Extreme downside risk and market turbulence

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We investigate the dynamics of the relationship between returns and extreme downside risk in different states of the market by combining the framework of Bali, Demirtas, and Levy (2009) with a Markov switching mechanism. We show that the risk-return relationship identified by Bali, Demirtas, and Levy (2009) is highly significant in the low volatility state but disappears during periods of market turbulence. This is puzzling since it is during such periods that downside risk should be most prominent. We show that the absence of the risk-return relationship in the high volatility state is due to leverage and volatility feedback effects arising from increased persistence in volatility. To better filter out these effects, we propose a simple modification that yields a positive tail risk-return relationship in all states of market volatility.

Keywords: Downside risk; Tail risk; Markov switching; Value-at-Risk; Leverage effect; Volatility feedback effect

JEL classification: C13, C14, C53, G10, G12

1. Introduction

The notion of tail risk, or extreme downside risk, has become increasingly prominent in the asset pricing literature. In particular, in contrast with the assumptions of the standard CAPM of Sharpe (1964) and Lintner (1965), in which portfolio risk is fully captured by the variance of the portfolio return distribution, asset returns display significant negative skewness and excess kurtosis, both of which increase the likelihood of extreme negative returns. A number of studies have examined the importance of these higher moments for asset pricing. Kraus and Litzenberger (1976) develop a three-moment CAPM, in which expected returns are determined, in part, by co-skewness with the market portfolio. This finding is supported by Harvey and Siddique (2000), who consider the role of co-skewness in a conditional asset pricing framework. Lamperiere *et al.* (2016) find evidence that skewness is a significant determinant of the risk premium using both international and cross-asset data. Dittmar (2002) develops a non-linear pricing kernel with an endogenously determined risk factor and shows that co-kurtosis is also priced. Using moments of the return distribution implied by option prices, Conrad *et al.* (2013) show that the risk-neutral skewness and kurtosis of individual securities are strongly related to their future returns. Ang *et al.* (2006) find that co-moment risks are still significant even after general downside risk is taken into account through a downside beta measure.

Other studies focus directly on the likelihood of extreme returns, rather than indirectly on the moments of the return distribution. For example, Chabi-Yo *et al.* (2018) use a copula-based approach to construct a systematic tail risk measure and show that stocks with high crash sensitivity, measured by lower tail dependence with the market, are associated with higher returns that cannot be explained by traditional risk factors, downside beta, co-skewness or co-kurtosis. Relatedly, Huang *et al.* (2012) propose a measure of idiosyncratic extreme downside risk based on the tail index of the generalised extreme value distribution, and show that it is associated with a premium in cross-section stock returns, even after controlling for market, size, value, momentum, and liquidity effects. Bali *et al.* (2014) note the difficulties in constructing robust measures of both systematic and idiosyncratic tail risk. They introduce a hybrid tail risk measure that incorporates both market-wide and firm-specific components and show that this yields a robust and significantly positive tail risk premium. Todorov and Bollerslev (2010) decompose the systematic risk of individual stocks into its continuous (diffusion) and discontinuous (jump) components and find that in most cases, jump betas are, on average, larger than diffusion betas. Moreover, jump betas display significant variation through time.

The studies described above examine the variation in expected returns caused by differences in tail risk across individual stocks. An alternative strand of the literature is concerned with the variation in tail risk over time, and its impact on aggregate equity returns. This is a more challenging objective owing to potential endogeneity in the measure of tail risk that serves to obscure the risk-return relation that would be predicted by asset pricing theory. For example, since investors prefer positive skewness, an investment with higher skewness should correspond to lower expected returns. However, skewness is, by construction, associated with large positive returns and so there will be a tendency for skewness to be positively related to returns. Additionally, owing to leverage and volatility feedback effects, high volatility tends to be associated with lower contemporaneous returns (see, for example, Black, 1976; Campbell and Hentschel, 1992). As a result, market tail risk measures such as Value-at-Risk (VaR) and Expected Tail Loss, which are positive functions of return volatility, will tend to have a negative relation with returns. Recognising this difficulty, Kelly and Jiang (2014) develop a measure of aggregate market tail risk that is based on the common component of the tail risk of individual stocks. They show that this tail risk measure is highly correlated with the tail risk implied by equity options, and that it has significant predictive power for aggregate market returns. Similarly, Allen *et al.* (2012) construct an aggregate systemic tail risk measure for the financial and banking system from the returns of financial firms and show that it can robustly predict economic downturns in the U.S., European and Asian markets.

Other studies, instead of extracting information from the cross section of individual stocks, investigate the impact of market tail risk on returns though the decomposition of variance or the variance risk premium into diffusion and jump components (see, for example, Bollerslev and Todorov, 2011; Bollerslev *et al.,* 2015; Guo *et al.,* 2014; Bandi and Reno, 2016, among others). Many of these studies specifically emphasize the impact of negative jumps on future returns and variance. For example, Guo *et al.* (2014) find evidence of a positive and significant equity premium attributable to negative jumps, while the premium for positive jumps is negative and insignificant. Bollerslev *et al.* (2015) show that left jump variation significantly predicts future returns at different horizons. More importantly, it accounts for most of the return predictability of the variance risk premium. Patton and Sheppard (2015) develop the framework of Barndorff-Nielsen *et al.* (2010) and show that the sign of jumps also affect future volatility.

A more direct approach to examining the intertemporal relation between stock market returns and tail risk is introduced in Bali *et al.* (2009) (hereafter BDL). In order to circumvent the inherent endogeneity of empirical measures of tail risk discussed above, they measure tail risk by the previous month's one-month ahead expectation of the VaR of the market return. Using monthly data over the period July 1962 to December 2005, they show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and is robust to different VaR measurement methods, different VaR confidence levels, alternative measures of tail risk, different measures of the market return and the inclusion of macroeconomic control variables to control for business cycle effects.

In this paper, we investigate the nature of the relation between returns and tail risk under different market conditions. This is motivated by empirical evidence that other, closely related risks, such as volatility and co-skewness risk, affect returns differently in alternative states of the world (see, for example, Friend and Westerfield, 1980; Guidolin and Timmermann, 2008). Feunou *et al.* (2013) show in an equilibrium consumption-based model that the price of risk is time variant and is a function of conditional skewness. Similarly, Li and Li (2015) develop a consumption CAPM within a jump-diffusion economy and show that the jump risk premium is time-varying and dependent on the jump times of aggregate consumption. Bekaert and Engstrom (2017) develop the Bad Environment-Good Environment (BEGE) model based on Campbell and Cochrane's (1999) habit formation framework and show that the risk premium associated with the bad environment is larger than that associated with the good environment. The cornerstone of the BEGE model is the introduction of a good shock and a bad shock simultaneously into the stochastic process for consumption growth. In other words, the consumption growth process is always the combination of these two simultaneous shocks, and the dominance of either shock at any time creates the good or bad environment. This idea is consistent with a two-state Markov switching process for the market, where the market condition at any time is the probability combination of a good state and a bad state. Motivated by this model, in order to model the state-dependent relation between tail risk and returns, we incorporate the BDL model into a two-state Markov switching framework.

We estimate the Markov switching model using an extended sample that covers the period July 1962 to December 2016, and which includes the last financial crisis. The two states in the estimated Markov switching model comprise a relatively infrequent high volatility state and a relatively frequent low volatility state. We find that the positive tail risk-return relation documented by BDL holds in the low volatility state, but disappears in the high volatility state. This result is consistent with Ghysels, Guerin and Marcellino (2014) who investigate the state-dependent relationship between returns and conditional volatility and find that the risk-return trade-off is positive in the good regime, but negative in the 'flight-to-quality' regime. We offer an explanation for this phenomenon, which is equally applicable to both volatility and tail risk.

The failure of the BDL model to capture the risk-return relationship in the turbulent state is counter-intuitive since tail risk is expected to be more relevant during such periods. In order to account for possible omitted variable bias, we expand the set of state variables that are included in the original BDL model to control for business cycle effects. This significantly improves the goodness of fit of the models as well as the implied Cochrane (1999) maximum unconditional Sharpe ratios, but the absence of a significantly positive tail risk-return relationship in the turbulent period remains. We also consider the possibility that the results are driven by the non-*iid* nature of the return generating process, and hence compute tail risk measures using returns that are standardised by time-varying conditional mean and volatility. This generates a positive risk-return relation in the turbulent period, but it is not statistically significant in any model.

The BDL model critically depends on the assumption that leverage and volatility feedback effects dissipate within one month so that the one-month ahead expectation of VaR, lagged by one month, can be considered pre-determined. We show, however, that leverage and volatility feedback effects take longer to dissipate during periods of high volatility and so the one-month ahead expectation of VaR is endogenous, even when lagged by one month. In order to circumvent the endogeneity of the tail risk measures in the high volatility state, we consider longer horizon expectations of market VaR at correspondingly longer lags. We show that using the two-month ahead expectation of VaR, lagged by two months, there is a statistically significant and positive relation between market returns and tail risk in both states.

This modification works consistently well with all the VaR-based tail risk measures that we consider, from the simple non-parametric measures to the more sophisticated non-*iid* parametric measures. It is also robust to the use of other systematic tail risk measures proposed in the literature, such as the Expected Tail Loss, the Left Jump Variation of Bollerslev *et al.* (2015) and the risk-neutral Expected Shortfall of Almeida *et al.* (2017).

Moreover, the modified measures are significant even after controlling for the conditional variance of returns in the regression model, implying that tail risk contains incremental information beyond that contained in dispersion risk. Exploiting the return predictability of the modified measures we construct an investment strategy using state-dependent tail risk to predict future returns and find that this strategy performs well, even in out-of-sample analysis.

Finally, we investigate the term structure of the tail risk-return trade-off in different states of the market by estimating the Markov switching predictive regression for future returns at different horizons. We find a consistent and significant positive tail risk premium in the calm state across all horizons, while the premium varies significantly from positive to negative across different horizons in the turbulent state. This result, again, emphasises the importance of market state information in shaping investment decisions, especially for those based on the tail risk premium.

The remainder of the paper is organised as follows. Section 2 describes the methodology and the data used in the empirical analysis. Sections 3 and 4 report the empirical results. Section 5 examines the robustness of our findings, while Section 6 examines the out-of-sample performance of the predictive regression. Section 7 presents the term structure of the tail risk-return trade-off. Section 8 provides a summary and offers some concluding remarks.

2. Methodology and data

2.1. Methodology

2.1.1. The BDL framework. In order to examine the dynamics of the relationship between tail risk and return, we utilise the framework of BDL, which we briefly summarise in this section. BDL measure tail risk by VaR, which, for a given cumulative distribution function of returns F_r and confidence level α, is defined as

$$\text{VaR} = -F_{\text{r}}^{-1}(1 - \alpha) \quad (1)$$

The impact of tail risk on returns is captured by regressing the value-weighted excess market return in month $t + 1$, R_{t+1}, on the month t forecast of VaR in month $t + 1$, $E_t(VaR_{t+1})$,[†] and a set of control variables X_t:

$$R_{t+1} = \alpha + \beta E_t(VaR_{t+1}) + \gamma X_t + \varepsilon_{t+1} \quad (2)$$

The control variables, X_t, include macroeconomic variables to proxy for business cycle fluctuations, the lagged excess market return and a dummy variable for the October 1987 crash. The risk-return relationship is reflected in the sign, magnitude, and significance of the coefficient β. When the market is in equilibrium, the realised future return for an asset equals its expected return. Thus, the BDL predictive regression framework essentially captures the expected risk-expected return relationship. As we show in Section 4, this is a simple but effective method to mitigate the large and confounding impact of the leverage and volatility feedback effects governing the negative relationship between realised risk and realised returns.

