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Frequency Response of a Moving Two-Dimensional Defect in Magnetic Flux Leakage Inspection

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Abstract—The magnetic fields produced by defects in magnetic flux leakage (MFL) enable the non-contact and non-destructive detection and characterisation of defects in magnetic structures for health and condition monitoring. The frequency response and bandwidth of the leakage magnetic fields have not been derived from physical field descriptions, which is fundamental for accurate sizing of defects from measured signals and for the appropriate design of MFL detection channels. In this letter, the Fourier transforms of the leakage fields from a two-dimensional surface defect are evaluated to produce analytical expressions for the frequency response of magnetic flux leakage (MFL) signals for flux sensitive elements. The derived expressions explicitly show the correlation between the spectral response of the leakage fields and defect dimensions and sensing element lift-off spacing, in the form of a product of frequency dependent defect width loss function, spacing loss function, and thickness enhancement function. The lower and upper bandedges of the band-limited leakage magnetic fields are theoretically identified. A spectral based method for sizing of defects is also proposed based on the frequency response derivation.

Index Terms—Electromagnetics, frequency response, magnetic flux leakage

I. INTRODUCTION

Magnetic flux leakage (MFL) inspection is a valuable non-destructive and non-contact technique for the detection of defects in magnetic structures for health and condition monitoring. The method involves applying a magnetising field in close proximity to the inspected magnetic medium and measuring the resulting leakage magnetic fields from defects, which are key for detection and characterisation of defects [Bray and Stanley 1997].

The physics of leakage fields are conventionally modelled analytically assuming simplified rectilinear defect geometry [Zatsepin and Shcherbinin 1966, Shcherbinin and Pashagin 1972] or cylindrical defect geometry [Mandache and Clapham 2003, Dutta et al. 2009], with magnetic charge sheets for the defect surfaces (under the assumption of constant material permeability) as sources for the stray magnetostatic fields. More detailed finite-element modelling of MFL fields from the solution of the quasi-static vector magnetic potential is also carried out to include the magnetizers, the nonlinear magnetic properties of the inspected material and to study eddy current effects (for example [Hwang and Lord 1975, Shin 1997]).

The derived leakage fields distributions using the magnetic charge sheet method provide good agreement with measured MFL signals (see cited references hitherto), and can be used in qualitative and quantitative methods to estimate physical defect dimensions from measured signals. However, the bandwidth and in particular the frequency response of the leakage fields in MFL is not known and has not been derived. The frequency response is fundamental to understand the correlation between the harmonic content of the leakage fields and physical defect dimensions and sensor/medium interface parameters. Moreover, the frequency response of the leakage fields is necessary for the optimal design of the MFL detection channel and accurate estimation of defect dimensions for the different scan speeds and defect feature sizes.

The Fourier transform is applied in this letter to derive analytically the frequency response of the leakage magnetic fields from a simplified two-dimensional, semi-infinite defect structure moving at a constant speed with respect to a fixed flux-sensitive sensing element. This approach has been previously used for the analysis of the harmonic content of fields from gapped magnetic recording heads (for example [Mallinson 1974, Lindholm 1975]). A spectral method is also proposed here for the sizing of defects based on the frequency response derivation. For simplicity and to keep the theory analytical, the geometry analysed is two-dimensional and ignores the presence of any permeable structures that modify the leakage fields in the vicinity of the sensing element (such as shields or flux concentrators). Moreover, the theory does not include transient phenomena including fields generated by eddy currents. The relative motion of the inspected medium with respect to the

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constant magnetizing field produce eddy current offset fields which are normally filtered out (electronically or through shielding). The inspected magnetic medium is assumed linear with constant permeability and ignores saturation effects, which are reduced in practice through control of the magnetizing field to avoid significant reductions in the local permeability in the defect region and altering the effective defect dimensions. Although the theoretical development here assumes flux-sensitive elements for sensing the leakage magnetic fields, the developed expressions can be easily differentiated to produce the frequency response in the case of inductive sensing elements.

II. THEORY

The theory presented here is for the two-dimensional, semi-infinite structure described in Fig. 1 for simplicity, which is applicable to defects with large extent in the *z*-direction compared to the other dimensions. The inspected medium (including the defect) is moving at a constant speed *v* relative to the fixed coordinate system shown in Fig. 1. The centre of this coordinate system corresponds to the fixed position of the flux-sensitive element, located at a distance *d* from the surface of the inspected medium. The defect has half-width *a* and depth *b*.

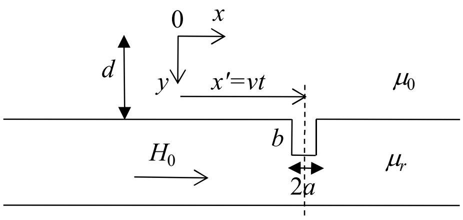

Fig. 1 Two-dimensional geometry used in the theory with a defect moving at velocity *v* with respect to the reference coordinate system corresponding to the fixed position of the flux-sensitive sensing element.

