEG
Var(Domestic Infl.)	0.0113	0.0062	0.0163
Var(Output Gap)	0.0006	0.0059	0.0020
Total	0.0119	0.0121	0.0183

.

2.7 Conclusion

In this paper, we developed a small open economy model in a dynamic general equilibrium framework which incorporates meaningful government consumption into the utility function in a non-separable form. We perform this exercise to show the effect of this introduction on the structure and dynamics of a standard New Keynesian model. The analysis of the model is twofold. First, we add a government sector to a standard open-economy New Keynesian model. Doing so allows us to make the analysis against two benchmark models. The first one is the closed-economy version of the model, while the second one is the Gali & Monacelli (2005) model, which is the workhorse model for small open economies. The second part of the analysis is the spillover effect of shocks in the foreign (closed) economy on the small open economy, and we make our analysis for this part against the analysis of Ganelli (2003).

In the open economy case, the effect of introducing government consumption in a non-separable form on monetary policy is reduced by the degree of openness in the economy. Monetary policy affects the internal balances in the domestic economy through its influence on the movement of the exchange rate and the terms of trade. Monetary policy behaves differently under the two imposed shocks on the model. Under a technology (supply) shock, the policy rate should be lowered to accommodate the expansion in output, and failure to do so will cause a high reduction in employment, resulting in higher welfare losses. Once interest rates are lowered, the purchasing power of the domestic currency will weaken, and

that will make domestic output more competitive. The decline in the purchasing power of the domestic currency will, as a result, direct domestic consumers to consume domestically produced goods. Under the government consumption (demand) shock, the policy rate is increased to offset the inflationary demand increase in the complementarity case. This increase in the interest rates will cause the domestic currency to appreciate, and that will increase the purchasing power of the domestic households. This increase, in return, will make domestic production less competitive and will direct private consumption towards imported goods. In the substitutability case, given the adverse effect of government consumption on private consumption, monetary policy will lower interest rates to minimise the decline in private consumption. This will negatively affect the purchasing power of the domestic consumers and direct their consumption towards domestically produced goods.

Under all of the evaluated shocks, the domestic inflation-targeting rule seems to outperform all of the other rules in the open economy version of the model. This finding is consistent with the findings of Gali & Monacelli (2005), where the difference between all the rules lies in the amount of external variability that they entail. Consequently, the exchange rate-peg regime was the worst performing rule amongst all the evaluated rules. Additionally, we have shown that the fiscal multiplier (the effect of government expenditure on output) will be subject to the response of monetary policy, the flexibility of the exchange rate, and the degree of openness of the economy which is consistent with the findings of Woodford (2011) and Koh (2017).

In the second part of this exercise, we evaluate the response of the domestic economy's authorities to shocks in the foreign economy. We find that the most significant transmission channel of these shocks is the interest rates differential channel. Given that the foreign economy's authorities will change the policy rates in response to any shock in the foreign economy, the exchange rate will immediately be affected by these changes. The role of the domestic economy's authorities, in this case, is to minimise the resulting depreciation or appreciation in the domestic currency to ensure internal stability. The dynamics of the model also highlight significant differences with the model used by Ganelli (2003) in regards to the spillover effect of foreign government consumption on domestic consumption. In our model, the international risk sharing condition shows how domestic government consumption and foreign government consumption have conflicting effects on domestic private consumption. Also, the interest rates differential affects the purchasing power of the domestic consumers due to the reaction of monetary policy to changes in aggregate demand. This further contributes to the contradicting effects of domestic government consumption and foreign government consumption on domestic consumption both in the complementarity

and substitutability case. While in Ganelli's model the two will have the same adverse effect on domestic private consumption.

We lastly derive the welfare loss function of the representative household to measure the costs of demand and supply shocks under different monetary policy rules. Among the three sub-optimal rules, the domestic inflation rule outperforms the other two under all the imposed shocks on the model. The welfare losses for the government consumption shock are higher than those of the TFP shock, and this is explained by the fact that the size of the standard deviation of the government shock is much higher than the size of the standard deviation of the TFP shock.

Chapter 3

Heterogeneity Among Commodity-Rich Economies: Beyond the Prices of Commodities

3.1 Introduction

Natural resources present a financial opportunity for developing countries to lift themselves out of poverty and achieve economic prosperity. For economic and technical reasons, these developing countries are prone to rent-seeking behaviour, as highlighted by Boschini et al. (2007). This phenomenon makes developing countries more dependent on primary commodity exports than high-income economies, which results in higher export concentration dominated by basic commodities. Consequently, this makes developing countries vulnerable to commodity-price cycles (super cycles), as noted by Rodrik (1999), Hausmann & Rigobon (2003), van der Ploeg & Poelhekke (2008) and Blattman et al. (2007). Moreover, natural resources might play an adversative role towards economic prosperity in these economies. Also, boom-bust cycles caused economies to suffer from shrinkage of their non-commodities tradables sector (Dutch Disease) or lower growth rates for commodity-rich economies than their non-resource rich counterparts (natural resource curse).

There exists a long and growing literature that investigates the effect of commodities on commodity-rich economies. The seminal paper by Sachs & Warner (1995) illustrated the adverse effect of the abundance of natural resources on economic growth. In addition, van der Ploeg & Poelhekke (2009) illustrate that the high volatility of commodity prices seems to be the quintessence of the resource curse since it generates large real exchange rate fluctuations and less investment, especially in countries where financial development is lagging (Aghion

et al. (2009)). Nevertheless, the above findings were challenged by numerous papers that have questioned the natural resource curse, pointing to examples of commodity-exporting countries that have done well, such as Chile, Norway and Botswana[1]. Moreover, Alexeev & Conrad (2009), Cotet & Tsui (2010) and Havranek et al. (2016) find very little evidence in support of the natural resource curse, while van der Ploeg (2011) showed empirical evidence that either outcome is possible, leading the literature to deviate from consensus on this issue. Another seminal paper by Mehlum et al. (2006) showed that institutions are a vital factor for the effect of resources on economic performance[2].

One possible explanation for the above disparity is that the literature mentioned above usually assumes that this group of countries is homogeneous. For instance, many studies that have been conducted on a single commodity-rich economy assumed that their results apply on all commodity-rich economies, labelling their case study as "prototypical" or "quintessential"[3]. In this paper, we try to contribute to the growing literature on natural resources and economic performance by highlighting one of the possible sources of heterogeneity among commodity-rich economies. We try to capture this heterogeneity by imposing the same commodity-price shock on a number of resource-rich economies. Doing so will allow us to show how the social capabilities of each economy and the characteristics of the commodity affect the response of key macroeconomic variables to a commodity-price shock.

Two findings in the literature motivate our approach. The first is Rodrik (1999)'s findings that the magnitude of a country's growth deceleration since the 1970's is a function of both the magnitude of the shocks and a country's social capability for adapting to shocks. Also, Fernndez et al. (2018) illustrate that there is strong comovement among the prices of commodities. Thus, this will enable us to isolate the two factors that affect the response of macroeconomic variables in each economy, and solely concentrate on the social capabilities and the characteristics of the commodity. To the extent of our knowledge, the existing literature has not yet addressed this phenomenon.

This paper proposes a small open economy model for a commodity-rich country to quantitatively study the triggers of business cycles in different commodity-rich economies. We extend the model used in Chapter 2 by adding some features to our model to make it more relevant for a commodity-rich economy. Moreover, three assumptions are imposed in this paper to guide the dynamics of the model. First, the supply of commodities is

[1] Larsen (2006) exhibited Norway as an example of an oil-rich country that was able to escape the Resource Curse. Englebert (2000), Sarraf & Jiwanji (2001), Acemoglu et al. (2003) and Iimi (2006) are among those noting Botswana's conspicuous escape from the Resource Curse.

[2] These findings refute the findings of Sachs & Warner (1995) of an insignificant role for institutions in overcoming the resource curse, and they show that the quality of institutions has to increase as the size of resources increase in the economy.

[3] See, for example, Isham et al. (2005), van der Ploeg & Poelhekke (2009), and Dauvin & Guerreiro (2017).

exogenous, and it is affected by political, geographical, and technical factors; Second, the small open economy is a small player in the world markets for the goods it exports and imports. Therefore, similar to the related literature, we assume that the small open economy takes the terms of trade as exogenously given; Third, the windfalls of commodities in the domestic economy are solely collected by the government, as it has full ownership of the natural resources of the domestic economy.

Moreover, our model allows for a quadruple role for commodities. First, the domestic government collects the windfalls of selling commodities to the rest of the world[4]. Second, commodities are consumed by households both in the domestic economy and the foreign economy. Third, firms both in the domestic economy and the foreign economy use commodities as an input in their production. Lastly, the domestic economy is affected by the second-round effect of an increase in the commodity prices in the form of high foreign inflation and low world demand. In addition, the main behavioural parameters that the paper focuses on are the elasticity of substitution between government consumption and private consumption and the response of government consumption to fluctuations in the commodity prices. The former parameter is an indicator of the efficiency of government consumption and its effect on private consumption (crowding-in versus crowding-out), while the latter captures the behaviour and the stance of fiscal policy during booms and busts of commodity prices, along with the size of the commodity windfalls in the government's revenue.

The analysis of this paper proceeds in four steps. First, we empirically estimate our behavioural parameters. Second, we generate the impulse response of the data using a structural VAR model. Third, we illustrate the full structure of our DSGE model. The model generates extra sources of stochastic processes that were proposed by the existing literature. The calibration of the parameters for our DSGE model is made for all of our economies of interest based on the empirical findings of this paper and the long-term averages found in the data. Fourth, we use Bayesian estimation techniques to calculate the variance decomposition of our variables of interest. The empirical and theoretical findings of this paper show that consumption is excessively volatile relative to output, which is consistent with the findings of the previous literature[5]. However, our findings show that this might also be the case for developed countries which are rich with natural resources, as in the case of Australia. The results also show that, once we control for the commodity prices, there is heterogeneity in the forces driving the business cycle within commodity-rich economies. The

[4] Introducing the fiscal sector was neglected by Fernndez et al. (2018), leaving out the most significant transmission channel for commodities-price shocks in commodity-rich economies.

[5] See for example, Neumeyer & Perri (2005), Aguiar & Gopinath (2007), Garcia-Cicco et al. (2010), Akinci (2014), and Drechsel & Tenreyro (2017).

fiscal sectors in these economies drive these forces, along with institutional factors and the share of commodity windfalls in the government's total revenue.

This paper relates to three strands of the literature. The first strand is the literature that studies the role of the fiscal sector in commodity-rich economies. Our results show the existence of a procyclical fiscal stance in commodity-rich countries. This is consistent with the findings of Kaminsky et al. (2005), Frankel (2011), and Bastourre et al. (2012). Nevertheless, we find that adopting the fiscal rule, as in the case of Chile, reverses this behaviour, which is consistent with the findings of Cespedes & Velasco (2014). Our findings also support the findings of Rodrik (1999) and Isham et al. (2005) of how the abundance of commodities erodes institutions, and that, in return, will affect how economies react to commodity shocks. The second strand of literature is the one that focuses on the effect of government consumption on private consumption. The results of this paper, at least regarding commodity-rich economies, strongly support the findings of Gali et al. (2007) and Bouakez & Rebei (2007) who show that government consumption has a crowding in effect on private consumption.

The third strand of the literature that this paper is relevant to is the literature on the contribution of external factors to the business cycles in developing economies. The first part of this strand of the literature studied the role of terms of trade in driving business cycles in developing economies[6]. Our work complements the work of Fernndez et al. (2018), Shousha (2016), Fernndez Martin et al. (2017), and Drechsel & Tenreyro (2017), who show a significant role for the proxy of terms of trade (commodity prices) in driving business cycles in developing economies[7]. We show significant heterogeneity in the contribution of terms of trade to business cycles among commodity-rich economies and illustrate that oil-rich economies are more vulnerable to these shocks. The second segment of this literature relates to the effect of external factors on small open economies. Neumeyer & Perri (2005), Uribe & Yue (2006), and Fernndez Martin et al. (2017) show a significant effect of external factors on business cycles in emerging economies. We contribute to this literature by showing that the effect of these shocks on commodity-rich economies is sensitive to the degree of openness in these economies and the adopted fiscal regime in each economy. This is attributed to the fact that the government is the main channel for the transmission of these fluctuations in commodity-rich economies as illustrated by Arezki & Ismail (2013). Our results also show that oil-rich countries, in this case as well, are more affected by external shocks than their commodity-rich counterparts.

[6]The standard reference papers in this part of the literature are Mendoza (1995) and Kose (2002) who show that terms of trade represent a significant source of fluctuation.

[7] The findings of Broda (2004) support these results, but they were challenged by Schmitt-Grohe & Uribe (2017) who undermine the role of commodities in driving business cycles.

The organisation of the remainder of this paper is as follows. In the second chapter, we illustrate our stylized facts and empirical findings for our economies of interest. In the third chapter, we build a DSGE model for a commodity-rich small open economy. We add some structural shocks that were suggested by the previous literature and calibrate the model based on our empirical findings and the long-term parameters found in the data. In the fourth chapter, we estimate the model using Bayesian estimation techniques. Chapter five concludes.

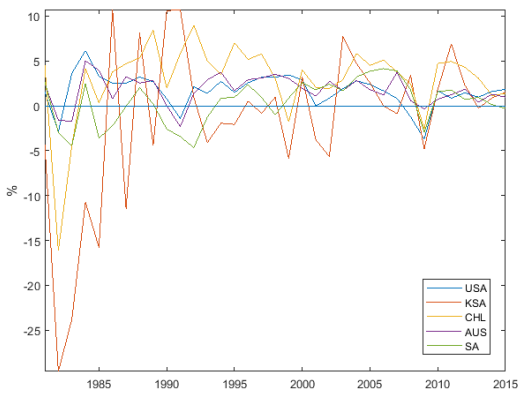
3.2 Stylized Facts

3.2.1 Data

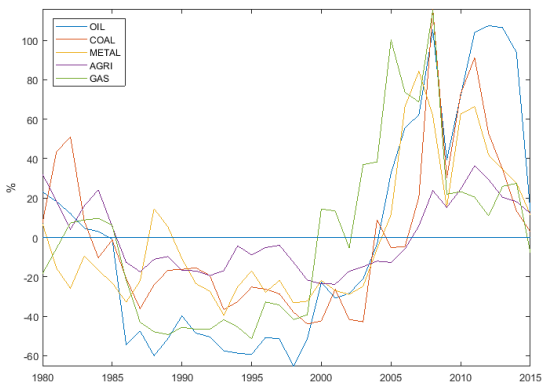
The variables that we use in this paper are real government consumption, real private consumption, and inflation for a selected number of commodity-rich economies[8] [9]. In addition to this, we add the same variables for the U.S economy. The latter will be used to calibrate the moments of the rest of the world, as shown below. The source of this data is World Bank's World Development Indicators (WDI) database, and all of the series are presented in annual per capita terms.

Figure 3.1 Real GDP Growth and Commodities Prices

(a) Real GDP per Capita Growth



(b) Commodities Prices



[8]The selected countries are Chile, a Copper-rich economy; Australia, a minerals-rich economy; Saudi Arabia, an oil-rich economy; and South Africa a coal and minerals-rich economy.

[9]Due to the unavailability of the required data for the Chilean CPI inflation at the WDI database, we use the series available at inflation.eu.

The commodity prices indices were retrieved from the World Bank commodities prices database (the pink sheet). All commodity prices were deflated using the U.S. CPI index. The deflation is done to reflect the real purchasing power of commodities windfalls. We also use mean deviation of real commodity prices rather than de-trending the series, in order to capture long persistence in commodity prices (super cycles). The data for the supply of commodities was downloaded from the IEA database.

The above graph shows significant heterogeneity in the growth rate of GDP per capita among the selected commodity-rich economies. We also include the US growth rate for reference. The above figure illustrates how the growth rates of commodity-rich economies deviate from the growth rate of GDP per capita in the US by different magnitudes. One possible explanation for this behaviour is the volatility of the prices of commodities in these economies[10], as shown in panel (b) of the above figure.

The above graph also shows comovement in the prices of commodities, consistent with the findings of Fernndez et al. (2018). As noted, the fluctuation of commodities prices results in high volatility in commodity-rich economies. In this paper, we impose the same commodity price on all of our selected economies to capture the heterogeneity among these economies beyond the different price fluctuation of each commodity. The price index that we impose in this paper is an average of both the energy and non-energy indices. The energy price index is a weighted average of crude oil prices, natural gas prices, and coal prices. Agricultural products and metal, on the other hand, represent almost 97 % of the non-energy price index.

3.2.2 What Affects Commodity Prices?

Our model makes some assumptions on what affects commodity prices in the world economy. Our framework assumes that commodity prices are determined by commodity supply[11], World output, World technology, and World government consumption. Therefore, the analysis of this section will not affect the structure nor the design of this model, as the parameters that govern the effect of our independent variables on real commodity prices are derived endogenously in the model and not estimated. Nevertheless, this exercise is useful as it will give us an indication of how real commodity prices are affected by developments in the macro variables of the world economy. The regression of this section is specified in the following from:

$$\tilde{P}^*_{O,t} = \beta_0 + \beta_1 Y^*_t + \beta_2 G^*_t + \beta_3 O^{*s}_t + \varepsilon_t \tag{3.1}$$

[10]See, for example, Rodrik (1999).

[11] We use energy supply as a proxy for commodity supply due to the unavailability of total commodities supply.

Where $\tilde{P}^*_{O,t} = \frac{P_{O,t} - \bar{P}_O}{\bar{P}_O} * 100$ is the mean deviation of real commodity prices. Y^*_t, G^*_t, O^{*s}_t are world output, world government consumption and the supply of commodities, respectively. The results of the regression are depicted in the below table and they show a significant effect of the supply of commodities and world output on real commodity prices. World government consumption, however, does not significantly affect commodity prices. The signs of the effect of the supply of commodities and world output are in line with the derivations of the DSGE model of this paper, as shown below.

Table 3.1 Regression Results for Commodity Prices

Commodity Prices	
World Output	1.46**
	(0.566)
World Government Consumption	-1.046
	(1.434)
Commodities Supply	-7.787***
	(2.753)
Constant	-614.7
	(800.355)
Observations	36
R-squared	0.38
Standard errors in parentheses	
*** p <0.01, ** p <0.05, * p <0.1	

3.2.3 The Effect of Government Consumption on Private Consumption

In this section, we empirically estimate the effect of government consumption on private consumption in our four commodity-rich economies and the U.S. economy, which represents the world economy in this model. For the U.S. economy, we estimate this effect while controlling for the commodity price index, U.S. output, and U.S. inflation. As for the other four economies, we control for world output, the commodity price index, domestic inflation, and domestic output. The regression of this section is specified in the following from:

$$ln(C_t) = \beta_0 + \chi ln(G_t) + \beta_1 ln(X_t) + \varepsilon_t \tag{3.2}$$

Where C_t is private consumption, G_t is government consumption, and X_t is a vector of control variables including world output, domestic output, domestic inflation and the real price of commodities. All variables are expressed in log forms. The key parameter of interest in this regression is χ, which denotes the effect of government consumption on private consumption.

The results of the regressions are shown in the below table and they display a significant positive effect of government consumption on private consumption for all five economies. As these results represent one of our behavioural parameters, we will use the below results in the baseline calibration part of our DSGE model, and they will be included as priors in the Bayesian estimation.

Table 3.2 Regression Results for the Effect of G on C

Domestic Consumption	USA	KSA	CHL	SA	AUS
World Output	1.151***	-0.701***	-0.378**	-0.037	0.781***
	(0.016)	(0.162)	(0.175)	(0.027)	(0.177)
World Government Consumption	0.056				
	(0.038)				
World Inflation	-0.524***				
	(0.143)				
Domestic Output		-0.073	1.119***	1.127***	-0.624***
		(0.202)	(0.114)	(0.106)	(0.176)
Domestic Government Consumption		0.736***	0.221*	0.145**	0.712***
		(0.170)	(0.121)	(0.062)	(0.135)
Domestic Inflation		0.642	-0.01	-0.074	-0.34
		(1.63)	(0.205)	(0.276)	(0.369)
Commodity Prices	0.035***	0.177	-0.053	-0.022	0.038*
	(0.005)	(12.45)	(0.04)	(0.027)	(0.02)
Constant	-250.58***	1116.506***	-138.52	-283.99***	172.256**
	(23.47)	(358.63)	(125.65)	(93.027)	(68.047)
Observations	36	36	36	36	36
R-squared	0.99	0.82	0.99	0.97	0.92

Bootstrap standard errors with 10,000 replications are in parentheses.

*** p <0.01, ** p <0.05, * p <0.1.

The below results also contribute to the divided literature on the effect of government consumption on private consumption[12]. Our results support the literature that shows government consumption as a complement to private consumption, at least in commodity-rich countries. Nevertheless, some of causality tests for all the regressions in this section show conflicting signs of the directions imposed by the regression assumptions. Also, we acknowledge the possibility of the presence of endogeneity. However, the adoption of the DSGE model in the next section will allow us to overcome these problems, because it takes into account the fact that these variables are simultaneously determined. Moreover, We will also further investigate this issue in the Bayesian estimation section and, as shown below, the Bayesian

[12] Coenen et al. (2013), Gali et al. (2007), and Fiorito & Kollintzas (2004) find that government consumption has a crowding in effect on private consumption. Aschauer (1985) and Ahmed (1986), on the other hand, show that government consumption has a crowding out effect on private consumption.

estimations show that the explanatory power of the data overcomes the prior values that we extract from the regression results in this section.

3.2.4 Business Cycle Moments

Table 3.3 Business Cycle Moments for Selected Economies

World	GDP growth	Gov. Growth	Cons. Growth	Inflation
Mean	1.68	0.57	2.01	2.03
Std. Deviation	1.86	1.68	1.74	0.86
Persistence	0.32	0.59	0.50	0.33
Correlation with GDP growth	1.00	-0.04	0.94	-0.04
Correlation with Gov. Growth	-0.04	1.00	0.06	-0.23
Correlation with Cons. Growth	0.94	0.06	1.00	-0.14
Correlation with inflation	-0.04	-0.23	-0.14	1.00
KSA	GDP growth	Gov. Growth	Cons. Growth	Inflation
Mean	-1.51	2.40	1.47	1.35
Std. Deviation	8.75	10.02	9.21	2.33
Persistence	0.31	0.12	0.36	0.72
Correlation with GDP growth	1.00	-0.05	-0.14	0.28
Correlation with Cons. Growth	-0.05	1.00	0.40	0.30
Correlation with Gov. Growth	-0.14	0.40	1.00	0.45
Correlation with inflation	0.28	0.30	0.45	1.00
CHL	GDP growth	Gov. Growth	Cons. Growth	Inflation
Mean	2.99	1.70	3.37	9.98
Std. Deviation	4.32	2.86	6.01	8.59
Persistence	0.25	0.44	-0.41	0.79
Correlation with GDP growth	1.00	0.35	0.92	-0.02
Correlation with Gov. Growth	0.35	1.00	0.25	-0.65
Correlation with Cons. Growth	0.92	0.25	1.00	-0.01
Correlation with inflation	-0.02	-0.65	-0.01	1.00
SA	GDP growth	Gov. Growth	Cons. Growth	Inflation
Mean	0.41	0.96	0.62	3.55
Std. Deviation	2.45	2.56	3.99	1.94
Persistence	0.43	0.35	0.03	0.77
Correlation with GDP growth	1.00	0.32	0.86	0.10
Correlation with Gov. Growth	0.32	1.00	0.26	0.14
Correlation with Cons. Growth	0.86	0.26	1.00	-0.03
Correlation with inflation	0.10	0.14	-0.03	1.00
AUS	GDP growth	Gov. Growth	Cons. Growth	Inflation
Mean	1.75	0.38	0.38	2.42
Std. Deviation	1.66	1.22	1.85	1.04
Persistence	0.21	0.58	0.02	0.42
Correlation with GDP growth	1.00	-0.03	-0.40	-0.37
Correlation with Gov. Growth	-0.03	1.00	0.19	-0.43
Correlation with Cons. Growth	-0.40	0.19	1.00	0.15
Correlation with inflation	-0.37	-0.43	0.15	1.00

The above table shows that consumption in developing countries fluctuates more than output which, as mentioned above, consistent with the existing literature on developing economies.

Nevertheless, the business cycle moments for Australia, which is a developed economy, show that consumption is fluctuating more than output, highlighting the possibility of commodities affecting the business cycle of developed economies the same way they affect developing economies. Also, the above persistence measures were estimated by fitting an AR(1) model for each variable.

The above table also shows that the behaviour of the growth rates of government consumption per capita demonstrates significant differences among the above economies. This variable shows more volatility in Saudi Arabia (an oil-rich economy). The growth rates of the same variable for South Africa and Chile, although three times less volatile than that of the Saudi economy, are still higher than the volatility of government consumption in the U.S. economy. Conversely, the growth of government consumption in Australia, which is a developed economy, showed less volatility than all of the above countries, including the U.S. Furthermore, the volatility in government consumption is positively correlated with the volatility of output per capita and the persistence of the growth of government consumption is negatively correlated with its volatility across all economies. These indicators demonstrate the different degrees of volatility among commodity-rich economies which might result from different factors that we aim to study in this paper after isolating the effect of commodity prices.

3.2.5 The Reaction of Government Consumption to Changes in Commodity Prices

In this section, we estimate the second and probably the most important behavioural parameter in the model. We empirically estimate the reaction of government consumption in our selected four economies to changes in the average commodity index. The magnitude of the response of government consumption to changes in commodity prices will be an indicator of two important factors. The first is the fiscal disciplines of the domestic government while the second is the size of the resource rent in the economy. We control for domestic output and domestic CPI inflation. The regression of this section is specified in the following from:

$$ln(G_t) = \beta_0 + \phi_g \tilde{p}^*_{O,t} + \beta_1 ln(X_t) + \varepsilon_t \tag{3.3}$$

Where G_t is is government consumption, $\tilde{p}^*_{O,t}$ is the mean deviation of the real prices of commodities, and X_t is a vector of control variables including domestic output and domestic inflation. All variables are expressed in log forms. The key parameter of interest in this regression is ϕ_g, which denotes the response of government consumption to changes in real commodity prices.

