Optimal Fiscal Policy in a Model of Firm Entry and Financial Frictions*

Dudley Cooke†

University of Exeter

Tatiana Damjanovic‡

Durham University

Abstract: This paper studies firm entry with financial frictions. We motivate our analysis by documenting that a fall in firm entry and a widening of the interest rate spread occur when there is a rise in idiosyncratic uncertainty. We then develop a model of firm entry and financial frictions - with fluctuations in the volatility of firm-level demand shocks - consistent with this empirical evidence. Finally, we study dividend and labor-income taxation. Financial frictions weaken the incentive to support firm entry, and in a calibrated version of our model, accounting for the increase in volatility observed during the 2007-09 recession, optimal fiscal policy raises (lowers) dividend (labor)-income taxes by up to 7 (1.5) percentage points.

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†Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, United Kingdom. Email: d.cooke@exeter.ac.uk

‡Department of Economics, Durham University, Mill Hill Lane, Durham DH1 3LB, United Kingdom. Email: tatiana.damjanovic@durham.ac.uk.

1. Introduction

Financial market frictions are an important part of the mechanism through which fluctuations in the volatility of firm-level shocks are transmitted to the real economy.[1] Similarly, firm entry has been shown to act as a propagation mechanism for business cycle dynamics.[2] In this paper, we bring these two ideas together. We first document the relationship between idiosyncratic uncertainty and firm entry. Using a vector autoregression, in which we allow for interaction between financial markets and the broader macroeconomy, we show that a fall in firm entry and a widening of the interest rate spread occur when there is a rise in idiosyncratic uncertainty. We then develop a model of firm entry and financial frictions - with fluctuations in the volatility of firm-level demand shocks - consistent with this empirical evidence.

We use our model to study dividend and labor-income taxation. We find that financial frictions weaken the incentive to support firm entry. Financial frictions generate a trade-off for fiscal policy when firm entry is endogenous. One the one hand, financial frictions lead to a reduction in firm entry, which the policymaker would like to mitigate. On the other hand, the policymaker accounts for the agency costs of default, which firms disregard due to limited liability. In a calibrated version of our model, accounting for the increase in volatility observed during the 2007-09 recession, we show that optimal dividend (labor)-income taxes rise (fall) by up to 7 (1.5) percentage points. Optimal fiscal policy therefore involves a switch away from supporting firm entry and towards supporting employment.

[1] Our focus on financial frictions and the volatility of firm-level shocks is similar to Christiano *et al.* (2014), Gilchrist *et al.* (2014), and Arellano *et al.* (2018). Other channels through which idiosyncratic uncertainty can affect the real economy are discussed in, for example, Bloom *et al.* (2018) and Senga (2018).

[2] For example, Clementi and Palazzo (2016) show that firm entry and exit play an significant role in the amplification and propagation of aggregate shocks. Gourio *et al.* (2016) show empirically that reduced firm entry leads to persistent negative effects on GDP.

The model of firm entry we develop builds on Bilbiie *et al.* (2012). New firms enter after paying a one-time cost, and each firm produces a differentiated good, under conditions of monopolistic competition, with a one-period lag.[3] We amend this setup in two directions. We suppose each firm receives an idiosyncratic demand shock, which occurs after labor has been hired, and production has taken place. This generates uncertainty over the revenue a firm can generate from the sale of its product.[4] We also assume firms finance their labor requirements by borrowing working capital from financial intermediaries who operate a monitoring technology similar to Carlstrom and Fuerst (1997).[5] In this environment, firms that produce goods with a relatively low level of ex-post demand default, and agency costs mean that default is costly.

To understand the key mechanism in our model, consider the production decision of an individual firm. Firms produce under limited liability, and they place zero weight on realizations of demand in which profits are negative, since they no longer carry the risk of losses from such realizations. With monopolistic competition, and uncertainty over firm-level revenue, limited liability creates an incentive for each firm to expand production, in an attempt to take advantage of a potentially good realization of demand. Expanding production, however, amounts to committing to a greater level of borrowing, in advance, and increased borrowing requires each firm to generate more revenue to avoid default.

New firms enter the market until their expected profit, conditional on not defaulting, is

[3]Chatterjee and Cooper (1993) and Devereux *et al.* (1996) develop general equilibrium models with (procyclical) firm entry and monopolistic competition which feature endogenous static entry and instantaneous zero profits.

[4]With idiosyncratic demand shocks, the price at which a good is sold (and hence, firm-level revenue) is unknown when the hiring decision is made. The effect of price uncertainty, for a competitive firm, is analysed in Sandmo (1971) and Hartman (1972).

[5]The financial frictions we consider are also similar to Bernanke *et al.* (1999), and firms require working capital, which is complementary to labor (Jermann and Quadrini, 2012).

sufficient to cover the cost of entry. The entry decision is subject to two opposing forces. Due to limited liability, expected revenue is higher, and this encourages entry. However, the minimum level of demand needed to avoid default is also higher, and this discourages entry. Whilst the possibility of default has no negative effect on the production decision of an individual firm, potential entrants weigh the possibility of increased revenues against the probability of default. The latter effect dominates, so, in equilibrium, the mass of firms is lower than when financial frictions are absent.

Now consider an increase in the volatility of firm-level demand shocks. In a more uncertain environment, and with limited liability, firms expand production. The probability of default rises, and this leads to a rise in the interest rate applied to working capital loans. With fewer firms, there is a fall in aggregate output and total employment. Conditional on an uncertainty shock, therefore, firm entry is procyclical, and firm default and the interest rate spread, are countercyclical.

Having established the role of financial frictions for firm entry, we study dividend and labor-income taxation. We show that financial frictions weaken the incentive to support entry. To understand this result, it helps to consider the case without financial frictions. Firm entry reduces profit per-firm (a profit destruction effect) but raises product variety (a consumer surplus effect).[6] Dividend-income should be subsidized - and firm entry encouraged - because the profit destruction effect is relatively weaker than the consumer surplus effect. In this case, with monopolistic competition, firm entry is inefficiently low.[7]

Since financial frictions reduce the mass of firms in the economy, it would appear likely that

[6] This result is discussed in Bilbiie *et al.* (2008, 2016). Chugh and Ghironi (2015) show that, in the long-run, this result is independent of the whether lump-sum taxes are available.

[7] Our analysis is based monopolistic competition and Dixit-Stiglitz preferences. We discuss the role of the consumption aggregator below, but leave the possibility of, for example, oligopolistic competition or translog preferences - both of which would generate an endogenous markup - to future research.

optimal policy should further subsidize dividend-income. In the absence of agency costs, it is optimal to increase the subsidy to dividend-income. With agency costs, however, there is a trade-off for fiscal policy, because, whilst firms neglect such costs, the government accounts for the societal cost of default. This means it is optimal to limit firm entry. In general, the socially optimal number of firms declines with agency costs, and so does the optimal subsidy to dividend-income. If we interpret this in terms of the standard result, with financial market frictions, firm entry is instead inefficiently high.

Our model also has a second margin: labor supply. When firm entry is endogenous, labor-income should receive a subsidy equal to price-markup (Bilbiie *et al.*, 2008). In our analysis, the subsidy to labor-income depends on the markup and the interest rate spread, and this introduces a second role for agency costs. As the volatility of firm-level demand rises, the markup falls - reducing the optimal subsidy to labor-income - but the interest spread rises - raising the subsidy. Taken together, the trade-off that characterizes the subsidy to dividend-income applies to labor-income. Therefore, during a recession, optimal fiscal policy involves a switch away from supporting firm entry and towards supporting employment.

We assess the main results of our paper quantitatively. To put our analysis into context, we focus on the Great Recession of 2007-2009 - a period characterized by an unprecedented increase in uncertainty and a drop in firm entry. Keeping dividend and labor-income taxes fixed, our model implies a (maximum) drop in firm entry of around 25 percent and a rise in the default rate of 1.5 percentage points. Optimal policy acts to raise (lower) dividend (labor)-income taxes by up to 7 (1.5) percentage points. As a point of comparison, we also consider a change in the labor-income tax, of 1 percent point, during the second quarter of 2009, consistent with the American Recovery and Reinvestment Act.[8] This policy indirectly

[8] Zubairy (2014) considers the Recovery Act of 2009, and focuses on the joint implications of a 1 percentage point cut in the labor-income tax, alongside anticipated rises in government spending, the latter from which we abstract.

supports firm entry, and thereby raises the agency costs of default.

Our paper contributes to the literature on the design of fiscal policy when product variety is endogenous and our results on optimal fiscal policy are closely related to Chugh and Ghironi (2015). An important finding, in the context of our analysis, is the result that dividend and labor-income taxes should not respond to aggregate shocks when preferences are of the Dixit-Stiglitz type.[9] This allows us to provide an analytical characterization of optimal fiscal policy when there are financial frictions. In general, we find that dividend and labor-income taxes should be time-varying. The design of fiscal policy with endogenous product variety has also been studied in environments with physical capital (Coto-Martinez *et al.*, 2007), long-run risk (Croce *et al.*, 2013), and oligopolistic competition (Colciago, 2016).[10]

The transmission mechanism through which fluctuations in the volatility of idiosyncratic shocks are propagated to the real economy has been discussed in a number of recent papers. For example, in Christiano *et al.* (2014), a widening in the distribution of productivity shocks increases the fraction of defaults, and in Gilchrist *et al.* (2014), financial frictions magnify shocks to firm-level uncertainty through movements in credit spreads. Arellano *et al.* (2018) argue that the majority of the decline in employment during the 2007-09 recession can be explained by an increase in firm-level volatility. Since firm entry plays an important role in aggregate fluctuations, our results provide a potentially different route through which financial frictions and idiosyncratic uncertainty can affect the macroeconomy.

Finally, our paper is related to a large literature on firm entry and exit. Our approach is most similar to Bilbiie *et al.* (2012). To their model of firm entry, we allow for endogenous

[9]In Chugh and Ghironi (2015), the extent to which profits should be taxed is also discussed in the context of preference aggregation. We choose to work with a form of preferences that lead to constant taxes to focus on the role of financial frictions.

[10]Lewis and Winkler (2015) analyse tax policy in a static model with firm entry. Edmond *et al.* (2018) analyze the cost of markups with firm heterogeneity and firm entry.

default by incorporating ex-post firm-level heterogeneity, a working capital constraint, and financial frictions. A complementary approach to studying firm entry and exit, which amends Hopenhayn's (1992) model with ex-ante heterogeneous firms to allow for investment in physical capital and aggregate shocks, is developed by Clementi and Palazzo (2016). Our modelling choices - which imply a symmetric employment decision by firms in equilibrium - are driven by the desire to generate analytical results. A general point, however, is that, in either setting, firm entry is a form of investment in which up-front costs incurred to start a business generate expected future profits.

The paper is organized as follows. In section 2, we motivate our theoretical work by analyzing the link between idiosyncratic uncertainty and firm entry using a vector autoregression. We study a static general equilibrium model of firm entry and financial frictions in section 3 and derive analytical expressions for the optimal mix of taxes on dividend and labor-income in section 4. In section 5, we develop a dynamic version of our model, and in section 6, we undertake a quantitative analysis, where we revisit the results on fiscal policy, motivated by the increase in uncertainty during 2007-09. A final section concludes.

2. Motivation and Empirical Evidence

This section provides motivation for the theoretical analysis to follow. We begin by plotting quarterly birth and death rates for US establishments and proxies for idiosyncratic uncertainty.

========== **Figure 1 Here** ==========

The upper panel of Figure 1 reports establishment birth and death rates over the period 1993:Q2-2015:Q1.[11] Birth and death rates are pro and countercyclical, respectively, and

[11]Data on establishment entry and exit is available from from the BLS's Business Employment Dynamics

change significantly during the 2007-09 period. The lower panel of Figure 1 reports three proxies for idiosyncratic uncertainty. The first proxy is based on the Federal Reserve Bank of Philadelphia's Business Outlook Survey, BOS; the second proxy is the interquartile range of firm-level sales growth, sIQR, and is based on Compustat data; the final proxy, iVOL, is that proposed by Gilchrist *et al.* (2014), and uses stock returns for US non-financial corporations. In all cases, uncertainty is countercyclical. To gauge the increase in uncertainty during the 2007-09 recession, we regressed each uncertainty proxy on a dummy variable covering this period. Our estimates suggest that, during the recession, uncertainty was about 20 percent above its long-run average.[12]

To understand the relationship between firm entry, uncertainty, and financial frictions, we use a vector autoregression model, with net firm entry, a measure of uncertainty, the interest rate spread, and real per capita GDP.[13]

========== **Figure 2 Here** ===========

In Figure 2, we report impulse responses to a one standard deviation innovation in uncertainty, for each proxy. Following an increase in uncertainty, net entry falls, with a peak response of around 0.3 and 0.4 percent, at between 10 and 12 quarters. There is a similar

database from 1993:Q2 onwards. The general cyclical pattern of entry and exit we report was first established by Campbell (1998), who studied (employment weighted) entry and exit rates for US manufacturing plants over 1972:Q2-1988:Q4.

[12]See Appendix A. In all cases, the correlation of these proxies with net entry is negative. The correlation of uncertainty with net entry varies between -0.06 (BOS) and -0.44 (sIQR) and the correlation between these two particular measures of uncertainty is 0.3.

[13]We order the variables $[uncertainty, spread, net\ entry, GDP]'$ and use a Cholesky decomposition to identify the uncertainty shock. The construction of these variables is discussed in Appendix A.

pattern of adjustment for GDP and a marked rise in the interest rate spread.[14] To a large extent, these results accord with Bachmann *et al.* (2013) and Gilchrist *et al.* (2014). Changes in uncertainty have an immediate effect on the macroeconomy and uncertainty plays a role in determining conditions in financial markets.

In our theoretical analysis, we also consider optimal dividend and labor-income taxes. One result we emphasize is that subsidies (taxes) on dividend-income should decrease (increase) when there are financial frictions. There is an established negative effect of corporate taxation on firm entry. For example, Da Rin *et al.* (2011), find a negative effect of corporate income taxation on entry rates, using industry-level panel data for 17 European countries, and Djankov *et al.* (2010) find a 10 percentage point increase in the first year effective corporate tax rate reduces the average entry rate by 1.4 percentage points, across 85 countries.[15]

3. Analytical Model

In this section, we develop a static general equilibrium model of firm entry and financial frictions. We explain how a change in the volatility of idiosyncratic demand can generate a reduction in firm entry, a drop in GDP, and a widening of the interest rate spread.

3.1. Model Economy

The economy is populated with a measure $n_t > 0$ of firms and a measure one of households and financial intermediaries. Each firm has a linear production technology and supplies a

[14]To check for robustness, we re-ordered the VAR, with the spread first, i.e., $[spread, uncertainty, net\ entry, GDP]'$. In this case, uncertainty shocks produced qualitatively similar results. We also experimented by replacing the spread with the GZ spread of Gilchrist and Zakrajsek (2012). Again, we found similar results.

