Optimal package pricing in healthcare services

Tushar Tanwar, U. Dinesh Kumar & Navonil Mustafee

Published online: 09 Sep 2019.

Submit your article to this journal ⌇

Article views: 36

View related articles ⌇

View Crossmark data ⌇

THE OPERATIONAL RESEARCH SOCIETY

Taylor & Francis
Taylor & Francis Group

ORIGINAL ARTICLE

OPEN ACCESS | Check for updates

Optimal package pricing in healthcare services

Tushar Tanwar[a], U. Dinesh Kumar[a] and Navonil Mustafee[b]

[a]Decision Sciences Area, Indian Institute of Management Bangalore, Bangalore, India; [b]The Centre for Simulation, Analytics and Modelling (CSAM), Department of Science, Innovation, Technology, and Entrepreneurship (SITE), University of Exeter Business School, Exeter, UK

ABSTRACT
Fixed pricing for healthcare services is emerging as an attractive business model for private healthcare service providers. Under fixed pricing (or flat rate) contract, the patient is charged a fixed price for the healthcare services irrespective of the actual cost incurred by the hospital. Such contracts increase the risks for the healthcare service provider, thus making pricing decision crucial. In this paper, we study uncertainty and analyse the flat rate pricing contract for a profit maximising hospital to find the optimal price of treatment and examined value-at-risk (VaR) associated with such contracts for a risk minimising hospital. Bounds on price were derived to support healthcare providers with price negotiations. We extended the basic models by adding constraints to obtain risk-adjusted optimal price. We proved analytically that the optimal price lies between profit maximisation value and risk minimisation value of price, which we refer to as the efficient pricing interval. Our models and insights provide practical support to private healthcare service providers for optimal pricing and keep them informed about their risk position.

1. Introduction

In recent years, fixed price strategy is being followed widely in which the healthcare service providers are paid a fixed price for the complete treatment rather than for each service that a patient receives. Taking the example of cardiac surgery, a fixed-price solution may include costs towards stay in general ward prior to surgery and subsequent stay in specialised wards such as the intensive cardiac care unit (ICCU) and the high dependency unit (HDU), costs towards operation theatre time, costs for the surgeons and anaesthetists, cost of medication and follow-up appointment with consultants, costs for nursing, and so on. A fixed price strategy, such as the one described above, is increasingly being adopted by both publicly funded healthcare systems, for e.g., the UK NHS, which also treats private patients, and those that are dominated by the private healthcare insurance sector, as in India and the US. To highlight the practical relevance of our work, we begin by discussing the importance of fixed-price strategy for both private and public healthcare systems.

In a private healthcare system, the intended users of the systems (the patients) usually purchase their health insurance. A certain percentage of the population, however, remain uninsured affecting their ability to access routine health care and having to pay out-of-pocket at the point of delivery (note, treatment in accident and emergency is usually free). The National Center for Health Statistics in the US analysed data from the 2015 National Health Interview Survey (NHIS) and reported that the number of persons under the age of 65 years that were uninsured at the time of the survey was 28.4 million; this accounted for 10.5% of persons under the age of 65 years (Ward, Clarke, Nugent, & Schiller, 2016). In the case of India, which spends around 5% of the GDP on health care, nearly 70% of health spending is borne by households, out-of-pocket sources (Garg & Karan, 2009). With fixed price services being offered for healthcare services, patients "shop" as consumers by comparing prices of various hospitals as providers (Richman, Udayakumar, Mitchell, & Schulman, 2008).