Table 3.4 Regression Results for the reaction of G to changes in Commodity prices

Government Consumption	KSA	CHL	SA	AUS
Commodity Prices	0.78***	0.25***	-0.04	-0.01
	(0.14)	(0.048)	(0.08)	(0.017)
Domestic Output	-0.713	0.44***	0.87***	0.29***
	(2.27)	(0.10)	(0.26)	(0.025)
Domestic Inflation	-0.212	0.11	2.24	-0.9*
	(0.375)	(0.314)	(0.59)	(0.48)
Constant	1161.3***	637.95***	-33.16	475.7***
	(418.13)	(162.69)	(278.30)	(26.9)
Observations	36	36	36	36
R-squared	0.63	0.94	0.77	0.83

Bootstrap standard errors with 10,000 replications are in parentheses.

*** p <0.01, ** p <0.05, * p <0.1.

The above results show that the reactions of the domestic governments display considerable differences among commodity-rich economies. While government consumption does not significantly react to changes in the prices of commodities in Australia and South Africa, it was significantly positive in Chile and Saudi Arabia with responses of differing degrees. The response of government consumption in Saudi Arabia is three times the response of government consumption in Chile. One possible explanation for this behaviour is the size of the resource rents in the economy. During our estimation period, resource rents as a percentage of GDP in Saudi Arabia, Chile, South Africa, and Australia averaged 34 %, 10.9 %, 6.25 % and 4.8 %, respectively (as shown in Appendix C.3)[13].

The above estimations of this behavioural parameter will also be used below in the baseline calibration of our model. These values will also be used as priors in the Bayesian estimation to be undertaken later.

3.2.6 Structural VAR Model

In this section, we address the effect of a commodity shock on the domestic economy by providing an empirical measure based on a Structural VAR model. Commodity shocks are easier to capture as they are observed, different from unobserved technology shocks. Thus, understanding the channels by which the effect of commodity prices affects economic activity is crucial from a policy perspective.

[13] In Appendix C.3 we report the resource rents averages for 88 countries. The stark finding in the data is the relatively higher share of natural resources, as a percentage of GDP, when the commodity is crude oil compared to other commodities.

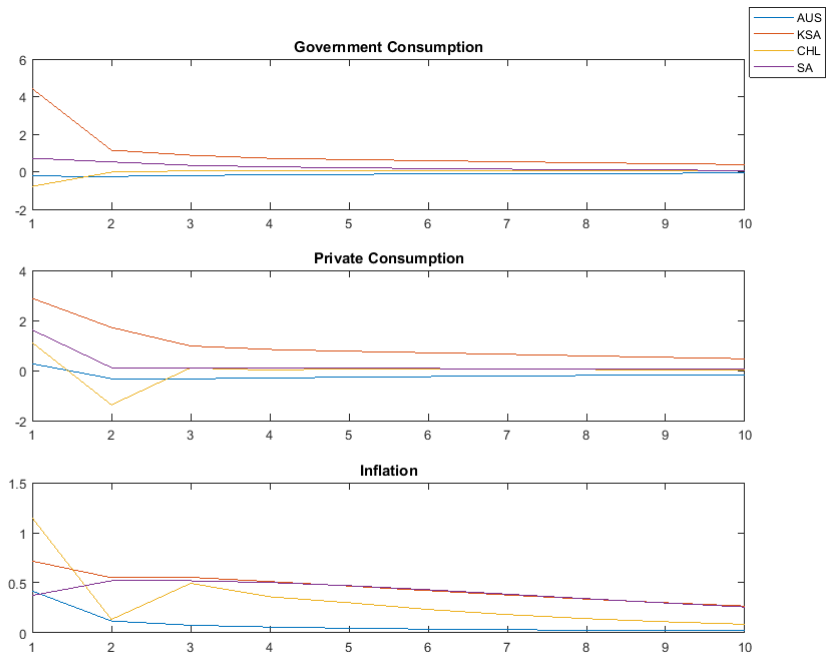
The Structural VAR model for each domestic economy includes four variables, namely the real commodity price index, the growth rate of real government consumption per capita, the growth rate of real private consumption per capita, and domestic CPI inflation, using annual data over the period 1980 to 2015 and defined as follows:

$$A_0 Y_t = \alpha_t + A_1 Y_{t-1} + ... + A_p Y_{t-p} + u_t \tag{3.4}$$

Y_t is a vector containing the four variables of interest for each economy. The underlying assumption that we make for this Structural VAR model is that real commodity prices are not contemporaneously affected by developments in the domestic economies. This is consistent with the small open economy framework that we adopt in this paper. Thus, having commodity prices first in the order of our variables in a Cholesky decomposition is a plausible assumption. In addition, the optimal lag criteria suggests that a lag of order 1 is the optimal choice for each of the four economies.

The economic principle behind the effect of a commodity price shock in our model is simple. When positive, it acts as an income shock that pushes up government consumption. In return, The increase in government consumption will boost private consumption and put inflationary pressure on domestic prices, if government consumption has a crowding in effect on private consumption.

Figure 3.2 Response to a Commodity Shock



The impulse responses illustrate how government consumption growth responds in a different manner among commodity-rich economies. The response of government consumption in Saudi Arabia, an oil-rich country, is the highest among its counterparts in this study. In addition, the insignificant response of Australia and Chile reflect the adopted fiscal policy objective or rule in these two economies. The reaction of the South African government consumption shows a positive reaction to a commodity-price shock. This contradicts with the findings of the previous estimations of this paper. Nevertheless, the Bayesian estimation section should confirm one of these findings.

The reaction of private consumption and domestic CPI inflation is determined by the crowding in effect of government consumption and the implemented subsidised schemes that are adopted in different commodity-rich economies. In this regard, the size of the consumption of commodities in the aggregate consumption bundle should reflect the size of these subsidies in our DSGE model below.

The next section builds a dynamic general equilibrium model guided by these stylized facts where we formally articulate a mechanism by which exogenous changes in commodity prices turn into fluctuations in real economic activity, along with other exogenous shocks that have been suggested by the previous literature.

3.3 The Model

In this section we construct a small open economy model for a commodity-rich economy by using the framework of Gali & Monacelli (2005). Moreover, we extend the model used in Chapter 2 by adding some features that were missing in the model to make it relevant for a commodity-rich economy. Our model allows for a quadruple role for commodities. First, the domestic government collects the windfalls from selling commodities to the rest of the world. Second, commodities are consumed by households both in the domestic economy and the foreign economy. Third, firms both in the domestic economy and the foreign economy use commodities as an input factor in their production. Lastly, the domestic economy is affected by the second-round effect of an increase in commodity prices in the form of high foreign inflation and low world demand or vice versa.

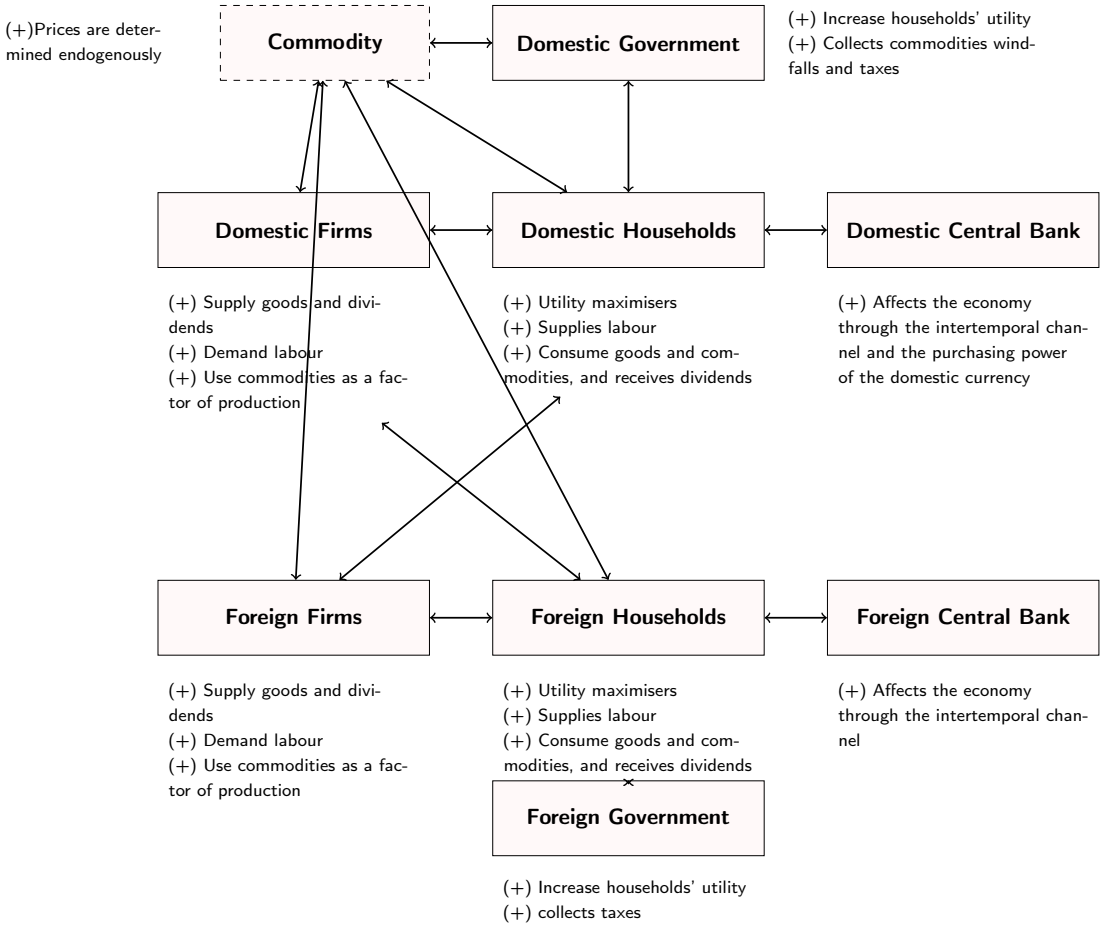


Figure 3.3 Structure of the Model

3.3.1 Domestic Economy

Household

The representative consumer in the domestic economy seeks to maximise the following discounted lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{C}_t, N_t) \tag{3.5}$$

The utility function is assumed to be continuous and twice differentiable. N_t is the number of hours worked; β is the discount factor; $\bar{C}_t$ is the aggregate consumption bundle. The aggregate consumption bundle is a constant elasticity of substitution aggregate that consists

of private consumption C_t and government consumption G_t:

$$\bar{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{3.6}$$

Where δ is the equilibrium share of private consumption in the aggregate consumption bundle and χ is the inverse elasticity of substitution between private consumption and government consumption. From equations (3.5) and (3.6) we can notice that the utility function is non-decreasing in government consumption G_t. The above utility function is subject to the following budget constraint:

$$\int_0^1 P_{H,t}(j) C_{H_t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i_t}(j) dj di + E_t Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t \tag{3.7}$$

Where D_t is the nominal payoff at period t+1 of bonds held at the end of period t, including shares in firms, government bonds, and deposits. $Q_{t,t+1}$ is a stochastic discount factor of nominal payoffs and it is equal to $\frac{1}{R_t}$; W_t is wages; T_t is lump-sum transfers to the households net of lump-sum taxes. All units are expressed in terms of domestic currency. In addition, the private consumption basket is an aggregate composition of core consumption and consumption of commodities:

$$C_t = \left[(1-\varpi)^{\frac{1}{\mu}} C_{Z,t}^{\frac{\mu-1}{\mu}} + \varpi^{\frac{1}{\mu}} C_{O,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{3.8}$$

In the above equation, $C_{O,t}$ is consumption of commodities by the domestic economy's households, and ϖ is the share of commodities consumption in the household's consumption bundle. $C_{Z,t}$ is the non-commodity consumption bundle (core consumption), and it has a size of $(1-\varpi)$ in the household's consumption bundle. μ is the elasticity of substitution between core consumption and consumption of commodities. The core consumption bundle $C_{Z,t}$ is a CES composite of home and foreign goods defined as follows:

$$C_{Z,t} = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.9}$$

The above equation is the same household's consumption bundle used by Gali & Monacelli (2005), which is the workhorse for small open economies. α here is the degree of openness in the economy which represents the share of imported goods $C_{F,t}$ in the household's consumption bundle. The home bias parameter $(1-\alpha)$ produces the possibility of a different consumption bundle in each economy. This is a consequence of having different consumption baskets in each country, despite the law of one price holding for each individual good. $\eta > 0$

is the elasticity of substitution between domestically produced goods and imported goods in the household's consumption bundle.

The utility function that we use assumes two separabilities. The first is the separation between consumption and the amount of hours worked, and the second is time separability. The household's problem is analysed in two stages here. We first deal with the expenditure minimisation problem faced by the representative household to derive the demand functions for commodity goods, non-commodity goods, domestic goods and foreign goods. In the second stage, the households choose the level of C_t and N_t, given the optimally chosen combination of goods. The standard optimality condition for households will be as follows:

$$\frac{W_t}{P_t} = N_t^{\varphi} \bar{C}_t^{\sigma} \left(\frac{C_t}{\bar{C}_t}\right)^{\chi} \delta^{-\chi} \tag{3.10}$$

The intertemporal optimality condition is:

$$\beta \left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{C_t}{C_{t+1}}\right)^{\chi} = Q_{t,t+1} \tag{3.11}$$

Taking the conditional expectation of equation (3.11) and rearranging the terms we get:

$$\beta R_t E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{C_t}{C_{t+1}}\right)^{\chi} \right] = 1 \tag{3.12}$$

3.3.2 Firms

Price Setting Behaviour

The firms in this model set their prices in a staggered manner following Calvo (1983)[14]. Under Calvo contracts, we have a random fraction $1-\theta$ of firms that are able to reset their prices at period t, while prices of the remaining firms of size θ are fixed at the previous period's price levels. Therefore, we can say that θ^k is the probability that a price set at period t will still be valid at period $t+k$. Also, the probability of the firm re-optimising its prices will be independent of the time passed since it last re-optimised its prices, and the average duration for prices not to change is $\frac{1}{1-\theta}$. Given the above information, the aggregate domestic price level will have the following form:

$$P_{H,t} = \left[\theta (P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t}^{1-\varepsilon}) \right]^{\frac{1}{1-\varepsilon}} \tag{3.13}$$

[14]The Calvo model makes aggregation easier because it gets rid of the heterogeneity in the economy. The alternative pricing scheme is the quadratic cost of price adjustment by Rotemberg (1982). The two dynamics are equivalent up to a first-order approximation.

Where $\bar{P}_{H,t}$ is the new price set by the optimising firms. From the derivations shown in Appendix C.2, we get the following form for inflation:

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{t-1}}\right)^{1-\varepsilon} \tag{3.14}$$

The above equation shows that the domestic inflation rate at any given period will be solely determined by the fraction of firms that reset their prices at that period. When a given firm in the economy sets its prices, it seeks to maximise the expected discounted value of its stream of profits, conditional that the price it sets remains effective:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} [c_{jt+k|t} (\bar{P}_{H,t} - \Psi_{t+k})] \right\} \tag{3.15}$$

The above equation is subject to a sequence of demand constraints: $c_{jt+k} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} C_t$. Solving this problem (also shown in Appendix C.2) yields the following optimal decision rule:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} C_{t+k} \left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathscr{M} MC_{t+k|t} \Pi_{t-1,t+k}^{H} \right] \right\} = 0 \tag{3.16}$$

Where $\mathscr{M}$ is the firm's markup at the steady state and MC_t is real marginal cost. As we can see from equation (3.16), in the sticky price scheme producers, given their forward-looking behaviour, adjust their prices at a random period to maximise the expected discounted value of their profits at that period and in the future. Thus, firms in this model will set their prices equal to a markup plus the present value of the future expected stream of their marginal costs. This is done because firms know that the price they set at period t will remain effective for a random period of time in the future. We also assume that all firms in the economy face the same marginal cost, given the constant return to scale assumption imposed on the model and the subsidy that the government pays to firms, as we will see in the following section. The firms also use the same discount factor β as the one used by households, and this is attributed to the fact that the households are the shareholders of these firms. Additionally, all the firms that optimise their prices in any given period will choose the same price which is also a consequence of the firms facing the same marginal cost. Equation (3.16) also shows that the inflation rate is proportional to the discounted sum of the future real marginal costs additional to a mark-up resulting from the monopolistic power of the firms.

Production

Firm (j) in the domestic economy produces a differentiated good following a linear production function:

$$Y_t(j) = [A_t N_t(j)]^{\nu} O_t^d(j)^{1-\nu} \tag{3.17}$$

In the above equation, $Y_t(j)$ is the output of final good (j) in the home economy. A_t is the level of technology in the production function. It evolves exogenously and is assumed to be common across all firms in the economy. $N_t(j)$ is the labour force employed by firm (j). O_t^d is the commodity used in the production process and $(1-\nu)$ is the size of commodities in the production function. The log form of total factor productivity $a_t = log(A_t)$ is assumed to follow an AR(1) process: $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$. Where ρ_a is the autocorrelation of the shock and the innovation to technology $\varepsilon_{a,t}$ is assumed to have a zero mean and a finite variance σ_a. The cost minimisation function for firm (j) has the following form:

$$(1-\tau)(1-\nu)W_t N_t(j) = \nu P_{o,t} O_t^d(j) \tag{3.18}$$

We note that in the above equation we left W_t without any firm specification, as we have a competitive labour market in this model. Also, τ is the subsidy that the government gives to firms in order to eliminate the markup distortion created by the firms' monopolistic power. The marginal cost equation takes the following form:

$$MC_t(j) = \frac{(1-\tau)W_t}{\nu A_t^{\nu} O_t^d(j)^{1-\nu} N_t(j)^{\nu-1}} \tag{3.19}$$

Using the above cost minimising equation, the above marginal cost equation is utilised to:

$$MC_t(j) = \frac{(1-\tau)^{\nu} W_t^{\nu} P_{o,t}^{1-\nu}}{\nu^{\nu}(1-\nu)^{(1-\nu)} A_t^{\nu}} \tag{3.20}$$

Lastly, given that aggregate output and aggregate employment in the domestic economy are defined by the Dixit & Stiglitz (1977) aggregator, the aggregate production function will take the following form:

$$Y_t = [A_t N_t]^{\nu} O_t^{d(1-\nu)} \tag{3.21}$$

3.3.3 Fiscal Policy

The government levies a lump sum tax on the agents of the economy and pays a subsidy to firms in order to eliminate its monopolistic power. The government also collects income from sales of its natural resources, and has access to the financial markets. Therefore, the

government budget constraint is defined as[15]:

$$G_t + (1 + R_{t-1})B_{t-1} + \tau = B_t + T_t + \phi_g P_{t,o} Y_{t,o} \tag{3.22}$$

Where B_t is the quantity of a riskless one-period bond maturing in the current period, which pays one unit. R_t denotes the gross nominal return on bonds purchased in period t. The government levies a non-distortionary lump-sum tax T_t to finance its consumption and pay a subsidy τ to firms. In addition, p_t^o is the price of commodities dominated in domestic currency and Y_t^o is the output of that commodity[16]. Given the above, G_t is government consumption will take the following form:

$$\frac{G_t}{G} = \left\{\frac{G_{t-1}}{G}\right\}^{\rho_g} \left\{\frac{P_{o,t}Y_o}{P_o Y_o}\right\}^{\phi_g} exp(\zeta_{G,t}) \tag{3.23}$$

Where $0 < \rho_g < 1$ is the autocorrelation of government consumption, and it captures the persistence of government consumption. ϕ_g captures the response of government consumption to changes in the prices of commodities. $\zeta_{G,t}$ represents an i.i.d. government spending shock with constant variance σ_g^2.

Monetary Policy

The monetary authorities in this model use a short-term interest rate as their policy tool. In our case, we have a cashless economy where money supply is implicitly determined to achieve the interest rate target. It is also assumed that the central bank will meet all the money demanded under the policy rate it sets.

$$\frac{R_t}{R} = \left\{\frac{\Pi_{Z,t}}{\Pi_Z}\right\}^{\phi_\pi} \left\{\frac{Y_t}{Y}\right\}^{\phi_x} exp(\zeta_{R,t}) \tag{3.24}$$

The parameters of the above equations (ϕ_π, ϕ_x) describe the strength of the response of the policy rate to deviations in the variables on the right-hand side. These parameters are assumed to be non-negative. The inflation response parameter ϕ_π in the above policy rule must be strictly greater than one in order for the solution of the model to be unique, as shown

[15]The definition of government consumption in this chapter is broader than the one used in the first two chapters, as it includes all government recurrent spending items, including salaries of state employees. We do this to establish consistency in the mapping of the observed government consumption variable.

[16]Given the fact that the production of natural resources is capital intensive,we follow the existing literature (e.g., Wills (2014), Berg et al. (2013) and Agénor (2014)) by assuming that production of natural resources is exogenous. moreover, the share of employment in the natural resource sector does not exceed 3 % of total employment in natural resource-rich economies, according to the ILO database, and the labour force lacks mobility between the two sectors.

by Bullard & Mitra (2002). Lastly, $\zeta_{R,t}$ represents an i.i.d. monetary policy shock with constant variance σ_R^2.

3.3.4 International Linkages

We first start by the defining the terms of trade as the ratio of imported prices to domestic prices. The bilateral terms of trade index between the domestic economy and any other small economy (country i) is defined as: $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$. The aggregate terms of trade index is defined as: $S_t = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$. Defining $P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ allows as us to define the aggregate effective terms of trade as:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{3.25}$$

If we plug in the log-linearised representation of the imported prices index from the above equation ($p_{F,t} = s_t + p_{H,t}$) in the log-linearised form of the CPI price index equation, we can derive the CPI index as a function of the domestic prices index and the terms of trade:

$$p_t = p_{H,t} + \alpha s_t \tag{3.26}$$

The above function shows the gap between the CPI index and the domestic price index which is filled with the terms of trade. The gap is parametrised by the degree of openness of the domestic economy. Before progressing on further derivations, we first define the bilateral exchange rate $\mathscr{E}_{i,t}$ as the value of country i's currency in terms of the domestic currency. Assuming that the law of one price holds, the price of any good in country (i) will be equal to:

$$P_{i,t}(j) = \mathscr{E}_{i,t} P_{i,t}^{i}(j) \tag{3.27}$$

Integrating the above equation yields the price index for country (i). Solving this integral for the imported prices index in the domestic economy yields:

$$P_{F,t} = \mathscr{E}_t P_t^* \tag{3.28}$$

The nominal effective exchange rate is equal to $\mathscr{E}_t \equiv \int_0^1 \mathscr{E}_{i,t} di$, and the world price index is defined as $P_t^* \equiv \int_0^1 P_{i,t} di$. Plugging the value of the imported prices index from the above

equation in the definition of the terms of trade yields:

$$S_t = \frac{\mathscr{E}_t P_t^*}{P_{H,t}} \tag{3.29}$$

We now define the bilateral real exchange rate as the ratio of the price index in country (i) to the CPI index in the domestic economy: $REER_{i,t} = \frac{\mathscr{E}_{i,t} P_t^i}{P_t}$. Integrating the bilateral real exchange rate equation yields the real effective exchange rate equation for the domestic economy: $REER_t = \frac{\mathscr{E}_t P_t^*}{P_t}$. From the definitions of the terms of trade and the real effective exchange rate, we can define the equation that links the two variables in a log-linearised form as follows:

$$q_t = (1-\alpha)s_t \tag{3.30}$$

Under the assumption of complete international financial markets, the price of a one-period riskless bond dominated in the domestic economy's currency from country (i) is equal to: $\mathscr{E}_{i,t} Q_t^i = E[\mathscr{E}_{i,t+1} Q_{t,t+1}]$. If we add this equation to the domestic bond's price equation ($Q_t = E[Q_{t,t+1}]$), we get the uncovered interest parity condition:

$$\frac{Q_t^i}{Q_t} = E_t\left(\frac{\mathscr{E}_{i,t+1}}{\mathscr{E}_{i,t}}\right) \tag{3.31}$$

The uncovered interest parity condition is crucial for the no-arbitrage condition to hold in the international bonds market. Under the uncovered interest parity we assume that foreign bonds are perfect substitutes to domestic bonds once both are expressed in the same currency. The uncovered interest parity equation also implies that higher foreign interest rates or a depreciation in the exchange rate will put upward pressure on domestic interest rates.

The last thing that we need do in this section is to derive the international risk condition. Under the assumptions of complete international markets and the identical preferences assumption, the foreign consumer's Euler equation can be presented as:

$$\beta\left(\frac{\bar{C}_{t+1}^*}{\bar{C}_t^*}\right)^{\chi-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\chi}\left(\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right) = Q_{t,t+1} \tag{3.32}$$

We divide the domestic inter-temporal optimality condition (eq. 3.11) by the foreign economy's inter-temporal optimality condition (eq. 3.32) to get:

$$1 = E_t\left(\frac{\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{\chi-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_t}{C_{t+1}}\right)^{\chi}}{\left(\frac{\bar{C}_{t+1}^*}{\bar{C}_t^*}\right)^{\chi-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)\left(\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right)\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\chi}}\right) \tag{3.33}$$

Plugging the definition of the real effective exchange rate in the above equation yields:

$$C_t = \mathscr{V}_t C_t^* (REER_t)^{\frac{1}{\chi}} \left(\frac{\bar{C}_t}{\bar{C}_t^*}\right)^{\frac{\chi-\sigma}{\chi}} \tag{3.34}$$

Where $\mathscr{V}_t = \frac{C_{t+1}\bar{C}_t^{*\frac{\chi-\sigma}{\chi}}}{C_{t+1}^*\bar{C}_{t+1}^{\frac{\chi-\sigma}{\chi}} REER_{t+1}^{\frac{1}{\chi}}}$ is a constant and it depends on the initial relative wealth position. We assume that we have a symmetric initial condition and set $\mathscr{V}_t = 1$; meaning that the net position of foreign assets is equal to zero. Thus, the international risk sharing condition simplifies to:

$$C_t = C_t^* (REER_t)^{\frac{1}{\chi}} \left(\frac{\bar{C}_t}{\bar{C}_t^*}\right)^{\frac{\chi-\sigma}{\chi}} \tag{3.35}$$

Complete security markets ensure that risk-averse consumers are able to trade away the risks and the shocks that they encounter. Under this setting, consumers are able to purchase contingent claims for realisations of all idiosyncratic shocks, and this will enable them to diversify all idiosyncratic risk through the capital markets. The above international risk sharing condition also shows how a depreciation in the real effective exchange rate boosts domestic consumption relative to the foreign economy's consumption. The log-linearised form of the above international risk sharing condition is:

$$c_t = c_t^* + \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}(g_t^* - g_t) + \frac{1}{\sigma_\delta} q_t. \tag{3.36}$$

Where $\sigma_\delta = \delta\sigma + (1-\delta)\chi$ is a weighted average of the intertemporal elasticity of substitution σ and the inverse elasticity of substitution between government consumption and private consumption χ.