[15]It is less common to study the impact of taxation on firm exit. However, Colciago *et al.* (2017) present evidence that a reduction in taxation results in an immediate drop in firm exit, with a delayed response of firm entry, using quarterly data for the US.

differentiated good. New firms are created each period by paying a one-time cost. Households consume a basket of goods and supply labor to firms. Financial intermediaries hold deposits from households and issue intra-period working capital loans to firms. If a firm has sufficient revenue, it repays it's loan to the financial intermediary. If not, the firm defaults, and the intermediary repossesses the assets of the firm, subject to a cost of receivership.[16]

At the beginning of the period, new firms are created, and households place deposits with financial intermediaries. Firms then make an employment decision and sign a contract with a financial intermediary to cover their working capital requirements.[17] Production takes place, and idiosyncratic demand, $\varepsilon \geq 0$, is realized. Firms with a sufficiently high level of demand, $\varepsilon \in [\varepsilon_t^\star, \infty)$, sell their goods to households. Firms with a low level of demand, $\varepsilon \in [0, \varepsilon_t^\star)$, default. Households receive net-of-tax dividend and labor-income, interest payments on deposits, and a lump-sum transfer from the government. At the end of the period all firms exit.[18]

Households Each household draws utility from a composite of goods, C_t, and disutility from aggregate labor, L_t, according to the following additively separable function,

$$u\left(C_t, L_t\right) \tag{1}$$

which is strictly increasing and strictly concave in C_t and strictly decreasing and strictly convex in L_t. Total consumption is, $C_t = \left\{n_t^{-\omega} \int_{i\in\Omega} \left[\varepsilon\left(i\right) \times c_t\left(i\right)\right]^\theta di\right\}^{1/\theta}$, where $c_t\left(i\right)$ is the consumption of good $i \in \Omega$, and $1/\left(1-\theta\right) > 1$ is the elasticity of substitution. The parameter ω controls consumer love of variety.[19] The integration over the probability space,

[16]Our formulation is equivalent to all firms selling their production and the financial intermediary bearing the burden of unpaid loans.

[17]The timing restriction we place on the firm is similar to Neumeyer and Perri (2005).

[18]At this point, we distinguish exogenous exit from default, because only the latter is associated with agency costs. We discuss this in more detail below.

[19]When $\omega = 0$, we have standard Dixit-Stiglitz preferences. When $\omega = 1-\theta$, we eliminate love of variety, and consumers are indifferent between consuming n_t units of a single good or 1 unit of n_t identical goods.

Ω, is $n_t \int dG(\varepsilon)$, and $G(\varepsilon)$ is the cumulative distribution function of idiosyncratic demand shocks.[20]

The representative household maximizes utility, subject to the budget constraint, $C_t = \left(1 - \tau_t^L\right) w_t L_t$, where $\tau_t^L < 1$ is a labor-income tax. This leads to a standard labor-leisure equation,

$$\frac{-u_L(t)}{u_C(t)} = \left(1 - \tau_t^L\right) w_t \tag{2}$$

where $u_C(t)$ and $u_L(t)$ denote the marginal utilities, evaluated at time-t. The household also chooses consumption, $c_t(i)$, to minimize the cost of acquiring C_t, taking prices and income as given. This leads to a downward-sloped demand curve for each good, which we express as,

$$c_t(i) = \left[\frac{p_t(i,\varepsilon)}{n_t^{-\omega}\varepsilon(i)^\theta}\right]^{-1/(1-\theta)} Y_t \tag{3}$$

where $p_t(i,\varepsilon)$ is the price of good i in units of consumption and Y_t is aggregate output.

Firms Each firm produces a differentiated good with technology,

$$y_t(i) = l_t(i) \tag{4}$$

where $y_t(i)$ is output and $l_t(i)$ is employment. Firms use working capital to finance production, and this requires a loan, at gross rate $r_t \geq 1$, equal to $w_t l_t(i)$.[21] The profit of firm i, with demand ε, is,

$$\pi_t(i,\varepsilon) = p_t(i,\varepsilon)\, y_t(i) - w_t r_t l_t(i) \tag{5}$$

[20]Similar to Bernard *et al.* (2011), the firm-level shock reflects product attributes, or product appeal. Midrigan (2011) refers to this shock as a quality shock.

[21]The interest rate on loans, r_t, is strictly greater than the interest rate on deposits. The deposit rate is exogenous in the static version of the model.

where $p_t(i,\varepsilon)\, y_t(i)$ are sales and $w_t r_t l_t(i)$ is debt.

Throughout the analysis, we assume firms operate under limited liability and act as though profit is bounded from below at zero. This implies a threshold level of demand, $\varepsilon_t^\star$, determines the mass of firms unable to meet their debt obligations ex-post. This endogenous threshold level of demand is defined as, $\varepsilon_t^\star \equiv \inf\{\varepsilon(i) : \pi_t(i,\varepsilon) > 0\}$, and the probability of default is $G(\varepsilon_t^\star) = \int_0^{\varepsilon_t^\star} dG(\varepsilon)$, where $G'(\varepsilon_t^\star) > 0$.

Each firm chooses an employment level, subject to demand and technological constraints, given by equations (3) and (4), and market clearing, $c_t(i) = y_t(i)$, to maximize conditional expected profits, $\pi(\varepsilon_t^\star) \equiv \int_{\varepsilon_t^\star}^{\infty} \pi(i,\varepsilon)\, dG(\varepsilon)$. To economize on notation, in what follows, we drop the i index. Profit maximization implies,

$$\left[\int_0^{\infty} \varepsilon^\theta dG(\varepsilon)\right]^{(1-\theta)/\theta} n_t^\alpha (\varepsilon_t^\star)^\theta = w_t r_t \quad \text{and} \quad \theta \int_{\varepsilon_t^\star}^{\infty} \frac{1}{1-G(\varepsilon_t^\star)} \varepsilon^\theta dG(\varepsilon) = (\varepsilon_t^\star)^\theta \tag{6}$$

where $\alpha \equiv [(1-\theta)-\omega]/\theta$.[22] The first condition in equations (6) determines the mass of firms, n_t, as a function of the marginal costs of production, $w_t r_t$ - the wage rate multiplied by the interest rate on working capital loans - the threshold level of demand, $\varepsilon_t^\star$, and the average demand for all goods, $\left[\int_0^{\infty} \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$, which is exogenous. The second condition implicitly determines the threshold. Although we have yet to determine the general equilibrium of the model, we see that, in this case, $\varepsilon_t^\star$ is independent of the macroeconomy. Thus, for given costs of production, a higher default rate, $G(\varepsilon_t^\star)$, is associated with a smaller mass of firms.

There is an unbounded mass of potential firms and the creation of a new firm is subject to an entry cost. Firms enter until the conditional expected profit, net of taxation, $\tau_t < 1$, is equal to the cost of entry. The free entry condition reads,

$$(1-\tau_t)\, \pi(\varepsilon_t^\star) = f_e \tag{7}$$

[22]The details of the firm optimization problem are presented in Appendix B.

where the cost of entry, $f_e > 0$, is specified in units of output, similar to Jaimovich and Floetotto (2008).[23]

Financial Intermediaries Each financial intermediary receives deposits and issues working capital loans. The expected assets of a financial intermediary are the revenue from the repayment of loans and the assets from liquidated firms, less the cost of receivership, $\phi > 0$. Financial intermediaries are competitive and earn zero profit, which leads to,

$$\left[\int_{\varepsilon_t^\star}^{\infty} dG(\varepsilon) + \int_0^{\varepsilon_t^\star} \left(\frac{\varepsilon}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon_t)\right] r_t = 1 + \phi\left[G(\varepsilon_t^\star)/w_t l_t\right] \tag{8}$$

where $\int_{\varepsilon_t^\star}^{\infty} dG(\varepsilon)$ is the survival probability of a firm and $\int_0^{\varepsilon_t^\star}\left(\frac{\varepsilon}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon_t)$ is the ratio of assets-to-loans of defaulting firms. The liabilities of financial intermediaries are given by $r_t^d w_t l_t$, where $r_t^d = 1$ is the normalized interest rate on deposits. Equation (8) defines the interest rate on working capital loans and the interest rate spread.

Equilibrium Labor market equilibrium requires,

$$L_t = n_t l_t \tag{9}$$

Equation (9) implies that, for given levels of aggregate employment, L_t, fewer operating firms, n_t, translates into an increase in firm-level employment, l_t.

The resource constraint of the economy is,

$$Y_t = C_t + f_e n_t + \phi\left[n_t G(\varepsilon_t^\star)\right] \tag{10}$$

where $Y_t = \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta} l_t n_t^{1+\alpha}$ is aggregate output, $f_e n_t$ represents investment at the extensive margin, and $\phi\left[n_t G(\varepsilon_t^\star)\right]$ is the resource cost associated with default.

[23] As emphasized by Djankov *et al.* (2002), entry costs not only reflect the time and effort of the entrepreneur, but also bureaucratic and transactions costs required for setting up a business. Also see Barseghyan and DiCecio (2011).

3.2. Changes in Volatility

In this section, we study a change in the volatility of idiosyncratic demand shocks. We show that a rise in volatility leads to a reduction in firm entry, a fall in GDP, and a widening of the spread applied to working capital loans.

To generate analytical results we assume demand shocks are lognormally distributed, with probability density function,

$$g(\varepsilon) = \frac{1}{\varepsilon \sigma \sqrt{2\pi}} \exp\left[-\frac{(\ln \varepsilon - m)^2}{2\sigma^2} \right] \tag{11}$$

where m and σ are the location and scale parameters and $E(\varepsilon) = 1$. From this point onwards, when we refer to the volatility of idiosyncratic demand shocks (or, volatility), this corresponds to the parameter σ. Since $E(\varepsilon) = 1$, a rise in volatility is also a mean preserving spread.

Proposition 1 When idiosyncratic shocks have a log normal distribution, the default threshold, $\varepsilon_t^\star$, and the probability of default, $G(\varepsilon_t^\star)$, increase with the volatility of idiosyncratic shocks, for $G(\varepsilon_t^\star) < 1/2$.

Proof See Appendix B. ■

We have already established a negative relationship between the mass of firms in the economy and the threshold level of demand, for given costs. Proposition 1 says that the same idea applies when we consider volatility. More volatile economies feature fewer firms and more defaults.

Before discussing the variables analyzed in section 2, we determine the aggregate markup. The average price of all goods in the economy (i.e., including those firms that default) is, $p_t \equiv \int_0^\infty p_t(\varepsilon)\, dG(\varepsilon)$. Using the demand curve, equation (3), we determine $p_t =$

$n_t^{\alpha}\left[\int_0^{\infty}\varepsilon^{\theta}dG\left(\varepsilon\right)\right]^{1/\theta}$. This expression says that the average price depends on the mass of firms and the average level of demand. We then link prices with marginal costs, $w_t r_t$, using equations (6).

$$p_t = \frac{\kappa\left(\varepsilon_t^{\star}\right)}{\theta} w_t r_t \quad ; \quad \kappa\left(\varepsilon_t^{\star}\right) \equiv \frac{\int_0^{\infty}\varepsilon^{\theta}dG\left(\varepsilon\right)}{\int_{\varepsilon_t^{\star}}^{\infty}\frac{1}{1-G\left(\varepsilon_t^{\star}\right)}\varepsilon^{\theta}dG\left(\varepsilon\right)} \tag{12}$$

where $\kappa\left(\varepsilon_t^{\star}\right) < 1$ and $\kappa'\left(\varepsilon_t^{\star}\right) < 0$ is an adjustment to the standard markup, $\frac{1}{\theta}$, which results from monopolistic competition and CES preferences.[24] Thus, financial frictions reduce the aggregate price-markup, a result we explain in the following way. Because firms operate under limited liability, when making an employment decision, they place zero weight on realizations in which profits will be negative. Since the loss from a low realization of demand is zero, there is an incentive for the firm to produce a higher level of output, with a lower expected price.

We also associate a higher threshold, $\varepsilon_t^{\star}$, with a volatility-induced recession, so, conditional on a shock to volatility, the aggregate markup is procyclical in our model (we show $\kappa\left(\varepsilon_t^{\star}\right)$ falls with volatility in Appendix B). Whilst there is considerable evidence for countercyclical markups (e.g., Bils (1987), Rotemberg and Woodford (1999)), Nekarda and Ramey (2013) argue that aggregate price markups are pro- to acyclical unconditionally, and Born and Pfeifer (2017) present evidence that price markups are procyclical, conditional on an uncertainty shock. Chevalier and Scharfstein (1996), and very recently, Gilchrist *et al.* (2017) study price-markups when there are financial frictions, and Etro and Colciago (2010) and Lewis and Poilly (2012) study price-markups and firm entry.[25] The mechanism in our model dif-

[24] We define the conditional price as, $p\left(\varepsilon_t^{\star}\right) \equiv \int_{\varepsilon_t^{\star}}^{\infty}\frac{1}{1-G\left(\varepsilon_t^{\star}\right)}p_t\left(\varepsilon\right)dG\left(\varepsilon\right)$, and we find, $p\left(\varepsilon_t^{\star}\right) = \frac{w_t r_t}{\theta}$, which says that financially unconstrained firms have higher (constant) markups.

[25] Chevalier and Scharfstein (1996) document that during regional and macroeconomic recessions, more financially constrained supermarket chains raise their prices relative to less financially constrained chains. Gilchrist *et al.* (2017) document that liquidity constrained firms increased prices in 2008, while their unconstrained counterparts cut prices.

fers considerably to these papers, the former of which assume customer markets, and latter, oligopolistic competition.

We now discuss the macroeconomic and financial variables analyzed in section 2. As a simplification, we assume $u_C(t) = 1/C_t$, which allows us to express total employment as a function of the threshold and dividend and labor-income taxes. Total employment is decreasing in the threshold and taxes.[26] Aggregate output and the mass of firms are determined by the following expressions,

$$Y_t = \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta} n_t^\alpha L_t \quad \text{and} \quad n_t = (1-\tau_t)\left(\frac{1-\theta}{f_e}\right)\frac{1-G(\varepsilon_t^\star)}{\kappa(\varepsilon_t^\star)}Y_t \tag{13}$$

The first condition in equations (13) is aggregate output. This condition is only dependent on financial frictions to the extent that frictions affect the mass of firms in the economy. The second condition in equations (13) is the free entry condition.

Consider the effect of an increase in volatility, σ. First, aggregate output depends on average demand, which is equal to $\exp\left[(\theta-1)\frac{\sigma^2}{2}\right]$.[27] Average demand is falling in volatility due to Jensen's inequality, which implies, $\int_0^\infty \varepsilon dG(\varepsilon) > \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$, for $\theta < 1$. Thus, due to the concavity of the aggregator function, a mean preserving spread will cause a fall in aggregate output, for a given mass of firms. To explain how financial frictions affect output, we use the entry equation, and note that as volatility rises, the term $[1-G(\varepsilon_t^\star)]/\kappa(\varepsilon_t^\star) = \int_0^\infty \varepsilon^\theta dG(\varepsilon) / \int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG(\varepsilon)$ falls. This term reflects the two opposing forces that act on the entry decision. Expected revenue is higher, because firms operate under limited liability, and this encourages entry.[28] However, potential entrants account for the increased probability

[26]The exact expression for $L_t = L(\tau_t^L, \tau_t; \varepsilon_t^\star)$ is derived in Appendix B. For the result $\partial L_t/\partial\varepsilon_t^\star < 0$, straightforward analytical solutions were unavailable, so we checked numerically using parameter values consistent with our calibrated model (section 6).