BDL measure VaR both parametrically and non-parametrically, using a rolling sample of daily returns over the most recent one to six months. Parametric VaR is obtained by fitting the Skewed Student-t distribution of Hansen (1994) to the rolling sample and calculating the

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[†] Strictly speaking, $E_t(VaR_{t+1})$ is the conditional expectation of VaR_{t+1}, which is not observed, and the forecast of VaR_{t+1} is the *estimated* conditional expectation. However, we retain this notation for the sake of simplicity.

corresponding quantile. Non-parametric VaR is measured as the quantile of the empirical distribution of returns over the rolling sample. For example, BDL use the lowest return over the last one month, which corresponds to a VaR confidence level of 95.24%, assuming that there are 21 trading days each month.[†]

BDL estimate the conditional expectation of VaR using two approaches. First, they assume that $E_t(VaR_{t+1}) = VaR_t$, which would be equal to the true conditional expectation if VaR follows a martingale-difference sequence. Second, they assume that VaR is mean-reverting and estimate an AR(4) model:

$$VaR_t = \theta_0 + \sum_{i=1}^{4} \theta_i VaR_{t-i} + v_t \tag{3}$$

The forecast of VaR in month $t+1$ is then given by $E(VaR_{t+1}) = \hat{\theta}_0 + \sum_{i=1}^{4} \hat{\theta}_i VaR_{t+1-i}$. We refer to these two measures as random walk (RW) VaR and AR4 VaR, respectively. BDL estimate the regression given by (2) using monthly data over the period July 1962 to December 2005, and show that there is a statistically and economically significant positive relation between market returns and tail risk. Moreover, the relationship between returns and tail risk is stronger than between returns and conditional volatility, and is robust to the different VaR measurement frameworks, different VaR confidence levels, alternative measures of tail risk and different measures of the market return.

An important aspect of the BDL approach is that they use the estimate of the conditional expectation of the risk measure, rather than its realisation, in order to offset the leverage and

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[†] While using the lowest return of the month could introduce considerable downward bias in the VaR estimation, the use of this over-simplified non-parametric measure is simply to highlight the prominence of the risk-return relationship that we investigate. The parametric Skewed Student-t VaR is used as an alternative in the analysis and produces quantitatively similar results

volatility feedback effects in returns. The use of the one-month ahead expectation, lagged by one month, implicitly assumes that these leverage and volatility feedback effects are short lived, lasting no longer than a month. This subtle but important observation is the basis of our modification of the BDL framework, as detailed in Section 4.

2.1.2. Tail risk in different market states: the Markov switching model. In order to examine the state-dependent dynamics of the tail risk-return relationship, we incorporate the BDL model in a Markov switching framework. The Markov switching framework has been applied in a number of different contexts to model changes in the behaviour of a time series with respect to different states of some underlying variable (see, among others, Hamilton, 1989; Hamilton, 1990; Gray, 1996; Nikolsko-Rzhevskyy and Prodan, 2012). Indeed, many studies have employed the Markov switching framework to examine the time-varying impact of volatility. For example, Turner *et al.* (1989) employ a Markov switching model to examine how the expectation of market volatility affects excess returns in different market conditions. Similarly, Chang-Jin *et al.* (2004) use Markov switching to directly model volatility feedback effect on returns. Given the large number of control variables in the BDL model, we choose the simplest setting with a first-order Markov process and two regimes. This is perhaps the most widely used variant of the Markov switching model in empirical studies (see, for example, Bansal and Hao, 2002; Guidolin and Timmermann, 2006; Bekaert *et al.,* 2015). The Markov switching BDL (hereafter MS-BDL) model is given by:

$$R_{t+1} = \alpha_{S_{t+1}} + \beta_{S_{t+1}} E_t(VaR_{t+1}) + \gamma_{S_{t+1}} X_t + \varepsilon_{S_{t+1}} \quad (4)$$

where $\varepsilon_{S_t} \sim N\left(0, \sigma_{S_t}^2\right)$ and $S_t = \begin{cases} 1, if\ state\ 1\ occurs\ at\ time\ t \\ 2, if\ state\ 2\ occurs\ at\ time\ t \end{cases}$.

Our framework captures the dynamic risk-return relationship through the coefficient β in the low volatility state ($\sigma_{S_t} = \sigma_1$) and the high volatility state ($\sigma_{S_t} = \sigma_2$).

2.2. Data

Following BDL, we use the value weighted index from the Center for Research in Security Prices (CRSP), which includes all stocks in the major US stock exchanges, to represent the return of the market.[†] The excess market return is computed as the difference between the market return and the one-month T-bill rate obtained from Kenneth French's website.[‡] Our sample period is July 1962 to December 2016, covering the original period of July 1962 to December 2005 studied by BDL, as well as the subsequent period that includes the financial crisis of 2007-08. In Table 1 we provide summary statistics (Panel A) and correlations (Panel B) for monthly excess returns and a range of realised risk measures, computed using daily returns within each month, over the full sample. The risk measures are standard deviation, mean absolute deviation, skewness, kurtosis, and maximum loss (which is the non-parametric estimate of VaR used by BDL). In Panel C, we report the estimated coefficients and corresponding t-statistics for the $AR(4)$ models of these risk measures.

[Table 1]

From Panel B of Table 1, it is clear that none of the commonly used realised risk measures can explain returns in a way that could be considered consistent with asset pricing theory. In particular, skewness is positively related to returns while the other measures are negatively related to returns. In unreported results, we show that these relationships hold even after controlling for state variables in a regression framework. The signs of the coefficients

[†] Available through the Wharton Research Data Services https://wrds-web.wharton.upenn.edu/wrds/
[‡] Available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

are not surprising: skewness is, by construction, associated with large positive returns, while the other risk measures are closely related to volatility, which is significantly negatively correlated with concurrent returns due to leverage and volatility feedback effects. It is these observations that motivate the use of expected risk measures, rather than realised risk measures, in the BDL framework. The estimated coefficients of the $AR(4)$ model shown in Panel C also support the use of this model in estimating expected VaR. The coefficients of the first, second, and third lags are highly significant, while that of the fourth lag is marginally significant.

In the regression analysis, we control for a range of state variables. The variables used by BDL are the detrended risk free rate (RFD), the change in the term structure risk premium (DTRP), the change in the credit risk premium (DCRP), and the dividend yield (DY). We construct these variables using exactly the same method and data sources as in BDL. To examine the robustness of our results, we also consider some additional macroeconomic variables that have been shown in the literature to be important determinants of aggregate equity returns, namely growth in industrial production (IPG), growth in the monetary base (MBG), the change in the inflation rate (DIF) and the change in the oil price (DO) (see, for example, Chen *et al.*, 1986; Kaul, 1990; Anoruo, 2011; Aburachis and Taylor, 2012). These variables are constructed as follows. We use the monthly growth rate in industrial production, the monthly growth rate of M2, the monthly change in inflation, and the monthly change in the oil price. The industrial production and money supply data are obtained from the Board of Governors of the Federal Reserve System database, while the inflation rate and oil price (series WPU0561) are obtained from the Bureau of Labor Statistics database.

3. The relationship between tail risk and returns in different states of the market

We first examine the tail risk-return relationship in different states of the market using the MS-BDL model given by (4). Table 2 presents the estimated coefficients and Newey-West (1987) HAC t-statistics for each of the states, the variance in each state and the duration of each state, using the four estimates of VaR employed by BDL: RW non-parametric VaR, RW Skewed Student-t VaR, AR4 non-parametric VaR and AR4 Skewed Student-t VaR. Parametric VaRs are calculated with 99% confidence level. We present the results of robustness checks in Section 5 using different VaR confidence levels. All measures are estimated using daily returns over the previous one month. We also estimate the model using a longer estimation sample for VaR ranging from two to six months, as in BDL. This yields very similar results to those reported here. It is clear that we can identify two distinct states of the market: a relatively frequent calm state of low volatility and a relatively infrequent turbulent state of high volatility. The variance in the turbulent state is about three times that in the calm state. This is consistent with the finding in Bekaert and Engstrom (2017) that the scale parameter of the Bad Environment is about three times that of the Good Environment in their BEGE model. The expected duration of the calm state is more than double that of the turbulent state.

We present in Figure 1 the smoothed probability of the turbulent state and the corresponding estimated state transitions in Panel A and Panel B, respectively, for the MS-BDL model using the RW Skew Student-t VaR tail risk measure. The state probabilities and transitions for the other models are very similar. It is clear that the turbulent state covers a number of periods of market distress, including the 1973-1974 oil crisis, the October 1987 crash, the burst of the dot-com bubble in the early 2000s, and the last financial crisis. We also show in Panel C the RW Skewed Student-t VaR measure and the Economic Policy

Uncertainty Index used in Amengual and Xiu (2017) and Baker *et al.* (2016).[†] This figure confirms the economic interpretation of VaR as a systematic risk measure since it tends to be higher when the market is in a turbulent state and when economic uncertainty is high. The correlation between VaR and the smoothed probability of state 2 is about 0.44, while its correlation with the uncertainty index is 0.41.

For all models, the coefficient on tail risk is positive and highly significant in the low volatility state. Thus, it appears that in periods of relatively calm market, there is a strong relationship between returns and tail risk, as implied by asset pricing theory. This is consistent with the results reported by BDL. However, in the high volatility state, the coefficient on VaR is *negative* for all VaR measures. In other words, in turbulent states of the market, it would appear that an increase in tail risk leads to *lower* returns in expectation.[‡]

[Table 2]

[Figure 1]

One possible explanation for the failure of the tail risk-return relation to hold across all market states is that it reflects a bias arising from the omission of state variables that are correlated with the tail risk measure. BDL include four control variables (the detrended risk free rate, the change in the term structure risk premium, the change in the credit risk premium and the dividend yield), but it could be argued that these may be insufficient to capture the

[†] This data is available from January 1985 at www.PolicyUncertainty.com

[‡] To shed further light on these results, we estimate the original BDL model (without Markov switching) using three samples: the original sample used by BDL (July 1962 to December 2005), the new sample (January 2006 to December 2016) and the full sample (July 1962 – December 2016). With the original BDL sample, we obtain results that are very close to those reported by BDL. In particular, in all cases, the estimated coefficient on the tail risk measure is significantly positive, suggesting that high tail risk is associated with high returns. However, for the new sample, the coefficient on tail risk is, in all cases, insignificantly positive, or even negative, suggesting a breakdown in the tail risk-return relation. As a result, using the full sample, the coefficient on tail risk is not significant using any of the four measures. To preserve space, we do not report these results.

full dynamics of the economic cycle during crisis periods. Indeed, we observe that half of the BDL control variables become insignificant in the new sample of the non-switching BDL regression. We therefore expand the set of state variables used by BDL to include four additional macro-variables that are commonly used in the asset pricing literature: growth in industrial production, growth in the monetary base, inflation and the change in the oil price.

In Table 3, we report the results of estimating the Markov switching BDL model with the expanded set of state variables. It is clear that the extended set of variables does not restore the positive risk-return relationship in the turbulent period. The negative or insignificant relationship between returns and tail risk in the high volatility state persists in most of the models. Additionally, we note that the inclusion of the additional state variables leads to a reduction in the estimated state variances (i.e., $\sigma^2_{S_t}$ in equation 4), especially in the second state. This reduction in the variance of the error term suggests that the additional state variables improve the overall goodness of fit of the Markov switching model. This is also confirmed by the reduction in the Akaike Information Criterion.[†] In the remaining empirical analysis, we therefore use the expanded set of state variables.