Market clearing conditions

We start by identifying the market clearing condition for the domestically produced products in the small open economy. Domestic output of good (j) is absorbed both by domestic demand and foreign demand:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)di \tag{3.37}$$

In the above equation, $C_{H,t}(j)$ is domestic demand for good (j) and $C_{H,t}^i$ is country (i)'s demand for good (j) in the domestic economy. We plug the domestic demand function for good (j). As for foreign demand for domestic good (j), we use the assumption of symmetric

preferences across all the countries of the world economy to get:

$$C^i_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\mathscr{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} \tag{3.38}$$

Plugging in the respective demand bundles transforms the market clearing condition for domestic production of good (j) to:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left((1-\alpha)\left(\frac{P_{H,t}}{P^i_t}\right)^{-\eta}C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\mathscr{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta}C^i_t(j)di\right) \tag{3.39}$$

Using the Dixit-Stiglitz aggregator of domestic output, we can write the above equation in aggregate terms:

$$Y_t = \left(\frac{P_{H,t}}{P^i_t}\right)^{-\eta}\left((1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathscr{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta}Q^\eta_{i,t}C^i_t di\right) \tag{3.40}$$

In the above equation, we took $\left(\frac{P_{H,t}}{P^i_t}\right)^{-\eta}$ as common factor. We have also used the definition of the bilateral real exchange rate. If we divide and multiply the term $\left(\frac{\mathscr{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta}$ by $P_{i,t}$ we get: $\left(\frac{P_{i,t}}{P_{H,t}}\frac{\mathscr{E}_{i,t}P^i_{F,t}}{P_{i,t}}\right)^{\gamma-\eta}$. The two terms that we get are basically the effective terms of trade for country (i) and the bilateral terms of trade between the domestic economy and country (i), and equation (3.40) simplifies to:

$$Y_t = \left(\frac{P_{H,t}}{P^i_t}\right)^{-\eta}\left((1-\alpha)C_t + \alpha\int_0^1\left(S^i_t Si,t\right)^{\gamma-\eta}Q^\eta_{i,t}C^i_t di\right) \tag{3.41}$$

Taking the first order log-linearisation of the above equation around a symmetric steady state yields:

$$y_t = (1-\alpha)c_t + \alpha c^*_t + \alpha[\gamma + \eta(1-\alpha)]s_t \tag{3.42}$$

Adding the log-linearised form of the international risk sharing condition to the above equation yields:

$$y_t = y^*_t + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}(g^*_t - g_t) + \frac{\omega_\alpha}{\sigma_\delta}s_t \tag{3.43}$$

where $\omega = \sigma_\delta\gamma + (1-\alpha)(\eta\sigma_\delta - 1)$ and $\omega_\alpha = (1-\alpha) + \alpha\omega$. The above equation links the actual rate of output to foreign and domestic government consumption, the rest of the world economy's output, and the terms of trade.

The Supply Side of the Economy

The log-linearised version of the real marginal cost equation could be written in the following format:

$$mc_t = \nu w_t + (1-\nu)p_{o,t} - \nu a_t - p_{H,t} \tag{3.44}$$

Adding and subtracting $(1-\nu)p_t$ yields:

$$mc_t = \nu(w_t - p_t) + (1-\nu)\tilde{p}_{o,t} + \alpha s_t - \nu a_t \tag{3.45}$$

Where $\tilde{p}_{o,t}$ is the real price of commodities and it is equal to: $p_{o,t} - p_t$. Using the log-linearised form of the labour supply equation, the international risk sharing condition, and replacing the domestic real commodity prices with international real commodity prices ($\tilde{p}_{o,t} = \tilde{p}^*_{o,t} + (1-\alpha)s_t$), the above equation transforms to:

$$mc_t = \frac{\nu\sigma_\delta}{1+\varphi(1-\nu)}y^*_t + \frac{\nu\varphi}{1+\varphi(1-\nu)}y_t + s_t - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)}a_t + \frac{(1-\nu)(1+\varphi)}{1+\varphi(1-\nu)}\tilde{p}^*_{o,t} + \frac{(\nu(\sigma-\sigma_\delta)}{1+\varphi(1-\nu)}g^*_t \tag{3.46}$$

Plugging in the value of the terms of trade from the international market clearing condition yields:

$$\begin{aligned} mc_t = {} & \frac{\nu\sigma_\delta\omega_\alpha - \sigma_\delta - \sigma_\delta\varphi(1-\nu)}{\omega_\alpha(1+\varphi(1-\nu))}y^*_t + \frac{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}{\omega_\alpha(1+\varphi(1-\nu))}y_t - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)}a_t \\ & + \frac{(1-\nu)(1+\varphi)}{1+\varphi(1-\nu)}\tilde{p}^*_{o,t} + \frac{(\sigma-\sigma_\delta)(\nu\omega_\alpha - (1-\alpha) - (1-\alpha)\varphi(1-\nu))}{\omega_\alpha(1+\varphi(1-\nu))}g^*_t + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha}g_t \end{aligned} \tag{3.47}$$

Setting $mc = -\mu$ and solving the above equation for output yields the equation of the natural rate of output:

$$\begin{aligned} \bar{y}_t = {} & -\frac{\nu\sigma_\delta\omega_\alpha - \sigma_\delta - \sigma_\delta\varphi(1-\nu)}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}y^*_t - \frac{((\sigma-\sigma_\delta)(\nu\omega_\alpha - (1-\alpha) - (1-\alpha)\varphi(1-\nu))}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}g^*_t \\ & - \frac{(1-\alpha)(\sigma-\sigma_\delta)(1+\varphi(1-\nu))}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}g_t + \frac{\nu(1+\varphi)\omega_\alpha}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}a_t - \frac{(1-\nu)(1+\varphi)\omega_\alpha}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}\tilde{p}^*_{o,t} \end{aligned} \tag{3.48}$$

Subtracting the above two equations from each other yields the marginal cost variable as a function of the output gap:

$$\hat{mc}_t = \frac{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}{\omega_\alpha(1+\varphi(1-\nu))}x_t \tag{3.49}$$

Adding the above equation to the derived Phillips curve in Appendix C.2 enables us to write domestic inflation as a function of the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa \frac{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}{\omega_\alpha(1+\varphi(1-\nu))} x_t \tag{3.50}$$

The Demand Side of the Economy

We start this section by adding the domestic economy's market clearing condition (eq. 3.42) to the log form of the Euler equation (eq. 3.11) to get:

$$\begin{aligned}
y_t =& E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\}) - \alpha[\gamma + \eta(1-\alpha)]\Delta E_t\{s_{t+1}\} - \alpha\Delta E_t\{y^*_{t+1}\} \\
&+ \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \frac{\alpha\omega}{\sigma_\delta}\Delta E_t\{s_{t+1}\} - \alpha\Delta E_t\{y^*_{t+1}\} \\
&+ \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \alpha(\omega-1)\Delta E_t\{y^*_{t+1}\} + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\
&+ \frac{\alpha(\sigma-\sigma_\delta)}{\sigma_\delta}\Delta E_t\{g^*_{t+1}\}
\end{aligned} \tag{3.51}$$

In the above system of equations, we made use of the CPI index equation in the domestic economy (eq. 3.26) and replaced the value of the terms of trade in equation (3.43). It is shown above that the effects of the domestic variables (government expenditure and real interest rates) on output are parametrised by the home-bias parameter $(1-\alpha)$, while the effects of the external variables are parametrised by the degree of openness in the economy α. This is inherited from the market clearing condition of the domestic economy. Solving the above IS curve for the output gap yields:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{r}r_t) \tag{3.52}$$

Where:

$$\begin{aligned}\bar{rr}_t &= \frac{\sigma_\delta}{\omega_\alpha}\Delta E_t\{\bar{y}_{t+1}\} - \frac{\sigma_\delta\alpha(\omega-1)}{\omega_\alpha}\Delta E_t\{y^*_{t+1}\} + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha}\Delta E_t\{g_{t+1}\} + \frac{\alpha(\sigma-\sigma_\delta)}{\omega_\alpha}\Delta E_t\{g^*_{t+1}\} \\ &= -\frac{\nu(1+\varphi)\sigma_\delta(1-\rho_a))}{\nu\varphi\omega_\alpha+\sigma_\delta+\sigma_\delta\varphi(1-\nu)}a_t + \frac{(1-\nu)(1+\varphi)\sigma_\delta}{\nu\varphi\omega_\alpha+\sigma_\delta+\sigma_\delta\varphi(1-\nu)}\Delta E_t\{\tilde{p}^*_{o,t+1}\} \\ &+ \frac{\nu\varphi(1-\alpha)(\sigma-\sigma_\delta)}{\nu\varphi\omega_\alpha+\sigma_\delta+\sigma_\delta\varphi(1-\nu)}\Delta E_t\{g_{t+1}\} + \frac{(\sigma-\sigma_\delta)(\nu\omega_\alpha(\alpha\varphi-\sigma_\alpha)+\sigma_\delta(1+\varphi-\varphi\nu))}{\omega_\alpha(\nu\varphi\omega_\alpha+\sigma_\delta+\sigma_\delta\varphi(1-\nu))}\Delta E_t\{g^*_{t+1}\} \\ &+ \frac{\sigma_\delta((1+\alpha)\sigma_\delta(1+\varphi(1-\nu))-\alpha\nu\varphi\omega_\alpha(\omega-1)-\alpha\omega\sigma_\delta(1+\varphi(1-\nu))-\nu\omega_\alpha\sigma_\delta)}{\omega_\alpha(\nu\varphi\omega_\alpha+\sigma_\delta+\sigma_\delta\varphi(1-\nu))}\Delta E_t\{y^*_{t+1}\}\end{aligned} \tag{3.53}$$

One of the contributions that this paper makes is adding real commodity prices to the reaction of the natural rate of interest function. The weight of commodities in the production function (ν) also affects the reaction of the natural rate of interest to all the possible shocks.

Lastly, to calculate domestic demand for commodities, we replace employment in the cost minimisation equation to get:

$$o^d_t = \frac{1+\varphi}{1+\varphi(1-\nu)}y_t + \frac{\nu\sigma_\delta}{1+\varphi(1-\nu)}y^*_t - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)}a_t + \frac{\nu(\sigma-\sigma_\delta)}{1+\varphi(1-\nu)}g^*_t - \frac{\nu}{1+\varphi(1-\nu)}\tilde{p}^*_{o,t} \tag{3.54}$$

The equation shows that increases in domestic output and world output have a positive effect on domestic demand for commodities. The effect of world government consumption, however, depends on whether world government consumption is a complement or a substitute to world private consumption, as the former's effect on the domestic economy varies under the two assumptions[17]. As for domestic technology, given that it is also a factor of production, it has a negative effect on domestic demand for commodities. Lastly, real international commodity prices have a negative effect on the demand of commodities in the domestic economy.

3.3.5 Rest of the World economy

Households

The representative household of the foreign economy seeks to maximise a similar utility function to the one shown above for the domestic economy:

$$E_0\sum_{t=0}^{\infty}\beta^t U(\bar{C}^*_t, N^*_t) \tag{3.55}$$

[17]see Chapter 2 for more details.

The utility function is assumed to be continuous and twice differentiable. N_t^* is the amount of hours worked; $\bar{C}_t^*$ is the aggregate consumption bundle, and it is a constant elasticity of substitution aggregate consisting of private consumption C_t^* and government consumption G_t^*:

$$\bar{C}_t^* = [\delta^{*\chi^*} C_t^{*1-\chi^*} + (1-\delta^*)^{\chi} G_t^{*1-\chi^*}]^{\frac{1}{1-\chi^*}} \tag{3.56}$$

Similar to the domestic economy, δ^* is the weight of private consumption C_t^* in the aggregate consumption bundle. C_t^* is our basic private consumption bundle, and it is a CES composite of core consumption and consumption of commodities, defined as follows:

$$C_t^* = \left[(1-\varpi)^{\frac{1}{\mu}} C_{Z,t}^{*\frac{\mu-1}{\mu}} + \varpi^{\frac{1}{\mu}} C_{O,t}^{*\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{3.57}$$

Where $C_{O,t}^*$ is consumption of commodities by the foreign economy's households, and ϖ^* is the share of oil consumption in the household's consumption bundle. $C_{Z,t}^*$ is the non-commodity consumption bundle (core consumption), and it has a size of $(1-\varpi^*)$ in the household's consumption bundle.

Using the world aggregate demand equation and plugging the foreign economy's consumption bundles, we get the aggregate CPI index for the foreign economy:

$$P_t^* = \left[(1-\varpi^*) P_{Z,t}^{*1-\mu} + \varpi^* P_{O,t}^{*1-\mu}\right]^{\frac{1}{1-\mu}} \tag{3.58}$$

Analogues to the domestic economy, the labour supply and the consumption intertemporal Euler equations take the following forms:

$$\frac{W_t^*}{P_t^*} = N_t^{*\varphi} \bar{C}_t^{*\sigma} \left(\frac{C_t^*}{\bar{C}_t^*}\right)^{\chi} \delta^{-\chi} \tag{3.59}$$

The intertemporal optimality condition is:

$$\beta \left(\frac{\bar{C}_{t+1}^*}{\bar{C}_t}^*\right)^{\chi-\sigma} \left(\frac{P_t^*}{P_{t+1}^*}\right) \left(\frac{C_t^*}{C_{t+1}^*}\right)^{\chi} = Q_{t,t+1} \tag{3.60}$$

Firms

Production

The representative firm in the foreign economy uses commodities and labour as inputs of production in the following form:

$$Y_t^*(i) = \left[A_t N_t^*(i)\right]^{\nu} O_t^{*d}(i)^{1-\nu} \tag{3.61}$$

In the above equation, N_t^* is labour input, and O_t^* is commodities input. ν is the share of non-commodity factors in the production function. Cost minimising with respect to the production

function yields the optimal resource allocation:

$$(1-\nu)(1-\tau)W_t^* N_t^*(i) = \nu P_{O,t}^* O_t^{*d}(i) \tag{3.62}$$

The optimal behaviour of firms requires the technical rate of substitution to equate the relative prices of the input factors. τ is an employment subsidy which the government in the foreign economy pays to firms to offset their monopolistic power distortion. The nominal marginal cost equation is defined as:

$$MC_t^* = \frac{(1-\tau)^{\nu} W_t^{*\nu} P_{O,t}^{*1-\nu}}{\nu^{\nu}(1-\nu)^{(1-\nu)} A_t^{*\nu}} \tag{3.63}$$

Price Setting

As for the price setting behaviour of the firms, we assume that the foreign economy firms also set their prices according to Calvo (1983) contracts. Thus, the resulting log-linearised New Keynesian Phillips Curve for the foreign economy is:

$$\hat{\pi}_t^* = \beta E_t[\hat{\pi}_{t+1}^*] + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{mc}_t^* \tag{3.64}$$

Fiscal Policy

The government in the foreign economy also levies a lump sum tax on the agents of the economy. It also pays a subsidy to firms in order to eliminate its monopolistic power and it has access to the financial markets. Therefore, the government budget constraint is given by:

$$G_t^* + (1+R_{t-1}^*)B_{t-1}^* + \tau^* = B_t^* + T_t^* \tag{3.65}$$

Where B_t^* is the quantity of a riskless one-period bond maturing in the current period , and it pays one unit. R_t^* denotes the gross nominal return on bonds purchased in period t. The government levies a non-distortionary lump-sum tax T_t^* to finance its consumption and pays a subsidy τ^* to firms. Given the above, G_t^* is government consumption and takes the following form:

$$\frac{G_t^*}{G^*} = \left\{\frac{G_{t-1}^*}{G^*}\right\}^{\rho_{g^*}} exp(\zeta_{G^*,t}) \tag{3.66}$$

Where $0 < \rho_g^* < 1$ is the autocorrelation of government consumption, and it captures the persistence of foreign government consumption. $\zeta_{G^*,t}$ represents an i.i.d. government consumption shock with constant variance $\sigma_{g^*}^2$.

Monetary Policy

The monetary authority in the foreign economy also uses a short-term interest rate as its policy tool:

$$\frac{R_t^*}{R^*} = \left\{\frac{\Pi_t^*}{\Pi^*}\right\}^{\phi_\pi} \left\{\frac{Y_t^*}{Y^*}\right\}^{\phi_x} exp(\zeta_{R^*,t}) \tag{3.67}$$

Monetary policy in the foreign economy reacts to deviations of inflation from its natural level and deviations of output from its natural level. $\zeta_{R^*,t}$ represents an i.i.d. monetary policy shock with constant variance $\sigma_{r^*}^2$

The Supply Side of The World Economy

We start this section by writing the log-linearised version of the real marginal cost equation in the foreign economy as follows:

$$mc_t^* = \nu w_t^* + (1-\nu)p_{o,t}^* - \nu a_t^* - p_t^* \tag{3.68}$$

Adding and subtracting νp_t^* yields:

$$mc_t^* = \nu(w_t^* - p_t^*) + (1-\nu)(\tilde{p}_{o,t}^* - p_t^*) - \nu a_t^* \tag{3.69}$$

Using the Euler equation yields:

$$mc_t^* = \nu(\sigma c_t^* + \varphi n_t^* + (\sigma - \sigma_\delta)g_t^*) + (1-\nu)\tilde{p}_{o,t}^* - \nu a_t^* \tag{3.70}$$

Using the production function, and the cost minimising equation yields:

$$mc_t^* = \frac{\nu\sigma + \nu\varphi}{1+(1-\nu)\varphi} y_t^* - \frac{\nu(1+\varphi)}{1+(1-\nu)\varphi} a_t^* + \frac{(1-\nu)(1+\varphi)}{1+(1-\nu)\varphi} \tilde{p_{o,t}}^* + \frac{\nu(\sigma-\sigma_\delta)}{1+(1-\nu)\varphi} g_t^* \tag{3.71}$$

Equating mc_t^* to the steady-state markup $(-\mu^*)$ and solving for output, yields the natural rate of output equation in the foreign economy:

$$\bar{y_t}^* = -\frac{1+(1-\nu)\varphi}{\nu\sigma+\nu\varphi}\mu^* + \frac{\nu(1+\varphi)}{\nu\sigma+\nu\varphi} a_t^* - \frac{(1-\nu)(1+\varphi)}{\nu\sigma+\nu\varphi}\tilde{p}_{o,t}^* - \frac{\nu(\sigma-\sigma_\delta)}{\nu\sigma+\nu\varphi} g_t^* \tag{3.72}$$

Subtracting the above two equations from each other yields the deviation of the marginal cost as a function of the output gap:

$$\hat{mc}_t^* = \frac{\nu\sigma+\nu\varphi}{1+(1-\nu)\varphi} x_t^* \tag{3.73}$$

Adding this to the NKPC equation gives us inflation as a function of the output gap:

$$\hat{\pi}_t^* = \beta E_t[\hat{\pi}_{t+1}^*] + \lambda \frac{\nu\sigma+\nu\varphi}{1+(1-\nu)\varphi} x_t^* \tag{3.74}$$

The Demand Side of The World Economy

Moving to the demand side of the foreign economy, we add the log-form of the Euler equation to the market clearing equation ($c_t^* = y_t^*$) to get:

$$y_t^* = E_t\{y_{t+1}^*\} - \frac{1}{\sigma_\delta}[r_t^* - E_t\{\pi_{t+1}^*\}] + \frac{\sigma - \sigma_\delta}{\sigma_\delta}\Delta E_t\{g_{t+1}^*\} \tag{3.75}$$

Solving the above IS curve for the output gap yields:

$$x_t^* = E_t\{x_{t+1}^*\} - \frac{1}{\sigma_\delta}[r_t^* - E_t\{\pi_{t+1}^*\} - \bar{rr}_t^*] \tag{3.76}$$

Where:

$$\begin{aligned}\bar{rr}_t^* &= \sigma_\delta \Delta \bar{y}_{t+1}^* + (\sigma - \sigma_\delta)\Delta g_{t+1}^* \\ &= -\frac{\sigma_\delta(1-\rho_a)(1+\varphi)\nu}{\nu(\varphi+\sigma)}a_t^* + \frac{\nu(\sigma-\sigma_\delta)(\sigma+\varphi-\sigma_\delta)}{\nu(\varphi+\sigma)}\Delta g_{t+1}^* - \frac{\sigma_\delta(1-\nu)(1+\varphi)}{\nu(\varphi+\sigma)}\Delta \tilde{p}_{o,t+1}^*\end{aligned} \tag{3.77}$$

In the above equation, similar to the natural rate of interest in the domestic economy, the natural rate of interest in the foreign economy also reacts to expected changes in the price of commodities.

Commodities Market Equilibrium

The supply of commodities is assumed to follow an AR(1) process:

$$\frac{O_t^{*s}}{O^{*s}} = \left\{\frac{O_{t-1}^{*s}}{O^{*s}}\right\}^{\rho_o^*} exp(\zeta_{o^*,t}) \tag{3.78}$$

Where $0 < \rho_o^* < 1$ is the autocorrelation parameter of the supply of commodities, and it captures the persistence of commodity supply. $\zeta_{o^*,t}$ represents an i.i.d. commodity supply shock with constant variance $\sigma_{o^*}^2$.

We solve for the demand of commodities from the cost minimisation equation and by plugging the value of the equilibrium level of employment to derive the demand of commodities in the world economy as a function of the world output, world technology, world government consumption, and real commodity prices:

$$o_t^{*d} = \frac{\nu\sigma+\varphi+1}{1+\varphi(1-\nu)}y_t^* - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)}a_t^* + \frac{\nu(\sigma-\sigma_\delta)}{1+\varphi(1-\nu)}g_t^* - \frac{\nu}{1+\varphi(1-\nu)}\tilde{p}_{o,t}^* \tag{3.79}$$

Using the commodities market equilibrium condition ($o_t^{*s} = o_t^{*d}$), the above equation can be solved for the equilibrium real commodity price:

$$\tilde{p}^*_{o,t} = \frac{\nu\sigma + \varphi + 1}{\nu} y^*_t - (1+\varphi)a^*_t + (\sigma - \sigma_\delta)g^*_t - \frac{1+\varphi(1-\nu)}{\nu} o^{*s}_t \tag{3.80}$$

The above equation illustrates how the real prices of commodities are driven by demand and supply factors in the world economy. Government consumption and world output are demand factors that have a positive effect on real commodity prices. Conversely, the supply of commodities and the world technology are supply factors that negatively affect the real prices of commodities.

3.3.6 Calibrated Parameters and Moments of the Model

Baseline Calibration

In this section, we illustrate the baseline calibration of the above model (table shown in Appendix C.4). The parameters set is divided into two sections. The first section illustrates the parameters that this model adopts from the standard literature[18], and the second section highlights the parameters that are specific to this model.

In the first section of the parameters set, we set θ equal to 0.75, which implies that firms only change their prices once a year. Our discount factor β is equal to 0.99. This implies that, given that $\beta = 1/r$ at the steady state, annual return is approximately equal to 4 percent. We set φ equal to 3, under the assumption that the labour supply elasticity is $\frac{1}{3}$. We set ϕ_π & ϕ_x equal to 1.5 and 0.5 following Taylor (1993). The size of household's private consumption in the aggregate consumption bundle δ equal to 0.95. The share of non-commodity inputs in the production functions are set to 0.95. The inverse elasticity of intertemporal substitution of consumption σ is set equal to 1, which implies log utility in consumption. The elasticity of substitution between domestic and foreign produced goods η is set to 1. This elasticity describes the change in consumption of imported goods in response to changes in the prices of foreign goods relative to domestic prices. The value of the parameter implies that demand of imported goods increases by exactly 1 % when the relative price of foreign goods declines by 1 %. The elasticity of substitution between the domestically produced goods ε equals 6 which corresponds to a steady state markup of 1.2. As for the standard deviations and persistence of the interest rates and productivity shock processes, we use the ones used by Smets & Wouters (2007) and Gali & Monacelli (2005), respectively.

As for the second section, the shares of foreign goods in the private consumption baskets of the domestic economies α are set equivalent to the average share of import to GDP over the sample period (1980-2015). The standard deviation and the persistence of the commodity supply variable are calculated by fitting an AR (1) model for the supply of energy which was extracted from the International Energy Agency's database. As for the rest of the standard deviations and persistence of the other shock processes, they were calculated in Table (3). The responses of government consumption to changes in commodity prices ϕ_g are adopted from the estimates shown in Table (4). This implies that a few of the results, which are based on the baseline calibration, will be inconsistent

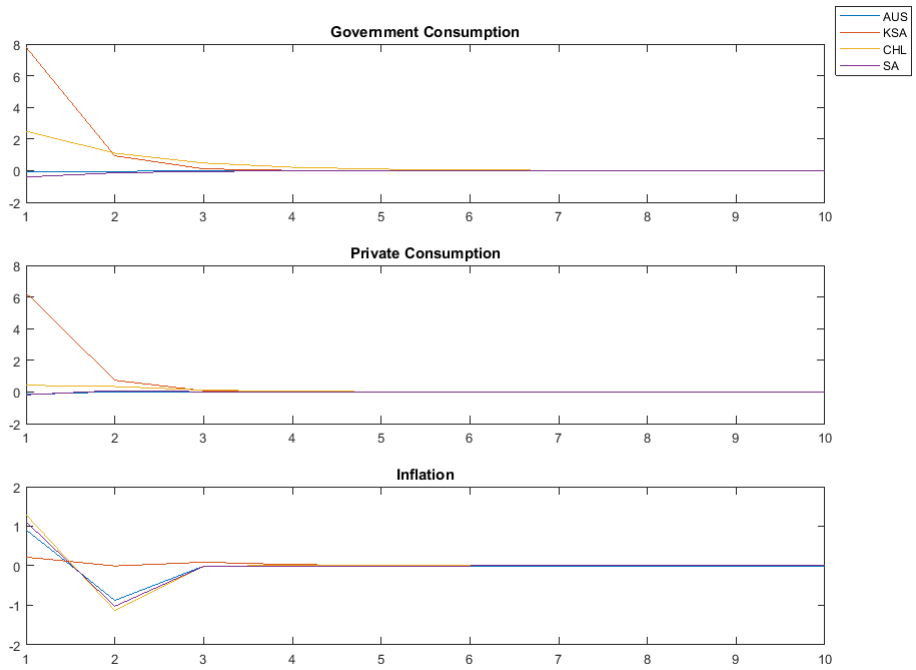
[18] See Taylor (1993), Gali & Monacelli (2005), Smets & Wouters (2007) and Shousha (2016)

with the results obtained in the structural VAR estimations. Nevertheless, the Bayesian estimations will be decisive in this matter.