For publicly funded healthcare services, demand for treatment usually exceeds the available capacity, and patients with non-urgent treatment requirements (e.g., those requiring elective procedures) are put into waiting list unless they are willing to pay for private care (Gutacker, Siciliani, & Cookson, 2016). The distinction here with co-payment models of health care is that a contribution towards the costs of care is sought from the patients (Sonnessa, Tànfani, & Testi, 2017). According to monitor, the sector regulator for health services in England, income generated by the NHS

from treating private patients (private patient income or PPI) has steadily increased from £98.4 million in 2005/06 to £389.4 million in 2013/14 (Monitor, 2014). With the 2012 Health and Social Care Act (HSCA), hospitals have been allowed to raise up to 49% of income from non-NHS work, which translates to more private patients. For these patients, although they are being treated in a publicly funded facility, greater transparency in healthcare pricing will enable them to make informed decisions as to the hospital they would choose for their treatment. Galetsi and Katsaliaki (2019) reported use of big data analytics in healthcare, and one of the main objectives is to reduce the cost of treatment and identification of high risk/cost patients.

In this paper, we analyse fixed pricing in healthcare services to find the optimal pricing under different scenarios. We have contributed to the healthcare service literature by analysing flat rate pricing under cost uncertainty which has not been studied in the literature to the best of our knowledge. We have also developed a risk-based pricing model for a healthcare provider using value-at-risk (VaR) analysis which helps hospital management to incorporate risk into their pricing decisions. Our optimisation models provide practical support while negotiating the price with the consumer. We believe that this work helps healthcare providers make an informed decision on pricing their services.

2. Pricing models and their relevance in pricing healthcare services

The basic pricing model used in the pricing of products is linear pricing, where the price is a linear function of the quantity purchased. The second kind of pricing strategy is called non-linear pricing, where the quantity purchased affects the price. The simplest example of non-linear pricing is the two-part pricing consisting of flat price and usage-based pricing when the usage exceeds a specified threshold. In the context of health care, the usage rate of healthcare service is not a function of price; rather, it depends on the individual consumer's health condition. Also, the quantity of the service consumed does not affect patients' valuation of the service as they are unaware of their consumption. In fact, the whole set of services is like one product, for example, heart surgery, wherein the patient does not have a choice of opting out from some treatment procedures which are an integral part of the surgery *per se*, neither can the hospital refuse to provide those services in order to save/optimise on cost. Therefore, we argue that non-linear pricing, such as two-part tariff, is not appropriate in healthcare services.

In recent years, fixed price strategy is being followed widely in the industry wherein the providers are paid a fixed price for services such as internet access (Odlyzko, 1999), information goods (Sundararajan, 2004) and telephone services (Mitchell, 1978). Research has focused on finding different conditions under which one should choose a flat rate, usage-based and two-part tariff (fixed fee along with per usage charges) as the pricing strategy. The existence of a flat rate pricing strategy has been attributed to flat-rate bias (Train, 1991). Empirical literature documenting the flat-rate bias points out that this bias is driven by consumers' uncertainty about future consumption (Herweg & Mierendorff, 2013). This fits perfectly in a healthcare setting where the patient or the consumer is uncertain of his future consumption of care required from the healthcare service providers.

The fixed price model in health care has mitigated the risk of cost uncertainty for the consumers and turned the risk tables on the healthcare service provider instead (Richman et al., 2008). The hospitals are therefore under huge financial risk since the entire monetary risk (which may occur due to failed internal processes, people and systems, or from external events) is transferred to them. Previously, in usage-based pricing, the hospital would receive payment for each of the services provided, wherein they earned a fixed profit. However, with a fixed package pricing, their revenue is fixed with the uncertain cost of treatment, so the profit is no more certain. However, along with this risk, there are benefits too: i.e., on the one hand, the hospital may make a loss when treating a high severity patient, but on the other hand, it may make a larger profit with a patient with less severity. The tradeoff between the two is what needs to be balanced. Therefore, it is important for a healthcare provider to study its costs and make better pricing decisions in the presence of uncertainty.