The two-dimensional magnetostatic fields beyond the defect surface, due to the application of the constant magnetizing field H_0 in Fig. 1, can be evaluated assuming that the two parallel surfaces of the defect are magnetic charge sheets. The magnetic fields due to a single charge sheet a distance $y = d$ from the reference coordinate system with uniform charge density σ_s and extending to infinity in the *z*-direction can be written as:

$$\boldsymbol{H}(x,y) = \frac{\sigma_s}{4\pi} \int_{y'=d}^{d+b} \frac{x\mathbf{i} - (y-d)\mathbf{j}}{x^2 + (y-y')^2} dy'$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the *x* and *y* directions respectively. Carrying out the integration for two parallel charge sheets located at $\pm a$ from the centre of the defect with charge densities $\pm\sigma_s$ and using superposition yields:

$$H_x(x';x,y) = \frac{\sigma_s}{2\pi}\left\{ \tan^{-1}\left[\frac{b(x-x'+a)}{(x-x'+a)^2 - (y-d)(b-(y-d))}\right] - \tan^{-1}\left[\frac{b(x-x'-a)}{(x-x'-a)^2 - (y-d)(b-(y-d))}\right]\right\} \quad (1)$$

$$H_y(x';x,y) = \frac{\sigma_s}{4\pi}\left\{ \ln\left[\frac{(x-x'+a)^2 + (b-(y-d))^2}{(x-x'+a)^2 + (y-d)^2}\right] + \ln\left[\frac{(x-x'-a)^2 + (y-d)^2}{(x-x'-a)^2 + (b-(y-d))^2}\right]\right\} \quad (2)$$

where $x' = vt$ defines the displacement of the centre of the defect from the centre of the reference coordinate system due to movement at constant velocity *v* at time *t*. Assuming the flux-sensitive element is located at the centre of the coordinate system ($x = y = 0$) reduces the two-dimensional defect fields to:

$$H_x(x') = \frac{\sigma_s}{2\pi}\left\{ -\tan^{-1}\left[\frac{b(x'-a)}{(x'-a)^2 + d(b+d)}\right] + \tan^{-1}\left[\frac{b(x'+a)}{(x'+a)^2 + d(b+d)}\right]\right\} \quad (3)$$

$$H_y(x') = \frac{\sigma_s}{4\pi}\left\{ \ln\left[\frac{(x'-a)^2 + (b+d)^2}{(x'-a)^2 + d^2}\right] + \ln\left[\frac{(x'+a)^2 + d^2}{(x'+a)^2 + (b+d)^2}\right]\right\} \quad (4)$$

The influence of the magnetizing field H_0 in the inspected medium of relative permeability μ_r may be included by expressing the surface charge density in terms of H_0 for an equivalent semi-elliptic cylindrical cavity following Edwards et al. [1986]:

$$\sigma_s = \frac{\pi\mu_r H_0}{\left(\mu_r - N_a(\mu_r - 1)\right)\tan^{-1}(b/a)} = \frac{n\pi(\mu_r + 1)H_0}{(\mu_r + n)\tan^{-1}(n)} \quad (5)$$

where $N_a = b/(a+b)$ is the demagnetising factor of the elliptic cylinder along the direction of the applied field, and $n = b/a$ is elliptical cavity aspect ratio. The elliptical cavity approximation was adopted here to yield simple analytical expressions for the field spectra, and provided good agreement with experimental MFL measurements [Edwards et al. 1986, Förster 1986]. A discussion on equivalent ellipsoid approximations and associated corrections is provided by Beleggia et al. [2006].

The normalised two-dimensional leakage field components are plotted in Fig. 2 for sensor-medium separation $d = 2a$ and when $\mu_r > n$. The longitudinal field component H_x is even with peak at the centre of the sensing element, while the vertical field component H_y is odd with peaks near the defect corners. Increasing the defect depth while keeping its width constant (with $b \geq a$) increases the magnitude of the leakage fields due to the increase of surface charge contribution with increasing depth.

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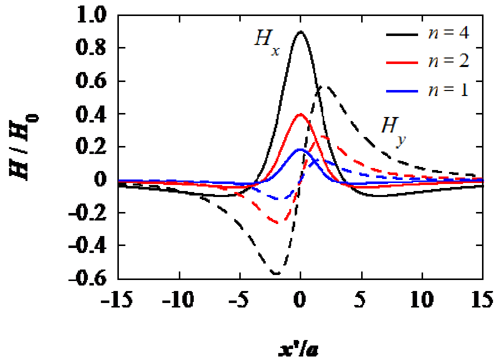

Fig. 2 Longitudinal (solid lines) and vertical (dashed lines) leakage field components for d = 2a and μ_r >> n.