The inverse elasticities of substitution between government consumption and private consumption for each of the five economies were all calibrated to values that generate responses of private consumption to changes in government consumption that are equivalent to the ones estimated in Table (2). As for the share of commodities consumption in the private consumption bundle, we use the share of energy consumption in the CPI basket from the OECD.stat database. In this regard, this share was not available for Saudi Arabia. Therefore, we use the lowest share of commodities in the KSA CPI given the generous subsidies scheme that was implemented in the country during our sample period, as highlighted by Abusaaq (2015).

Moments of the Model

Figure 3.4 Response to a Commodity Shock



In the above graph, we show how the domestic economies react to a commodity shock under the model structure. In this regard, we add a commodity i.i.d shock to our framework specifically for this part of the analysis[19]. The graph illustrates how the four economies react in a different manner to a commodity shock, similar to all of the results above. It also shows how government consumption in

[19]The shock process for the average commodity index was constructed by fitting the series to an AR(1) model to capture the persistence of the index (0.9).

Saudi Arabia, an oil-rich economy, reacts to the shock more than its counterparts and how inflation in Saudi Arabia moderately reacts to the commodity shocks given the low share of commodities in the Saudi private consumption basket[20].

Nevertheless, the model seems to overstate the reaction of the Chilean government consumption to the commodity shock. In addition, government consumption in South Africa behaves countercyclically which also contradicts with the Structural VAR estimations. These differences reflect the difference between the previous estimations, as noted above. Nevertheless, we will re-estimate each of the behaviour parameters below using Bayesian estimation techniques. In addition, the below table shows that the theoretical moments, under the seven imposed shocks of the model, qualitatively resemble the moments found in the data.

Table 3.5 Theoretical Moments of the Model

KSA	Gov. Consumption	Private Consumption	Inflation
Std. Deviation	1.01	1.27	0.77
Persistence	0.12	0.34	0.26
Correlation with Gov. Consumption	1.00	0.64	0.01
Correlation with Private consumption	0.64	1.00	0.08
Correlation with inflation	0.01	0.08	1.00
CHL	Gov. Consumption	Private Consumption	Inflation
Std. Deviation	1.11	0.75	0.41
Persistence	0.44	0.16	-0.18
Correlation with Gov. Consumption	1.00	0.32	0.01
Correlation with Private consumption	0.32	1.00	-0.04
Correlation with inflation	0.01	-0.04	1.00
SA	Gov. Consumption	Private Consumption	Inflation
Std. Deviation	1.08	0.78	0.46
Persistence	0.36	0.61	-0.09
Correlation with Gov. Consumption	1.00	0.19	-0.00
Correlation with Private consumption	0.19	1.00	0.00
Correlation with inflation	0.00	0.00	1.00
AUS	Gov. Consumption	Private Consumption	Inflation
Std. Deviation	1.11	1.10	0.51
Persistence	0.44	0.46	0.23
Correlation with Gov. Consumption	1.00	0.71	0.10
Correlation with Private consumption	0.71	1.00	0.02
Correlation with inflation	0.10	0.02	1.00

[20] We show the theoretical effect of all the seven shock of the model on our variables of interest for all of the four economies in Appendix C.4.

3.4 Estimation

Our Bayesian estimations in this section are conducted using three observable variables for each of our domestic economies in addition to the commodities index. All observables are directly mapped to variables in the structural model using the following equations:

$$\Delta lnG_t^{obs} = lnG_t - lnG_{t-1} + Trend_g \tag{3.81}$$

$$\Delta lnC_t^{obs} = lnC_t - lnC_{t-1} + Trend_c \tag{3.82}$$

$$\pi_t^{obs} = \pi_t + Trend \tag{3.83}$$

$$p_{o,t}^{obs} = p_{o,t} \tag{3.84}$$

The Bayesian estimations are conducted on our selected parameters using an MCMC algorithm to obtain draws from the marginal posterior distribution of the parameters[21]. We estimate the stochastic processes of each of the exogenous disturbances of the model, along with the parameter that governs the response of government consumption to changes in the commodity prices ϕ_g and the parameter that shows the effect of government consumption on private consumption χ, as shown in the below table. The estimation of these two parameters using the data allows us to capture the size of these two parameters within the framework of our model.

In this regard, we use the calibrated values for χ and ϕ_g as the prior values for those two parameters while obtaining the values of the standard deviation from the regression results of this paper for ϕ_g. As for χ, we used standard deviations that are equivalent to the standard deviations of the regression results in percentage terms. Moreover, we impose a non-negativity assumption on χ by assuming an inverse gamma prior distribution. As for ϕ_g, we assume a prior normal distribution in order to give the parameter the freedom to move between negative and positive values. In addition, the prior values and the standard deviations for the stochastic processes of all the shocks were taken from Smets & Wouters (2007). Also, we impose the same prior values and standard deviations for all the shocks to have the same relative importance for all the shocks at the starting point.

[21] We take 1,000,000 draws with an acceptance rate of 32.4 for Chile, 34.2 for Saudi Arabia, 34.5 for Australia, and 33.3 for South Africa. We also discard 25 percent of the draws and keep the remaining ones for inference.

Table 3.6 Estimated Parameters

Parameter	Prior	Mean	std. dev.
AUS			
χ	Inverse-Gamma	67.6	12.82
ϕ_g	Normal	-0.01	0.017
CHL			
χ	Inverse-Gamma	8.19	4.84
ϕ_g	Normal	0.25	0.048
SA			
χ	Inverse-Gamma	5.3	2.266
ϕ_g	Normal	-0.04	
KSA			
χ	Inverse-Gamma	102.4	19.42
ϕ_g	Normal	0.78	0.14
ρ_i	Beta	0.5	0.2
σ_i	Inverse-Gamma	0.05	2
	$i = a, a^*, g^*, o, g, r, r^*$		

Table 3.7 Historical Decomposition

KSA							
Observed Variable	ε_a	ε_{a^*}	ε_{o^*}	ε_g	ε_{g^*}	ε_r	ε_{r^*}
ΔlnG_t^{obs}	0.00%	18.86%	0.00%	67.32%	5.92%	0.00%	7.89%
ΔlnC_t^{obs}	0.00%	28.70%	0.00%	35.67%	12.61%	0.00%	23.03%
π_t^{obs}	0.00%	44.97%	0.00%	3.10%	44.71%	0.00%	7.22%
CHL							
Observed Variable	ε_a	ε_{a^*}	ε_{o^*}	ε_g	ε_{g^*}	ε_r	ε_{r^*}
ΔlnG_t^{obs}	15.48%	0.85%	0.00%	57.37 %	0.00%	5.68%	20.63%
ΔlnC_t^{obs}	55.95%	0.23%	0.00%	0.21%	0.00%	40.54%	3.08%
π_t^{obs}	13.35%	0.21%	0.00%	0.00 %	0.00%	85.85%	0.59%
AUS							
Observed Variable	ε_a	ε_{a^*}	ε_{o^*}	ε_g	ε_{g^*}	ε_r	ε_{r^*}
ΔlnG_t^{obs}	0.00%	3.35%	0.00%	94.67%	0.00%	0.04%	1.94%
ΔlnC_t^{obs}	0.00%	22.70%	0.00%	5.89%	0.01%	1.96%	69.44%
π_t^{obs}	0.00%	26.93%	0.00%	0.27%	0.01%	53.66%	19.12%
SA							
Observed Variable	ε_a	ε_{a^*}	ε_{o^*}	ε_g	ε_{g^*}	ε_r	ε_{r^*}
ΔlnG_t^{obs}	2.25%	0.00%	0.00%	84.15%	0.00%	0.27%	13.33%
ΔlnC_t^{obs}	74.20%	0.00%	0.00%	0.20%	0.00%	17.74%	7.86%
π_t^{obs}	20.40%	0.00%	0.00%	0.01%	0.00%	76.50%	3.09%

The estimation results (shown in Appendix C.5) indicate no significant change in the elasticity of substitution between government consumption and private consumption χ for each of the four

economies from the baseline calibrated values. These results support the literature that shows the crowding in effect of government consumption on private consumption in our selected economies. As for the response of government consumption to changes in the commodity index ϕ_g, the response of the Australian government consumption is in line with the baseline calibrations, indicating a countercyclical fiscal stance. As for the Chilean government consumption, the results show a significant drop in ϕ_g, contradicting the results shown in Table (4) but supporting the SVAR impulse responses and are backed by the adopted fiscal rule by the Chilean government. The results for ϕ_g of the South African economy indicate a change in the response from a negative value of -0.04 to a positive response of 0.032, which is also in line with the SVAR impulse responses, and indicating a procyclical fiscal stance by the South African government. Lastly, the response of ϕ_g in the Saudi economy is reduced to 0.13, but it is not robust for changes in the standard deviation, and it is biased towards increasing whenever the standard deviation is increased. Overall the posterior densities are considerably different from the loose priors that we choose, implying that the data is informative regarding this estimated parameter. As a robustness check, we re-estimate the model while increasing the standard deviations of the structural parameters by 50 %. The robustness checks show that the values of the structural parameters for all four economies are not sensitive to these changes in the standard deviations. The only exception to this, as noted above, is the response of the Saudi government consumption to changes in the commodity price index. The response increased from 0.13 to 0.22, which is still the highest among the government response of all selected economies.

The above results, which are based on an infinite horizon forecast-error variance decomposition, show some results which are worth highlighting. First, the results show no role for shocks in the supply of commodities on any of the domestic variables across all four economies. This is also shown in the posterior estimation of the standard deviations of the supply of commodities across all economies (see Appendix C.5). Second, domestic interest rates explain the behaviour of all the domestic variables except in the Saudi economy. This is apparently a result of the policy crowding-out effect of fiscal policy on monetary policy. In the other three economies, interest rates explain a significant percentage of domestic CPI inflation, showing an indication of an active monetary policy stance in these economies. Third, foreign government consumption has no effect on all economies except for the Saudi economy in this case as well, where foreign government consumption has a significant effect on all of the domestic variables of the Saudi economy. Lastly, the behaviour of private consumption and CPI inflation in Saudi Arabia are mostly explained by external shocks, unlike the two variables in Chile and South Africa which are mainly explained by domestic factors. The behaviour of those two variables in Australia, on the other hand, shows dominance in the effect of external shocks. Nevertheless, the Australian economy, unlike its counterparts, has a well-developed financial market which makes it more linked to the rest of the world economy.

3.5 Conclusion

The previous literature on commodity-rich economies has always assumed that this group of countries was homogeneous and that the only source of heterogeneity among those countries comes from the difference in the volatility of the prices of commodities. This paper sought to investigate the heterogeneity among commodity-rich countries beyond the prices of commodities. We achieved this by imposing the same commodity-price index on four economies which are rich with different types of commodities. We build a model that nests different sources of shocks that were proposed in the previous literature and add a central role for commodities in the model. Our model allows for a quadruple role for commodities. First, the domestic government collects the windfalls of selling commodities to the rest of the world. Second, commodities are consumed by households both in the domestic economy and the foreign economy. Third, firms both in the domestic economy and the foreign economy use commodities as an input in their production. Lastly, the domestic economy is affected by the second-round effect of an increase in the commodity prices in the form of high foreign inflation and low world demand. Government consumption is included in the utility function as a complement to private consumption in a non-separable form. This results in comovement between government consumption and private consumption and it generates volatility in private consumption which is qualitatively similar to the one shown in the data.

We focus on two key behavioural parameters in the model: the elasticity of substitution between government consumption and private consumption and the response of government consumption to changes in the prices of commodities. Our results show that government consumption in all four economies has a crowding in effect on private consumption with differing degrees of complementarity. These results support the findings of Bouakez & Rebei (2007) and Gali et al. (2007) who show similar results to ours. In addition, the crowding in effect of government consumption on private consumption affects the transmission mechanism of monetary policy and leads to a policy crowding out effect of fiscal policy towards monetary policy, as in the case of Saudi Arabia. The response of government consumption to changes in commodity prices, on the other hand, was an indication of two essential factors. The first one is the fiscal stance of government consumption (Institutional factors), while the second one is the size of natural resources rents in the domestic economy. The latter shows discrepancy across commodity-rich countries. The data (shown in Appendix C.3) and our estimations show that, given the significantly larger share of oil rents relative to other commodities, oil-rich countries are more vulnerable to external shocks.

The model is estimated using Australian, Chilean, Saudi and South African data from 1980 to 2015. Doing so allows us to evaluate the contribution of different sources of shock in the behaviour of the model's three domestic variables over a long time horizon. The results show that our key variables in Saudi Arabia, an oil-rich country, were more vulnerable to developments in the foreign economy than its developing counterparts, namely Chile and South Africa. The macro variables in Australia also appeared to be affected by developments in the foreign economy, but this is mainly attributed to its developed financial sector which is highly linked to the rest of the world economy. Nevertheless,

Australia, despite being a developed country, showed higher fluctuations in private consumption relative to output, a characteristic which is normally assigned to developing and emerging economies.

The main results of the paper illustrate that the proposed solutions for commodity-rich economies by various articles in the literature, which are based on a "quintessential" single economy, should be read with caution.

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Appendix A

Appendices For Chapter One

Log-linearisation

- Aggregate consumption bundle:

$$\bar{c}_t = \delta c_t + (1-\delta) g_t \tag{A.1}$$

- IS curve :

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\delta}[r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t] \tag{A.2}$$

- Natural rate of interest:

$$\bar{rr}_t = -\frac{\sigma_\delta(1-\rho_a)(1+\varphi)}{\varphi+\sigma_\delta} a_t + -\frac{((\sigma-\sigma_\delta)(1-\rho_g)\varphi}{\varphi+\sigma_\delta} g_t \tag{A.3}$$

- Phillips curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(\varphi+\sigma_\delta) x_t \tag{A.4}$$

- Flexible-price output:

$$\bar{y}_t = -\Big(\frac{\sigma-\sigma_\delta}{\varphi+\sigma_\delta}\Big) g_t + \Big(\frac{1+\varphi}{\varphi+\sigma_\delta}\Big) a_t \tag{A.5}$$

- Output gap:

$$x_t = y_t - \bar{y}_t. \tag{A.6}$$

- Production function:

$$y_t = a_t + n_t. \tag{A.7}$$

- Labour supply:

$$w_t - p_t = \varphi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t. \tag{A.8}$$

- Monetary policy:

$$\begin{cases} r_t = \overline{rr_t} + \phi_\pi \pi_t + \phi_x x_t, & \text{Optimal policy,} \\ r_t = \phi_\pi \pi_t + \phi_x x_t, & \text{Taylor rule,} \\ r_t = \rho + \phi_\pi \pi_t, & \text{CPI inflation targeting,} \end{cases} \tag{A.9}$$

- Market clearing condition:

$$y_t = c_t \tag{A.10}$$

- Exogenous process:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{A.11}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}. \tag{A.12}$$

Appendix A.2

To understand the inflation dynamics in the model, we start by analysing the price-setting behaviour of firms. We follow the steps of Gali & Monacelli (2005), and the 3rd chapter of Gali (2008) to derive the price-setting behaviour of firms in the model under a sticky prices framework. The aggregate domestic price index in the model is a weighted average of prices that have been adjusted at period t and prices that have not been adjusted:

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_t)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.13}$$

$\bar{P}_t$ is the re-optimised price that a fraction of the firms $(1-\theta)$ choose at period t, and this is normally higher than the prevailing price during the last period before. P_{t-1} is the price imposed by the other fraction of firms who have not been able to adjust their prices, and this is why we keep last period's prices as the prevailing prices for those firms. We divide the above equation by P_{t-1} to get:

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\bar{P}_t}{P_{t-1}}\right)^{1-\varepsilon} \tag{A.14}$$

Log-linearising the above equation around a steady state with zero inflation yields[1]

$$\pi_t = (1-\theta)(\bar{p}_t - p_{t-1}) \tag{A.15}$$

In the above equation, inflation at the current period is affected by the price adjustment that a fraction of the firms in the economy makes to their prices. Therefore, as mentioned above, we start deriving the price-setting behaviour of firms to capture the dynamics of prices in the economy. When firms set their prices according to Calvo (1983) contract scheme, they aim to maximise the expected discounted value of their profits under the assumption that the newly set price will still be effective:

$$max_{\bar{P}_t} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} [c_{jt+k|t} (\bar{P}_t - \Psi_{t+k})] \} \tag{A.16}$$

Ψ is the cost function, θ^k is the probability that the re-optimised price at period t will remain effective at period t+k, and $Q_{t,t+k}$ is a the discount factor of nominal pay off and it is defined in equation (1.9). $c_{jt+k|t}$ is the Expected demand/production for period t+k at period t. The equation is subject to the following demand constraint: $c_{jt+k} = \left(\frac{\bar{P}_t}{P_{t+k}}\right)^{-\varepsilon} C_t$. Plugging in the demand function into the firm's maximisation problem yields:

$$max_{\bar{P}_t} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} [\left(\frac{\bar{P}_t}{P_{t+k}}\right)^{-\varepsilon} C_{t+k} (\bar{P}_t - \Psi_{t+k})] \} \tag{A.17}$$

Taking the first order condition of the above equation yields:

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} C_{t+k} [\bar{P}_t - \mathscr{M} \psi_{t+k|t}] \} = 0 \tag{A.18}$$

ψ is the nominal marginal cost, and $\mathscr{M}$ is the gross mark-up and its equal to $\frac{\varepsilon}{\varepsilon-1}$. Now, we divide the above equation by $P_{H,t-1}$ and divide and multiply the second term by $P_{H,t+k}$:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} C_{t+k} \left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathscr{M} MC_{t+k|t} \Pi^H_{t-1,t+k} \right] \right\} = 0 \tag{A.19}$$

Where $\Pi^H_{t-1,t+k} = \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{H,t+k}}$. We log-linearise the above equation around a zero-inflation steady state. Noting that $Q_{t,t+k}$ in the steady state will equal β^k:

$$\bar{p}_t - p_{t-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + \mu + (p_{t+k} - p_{t-1}) \} \tag{A.20}$$

We notice from the above equation that the firms discount the expected stream of their future profits using the household's discount factor. This is simply attributed to the fact that the households are the

[1] Log-linearising around a steady state of zero inflation allows us to get rid of the price dispersion created by the nominal friction in the model.

share holders of those firms. Rearranging the above equation gives:

$$\bar{p}_t = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{t+k}\} \tag{A.21}$$

The above equation is describing how firm set their prices with a certain mark-up and the discounted present value of the stream of marginal costs, and it is the one we use in the text. In the case when $\theta = 0$, all firms will be able to adjust their prices in each period (flexible prices scheme), and the above equation will simplify to:

$$\bar{p}_t = \mu + mc_t \tag{A.22}$$

The price the firms set in this case is equal to their markup over the nominal marginal cost. Of course, this shows that the price set by the firms is above their marginal cost since the markup is greater than 1. As a result, output will be lower than its level under perfect competition. It will be shown how the government can offset this distortion by giving the firms a certain employment subsidy. Now going back to equation (A.20), if we rewrite down the equation in a compact form we get:

$$\bar{p}_t - p_{t-1} = \beta\theta E_t\{p_{t+1} - p_t\} + \pi_t + (1-\beta\theta)\hat{mc}_t \tag{A.23}$$

Where $\hat{mc}_t = mc_t - p_t + \mu$. Adding the above equation to the price setting equation gives us:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa\hat{mc}_t \tag{A.24}$$

Where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The above equation is the core New Keynesian Phillips Curve. We develop it in the text to link inflation to the output gap through the relationship between the $\hat{mc}$ and the output gap (x_t). κ in the Phillips curve equation is strictly decreasing in the stickiness parameter θ. From the above equation, we see that inflation in this type of models is a result of aggregate price-setting of the firms who adjust their prices based on current and future stream of their marginal costs.

Appendix A.3

The utility function of the representative household is:

$$E_0\sum_{t=0}^{\infty}\beta^t U(\bar{C}_t, N_t) \tag{A.25}$$

The aggregate consumption bundle is represented as follows:

$$\bar{C}_t = \left[\delta^{\chi}C_t^{1-\chi} + (1-\delta)^{\chi}G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{A.26}$$

The above utility function is separable in consumption and hours worked ($U_{CN} = 0$). But it is not separable in consumption and government expenditure ($U_{CG} \neq 0$). The quadratic approximation of the above utility function will be:

$$\begin{aligned}
U_t - U =& U_c C\Big(\frac{C_t - C}{C}\Big) + U_N N\Big(\frac{N_t - N}{N}\Big) + \frac{1}{2}U_{cc}C^2\Big(\frac{C_t - C}{C}\Big)^2 + \frac{1}{2}U_{NN}N^2\Big(\frac{N_t - N}{N}\Big)^2 \\
&+ U_{CG}CG\Big(\frac{C_t - C}{C}\Big)\Big(\frac{G_t - G}{G}\Big) + o(||a||^3) + t.i.p \\
=& U_c C(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + U_N N(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2}U_{cc}C^2(\hat{c}_t^2) + \frac{1}{2}U_{NN}N^2(\hat{n}_t^2) \\
&+ U_{CG}CG(\hat{c}_t + \frac{1}{2}\hat{c}_t^2)(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) + o(||a||)^3 + t.i.p
\end{aligned} \tag{A.27}$$

$t.i.p$ is terms independent of monetary policy. dividing both sides by $U_c C$ yields:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& (\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + \frac{U_N N}{U_c C}(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2}\frac{U_{cc}C}{U_c}(\hat{c}_t^2) + \frac{1}{2}\frac{U_{NN}N^2}{U_c C}(\hat{n}_t^2) \\
&+ \frac{U_{CG}G}{U_c}(\hat{c}_t + \frac{1}{2}\hat{c}_t^2)(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) + o(||a||^3 + t.i.p \\
=& (\hat{c}_t + \frac{1}{2}\hat{c}_t^2 + \frac{1}{2}\frac{U_{cc}C}{U_c}\hat{c}_t^2) + \frac{U_N N}{U_c C}(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2}\frac{U_{NN}N^2}{U_c C}(\hat{n}_t^2) \\
&+ \frac{U_{CG}G}{U_c}(\hat{c}_t\hat{g}_t) + o(||a||^3 + t.i.p \\
=& \hat{c}_t + \Big[\frac{1}{2} + \frac{1}{2}\frac{U_{cc}C}{U_c}\Big]\hat{c}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{n}_t + \Big(\frac{1}{2} + \frac{1}{2}\frac{U_{NN}N^2}{U_c C}\Big)\hat{n}_t^2\Big] \\
&+ \frac{U_{CG}G}{U_c}(\hat{c}_t\hat{g}_t) + o(||a||^3 + t.i.p
\end{aligned} \tag{A.28}$$

Now, to simplify the above equations, we use the first and second order derivatives of the utility function with respect to consumption, labour, and government expenditure:

$$\begin{aligned}
U_C &= \frac{\partial U}{\partial C} = \bar{C}^{-\sigma} \\
U_{CC} &= \frac{\partial^2 U}{\partial C^2} = -\sigma\bar{C}^{-\sigma-1} \\
U_N &= \frac{\partial U}{\partial N} = -N^{\varphi} \\
U_{NN} &= \frac{\partial^2 U}{\partial N^2} = -\varphi N^{\varphi-1} \\
U_{CG} &= \frac{\partial^2 U}{\partial C \partial G} = -\sigma\bar{C}^{-\sigma-1}
\end{aligned} \tag{A.29}$$

Plugging the above derivations in the second-order derivations yields:

$$\frac{U_t - U}{U_c C} = \hat{c}_t + \Big[\frac{1 - \sigma\delta}{2}\Big]\hat{c}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{n}_t + \Big(\frac{1 + \varphi}{2}\Big)\hat{n}_t^2\Big] - \sigma(1 - \delta)(\hat{c}_t\hat{g}_t) + o(||a||^3) + t.i.p \tag{A.30}$$

Plugging in the value of consumption by using the domestic market's clearing condition , and the fact that we have a log-utility in consumption $\sigma = 1$ we get:

$$\frac{U_t - U}{U_c C} = \hat{y}_t + \Big[\frac{1-\delta}{2}\Big]\hat{y}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{n}_t + \Big(\frac{1+\varphi}{2}\Big)\hat{n}_t^2\Big] - (1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \tag{A.31}$$

We next turn our attention to the hours worked variable (n_t). From the production function and the price dispersion equation we get: $\hat{n}_t = \hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\}$, following the work of Gali (2008). Plugging this in the above equation yields:

$$\begin{aligned}\frac{U_t - U}{U_c C} =& \hat{y}_t + \Big[\frac{1-\delta}{2}\Big]\hat{y}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_t(j)\} \\ &+ \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\})^2\Big] - (1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \\ =& \hat{y}_t + \Big[\frac{1-\delta}{2}\Big]\hat{y}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{y}_t + \frac{\varepsilon}{2} var_j\{p_t(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t + a_t)^2\Big] \\ &- (1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p\end{aligned} \tag{A.32}$$

Under the optimal subsidy scheme, the following condition holds: $\frac{U_N N}{U_c C} = -1$. Using this condition in the above equation yields:

$$\begin{aligned}\frac{U_t - U}{U_c C} =& \hat{y}_t + \Big[\frac{1-\delta}{2}\Big]\hat{y}_t^2 - \Big[\hat{y}_t + \frac{\varepsilon}{2} var_j\{p_t(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t + a_t)^2\Big] \\ &- (1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \\ =& \Big[\frac{(1-\delta)}{2}\Big]\hat{y}_t^2 - \Big[\frac{\varepsilon}{2} var_j\{p_t(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2a_t\hat{y}_t + a_t^2)\Big] \\ &- (\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \\ =& \Big[\frac{(1-\delta)}{2}\Big]\hat{y}_t^2 - \Big[\frac{\varepsilon}{2} var_j\{p_t(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2a_t\hat{y}_t)\Big] \\ &- (\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p\end{aligned} \tag{A.33}$$

Using the fact that $\hat{y}_t^n = a_t$:

$$\frac{U_t - U}{U_c C} = (1-\delta)\hat{y}_t^2 - \Big[\frac{\varepsilon}{2} var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2\hat{y}_t^n\hat{y}_t)\Big] - (1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \tag{A.34}$$

In the above equation, I will use one simple trick to simplify the following term: $\hat{y}_t^2 - 2\hat{y}_t^n\hat{y}_t$. The term is simplified to: $(\hat{y}_t - \hat{y}_t^n)^2 - \hat{y}_t^{n^2}$. Where $\hat{y}_t^{n^2}$ eliminates the additional third term that we get from the quadratic expression. The same also applies to $\frac{(1-\delta)}{2}\hat{y}_t^2 - (1-\delta)(\hat{y}_t\hat{g}_t)$. Doing so will simplify the above equation to:

$$\begin{aligned}\frac{U_t - U}{U_c C} &= -\left[\frac{\varepsilon}{2} var_j\{p_t(j)\} + \left(\frac{1+\varphi}{2}\right)[(\hat{y}_t - \hat{y}_t^n)^2 - \hat{y}_t^{n^2}]\right] \\ &+ \left[\frac{(1-\delta)}{2}\right]\left[(\hat{y}_t - \hat{g}_t)^2 - \hat{g}_t^2\right] + o(||a||^3) + t.i.p \\ &= -\frac{1}{2}\left[\varepsilon.var_j\{p_t(j)\} + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\right] + o(||a||^3) + t.i.p\end{aligned} \tag{A.35}$$

We can write the above equation as the discounted sum of all periods:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{U_t - U}{U_c C}\right] = -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left[\varepsilon.var_j\{p_t(j)\} + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\right] + o(||a||^3) + t.i.p \tag{A.36}$$

Following the work of Woodford (2001), we make use of the following Calvo property:

$$\sum_{t=0}^{\infty} \beta^t \varepsilon.var_j\{p_t(j)\} = \frac{1}{\lambda}\sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2. \tag{A.37}$$

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{U_t - U}{U_c C}\right] = -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\lambda}\pi_t^2 + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\right] + o(||a||^3) + t.i.p \tag{A.38}$$

The above second-order approximation shows the utility loss resulting from deviation from the steady state (optimal) policy, and it is represented as a fraction of the steady-state consumption of the representative household. The per-period welfare-loss function is:

$$L = -\frac{1}{2}\left[\frac{\varepsilon}{\lambda} var.\pi_{H,t} + (1+\varphi) var.\hat{x}_t + (1-\delta) var.(\hat{g}_t - \hat{y}_t)\right] \tag{A.39}$$

Appendix A.4

Following Bullard & Mitra (2002), we try to explain how determinacy of monetary policy rules is derived from the below matrix:

$A = \Omega \begin{bmatrix} \sigma_\delta & (1-\beta\phi_\pi) \\ \kappa_g\sigma_\delta & \beta(\sigma_\delta+\phi_x)+\kappa_g \end{bmatrix}$ and $\Omega = \frac{1}{\sigma_\delta+\phi_x+\kappa_g\phi_\pi}$.