[Table 3]

A second possible explanation for the failure of the risk-return relation to hold in both market states is that the estimators of tail risk employed by BDL are based on the

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[†] We also conduct non-switching BDL regressions in the three sub-samples. The tail risk coefficient remains insignificant in most cases in both the full sample and the new sample. Nevertheless, the additional state variables clearly improve the overall fit of the BDL model. In particular, the adjusted R-squared coefficient increases by a factor of four in the new sample, and by a factor of two in the full sample. We also compute the Cochrane (1999) maximum unconditional Sharpe ratio of the predictive regression to show the improved economic significance of the extended set of regressors. The extended set of regressors helps to increase the Sharpe ratio by 0.2 in the full sample, and by as much as 0.7 in the new sample. To preserve space, we do not report these results.

unconditional distribution of returns, and therefore implicitly assume that returns are *iid*. Ignoring the characteristics of the true dependence structure in returns, such as autocorrelation and volatility clustering, is likely to reduce the power of the regression-based tests used to identify the risk-return relation. We therefore relax the *iid* assumption and estimate tail risk using a location-scale VaR model, in which VaR is estimated using the standardised residuals of a location-scale filtering model for daily market returns. In the literature, this method has been extensively demonstrated to deliver superior VaR estimates (see, for example, Berkowitz and O'Brien, 2002; Kuester *et al.*, 2006). We use the AR(1)-GJR GARCH(1,1) model for the location-scale filtering, which not only accounts for autocorrelation and time-varying volatility, but also captures leverage effects in volatility. These are important features that must be captured in a good volatility model, as argued by Engle and Patton (2001). Specifically, to estimate market VaR for day d, we first estimate the location-scale model using information up to day $d-1$ as:

$$r_d = \mu_d + \varepsilon_d = \mu_d + \sigma_d z_d, \quad z_d \sim Skewed\ Student - t(0,1,\upsilon,\lambda) \quad (5)$$

$$\mu_d = a_0 + a_1 r_{d-1} \quad (6)$$

$$\sigma_d^2 = c_0 + c_1\sigma_{d-1}^2 + c_2\varepsilon_{d-1}^2 + c_3 I[\varepsilon_{d-1} < 0]\varepsilon_{d-1}^2 \quad (7)$$

where r_d is the market return on day d with expected return μ_d and variance σ_d^2; ε_d is the residual term on day d; and $I[\varepsilon_{d-1} < 0]$ is the indicator function which takes value of 1 if $\varepsilon_{d-1} < 0$ and 0 otherwise; υ and λ are the degree of freedom and skew parameters of the Hansen (1994) Skewed Student-t distribution. The quantile of the standardised residuals $z_d = \varepsilon_d/\sigma_d$ is transformed into an estimate of VaR using the one-step ahead forecast of the mean and volatility of returns for day *d*. After obtaining VaR estimates for each day, we take the average of these within a month to be the RW non-*iid* risk measure. This corresponds to the RW VaRs in the original BDL model as their measure is essentially the estimated daily tail risk every month. We apply an AR(4) process to these RW non-*iid* measures to estimate the corresponding AR4 non-*iid* measures.

We estimate the AR(1)-GJR GARCH(1,1) model using a five-year rolling window (1260 daily observations), and employ the Skewed Student-t distributions for the residuals. The use of non-Gaussian error term is motivated by Gerlach and Wang (2016), who show that incorporating the Student-t error in the GARCH framework increases the accuracy of volatility and tail risk forecasts. Since we must specify a distribution for the error term in the location-scale estimation, we are not able to compute a non-*iid* version of the non-parametric VaR measure. The results of estimating the Markov switching BDL model using the non-*iid* VaR measures are reported in Table 4. The dependence structure of returns in the estimation of VaR does not explain the breakdown of the risk-return relationship in the high volatility state. In particular, the coefficient on tail risk, although consistently positive, is still insignificant for both the RW and AR4 VaR measures.[†]

[Table 4]

4. A modified measure of expected tail risk

The preceding results show that the inclusion of additional state variables in the BDL model and the use of VaR measures that explicitly allow for the dependence structure in returns serve to improve the fit of the model, but are not able to rescue the relationship between returns and tail risk in the high volatility state. In this section, we investigate the role of

[†] We obtain similar results for the sub-sample BDL regression. To preserve space, we do not report these results.

leverage and volatility feedback effects, which lead to endogeneity in realised measures of tail risk. In particular, while asset pricing theory predicts a positive relationship between returns and tail risk, realised tail risk is, by construction, associated with negative returns because high volatility (and hence high tail risk) is associated with negative returns through leverage and feedback effects. It is this endogeneity that motivates the use of lagged rather than concurrent measures of tail risk in the BDL framework. However, BDL construct expected tail risk in month t by conditioning on the information set in month $t-1$, and so implicitly assume that the leverage and feedback effects dissipate within one month. While this may be a reasonable assumption in low volatility periods, it is less likely to hold in high volatility periods. This is because high volatility is associated with higher persistence in volatility, and so leverage and volatility feedback effects take longer to dissipate. In this case, the expected risk measure used in the BDL framework will be endogenous, thus obscuring the true relation between returns and tail risk in the high volatility state.

To investigate this idea, in Table 5 we regress the lag-one autocorrelation of the three consecutive variance values from month t to month $t+2$ (which measures variance persistence) on the variance of the market return in month t, with and without the full set of control variables. Following BDL, we use the French *et al.* (1987) measure of realised variance:

$$\tilde{\sigma}_t^2 = \sum_{d=1}^{D} r_d^2 + 2\sum_{d=2}^{D} r_d\, r_{d-1} \tag{8}$$

where $\tilde{\sigma}_t^2$ is the realised variance of month t, D is the number of trading days in the month (assumed to be 21), and r_d is the return of the market on day d. The coefficient on the market variance in month t is positive and highly significant in all specifications, implying that high variance is indeed associated with high persistence in variance. As leverage and volatility feedback effects are associated with high variance, this implies that these effects will also persist for multiple periods. We will therefore observe successive periods of high tail risk and low returns. As a result, the expected tail risk measures used by BDL will still be endogenous and negatively correlated to returns.

[Table 5]

These results suggest a simple modification of the BDL framework to account for the persistence of leverage and volatility feedback effects. In particular, we construct the following modified tail risk measure:

$$E_t(VaR_{t+1}) = \theta_0 + \theta_1 E_{t-1}(VaR_t) + \sum_{i=2}^{4} \theta_i VaR_{t-i} \tag{9}$$

where θ_i $(i = 0, \dots, 4)$ are the estimated coefficients of an AR(4) model of the VaR series and $E_{t-1}(VaR_t) = \theta_0 + \sum_{i=1}^{4} \theta_i VaR_{t-1-i}$. This is similar to the AR(4) measure used by BDL, and differs only in that the first term on the right hand side, VaR_t, is replaced by its time $t-1$ forecast value. In Table 6, we report the results of estimating the MS-BDL model using this modified measure of expected tail risk. The estimated relationship between returns and tail risk is positive and, in contrast with the results in Tables 3 and 4, highly significant in both states of the market. It is important to note that the significant positive tail risk-return trade-off is observed for all modified VaR measures, regardless of whether they are estimated parametrically or non-parametrically, and whether or not the non-*iid* features of returns are accounted for. Although not reported, we also observe an improvement in the log likelihood and AIC statistics of models using the modified expected tail risk measure relative to those obtained using the RW and AR4 measures.† Our results are consistent with the findings of Drechsler and Yaron (2011) who show that the variance risk premium at time $t-1$ performs better than the variance risk premium at time t in predicting returns at time $t+1$.

[Table 6]

5. Robustness checks

The framework used in the previous sections is based on VaR as the tail risk measure for the market. Although VaR is the most popular tail risk measure in practice, it does have a number of shortcomings. Firstly, it does not contain information about the severity of the tail event given that the loss has exceeded the VaR level. Secondly, it is not a coherent measure of risk (see Artzner *et al.,* 1999). In this section, we examine the robustness of our results and inferences to the use of alternative risk measures.

5.1. Expected tail loss

Both of the shortcomings of VaR described above are addressed by Expected Tail Loss (ETL), a measure closely related to VaR. Specifically, ETL is the expected value of the loss given that it has exceeded the VaR level. Therefore, ETL contains information about the severity of the tail event. Moreover, Artzner *et al.* (1999) demonstrate that ETL is, in contrast with VaR, a coherent risk measure. Here, we use a simple Gaussian ETL as the tail measure. Under the assumption of Normally distributed daily market returns, $r_d \sim \mathrm{N}(\mu_d, \sigma_d^2)$, the ETL at the 100α percent confidence level is given by:

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† We also find that in sub-sample non-switching BDL regressions, the use of the modified expected tail risk measure yields a positive and statistically significant relation between returns and risk in all cases.

$$ETL_\alpha = \frac{1}{1-\alpha}\varphi\left(\Phi^{-1}(1-\alpha)\right)\sigma_d - \mu_d \tag{10}$$

where φ is the Standard Normal probability density function and $\Phi^{-1}(1-\alpha)$ is the $(1-\alpha)$ quantile. Analogous to the *iid* and non-*iid* VaR-based measures of tail risk, we construct the RW, AR4, and modified ETL, under both the *iid* and non-*iid* return assumption. Table 7 presents the results of estimating the MS-BDL model with these six ETL-based measures. The conclusions are similar to those obtained using the corresponding VaR-based measures. In particular, the RW and AR4 measures are only positive and statistically significant in the low volatility state, while the modified measures are positive and statistically significant in the both states. These results are consistent with BDL, who show that VaR-based and ETL-based measures of tail risk produce similar results.

[Table 7]

5.2. Alternative VaR significance levels

An alternative way of accounting for information in the tail of returns beyond VaR is to vary the VaR significance level. In the analysis above, parametric VaR is estimated using the 99 percent confidence level. As a robustness check, we use alternative confidence levels of 97.5 percent and 95 percent, and obtain qualitatively similar results to those reported above. As shown in Table 8, the original VaR measures have a positive relationship with returns in the low volatility state, but in the high volatility state, the relationship is insignificant or even negative. In contrast, the modified measures yield a significantly positive tail risk-return relationship in both states of the market.[†]

[†] To preserve space, we report the results only for 95% VaR.

[Table 8]

5.3. Other systematic tail risk measures

In this section, we examine the performance of some alternative tail risk measures available in the literature. First, we examine the risk-neutral Expected Shortfall (ES) proposed by Almeida *et al.* (2017). Since it is calculated from the risk-neutral probability density of stock returns, this measure takes into account economic conditions as well as investors' risk aversion and preferences. It is shown to predict future market returns at different (primarily short-run) horizons as well as future economic conditions. The estimation of the measure does not require the use of options data and instead can be estimated non-parametrically using only stock return data. This measure is therefore applicable to all markets with a reasonably long history of data. Almeida *et al.* (2017) verify that their measure is highly correlated with other risk-neutral tail risk measures estimated from options data in the literature. Second, since we use VaR as the primary risk measure in our analysis, we also examine the performance of risk-neutral VaR which we estimate using the same method as risk-neutral ES.[†]

For each of these alternative measures, we estimate the corresponding AR4 measure and modified measure in the same manner as the AR4 and modified VaR measures. The original measures are referred to as RW risk-neutral ES and RW risk-neutral VaR. Their corresponding AR4 and modified measures are referred to as AR4 risk-neutral ES, AR4 risk-neutral VaR, modified risk-neutral ES, and modified risk-neutral VaR, respectively. We report the results of the analysis using these measures in Table 9. Results for the risk-neutral

[†] We also examine the Left Jump Variation (LJV) developed by Bollerslev, Todorov and Xu (2015). LJV represents the part of the variance risk premium that is due to the left tail jump, which can be estimated non-parametrically from data in the options and futures markets. These results show that using LJV yields the same conclusions. To preserve space, we do not report them.

ES and VaR measures are reported in Panels A and B, respectively. The data for these measures cover the period from July 1962 to April 2014. We again find that the relationship between tail risk and returns in the high volatility state breaks down. The coefficient on the tail risk measure in this state is positive, but not significant in the case of the RW and AR4 measures. However, using the corresponding modified measures, there is again a significantly positive tail risk-return relationship in both states of the market.