[27]Given the distribution of idiosyncratic demand, $\int_0^\infty \varepsilon^\theta dG(\varepsilon)$ is equal to $\exp\left[\theta m + \frac{(\theta\sigma)^2}{2}\right]$, where $m = -\sigma^2/2$, since $E(\varepsilon) = 1$.

[28]Firm sales are, $s(\varepsilon_t) \equiv p(\varepsilon_t) y_t = \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{(1-\theta)/\theta} n_t^\alpha \varepsilon^\theta y_t$, and average sales, conditional on not

of default, which depresses entry, and this effect dominates. Thus, in a volatility-induced recession, firm entry (default) is pro (counter)-cyclical.

The overall implications of a change volatility also depend on consumer love of variety, which is controlled by the parameter ω. A standard approach is to assume values $\omega \in \{0, 1-\theta\}$, where the former (latter) case corresponds to Dixit-Stiglitz (scale free) preferences. This parameterization is of independent interest in our analysis because, given total employment, it determines how financial frictions affect output. In particular, since output is unaffected by the mass of firms once preferences are scale free, output is insulated from financial frictions, and falls with volatility only through lower average demand, $\left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$. However, this is not the case for firm entry, and in all cases, as volatility rises, there are fewer firms.

Finally, we consider the interest rate spread, $r_t > 1$, which is determined by the zero-profit condition for financial intermediaries. Re-writing equation (8),

$$r_t = \frac{1}{1-G(\varepsilon_t^\star)} \left\{1 + \frac{1}{\theta}\left[\frac{\kappa(\varepsilon_t^\star)}{1-G(\varepsilon_t^\star)} - 1 - \frac{\phi(1-\tau_t)(1-\theta)}{f_e} G(\varepsilon_t^\star)\right]\right\}^{-1} \tag{14}$$

which makes it clear that the interest rate spread is increasing with volatility. This is because the agency costs of default - which are controlled by the parameter $\phi \geq 0$ - rise with volatility, and the probability of survival, $\int_{\varepsilon_t^\star}^\infty dG(\varepsilon)$, falls with volatility.[29]

4. Optimal Fiscal Policy

In this section, we study optimal dividend and labor-income taxes. We demonstrate that financial frictions weaken the incentive to support entry.

Proposition 2

defaulting are, $s(\varepsilon_t^\star) = \int_{\varepsilon_t^\star}^\infty \frac{1}{1-G(\varepsilon_t^\star)} s(\varepsilon_t)\, dG(\varepsilon)$. Using the definition for the markup adjustment, we have, $s(\varepsilon_t^\star) = \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta} \frac{n_t^\alpha l_t}{\kappa(\varepsilon_t^\star)}$.

[29] It is possible to generate a spread, without agency costs (i.e., as $\phi \to 0$), because the spread also depends on the ratio of assets-to-loans of defaulting firms, $\int_0^{\varepsilon_t^\star} \left(\frac{\varepsilon}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon_t)$.

1. The optimal dividend-income tax is,

$$\tau_t = 1 - \frac{\alpha}{1-\theta}\frac{\kappa\left(\varepsilon_t^{\star}\right)}{1-G\left(\varepsilon_t^{\star}\right)}\frac{1}{1+fG\left(\varepsilon_t^{\star}\right)} \tag{15}$$

where $\alpha \equiv \left[(1-\theta)-\omega\right]/\theta$ and $f \equiv \phi/f_e$.

2. The optimal labor-income tax is,

$$\tau_t^L = 1 - \frac{\kappa\left(\varepsilon_t^{\star}\right)}{\theta} r_t \tag{16}$$

where $r_t \geq 1$ is determined by equation (8).

Proof See Appendix B. ■

Proposition 2 says that, absent financial frictions (and with Dixit-Stiglitz preferences; $\omega = 0$), both dividend and labor-income should be subsidized at the monopolistic markup, with $\tau_t = \tau_t^L = 1 - \frac{1}{\theta} < 0$. Dividend-income should be subsidized because the returns-to-variety outweigh the reduction in profit per-firm implied by additional firm entry. This is a result of a profit destruction effect and a consumer surplus effect when firm entry is endogenous.[30] Labor-income should be subsidized because leisure is not subject to a markup and there is a wedge between the marginal rates of substitution and transformation of consumption and leisure. From the perspective of our analysis, an equal subsidy to dividend and labor-income is an important benchmark, and without financial frictions, equations (15) and (16) imply there is no role for short-run stabilization policy.

We now characterize optimal policy with financial frictions. We start with the subsidy to dividend-income. To understand the trade-off for fiscal policy when there are financial frictions and firm entry is endogenous we define two wedges. First, at the societal level, we

[30] This terminology is taken from Grossman and Helpman (1991) and this trade-off is discussed in Bilbiie *et al.* (2008, 2016) and Chugh and Ghironi (2015) both of which analyse a dynamic model without default.

use the resource constraint, given by equation (10), to express the ratio of the marginal cost to marginal benefit of firm entry in the following way.[31]

$$\Lambda_t^s \equiv \frac{1}{1+fG\left(\varepsilon_t^\star\right)}\frac{\alpha}{1-\theta}\frac{\pi\left(\varepsilon_t^\star\right)}{f_e\left[1-G\left(\varepsilon_t^\star\right)\right]/\kappa\left(\varepsilon_t^\star\right)} \tag{17}$$

Second, at the decentralized level, because each potential entrant does not internalize the cost of default, the marginal benefit of firm entry is the net-of-tax expected profit, $(1-\tau_t)\,\pi\left(\varepsilon_t^\star\right)$, and the cost of entry is $f_e > 0$. The ratio marginal cost to marginal benefit is,

$$\Lambda_t^d \equiv (1-\tau_t)\,\frac{\pi\left(\varepsilon_t^\star\right)}{f_e} \tag{18}$$

The optimal dividend-income tax equalizes the societal and private margins on firm entry. Since equations (17) and (18) only depend on $\varepsilon_t^\star$, consistent with our discussion above, we start by analyzing how a change in $\varepsilon_t^\star$ affects the optimal tax. An increase in $\varepsilon_t^\star$ implies a higher default rate, $G\left(\varepsilon_t^\star\right)$. As the default rate rises, the subsidy to dividend-income falls (lower Λ_t^s), and it is optimal to restrict entry, relative to the case without financial frictions. An increase in $\varepsilon_t^\star$ also leads to a fall in $\left[1-G\left(\varepsilon_t^\star\right)\right]/\kappa\left(\varepsilon_t^\star\right)$, which depresses entry. This has the opposite effect to the change in the default rate, as far as optimal taxation is concerned, since it requires raising the subsidy to dividend income (higher Λ_t^s).

Notice, as $\omega \to 1-\theta$, and love of variety is eliminated, equation (15) implies $\tau_t \to 1$; i.e., a 100 percent tax can be optimal, a possibility discussed in Chugh and Ghironi (2015).[32] One interpretation of this result, is that, for any $\omega \in [0, 1-\theta)$, the policymaker faces an additional trade-off when there are financial frictions, but without love of variety, since there

[31] The social benefit of firm entry is output. At a given level of employment, output increases with the number of firms, and the marginal benefit is, $\partial Y_t/\partial n_t = \alpha Y_t/n_t$. The social costs of entry are the cost of entry and default, and the marginal costs are, $f_e + \phi G\left(\varepsilon_t^\star\right)$. Thus, the ratio of marginal social benefits to marginal costs is, $\left(\alpha Y_t/n_t\right)/\left[1+fG\left(\varepsilon_t^\star\right)\right]f_e$. In appendix B, we show that $\pi\left(\varepsilon_t^\star\right) = (1-\theta)\frac{Y_t}{n_t}\left[1-G\left(\varepsilon_t^\star\right)\right]/\kappa\left(\varepsilon_t^\star\right)$.

[32] In this case, absent financial frictions, we are left with only the profit destruction effects (the reduction in profit per-firm) from additional firm entry.

is no reason to support entry, the trade-off is eliminated. There is a second case in which taxation can be optimal. When f is relatively high, so are the costs of receivership, and the trade-off generated by financial frictions worsens. However, whilst a fall in love of variety and a rise in the cost of receivership have similar normative implications (at least from the perspective of the taxation of dividend-income), the latter directly affects the interest rate applied to working capital loans.

Proposition 3 The dividend-income tax is procyclical for $f > f^{\star}$, where $f^{\star} > 0$ is defined in Appendix B.

Proof See Appendix B. ■

Proposition 3 characterizes the response of the subsidy to dividend-income to a change in volatility. Whist, for relatively low costs of receivership, it may be optimal to rise subsidies during a recession, as these costs rise, it becomes optimal to restrict firm entry. We emphasize the following point. Even in a recession, with reduced firm entry, it may not be optimal to encourage entry because the endogenous component of default costs are relatively high. These societal costs are disregarded by firms because they act under limited liability.

We now consider the subsidy to labor-income. The presence of financial frictions lowers the markup charged by firms. Without financial frictions, $\kappa\left(\varepsilon_t^{\star}\right) \rightarrow 1$, and so the adjustment to the optimal labor-income subsidy is a reflection of a change in the markup under financial frictions. Despite this, due to the presence of the interest rate spread, the trade-off identified for dividend-income policy is also relevant for labor-income policy. Eliminating the spread in equation (16),

$$\tau_t^L = 1 - \frac{1}{1-(1-\theta)\left[1-G\left(\varepsilon_t^{\star}\right)\right]/\kappa\left(\varepsilon_t^{\star}\right) - \alpha/\left[1+1/fG\left(\varepsilon_t^{\star}\right)\right]} \tag{19}$$

When there is a rise in $\varepsilon_t^{\star}$, there are offsetting effects. Since $\left[1-G\left(\varepsilon_t^{\star}\right)\right]/\kappa\left(\varepsilon_t^{\star}\right)$ falls, this implies a reduction in the subsidy to labor-income. However, since $\alpha > 0$, and $G\left(\varepsilon_t^{\star}\right)$ rises with $\varepsilon_t^\star$, this implies a higher subsidy to labor-income. This result also holds when we consider a change in volatility, and the relative strength of these effects depends on agency costs. For relatively larger values of f, the subsidy to labor-income should rise with volatility. Therefore, during a volatility-induced recession, optimal fiscal policy involves a switch away from supporting firm entry and towards supporting employment.

5. Dynamic Model

In this section, we develop a dynamic version of our model. In doing so, we allow for a gradual adjustment in the number of firms, with noninstantaneous zero profits. We also assume the government finances an exogenous stream of public spending through dividend and labor-income taxes, and by issuing state-contingent government debt.[33]

Time is indexed by, $t = 0, 1, 2...$ In each period, there is a measure of $n_t > 0$ firms, each producing a differentiated good, and $n_{e,t} > 0$ entrants, which produce in period $t + 1$. Preexisting firms and new entrants have a probability δ of exiting. Since exit occurs after production and entry have taken place, and the total mass of firms in period $t + 1$ is, $n_{t+1} = (1 - \delta)(n_t + n_{e,t})$. Firms require an intra-period working capital loan to produce their good. Firms that receive a low realization of demand, $\varepsilon \in [0, \varepsilon_t^\star)$, are financially constrained, and default. Defaulting firms generate agency costs, whilst firms that exit for exogenous reasons do not.

5.1. Households

Lifetime utility is, $\sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$, where $\beta \in (0, 1)$ is the subjective discount factor. Households place deposits, d_t, with financial intermediaries, and purchase shares, x_t, in firms. They also have access to a complete set of state-contingent government bonds, B_{t+1}^s.

[33]These two changes bring our analysis closer to that of Bilbiie *et al.* (2012) and Chugh and Ghironi (2015).

Households maximize their expected lifetime utility, subject to the following flow budget constraint,

$$d_t + \sum_s \frac{1}{r_t^s} B_{t+1}^s + C_t + (n_t + n_{e,t}) x_t v_t = \left(1 - \tau_t^L\right) w_t L_t + \left[(1 - \tau_t) \pi \left(\varepsilon_t^\star\right) + v_t\right] n_t x_{t-1} + r_{t-1}^d d_{t-1} + B_t^s \tag{20}$$

where r_t^d and r_t^s are the rates of return on deposits and bonds, and v_t is the price of the firm at the end-of-period t, after the realization of uncertainty. Household decisions over bonds and deposits are characterized by the following consumption Euler equations,

$$u_C(t) = \beta E_t r_t^s u_C(t+1) \quad \text{and} \quad u_C(t) = \beta E_t r_t^d u_C(t+1) \tag{21}$$

The optimal condition for equity is,

$$v_t = \beta (1 - \delta) E_t \left\{ \frac{u_C(t+1)}{u_C(t)} \left[(1 - \tau_{t+1}) \pi \left(\varepsilon_{t+1}^\star\right) + v_{t+1}\right] \right\} \tag{22}$$

Under this formulation, once a firm defaults, its value is retained, and sold in the following period.

5.2. Firms

Each firm has production function, $y_t = a_t l_t$, where a_t is aggregate technology. We write the instantaneous profit function of a firm as,

$$\pi_t = \max \left\{ \varepsilon^\theta n_t^\alpha \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{(1-\theta)/\theta} \bar{l}_t^{1-\theta} y_t - w_t r_t \left(l_t + \frac{f_o}{a_t}\right), 0 \right\} \tag{23}$$

where $f_o > 0$ is a quasi-fixed overhead cost, denominated in units of labor, and $\bar{l}_t$ is average firm-level employment, which is taken as given by the firm.[34] The introduction of an overhead cost - a cost distinct from firm entry - has two roles. First, there is an interaction

[34] In our specification the entire wage bill is borrowed in advance. Evidence for this assumption is presented in Lewis and Poilly (2012).

between the threshold level of demand and firm-level employment, which is absent in the static model. All else equal, higher overhead costs imply higher default rates, so we use overhead costs to help match default rates in the data.[35] Overhead costs also play a role in total employment. Bartelsman *et al.* (2013) suggest that firms' use of overhead labor accounts for approximately 14 percent of total employment in US manufacturing establishments.

Firms maximize conditional expected net worth, $z_t\left(\varepsilon_t^\star\right) \equiv \max\left[\left(1-\tau_t\right)\pi\left(\varepsilon_t^\star\right), 0\right] + v_t$, subject to demand and technological constraints, and the optimal level of employment is,

$$\int_{\varepsilon_t^\star}^{\infty}\left[\theta\varepsilon^\theta - \left(\varepsilon_t^\star\right)^\theta\left(\frac{l_t}{l_t + f_o/a_t}\right)\right]dG(\varepsilon) = 0 \tag{24}$$

In the static model, this condition was self-contained, and it implicitly determined the threshold, $\varepsilon_t^\star$ (see equation (6)). With overhead costs, $f_o > 0$, this is no longer the case.

There is an unbounded mass of potential firms each period, and the creation of a new firm is subject to an entry cost, $f_e > 0$, specified in units of output. Since prospective entrants are forward-looking, firm entry occurs until the expected present discounted value of post-entry profits, net of taxation, $\tau_t < 1$, is equal to the cost of entry. The free entry condition reads,

$$v_t = E_t\sum_{j=1}^{\infty} M_{t,t+j}\left(1-\tau_{t+j}\right)\pi\left(\varepsilon_{t+j}^\star\right) = f_e \tag{25}$$

where $M_{t,t+j} \equiv \left[\beta\left(1-\delta\right)\right]^j\left[u_C\left(t+j\right)/u_C\left(t\right)\right]$ is a stochastic discount factor.