The characteristic polynomial of the above matrix is:

$$p(\lambda) = \lambda^2 - \lambda a_1 + a_2 \tag{A.40}$$

For the eigenvalues of matrix A to be inside the unit circle, the following conditions have to be met:

$$|a_2| < 1 \tag{A.41}$$

$$|a_1| < 1 + |a_2| \tag{A.42}$$

Where a_1 is the trace of matrix A and a_2 is the determinant of the same matrix:

$$a_2 = \frac{\beta\sigma_\delta}{\sigma_\delta+\phi_x+\kappa_g\phi_\pi} \tag{A.43}$$

The determinant of this matrix should satisfy the first condition: $\beta\sigma_\delta < \sigma_\delta+\phi_x+\kappa_g\phi_\pi$. This rule is easily satisfied since $0 < \beta < 1$ and given the fact that all of the parameters have positive values. The trace of the matrix a_1 is:

$$a_1 = \frac{-(\sigma_\delta+\beta\sigma_\delta+\beta\phi_x+\kappa_g)}{\sigma_\delta+\phi_x+\kappa_g\phi_\pi} \tag{A.44}$$

From the second condition, the following inequality must hold for the eigenvalues of the matrix to lie inside the unit circle: $\kappa_g(\phi_\pi-1)+(1-\beta)\phi_x > 0$. The inflation parameter has to be greater than 1 for this rule to hold along with the first condition.

Appendix B

Appendices For Chapter Two

Appendix B.1

- Derivation of the natural rate of output:

We start the derivation of the natural rate of output from the log form of the marginal cost equation:

$$\begin{aligned} mc &= -v + w_t - p_{H,t} - a_t \\ &= -v + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -v + \varphi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t + \alpha s_t - a_t \\ &= -v + \varphi y_t + \sigma_\delta y_t^* + (\sigma - \sigma_\delta) g_t^* + s_t - (1+\varphi) a_t \end{aligned} \tag{B.1}$$

In the above system of equations we made use of the log form of the domestic economy's labour supply equation (eq.2.9), the CPI index equation (eq.2.23), and the international risk sharing condition (eq.2.33). Plugging in the value of the terms of trade from the market clearing condition of the domestic economy (eq.2.40) yields:

$$mc_t = \Big(\frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha}\Big) y_t + \Big(\frac{\sigma_\delta(\omega_\alpha - 1)}{\omega_\alpha}\Big) y_t^* + \Big(\frac{\alpha\omega(\sigma - \sigma_\delta)}{\omega_\alpha}\Big) g_t^* + \Big(\frac{(1-\alpha)(\sigma - \sigma_\delta)}{\omega_\alpha}\Big) g_t - (1+\varphi) a_t \tag{B.2}$$

To solve for the natural rate of output, we first set $mc_t = -\mu$. Where μ is the markup under flexible prices. Solving the above equation for output yields:

$$\bar{y}_t = \Big(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi + \sigma_\delta}\Big) a_t + \Big(\frac{\sigma_\delta(\omega_\alpha - 1)}{\omega_\alpha\varphi + \sigma_\delta}\Big) y_t - \Big(\frac{\alpha\omega(\sigma - \sigma_\delta)}{\omega_\alpha\varphi + \sigma_\delta}\Big) g_t^* - \Big(\frac{(1-\alpha)(\sigma - \sigma_\delta)}{\omega_\alpha\varphi + \sigma_\delta}\Big) g_t \tag{B.3}$$

- Log-linearised model:

Domestic economy

- Aggregate consumption bundle:

$$\bar{c}_t = \delta c_t + (1-\delta) g_t \tag{B.4}$$

- IS curve:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t) \tag{B.5}$$

- Natural rate of interest:

$$\begin{aligned}\bar{rr}_t = &-\frac{\sigma_\delta(1+\varphi)(1-\rho_a)}{\varphi\omega_\alpha}a_t + \frac{\alpha(\omega-1)\varphi\sigma_\alpha}{\varphi\omega_\alpha}\Delta E_t\{y^*_{t+1}\} - \frac{\alpha\omega(\sigma-\sigma_\delta)\varphi(1-\rho_{g^*})}{\varphi\omega_\alpha}g^*_t \\ &- \frac{(1-\alpha)\omega(\sigma-\sigma_\delta)\varphi(1-\rho_g)}{\varphi\omega_\alpha}g_t\end{aligned} \tag{B.6}$$

- Phillips curve:

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \frac{\kappa(\varphi\omega_\alpha + \sigma_\alpha)}{\omega_\alpha}\hat{x}_t, \tag{B.7}$$

- Flexible-price output:

$$\bar{y}_t = \Big(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi+\sigma_\delta}\Big)a_t - \Big(\frac{\sigma_\delta(\omega_\alpha-1)}{\omega_\alpha\varphi+\sigma_\delta}\Big)y^*_t - \Big(\frac{\alpha\omega(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g^*_t - \Big(\frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g_t \tag{B.8}$$

- Output gap:

$$x_t = y_t - \bar{y}_t. \tag{B.9}$$

- CPI inflation:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \alpha\left(\hat{s}_t - \hat{s}_{t-1}\right). \tag{B.10}$$

- Monetary policy:

$$\begin{cases} r_t = \overline{rr_t} + \phi_\pi\pi_t + \phi_x x_t, & \text{Optimal policy,} \\ r_t = \phi_\pi\pi_{H,t}, & \text{Domestic inflation targeting,} \\ r_t = \phi_\pi\pi_t, & \text{CPI inflation targeting,} \\ e_t = 0, & \text{Exchange rate peg,} \end{cases} \tag{B.11}$$

- Exogenous processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{B.12}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{B.13}$$

- Labour Supply:

$$w_t - p_t = \phi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t \tag{B.14}$$

- Production:

$$y_t = a_t + n_t \tag{B.15}$$

Rest of the world

- Aggregate consumption bundle in the rest of the world:

$$\bar{c}_t^* = \delta c_t^* + (1-\delta) g_t^* \tag{B.16}$$

- IS curve in the rest of the world:

$$x_t^* = E_t\{x_{t+1}^*\} - \frac{1}{\sigma_\delta}[r_t^* - E_t\{\pi_{t+1}^*\} - \bar{rr}_t^*] \tag{B.17}$$

- Natural rate of interest:

$$\bar{rr}_t^* = -\frac{\sigma_\delta(1-\rho_{a^*})(1+\varphi)}{\varphi+\sigma_\delta} a_t^* + -\frac{((\sigma-\sigma_\delta)(1-\rho_{g^*})\varphi}{\varphi+\sigma_\delta} g_t^* \tag{B.18}$$

- Phillips curve:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \kappa(\varphi+\sigma_\delta) x_t^* \tag{B.19}$$

- Flexible-price output:

$$\bar{y}_t^* = -\Big(\frac{\sigma-\sigma_\delta}{\varphi+\sigma_\delta}\Big) g_t^* + \Big(\frac{1+\varphi}{\varphi+\sigma_\delta}\Big) a_t^* \tag{B.20}$$

- Output gap:

$$x_t^* = y_t^* - \bar{y}_t^*. \tag{B.21}$$

- Monetary policy:

$$r_t^* = \overline{rr_t}^* + \phi_\pi^* \pi_t{}^* + \phi_x^* x_t^* \tag{B.22}$$

- Rest of the world market clearing condition:

$$y_t^* = c_t^* \tag{B.23}$$

- Exogenous process:

$$a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^*, \tag{B.24}$$
$$g_t^* = \rho_g^* g_{t-1}^* + \varepsilon_{g,t}^*. \tag{B.25}$$

- Labour Supply:

$$w_t^* - p_t^* = \phi n_t^* + \sigma_\delta c_t^* + (\sigma - \sigma_\delta) g_t^* \tag{B.26}$$

- Production:

$$y_t^* = a_t^* + n_t^* \tag{B.27}$$

International linkages

- Goods market clearing:

$$y_t = (1-\alpha)c_t + \alpha c_t^* + \alpha[\gamma + \eta(1-\alpha)]s_t \tag{B.28}$$

- Terms of trade:

$$s_t - s_{t-1} = e_t - e_{t-1} + \pi_t^* - \pi_{H,t}. \tag{B.29}$$

- Real exchange rate:

$$q_t = (1-\alpha)s_t. \tag{B.30}$$

- International risk sharing:

$$c_t = c_t^* + \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}(g_t^* - g_t) + \frac{1}{\sigma_\delta} q_t. \tag{B.31}$$

Appendix B.2

To understand the inflation dynamics in the model, we start by analysing the price-setting behaviour of firms. We follow the steps of Gali & Monacelli (2005), and the 3rd chapter of Gali (2008) to derive the price-setting behaviour of firms in the model under a sticky prices framework. The aggregate domestic price index in the model is a weighted average of prices that have been adjusted at period t and prices that have not been adjusted:

$$P_{H,t} = \left[\theta (P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{B.32}$$

$\bar{P}_{H,t}$ is the re-optimised price that a fraction of the firms $(1-\theta)$ choose at period t, and this is normally higher than the prevailing price during the last period before. P_{t-1} is the price imposed by the other fraction of firms who have not been able to adjust their prices, and this is why we keep last period's

prices as the prevailing prices for those firms. We divide the above equation by $P_{H,t-1}$ to get:

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta)\Big(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\Big)^{1-\varepsilon} \tag{B.33}$$

Log-linearising the above equation around a steady state with zero inflation yields[1]

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t-1}) \tag{B.34}$$

In the above equation, inflation at the current period is affected by the price adjustment that a fraction of the firms in the economy makes to their prices. Therefore, as mentioned above, we start deriving the price-setting behaviour of firms to capture the dynamics of prices in the economy. When firms set their prices according to Calvo (1983) contract scheme, they aim to maximise the expected discounted value of their profits under the assumption that the newly set price will still be effective:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\big\{Q_{t,t+k}[c_{jt+k|t}(\bar{P}_{H,t} - \Psi_{t+k})]\big\} \tag{B.35}$$

Ψ is the cost function, θ^k is the probability that the re-optimised price at period t will remain effective at period t+k, and $Q_{t,t+k}$ is a the discount factor of nominal pay off and it is defined in equation (2.3). $c_{jt+k|t}$ is the Expected demand/production for period t+k at period t. The equation is subject to the following demand constraint: $c_{jt+k} = \Big(\frac{\bar{P}_t}{P_{t+k}}\Big)^{-\varepsilon} C_t$. Plugging in the demand function into the firm's maximisation problem yields:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\big\{Q_{t,t+k}[\Big(\frac{\bar{P}_{H,t}}{P_{t+k}}\Big)^{-\varepsilon} C_{t+k}(\bar{P}_t - \Psi_{t+k})]\big\} \tag{B.36}$$

Taking the first order condition of the above equation yields:

$$\sum_{k=0}^{\infty} \theta^k E_t\big\{Q_{t,t+k} C_{t+k}[\bar{P}_{H,t} - \mathscr{M}\psi_{t+k|t}]\big\} = 0 \tag{B.37}$$

ψ is the nominal marginal cost, and $\mathscr{M}$ is the gross mark-up and its equal to $\frac{\varepsilon}{\varepsilon-1}$. Now, we divide the above equation by $P_{H,t-1}$ and divide and multiply the second term by $P_{H,t+k}$:

$$\sum_{k=0}^{\infty} \theta^k E_t\Big\{Q_{t,t+k} C_{t+k}\Big[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathscr{M} MC_{t+k|t}\Pi_{t-1,t+k}^{H}\Big]\Big\} = 0 \tag{B.38}$$

Where $\Pi_{t-1,t+k}^{H} = \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{H,t+k}}$. We log-linearise the above equation around a zero-inflation steady state. Noting that $Q_{t,t+k}$ in the steady state will equal β^k:

[1] Log-linearising around a steady state of zero inflation allows us to get rid of the price dispersion created by the nominal friction in the model.

$$\bar{p}_{H,t} - p_{H,t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + \mu + (p_{H,t+k} - p_{H,t-1})\} \tag{B.39}$$

We notice from the above equation that the firms discount the expected stream of their future profits using the household's discount factor. This is simply attributed to the fact that the households are the share holders of those firms. Rearranging the above equation gives:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{H,t+k}\} \tag{B.40}$$

The above equation is describing how firm set their prices with a certain mark-up and the discounted present value of the stream of marginal costs. In the case when $\theta = 0$, all firms will be able to adjust their prices in each period (flexible prices scheme), and the above equation will simplify to:

$$\bar{p}_{H,t} = \mu + mc_t \tag{B.41}$$

The price the firms set in this case is equal to their markup over the nominal marginal cost. Of course, this shows that the price set by the firms is above their marginal cost since the markup is greater than 1. As a result, output will be lower than its level under perfect competition. It will be shown how the government can offset this distortion by giving the firms a certain employment subsidy. Now going back to equation (B.39), if we rewrite down the equation in a compact form we get:

$$\bar{p}_{H,t} - p_{H,t-1} = \beta\theta E_t\{p_{H,t+1} - p_{H,t}\} + \pi_t + (1-\beta\theta)\hat{mc}_t \tag{B.42}$$

Where $\hat{mc}_t = mc_t - p_{H,t} + \mu$. Adding the above equation to the price setting equation gives us:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa\hat{mc}_t \tag{B.43}$$

Where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The above equation is the core New Keynesian Phillips Curve. We develop it in the text to link inflation to the output gap through the relationship between the $\hat{mc}$ and the output gap x_t. κ in the Phillips curve equation is strictly decreasing in the stickiness parameter θ. From the above equation, we see that inflation in this type of models is a result of aggregate price-setting of the firms who adjust their prices based on current and future stream of their marginal costs.

Appendix B.3

In this section we aim to exploit how the effect of government consumption on private consumption changes under different values of the elasticity of substitution between the two. We show this under three different values for the inverse elasticity of substitution between government expenditure and private consumption χ: a) The Cobb-Douglas scenario when the the elasticity between government

consumption and private consumption is equal to the inverse elasticity of intertemporal substitution $\chi = \sigma$, b) The complementarity case when $\chi > \sigma$, and c) and the last case when $\chi < \sigma$ when the two items are substitutes. Maximising the utility function with respect to the budget constraint:

$$\ell = \frac{\left[\left(\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right)^{\frac{1}{1-\chi}}\right]^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t (D_t + W_t N_t + P_t T_t - P_t C_t - E_t[Q_{t,t+1} D_{t+1}]) \tag{B.44}$$

We take the F.O.C with respect to consumption to show how the marginal utility of consumption reacts to changes in government spending under different elasticities of substitution:

$$\frac{\partial \ell}{\partial C_t} = \delta^{\chi} \bar{C}_t^{-\sigma} \left(\frac{C_t}{\bar{C}_t}\right)^{-\chi} - P_t \lambda_t = 0 \tag{B.45}$$

Now we check the response of the marginal utility of consumption to changes in G_t:

$$\frac{\partial \lambda_t}{\partial G_t} = \chi - \sigma \bar{C}_t^{\chi-\sigma-1} \left(\frac{C_t G_t}{\bar{C}_t}\right)^{-\chi} \left(\frac{\delta^{\chi}(1-\delta)^{\chi}}{P_t}\right) \tag{B.46}$$

In the above equation, it is obvious that the reaction of the marginal utility of consumption for a given level of consumption will depend on $\chi - \sigma$:

a) $\chi = \sigma$: In the Cobb-Douglas case when $\chi = 1$, the above ratio will collapse to 0, regardless of the size of δ in the utility function.

b) $\chi > \sigma$: In this case, the effect of government expenditure will be positive, and as $\chi \to \infty$ the two items will be perfect complements.

c) $\chi < \sigma$, In this case, the sign of the term above will turn into negative, and any changes in government expenditure will have an adverse effect on consumption. Also, as $\chi \to 0$ the two items will be perfect substitutes.

It is easy to see from above that once we change the size of χ, the dynamics of the whole model will follow. In the separable case when $\chi = \sigma$, the whole model collapses to the basic model of Gali (2008) since the government consumes different goods than the ones consumed by the representative consumer.

When $\chi < \sigma$ an increase in government consumption will cause a drop in private consumption, and this will cause a negative output gap which will push domestic prices down. The response of monetary policy, in this case, will be a reduction of the policy rate to minimise the decline in private consumption. In the substitutability case, government expenditure will have an adverse effect on the natural rate of output, and this will cause a reduction in the natural rate of interest. Also, when $\chi < \sigma$, the slope of the IS curve will be steeper than the other two case. This translates to a higher response of consumption to changes in the real interest rate.

Appendix B.4

The utility function of the representative household is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{C}_t, N_t) \tag{B.47}$$

The aggregate consumption bundle is represented as follows:

$$\bar{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{B.48}$$

The above utility function is separable in consumption and hours worked ($U_{CN} = 0$). But it is not separable in consumption and government expenditure ($U_{CG} \neq 0$). The quadratic approximation of the above utility function will be:

$$\begin{aligned}
U_t - U =& U_c C\Big(\frac{C_t - C}{C}\Big) + U_N N\Big(\frac{N_t - N}{N}\Big) + \frac{1}{2} U_{cc} C^2 \Big(\frac{C_t - C}{C}\Big)^2 + \frac{1}{2} U_{NN} N^2 \Big(\frac{N_t - N}{N}\Big)^2 \\
&+ U_{CG} CG\Big(\frac{C_t - C}{C}\Big)\Big(\frac{G_t - G}{G}\Big) + o(||a||^3) + t.i.p \\
=& U_c C(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + U_N N(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2} U_{cc} C^2 (\hat{c}_t^2) + \frac{1}{2} U_{NN} N^2 (\hat{n}_t^2) \\
&+ U_{CG} CG(\hat{c}_t + \frac{1}{2}\hat{c}_t^2)(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) + o(||a||)^3 + t.i.p
\end{aligned} \tag{B.49}$$

$t.i.p$ is terms independent of monetary policy. dividing both sides by $U_c C$ yields:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& (\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + \frac{U_N N}{U_c C}(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2}\frac{U_{cc} C}{U_c}(\hat{c}_t^2) + \frac{1}{2}\frac{U_{NN} N^2}{U_c C}(\hat{n}_t^2) \\
&+ \frac{U_{CG} G}{U_c}(\hat{c}_t + \frac{1}{2}\hat{c}_t^2)(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) + o(||a||^3 + t.i.p \\
=& (\hat{c}_t + \frac{1}{2}\hat{c}_t^2 + \frac{1}{2}\frac{U_{cc} C}{U_c}\hat{c}_t^2) + \frac{U_N N}{U_c C}(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2}\frac{U_{NN} N^2}{U_c C}(\hat{n}_t^2) \\
&+ \frac{U_{CG} G}{U_c}(\hat{c}_t \hat{g}_t) + o(||a||^3 + t.i.p \\
=& \hat{c}_t + \Big[\frac{1}{2} + \frac{1}{2}\frac{U_{cc} C}{U_c}\Big]\hat{c}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{n}_t + \Big(\frac{1}{2} + \frac{1}{2}\frac{U_{NN} N^2}{U_c C}\Big)\hat{n}_t^2\Big] \\
&+ \frac{U_{CG} G}{U_c}(\hat{c}_t \hat{g}_t) + o(||a||^3 + t.i.p
\end{aligned} \tag{B.50}$$

Now, to simplify the above equations, we use the first and second order derivatives of the utility function with respect to consumption, labour, and government expenditure:

$$\begin{aligned}
U_C &= \frac{\partial U}{\partial C} = \bar{C}^{-\sigma} \\
U_{CC} &= \frac{\partial^2 U}{\partial C^2} = -\sigma \bar{C}^{-\sigma-1} \\
U_N &= \frac{\partial U}{\partial N} = -N^{\varphi} \\
U_{NN} &= \frac{\partial^2 U}{\partial N^2} = -\varphi N^{\varphi-1} \\
U_{CG} &= \frac{\partial^2 U}{\partial C \partial G} = -\sigma \bar{C}^{-\sigma-1}
\end{aligned} \tag{B.51}$$

Plugging the above derivations in the second-order derivations yields:

$$\frac{U_t - U}{U_c C} = \hat{c}_t + \left[\frac{1-\sigma\delta}{2}\right]\hat{c}_t^2 + \frac{U_N N}{U_c C}\left[\hat{n}_t + \left(\frac{1+\varphi}{2}\right)\hat{n}_t^2\right] - \sigma(1-\delta)(\hat{c}_t \hat{g}_t) + o(||a||^3) + t.i.p \tag{B.52}$$

Plugging in the value of consumption by combining the domestic market's clearing condition with the international risk sharing condition, and using the fact that $\sigma = 1$ we get:

$$\frac{U_t - U}{U_c C} = \frac{1-\alpha}{\omega_\alpha}\hat{y}_t + \left[\frac{1-\delta}{2}\right]\frac{1-\alpha}{\omega_\alpha}\hat{y}_t^2 + \frac{U_N N}{U_c C}\left[\hat{n}_t + \left(\frac{1+\varphi}{2}\right)\hat{n}_t^2\right] - \frac{1-\alpha}{\omega_\alpha}(1-\delta)(\hat{y}_t \hat{g}_t) + o(||a||^3) + t.i.p \tag{B.53}$$

We next turn our attention to the hours worked variable (n_t). From the production function and the price dispersion equation we get: $\hat{n}_t = \hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\}$. Plugging this in the above equation yields:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& \frac{1-\alpha}{\omega_\alpha}\hat{y}_t + \left[\frac{1-\delta}{2}\right]\frac{1-\alpha}{\omega_\alpha}\hat{y}_t^2 + \frac{U_N N}{U_c C}\Big[\hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\} \\
&+ \left(\frac{1+\varphi}{2}\right)(\hat{y}_t + a_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\})^2\Big] - \frac{1-\alpha}{\omega_\alpha}(1-\delta)(\hat{y}_t \hat{g}_t) + o(||a||^3) + t.i.p \\
=& \frac{1-\alpha}{\omega_\alpha}\hat{y}_t + \left[\frac{1-\delta}{2}\right]\frac{1-\alpha}{\omega_\alpha}\hat{y}_t^2 + \frac{U_N N}{U_c C}\left[\hat{y}_t + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\} + \left(\frac{1+\varphi}{2}\right)(\hat{y}_t + a_t)^2\right] \\
&- \frac{1-\alpha}{\omega_\alpha}(1-\delta)(\hat{y}_t \hat{g}_t) + o(||a||^3) + t.i.p
\end{aligned} \tag{B.54}$$

Under the optimal subsidy scheme, the following condition holds: $\frac{U_N N}{U_C C} = -\frac{1-\alpha}{\omega_\alpha}$. Using this condition in the above equation yields:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& \frac{1-\alpha}{\omega_\alpha}\hat{y}_t + \Big[\frac{1-\delta}{2}\Big]\frac{1-\alpha}{\omega_\alpha}\hat{y}_t^2 - \frac{1-\alpha}{\omega_\alpha}\Big[\hat{y}_t + \frac{\varepsilon}{2}var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t + a_t)^2\Big] \\
&- \frac{1-\alpha}{\omega_\alpha}(1-\delta)(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \\
=& \Big[\frac{(1-\delta)(1-\alpha)}{2\omega_\alpha}\Big]\hat{y}_t^2 - \frac{1-\alpha}{\omega_\alpha}\Big[\frac{\varepsilon}{2}var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2a_t\hat{y}_t + a_t^2)\Big] \\
&- \frac{(1-\alpha)(1-\delta)}{\omega_\alpha}(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p \\
=& \Big[\frac{(1-\delta)(1-\alpha)}{2\omega_\alpha}\Big]\hat{y}_t^2 - \frac{1-\alpha}{\omega_\alpha}\Big[\frac{\varepsilon}{2}var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2a_t\hat{y}_t)\Big] \\
&- \frac{(1-\alpha)(1-\delta)}{\omega_\alpha}(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p
\end{aligned} \tag{B.55}$$

Using the fact that $\hat{y}_t^n = a_t$:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& \Big[\frac{(1-\delta)(1-\alpha)}{2\omega_\alpha}\Big]\hat{y}_t^2 - \frac{1-\alpha}{\omega_\alpha}\Big[\frac{\varepsilon}{2}var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)(\hat{y}_t^2 - 2\hat{y}_t^n\hat{y}_t)\Big] \\
&- \frac{(1-\alpha)(1-\delta)}{\omega_\alpha}(\hat{y}_t\hat{g}_t) + o(||a||^3) + t.i.p
\end{aligned} \tag{B.56}$$