The frequency response of the two-dimensional fields can be derived from evaluating their Fourier transform in the direction of movement. The Fourier transform of function $g(t)$ defined in terms of the angular frequency ω is:

$$G(\omega)=\int_{t=-\infty}^{\infty} g(t)e^{-j\omega t}dt$$

Using $x'=vt$ yields the spatial Fourier transform $G(k)$ with wavenumber $k=\omega/v=2\pi/\lambda$ where λ is the wavelength:

$$\begin{aligned}G(\omega)&=\int_{x'=-\infty}^{\infty} g(x')e^{-j\omega x'/v}dx'/v\\&=\int_{x'=-\infty}^{\infty} g(x')e^{-jkx'}dx'/v=\frac{G(k)}{v}\end{aligned} \quad (6)$$

which is more conveniently used to determine the frequency/wavelength response of the two-dimensional defect leakage fields next.

Applying the spatial Fourier transform to H_x in (3) [Mansuripur 1998] noting that the arctangent function is odd and utilising the translation property of the Fourier transform [Brigham 1988], the Fourier integral may be evaluated exactly using [Gradshteyn and Ryzhik 2015] to yield the frequency response:

$$H_x(k)=\sigma_s a\left[\frac{\sin(ka)}{ka}\right]\cdot\left[\exp(-kd)\right]\cdot\left[1-\exp(-kb)\right] \quad (7)$$

which is real since H_x is even (see Fig. 2). The Fourier transform of H_y can be determined directly from (7) noting the orthogonal nature of two-dimensional magnetostatic fields beyond semi-infinite surfaces [Bertram 1994], which are shifted in phase by $\pm\pi/2$. Hence the Fourier transform of the vertical field component is given by:

$$H_y(k)=\pm j\,\mathrm{sgn}(k)H_x(k) \quad (8)$$

where sgn(k) is the Signum function, and the spectrum in (8) is imaginary due to the odd symmetry of the vertical defect field as shown in Fig. 2.

III. RESULTS AND DISCUSSION

The terms in square brackets in the Fourier transform of the leakage field in (7) provide direct correlation between the losses/enhancement in the MFL signal and the physical dimensions of the defect and interface parameters. The first term is a sinc function responsible for producing nulls in the spectrum at integer multiples of the wavenumber $k = \pi/a$ (or wavelength $\lambda = 2a$) and represent the defect width loss in the MFL spectrum. Hence the first null practically defines the upper bandedge of the leakage signal at $\omega = \pi v/a$. The second term in square brackets $\exp(-kd)$ is the spacing loss term responsible for the exponential field attenuation with increasing separation d per wavelength from the defect surface. The final term in square brackets describes the signal enhancement with increasing defect depth b. For a constant defect depth b this term approaches unity at high frequencies therefore affecting mainly the low frequency (wavenumber) part of the MFL spectrum. It is interesting to note that the frequency response in (7) share similar features to the readout spectrum in magnetic recording, which also involves the detection of spatial changes of the recorded magnetisation in the moving recording medium with a gapped head structure [Bertram 1994].

The normalised magnitude of the leakage field spectrum of H_x in (7) is plotted in Fig. 3 for increasing defect depth (and constant defect width). This plot shows the first three nulls in the spectrum at integer multiples of the wavenumber π/a, and illustrates the enhancement in leakage field magnitude with increasing defect depth b at low frequencies. Fig. 4 illustrates the effect of the spacing loss term in increasing the roll-off of the MFL spectrum at high frequencies (large wavenumbers) with increasing distance from the inspected medium surface.

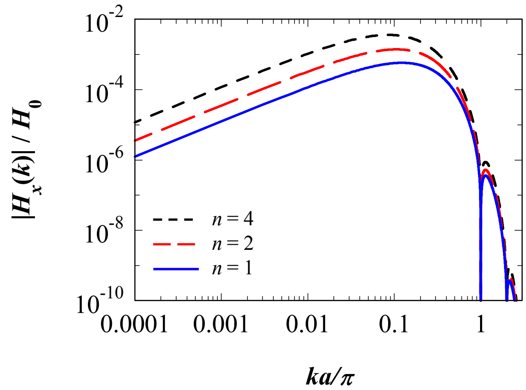

Fig. 3 Spectrum of two-dimensional leakage field determined using (7) for d = 2a, μ_r >> n and constant defect width for different defect depths.