[Table 9]

5.4. Accounting for volatility

Finally, we investigate the incremental information content of our modified measures of expected tail risk after controlling for volatility. Similarly to BDL, we include the realised variance calculated using equation (8) in the MS-BDL regression. The results from estimating the MS-BDL model including both the tail risk measure and the variance are reported in Table 10. Consistent with the results reported by BDL, there is no statistically significant positive relationship between returns and variance. This is consistent with evidence of a weak relationship between variance and short-term future returns reported in many other studies in the literature (see, for example, Bandi and Perron, 2008; Bollerslev *et al.*, 2009; among others). Moreover, the coefficient of variance is always positive in the calm state and negative in the turbulent state. This further supports our finding that the leverage and feedback effects are more persistent during turbulent periods. Meanwhile, the coefficients of the modified measures are positive and significant in both states in all cases. Thus, our results suggest that tail risk contains incremental price-determining information and may even be more important than variance, as documented by Bollerslev *et al.* (2015).

[Table 10]

6. Out-of-sample analysis

Our in-sample analysis has shown that using a modified tail risk measure yields a significantly positive tail risk-return relationship in all states of the market. However, it is important to determine whether investors can obtain improved return predictability using the modified method to estimate tail risk, conditional on information available to them at any point in time. We therefore examine the out-of-sample predictive performance of the modified measures. The in-sample analysis suggests that in the calm state, the previous month's expectation of tail risk predicts returns, while in the turbulent state, predictability arises from the expectation two months before. We therefore construct a strategy based on the combination of the AR4 measure and the modified measure using the out-of-sample estimated probability of each state to be realised in the future to predict returns out-of-sample.[†] At the beginning of every month, an investor estimates a Markov switching model for the monthly excess returns of the market using the macroeconomic variables available at that date. The probability of the realisation of each state in the next month is then calculated from the product of the estimated smoothed probabilities in the last period of the available sample and the probability transition matrix. These probabilities are used to combine the AR4 measure and the modified measure to create a new Hybrid measure:

$$Hybrid\ VaR = p_1 \times AR4\ VaR + \ p_2 \times modified\ VaR \tag{11}$$

where p_i is the estimated probability of state i to be realised in the next month. As this analysis is performed out-of-sample, the AR4 measure and the modified measure are both

[†] We use the modified measure to capture the tail risk-return relationship in both states throughout our analysis. However, in this section, the aim is to propose a good predictive measure rather than to demonstrate the risk-return relationship. We argue that the AR4 measure should be a better predictor in the calm state since it contains the most recent information and its impact on returns is not undermined by the leverage and feedback effects. Therefore, the Hybrid measure combines the AR4 measure with the modified measure based on the probability of the subsequent state of the market.

estimated using data available only up to that date. We restrict the minimum sample length for each Markov switching estimation to be 60 months. Therefore, the first estimated future state probability is available in July 1967, and thereafter we use an expanding estimation window. Since the results of the in-sample analysis are consistent across different VaR measures, we only report the out-of-sample results for the AR4 measure and the modified measure based on the simple non-parametric VaR. We obtain qualitatively similar results for *iid* Skewed Student-t VaR and non-*iid* Skewed Student-t VaR. The results of the out-of-sample analysis are reported in Table 11.

Table 11 shows that the Hybrid measure has a consistent relationship with future returns in both market states. It has significant predictive power for one month ahead returns in both the full sample covering the period from July 1967 to December 2016 and the new sample covering the period from January 2006 to December 2016. Its coefficient is also consistently positive in both states of the market in the MS-BDL regression. Moreover, we obtain a smaller sum of squared forecast errors for the predictive regression using the Hybrid measure than the regression using the AR4 measure. We also calculate the Diebold and Mariano (1995) test statistic to compare the predictive performance of the AR4 measure and the Hybrid measure. The value of the statistic is 0.96, which is not significant at the 10 percent significance level. However, given that a part of the information contained in the Hybrid measure is from the AR4 measure, and that the tail risk measure is only one predictive variable among a number of additional macroeconomic variables used in the predictive regression, the improvement in the predictive performance of the Hybrid measure relative to that of AR4 measure is not trivial.

[Table 11]

7. The term structure of the tail risk-return trade-off in different market states

In this section, we examine the pattern of the term structure of the predictability of tail risk with respect to future returns using the Markov switching framework. Since investors have different investment horizons, understanding the implication of tail risk on returns over different horizons is important for shaping their investment strategies. To conduct this investigation, we regress future excess returns of one to twelve months on last month's tail risk measures and other state variables in the MS-BDL framework. Specifically, the predictive Markov switching regression is given by:

$$R_{t,t+h} = \alpha^h_{S_{t,t+h}} + \beta^h_{S_{t,t+h}} VaR_t + \gamma^h_{S_{t,t+h}} X_t + \varepsilon_{S_{t,t+h}} \quad (12)$$

where $R_{t,t+h}$ is the next *h* month excess return, $h = 1,2, \dots, 12$, $\varepsilon_{S_{t,t+h}} \sim N\left(0, \sigma^2_{S_{t,t+h}}\right)$ and

$$S_{t,t+h} = \begin{cases} 1 \text{ given next } h \text{ month return is in state 1} \\ 2 \text{ given next } h \text{ month return is in state 2} \end{cases}.$$

For this analysis, both parametric and non-parametric VaR, which are estimated with either *iid* or non-*iid* returns assumption, can be used as the tail risk measure. Table 12 shows the estimated coefficients of these tail risk measures in explaining future returns of one to twelve month horizons. Similar to Ang *et al.* (2006) and Bollerslev *et al.* (2015), we use Newey-West (1987) standard errors in evaluating the significance of the estimated coefficients to account for the fact that the dependent variable is overlapping. To visualise the term structure of the tail risk-return relationship, we plot the estimated values of the tail risk coefficient in Figure 2.

Some interesting findings are clear from Table 12 and Figure 2. Firstly, given that the market is in the calm state, tail risk significantly positively predicts returns at all horizons.

The impact is most prominent for mid-term horizons from 4 to 8 months. However, in the turbulent state, tail risk significantly positively predicts future returns at the two-month horizon only. Tail risk has a negative relationship with returns at the one-month horizon. This supports the rationale underlying our modified measure. Interestingly, the tail risk impact starts to decline and eventually becomes progressively more negative for horizons longer than two months. A possible explanation for this finding is that given the high level of uncertainty in a turbulent market, tail risk cannot reliably predict medium and long-term returns.

[Table 12]

[Figure 2]

8. Conclusion

In this paper, we implement a Markov switching model to estimate the relationship between returns and tail risk documented by Bali *et al.* (2009), in different states of the market. We show that the relationship breaks down in the high volatility state that covers a number of crises. This is surprising since it is under such conditions that tail risk is expected to be most important. We show that this result is robust to a range of features of the model, including expansion of the set of control variables, and the use of tail risk measures that accounts for the non-*iid* nature of market returns.

We show that the underlying reason for this finding is the heightened leverage and volatility feedback effects during crisis periods that arise as a result of increased persistence in volatility during such times. We propose a modified tail risk measure that better filters out these effects, and show that it yields a positive relation between returns and tail risk in both the low volatility and high volatility states. Moreover, this relation is robust to the use of different VaR confidence levels, alternative measures of tail risk, and after controlling for volatility. It would be interesting to consider the implications of our findings at the individual stock level. In particular, accounting for leverage and volatility feedback effects in the construction of tail risk measures for individual stocks could help to strengthen the evidence in support of a systematic tail risk premium.

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REFERENCES

Aburachis, A.T., Taylor, D.R., Inflation and stock prices. *Journal of the Academy of Business & Economics*, 2012, **12**, 13-20.

Alexander, C., *Market risk analysis: Practical Financial Econometrics*, 2008 (John Wiley & Sons).

Allen, L., Bali, T.G., Tang, Y., Does systemic risk in the financial sector predict future economic downturns? *Review of Financial Studies*, 2012, **25**, 3000-3036.

Almeida, C., Ardison, K., Garcia, R., Vicente, J., Nonparametric tail risk, stock returns, and the macroeconomy. *Journal of Financial Econometrics*, 2017, **15**, 333-376.

Amengual, D., Xiu, D., Resolution of policy uncertainty and sudden declines in volatility. *Journal of Econometrics*, 2018, **203**, 297-315.

Andreou, E., Ghysels, E., Kourtellos, A., Regression Models with Mixed Sampling Frequencies. *Journal of Econometrics*, 2010, **158**, 246-261.

Ang, A., Chen, J., Yuhang, X., Downside Risk. *Review of Financial Studies*, 2006, **19**, 1191-1239.

Anoruo, E., Testing for Linear and Nonlinear Causality between Crude Oil Price Changes and Stock Market Returns. *International Journal of Economic Sciences & Applied Research*, 2011, **4**, 75-92.

Artzner, P., Delbaen, F., Eber, J., Heath, D., Coherent measures of risk. *Mathematical Finance*, 1999, **9**, 203-228.

Baker, S. R., Bloom, N., Davis, S. J., Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, 2016, **131**, 1593-1636.

Bali, T. G., Cakici, N., Whitelaw, R. F., Hybrid Tail Risk and Expected Stock Returns: When Does the Tail Wag the Dog?. *Review of Asset Pricing Studies*, 2014, **4**, 206-246.

Bali, T.G., Demirtas, K.O., Levy, H., Is There an Intertemporal Relation between Downside Risk and Expected Returns? *Journal of Financial & Quantitative Analysis*, 2009, **44**, 883-909.

Barndorff-Nielsen, O., Kinnebrock, S., Shephard, N. Measuring downside risk: Realised semivariance. In *Volatility and Time Series Econometrics: Essays in Honor of Robert Engle*, 2010 (Oxford University Press).

Bandi, F. M., Perron, B., Long-run risk-return trade-offs. *Journal of Econometrics*, 2008, **143**, 349-374.

Bandi, F. M., Reno, R., Price and volatility co-jumps. *Journal of Financial Economics*, 2016, **119**, 107-146.

Bansal, R., Hao, Z., Term Structure of Interest Rates with Regime Shifts. *The Journal of Finance*, 2002, **57**, 1997-2043.

Behr, A., Poetter, U., Modeling Marginal Distributions of Ten European Stock Market Index Returns. *International Research Journal of Finance & Economics*, 2009, 104-119.

Bekaert, G., Engstrom, E., Ermolov, A., Bad environments, good environments: A non-Gaussian asymmetric volatility model. *Journal of Econometrics*, 2015, **186**, 258-275.

Bekaert, G., Engstrom, E., Asset Return Dynamics under Habits and Bad Environment-Good Environment Fundamentals. *Journal of Political Economy*, 2017, **125**, 713-760.

Bollerslev, T., Todorov, V., Tails, Fears, and Risk Premia. *The Journal of Finance*, 2011, **66**, 2165-2211.

Bollerslev, T., Todorov, V., Xu, L., Tail risk premia and return predictability. *Journal of Financial Economics*, 2015, **118**, 113-134.

Bollerslev, T., Tauchen, G., Zhou, H., Expected Stock Returns and Variance Risk Premia. *Review of Financial Studies*, 2009, **22**, 4463-4492.

Campbell, J. Y., Cochrane, J. H., By Force of Habit: A consumption based explanation of aggregate stock market behavior. *Journal of Policital Economy*, 1999, **107**, 205-251.

Chabi-Yo, F., Ruenzi, S., Weigert, F., Crash sensitivity and the cross section of expected stock returns. *Journal of Financial and Quantitative Analysis*, 2018, **53**, 1059-1100.

Chang-Jin, K.I.M., Morley, J.C., Nelson, C.R., Is There a Positive Relationship between Stock Market Volatility and the Equity Premium? *Journal of Money, Credit & Banking*, 2004, **36**, 339-360.

Chen, N.-F., Roll, R., Ross, S.A., Economic Forces and the Stock Market. *Journal of Business*, 1986, **59**, 383-403.

Cochrane, J., Portfolio advice for a multifactor world. *Economic Perspectives* (Federal Reserve Bank of Chicago), 1999, **23**, 59-78.

Conrad, J., Dittmar, R.F., Ghysels, E., Ex Ante Skewness and Expected Stock Returns. *The Journal of Finance*, 2013, **68**, 85-124.

Corsi, F., A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 2009, **7**, 174-196.