In the above equation, I will use one simple trick to simplify the following term: $\hat{y}_t^2 - 2\hat{y}_t^n\hat{y}_t$. The term is simplified to: $(\hat{y}_t - \hat{y}_t^n)^2 - \hat{y}_t^{n^2}$. Where $\hat{y}_t^{n^2}$ eliminates the additional third term that we get from the quadratic expression. The same also applies to $\Big[\frac{(1-\delta)(1-\alpha)}{2\omega_\alpha}\Big]\hat{y}_t^2 - \frac{(1-\alpha)(1-\delta)}{\omega_\alpha}(\hat{y}_t\hat{g}_t)$. Doing so will simplify the above equation to:

$$\begin{aligned}
\frac{U_t - U}{U_c C} =& -\frac{1-\alpha}{\omega_\alpha}\Big[\frac{\varepsilon}{2}var_j\{p_{H,t}(j)\} + \Big(\frac{1+\varphi}{2}\Big)[(\hat{y}_t - \hat{y}_t^n)^2 - \hat{y}_t^{n^2}]\Big] \\
&+ \Big[\frac{(1-\delta)(1-\alpha)}{2\omega_\alpha}\Big]\Big[(\hat{y}_t - \hat{g}_t)^2 - \hat{g}_t^2\Big] + o(||a||^3) + t.i.p \\
=& -\frac{1-\alpha}{2\omega_\alpha}\Big[\varepsilon.var_j\{p_{H,t}(j)\} + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\Big] + o(||a||^3) + t.i.p
\end{aligned} \tag{B.57}$$

We can write the above equation as the discounted sum of all periods:

$$\sum_{t=0}^{\infty}\beta^t\Big[\frac{U_t - U}{U_c C}\Big] = -\frac{1-\alpha}{2\omega_\alpha}\sum_{t=0}^{\infty}\beta^t\Big[\varepsilon.var_j\{p_{H,t}(j)\} + (1+\varphi)\hat{x}_t^2 + (1-\delta)(\hat{g}_t - \hat{y}_t)^2\Big] + o(||a||^3) + t.i.p \tag{B.58}$$

Following the work of Woodford (2001), we make use of the following Calvo property:

$$\sum_{t=0}^{\infty}\beta^t\varepsilon.var_j\{p_{H,t}(j)\} = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2. \tag{B.59}$$

$$\sum_{t=0}^{\infty}\beta^t\left[\frac{U_t-U}{U_cC}\right]=-\frac{1-\alpha}{2\omega_\alpha}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{H,t}^2+(1+\varphi)\hat{x}_t^2+(1-\delta)(\hat{g}_t-\hat{y}_t)^2\right]+o(||a||^3)+t.i.p \quad \text{(B.60)}$$

The above second-order approximation shows the utility loss resulting from deviation from the steady state (optimal) policy, and it is represented as a fraction of the steady-state consumption of the representative household. The per-period welfare-loss function is:

$$L=-\frac{1-\alpha}{2\omega_\alpha}\left[\frac{\varepsilon}{\lambda}var.\pi_{H,t}+(1+\varphi)var.\hat{x}_t+(1-\delta)var.(\hat{g}_t-\hat{y}_t)\right] \quad \text{(B.61)}$$

Appendix B.5

Table B.1 Contribution to Welfare Losses in the Domestic Economy

A. Under a government shock			
	DIT	CIT	PEG
Var(Domestic Infl.)	0.6358	2.0065	1.2086
Var(Output Gap)	0.012	0.0438	0.2072
Var(Fiscal Gap)	0.0047	0.0038	0.001
Total	0.6525	2.0541	1.4168
B. Under a TFP shock			
	DIT	CIT	PEG
Var(Domestic Infl.)	0.0096	0.0223	0.0163
Var(Output Gap)	0.0005	0.0040	0.0020
Total	0.0101	0.0121	0.0183

Table B.2 Cyclical Properties of Alternative Policy Regimes in the Domestic Economy

A. Under a TFP shock				
	Optimal	DIT	CIT	PEG
Domestic inflation	0.00	0.27	0.41	0.35
Output gap	0.00	0.08	0.24	0.52
CPI inflation	0.34	0.49	0.48	0.21
Nominal interest rate	0.17	0.41	0.72	0.36
Terms of trade	1.88	1.97	1.87	1.51
Nominal depr. rate	0.84	0.98	0.72	0.00
REER	1.13	1.18	1.12	0.91
Fiscal Gap	0.23	0.31	0.31	0.40
B. Under a government shock				
	Optimal	DIT	CIT	PEG
Domestic inflation	0.00	3.24	2.34	3.04
Output gap	0.00	1.86	3.29	5.27
CPI inflation	3.44	3.58	1.50	1.83
Nominal interest rate	2.72	4.86	2.26	0.00
Terms of trade	13.60	11.33	10.50	9.06
Nominal depr. rate	8.59	6.87	3.36	0.0
REER	8.15	6.80	6.30	5.43
Fiscal Gap	49.3	47.44	46.60	45.04

Appendix B.6

Following Bullard & Mitra (2002), we try to explain how determinacy of monetary policy rules is derived from the below matrix:

$$A = \Omega \begin{bmatrix} \sigma_\delta & \omega_\alpha(1-\beta\phi_\pi) \\ \kappa_g\sigma_\delta & \beta(\sigma_\delta+\omega_\alpha\phi_x)+\kappa_g\omega_\alpha \end{bmatrix} \text{ and } \Omega = \frac{1}{\sigma_\delta+\omega_\alpha\phi_x+\kappa_g\omega_\alpha\phi_\pi}.$$

The characteristic polynomial of the above matrix is:

$$p(\lambda) = \lambda^2 - \lambda a_1 + a_2 \tag{B.62}$$

For the eigenvalues of matrix A to be inside the unit circle, the following conditions have to be met:

$$|a_2| < 1 \tag{B.63}$$

$$|a_1| < 1 + |a_2| \tag{B.64}$$

Where a_1 is the trace of matrix A and a_2 is the determinant of the same matrix:

$$a_2 = \frac{\beta\sigma_\delta}{\sigma_\delta+\omega_\alpha\phi_x+\kappa_g\omega_\alpha\phi_\pi} \tag{B.65}$$

The determinant of this matrix should satisfy the first condition: $\beta\sigma_\delta < \sigma_\delta+\omega_\alpha\phi_x+\kappa_g\omega_\alpha\phi_\pi$. This rule is easily satisfied since $0 < \beta < 1$ and given the fact that all of the parameters have positive values. The trace of the matrix a_1 is:

$$a_1 = \frac{-(\sigma_\delta+\beta\sigma_\delta+\beta\omega_\alpha\phi_x+\kappa_g\omega_\alpha)}{\sigma_\delta+\omega_\alpha\phi_x+\kappa_g\omega_\alpha\phi_\pi} \tag{B.66}$$

From the second condition, the following inequality must hold for the eigenvalues of the matrix to lie inside the unit circle: $\kappa_g\omega_\alpha(\phi_\pi-1)+(1-\beta)\omega_\alpha\phi_x > 0$. The inflation parameter has to be greater than 1 for this rule to hold along with the first condition.

Appendix B.7

Figure B.1 Response to a Domestic TFP shock (Substitutability)

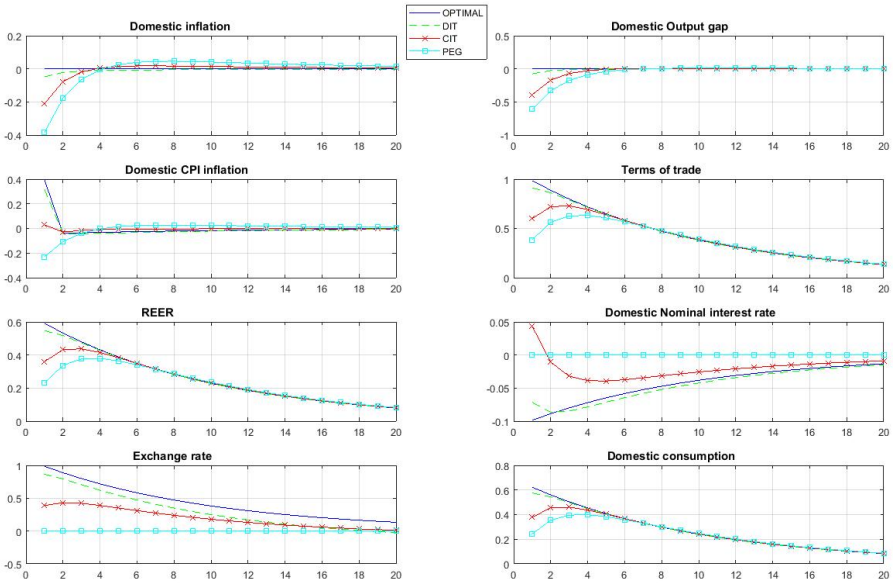


Figure B.2 Response to a Domestic Government Consumption Shock (Substitutability)

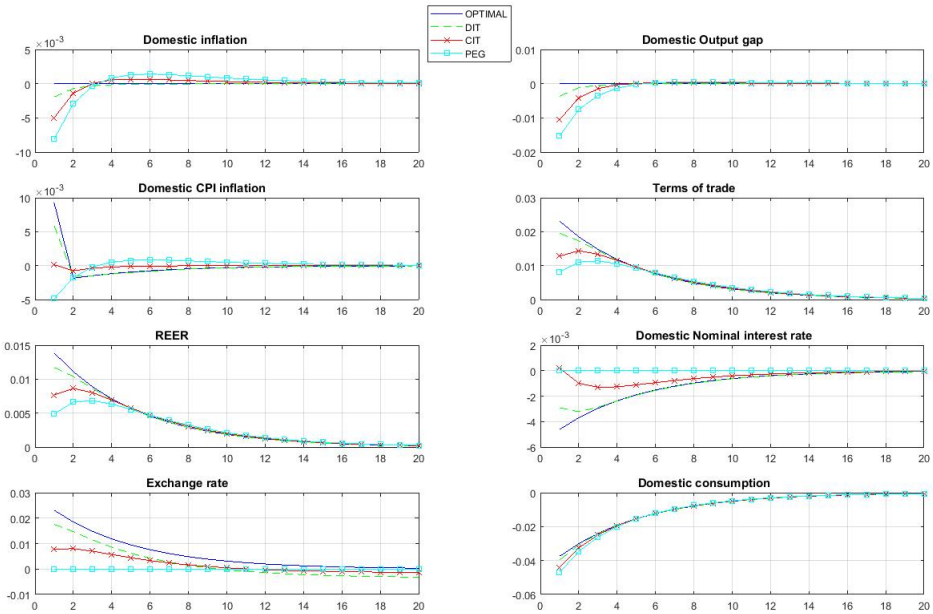


Figure B.3 Response to a TFP Shock in the Foreign Economy (Substitutability)

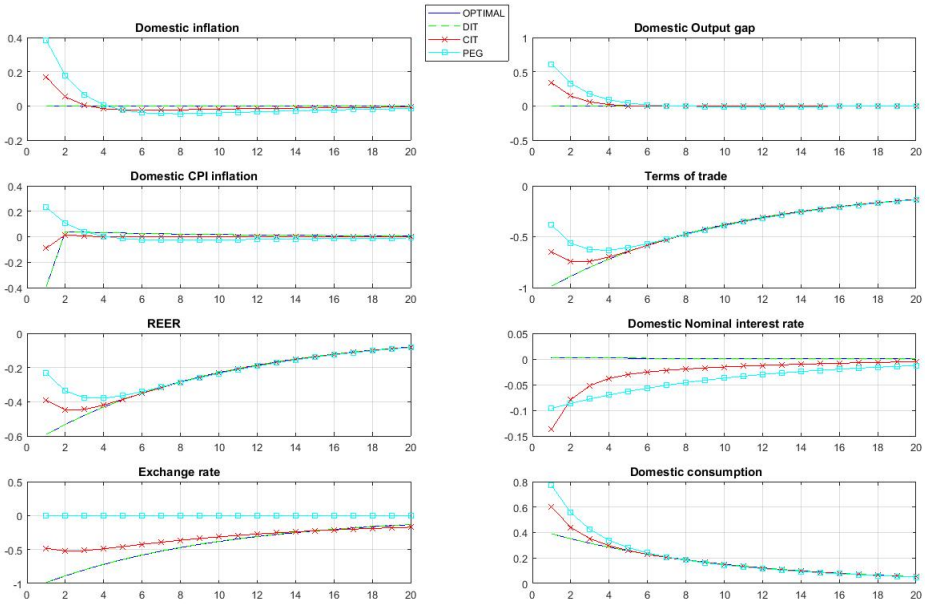
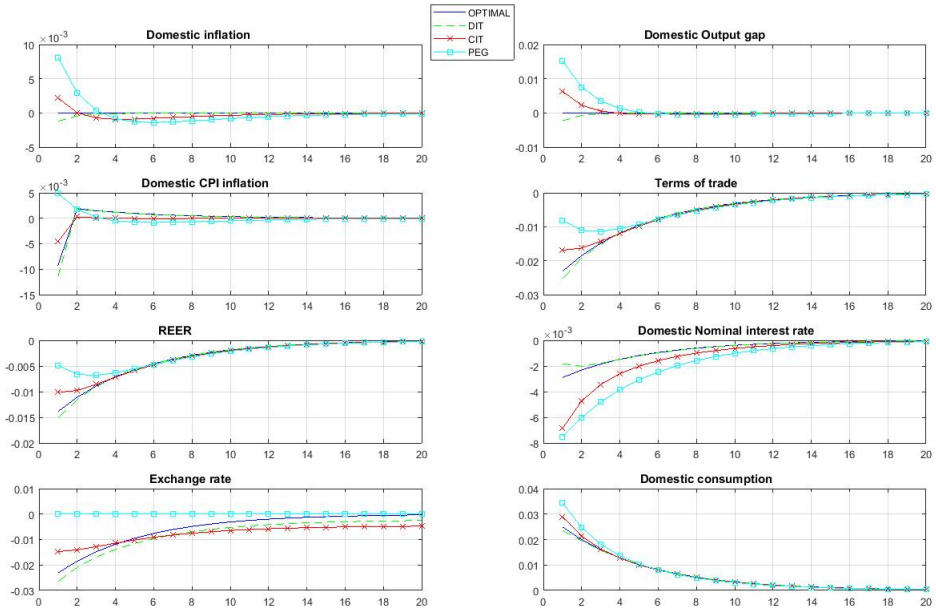


Figure B.4 Response to a Government Consumption Shock in the Foreign Economy (Substitutability)



Appendix C

Appendices For Chapter Three

Appendix C.1

- Domestic economy

- Optimal consumption bundles:

$$c_{Z,t} = c_t - mu * p_{Z,t} \tag{C.1}$$

$$c_{O,t} = c_t - mu * p_{O,t} \tag{C.2}$$

$$c_{H,t} = c_{Z,t} - mu * (p_{H,t} - p_{Z,t}) \tag{C.3}$$

$$c_{F,t} = c_t - mu * (p_{F,t} - p_{Z,t}) \tag{C.4}$$

- Relative prices and inflation rates:

$$0 = (1 - \varpi) * p_{Z,t} + \varpi * p_{O,t} \tag{C.5}$$

$$p_{Z,t} = (1 - \alpha) * p_{H,t} + \alpha * p_{F,t} \tag{C.6}$$

$$\pi_{Z,t} = p_{Z,t} - p_{Z,t-1} + \pi_t \tag{C.7}$$

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} + \pi_t \tag{C.8}$$

$$\pi_{F,t} = p_{F,t} - p_{F,t-1} + \pi_t \tag{C.9}$$

- Labour supply:

$$\text{Real wages} = \sigma * c_t + \varphi * n_t + (\sigma - \sigma_\delta) * g_t \tag{C.10}$$

- IS curve:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t) \tag{C.11}$$

- Phillips curve

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa \frac{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)}{\omega_\alpha(1+\varphi(1-\nu))} x_t \tag{C.12}$$

- Natural rate of interest:

$$\begin{aligned}\bar{r}r_t = &-\frac{\nu(1+\varphi)\sigma_\delta(1-\rho_a))}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} a_t + \frac{(1-\nu)(1+\varphi)\sigma_\delta}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} \Delta E_t\{\tilde{p}^*_{o,t+1}\} \\ &+ \frac{\nu\varphi(1-\alpha)(\sigma-\sigma_\delta)}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} \Delta E_t\{g_{t+1}\} + \frac{(\sigma-\sigma_\delta)(\nu\omega_\alpha(\alpha\varphi-\sigma_\alpha) + \sigma_\delta(1+\varphi-\varphi\nu))}{\omega_\alpha(\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu))} \Delta E_t\{g^*_{t+1}\} \\ &+ \frac{\sigma_\delta((1+\alpha)\sigma_\delta(1+\varphi(1-\nu)) - \alpha\nu\varphi\omega_\alpha(\omega-1) - \alpha\omega\sigma_\delta(1+\varphi(1-\nu)) - \nu\omega_\alpha\sigma_\delta)}{\omega_\alpha(\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu))} \Delta E_t\{y^*_{t+1}\}\end{aligned} \tag{C.13}$$

- Flexible-price output:

$$\begin{aligned}\bar{y}_t = &-\frac{\nu\sigma_\delta\omega_\alpha - \sigma_\delta - \sigma_\delta\varphi(1-\nu)}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} y^*_t - \frac{((\sigma-\sigma_\delta)(\nu\omega_\alpha - (1-\alpha) - (1-\alpha)\varphi(1-\nu))}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} g^*_t \\ &- \frac{(1-\alpha)(\sigma-\sigma_\delta)(1+\varphi(1-\nu))}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} g_t + \frac{\nu(1+\varphi)\omega_\alpha}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} a_t - \frac{(1-\nu)(1+\varphi)\omega_\alpha}{\nu\varphi\omega_\alpha + \sigma_\delta + \sigma_\delta\varphi(1-\nu)} \tilde{p}^*_{o,t}\end{aligned} \tag{C.14}$$

- Output gap:

$$x_t = y_t - \bar{y}_t. \tag{C.15}$$

- Production function:

$$y_t = \nu a_t + \nu n_t + (1-\nu) o_d \tag{C.16}$$

- Domestic commodity demand:

$$o^d_t = \frac{1+\varphi}{1+\varphi(1-\nu)} y_t + \frac{\nu\sigma_\delta}{1+\varphi(1-\nu)} y^*_t - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)} a_t + \frac{\nu(\sigma-\sigma_\delta)}{1+\varphi(1-\nu)} g^*_t - \frac{\nu}{1+\varphi(1-\nu)} \tilde{p}^*_{o,t} \tag{C.17}$$

- Monetary policy:

$$r_t = \rho_r r_{t-1} + \phi_\pi \pi_{Z,t} + \phi_y x_t + \varepsilon_t \tag{C.18}$$

- Fiscal policy

$$g_t = \rho_g g_{t-1} + \phi_g * (p_O) + \varepsilon_g \tag{C.19}$$

- Rest of the world

- Optimal consumption bundles:

$$c^*_{Z,t} = c^*_t - mu^* * p^*_{Z,t} \tag{C.20}$$

$$c^*_{O,t} = c^*_t - mu^* * p^*_{O,t} \tag{C.21}$$

- Relative prices and inflation rates:

$$0 = (1-\varpi^*) * p^*_{Z,t} + \varpi^* * p^*_{O,t} \tag{C.22}$$

$$\pi^*_{Z,t} = p^*_{Z,t} - p^*_{Z,t-1} + \pi^*_t \tag{C.23}$$

- Labour supply:

$$\text{Real wages} = \sigma * c^*_t + \varphi * n^*_t + (\sigma - \sigma_\delta) * g^*_t \tag{C.24}$$

- IS curve:

$$x^*_t = E_t\{x^*_{t+1}\} - \frac{1}{\sigma_\delta}[r^*_t - E_t\{\pi^*_{t+1}\} - \bar{rr}^*_t] \tag{C.25}$$

- Natural rate of interest:

$$\bar{rr}^*_t = -\frac{\sigma_\delta(1-\rho_a)(1+\varphi)\nu}{\nu(\varphi+\sigma)}a^*_t + \frac{\nu(\sigma-\sigma_\delta)(\sigma+\varphi-\sigma_\delta)}{\nu(\varphi+\sigma)}\Delta g^*_{t+1} - \frac{\sigma_\delta(1-\nu)(1+\varphi)}{\nu(\varphi+\sigma)}\Delta\tilde{p}^*_{o,t+1} \tag{C.26}$$

- Phillips curve

$$\hat{\pi}^*_t = \beta E_t[\hat{\pi}^*_{t+1}] + \lambda\frac{\nu\sigma+\nu\varphi}{1+(1-\nu)\varphi}x^*_t \tag{C.27}$$

- Flexible-price output:

$$\bar{y}_t{}^* = -\frac{1+(1-\nu)\varphi}{\nu\sigma+\nu\varphi}\mu^* + \frac{\nu(1+\varphi)}{\nu\sigma+\nu\varphi}a^*_t - \frac{(1-\nu)(1+\varphi)}{\nu\sigma+\nu\varphi}\tilde{p}^*_{o,t} - \frac{\nu(\sigma-\sigma_\delta)}{\nu\sigma+\nu\varphi}g^*_t \tag{C.28}$$

- Output gap:

$$x^*_t = y^*_t - \bar{y}_t{}^* \tag{C.29}$$

- Production function:

$$y^*_t = \nu a^*_t + \nu n^*_t + (1-\nu)o^*_d \tag{C.30}$$

- Market clearing condition

$$y^*_t = c^*_{Z,t} \tag{C.31}$$

- World commodity demand:

$$o^{*d}_t = \frac{\nu\sigma+\varphi+1}{1+\varphi(1-\nu)}y^*_t - \frac{\nu(1+\varphi)}{1+\varphi(1-\nu)}a^*_t + \frac{\nu(\sigma-\sigma_\delta)}{1+\varphi(1-\nu)}g^*_t - \frac{\nu}{1+\varphi(1-\nu)}\tilde{p}^*_{o,t} \tag{C.32}$$

- Real commodity prices:

$$\tilde{p}^*_{o,t} = \frac{\nu\sigma + \varphi + 1}{\nu} y^*_t - (1+\varphi)a^*_t + (\sigma - \sigma_\delta)g^*_t - \frac{1+\varphi(1-\nu)}{\nu} o^{*s}_t \tag{C.33}$$

- Monetary policy:

$$r^*_t = \rho_r r^*_{t-1} + \phi_\pi \pi^*_t + \phi_y x^*_t + \varepsilon_{r^*,t} \tag{C.34}$$

- International linkages

- Goods market clearing:

$$y_t = (1-\alpha)c_t + \alpha c^*_t + \alpha[\gamma + \eta(1-\alpha)]s_t \tag{C.35}$$

- Domestic output as a function of world output and the terms of trade

$$y_t = y^*_t + \frac{(1-\alpha)(\sigma - \sigma_\delta)}{\sigma_\delta}(g^*_t - g_t) + \frac{\omega_\alpha}{\sigma_\delta} s_t \tag{C.36}$$

- Real exchange rate:

$$q_t = (1-\alpha)s_t. \tag{C.37}$$

- Exogenous processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{C.38}$$

$$a^*_t = \rho_a a^*_{t-1} + \varepsilon_{a^*,t} \tag{C.39}$$

$$o^*_t = \rho_o o^*_{t-1} + \varepsilon_{o^*,t} \tag{C.40}$$

$$g^*_t = \rho_{g^*} g^*_{t-1} + \varepsilon_{g^*,t} \tag{C.41}$$

- Measurement equations:

$$dg^{obs}_t = g_t - g_{t-1} + Trend_g \tag{C.42}$$

$$dc^{obs}_t = c_t - c_{t-1} + Trend_c \tag{C.43}$$

$$\pi^{obs}_t = \pi_t + Trend \tag{C.44}$$

$$p^{obs}_{o,t} = p_{o,t} \tag{C.45}$$

Appendix C.2

To understand the inflation dynamics in the model, we start by analysing the price-setting behaviour of firms. We follow the steps of Gali & Monacelli (2005), and the 3rd chapter of Gali (2008) to derive the price-setting behaviour of firms in the model under a sticky prices framework. The aggregate domestic price index in the model is a weighted average of prices that have been adjusted at period t and prices that have not been adjusted:

$$P_{H,t} = \left[\theta(P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{C.46}$$

$\bar{P}_{H,t}$ is the re-optimised price that a fraction of the firms $(1-\theta)$ choose at period t, and this is normally higher than the prevailing price during the last period before. P_{t-1} is the price imposed by the other fraction of firms who have not been able to adjust their prices, and this is why we keep last period's prices as the prevailing prices for those firms. We divide the above equation by $P_{H,t-1}$ to get:

$$\Pi_{H,t}^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\right)^{1-\varepsilon} \tag{C.47}$$

Log-linearising the above equation around a steady state with zero inflation yields[1]

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t-1}) \tag{C.48}$$

In the above equation, inflation at the current period is affected by the price adjustment that a fraction of the firms in the economy make to their prices. Therefore, as mentioned above, we start deriving the price-setting behaviour of firms to capture the dynamics of prices in the economy. When firms set their prices according to Calvo (1983) contract scheme, they aim to maximise the expected discounted value of their profits under the assumption that the newly set price will still be effective:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k}[c_{jt+k|t}(\bar{P}_{H,t} - \Psi_{t+k})]\right\} \tag{C.49}$$

Ψ is the cost function, θ^k is the probability that the re-optimised price at period t will remain effective at period t+k, and $Q_{t,t+k}$ is a the discount factor of nominal pay off and it is defined in equation (3.9). $c_{jt+k|t}$ is the Expected demand/production for period t+k at period t. The equation is subject to the following demand constraint: $c_{jt+k} = \left(\frac{\bar{P}_t}{P_{t+k}}\right)^{-\varepsilon} C_t$. Plugging in the demand function into the firm's maximisation problem yields:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k}[\left(\frac{\bar{P}_{H,t}}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}(\bar{P}_t - \Psi_{t+k})]\right\} \tag{C.50}$$

[1] Log-linearising around a steady state of zero inflation allows us to get rid of the price dispersion created by the nominal friction in the model.