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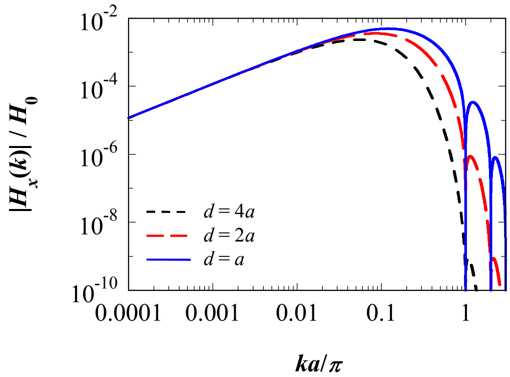

Fig. 4 Two-dimensional spectrum of leakage field determined using (7) for $b = 4a$ and $\mu_r >> n$ for different surface separations d.

The frequency response expressions in (7) and (8) are in the form of products of frequency dependent surface field terms (related to defect dimensions) multiplied by the spacing loss term. The spacing loss term is a natural outcome to the solution to Laplace's equation for potentials or fields beyond semi-infinite surfaces in two-dimensions and therefore a common term for any defect shape. Thus the leakage fields can be written as the convolution of the magnetic fields at the surface of the inspected medium, which are directly related to the physical defect dimensions, and the spacing loss kernel in the from:

$$\boldsymbol{H}(x,y) = \frac{1}{\pi} \int_{x'=-\infty}^{\infty} H_x(\bar{x},0) \frac{y\mathbf{i} - (\bar{x}-x)\mathbf{j}}{(\bar{x}-x)^2 + y^2} d\bar{x} \tag{9}$$

This form allows the spacing loss term to be factored out from MFL measurements, and enable the application of spectral methods (such as inverse filtering [Wells 1985]) to extract the spatial distribution of the surface fields of defects from MFL measurements and estimating defect sizes [Lukyanets et al. 2003].

At low frequencies or wavenumbers, the leakage field spectrum in (7) can be expanded for small values of k to first-order to yield:

$$H_x(k)\big|_{k\to 0} \approx \sigma_s ab \cdot k \tag{10}$$

which is zero at DC, linear with k (or ω/v) and with slope determined by the defect dimensions. Thus the spectrum of the defect leakage field is band-pass with the lower bandedge governed by the defect dimension a and b, and the upper bandedge controlled by the defect width a in the first spectral null.

The band-pass nature of the spectrum of the two-dimensional leakage fields and the explicit dependence of the bandedges on defect dimensions suggest a method of extraction of defect dimension from measured MFL spectra. The first null in the MFL spectrum enable estimation of the defect width according to $a \sim \pi / k_{null}$. The defect depth can then be estimated from the slope of the spectrum at low frequencies from (10) where $b \sim 1/(\sigma_s a) dH(k)/dk$, using knowledge of the surface charge density. Further work is needed to validate this proposed sizing approach and its sensitivity to the assumed surface charge density (for example in (5)). Alternatively, the slope of the spectrum at long wavelengths can be normalised by the peak field in (3), $H_x(x' = 0)$, to eliminate σ_s and solve the resulting transcendental equation for b using knowledge of the lift-off distance d.

The leakage fields in (1) and (2) were derived assuming uniform magnetic charge distributions on the inner defect surfaces (normal to the in-plane magnetization). This approximation yielded good agreement with MFL measurements at practical distances from the defect surface (for example [Zatsepin and Shcherbinin 1966, Edwards et al. 1986]). The charge distribution on the defect surfaces is however not uniform and becomes very large near to the corners [Bertram 1994]. Rigorous Fourier series solution of Laplace's equation for the two-dimensional gapped semi-infinite structure (equivalent to the defect structure considered here), with non-uniform surface charges, indicated enhancement of the magnetic fields near the corners of the structure at close separations from the surface of the defect [Middleton et al. 2000]. At increasing separations, the fields become identical to the uniform charge solutions. The non-uniform surface charge in this case cause slight shifting of the defect width (gap) nulls in the field spectrum towards longer wavelengths (lower frequencies) in comparison to uniform charge case. Thus the frequency response developed in this article is expected to be applicable to a wide class of two-dimensional symmetrical defect shapes with similar magnetic charge concentrations near the defect corners, producing spectral nulls at integer multiples of $k = \pi/a$ [Aziz et al. 2016a]. Asymmetrical (tilted) defects with small tilt angles can still produce detectable width nulls in the leakage field spectrum due to the large concentration of charges in the corner regions. However large tilt angles can lead to reduction of the effective defect width and smearing of the nulls in the frequency response [Aziz et al. 2016b]. Further investigation is needed to understand the dependence of the frequency response of the leakage fields from asymmetrical defects with finite depth and realistic charge distribution on the tilt angle.

IV. CONCLUSION

The frequency response of the leakage fields for a two-dimensional defect moving at constant speed is derived analytically in this letter. The derived expressions show explicitly the relation between the spectral content of the defect fields and physical defect dimensions and lift-off distance. The bandwidth and bandedges of the MFL spectrum were also identified as functions of the defect dimensions. A method is proposed for the sizing of defects using the frequency response derivation.

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