5.3. Government

The government collects dividend and labor-income taxes and issues state-contingent real debt to finance an exogenous constant stream of government spending, $\mathcal{G} > 0$. The flow government budget constraint is,

$$\tau_t n_t x_{t-1}\pi\left(\varepsilon_t^\star\right) + \tau_t^L w_t L_t + \sum_s \frac{1}{r_t^s}B_{t+1}^s = B_t + \mathcal{G} \tag{26}$$

[35]Absent overhead costs, our model still generates default. However, matching default rates, in this case, requires higher firm-level volatility.

where $n_t x_{t-1} \pi \left(\varepsilon_t^{\star} \right) + \tau_t^L w_t L_t \tau_t$ is government income from taxation.

6. Quantitative Analysis

In this section, we undertake a quantitative analysis of the model developed in section 5. We first outline the calibration of the steady-state. We then consider a one-time shock to the volatility of idiosyncratic demand and aggregate technology. Finally, we study optimal fiscal policy, focusing on the increase in volatility observed during the 2007-09 recession.

6.1. Parameterization and Calibration

This section discusses the parameterization and calibration of the steady-state of the model.[36] Each period is a quarter and we set $\beta = 0.99$. This implies the annualized real interest rate is 4.1%. We adopt the following functional form for period utility,

$$u \left(C_t, L_t \right) = \ln C_t + \chi \frac{\left(1 - L_t \right)^{1-\upsilon} - 1}{1 - \upsilon} \tag{27}$$

The scale parameter, $\chi > 0$ in equation (27), is set such that that households allocate 20 percent of their time to work in the steady-state, and the Frisch elasticity of labor supply with respect to wages - here equal to $u_L / u_{LL} L > 0$ - is assumed to be $1/\upsilon = 0.72$, based on the empirical evidence in Heathcote *et al.* (2010). The elasticity of substitution between differentiated goods is set at $\theta = 3.8$.[37] This value is based on Bernard *et al.* (2003) and implies a markup of 35.7 percent.[38] For fiscal variables, we assume the government expenditure-to-output is $\mathcal{G}/Y = 0.2$, and dividend and labor-income taxes are assumed to

[36] For the purposes of calibration, we revert to the assumption that the government has a lump-sum transfer available to balance its budget.

[37] We assume Dixit-Stiglitz preferences throughout this section.

[38] Hall (2018) finds that the average US markup (weighted by value-added shares) increased from 1.12 in 1988 to 1.38 in 2015 in KLEMS data. A similar rise in markups is reported in De Loecker and Eeckhout (2017).

be $\tau^L = 0.25$ and $\tau = 20$, based on values reported in Arseneau and Chugh (2010) and Gourio and Miao (2010).

We calibrate the remaining parameters of the model to steady-state targets. Table 2 presents the parameters and their respective targets.

===== Table 1 Here =====

An important concern in our analysis is the volatility of idiosyncratic demand shocks. We calibrate volatility using sales data from Compustat. In Figure 1, we reported the time series of the interquartile range of sales growth. We map this interquartile range into the distribution of sales produced by our model. Recall that firm-level demand is log-normally distributed, and period t sales are given by, $\varepsilon^{\theta} n_t^{\alpha} \left[\int_0^{\infty} \varepsilon^{\theta} dG(\varepsilon)\right]^{(1-\theta)/\theta} l_t$. This implies the logarithm of period t sales are normally distributed, with mean $-\theta\sigma_t^2/2 + \ln\left[\int_0^{\infty} \varepsilon^{\theta} dG(\varepsilon)\right]^{(1-\theta)/\theta} n_t^{\alpha} l_t$, and standard deviation $\theta^2\sigma_t^2$. Since sales growth is the ratio of two log-normals, it is also log-normal, and at the steady-state has mean zero and standard deviation $\theta^2 2\sigma^2$. With θ chosen, we set σ to match the average interquartile range in the data.

The annual rate of firm exit, over the period 1993-2015, is 11.78 percent, based on BED data. Given the law of motion for firms, we use the parameter $\delta > 0$ to target this exit rate.[39] Giesecke *et al.* (2011) report that historical annual value-weighted mean default rate for US non-financial firms is 1.52 percent (Table 1, page 237). For the 1993-2015 period, the all-rated, issuer-weighted default rate, reported in the 2018 annual report of Moody's Investors Service (Ou *et al.* (2018), Exhibit 30), was also 1.52 percent. Given the interquartile range

[39] Since default occurs prior to exit shock, the parameter $\delta > 0$ captures the change in the total mass of products consumed in the economy.

of sales growth, we use overhead costs, f_o, to target this default rate.[40] Finally, we target an interest rate spread of 241 basis points using the agency cost parameter, ϕ.[41] The implied cost of default in our model 0.45 percent of steady-state GDP and the use of overhead labor is 4.3 percent of total employment.

6.2. Macroeconomic Implications of Aggregate Shocks

In this section, we analyze the dynamics of the model's endogenous variables in response to two aggregate shocks: a shock to volatility (σ) and a shock to aggregate technology (a).[42] We assume volatility and technology follow independent AR(1) processes,

$$\eta_{t+1} = \Lambda \eta_t + \varepsilon_{t+1} \quad ; \quad \omega_{t+1} \sim N(0, V) \tag{28}$$

where $\eta_t = [\ln(\sigma_t), \ln(a_t)]^T$ and $\omega = [\varepsilon^\sigma, \varepsilon^a]^T$ is the vector of shocks. We set the persistence of technology to 0.979 and the standard deviation of innovations to technology at 0.0072. Based on the estimates of Glichrist *et al.* (2014), we set the persistence of volatility to 0.9 and the standard deviation of innovations to volatility at 0.04.[43]

Figure 3 depicts the behavior of key endogenous variables in response to a positive shock to volatility of one standard deviation.

===== Figure 3 Here =====

[40]Dropping overhead costs, default falls to 0.34 percent, based on the other calibrated parameter values.

[41]Since firms make identical employment decisions, they face the same interest rate spreads, and we use the agency cost parameter in a similar way to Carlstrom and Fuerst (1997).

[42]We study model dynamics by log-linearizing the equilibrium conditions around the steady-state.

[43]Glichrist *et al.* (2014) use firm-level sales data from Compustat to estimate idiosyncratic technology shocks, which, in our model, are observationally equivalent to demand shocks. Using an estimated DGSE model, with data covering 1985-2010, Christiano *et al.* (2014) find the volatility of innovations to idiosyncratic productivity to be 0.07.

The immediate effect of an increase in volatility is a reduction in firm entry ($n_{e,t}$) of around 1 percent. The number of operating firms begins to fall one period (quarter) after the shock. With constant exogenous exit, net entry is negative ($n_{t+1} - n_t = n_{e,t} - \delta\left(n_t + n_{e,t}\right) < 0$), until quarter 12, when the number of operating firms begins to recover ($n_{t+1} > n_t$). Firm entry, however, is below its long-run level until quarter 18.[44] The change in the mass of operating firms is also consistent with a variety effect, which is reflected in the average price, $p_t = n_t^{\alpha}\left[\int_0^{\infty} \varepsilon^{\theta} dG\left(\varepsilon\right)\right]^{1/\theta}$.[45]

In the static model (section 3), the impetus for a reduction in firm entry came from a fall in profits. In the dynamic model, the drop in firm entry also reflects the change in expected future profits. Although firm-level profits fall sharply, upon impact, they rise quickly back to their long-run level, and then change sign. This feature of the model is consistent with the idea that it is possible for a firm to benefit from increased volatility by a good realization of demand. This positive effect of volatility on firm entry manifests itself in an expansion in firm-level production, and is consistent with an Oi-Hartman-Abel effect. As Bloom (2014) discusses, the Oi-Hartman-Abel effect implies that firms can expand to exploit good outcomes and contract to insure against bad outcomes, making them potentially risk loving. In our model, we see this as a rise in the conditional demand for goods, which is defined as, $\Delta\left(\varepsilon_t^{\star}\right) \equiv \left[\frac{1}{1-G\left(\varepsilon_t^{\star}\right)}\int_{\varepsilon_t^{\star}}^{\infty} \varepsilon^{\theta} dG\left(\varepsilon\right)\right]^{1/\theta}$.[46]

The negative effect of volatility on firm entry (and that which dominates) is the increase in the default rate, $G\left(\varepsilon_t^{\star}\right) = \int_0^{\varepsilon_t^{\star}} dG\left(\varepsilon\right)$. This occurs endogenously, via the threshold, $\varepsilon_t^{\star}$,

[44] Using a circumflex denotes the log-deviation of a variable its steady-state value, net entry is, $\widehat{n}_{t+1} - \widehat{n}_t = \delta\left(\widehat{n}_{e,t} - \widehat{n}_t\right)$, and so in quarter 12, both entry and the mass of firms are $\widehat{n}_{e,t}, \widehat{n}_t < 0$.

[45] Variety effects are also present in the analysis of Bilbiie *et al.* (2012) and Chugh and Ghironi (2015).

[46] Average sales, conditional on not defaulting are, $s\left(\varepsilon_t^{\star}\right) = \int_{\varepsilon_t^{\star}}^{\infty} \frac{1}{1-G\left(\varepsilon_t^{\star}\right)} s\left(\varepsilon_t\right) dG\left(\varepsilon\right)$, or, $s\left(\varepsilon_t^{\star}\right) = \left[\Delta\left(0\right)\right]^{1-\theta} n_t^{1/\theta - 1}\left[\Delta\left(\varepsilon_t^{\star}\right)\right]^{\theta} y_t$, where $\Delta\left(\varepsilon_t^{\star}\right)$ is a revenue shifter, which is exogenous from the perspective of the individual firm, and $\Delta\left(0\right) \equiv \left[\int_0^{\infty} \varepsilon^{\theta} dG\left(\varepsilon\right)\right]^{1/\theta}$ is average demand.

and for exogenous reasons, via the process for volatility, σ_t.[47] In the static model, the threshold level of demand, $\varepsilon_t^\star$, was proportional to conditional demand, whilst in this case, using equation (24), we have, $(\varepsilon_t^\star)^\theta l_t = \theta (l_t + f_o) [\Delta (\varepsilon^\star)]^\theta$. Despite this interaction, terms associated with the threshold - i.e., the default rate and the interest rate spread - return back to their long-run levels relatively quickly, whilst the nature of firm entry and exit is such that expansion of firm production is longer-lasting.

We started with an assertion that financial frictions are an important part of the transmission mechanism through which fluctuations in uncertainty are propagated to the real economy. In Figure 3, shocks to volatility cause recessions; GDP and consumption both fall, and the default rate and interest rate spread both rise.[48] A natural question is, to what extent do financial frictions matter for our results. Absent financial frictions, a change in volatility works entirely through average demand, $\left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$, which acts like aggregate technology, since it only enters the equilibrium conditions of the economy though aggregate production. However, changes in volatility have a small effect on demand, meaning that the fall in firm entry, for example, is a magnitude smaller than the when there are financial frictions.

Figure 4 depicts the behavior of key endogenous variables in response to negative technology shock of one standard deviation.

[47] We have, $G(\varepsilon_t^\star) = \int_0^{\varepsilon_t^\star} g(x)dx$, with the PDF defined in equation (12). In linear terms, this is the sum of three components. $G(\varepsilon^\star)\widehat{G}_t = [g(\varepsilon^\star)\varepsilon^\star]\widehat{\varepsilon}_t^\star + \left[\int_0^{\varepsilon_t^\star} \sigma \frac{dg}{d\sigma}(x)dx\right]\widehat{\sigma}_t + \left[\int_0^{\varepsilon_t^\star} m \frac{dg}{dm}(x)dx\right]\widehat{m}_t$, where the final two terms are exogenous. In Figure 3, we plot the path of $G(\varepsilon^\star)\widehat{G}_t$ and $\widehat{\varepsilon}_t^\star$. Since $g(\varepsilon^\star)\varepsilon^\star$ is a small number, any change in $\widehat{\varepsilon}_t^\star$ will only cause a small change in $G(\varepsilon^\star)\widehat{G}_t$. Since, $\sigma_t^2/2 = -m_t$, a rise in volatility will generate a fall in m_t, depressing any change the default rate.

[48] Wages also fall in recessions. The response of wages and the specification of entry costs is an important consideration for our analysis. We experimented with entry costs specified in units of labor. Despite higher default, this can encourage firm entry, because entry costs also fall.

===== Figure 4 Here =====

A negative technology shock leads to a large and persistent fall in firm entry and GDP. The interest rate spread rises, but only in the initial period, and the default rate, which operates via the threshold, $\varepsilon_t^{\star}$, rises slightly, upon impact. As in Bilbiie *et al.* (2012), our model can be interpreted as a real business cycle (RBC) model, where the number of operating firms acts as the capital stock of the economy. In this sense, the results in Figure 4 can be compared to the analysis of Carlstrom and Fuerst (1997): whereas Carlstrom and Fuerst (1997) seeks to understand the role of agency costs using an RBC model, we use a model of firm entry. In the RBC setting, agency costs generate persistence in output, but defaults are procyclical; with endogenous firm entry, defaults are countercyclical, but there is relatively less additional persistence.

6.3. Tax Polices and the 2007-09 Recession

Using the dynamic model developed above, we now revisit the results on dividend and labor-income tax policies. To put our analysis into context, we analyze the response of optimal taxes, accounting for the increase in volatility observed during the 2007-09 recession. In particular, we map the rise in volatility during this period into our model by specifying a series of unanticipated shocks to volatility that reproduces the path of the interest rate spread used in our empirical analysis (section 2). Absent changes in fiscal policy, this rise in volatility causes a maximum drop in GDP of around a 0.5 percent, considerably less than during 2007-09.[49] We therefore also specify a series of unanticipated shocks to technology (over the same time horizon) that generate a drop in GDP, consistent with the 2007-09

[49] Our model does not contain many of the ingredients needed to provide such a drop in output. When Basu and Bundick (2017) feed a macro-uncertainty shock (of a size consistent with the 2007-09 recession) into their baseline model, the drop in output is around 0.6 percent, which is comparable with our results.

period.[50]

Our analysis of fiscal policy also differs from section 4 because we study optimal policy without the availability of a lump-sum transfer. In this case, we solve for optimal dividend and labor-income taxes using the following reduced policy problem.

Definition 1 Plans $\Xi_t \equiv \{C_t, L_t, n_{e,t}, n_{t+1}, l_t\}_{t=0}^{\infty}$ and $\{\varepsilon_t^\star\}_{t=0}^{\infty}$ represent the optimal allocation if they solve the following problem.

$$\begin{aligned}
&\max_{\{\Xi_t,\varepsilon_t^\star\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t, L_t, \xi) \\
&+\beta^t \lambda_{1,t} \left\{ n_t^\alpha \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon) \right]^{1/\theta} a_t l_t n_t - C_t - \mathcal{G} - n_{e,t} f_e - \phi \left[n_t G(\varepsilon_t^\star) \right] \right\} \\
&+\beta^t \lambda_{2,t} \left[L_t - n_t \left(l_t + \frac{f_o}{a_t} \right) \right] + \beta^t \lambda_{3,t} \left[(1-\delta)(n_t + n_{e,t}) - n_{t+1} \right] \\
&+\beta^t \lambda_{4,t} \left\{ l_t \left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)} \right]^\theta - \theta \left(l_t + \frac{f_o}{a_t} \right) \right\} - \xi A
\end{aligned} \tag{29}$$

where,

$$\mathcal{U}(C_t, L_t, \xi) \equiv u(C_t, L_t) + \xi \left[u_C(t) C_t + u_L(t) L_t \right] \tag{30}$$

and given,

$$A \equiv u_C(0) \left[r_{-1}^d d_{-1} + b_0 + n_0 z(\varepsilon_0^\star) \right] \tag{31}$$

where $\{\lambda_{j,t}\}_{j=1}^4$ are lagrange multipliers associated with constraints, ξ is a (constant) lagrange multiplier associated with the implementability constraint.[51]

[50] Throughout this exercise, it is assumed that the process for volatility (technology) has persistence equal to 0.9 (0.8).