In this paper, we analysed the pricing problem faced by a monopolist hospital analytically and derive optimal price under flat-rate pricing strategy. To our best knowledge, the literature on optimal pricing is scarce in the healthcare industry. Newhouse (1970) and Margolis (1983) solved the profit maximisation problem to find optimal pricing of physician services. Pricing model by Dittman and Morey (1981) sets prices which maximise the hospital's profit while curtailing total hospital resources. These papers assumed that costs are known to the hospital. Our pricing model extends these models to find the optimal price under cost uncertainty. We use VaR analysis to incorporate risk due to uncertainty to make risk-adjusted pricing decisions.

3. Flat rate pricing model – theory development

We make two assumptions for the flat rate pricing model: (a) "Demand" is a function of price, and (b) Hospital does not change the quality of care depending on the severity of the patient. We offer justification in support of our two assumptions as follows. In relation to (a), Ringer and Hosek (2000) have stated that demand, in general, is price and income inelastic, but certain types of care are more sensitive to price such as preventive care and pharmacy. This is mainly due to multiple substitutes present in these cases. Therefore, it is arguable that the assumption "demand is price elastic" can hold for countries where there is a lack of insurance and dominance of multiple private healthcare providers. Concerning (b), Richman et al. (2008) have stated that Indian hospitals, for example, have managed to give quality treatment within this constraint of a flat fee.

We define the total cost (TC) function as follows:

$$TC = \beta_0 X^{\beta_1}, \qquad 0 < \beta_1 < 1 \qquad (1)$$

The function in Equation (1) is similar to Cobb–Douglas production function in economics literature which is widely used to represent the relationship between inputs and output produced. In our case, the output is the TC incurred, and the input is the length of stay (X) of the patient. Length of stay (LoS) is considered as a reasonable proxy of resource consumption (Marazzi, Paccaud, Ruffieux, & Beguin, 1998), and in our model, we introduce uncertainty in the cost of treatment by assuming the cost to be a function of LoS. Ayer, Ayvaci, Karaca, and Vlachy (2019) claimed that there is a significant difference in LoS distribution across cohorts. There is empirical evidence that the treatment cost is a concave function of LoS (Polverejan, Gardiner, Bradley, Holmes Rovner, & Rovner, 2003). This is also rather intuitive since a major part of the total cost (high utilisation of resources) is incurred during the initial period of stay in the hospital. This function has decreasing returns to scale since $0 < \beta_1 < 1$. We know that the LoS of a patient is random due to the difference in the severity of individual patients and in most cases an unknown quantity. Kumar, Costa, Fackrell, and Taylor (2018) claim that the range of LoS is large and generally positively skewed.

In literature, both log-normal and phase type distributions are used for modelling the LoS. Marazzi et al. (1998) compared lognormal, Weibull, and gamma distributions for LOS distributions for multiple countries. They suggested both log-normal and Weibull distributions depending on country and diagnosis/illness. Exponential and phase type distributions have been used for fitting LoS for geriatric/

elderly patients (Marshall, McClean, Shapcott, & Millard, 2002; Marshall & McClean, 2003, 2004; Vasilakis & Marshall, 2005). Faddy, Graves, and Pettitt (2009) compared phase type, gamma and log-normal distributions for LoS distribution. Though they show that phase type is a better fit in comparison but log-normal too gave a good fit (BIC = −9927.85 and −9936.21, respectively). Also, they found similar estimates of the covariates for both the distributions. Demir, Lebcir, and Adeyemi (2014) modelled length of stay using continuous time Markov chain in the context of neonatal care. Gillespie et al. (2016) used a phase type distribution (Markov chain) to model the entire patient journey for a specific treatment.

In this paper, we use both log-normal and phase-type distribution (Erlang) to study the problem. In the first model, we assume X as a random variable denoting LoS, which follows a log-normal distribution with parameters μ and σ. The parameter σ is the shape parameter of X, while e^μ is the scale parameter of X, where $X \geq 1$. It has been widely known that distribution of LoS is rightly skewed (Atienza, 2005; Atienza, García-Heras, Muñoz-Pichardo, & Villa, 2008; Hellervik & Rodgers, 2007; Lim & Tongkumchum, 2009) and empirical evidence for log-normal distribution for LoS in a hospital is provided by Marazzi et al. (1998). In the second model, we assume that the LoS follows an Erlang-K distribution with scale parameter k and shape parameter θ.