Taking the first order condition of the above equation yields:

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} C_{t+k} [\bar{P}_{H,t} - \mathscr{M} \psi_{t+k|t}] \} = 0 \tag{C.51}$$

ψ is the nominal marginal cost, and $\mathscr{M}$ is the gross mark-up and its equal to $\frac{\varepsilon}{\varepsilon-1}$. Now, we divide the above equation by $P_{H,t-1}$ and divide and multiply the second term by $P_{H,t+k}$:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} C_{t+k} \left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathscr{M} MC_{t+k|t} \Pi^H_{t-1,t+k} \right] \right\} = 0 \tag{C.52}$$

Where $\Pi^H_{t-1,t+k} = \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{H,t+k}}$. We log-linearise the above equation around a zero-inflation steady state. Noting that $Q_{t,t+k}$ in the steady state will equal β^k:

$$\bar{p}_{H,t} - p_{H,t-1} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + \mu + (p_{H,t+k} - p_{H,t-1}) \} \tag{C.53}$$

We notice from the above equation that the firms discount the expected stream of their future profits using the household's discount factor. This is simply attributed to the fact that the households are the share holders of those firms. Rearranging the above equation gives:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + p_{H,t+k} \} \tag{C.54}$$

The above equation is describing how firm set their prices with a certain mark-up and the discounted present value of the stream of marginal costs. In the case when $\theta = 0$ all firms will be able to adjust their prices in each period (flexible prices scheme), and the above equation will simplify to:

$$\bar{p}_{H,t} = \mu + mc_t \tag{C.55}$$

The price the firms set in this case is equal to their markup over the nominal marginal cost. Of course, this shows that the price set by the firms is above their marginal cost since the markup is greater than 1. As a result, output will be lower than its level under perfect competition. It will be shown how the government can offset this distortion by giving the firms a certain employment subsidy. Now going back to equation (C.53), if we rewrite down the equation in a compact form we get:

$$\bar{p}_{H,t} - p_{H,t-1} = \beta\theta E_t \{ p_{H,t+1} - p_{H,t} \} + \pi_t + (1-\beta\theta) \hat{mc}_t \tag{C.56}$$

Where $\hat{mc}_t = mc_t - p_{H,t} + \mu$. Adding the above equation to the price setting equation gives us:

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa \hat{mc}_t \tag{C.57}$$

Where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The above equation is the core New Keynesian Phillips Curve. We develop it in the text to link inflation to the output gap through the relationship between the $\hat{mc}$ and the output gap x_t. κ in the Phillips curve equation is strictly decreasing in the stickiness parameter θ. From the above equation, we see that inflation in this type of models is a result of aggregate price-setting of the firms who adjust their prices based on current and future stream of their marginal costs.

Appendix C.3

Table C.1 Averages of Natural Resources Rents During 1980-2015

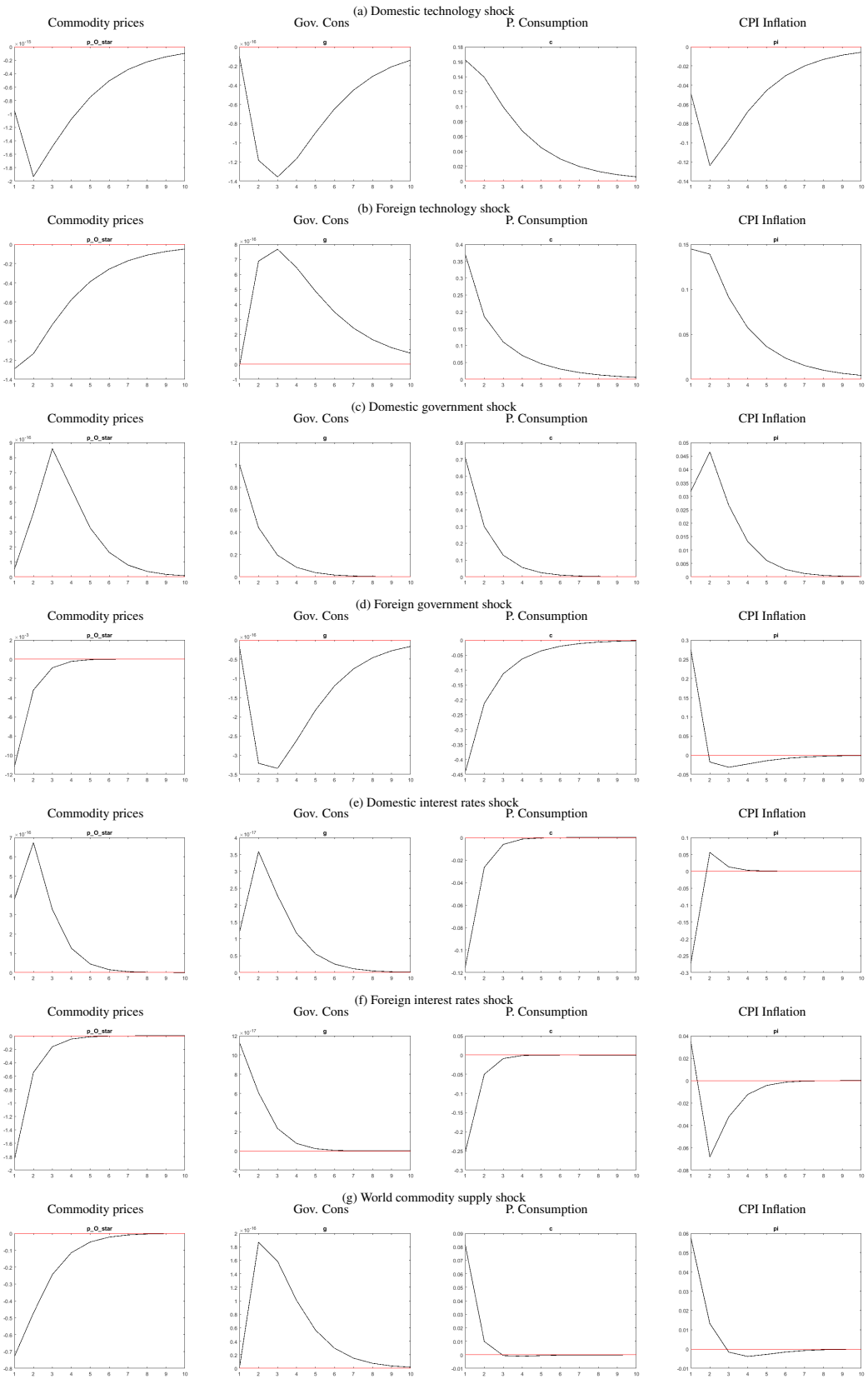
Country	Dominant Resource	Natural Resources Rents (% of GDP)	Country	Dominant Resource	Natural Resources Rents (% of GDP)
Albania	Oil	4.44	Madagascar	Forests	6.15
Algeria	Oil	14.9	Malawi	Forests	8.27
Angola	Oil	38.64	Malaysia	Forests and Oil	15.4
Australia	Minerals	4.49	Mali	Forests	6.78
Bahrain	Forests	6.53	Mauritania	Minerals	19.43
Benin	Forests	6.53	Mongolia	Coal and Minerals	14.93
Botswana	Minerals	3.09	Mozambique	Forests	10.41
Brunei Darussalam	Oil	23.47	Myanmar	Forests	8.07
Burkina Faso	Forests and Minerals	10.06	Namibia	Minerals	4.3
Burundi	Forests	18.45	Niger	Forests	8.61
Cambodia	Forests	3.76	Nigeria	Oil	28.58
Cameroon	Forests and Oil	9.36	Norway	Oil	7.16
Canada	Oil	3.52	Oman	Oil	34.7
Central Africa	Forests	9.69	Papa New Guinea	Minerals	29.21
Chad	Forests and Oil	16.16	Qatar	Natural Gas and Oil	27.5
Chile	Minerals	10.86	Republic of Congo	Oil	41.37
China	Coal and Oil	5.93	Russia	Natural Gas and Oil	13.3
Comoros	Forests	2.18	Rwanda	Forests	7.34
Dem. Congo	Forests and Minerals	24.19	Senegal	Forests	3.43
Egypt	Oil	11.2	Sierra Leone	Forests	14.29
Equatorial Guinea	Oil	11.2	Solomon Islands	Forests and Minerals	11.26
Eritrea	Forests	5.59	South Africa	Coal and Minerals	6.25
Ethiopia	Forests	17.94	South Sudan	Oil	32.77
Gabon	Oil	31.34	Sudan	Oil	5.51
Gambia	Forests	3.83	Suriname	Minerals	14.56
Ghana	Forests and Oil	10.19	Swaziland	Forests	5.32
Guinea	Forests and Minerals	20.01	Syria	Oil	16.7
Guinea-Bissau	Forests	18.55	Tanzania	Forests	8.05
Guyana	Forests and Minerals	20.5	Togo	Forests and Minerals	11.58
India	Coal and Oil	3.16	Trinidad and Tobago	Oil	13.02
Indonesia	Oil	8.62	Tunisia	Oil	5.38
Iran	Oil	19.44	Turkmenistan	Natural Gas	41.1
Iraq	Oil	32.3	UAE	Oil	19.7
Ivory Coast	Forests	5.02	Uganda	Forests	16.6
Jamaica	Minerals	4.12	Ukraine	Minerals	4.39
Kazakhstan	Oil	15.98	Uzbekistan	Natural Gas	14.66
Kenya	Forests	4.21	Venezuela	Oil	15.67
Kuwait	Oil	38.8	Vietnam	Forests and Oil	7.04
Kyrgyzstan	Minerals	3.98	Yemen	Oil	25.2
Lao	Forests and Minerals	9.03	Zambia	Minerals	12.83
Lesotho	Forests	4.98	Zimbabwe	Coal and Minerals	7.45
Liberia	Forests and Minerals	40.98	KSA	Oil	34.8
Libya	Oil	39.08			
Macedonia	Minerals	2.19			

Appendix C.4

Table C.2 Baseline Calibration

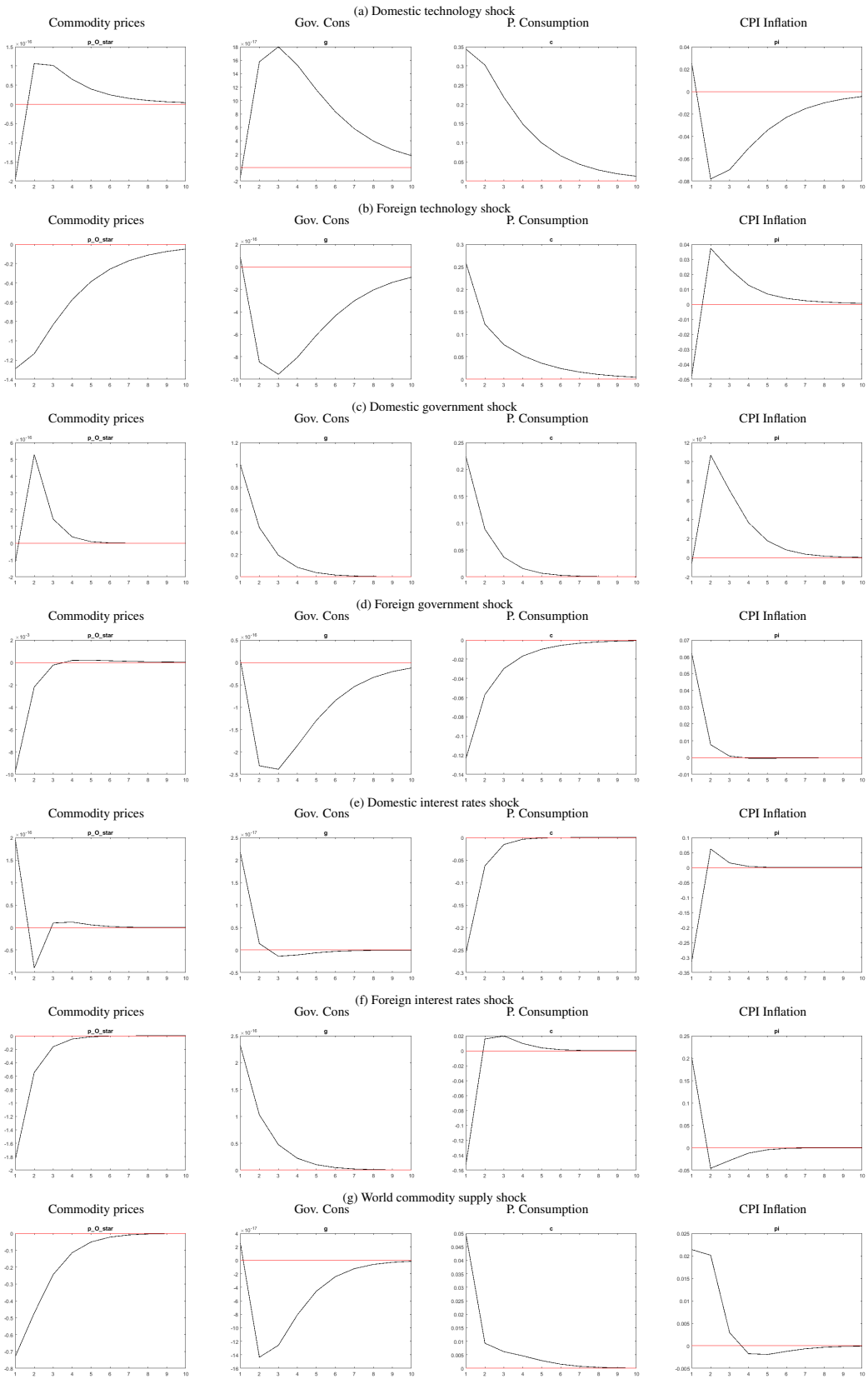
Fixed		Value
β	Discount factor	0.99
δ	share of private consumption in the aggregate consumption bundle	0.95
ε	Elasticity of substitution	6
η	elasticity of substitution between domestic and foreign goods	1
γ	Elasticity of substitution between goods in the world economy	1
ν	Share of non-commodity factors in the production function	0.95
σ	Inverse elasticity of intertemporal substitution	1
$\varphi\&\varphi^*$	Inverse Frisch labour supply elasticity	3
μ	Elasticity of substitution between commodity and core consumption	0.3
θ	Calvo probability	0.75
ϕ_π	Inflation elasticity of the nominal interest rate	1.5
ϕ_x	Output gap elasticity of the nominal interest rate	0.5
ρ_{a,a^*}	AR(1) coefficient of domestic and foreign productivity	0.66
ρ_{r,r^*}	AR(1) coefficient of domestic and foreign interest rates	0.12
σ_{a,a^*}	standard deviation of a domestic and foreign productivity shocks	0.0071
σ_{r,r^*}	standard deviation of a domestic and foreign interest rates shocks	0.24
Calibrated		
α	Degree of openness in the domestic economy	
	CHL	28.6
	AUS	19.2
	SA	25.0
	KSA	31.7
ϖ	share of consumption of commodities in the consumption basket of the domestic economy	
	CHL	8.7
	AUS	7.2
	SA	7.9
	KSA	2.2
ϖ^*	share of consumption of commodities in the consumption basket of the foreign economy	8.8
χ	inverse elasticity of substitution between *C&G* in the domestic economy	
	CHL	8.19
	AUS	67.6
	SA	5.3
	KSA	102.4
χ^*	inverse elasticity of substitution between *C&G* in the foreign economy	7.25
ϕ_g	response of domestic government consumption to changes in commodity prices	
	CHL	0.25
	AUS	-0.01
	SA	-0.04
	KSA	0.78
ρ_o	AR(1) coefficient of aggregate commodity supply	0.376
ρ_g	AR(1) coefficient of domestic government consumption	
	CHL	0.44
	AUS	0.58
	SA	0.35
	KSA	0.12
ρ_{g^*}	AR(1) coefficient of foreign government consumption	0.59
σ_o	standard deviation of an aggregate commodity supply shock	0.0166
σ_g	standard deviation of domestic government consumption	
	CHL	0.0286
	AUS	0.0122
	SA	0.0256
	KSA	0.1002
σ_{g^*}	standard deviation of foreign government consumption	0.0168

Figure C.1 Impulse Response Functions in the Australian



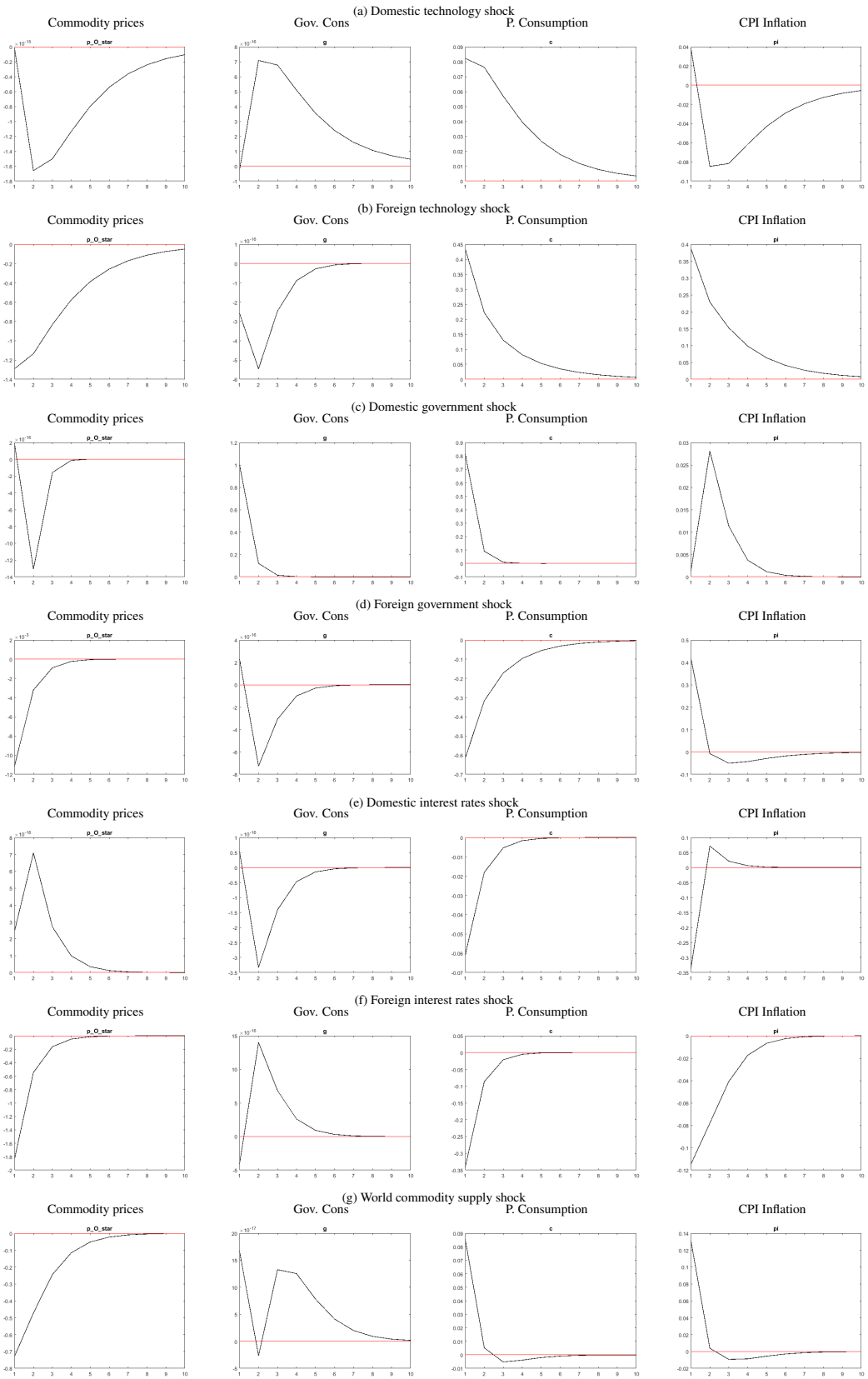
Note: all seven shocks are normalised to 1.

Figure C.2 Impulse Response Functions in the Chilean Economy



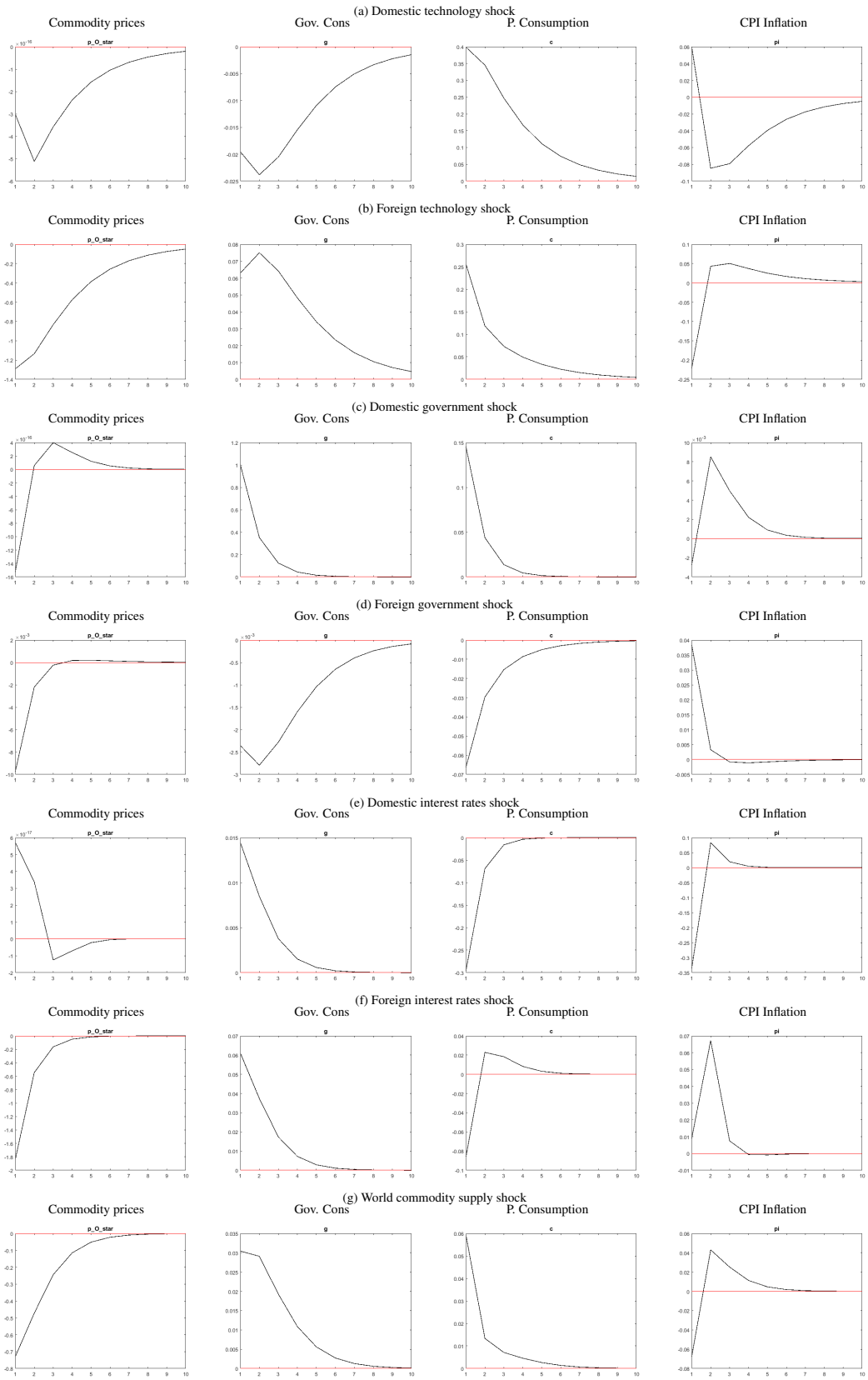
Note: all seven shocks are normalised to 1.

Figure C.3 Impulse Response Functions in the Saudi Economy



Note: all seven shocks are normalised to 1.

Figure C.4 Impulse Response Functions in the South African Economy



Note: all seven shocks are normalised to 1.