[51] As in standard Ramsey taxation problems, the government is assumed to commit, as of period zero, to time invariant policy functions for $t \geq 1$. Following Chugh and Ghironi (2015), we also assume that the schedule of state-contingent profit taxes is posted one period in advance.

Finally, we consider the implied tax change that resulted from the American Recovery and Reinvestment Act of 2009 (ARRA), which was enacted as a direct result of the 2007-09 recession.[52] We model this change in taxation as a 1 percent point unanticipated drop in the labor-income tax during 2009:Q2, a period in which volatility (in our model) is 12.8 percent above its steady-state value.[53]

Figure 5 depicts the behavior of the model's endogenous variables, beginning in 2007:Q3, for three cases; optimal policy, an unanticipated cut in the labor-income tax in 2009:Q2, and fixed dividend and labor-income taxes.

===== Figure 5 Here =====

By construction, the path of the interest rate spread and GDP (with fixed taxes) in Figure 5 corresponds to the data, for the first 12 quarters.[54] There are two important points to note from this exercise. First, both firm entry and the mass of firms fall strongly in 2009, and have a close match with the data, in terms of their size. As Siemer (2014) reports, the 2007-09 recession exhibited a 5 percent decline in the number of firms, which was driven by a 25 percent decline in the number of entrants. Second, the default rate also rises (at its maximum) by 1.5 percentage points, although this is somewhat below that which occurred during the 2007-09 period. Both of these variables matter for the optimal tax decision because they determine agency costs.

[52] In early 2009, the US Congress passed a 787 billion USD package to stimulate the economy, of which tax incentives to individuals (companies) comprised around one third (one sixteenth). Since the largest component of tax incentives to individuals, close to 116 billion USD, was in the form of payroll tax credits, we model policy under the ARRA as a cut in the labor-income tax rate.

[53] Zubairy (2014) also models ARRA payroll tax credits as a 1 percent point cut in labor-income taxes. Mertens and Ravn (2011) discuss the role of anticipated and unanticipated tax policy shocks.

[54] We choose 12 quarters to capture the rise and fall of volatility over this period. The results in Figure 5 are robust to changes in the length of this mapping.

Figure 5 shows the extent to which financial frictions weaken the support for firm entry. As with the static model, during a recession, the default rate rises, and this acts to raise agency costs. Whilst firms disregard such costs, the policymaker accounts for the societal costs of default by discouraging entry, raising the tax applied to dividend-income. With volatility at levels consistent with the 2007-09 period, this translates into a rise in the tax rate of up to 7 percentage points.[55] Whilst raising taxes on dividend-income discourages firm entry, the drop in the tax applied to labor-income (of up to 1.5 percentage points) supports employment. In doing so, it provides indirect support to firm entry - this also occurred in the static model (see the discussion prior to equations (13)). Despite lower taxes on labor-income, overall, under optimal policy, the drop in firm entry is around 10 percentage points higher than if taxes remain unchanged.

Now consider an exogenous (and unanticipated) drop in the labor-income tax in 2009:Q2, which we interpret as a ARRA-type tax cut. Although this experiment is designed to understand policies taken in response the 2007-09 recession, it also helps to explain how a cut in labor-income taxes supports firm entry.[56] In Figure 5, a 1 percent drop in the labor-income tax generates a relatively large effect on employment, and this results in a smaller fall in firm entry, upon implementation. Over time, this feature leads to a sizable difference in the mass of firms operating in the economy. Since the default rates are similar across tax policies, this means agency costs are relatively higher than when only labor-income taxes are cut.

[55]The default rate (and interest rate spread) are relatively less sensitive to changes in dividend-income taxation than they are to changes in volatility, which explains why, in Figure 5, the lines depicting default rates, for fixed taxes and optimal policy, are close to one another.

[56]For the purposes of this experiment, we assumed a lump-sum transfer was available. This explains the relatively larger fall in consumption, after period 9, with optimal taxes, since this policy accounts for the implementability constraint. We set the persistence of the labor-income tax at 0.87, to be comparable with Chugh and Ghironi (2015).

7. Conclusion

This paper studies financial market frictions and firm entry over the business cycle. We document empirically that a reduction in firm entry and a widening of the interest rate spread occur when there is a rise in idiosyncratic uncertainty. We then develop a model of endogenous firm entry and financial frictions, which features shocks to the volatility of firm-level demand, and is consistent with this empirical evidence. Analyzing optimal dividend and labor-income taxation, we find that financial frictions weaken the incentive to support firm entry. In a calibrated version of our model, accounting for the increase in volatility observed during the 2007-09 recession, optimal policy acts to raise (lower) dividend (labor)-income taxes by around 7 (1.5) percentage points.

Appendix A: Empirical Analysis

A1. Proxies for Idiosyncratic Uncertainty

We use three proxies for idiosyncratic uncertainty at the aggregate level.

1. *BOS*: We use the series "Future Activity Index" from the Federal Reserve Bank of Philadelphia's Business Outlook Survey and apply, $\sqrt{F^{i}_{i,t} + F^{+}_{i,t} - \left(F^{+}_{i,t} - F^{-}_{i,t}\right)^2}$ for each month, similar to Bachmann *et al.* (2013), where $F^{+}_{i,t}$ ($F^{-}_{i,t}$) is defined as the fraction of firms in the cross section with increase (decrease) responses at month t. We then make the series quarterly by averaging it across the three months within each quarter.

2. *sIQR*: We use the series SALEQ from Compustat. We keep firms with at least 100 quarters of observations, starting from 1970:Q1. We drop firms with negative sales and use the observations since 1993:Q2. This gives an unbalanced panel of 2578 firms. We calculate the growth rate of sales as $(s_{i,t} - s_{i,t-4}) / \frac{1}{2}(s_{i,t} + s_{i,t-4})$, following Davis and Haltiwanger (1992), where $s_{i,t}$ are the sales of firm i, in quarter t, deflated by the consumer price index.

3. *iVOL*: Caldara *et al.* (2016) construct a monthly series based in Gilchrist *et al.* (2014). We make this series quarterly by averaging it across the three months within each quarter.

A2. Uncertainty in the 2007-09 Recession

We regress each of the uncertainty proxies on a dummy variable for the 2007-09 recession (NBER recession dates). Table A reports time-series OLS regression point estimates (with standard errors below in parentheses).

===== Table A Here =====

The rise in uncertainty over the 2007-09 period was calculated by dividing the estimated value of the coefficient on the dummy variable by the mean. For example, $0.035/0.187 = 18.7$ percent.

A3. Other Aggregate Variables

Data cover the period 1993:Q2 to 2015:Q1. We use establishment births ($n_{e,t}$) and establishment deaths ($n_{x,t}$) from the BLS's Business Employment Dynamics (BED) program; https://www.bls.gov/web/cewbd/table9_1.txt. We use the Census Bureau's Business Dynamics Statistics (BDS) so that the number of establishments in period $t = 0$ (i.e., n_0) is $5,630,195$. The total number of establishment is, $n_t = (n_{e,t} - n_{x,t}) + n_{t-1}$. The quarterly exit rate is ($n_{x,t}/n_t$) and the entry rate is ($n_{e,t}/n_t$). In the VAR, we use an index ($2005 = 100$) of net entry index and per-capita GDP. We measure the interest rate spread by the difference between the interest rate on BAA-rated corporate bonds and the 10 year US government bond rate. These data are downloaded from the Federal Reserve Bank of St. Louis (FRED); mnemonics GDPC1, CNP16OV, and BAA10Y. For robustness, we also used the GZ spread downloaded from http://people.bu.edu/sgilchri/Data/data.htm.

Appendix B: Analytical Model

Appendix B.1 (Firm i profit maximization and average prices)

The profit of firm i with demand level ε is given by, $\pi_t(i,\varepsilon) = p_t(i,\varepsilon)\, y_t(i) - w_t r_t l_t(i)$. Firm i maximizes conditional expected profit, $\int_{\varepsilon_t^\star}^{\infty} \pi_t(i,\varepsilon)\, dG(\varepsilon)$, choosing an employment level, $l_t(i)$, subject to technology, $y_t(i) = l_t(i)$, demand, $c_t(i) = \left[n_t^\omega \frac{p_t(i,\varepsilon)}{\varepsilon_t^\theta}\right]^{-1/(1-\theta)} Y_t$, and market clearing, $c_t(i) = y_t(i)$, where $\varepsilon_t^\star = \inf\{\varepsilon_t : \pi(i,\varepsilon_t) > 0\}$. The maximization problem of firm i is,

$$\max_{l_t(i),\varepsilon_t^\star} \int_{\varepsilon_t^\star}^{\infty} \left\{ \frac{\varepsilon^\theta}{n_t^\omega} \left[\frac{l_t(i)}{Y_t}\right]^{\theta-1} l_t(i) - w_t r_t l_t(i) \right\} dG(\varepsilon) - \lambda_t \left\{ \frac{(\varepsilon_t^\star)^\theta}{n_t^\omega} \left[\frac{l_t(i)}{Y_t}\right]^{\theta-1} l_t(i) - w_t r_t l_t(i) \right\}$$

with the wage and interest rate, the mass of firms, and total output, given. The first-order conditions imply,

$$\int_{\varepsilon_t^\star}^{\infty} \theta \frac{\varepsilon^\theta}{n_t^\omega} \left[\frac{l_t(i)}{Y_t}\right]^{\theta-1} dG(\varepsilon) - \left[1 - G(\varepsilon_t^\star)\right] w_t r_t = 0 \tag{32}$$

Using the zero-profit condition with the first order condition, we have, $\theta \int_{\varepsilon_t^\star}^{\infty} \varepsilon^\theta dG(\varepsilon) = \left[1 - G(\varepsilon_t^\star)\right] (\varepsilon_t^\star)^\theta$. In equilibrium, all firms make the same decision, so $l_t(i) = l_t$, and, $\frac{(\varepsilon_t^\star)^\theta}{n_t^\omega} \left(\frac{l_t}{Y_t}\right)^{\theta-1} = w_t r_t$. Using total production, $Y_t = n_t^{(1-\omega)/\theta} \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta} l_t$, in the previous equation, we have, $\left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{(1-\theta)/\theta} n_t^{[(1-\theta)-\omega]/\theta} (\varepsilon_t^\star)^\theta = w_t r_t$. In the text we define, $\alpha = \left[(1-\theta) - \omega\right]/\theta$.

Define $\Delta(\varepsilon_t^\star) \equiv \left[\frac{1}{1-G(\varepsilon_t^\star)} \int_{\varepsilon_t^\star}^{\infty} \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$ and $\Delta(0) \equiv \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$ as conditional demand and average demand. The price of a good with demand ε equals, $p_t(\varepsilon) = \frac{\varepsilon^\theta}{n_t^\omega} \left(\frac{l_t}{Y_t}\right)^{\theta-1} = \varepsilon^\theta \left[\Delta(0)\right]^{1-\theta} n_t^\alpha$. The average price of all goods is defined as, $p_t \equiv \int_0^\infty p_t(\varepsilon)\, dG(\varepsilon)$. Applying this definition to demand, we have, $p_t = n_t^\alpha \Delta(0)$. We use the firm optimization conditions to link prices with marginal costs. Specifically, $n_t^\alpha \Delta(0) = \left[\Delta(0)\right]^\theta w_t r_t / (\varepsilon_t^\star)^\theta$ and $\left[\varepsilon_t^\star / \Delta(\varepsilon_t^\star)\right]^\theta = \theta$ imply,

$$p_t = \left[\frac{\Delta(0)}{\Delta(\varepsilon_t^\star)}\right]^\theta \frac{w_t r_t}{\theta} \tag{33}$$

In the main text, we define $\kappa\left(\varepsilon_t^\star\right) \equiv \left[\frac{\Delta(0)}{\Delta(\varepsilon_t^\star)}\right]^\theta$, which is expressed in equation (12). Formally, we can show, $\frac{\partial[\Delta(\varepsilon_t^\star)]^\theta}{\partial\varepsilon_t^\star} = \frac{dG(\varepsilon_t^\star)}{[1-G(\varepsilon_t^\star)]^2}\int_{\varepsilon_t^\star}^{\infty}\left[\varepsilon^\theta - \left(\varepsilon_t^\star\right)^\theta\right] dG\left(\varepsilon\right) > 0$ where $\varepsilon > \varepsilon_t^\star$ and $\Delta\left(\varepsilon_t^\star\right) > \Delta\left(0\right)$. This implies that the markup is less than one, $\kappa\left(\varepsilon_t^\star\right) < 1$, and is falling in the default rate, $\kappa'\left(\varepsilon_t^\star\right) < 0$, where the default-threshold level of demand is implicitly determined by $\left[\varepsilon_t^\star/\Delta\left(\varepsilon_t^\star\right)\right]^\theta = \theta$.

Finally, define, $p\left(\varepsilon_t^\star\right) \equiv \int_{\varepsilon_t^\star}^{\infty}\frac{1}{1-G(\varepsilon_t^\star)}p_t\left(\varepsilon\right)dG\left(\varepsilon\right)$, which is the conditional price per-firm. Applying this definition to the price equation, $p_t\left(\varepsilon\right) = \varepsilon_t^\theta\left[\Delta\left(0\right)\right]^{1-\theta} n_t^\alpha$, generates, $p\left(\varepsilon_t^\star\right) = \frac{w_t r_t}{\theta}$. Thus, $p_t < p\left(\varepsilon_t^\star\right)$ and the conditional mark up is a constant. Now consider the firm problem with unlimited liability. In this case, the average price is also equal to $\frac{w_t r_t}{\theta}$, so that we confirm limited liability acts to depress the expected price.