Notations

a: Market size for the service offered	$NP(P)$: Hospitals net profit
b: Price sensitivity of demand	α: Probability of not exceeding a threshold profit
$D(P)$: Demand at price P	$E[NP(P)]$: Expected net profit
β_0: Accounts for factors other than LoS	VaR_α: Value at risk with probability α
β_1: Cost elasticity of LoS	P^*: Optimal price for profit maximising hospital
X: Random variable denoting length of stay	P_{VaR}: Optimal price for risk minimising hospital

4. Optimal pricing and net profit for profit maximising (risk neutral) hospital

To model the market for a healthcare service, we assume that the customers/patients make the decisions based on the price of the service. We assume the aggregate inverse demand function, $D(P)$ to be a linear function of price.

$$D(P) = a - bP \qquad (2)$$

$D(P)$ is the number of customers buying the service at price P, given the total market size of the service is a. So, the pricing interval is $[0, \frac{a}{b}]$.

The net profit at price P is given by:

$$NP(P) = \left[(P - \beta_0 x^{\beta_1})(a - bP) \right] \qquad (3)$$

The primary objective of the hospital is to offer a price which maximises its expected returns. Theorem 1a gives the optimal price, which maximises the expected profit, under the assumption that LoS follows a log-normal distribution. Theorem 1b gives the optimal price, which maximises the expected profit under the assumption that the LoS follows an Erlang-K distribution.

Theorem 1a: *The optimal price for the hospital is given by* $P^* = \dfrac{a + \left(\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) b}{2b}$, *which maximises the expected return for the hospital and the maximum profit is* $\pi^* = \dfrac{\left(a - \beta_0 b e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)^2}{4b}$, *where a and b are such that* $\frac{a}{b} > \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}}$ *(i.e. the maximum price is more than the expected cost).*

Proof: The expected net profit is given by

$$
\begin{aligned}
E[NP(P)] &= E\left[(P - \beta_0 x^{\beta_1})(a - bP) \right] \\
&= \left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP)
\end{aligned}
\qquad (4)
$$

From the equation, we solve the following first-order condition to get the critical point:

$$\frac{\partial E[NP(P)]}{\partial P} = a - 2bP + \left(\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) b = 0$$

$$\iff P^* = \frac{a + \left(\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) b}{2b} \qquad (5)$$

As the second derivative $\frac{\partial E^2[NP(P)]}{\partial P^2} = -2b$ is negative, P^* defined in (5) attains the maximum value of the expected profit. Substituting the optimal price in Equation (4), we get expected total profit as $\dfrac{\left(a - b\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)^2}{4b}$, which can also be written as $\frac{1}{b}(a - bP^*)^2$.

From Equation (5), we can conclude that the optimal price decreases with price sensitivity of demand and increases with scaling power of LoS, mean and variance.

Theorem 1b: *The optimal price for the hospital is given by* $P^* = \dfrac{a + \left(\frac{\Gamma(k + \beta_1) \beta_0 \theta^{\beta_1}}{(k-1)!} \right) b}{2b}$, *which maximises the expected return for the hospital, where a and b are such that* $\frac{a}{b} > \left(\frac{\Gamma(k + \beta_1) \beta_0 \theta^{\beta_1}}{(k-1)!} \right)$ *(i.e., the maximum price is more than the expected cost).*

Proof: We assume X as a random variable denoting LoS, which follows Erlang-K distribution with

parameters k and θ. The parameter k is the shape parameter of X, while θ is the scale parameter of X.