Appendix C.5

- Saudi Arabia

Table C.3 KSA Estimation Output

	Prior Mean	Posterior Mean	Posterior Mode	90% HPD Interval	Prior	Prior. Stdev
Parameter						
ρ_a	0.5	0.4957	0.5	0.1680-0.8196	Beta	0.2
ρ_{a^*}	0.5	0.6123	0.6213	0.5439-0.6809	Beta	0.2
ρ_r	0.5	0.977	0.9851	0.9590-0.9969	Beta	0.2
ρ_{r^*}	0.5	0.6478	0.7121	0.4002-0.9132	Beta	0.2
ρ_g	0.5	0.8666	0.8841	0.7762-0.9596	Beta	0.2
ρ_{g^*}	0.5	0.9345	0.9570	0.8792-0.9921	Beta	0.2
ρ_{o^*}	0.5	0.5086	0.5	0.1835-0.8377	Beta	0.2
χ	102.4	102.6194	94.6648	72.4744-134.1410	inv-Gamma	19.42
ϕ_g	0.78	0.1994	0.1826	0.1128-0.2866	Normal	0.14
Standard deviations						
ε_a	0.05	0.0474	0.0230	0.0115-0.0879	inv-Gamma	2.0
ε_{a^*}	0.05	14.8362	14.1357	11.7893-17.8101	inv-Gamma	2.0
ε_r	0.05	0.0476	0.0230	0.0114-0.0864	inv-Gamma	2.0
ε_{r^*}	0.05	9.8784	9.2001	7.7125-12.0271	inv-Gamma	2.0
ε_g	0.05	9.9372	9.1847	7.6239-12.1326	inv-Gamma	2.0
ε_{g^*}	0.05	8.2126	7.9655	6.5845-9.7930	inv-Gamma	2.0
ε_{o^*}	0.05	0.0449	0.0230	0.0118-0.0826	inv-Gamma	2.0

Figure C.5 Historical Decomposition of government consumption in KSA 1981-2015

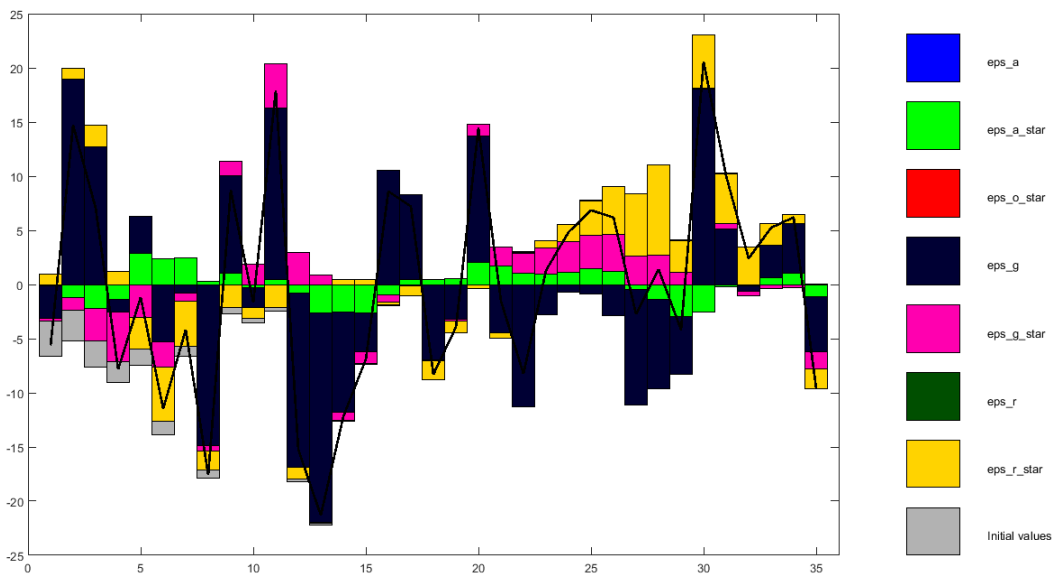


Figure C.6 Historical Decomposition of private consumption in KSA 1981-2015

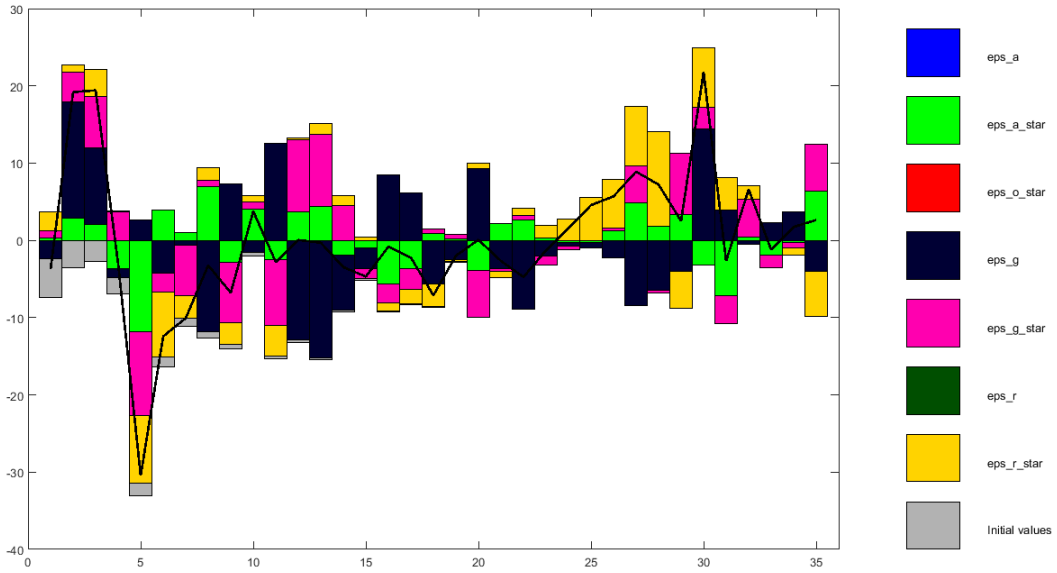
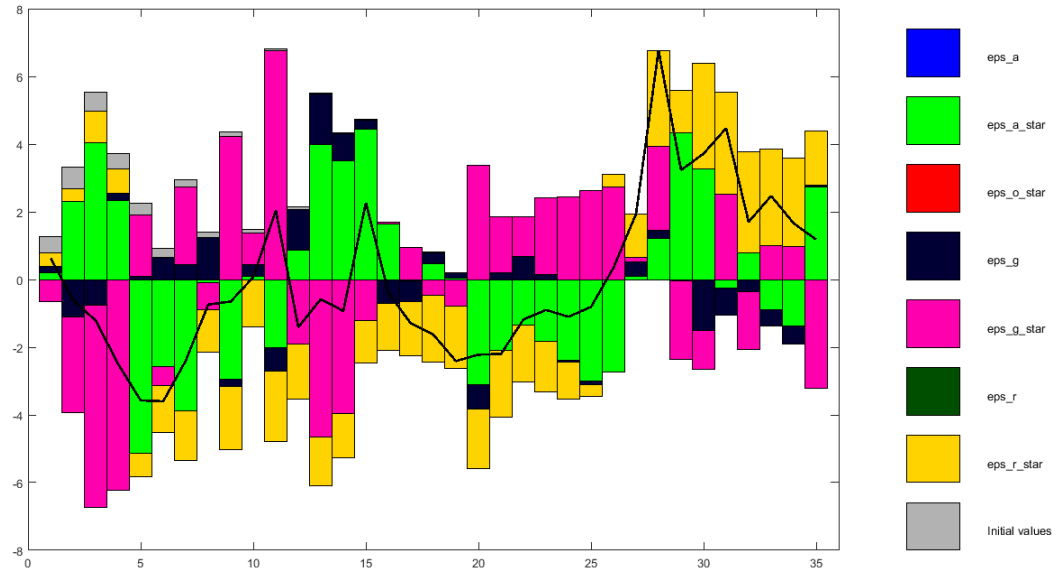


Figure C.7 Historical Decomposition of inflation in KSA 1981-2015



Australia

Table C.4 AUS Estimation Output

	Prior Mean	Posterior Mean	Posterior Mode	90% HPD Interval	Prior	Prior. Stdev
Parameter						
ρ_a	0.5	0.4930	0.5000	0.1703-0.8199	Beta	0.2
ρ_{a^*}	0.5	0.8371	0.8371	0.7444-0.9237	Beta	0.2
ρ_r	0.5	0.8765	0.9049	0.7822-0.9762	Beta	0.2
ρ_{r^*}	0.5	0.6877	0.7603	0.4524-0.9400	Beta	0.2
ρ_g	0.5	0.9035	0.9136	0.8402-0.9724	Beta	0.2
ρ_{g^*}	0.5	0.4949	0.5001	0.1701-0.8245	Beta	0.2
ρ_{o^*}	0.5	0.5007	0.5000	0.1768-0.8298	Beta	0.2
χ	67.6	67.8278	62.4937	47.6358-87.3705	inv-Gamma	12.82
ϕ_g	-0.01	-0.0122	-0.0124	-0.0206 - -0.0040	Normal	0.017
Standard deviations						
ε_a	0.05	0.0581	0.0230	0.0113-0.0973	inv-Gamma	2.0
ε_{a^*}	0.05	4.4787	4.3014	3.5909-5.3540	inv-Gamma	2.0
ε_r	0.05	3.0546	2.8909	2.4017-3.6755	inv-Gamma	2.0
ε_{r^*}	0.05	8.7136	8.2294	6.9810-10.4489	inv-Gamma	2.0
ε_g	0.05	1.1069	1.0447	0.8834-1.3229	inv-Gamma	2.0
ε_{g^*}	0.05	0.0589	0.0230	0.0111-0.0907	inv-Gamma	2.0
ε_{o^*}	0.05	0.0509	0.0230	0.0110-0.0899	inv-Gamma	2.0

Figure C.8 Historical Decomposition of government consumption in Australia 1981-2015

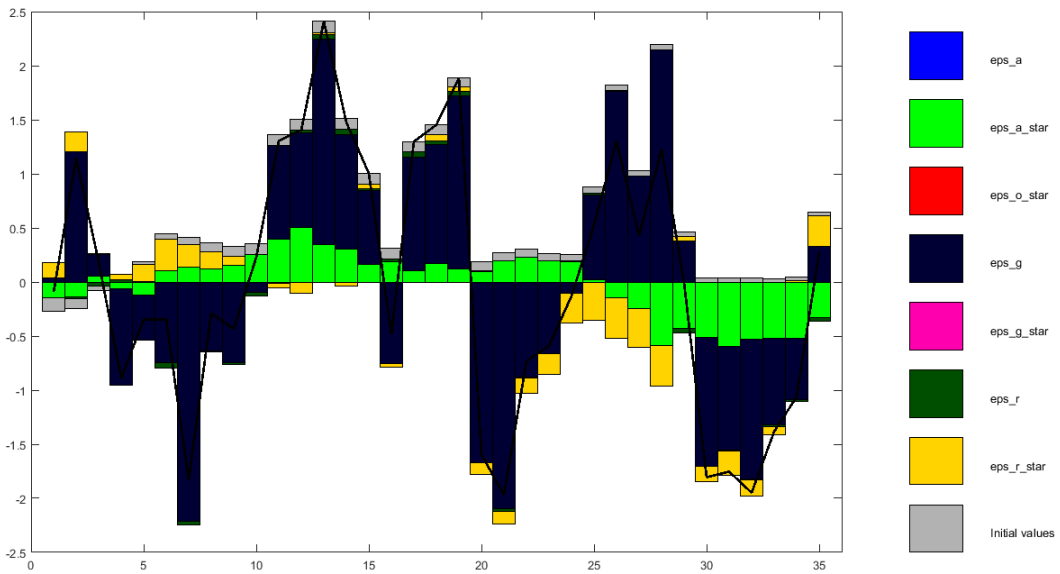


Figure C.9 Historical Decomposition of private consumption in Australia 1981-2015

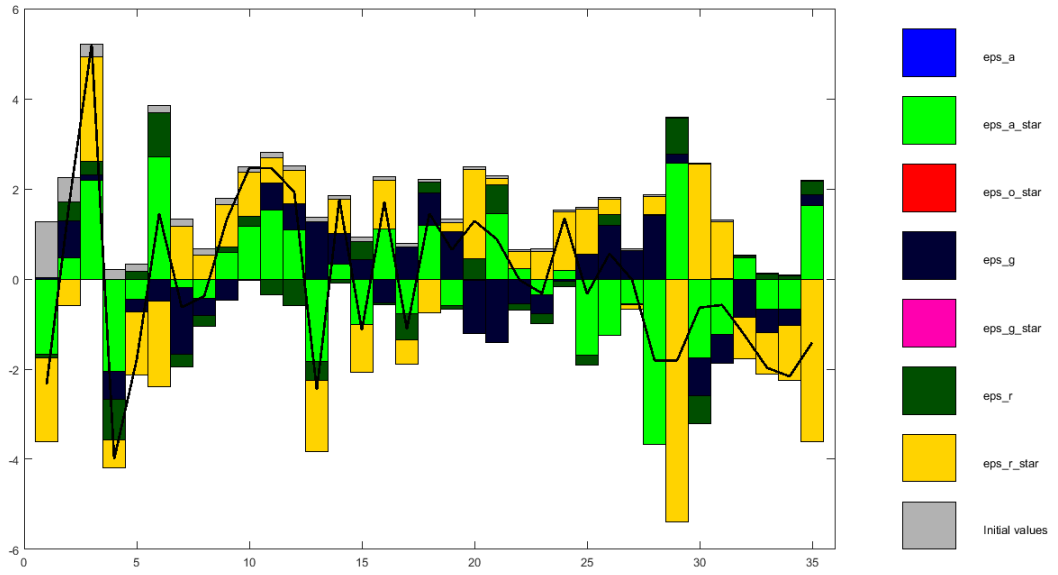
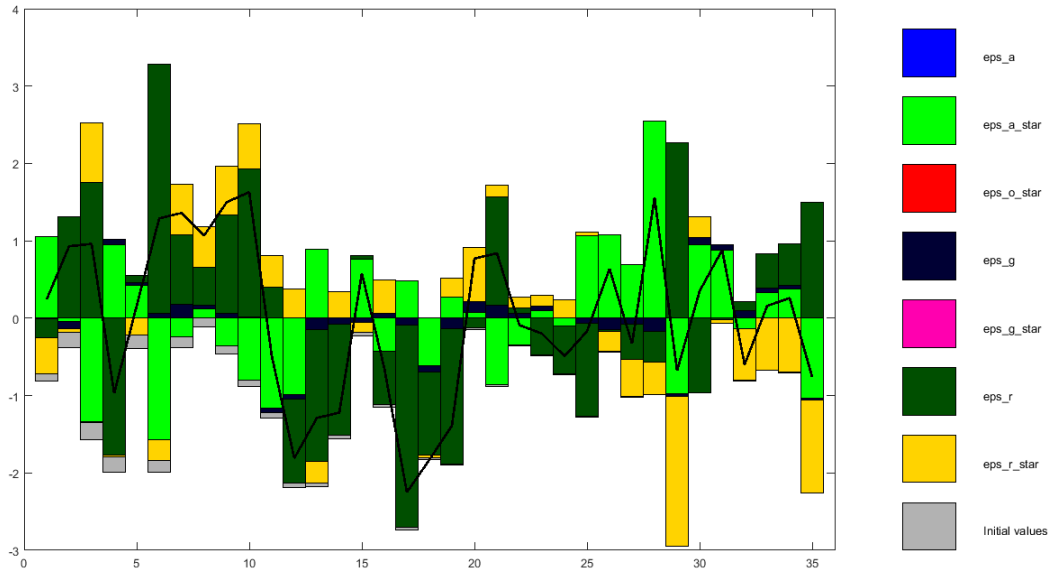


Figure C.10 Historical Decomposition of inflation in Australia 1981-2015



Chile

Table C.5 CHL Estimation Output

	Prior Mean	Posterior Mean	Posterior Mode	90% HPD Interval	Prior	Prior. Stdev
Parameter						
ρ_a	0.5	0.6155	0.6466	0.4147-0.7960	Beta	0.2
ρ_{a^*}	0.5	0.6640	0.9309	0.3015-0.9794	Beta	0.2
ρ_r	0.5	0.2166	0.0860	0.0078-0.4871	Beta	0.2
ρ_{r^*}	0.5	0.6416	0.5141	0.3480-0.9520	Beta	0.2
ρ_g	0.5	0.7512	0.7493	0.6465-0.8582	Beta	0.2
ρ_{g^*}	0.5	0.5157	0.5	0.1865-0.8364	Beta	0.2
ρ_{o^*}	0.5	0.5062	0.5	0.1731-0.8238	Beta	0.2
χ	8.19	7.6746	5.5958	3.3082-12.1583	inv-Gamma	4.84
ϕ_g	0.25	0.0921	0.0750	0.0565-0.1261	Normal	0.048
Standard deviations						
ε_a	0.05	25.7249	22.4146	18.3701-33.3067	inv-Gamma	2.0
ε_{a^*}	0.05	1.8607	4.1777	0.0089-5.7465	inv-Gamma	2.0
ε_r	0.05	24.4385	24.5309	19.2848-29.5408	inv-Gamma	2.0
ε_{r^*}	0.05	9.9686	5.5864	4.1676-15.2937	inv-Gamma	2.0
ε_g	0.05	2.4501	2.2099	1.9075-2.9527	inv-Gamma	2.0
ε_{g^*}	0.05	0.0472	0.0230	0.0111-0.0923	inv-Gamma	2.0
ε_{o^*}	0.05	0.0466	0.0230	0.0118-0.0837	inv-Gamma	2.0

Figure C.11 Historical Decomposition of government consumption in Chile 1981-2015

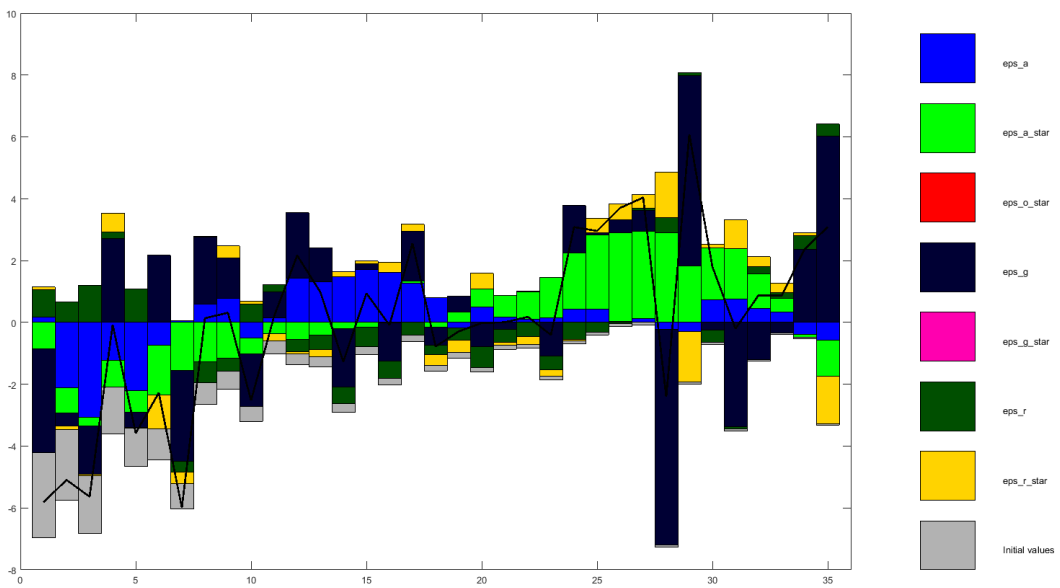


Figure C.12 Historical Decomposition of private consumption in Chile 1981-2015

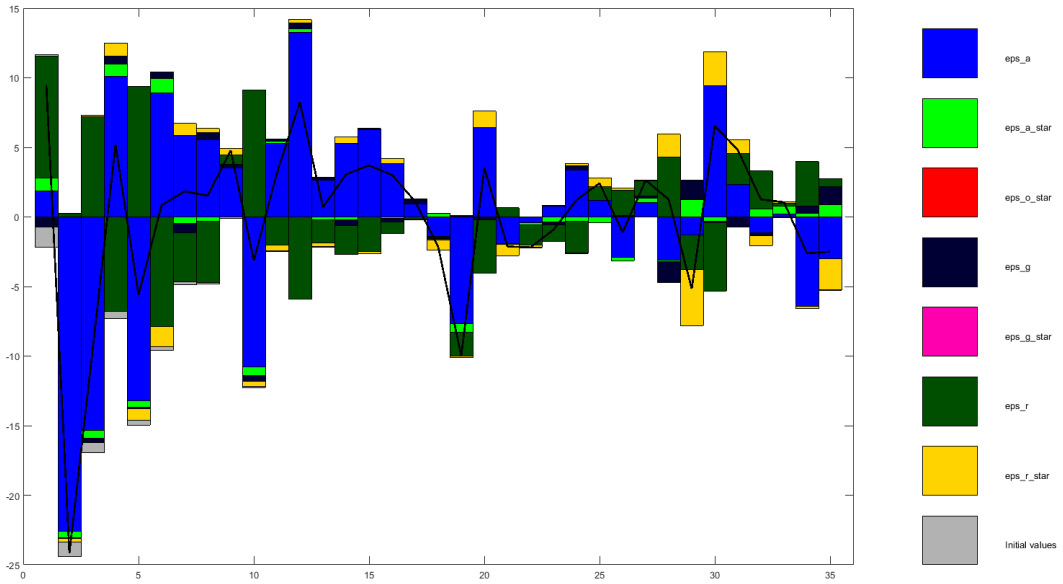
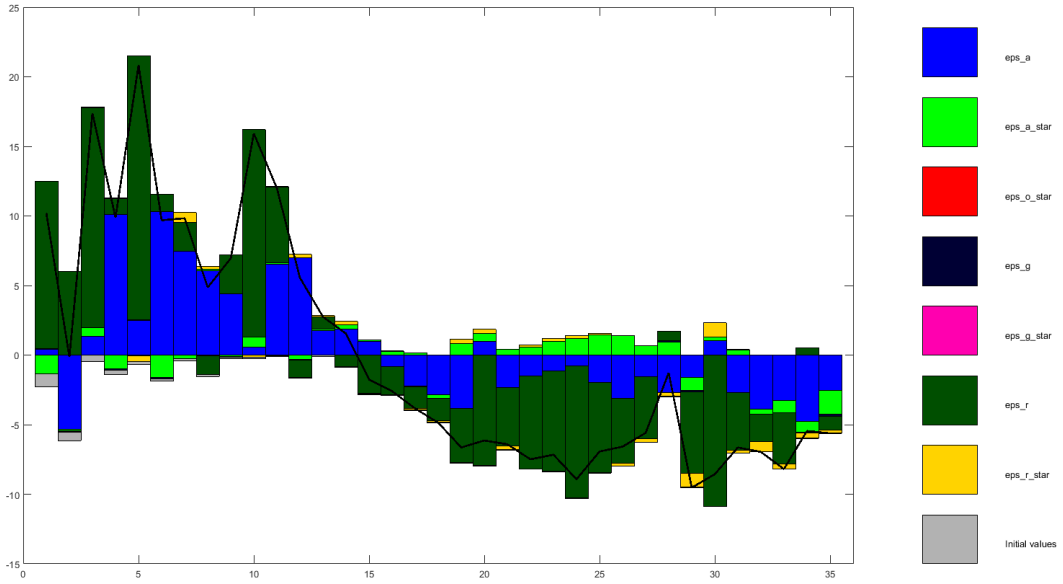


Figure C.13 Historical Decomposition of inflation in Chile 1981-2015



South Africa

Table C.6 SA Estimation Output

	Prior Mean	Posterior Mean	Posterior Mode	90% HPD Interval	Prior	Prior. Stdev
Parameter						
ρ_a	0.5	0.5689	0.5825	0.3897-0.7525	Beta	0.2
ρ_{a^*}	0.5	0.4915	0.5	0.1543-0.8121	Beta	0.2
ρ_r	0.5	0.7647	0.7955	0.6015-0.9364	Beta	0.2
ρ_{r^*}	0.5	0.7129	0.7835	0.4902-0.9498	Beta	0.2
ρ_g	0.5	0.8729	0.8773	0.8075-0.9433	Beta	0.2
ρ_{g^*}	0.5	0.4987	0.5	0.1724-0.8253	Beta	0.2
ρ_{o^*}	0.5	0.4967	0.5	0.1675-0.8263	Beta	0.2
χ	5.3	5.2948	4.0505	2.5256-8.1299	inv-Gamma	2.266
ϕ_g	-0.04	0.0419	0.0415	0.0244-0.0597	Normal	0.08
Standard deviations						
ε_a	0.05	13.8192	13.2063	10.6822-16.8417	inv-Gamma	2.0
ε_{a^*}	0.05	0.0487	0.0230	0.0113-0.0881	inv-Gamma	2.0
ε_r	0.05	7.3169	6.9333	5.7996-8.7983	inv-Gamma	2.0
ε_{r^*}	0.05	12.2104	11.5483	9.7760-14.5750	inv-Gamma	2.0
ε_g	0.05	2.1457	2.0189	1.7181-2.5694	inv-Gamma	2.0
ε_{g^*}	0.05	0.0470	0.0230	0.0116-0.0870	inv-Gamma	2.0
ε_{o^*}	0.05	0.0455	0.0230	0.0113-0.0837	inv-Gamma	2.0

Figure C.14 Historical Decomposition of government consumption in South Africa 1981-2015

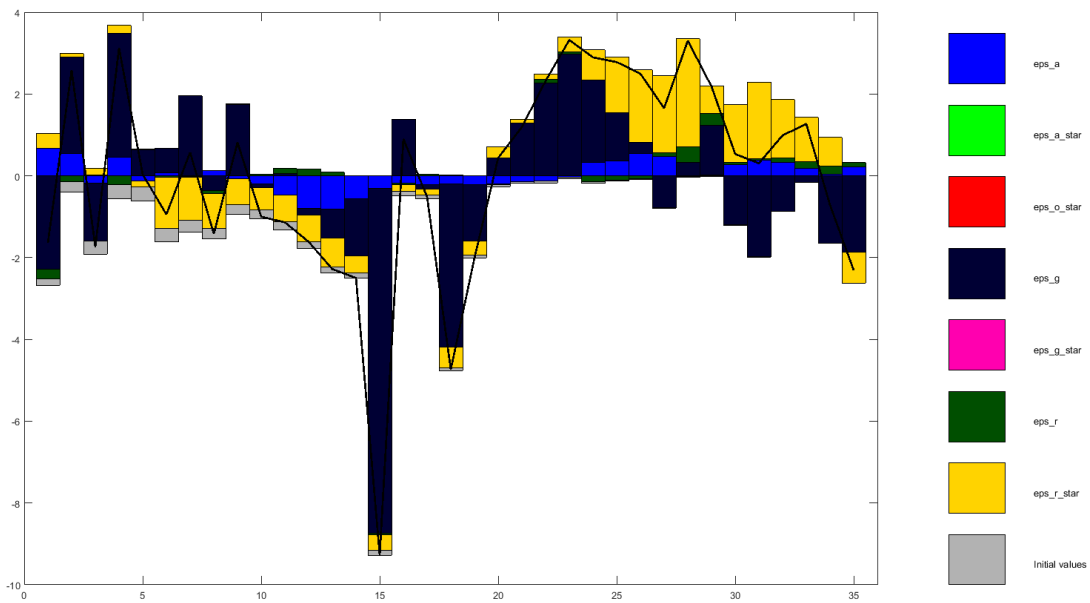


Figure C.15 Historical Decomposition of private consumption in South Africa 1981-2015

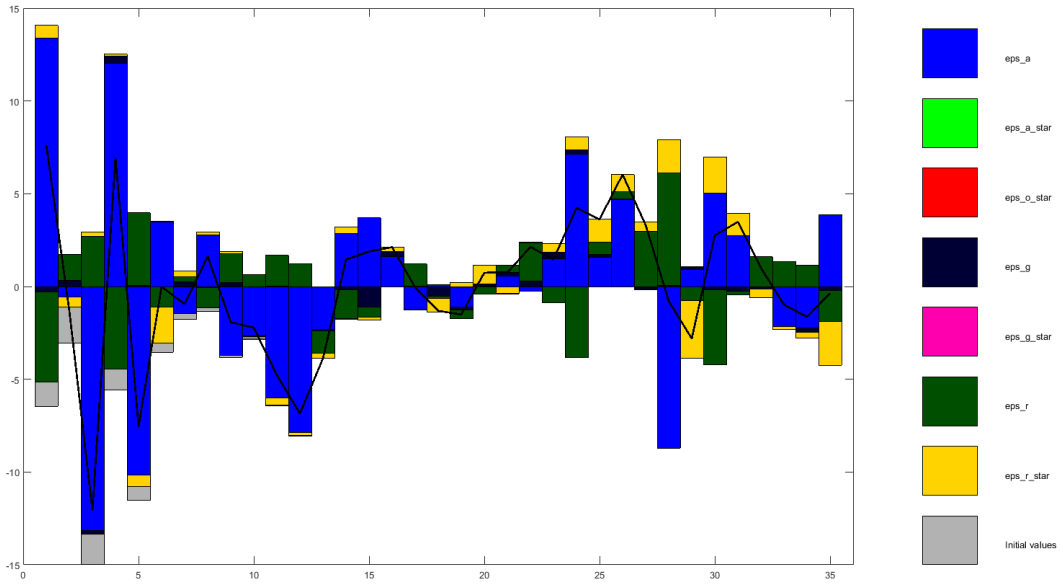


Figure C.16 Historical Decomposition of inflation in South Africa 1981-2015