Appendix B.2 (Proposition 1)

We start by proving there is a unique threshold level of demand, $\varepsilon_t^\star > 0$. Our analysis is based on the condition, $\theta\Delta\left(\varepsilon_t^\star\right)^\theta = \left(\varepsilon_t^\star\right)^\theta$. Drop time-subscripts and define $H\left(\varepsilon^\star\right) \equiv \frac{1}{1-G(\varepsilon^\star)}\int_{\varepsilon^\star}^{\infty}\left(\frac{\varepsilon}{\varepsilon^\star}\right)^\theta dG\left(\varepsilon\right)$. There is a unique solution to $H\left(\varepsilon^\star\right) = \frac{1}{\theta}$, if $\frac{-g'(\varepsilon^\star)\varepsilon^\star}{g(\varepsilon^\star)}$ is an increasing function, and if $\lim\limits_{\varepsilon^\star\to+\infty}\frac{-g'(\varepsilon^\star)\varepsilon^\star}{g(\varepsilon^\star)} = +\infty$, where $g\left(\varepsilon^\star\right)$ is the PDF. To show this, note $1 - G\left(\varepsilon^\star\right) = \int_{\varepsilon_t^\star}^{\infty} dG\left(\varepsilon\right)$, and make the change of variables, $u = \varepsilon/\varepsilon^\star$, such that, $H\left(\varepsilon^\star\right) \equiv \frac{\int_1^\infty u^\theta g(\varepsilon^\star u)du}{\int_1^\infty g(\varepsilon^\star u)du}$. We then have,

$$H'\left(\varepsilon^\star\right) = \frac{\int_1^\infty u^{\theta+1}g'\left(\varepsilon^\star u\right)du \times \int_1^\infty g\left(\varepsilon^\star u\right)du - \int_1^\infty u^\theta g\left(\varepsilon^\star u\right)du \times \int_1^\infty u g'\left(\varepsilon^\star u\right)du}{\left[\int_1^\infty g\left(\varepsilon^\star u\right)du\right]^2} \tag{34}$$

Now define a new CDF as $G_1(x) = \frac{\int_1^x g(\varepsilon^\star u)du}{\int_1^\infty g(\varepsilon^\star u)du}$ and the elasticity of the PDF as $\eta\left(\varepsilon^\star u\right) = -\frac{g'(\varepsilon^\star u)\varepsilon^\star u}{g(\varepsilon^\star u)}$. We use this to re-write $H'\left(\varepsilon^\star\right)$ as,

$$H'\left(\varepsilon^\star\right) = E_{G_1}(u^\theta)E_{G_1}(\eta) - E_{G_1}(u^\theta \times \eta) = -COV_{G_1}(u^\theta, \eta(u))$$

where COV_{G_1} defines as covariance with respect to measure G_1. As the covariance of two increasing functions is positive, we have proved that $H'\left(\varepsilon^\star\right) < 0$.

We can now verify that $\lim\limits_{\varepsilon^\star \to 0} H(\varepsilon^\star) \equiv \frac{\int_{\varepsilon^\star}^{\infty} \varepsilon^\theta dG(\varepsilon)/[1-G(\varepsilon^\star)]}{(\varepsilon^\star)^\theta} = +\infty > 1/\theta$, since the nominator $\frac{\int_{\varepsilon^\star}^{\infty} (\varepsilon)^\theta dG(\varepsilon)}{1-G(\varepsilon^\star)}$ is a regular function, and the denominator converges to zero.

Finally, we need to compute $\lim\limits_{\varepsilon^\star \to +\infty} H(\varepsilon^\star)$. To do so, we use L'Hospital's Rule, as both nominator and denominator converge to 0. We find,

$$\begin{aligned}
\lim_{\varepsilon^\star \to +\infty} H(\varepsilon^\star) &= \lim_{\varepsilon^\star \to +\infty} \frac{\frac{\partial}{\partial \varepsilon^\star} \int_{\varepsilon^\star}^{\infty} \left(\frac{\varepsilon}{\varepsilon^\star}\right)^\theta dG(\varepsilon)}{\frac{\partial}{\partial \varepsilon^\star}(1 - G(\varepsilon^\star)} = \lim_{\varepsilon^\star \to +\infty} \frac{-g(\varepsilon^\star) - \frac{\theta}{\varepsilon^\star} \int_{\varepsilon^\star}^{\infty} \left(\frac{\varepsilon}{\varepsilon^\star}\right)^\theta dG(\varepsilon)}{-g(\varepsilon^\star)} \\
&= \lim_{\varepsilon^\star \to +\infty} \left[1 + \theta \frac{(1 - G(\varepsilon^\star))}{g(\varepsilon^\star)\,\varepsilon^\star} H(\varepsilon^\star)\right]
\end{aligned}$$

Therefore, the sufficient condition for $\lim\limits_{\varepsilon^\star \to +\infty} H(\varepsilon^\star) = 1 < 1/\theta$ is that $\lim\limits_{\varepsilon^\star \to +\infty} \frac{1-G(\varepsilon^\star)}{g(\varepsilon^\star)\varepsilon^\star} = 0$, which is an infinitely increasing log hazard ratio, $\lim\limits_{\varepsilon^\star \to +\infty} \frac{g(\varepsilon^\star)\varepsilon^\star}{1-G(\varepsilon^\star)} = +\infty$.

We now describe the properties of $\varepsilon^\star$, $G(\varepsilon^\star)$, and $\mu(\varepsilon_t^\star) = [\Delta(0)/\Delta(\varepsilon_t^\star)]^\theta$. In the text, we adopt a log normal distribution, with PDF defined in equation (11). Define a transformed threshold, $x^\star = \frac{\ln(\varepsilon^\star) - m}{\sigma}$. Our main variables of interest are,

$$\int_{\varepsilon^\star}^{\infty} \varepsilon^\theta dG(\varepsilon) = \exp\left(\frac{\theta^2\sigma^2}{2}\right)\Phi(\theta\sigma - x^\star);\;\; G(\varepsilon^\star) = \Phi(x^\star);\;\; \left[\frac{\Delta(0)}{\Delta(\varepsilon_t^\star)}\right]^\theta = \frac{\Phi(-x^\star)}{\Phi(\theta\sigma - x^\star)}$$

where $\Phi(x^\star)$ is the CDF of the normal distribution,

$$\Phi(x^\star) = \frac{1}{\sqrt{2\pi}} \int_0^{x^\star} \exp\left(-\frac{x^2}{2}\right) dx \quad \text{and} \quad \Phi'(x^\star) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

Using $1 - G(\varepsilon^\star) = \Phi(-x^\star)$, we re-write the threshold equation as, $\theta \int_{\varepsilon^\star}^{\infty} \varepsilon^\theta dG(\varepsilon) = \Phi(-x^\star)(\varepsilon^\star)^\theta$, and then,

$$\exp(\theta\sigma x)\,\Phi(-x^\star) = \theta \exp\left(\frac{\theta^2\sigma^2}{2}\right)\Phi(\theta\sigma - x^\star) \tag{35}$$

where $1 - G(\varepsilon^\star) = \Phi(-x^\star)$. Now consider, $\Phi'(x^\star) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x^\star)^2}{2}\right]$, and note that, $\Phi'(\theta\sigma - x^\star) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(\theta\sigma - x^\star)^2}{2}\right]$, which implies,

$$\Phi'(x^\star) = \exp\left(\frac{\sigma^2\theta^2}{2} - \theta\sigma x^\star\right)\Phi'(\theta\sigma - x^\star) \tag{36}$$

Equations (35) and (36) generate,

$$\frac{\Phi(\theta\sigma - x^\star)}{\Phi'(\theta\sigma - x^\star)} = \frac{1}{\theta}\frac{\Phi(-x^\star)}{\Phi'(x^\star)} \quad ; \quad \theta < 1 \tag{37}$$

We use equation (37) in what follows. To evaluate $\frac{dx^\star}{d\sigma}$ we use threshold equation, as in equation (35), and apply the implicit function theorem. Multiplying through by $\exp(-\theta\sigma x^\star)$, using equations (35) and (36) to eliminating $\Phi(-x^\star)$ and $\Phi'(x^\star)$, respectively, implies,

$$\frac{dx^\star}{d\sigma} = \theta\frac{(\sigma\theta - x^\star)\,\Phi(-x^\star) + \theta\Phi'(x^\star)}{\theta\sigma\Phi(-x^\star) - (1-\theta)\,\Phi'(x^\star)} \tag{38}$$

The numerator is positive and the denominator is positive, if $\sigma\theta > (1-\theta)\,\frac{\Phi'(x^\star)}{\Phi(-x^\star)}$. For this condition to hold, it is sufficient that $x^\star < 0$, which is equivalent to $\Phi(x^\star) = G(\varepsilon^\star) < 1/2$, as claimed in the text. This also implies the probability of default increases with volatility.

Now consider how the markup depends on σ. Using equations (35), we have,

$$\begin{aligned}\frac{d}{d\sigma}\left[\frac{\Delta(\varepsilon_t^\star)}{\Delta(0)}\right]^\theta &= \frac{d}{d\sigma}\frac{\Phi(\theta\sigma - x^\star)}{\Phi(-x^\star)} = \frac{d}{d\sigma}\frac{1}{\theta}\exp\left(\theta\sigma x^\star - \frac{\theta^2\sigma^2}{2}\right) \\ &= \sigma\exp\left(\theta\sigma x^\star - \frac{\theta^2\sigma^2}{2}\right)\left[\frac{dx^\star}{d\sigma} + x^\star - \theta\right]\end{aligned}$$

The following calculation shows that $\frac{dx^\star}{d\sigma} + x^\star - \theta > 0$, when $\sigma < 1$,

$$\frac{dx^\star}{d\sigma} + x^\star - \theta = \frac{-x^\star\theta\Phi(-x^\star)\,(1-\sigma) + (-x^\star\,(1-\theta) + \theta)\,\Phi'(x^\star)}{\theta\sigma\Phi(-x^\star) - (1-\theta)\,\Phi'(x^\star)}$$

This proves $\frac{d\kappa(\varepsilon_t^\star)}{d\sigma} < 0$.

Appendix B.3 (Solution with Given Taxes)

At this point, we define $D\left(\varepsilon_t^\star\right) \equiv \left[1 - G\left(\varepsilon_t^\star\right)\right]/\kappa\left(\varepsilon_t^\star\right)$. Now recall, $\pi\left(\varepsilon_t^\star\right) \equiv \int_{\varepsilon_t^\star}^\infty \pi_t\left(\varepsilon\right)dG(\varepsilon) = \int_{\varepsilon_t^\star}^\infty \left[p_t\left(i\right)y_t - w_t r_t l_t\right]dG(\varepsilon) = \left[1 - G\left(\varepsilon_t^\star\right)\right]\left[p\left(\varepsilon_t^\star\right)l_t - w_t r_t l_t\right]$. Using the result above, that $p\left(\varepsilon_t^\star\right) = \frac{w_t r_t}{\theta}$, then $\pi\left(\varepsilon_t^\star\right) = \frac{1-\theta}{\theta}\left[1 - G\left(\varepsilon_t^\star\right)\right]w_t r_t l_t$. Total production is, $\frac{Y_t}{n_t} = n_t^\alpha\Delta(0)l_t$, where, $\Delta(0) \equiv \left[\int_0^\infty \varepsilon^\theta dG(\varepsilon)\right]^{1/\theta}$, and the firm optimization condition implies, $w_t r_t = n_t^\alpha\left[\Delta(0)\right]^{1-\theta}\left(\varepsilon_t^\star\right)^\theta$.

So, $w_t r_t l_t = \frac{Y_t}{n_t}\left[\frac{\varepsilon_t^\star}{\Delta(0)}\right]^\theta = \theta\frac{Y_t}{n_t}\frac{D(\varepsilon_t^\star)}{1-G(\varepsilon_t^\star)}$, and $\pi\left(\varepsilon_t^\star\right) = (1-\theta)\frac{Y_t}{n_t}D\left(\varepsilon_t^\star\right) = f_e/\left(1-\tau_t\right)$, by free entry. Using the resource constraint,

$$Y_t = C_t + f_e n_t \left[1 + fG\left(\varepsilon_t^\star\right)\right] \Leftrightarrow C_t = Y_t\left\{1 - D\left(\varepsilon_t^\star\right)(1-\theta)\left(1-\tau_t\right)\left[1+fG\left(\varepsilon_t^\star\right)\right]\right\} \tag{39}$$

Equating labor supply and demand, labor market equilibrium implies,

$$\begin{aligned} -u_L(t)\,L_t &= Y_t\left[\frac{D\left(\varepsilon_t^\star\right)}{1-G\left(\varepsilon_t^\star\right)}\frac{\left(1-\tau_t^L\right)\theta}{r_t}\right]u_C(t) \\ &= \left(\frac{Y_t}{C_t}\right)\frac{1}{r_t}\left[\frac{D\left(\varepsilon_t^\star\right)\left(1-\tau_t^L\right)\theta}{1-G\left(\varepsilon_t^\star\right)}\right]C_t u_C(t) \end{aligned}$$

Finally, we determine interest rates using equation (8). Eliminate wages using the firms optimal condition, apply $L_t = Y_t/n_t^\alpha\Delta(0)$, and eliminate the mass of firm with free entry,

$$\frac{1}{r_t} = \int_{\varepsilon_t^\star}^\infty dG(\varepsilon) + \int_0^{\varepsilon_t^\star}\left(\frac{\varepsilon_t}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon) - \frac{\phi}{f_e}G\left(\varepsilon_t^\star\right)\frac{(1-\theta)\left(1-\tau_t\right)\left[1-G\left(\varepsilon_t^\star\right)\right]}{\theta}$$

Note that, by the threshold condition, $\left(\varepsilon_t^\star\right)^{-\theta} = \frac{1-G(\varepsilon_t^\star)}{\theta\int_{\varepsilon_t^\star}^\infty \varepsilon_t^\theta dG(\varepsilon)}$, and so, $\int_0^{\varepsilon_t^\star}\left(\frac{\varepsilon_t}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon) = \frac{1-G(\varepsilon_t^\star)}{\theta}\left[\frac{1}{D(\varepsilon_t^\star)}-1\right]$. This leads to,

$$\frac{1}{r_t}\frac{1}{1-G\left(\varepsilon_t^\star\right)} = 1 + \frac{1}{\theta}\left[\frac{1}{D\left(\varepsilon_t^\star\right)}-1\right] - fG\left(\varepsilon_t^\star\right)\frac{(1-\theta)\left(1-\tau_t\right)}{\theta} \quad ; \quad f = \frac{\phi}{f_e} \tag{40}$$

which is reported in the main text as equation (14) for the case in which $\tau_t = 0$. Notice that the term $\frac{1}{r_t}\frac{1}{1-G(\varepsilon_t^\star)}$ appears in the labor market condition. Eliminating this term and eliminating Y_t using the resource constraint,

$$-\frac{u_L(t)\,L_t}{C_t u_C(t)} = \left(1-\tau_t^L\right)\frac{1 - D\left(\varepsilon_t^\star\right)(1-\theta)\left[1+\left(1-\tau_t\right)fG\left(\varepsilon_t^\star\right)\right]}{1 - D\left(\varepsilon_t^\star\right)(1-\theta)\left(1-\tau_t\right)\left[1+fG\left(\varepsilon_t^\star\right)\right]}$$

If we further assume $C_t u_C(t) = 1$ then total employment is constant. Recall, $u_L(t) < 0$, so the left-hand side of this expression is positive. Therefore, employment, $L_t = L\left(\tau_t^L, \tau_t, \varepsilon_t^\star\right)$, is decreasing in τ_t^L and τ_t, and for $\tau_t = 0$, is independent of $\varepsilon_t^\star$. In this case, the solution to our model is summarized in the following way. The threshold equation, determines $\varepsilon_t^\star$, and equation (40) determines the interest rate spread, r_t. Since total labor supply is fixed, the mass of firms and output are determined by free entry and the equation $Y_t = \Delta(0) n_t^{\alpha} L$, where $\alpha \equiv [(1-\theta) - \omega]/\theta$. In this case, firm-level employment is also determined. Finally, equation (39) determines consumption, given output.