The expected net profit is given by

$$
\begin{aligned}
E[NP(P)] &= E\left[(P - \beta_0 x^{\beta_1})(a - bP) \right] \\
&= (P - \beta_0 E[x^{\beta_1}])(a - bP)
\end{aligned}
\qquad (6)
$$

$$E[x^{\beta_1}] = \int_0^\infty x^{\beta_1} f(x) \text{ where } f(x) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{(k-1)! \theta^k},$$

$$= \int_0^\infty \frac{x^{k + \beta_1 - 1} e^{-\frac{x}{\theta}}}{(k-1)! \theta^k}$$

Using Gamma function, we get

$$= \frac{\Gamma(k + \beta_1) \theta^{\beta_1}}{(k-1)!}$$

From the equation, we solve the following first-order condition to get the critical point:

$$E[NP(P)] = \left(P - \frac{\Gamma(k + \beta_1) \beta_0 \theta^{\beta_1}}{(k-1)!} \right)(a - bP)$$

$$\frac{\partial E[NP(P)]}{\partial P} = a - 2bP + \left(\frac{\Gamma(k + \beta_1) \beta_0 \theta^{\beta_1}}{(k-1)!} \right) b = 0$$

$$\iff P^* = \frac{a + \left(\frac{\Gamma(k + \beta_1) \beta_0 \theta^{\beta_1}}{(k-1)!} \right) b}{2b} \qquad (7)$$

4.1. Feasibility of offering a flat rate price

To establish if offering the treatment at a price P is feasible for the hospital, the hospital needs to evaluate the risk of offering the treatment at the price. We define it as the probability of profit exceeding a threshold, which gives the risk that the hospital will take when offering a flat rate price P for a given treatment. Theorem 2 gives the possibility of offering the treatment at price P.

Theorem 2: *The probability of the hospital's profit exceeding a threshold value π for demand $D(P)$ is given as:*

$$\text{Prob}_\pi = \Phi \left(\frac{\log \left[\frac{1}{\beta_0} \left(P - \frac{\pi}{(a - bP)} \right) \right] - \beta_1 \mu}{\beta_1 \sigma} \right).$$

Proof: Substituting the value of $NP(P)$ from (3), we have:

$$\text{Prob}\{ NP(x) \geq \pi \} = \text{Prob}\{ (P - \beta_0 x^{\beta_1})(a - bP) \geq \pi \}$$

$$\iff \text{Prob}\left\{ \beta_0 x^{\beta_1} \leq P - \frac{\pi}{(a - bP)} \right\}$$

$$\iff \text{Prob}\left\{ x^{\beta_1} \leq \frac{1}{\beta_0} \left(P - \frac{\pi}{(a - bP)} \right) \right\}$$

$\Longleftrightarrow \text{Prob}\left\{\beta_1 \log x \leq \log\left[\frac{1}{\beta_0}\left(P - \frac{\pi}{(a-bP)}\right)\right]\right\}$,   here $\beta_1 \log x$ follows $N(\beta_1\mu, \beta_1^2\sigma^2)$

$\Longleftrightarrow \text{Prob}\left\{Z \leq \frac{\left(\log\left[\frac{1}{\beta_0}\left(P - \frac{\pi}{(a-bP)}\right)\right] - \beta_1\mu\right)}{\beta_1\sigma}\right\}$, here Z is a standard normal variable.

$\Longleftrightarrow \Phi\left(\frac{\log\left[\frac{1}{\beta_0}\left(P - \frac{\pi}{(a-bP)}\right)\right] - \beta_1\mu}{\beta_1\sigma}\right)$

$\Longleftrightarrow \Phi\left(\frac{\log\left[\left(P - \frac{\pi}{(a-bP)}\right)\right] - (\beta_1\mu + \log\beta_0)}{\beta_1\sigma}\right)$ (8)

Equation (8) gives the probability that the profit will exceed the threshold value π. The hospital would prefer to have the probability of exceeding its threshold profit as high as possible. Corollary 2(a) gives the price at which the probability obtained in Theorem 2 is maximised. This is the price a hospital with the objective of earning a threshold profit should quote.