Diebold, F., Mariano, R., Compary predictive accuracy. *Journal of Business and Economic Statistics*, 1995, **13**, 253-263.

Dittmar, R.F., Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns. *The Journal of Finance*, 2002, **57**, 369-403.

Drechsler, I., Yaron, A., What's vol got to do with it. *Review of Financial Studies*, 2011, **24**, 1-45.

Engle, R.F., Patton, A.J., What good is a volatility model?. *Quantitative Finance*, 2001, **1**, 237-245.

Feunou, B., Jahan-Parvar, M., Tedongap, R., Modeling market downside volatility. *Review of Finance*, 2013, **17**, 443-481.

French, K.R., Schwert, G.W., Stambaugh, R.F., Expected stock returns and volatility. *Journal of Financial Economics*, 1987, **19**, 3-29.

Friend, I., Westerfield, R., Co□skewness and capital asset pricing. *The Journal of Finance*, 1980, **35**, 897-913.

Galeano, P., Ausín, M.C., The Gaussian Mixture Dynamic Conditional Correlation Model: Parameter Estimation, Value at Risk Calculation, and Portfolio Selection. *Journal of Business & Economic Statistics*, 2010, **28**, 559-571.

Gerlach, R., Wang, C., Forecasting risk via realized GARCH, incorporating the realized range. *Quantitative Finance*, 2016, **16**, 501-511.

Ghysels, E., Guerin, P., Marcellino, M., Regime switches in the risk-return trade-off. *Jounral of Empirical Finance*, 2014, **28**, 118-138.

Glosten, L., Jagannathan, R., Runkle, D.E., On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, 1993, **48**, 1779-1801.

Gray, S.F., Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics*, 1996, **42**, 27-62.

Guidolin, M., Timmermann, A., An econometric model of nonlinear dynamics in the joint distribution of stock and bond returns. *Journal of Applied Econometrics*, 2006, **21**, 1-22.

Guidolin, M., Timmermann, A., International Asset Allocation under Regime Switching, Skew, and Kurtosis Preferences. *Review of Financial Studies*, 2008, **21**, 889-935.

Guo, H., Wang, K., Zhou, H., Good Jumps, Bad Jumps, and Conditional Equity Premium. *Asian Finance Association (AsianFA) 2014 Conference Paper*; PBCSF-NIFR Research Paper No. 14-05, 2014. Available at SSRN: https://ssrn.com/abstract=2515212

Hamilton, J.D., A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 1989, **57**, 357-384.

Hamilton, J.D., Analysis of time series subject to changes in regime. *Journal of Econometrics*, 1990, **45**, 39-70.

Hansen, B.E., Autoregressive conditional density estimation. *International Economic Review*, 1994, **35**, 705-730.

Harvey, C.R., Siddique, A., Conditional Skewness in Asset Pricing Tests. *The Journal of Finance*, 2000, **55**, 1263-1295.

Hodrick, R., Dividend yields and expected stock returns: alternative procedures for inference and measurement. *Review of Financial Studies*, 1992, **5**, 357-386.

Huang, W., Liu, Q., Ghon Rhee, S., Wu, F., Extreme downside risk and expected stock returns. *Journal of Banking & Finance*, 2012, **36**, 1492-1502.

Kaul, G., Monetary Regimes and the Relation between Stock Return and Inflationary Expectations. *Journal of Financial & Quantitative Analysis*, 1990, **25**, 307-321.

Kelly, B., Jiang, H., Tail Risk and Asset Prices. *Review of Financial Studies*, 2014, **27**, 2841-2871.

Kon, S.J., Models of Stock Returns - A Comparison. *The Journal of Finance*, 1984, **39**, 147-165.

Kraus, A., Litzenberger, R.H., Skewness preference and the valuation of risk assets. *The Journal of Finance*, 1976, **31**, 1085-1100.

Kuester, K., Mittnik, S., Paolella, M.S., Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 2006, **4**, 53-89.

Lemperiere, Y., Deremble, C., Nguyen, T.T., Seager, P., Potters, M., Bouchaud, J.P., Risk premia: asymmetric tail risks and excess returns. *Quantitative Finance*, 2017, **17**, 1-14.

Li, O.X., Li, W., Hedging jump risk, expected returns and risk premia in jump-diffusion economies. *Quantitative Finance*, 2015, **15**, 873-888.

Lintner, J., The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics & Statistics*, 1965, **47**, 13-37.

Mitton, T., Vorkink, K., Equilibrium Underdiversification and the Preference for Skewness. *Review of Financial Studies*, 2007, **20**, 1255-1288.

Nelson, D.B., Conditional heteroskedasticity in asset returns: a new approach. *Econometrica*, 1991, **59**, 347-370.

Newey, W. K., West, K. D., A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 1987, **55**, 703-708.

Nikolsko-Rzhevskyy, A., Prodan, R., Markov switching and exchange rate predictability. *International Journal of Forecasting*, 2012, **28**, 353-365.

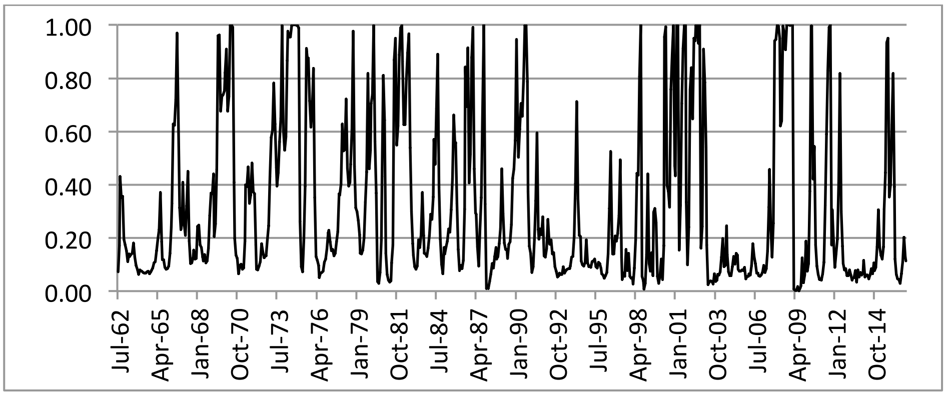
Patton, A., Sheppard, K. Good volatility, bad volatility: signed jumps and the persistence of volatility. *Review of Economics and Statistics*, 2015, **97**, 683-697.

Sharpe, W.F., Capital asset prices: a theory of market equilibrium under conditions of risk. *The Journal of Finance*, 1964, **19**, 425-442.

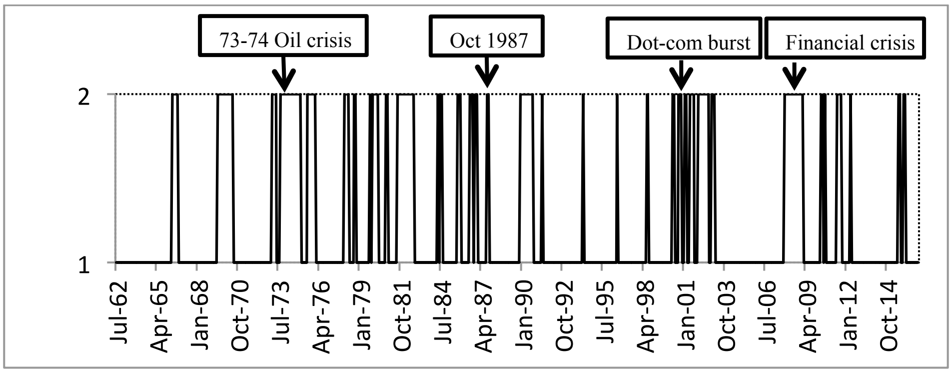
Todorov, V., Bollerslev, T., Jumps and betas: a new framework for disentangling and estimating systematic risks. *Journal of Econometrics*, 2010, **157**, 220-235.

Turner, C.M., Startz, R., Nelson, C.R., A Markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics*, 1989, **25**, 3-22.

Panel A: Smoothed probability of turbulent state



Panel B: Markov switching state timing



Panel C: VaR and Economic Uncertainty Index

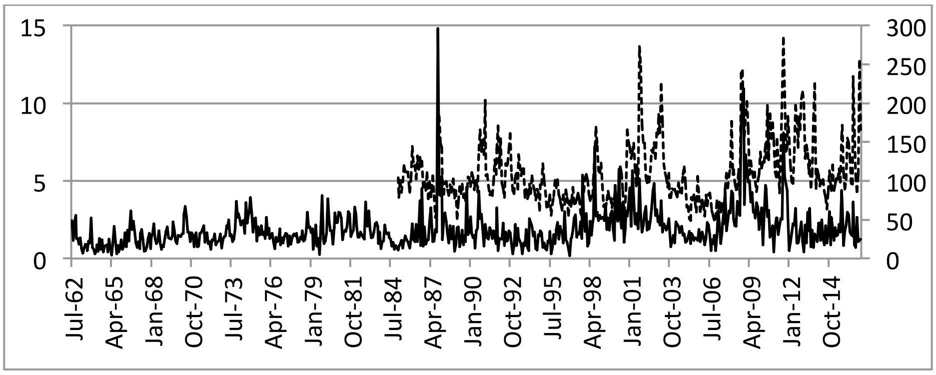


Figure 1: Tail risk measure and estimated states over time. Panel A and B show the smoothed probability of the turbulent state and the corresponding estimated state transitions using a threshold probability of 0.5, for the estimated MS-BDL model using the RW Skewed Student-t tail risk measure. Panel C shows the over time evolution of this tail risk (solid line), as well as that of the Economic Policy Uncertainty Index developed by Baker *et al.* (2016) (dashed line).

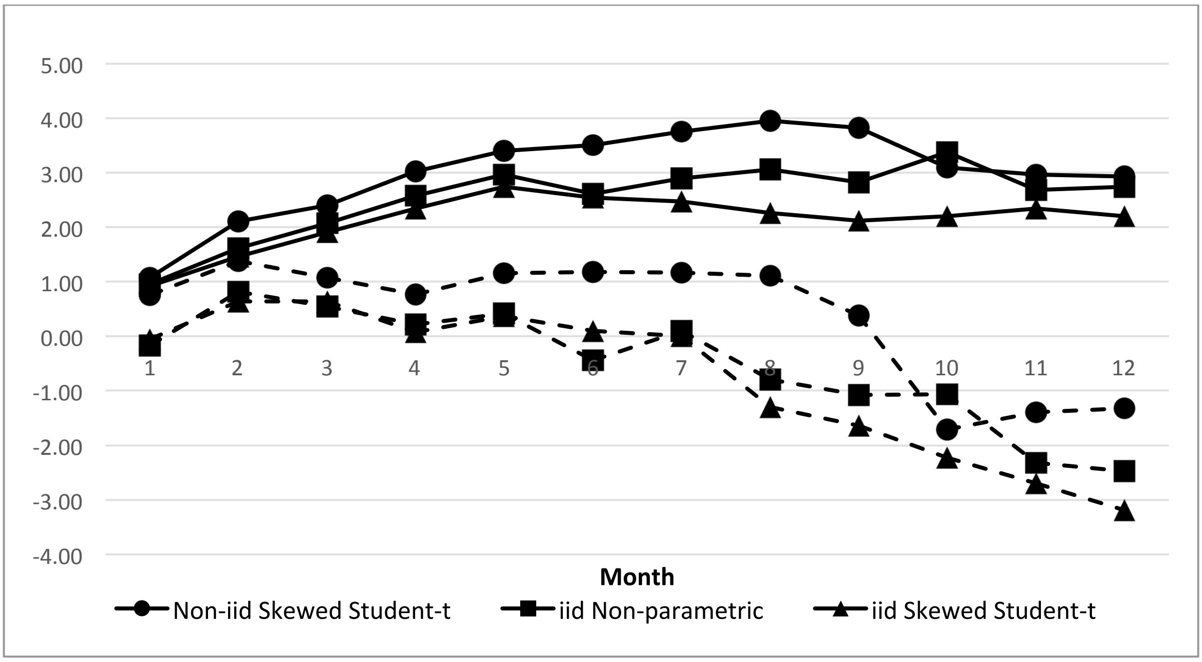


Figure 2: Predictability term structure of tail risk in different market states. This figure shows the estimated coefficients of different tail risk measures in predicting future returns at different horizons. The solid lines show the estimated coefficients in the calm state of the market, while the dashed lines show those in the turbulent state.