Appendix B.4 (Proposition 2)

The policy problem is find the set of taxes $\left\{\tau_t, \tau_t^L\right\}$ that maximize utility, subject to the equilibrium conditions of the model, as described in Appendix B.3. An immediate implication of the threshold condition is that $\varepsilon_t^{\star} > 0$ is not a choice variable for government and, by equation (40), neither is r_t. Moreover, since we can pick τ_t^L (τ_t), labor supply (free entry) constraints do not bind. Finally, the choice over wages means the labor demand constraint does not bind. The policy problem reduces to,

$$\max_{C_t, L_t, n_t} u\left(C_t, L_t\right) + \lambda_t \left\{\Delta\left(0\right) n_t^{\alpha} L_t - n_t \left[f_e + \phi G\left(\varepsilon_t^{\star}\right)\right] - C_t\right\} \tag{41}$$

with first-order conditions,

$$u_C\left(t\right) = \lambda_t \quad ; \quad -u_L\left(t\right) = \lambda_t \Delta\left(0\right) n_t^{\alpha} \quad ; \quad \left[f_e + \phi G\left(\varepsilon_t^{\star}\right)\right] n_t = \alpha \Delta\left(0\right) n_t^{\alpha} L_t$$

We determine the optimal tax on dividend-income using free entry, which implies, $1 - \tau_t = f_e / \pi\left(\varepsilon_t^{\star}\right)$. Using the first-order condition for the mass of varieties,

$$\left[f_e + \phi G\left(\varepsilon_t^{\star}\right)\right] n_t = \alpha Y_t \Leftrightarrow 1 - \tau_t = \frac{\alpha f_e}{\left(1-\theta\right)\left[f_e + \phi G\left(\varepsilon_t^{\star}\right)\right] D\left(\varepsilon_t^{\star}\right)} \tag{42}$$

which is reported as equation (15) in Proposition 2, replacing $D\left(\varepsilon_t^{\star}\right) = \left[1 - G\left(\varepsilon_t^{\star}\right)\right] / \kappa\left(\varepsilon_t^{\star}\right)$. To determine the tax on labor-income, we use the labor-leisure equation. Using the optimal condition for labor, this implies,

$$-u_L\left(t\right) = \lambda_t \Delta\left(0\right) n_t^{\alpha} \Leftrightarrow 1 - \tau_t^L = \frac{1 - G\left(\varepsilon_t^{\star}\right)}{D\left(\varepsilon_t^{\star}\right)} \frac{r_t}{\theta} \tag{43}$$

which is reported in the main text as equation (16), again replacing $D\left(\varepsilon_t^{\star}\right)$. This condition also contains the interest rate, which is only a (increasing) function of $\varepsilon_t^{\star}$. To see why, start with equation (8), from the main text, and eliminate wages using the firms optimal condition, and labor, using $L_t = Y_t/n_t^\alpha \Delta(0)$. Then use the optimal mass of firms, from equation (42),

$$\frac{1}{r_t}\frac{1}{1-G\left(\varepsilon_t^\star\right)} = 1+\frac{1}{\theta}\left[\frac{1}{D\left(\varepsilon_t^\star\right)}-1\right] - fG\left(\varepsilon_t^\star\right)\frac{\alpha}{D\left(\varepsilon_t^\star\right)}\frac{1}{\theta\left[1+fG\left(\varepsilon_t^\star\right)\right]} \tag{44}$$

which is directly comparable to equation (40), in Appendix B.3. Eliminating the spread from the optimal tax on labor-income results in equation (19), which is reported and discussed in the text.

Appendix B.5 (Proposition 3)

We drop time subscripts and suppress the $\varepsilon^\star$ index to re-write the optimal tax on dividend-income tax as, $\frac{1}{1-\tau} = \frac{1-\theta}{\alpha}D\left(1+fG\right)$, where $D = \left(1-G\right)/\kappa$. The optimal dividend tax is the product of declining and increasing function, for which there is only one point of inflection. We therefore evaluate,

$$\frac{\alpha}{1-\theta}\frac{d\frac{1}{1-\tau_t}}{d\sigma} = 0 \Leftrightarrow -\frac{dD}{d\sigma} = f^\star\left(D\frac{dG}{d\sigma}+G\frac{dD}{d\sigma}\right) = f^\star\frac{d\left(GD\right)}{d\sigma} \tag{45}$$

where,

$$\frac{dG}{d\sigma} = \Phi'(x^\star)\frac{dx^\star}{d\sigma} > 0 \quad \text{and} \quad \frac{dD}{d\sigma} = \Phi'(\theta\sigma - x^\star)\left(\theta - \frac{dx^\star}{d\sigma}\right) < 0$$

In what follows, we characterize $f^\star > 0$. Applying these conditions to equation (45),

$$-\Phi'(\theta\sigma - x^\star)\left(\theta - \frac{dx^\star}{d\sigma}\right) = f^\star\left[\Phi(\theta\sigma - x^\star)\Phi'(x^\star)\frac{dx^\star}{d\sigma} + \Phi(x^\star)\Phi'(\theta\sigma - x^\star)\left(\theta - \frac{dx^\star}{d\sigma}\right)\right]$$

Recall equations (35) and (36), and use them to eliminate $\Phi(\theta\sigma - x_t^\star)$ and $\Phi'(\theta\sigma - x^\star)$, respectively,

$$f^\star = \left(\frac{dx^\star}{d\sigma} - \theta\right)\left\{\left[1-\Phi(x^\star)\right]\frac{1}{\theta}\frac{dx^\star}{d\sigma} - \Phi(x^\star)\left(\frac{dx^\star}{d\sigma} - \theta\right)\right\}^{-1} \tag{46}$$

where has been used $\Phi(-x^\star) = 1 - \Phi(x^\star)$. Note that equation (38) implies $\frac{dx^\star}{d\sigma} > \theta$ under the condition $G < 1/2$.

Appendix C: Quantitative Model

Appendix C.1 (Model Summary)

The equations for the model economy are

===== Table C Here =====

The conditions in Table 1 form a system of equations which solve for $\{C_t, L_t, l_t, n_{t+1}, n_{e,t}\}$ and $\{z(\varepsilon_t^\star), \pi(\varepsilon_t^\star), \varepsilon_t^\star\}$ and $\left\{v_t, w_t, r_{t-1}^d, r_t, r_{t+1}^s\right\}$, where $Y_t = \Delta(0)\, n_t^{(1-\omega)/\theta} l_t$, with government expenditure, $\mathcal{G} > 0$, and dividend and labor-income, $\left\{\tau_t, \tau_t^L\right\}$, given.

Appendix C.2 (Derivation of the Implementability Constraint)

Multiply the household budget constraint - given by equation (20) in the text - by the marginal utility of consumption, $u_C(t)$, impose the equilibrium condition $x_{t-1} = 1$, and integrate forward. Use the labour supply and the dynamic Euler equations, $u_C(t) = \beta E_t\left[u_C(t+1) r_t^s\right]$; $u_C(t) = \beta E_t\left[u_C(t+1) r_t^d\right]$ and $v_t u_C(t) = \beta(1-\delta) E_t\left[z(\varepsilon_{t+1}^\star) u_C(t+1)\right]$. Finally, use dynamic equation for product creation, $n_t = (1-\delta)(n_{t-1} + n_{e,t-1})$, to write the implementability constraint as,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_C(t) C_t + u_L(t) L_t\right] = A \tag{47}$$

where $A \equiv u_C(0)\left[r_{-1}^d d_{-1} + b_0 + n_0 z(\varepsilon_0^\star)\right]$ is assumed to be exogenous.

Appendix C.3 (Reduced Optimal Policy Problem)

Following Chugh and Ghironi (2015), the policy maker picks τ_t^L and commits to pick τ_{t+1}^d in period t. The problem can be written as one of maximizing $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$, subject to all the conditions presented in Table 1, the present value constraint, given by equation

(47), and $\varepsilon_t^\star > 0$. Plans are made over $\{n_{e,t}, n_{t+1}, l_t, C_t, L_t, \pi(\varepsilon_t^\star)\}_{t=0}^\infty$, prices $\left\{w_t, r_t, r_t^d\right\}_{t=0}^\infty$, tax rates, $\left\{\tau_{t+1}, \tau_t^L\right\}_{t=0}^\infty$ and the default threshold, $\{\varepsilon_t^\star\}_{t=0}^\infty$. By choosing tax rates, however, the constraints on the labor-leisure and the Euler equation for shares (i.e., product creation) do not bind. Similarly, by picking wages and interest rates directly, the constraints on firm pricing, the zero profit condition for financial intermediaries, and the Euler equation for deposits do not bind. This allows us to re-write the reduced Ramsey policy problem as in the text, where $\varepsilon_t^\star > 0$.

Appendix C.4 (First-Order Conditions for Optimal Policy)

From the policy problem defined in the text, the conditions for $\{C_t, L_t, n_{e,t}\}_{t=0}^\infty$ are,

$$0 = u_C(t) - \lambda_{1,t} + \xi\left[u_{CC}(t)C_t + u_C(t)\right]$$

$$0 = u_L(t) + \lambda_{2,t} + \xi\left[u_{LL}(t)L_t + u_L(t)\right]$$

$$0 = -\lambda_{1,t} f_e + \lambda_{3,t}(1-\delta)$$

The conditions for $\{n_{t+1}, l_t\}_{t=0}^\infty$ are,

$$0 = \beta\lambda_{1,t+1}\left[(1+\alpha)\frac{Y_{t+1}}{n_{t+1}} - \phi G\left(\varepsilon_{t+1}^\star\right)\right] - \beta\lambda_{2,t+1}\left(l_{t+1} + \frac{f_o}{a_{t+1}}\right) + \lambda_{3,t+1}\left[\beta(1-\delta)\right] - \lambda_{3,t}$$

$$0 = \lambda_{1,t}\left(\frac{Y_t}{l_t}\right) - \lambda_{2,t} n_t + \lambda_{4,t}\left\{\left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)}\right]^\theta - \theta\right\}$$

Finally, the condition for $\{\varepsilon_t^\star\}_{t=0}^\infty$ is,

$$0 = -\beta^t\lambda_{1,t}\left[\phi n_t g(\varepsilon_t^\star)\right] + \beta^t l_t \lambda_{4,t}\frac{d}{d\varepsilon_t^\star}\left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)}\right]^\theta$$

where we assume $\eta_t = 0$ as $\varepsilon_t^\star > 0$. The final term in this expression is,

$$\frac{d}{d\varepsilon_t^\star}(\varepsilon_t^\star)^\theta\left[\Delta(\varepsilon_t^\star)\right]^{-\theta} = \frac{\theta}{\varepsilon_t^\star}\left[1 - \frac{\varepsilon_t^\star\Delta'(\varepsilon_t^\star)}{\Delta(\varepsilon_t^\star)}\right]\left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)}\right]^\theta$$

and $\frac{\varepsilon_t^\star\Delta'(\varepsilon_t^\star)}{\Delta(\varepsilon_t^\star)} < 1$.

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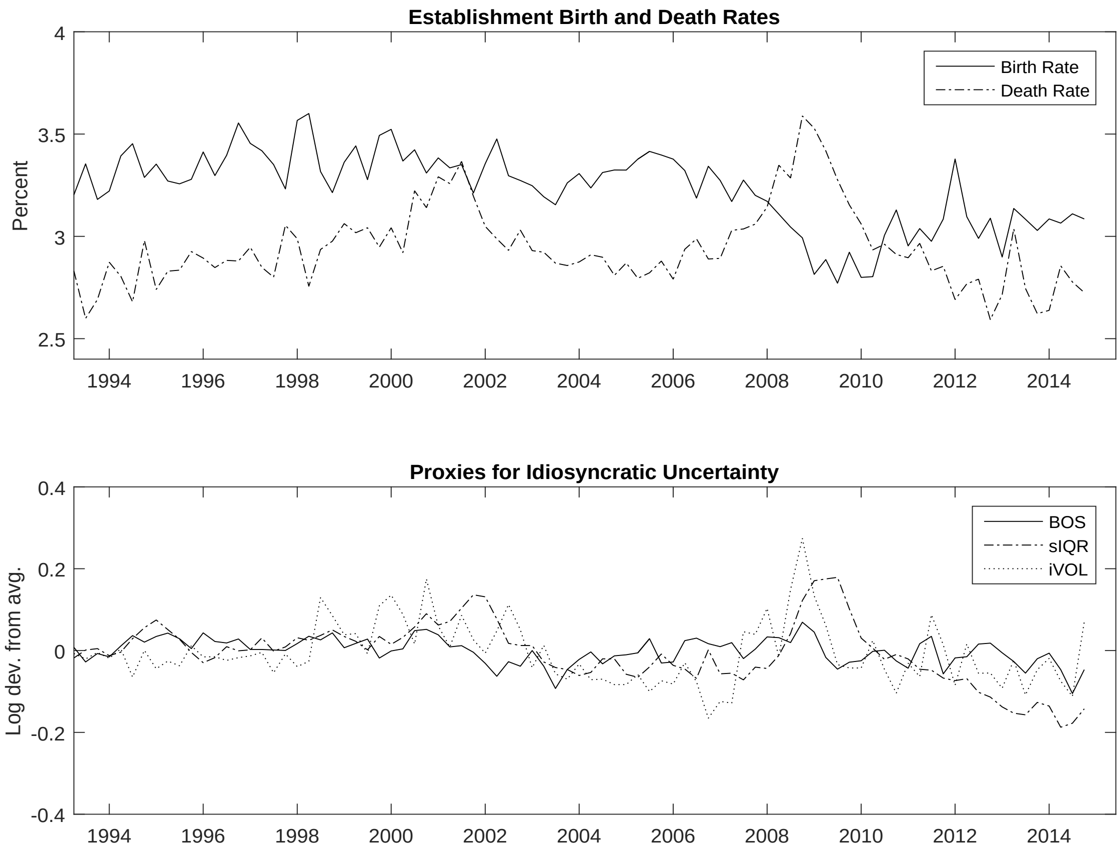
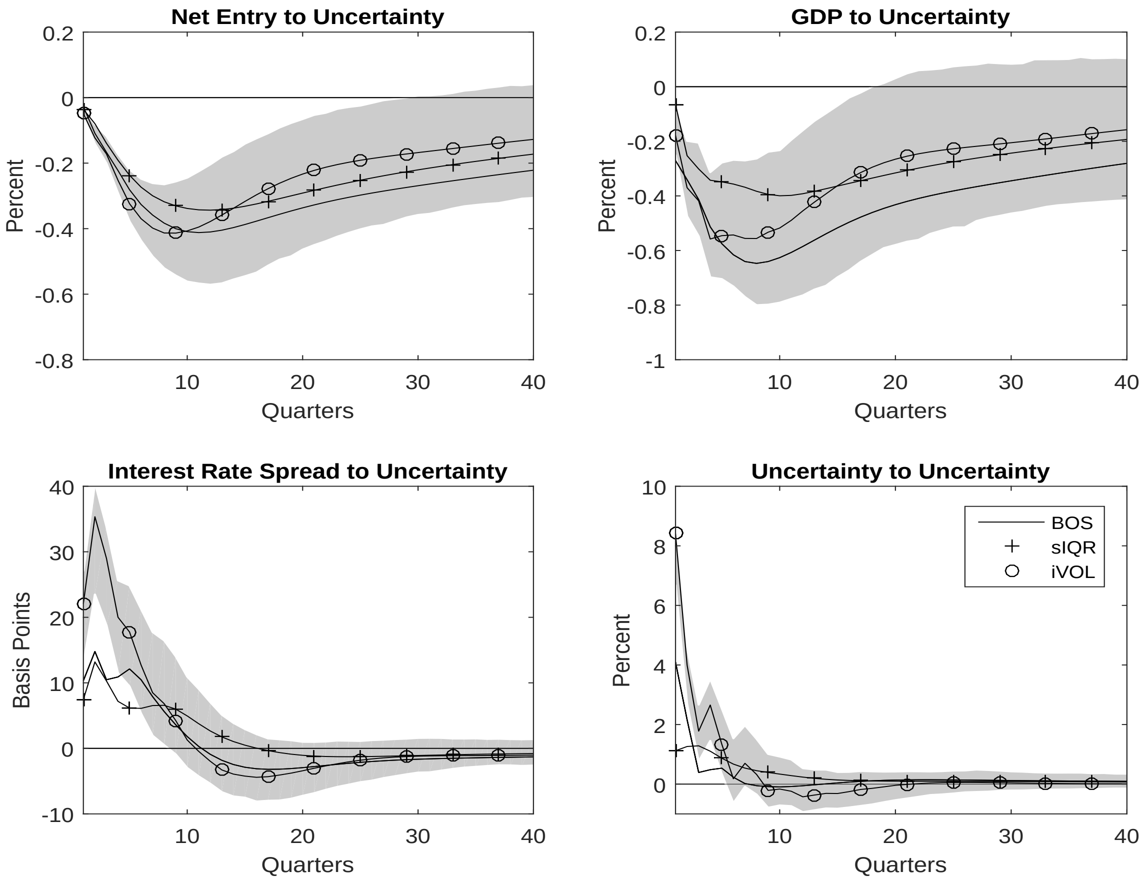
Figure 1: Firm Entry and Exit Rates and Proxies for Idiosyncratic Uncertainty[57]