Corollary 2 (a): *Given a threshold profit π, the upper bound on price negotiation is given by $P_\pi = \frac{a}{b} - \sqrt{\frac{\pi}{b}}$, where $\pi < \frac{a^2}{b}$*

Proof: Given a threshold profit, the hospital would like to maximise the probability of exceeding it, i.e.,

$$\underset{P}{\text{Max}}\,\Phi\left(\frac{\log\left[\frac{1}{\beta_0}\left(P - \frac{\pi}{(a-bP)}\right)\right] - \beta_1\mu}{\beta_1\sigma}\right).$$

The above problem is equivalent to $\text{Max}_P\left[\frac{1}{\beta_0}\left(P - \frac{\pi}{(a-bP)}\right)\right]$ since log and cumulative distribution functions are increasing functions. We solve the following first-order condition to get the critical point:

$$\frac{1}{\beta_0}\left(1 - \frac{\pi b}{(a-bP)^2}\right) = 0$$

$$\Longleftrightarrow P_\pi = \frac{a}{b} - \sqrt{\frac{\pi}{b}} \qquad (9)$$

As the second derivative at $P_\pi = \frac{a}{b} - \sqrt{\frac{\pi}{b}}$ given by $\left(-\sqrt{\frac{b}{\pi}}\right)$ is negative, P_π attains a maxima and the maximum probability is given by $\Phi\left(\frac{\log\left[\left(\frac{a}{b} - 2\sqrt{\frac{\pi}{b}}\right)\right] - (\beta_1\mu + \log\beta_0)}{\beta_1\sigma}\right)$.

From Corollary 2(a), we observe that there is a tradeoff between the threshold profit and maximum probability of achieving it. Corollary 2(b) shows this tradeoff by varying the threshold around the maximum expected profit.

Corollary 2(b): *For threshold $\pi = \pi^* = \frac{1}{b}(a - bP^*)^2$ (expected profit), optimal price P^* achieves the maximum probability of exceeding it given by $\Phi\left(\frac{\beta_1\sigma}{2}\right)$.*

Proof: Substituting the value of threshold $\pi = \frac{1}{b}(a - bP^*)^2$ in Equation (7), we get $P_\pi = \frac{a}{b} - \sqrt{\frac{\frac{1}{b}(a-bP^*)^2}{b}}$ $= P^* = \frac{a + \left(\beta_0 e^{\beta_1\mu + \frac{\beta_1^2\sigma^2}{2}}\right)b}{2b}$, and the maximum probability of exceeding the expected profit is obtained by substituting P^* in Equation (6) and is given by $\Phi\left(\frac{\beta_1\sigma}{2}\right)$. In this corollary, we establish that the hospital would earn the expected profit, which is not enough as it will happen with a certain probability. As both, P_π and the probability of exceeding threshold have an inverse relationship with the threshold, we can conclude that for $\pi < \pi^*$, we have $P_\pi > P^*$ and the probability of exceeding threshold is greater than $\Phi\left(\frac{\beta_1\sigma}{2}\right)$. Also, for $\pi > \pi^*$, we have $P_\pi < P^*$ and the probability of exceeding threshold is smaller than $\Phi\left(\frac{\beta_1\sigma}{2}\right)$. Using this information on the tradeoff of threshold and chances of exceeding it, the hospital can set its threshold around the maximum expected profit. The probability of exceeding the threshold profit decreases with the price sensitivity of demand, the variance of LoS and scaling factor of LoS.

We propose that the hospital should offer the treatment if it makes a positive profit with the desired confidence level. Corollary 2(c) gives the minimum price at which a hospital should offer the treatment.

Corollary 2(c): *Lower bound on price*
The hospital should offer the treatment at price P, if their threshold profit with confidence level $1-\alpha$ is positive, i.e. if $(P - \beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)})(a-bP)$ is positive.