Table 1. The table reports summary statistics for the CRSP value weighted monthly excess return, and the realised standard deviation, mean absolute deviation (MAD), skewness, kurtosis and non-parametric VaR. The realised risk measures are calculated using daily returns over one month. t-statistics are reported in parentheses. The sample period is July 1962 to December 2016.

	Monthly Excess return	Standard deviation	MAD	Skewness	Kurtosis	Non-parametric VaR
Panel A: Basic statistics						
Mean	0.52	0.83	0.64	-0.05	3.06	1.62
Median	0.87	0.69	0.53	-0.04	2.82	1.36
Standard deviation	4.42	0.51	0.39	0.58	1.13	1.24
Minimum	-23.14	0.18	0.14	-2.97	1.63	0.18
Maximum	16.05	4.96	3.79	2.51	12.48	17.13
Panel B: Cross correlation						
Monthly Excess return	1.00	-0.32	-0.30	0.08	-0.03	-0.45
Standard deviation	-0.32	1.00	0.99	0.03	0.05	0.89
MAD	-0.30	0.99	1.00	0.06	-0.05	0.84
Skewness	0.08	0.03	0.06	1.00	-0.18	-0.25
Kurtosis	-0.03	0.05	-0.05	-0.18	1.00	0.24
Non-parametric VaR	-0.45	0.89	0.84	-0.25	0.24	1.00
Panel C: Lags' coefficients in AR(4)						
Lag 1	0.08	0.55	0.59	0.07	0.00	0.30
(t-statistic)	(2.231)	(33.872)	(30.247)	(1.865)	(-0.035)	(13.516)
Lag 2	-0.05	0.12	0.13	0.08	0.00	0.17
(t-statistic)	(-1.328)	(3.241)	(4.005)	(2.104)	(-0.027)	(5.403)
Lag 3	0.02	0.12	0.04	0.13	0.11	0.22
(t-statistic)	(0.754)	(2.382)	(1.030)	(3.776)	(2.986)	(4.903)
Lag 4	0.02	-0.03	0.01	0.04	0.00	-0.06
(t-statistic)	(0.479)	(-0.840)	(0.371)	(0.861)	(-0.052)	(-1.505)

Table 2. The table reports the results of estimating the MS-BDL model using different measures of extreme downside risk. The risk measures are calculated using daily returns over the last month. The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$ and the following control variables measured at time *t*: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). The Skewed Student-t VaR is estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged return	RFD	DTRP	DCRP	DY	State variance	Expected Duration
				RW Non-parametric VaR						
1	0.282	1.019	-24.449	-0.024	-0.531	0.058	2.167	-0.077	8.957	10.736
	(0.243)	(3.799)	(-4.705)	(-0.326)	(-2.404)	(0.181)	(0.997)	(-0.256)		
2	-2.190	-0.862	3.681	-0.030	-0.382	-1.986	5.562	0.705	28.566	4.919
	(-1.676)	(-0.886)	(0.267)	(-0.287)	(-0.740)	(-2.759)	(1.520)	(1.881)		
				RW Skewed Student-t VaR						
1	0.114	1.014	-21.870	-0.028	-0.523	0.086	2.338	-0.087	8.590	7.924
	(0.028)	(1.395)	(-3.664)	(-0.222)	(-0.652)	(0.053)	(0.883)	(-0.103)		
2	-2.424	-0.846	1.712	-0.015	-0.441	-2.007	5.755	0.815	26.760	3.757
	(-1.930)	(-1.414)	(0.093)	(-0.124)	(-0.912)	(-2.537)	(3.005)	(2.276)		
				AR4 Non-parametric VaR						
1	-1.386	2.059	-19.575	-0.083	-0.590	-0.023	2.678	-0.059	8.788	13.147
	(-1.368)	(4.167)	(-7.560)	(-2.104)	(-2.556)	(-0.215)	(1.681)	(-0.340)		
2	-3.157	-0.353	-6.331	0.051	-0.322	-1.917	3.555	0.766	30.274	5.993
	(-0.944)	(-0.189)	(-0.807)	(0.761)	(-1.056)	(-3.141)	(1.445)	(2.593)		
				AR4 Skewed Student-t VaR						
1	-1.330	1.752	-18.084	-0.083	-0.573	-0.004	2.536	-0.053	8.441	9.338
	(-1.571)	(5.091)	(-7.321)	(-1.524)	(-2.109)	(-0.441)	(1.580)	(-0.317)		
2	-2.399	-0.785	-4.384	0.043	-0.401	-1.961	4.365	0.779	28.453	4.389
	(-1.342)	(-0.538)	(-0.479)	(0.745)	(-0.707)	(-2.965)	(1.784)	(2.182)		

Table 3. The table reports the results of estimating the MS-BDL model using the expanded set of state variables. The risk measures are calculated using daily returns over one month. The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$ and the following control variables measured at time t: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). The Skewed Student-t VaR is estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
RW Nonparam	1	0.741	0.955	-23.809	-0.055	-0.534	-0.077	1.919	-0.131	-64.284	6.655	-1.632	0.017	8.799	11.352
		(0.801)	(4.558)	(-6.636)	(-1.071)	(-1.691)	(-0.108)	(1.063)	(-0.536)	(-1.534)	(0.087)	(-1.906)	(0.769)		
	2	-3.553	-0.170	-7.818	-0.022	-0.352	-1.775	7.978	0.771	113.440	-43.731	-0.278	0.097	25.685	5.541
		(-2.805)	(-0.365)	(-1.324)	(-0.197)	(-0.903)	(-1.812)	(2.211)	(2.697)	(0.679)	(-0.331)	(-0.052)	(1.852)		
RW Skewed Student-t	1	0.626	0.914	-20.794	-0.054	-0.552	-0.084	2.155	-0.146	-62.533	5.891	-1.503	0.016	8.643	10.683
		(0.870)	(5.047)	(-6.936)	(-0.816)	(-1.801)	(-0.184)	(0.755)	(-0.645)	(-1.173)	(0.111)	(-1.322)	(0.340)		
	2	-3.841	-0.055	-9.357	-0.005	-0.366	-1.745	7.762	0.824	118.861	-55.201	-0.239	0.097	25.409	5.308
		(-1.601)	(-0.082)	(-1.019)	(-0.159)	(-0.666)	(-1.544)	(1.907)	(1.601)	(0.766)	(-0.104)	(-0.083)	(3.681)		
AR4 Nonparam	1	-0.929	1.962	-19.254	-0.103	-0.582	-0.150	2.453	-0.119	-61.070	19.298	-1.207	0.010	8.676	13.697
		(-0.794)	(3.880)	(-6.499)	(-2.462)	(-2.428)	(-0.401)	(1.642)	(-0.618)	(-1.719)	(0.414)	(-0.988)	(0.490)		
	2	-7.080	1.599	-17.159	0.022	-0.266	-1.698	5.958	1.023	134.157	-93.692	-0.690	0.119	26.133	6.593
		(-3.857)	(2.081)	(-4.603)	(0.445)	(-0.712)	(-2.346)	(2.006)	(3.764)	(1.447)	(-0.777)	(-0.307)	(3.632)		
AR4 Skewed Student-t	1	-0.819	1.638	-17.532	-0.098	-0.593	-0.164	2.439	-0.123	-59.333	16.596	-1.150	0.010	8.597	12.844
		(-0.594)	(1.990)	(-5.900)	(-0.765)	(-0.581)	(-0.065)	(0.455)	(-0.263)	(-1.037)	(0.061)	(-0.117)	(0.033)		
	2	-6.417	1.041	-14.358	0.026	-0.267	-1.692	6.197	0.984	136.146	-88.540	-0.563	0.117	25.828	6.189
		(-2.132)	(0.776)	(-4.570)	(0.073)	(-0.136)	(-0.411)	(0.284)	(2.102)	(0.426)	(-0.444)	(-0.021)	(0.720)		

Table 4. The table reports the results of estimating the MS-BDL model using the non-*iid* risk measures. The risk measures are calculated using average daily VaR over a month. The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$ and the following control variables measured at time *t*: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). Skewed Student-t VaR is estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
RW Skewed Student-t	1	-0.150	1.076	-15.151	-0.084	-0.577	0.008	2.849	-0.098	-37.840	16.386	-0.537	0.007	8.568	13.283
		(-0.070)	(2.549)	(-5.573)	(-1.979)	(-1.667)	(0.117)	(1.462)	(-0.273)	(-0.874)	(0.069)	(-0.417)	(0.141)		
	2	-6.244	0.761	-13.187	0.037	-0.397	-1.997	3.611	1.153	83.015	-118.829	-2.115	0.134	26.866	6.365
		(-2.248)	(0.803)	(-3.238)	(0.218)	(-1.061)	(-2.660)	(0.617)	(2.555)	(0.897)	(-0.668)	(-0.681)	(3.362)		
AR4 Skewed Student-t	1	-0.625	1.304	-15.157	-0.083	-0.578	0.032	3.101	-0.099	-35.464	14.610	-0.500	0.007	8.500	12.911
		(-0.588)	(5.223)	(-8.933)	(-2.162)	(-2.581)	(0.193)	(1.847)	(-0.267)	(-1.072)	(0.139)	(-0.246)	(0.181)		
	2	-6.177	0.824	-12.908	0.035	-0.409	-2.007	3.862	1.104	80.323	-113.748	-2.297	0.131	26.950	6.339
		(-2.626)	(1.341)	(-4.290)	(0.658)	(-0.622)	(-2.485)	(0.639)	(2.618)	(1.239)	(-0.907)	(-0.573)	(3.301)		

Table 5. The table reports the results of estimating the regression of the lag-one autocorrelation of the three variance values from month t to month t+2 on the realised variance in month t, with and without the following state variables: market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). The sample period is July 1962 to December 2016.

	Const	Realised variance	Market return	Dummy	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adjusted R^2
with no state variable	-31.708	0.052											0.67%
	(-35.218)	(3.880)											
with state variables at t	-36.349	0.108	0.337	-47.209	1.283	1.037	-5.845	1.161	-7.682	-35.931	-1.906	0.095	0.41%
	(-13.124)	(4.307)	(1.566)	(-2.967)	(1.556)	(0.843)	(-0.637)	(1.481)	(-0.061)	(-0.159)	(-0.480)	(1.071)	
with state variables at t+1	-33.884	0.071	0.239	-33.271	1.214	-0.003	-19.408	0.946	-30.787	-194.666	0.000	-0.066	1.01%
	(-12.749)	(3.527)	(1.042)	(-5.611)	(1.451)	(-0.002)	(-2.452)	(1.197)	(-0.248)	(-0.748)	(-0.000)	(-0.621)	
with state variables at t+2	-35.833	0.062	-0.568	-1.544	1.428	0.593	0.716	0.388	47.977	533.619	-2.320	-0.048	1.53%
	(-13.174)	(3.554)	(-2.725)	(-0.280)	(1.546)	(0.465)	(0.099)	(0.483)	(0.438)	(2.295)	(-0.605)	(-0.470)	