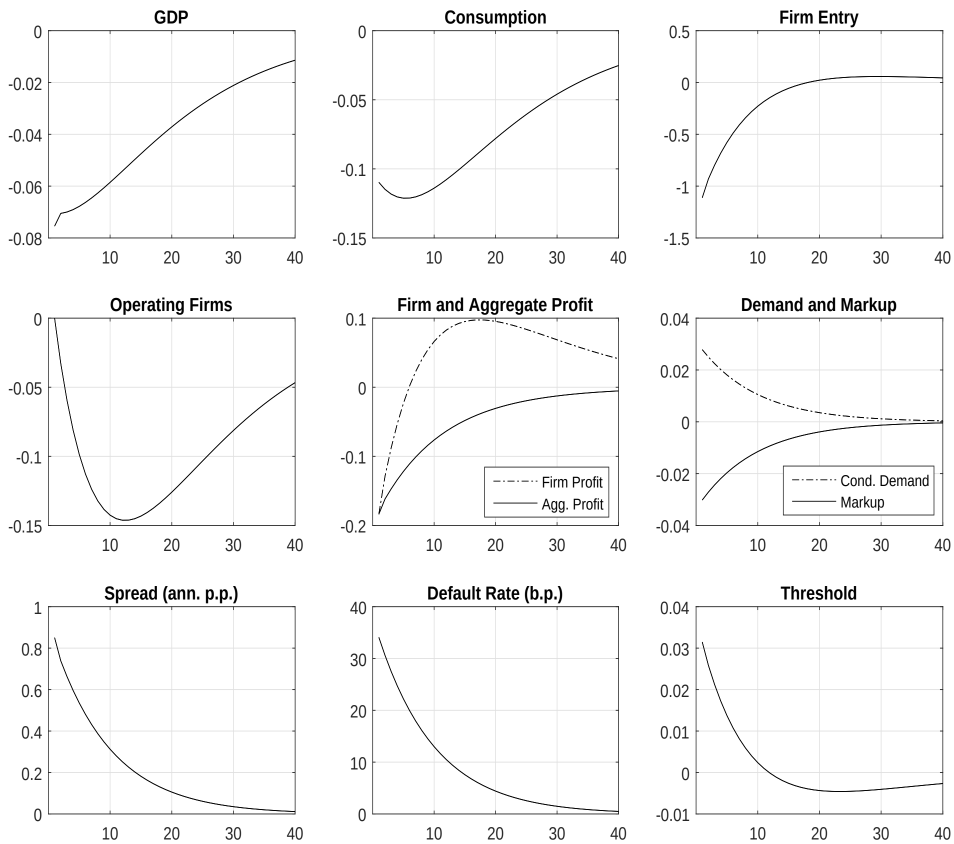
[57]Upper Panel. Private sector establishment births and deaths, seasonally adjusted. Source: Bureau of Census and Bureau of Labor Statistics. Lower Panel. Uncertainty Proxies (for calculation, see Appendix A). Sources: Federal Reserve Bank of Philadelphia's Business Outlook Survey (BOS), Compustat (sIQR), and Caldara *et al.* (2016) (iVOL).

Figure 2: Empirical Impulse Responses for an Uncertainty Shock[58]



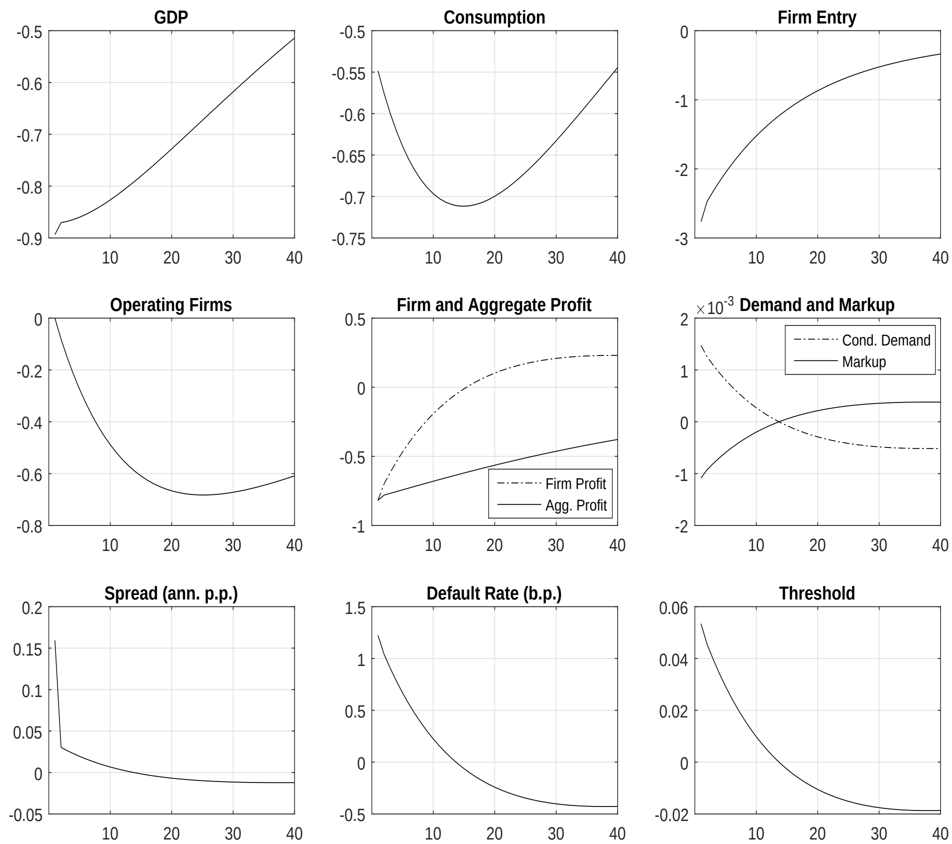
[58]Estimation is over the period 1993:Q2-2015:Q1. The VAR is specified with three lags. Each impulse response is estimated from a separate VAR system (corresponding to an uncertainty proxy). Identification is by Choleski decomposition with the uncertainty measure placed first. Both net entry and GDP are used in log terms from an index in which 2005:Q1=100. The shaded areas are one standard error bootstrapped confidence intervals using the system with iVOL. Appendix A contains details of the uncertainty proxies.

Figure 3: Impulse Responses for a Volatility Shock[59]



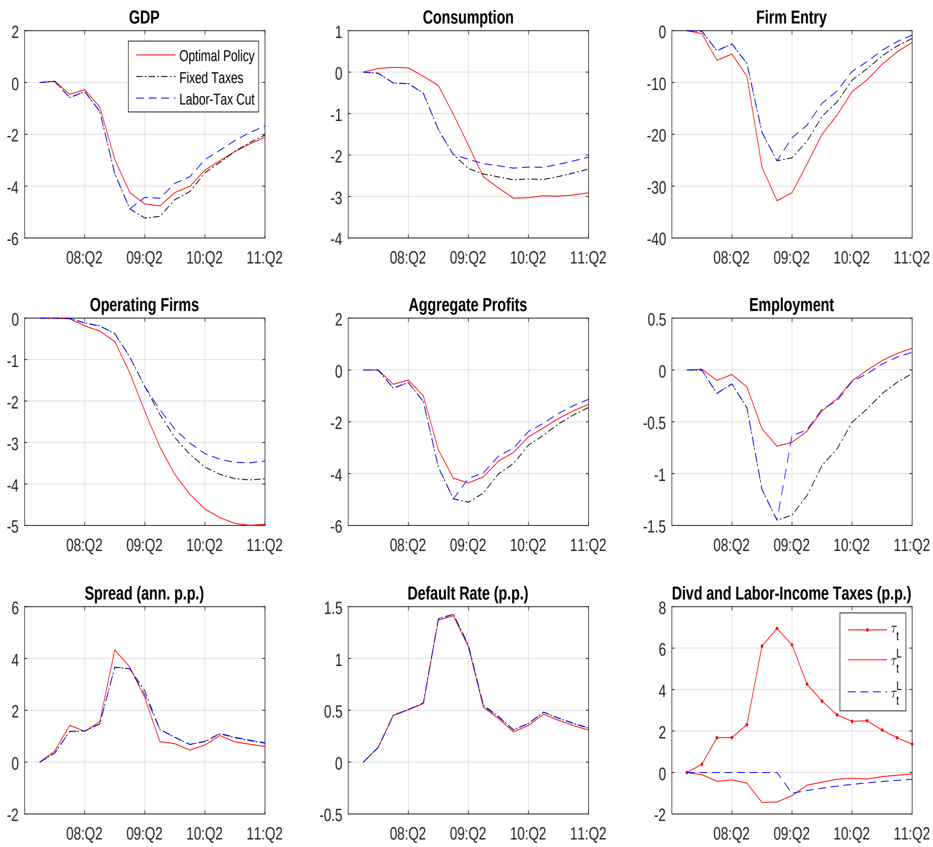
[59]Notes: Percent deviations from steady state reported on the vertical axis (unless otherwise stated). Quarters reported on the horizontal axis.

Figure 4: Impulse Responses for a Negative Technology Shock[60]



[60]Notes: Percent deviations from steady state reported on the vertical axis (unless otherwise stated). Quarters reported on the horizontal axis.

Figure 5: Impulse Responses with Different Tax Policies[61]



[61]Notes: Percent deviations from steady state reported on the vertical axis (unless otherwise stated). Quarters reported on the horizontal axis.

Table A: Uncertainty Regressions

Dependent Variable	BOS	sIQR	iVOL
Recession Indicator	0.050 (0.020)	0.035 (0.012)	17.266 (3.441)
Mean of Dep. Var.	0.700	0.187	50.823
	Rise in Uncertainty		
	7.2%	18.7%	34.0%

Table C: Summary of Quantitative Model

Description	Equation
Labor market clearing	$L_t = n_t \left(l_t + \frac{f_o}{a_t} \right)$
Resource constraint	$a_t \Delta\left(0\right) n_t^{(1-\omega)/\theta} l_t - \mathcal{G} = C_t + f_e n_{e,t} + \phi n_t G\left(\varepsilon_t^\star\right)$
Labor demand	$\Delta(0) n_t^\alpha = \left[\frac{\Delta(0)}{\Delta(\varepsilon_t^\star)}\right]^\theta \frac{w_t r_t}{a_t \theta}$
Labor supply	$w_t\left(1 - \tau_t^L\right) = -\frac{u_L(t)}{u_C(t)}$
Net worth	$z\left(\varepsilon_t^\star\right) = \left(1 - \tau_t\right) \pi\left(\varepsilon_t^\star\right) + v_t$
Conditional expected profit	$\pi\left(\varepsilon_t^\star\right) = \left[\frac{1-G(\varepsilon_t^\star)}{\kappa(\varepsilon_t^\star)}\right] n_t^\alpha \Delta\left(0\right) a_t \left[\left(1-\theta\right) l_t - \theta \frac{f_o}{a_t}\right]$
Mass of firms	$n_{t+1} = \left(1 - \delta\right)\left(n_t + n_{e,t}\right)$
Default Threshold	$\int_{\varepsilon_t^\star}^\infty \left[\theta \varepsilon_t^\theta - \left(\varepsilon_t^\star\right)^\theta \left(\frac{l_t}{l_t + f_o}\right)\right] dG(\varepsilon) = 0$
Financial intermediaries	$w_t r_{t-1}^d + f_m \frac{G(\varepsilon_t^\star)}{l_t + \phi/a_t} = \left[\int_{\varepsilon_t^\star}^\infty dG\left(\varepsilon\right) + \int_0^{\varepsilon_t^\star} \left(\frac{\varepsilon}{\varepsilon_t^\star}\right)^\theta dG\left(\varepsilon\right)\right] w_t r_t$
Euler equation (equity) and free entry	$v_t = \beta(1-\delta) E_t \left[\frac{u_C(t+1)}{u_C(t)}\right] z\left(\varepsilon_{t+1}^\star\right)$ and $v_t = f_e$
Euler equations (deposits and bonds)	$1 = \beta E_t \left[\frac{u_C(t+1)}{u_C(t)}\right] r_t^d$ and $1 = \beta E_t \left[\frac{u_C(t+1)}{u_C(t)}\right] r_t^s$

where $\Delta\left(\varepsilon_t^\star\right) \equiv \left[\frac{1}{1-G(\varepsilon_t^\star)} \int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG\left(\varepsilon\right)\right]^{1/\theta}$ and $\Delta\left(0\right) \equiv \left[\int_0^\infty \varepsilon^\theta dG\left(\varepsilon\right)\right]^{1/\theta}$.

Table 1: Parameter Values used in Quantitative Analysis

Parameters Set Exogenously			
	Parameter	Value	Target/Source
Discount factor	β	0.99	$(\beta^{-4} - 1) \times 100 = 4.01\%$
Frisch elasticity	$\upsilon\left(\frac{1-L}{L}\right)$	0.74	Heathcote *et al.* (2010)
Elasticity of Substitution	$1/(1-\theta)$	3.8	Bernard *et al.* (2003)
Sunk cost	f_e	1	Normalization

Calibrated Parameters				
	Parameter	Value	Target	Source
IQR sales growth	σ	0.130	18.68%	Compustat (see text)
Default rate	f_o	0.012	1.52%	Moody's (see text)
Exit rate (annual)	δ	0.029	11.78%	BLS (see text)
Spread (annual)	ϕ	0.285	241 b.p.	FRED (BAA10Y)
Hours worked	χ	1.381	20%	-