Table 6. The table reports the results of estimating the MS-BDL model using the modified risk measures. The *iid* measures are RW measures calculated using daily returns over one month. The non-*iid* measures are RW measures calculated using the daily VaR over one month. These RW measures are used to estimate the corresponding modified measures according to formula (9). The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$ and the following control variables measured at time t: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). Skewed Student-t VaRs are estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
iid	1	-2.329	2.934	-11.906	-0.185	-0.710	-0.230	3.435	-0.081	-54.472	8.959	-0.458	0.000	8.327	11.748
Nonparam		(-1.505)	(3.984)	(-8.392)	(-4.470)	(-2.880)	(-0.881)	(2.428)	(-0.350)	(-1.973)	(0.175)	(-0.471)	(0.367)		
	2	-8.949	2.476	-11.229	-0.051	-0.211	-1.542	4.480	1.085	141.533	-60.086	-1.994	0.124	24.165	6.315
		(-4.938)	(4.680)	(-7.071)	(-0.954)	(-0.607)	(-2.960)	(1.487)	(3.679)	(2.779)	(-0.588)	(-0.800)	(4.239)		
iid	1	-0.729	1.673	-11.323	-0.165	-0.681	-0.197	2.930	-0.153	-63.205	13.773	-0.542	0.004	8.641	12.088
Parametric		(-0.500)	(2.737)	(-7.450)	(-3.594)	(-2.543)	(-0.587)	(1.721)	(-0.719)	(-2.049)	(0.288)	(-0.541)	(0.251)		
Skewed	2	-8.829	1.982	-11.157	-0.028	-0.270	-1.773	4.148	1.132	146.047	-77.576	-2.251	0.127	24.888	5.861
Student-t		(-4.171)	(3.193)	(-6.420)	(-0.406)	(-0.663)	(-2.632)	(1.185)	(3.499)	(2.104)	(-0.812)	(-0.872)	(3.811)		
Non- *iid*	1	-1.052	1.522	-11.040	-0.152	-0.626	-0.095	3.296	-0.107	-45.056	23.087	-0.468	0.002	8.551	13.067
Parametric		(-0.995)	(5.511)	(-7.115)	(-2.732)	(-2.779)	(-0.244)	(1.980)	(-0.449)	(-0.678)	(0.424)	(-0.629)	(0.105)		
Skewed	2	-9.093	1.649	-11.363	-0.031	-0.275	-1.976	4.606	1.308	135.082	-86.474	-2.771	0.131	25.373	6.106
Student-t		(-3.980)	(2.974)	(-4.105)	(-0.275)	(-0.746)	(-1.933)	(1.205)	(3.472)	(0.835)	(-0.773)	(-1.022)	(3.582)		

Table 7. The table reports the results of estimating the MS-BDL model using the Gaussian-ETL tail risk measures. The *iid* measures are calculated using daily returns over one month. The non-*iid* measures are calculated using daily ETL over one month. The monthly market excess return at time t+1 is regressed on $E_t(ETL_{t+1})$ and the following control variables measured at time t: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). Parametric ETLs are estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel A: *iid* measures															
	1	0.322	1.003	-21.025	-0.045	-0.522	0.033	1.918	-0.142	-47.039	-2.447	-1.102	0.011	8.360	11.041
RW *iid*		(0.414)	(5.147)	(-6.390)	(-0.691)	(-1.831)	(0.154)	(1.067)	(-0.611)	(-0.633)	(-0.084)	(-0.670)	(0.562)		
	2	-3.551	-0.066	-9.547	-0.016	-0.390	-1.855	7.392	0.798	96.792	-60.260	-0.685	0.100	26.028	5.695
		(-1.510)	(-0.059)	(-0.792)	(-0.205)	(-1.036)	(-1.729)	(1.028)	(1.503)	(0.570)	(-0.261)	(-0.147)	(1.503)		
	1	-0.732	1.491	-19.676	-0.074	-0.525	-0.009	2.217	-0.125	-46.976	1.557	-0.991	0.009	8.324	11.642
AR4		(-0.819)	(6.472)	(-9.682)	(-1.604)	(-2.300)	(-0.139)	(1.360)	(-0.557)	(-1.004)	(0.049)	(-0.741)	(0.469)		
iid	2	-4.517	0.263	-11.616	0.007	-0.385	-1.804	6.604	0.877	109.005	-76.167	-0.824	0.107	26.120	5.904
		(-2.841)	(0.571)	(-2.883)	(0.151)	(-1.196)	(-2.339)	(1.944)	(2.597)	(0.913)	(-0.617)	(-0.345)	(3.591)		
	1	-1.423	1.787	-11.897	-0.169	-0.615	-0.180	2.951	-0.103	-48.752	7.706	-0.490	0.003	8.548	13.283
Modified		(-1.059)	(4.015)	(-7.937)	(-4.060)	(-2.545)	(-0.576)	(1.933)	(-0.364)	(-1.671)	(0.245)	(-0.446)	(0.267)		
iid	2	-9.791	1.972	-11.542	-0.032	-0.295	-1.839	3.835	1.264	158.901	-84.344	-2.452	0.135	24.954	6.085
		(-4.838)	(3.894)	(-8.021)	(-0.706)	(-0.658)	(-2.487)	(1.114)	(3.920)	(2.316)	(-0.866)	(-0.882)	(3.895)		

(*Continued*)

Table 7. Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: non-*iid* measures															
	1	-0.252	1.112	-17.097	-0.085	-0.587	-0.046	2.900	-0.117	-41.372	21.084	-0.541	0.007	8.717	14.004
RW		(-0.076)	(1.434)	(-3.054)	(-1.458)	(-2.686)	(-0.173)	(1.607)	(-0.156)	(-1.095)	(0.174)	(-0.489)	(0.136)		
non-*iid*	2	-6.584	0.894	-14.900	0.050	-0.411	-1.954	3.313	1.142	91.914	-124.484	-2.087	0.136	26.910	6.513
		(-1.991)	(0.855)	(-4.674)	(0.180)	(-0.400)	(-2.228)	(0.618)	(2.685)	(0.975)	(-0.926)	(-0.517)	(2.746)		
	1	-0.864	1.388	-16.911	-0.086	-0.587	-0.016	3.200	-0.117	-37.798	20.996	-0.460	0.006	8.639	13.893
AR4		(-0.597)	(5.332)	(-7.586)	(-2.087)	(-2.458)	(-0.138)	(1.739)	(-0.522)	(-1.085)	(0.168)	(-0.469)	(0.165)		
non-*iid*	2	-6.945	1.111	-14.797	0.049	-0.428	-1.977	3.231	1.130	87.596	-124.842	-2.483	0.136	27.072	6.615
		(-2.295)	(0.970)	(-3.928)	(0.263)	(-1.226)	(-2.820)	(0.559)	(3.044)	(0.995)	(-1.046)	(-0.905)	(3.330)		
	1	-1.276	1.604	-11.300	-0.157	-0.641	-0.164	3.225	-0.123	-49.339	24.977	-0.501	0.004	8.720	13.150
Modified		(-0.863)	(4.292)	(-8.666)	(-3.863)	(-2.028)	(-0.362)	(1.977)	(-0.482)	(-1.272)	(0.394)	(-0.500)	(0.099)		
non-*iid*	2	-9.443	1.793	-11.450	-0.032	-0.283	-1.899	4.857	1.249	152.520	-84.318	-2.695	0.130	25.186	5.935
		(-4.024)	(3.092)	(-4.581)	(-0.356)	(-0.350)	(-2.066)	(1.523)	(2.848)	(1.752)	(-0.543)	(-1.023)	(4.241)		

Table 8. The table reports the results of estimating the MS-BDL model using the 95% VaR tail risk measures. The *iid* measures are calculated using daily returns over one month. The non-*iid* measures are calculated using daily VaR over one month. The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$ and the following control variables measured at time *t*: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: *iid* measures								
	1	0.487	1.530	-17.676	-0.023	-0.521	-0.030	2.032	-0.149	-58.629	2.062	-1.306	0.014	8.461	10.158
RW *iid*		(0.252)	(2.968)	(-3.439)	(-0.062)	(-0.994)	(-0.029)	(0.693)	(-0.621)	(-0.529)	(0.064)	(-0.814)	(0.359)		
	2	-3.812	-0.092	-9.592	-0.008	-0.400	-1.771	7.612	0.829	117.711	-56.481	-0.381	0.097	25.264	5.177
		(-0.737)	(-0.024)	(-0.387)	(-0.121)	(-0.165)	(-1.867)	(1.094)	(1.013)	(0.492)	(-0.094)	(-0.041)	(0.406)		
	1	-0.735	2.446	-16.453	-0.059	-0.544	-0.075	2.284	-0.128	-52.130	8.972	-1.076	0.010	8.395	12.047
AR4		(-0.587)	(5.267)	(-7.682)	(-0.835)	(-2.049)	(-0.150)	(1.528)	(-0.517)	(-0.902)	(0.159)	(-0.917)	(0.473)		
iid	2	-5.265	0.842	-11.974	0.025	-0.340	-1.758	6.484	0.933	119.920	-81.806	-0.673	0.112	25.981	6.002
		(-2.764)	(0.763)	(-3.780)	(0.250)	(-0.811)	(-1.842)	(1.935)	(2.227)	(0.956)	(-0.577)	(-0.330)	(3.429)		
	1	-0.949	2.674	-11.672	-0.168	-0.654	-0.176	2.766	-0.126	-54.667	8.450	-0.480	0.003	8.496	12.364
Modified		(-0.841)	(4.909)	(-9.052)	(-4.062)	(-2.572)	(-0.723)	(1.816)	(-0.477)	(-1.739)	(0.297)	(-0.688)	(0.110)		
iid	2	-8.669	2.753	-11.174	-0.026	-0.321	-1.833	3.941	1.170	144.801	-85.620	-2.278	0.129	25.106	5.929
		(-3.993)	(2.995)	(-6.948)	(-0.476)	(-0.831)	(-2.730)	(1.301)	(3.708)	(1.963)	(-0.865)	(-1.026)	(4.153)		

(Continued)

Table 8. Continued

Measure	State	Const	$E_t(VaR_{t+1})$	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: non-*iid* measures															
	1	-0.223	1.800	-15.223	-0.118	-0.578	-0.040	2.812	-0.078	-33.579	5.025	-0.520	0.004	8.504	13.320
RW		(-0.138)	(4.036)	(-9.552)	(-2.419)	(-1.455)	(-0.057)	(1.407)	(-0.243)	(-0.700)	(0.039)	(-0.245)	(0.115)		
non-*iid*	2	-6.108	1.121	-12.495	0.021	-0.399	-1.992	3.931	1.113	86.566	-110.200	-2.159	0.134	26.918	6.200
		(-3.090)	(1.393)	(-5.065)	(0.175)	(-0.789)	(-2.155)	(0.755)	(3.108)	(0.938)	(-0.913)	(-0.477)	(2.745)		
	1	-0.573	2.065	-15.386	-0.111	-0.579	-0.013	2.994	-0.080	-31.337	1.820	-0.493	0.005	8.489	13.068
AR4		(-0.616)	(6.948)	(-9.529)	(-2.817)	(-2.734)	(-0.118)	(1.869)	(-0.369)	(-1.136)	(0.106)	(-0.419)	(0.247)		
non-*iid*	2	-5.874	1.112	-12.230	0.022	-0.406	-2.002	4.253	1.060	79.419	-105.540	-2.179	0.130	27.085	6.167
		(-3.130)	(1.458)	(-5.333)	(0.310)	(-1.149)	(-2.922)	(1.087)	(3.240)	(1.193)	(-0.948)	(-0.812)	(3.741)		
	1	-0.751	2.262	-11.467	-0.166	-0.635	-0.131	3.616	-0.104	-47.242	10.415	-0.536	0.001	8.632	12.296
Modified		(-0.856)	(6.527)	(-9.458)	(-4.515)	(-2.797)	(-0.574)	(2.287)	(-0.483)	(-1.623)	(0.195)	(-0.538)	(0.151)		
non-*iid*	2	-8.321	2.273	-11.080	-0.014	-0.289	-1.939	5.146	1.199	148.904	-82.935	-2.429	0.129	25.236	5.622
		(-3.880)	(2.793)	(-3.970)	(-0.119)	(-0.912)	(-2.615)	(1.560)	(3.714)	(1.894)	(-0.681)	(-0.934)	(4.098)		

Table 9. The table reports the results of estimating the MS-BDL model using alternative tail risk measures, including risk-neutral ES, and risk-neutral VaR. The RW measures are the original measures of risk-neutral ES and risk-neutral VaR. The AR4 and modified measures are one and two period ahead expected value, lagged by one and two months, correspondingly, from the AR(4) model of the RW measures. The monthly market excess return at time *t*+1 is regressed on a tail measure and the following control variables measured at time *t*: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses).

Measure	State	Const	Tail measure	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
							Panel A: risk-neutral ES								
	1	-0.239	1.277	-14.241	-0.047	-0.584	-0.070	2.261	-0.107	-48.730	18.498	-0.647	0.003	8.711	12.205
RW		(-0.260)	(6.527)	(-8.836)	(-0.922)	(-2.140)	(-0.203)	(1.171)	(-0.499)	(-1.252)	(0.415)	(-0.502)	(0.258)		
	2	-4.539	0.137	-10.637	-0.001	-0.469	-2.002	6.558	1.011	91.355	-95.816	-1.220	0.108	27.346	5.855
		(-2.171)	(0.304)	(-5.209)	(-0.047)	(-1.047)	(-2.276)	(1.889)	(2.240)	(1.026)	(-0.791)	(-0.442)	(2.905)		
	1	-0.916	1.573	-13.684	-0.081	-0.554	-0.027	2.723	-0.065	-42.307	14.757	-0.370	-0.005	8.554	11.025
AR4		(-0.853)	(6.519)	(-7.807)	(-1.653)	(-2.075)	(-0.077)	(1.380)	(-0.232)	(-1.028)	(0.148)	(-0.430)	(-0.250)		
	2	-6.251	0.630	-11.310	0.018	-0.574	-2.101	5.358	1.237	99.687	-117.929	-2.197	0.124	26.721	5.237
		(-1.921)	(0.765)	(-4.485)	(0.103)	(-1.341)	(-2.641)	(1.715)	(2.560)	(0.888)	(-0.716)	(-0.930)	(3.133)		
	1	-0.740	1.552	-11.289	-0.172	-0.612	-0.121	3.288	-0.063	-54.139	8.593	-0.428	-0.011	8.554	9.719
Modified		(-0.874)	(6.925)	(-8.833)	(-4.312)	(-2.400)	(-0.332)	(1.935)	(-0.272)	(-1.754)	(0.213)	(-0.550)	(-0.542)		
	2	-9.228	1.499	-10.076	-0.002	-0.579	-2.039	5.634	1.475	165.733	-108.162	-2.403	0.134	24.298	4.494
		(-3.919)	(2.739)	(-7.523)	(-0.175)	(-1.202)	(-2.962)	(2.090)	(3.459)	(2.447)	(-1.013)	(-1.335)	(4.768)		

(Continued)

Table 9. Continued

Measure	State	Const	Tail measure	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
Panel B: risk-neutral VaR															
	1	-0.215	1.301	-14.061	-0.048	-0.589	-0.061	2.264	-0.109	-48.750	17.336	-0.640	0.003	8.682	11.927
RW		(-0.064)	(1.304)	(-5.397)	(-0.266)	(-0.365)	(-0.015)	(0.397)	(-0.224)	(-0.313)	(0.298)	(-0.061)	(0.009)		
	2	-4.524	0.132	-10.605	-0.001	-0.472	-2.017	6.549	1.013	90.593	-96.391	-1.241	0.107	27.263	5.759
		(-1.618)	(0.271)	(-4.948)	(-0.191)	(-0.260)	(-0.461)	(0.792)	(1.086)	(0.258)	(-0.643)	(-0.058)	(0.559)		
	1	-0.908	1.608	-13.559	-0.082	-0.557	-0.025	2.709	-0.064	-42.748	14.305	-0.378	-0.005	8.539	10.888
AR4		(-0.869)	(6.614)	(-8.047)	(-1.568)	(-1.549)	(-0.065)	(1.522)	(-0.348)	(-1.134)	(0.257)	(-0.856)	(-0.246)		
	2	-6.232	0.634	-11.228	0.018	-0.575	-2.102	5.379	1.237	100.258	-118.443	-2.171	0.124	26.647	5.188
		(-2.921)	(1.385)	(-2.825)	(0.089)	(-0.874)	(-2.806)	(1.718)	(2.806)	(1.026)	(-0.639)	(-1.167)	(3.183)		
	1	-0.755	1.592	-11.291	-0.172	-0.612	-0.123	3.265	-0.059	-54.075	8.430	-0.432	-0.011	8.553	9.713
Modified		(-0.440)	(4.897)	(-7.157)	(-4.025)	(-2.650)	(-0.379)	(2.096)	(-0.143)	(-1.544)	(0.288)	(-0.529)	(-0.645)		
	2	-9.158	1.505	-10.080	-0.002	-0.580	-2.038	5.651	1.469	165.923	-107.831	-2.390	0.134	24.293	4.493
		(-3.869)	(2.695)	(-7.108)	(-0.190)	(-1.180)	(-2.897)	(2.168)	(3.485)	(2.341)	(-0.950)	(-1.358)	(4.700)		

Table 10. The table reports the results of estimating the MS-BDL model using both the modified measures and the realised variance. The *iid* measures are calculated using daily returns over one month. The non-*iid* measures are calculated using daily VaR over one month. The monthly market excess return at time t+1 is regressed on $E_t(VaR_{t+1})$, and the following control variables measured at time t: realised variance, lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses). Skewed Student-t VaRs are estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	Const	$E_t(VaR_{t+1})$	Variance	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
iid	1	0.028	1.229	0.029	-29.594	-0.135	-0.539	-0.059	1.929	-0.198	-64.505	10.833	-0.682	0.007	8.527	12.636
Nonparam		(0.191)	(2.777)	(1.393)	(-2.187)	(-2.811)	(-1.255)	(-0.186)	(0.842)	(-0.981)	(-2.405)	(0.108)	(-0.381)	(0.126)		
	2	-10.494	3.313	-0.006	-8.038	-0.063	-0.255	-1.758	5.233	1.157	157.441	-53.009	-2.063	0.124	24.859	6.205
		(-2.974)	(2.643)	(-0.613)	(-1.287)	(-1.051)	(-0.188)	(-1.712)	(1.080)	(2.509)	(2.243)	(-0.205)	(-0.416)	(2.097)		
iid	1	-0.032	1.106	0.026	-27.631	-0.138	-0.560	-0.103	1.922	-0.189	-63.148	11.764	-0.859	0.008	8.470	12.376
Skewed		(-0.306)	(3.267)	(1.380)	(-2.311)	(-2.811)	(-1.999)	(-0.394)	(1.020)	(-1.337)	(-2.535)	(0.382)	(-0.378)	(0.218)		
Student-t	2	-8.414	1.924	-0.004	-8.971	-0.044	-0.260	-1.662	5.625	1.073	153.699	-78.881	-1.389	0.116	25.174	6.148
		(-3.835)	(3.314)	(-0.276)	(-1.138)	(-0.594)	(-0.472)	(-2.236)	(1.222)	(2.842)	(2.473)	(-0.705)	(-0.342)	(2.443)		
Non-*iid*	1	-0.280	1.053	0.021	-24.434	-0.141	-0.513	-0.032	2.528	-0.144	-52.858	14.019	-0.652	0.004	8.435	11.908
Skewed		(-0.209)	(2.474)	(1.001)	(-1.833)	(-3.235)	(-1.866)	(-0.147)	(1.106)	(-0.388)	(-1.522)	(0.199)	(-0.333)	(0.170)		
Student-t	2	-8.509	1.519	-0.005	-8.012	-0.039	-0.303	-1.834	6.457	1.239	159.095	-84.907	-2.104	0.120	25.078	5.694
		(-3.007)	(2.291)	(-0.510)	(-1.351)	(-0.603)	(-0.638)	(-2.294)	(1.229)	(3.185)	(1.931)	(-0.661)	(-0.364)	(2.448)		

Table 11. The table reports the results of estimating the BDL and MS-BDL models using the out-of-sample Hybrid VaR measure. The monthly market excess return at time t+1 is regressed on the Hybrid VaR measure and the following control variables measured at time t: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). For each regression, the first line shows the estimated regression coefficients, while the second line shows the corresponding HAC t-statistics (in parentheses).

Const	Hybrid VaR	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	Adj R2 & Sharpe
Panel A: New period January 2006 - December 2016												
-7.693	2.740		-0.017	1.439	-0.822	-3.436	1.912	122.371	-131.506	-2.577	0.070	23.78%
(-2.461)	(3.454)		(-0.219)	(2.438)	(-0.927)	(-1.840)	(1.828)	(1.349)	(-0.855)	(-1.993)	(2.402)	2.32
Panel B: Full period July 1967 - December 2016												
-2.322	1.260	-12.089	0.052	-0.392	-0.709	1.278	0.392	48.183	-73.766	-1.160	0.059	4.55%
(-2.519)	(3.332)	(-6.978)	(1.229)	(-2.081)	(-2.285)	(0.888)	(2.290)	(1.270)	(-1.421)	(-1.464)	(2.243)	1.04

Markov switching estimation

State	Const	Hybrid VaR	Dummy	Lagged Return	RFD	DTRP	DCRP	DY	IPG	MBG	DIF	DO	State variance	Expected Duration
1	-1.860	2.522	-18.364	-0.120	-0.623	-0.071	3.346	0.011	-52.540	-3.864	-0.394	-0.002	9.097	13.077
	(-1.662)	(3.495)	(-5.818)	(-2.464)	(-2.615)	(-0.242)	(2.053)	(0.096)	(-1.196)	(-0.102)	(-0.433)	(-0.095)		
2	-8.667	2.508	-18.841	-0.008	-0.238	-1.846	5.311	1.187	134.707	-118.339	-1.804	0.124	26.091	6.490
	(-3.766)	(2.765)	(-4.785)	(-0.169)	(-0.718)	(-2.641)	(1.506)	(3.480)	(1.357)	(-1.143)	(-0.728)	(3.507)		

Table 12. The table reports the estimated coefficients of the tail risk measures in predicting future excess returns of different horizon. Each column shows the results of the regression where the horizon of the returns used as the dependent variable is given on the top of the column. The regressions also control for other state variables in the MS-BDL framework which are not reported here due to space scarcity. The *iid* measures are calculated using daily returns over one month. The non-*iid* measures are calculated using daily VaR over one month. The future market excess return is regressed on VaR_t and the following control variables measured at time t: lagged market excess return, October 1987 dummy, detrended risk free rate (RFD), change in the term structure risk premium (DTRP), change in the credit risk premium (DCRP), dividend yield (DY), growth in the industrial production (IPG), growth in the monetary base M2 (MGB), change in the inflation rate (DIF), change in the oil price (DO). Coefficients with * are the ones which are significant at 10 percent significant level or less, using HAC standard error. Skewed Student-t VaRs are estimated at the 99% confidence level. The sample period is July 1962 to December 2016.

Measure	State	1	2	3	4	5	6	7	8	9	10	11	12
iid	1	0.955*	1.608*	2.070*	2.574*	2.959*	2.614*	2.897*	3.061*	2.827*	3.376*	2.681*	2.743*
Non-parametric	2	-0.170	0.814*	0.545	0.216	0.417	-0.445	0.102	-0.798	-1.077	-1.067	-2.324*	-2.475*
iid	1	0.914*	1.465*	1.901*	2.345*	2.744*	2.540*	2.466*	2.255*	2.113*	2.206*	2.343*	2.198*
Skew Student-t	2	-0.055	0.643	0.637	0.079	0.363	0.101	0.006	-1.299	-1.635	-2.227*	-2.699*	-3.193*
Non-*iid*	1	1.076*	2.111*	2.398*	3.028*	3.394*	3.504*	3.746*	3.947*	3.818*	3.094*	2.964*	2.925*
Skewed Student-t	2	0.761*	1.383*	1.076	0.769	1.150	1.173	1.169	1.102	0.375	-1.711*	-1.393	-1.327