Proof: The hospital would like to restrict the risk with probability α, which can be written as:

$\text{Prob}\left\{(P - \beta_0 x^{\beta_1})(a-bP) \geq \pi_\alpha\right\} = 1 - \alpha$

$\Longleftrightarrow \Phi\left\{\frac{\log\left[\frac{1}{\beta_0}\left(P - \frac{\pi_\alpha}{(a-bP)}\right)\right] - \beta_1\mu}{\beta_1\sigma}\right\} = 1 - \alpha$ (from Equation (6))

$\Longleftrightarrow \log\left[\frac{1}{\beta_0}\left(P - \frac{\pi_\alpha}{(a-bP)}\right)\right] = \beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)$

$\Longleftrightarrow P - \frac{\pi_\alpha}{(a-bP)} = \beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)}$

$\Longleftrightarrow P(a-bP) - \pi_\alpha = (a-bP)\beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)}$

$\Longleftrightarrow \left[P - \beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)}\right](a-bP) = \pi_\alpha$ (10)

For earning positive profit $(\pi_\alpha > 0)$ with confidence $1-\alpha$, the lower bound on P is given by $\beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)}$, where a and b are such that $\frac{a}{b} > \beta_0 e^{\beta_1\mu + \beta_1\sigma\Phi^{-1}(1-\alpha)}$.

A healthcare provider with a target profit can use the results from Theorem 2 to analyse the
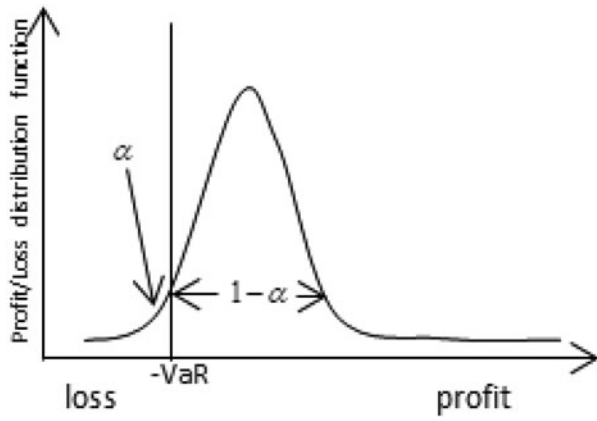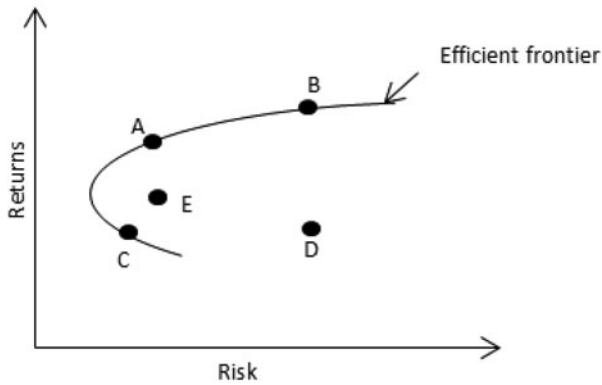
Figure 1. Value-at-risk.

Figure 2. Efficient frontier.

probability of not making it to their target at a given price. This result can be used to compare different prices and threshold. However, with different thresholds comes along the different probabilities of achieving it and profit is meaningless without the probability of achieving it. So, to make an optimal decision, the hospital would require a better criterion to be optimised. So, we suggest that the hospital fix the probability of making losses and minimise their value-at-risk (VaR_α) to find the optimal price. Let us introduce VaR with a formal definition.

4.2. Optimal pricing and net profit for a risk minimising hospital

In this section, we model the problem where the hospital would like to minimise its risk, and where the risk is characterised as the VaR. In the late 1980s, VaR was used by financial firms to measure their assets portfolio risk. VaR methods have now become a common benchmark for risk management in the financial industry. It is a statistical technique used to quantify potential losses/risk. Risk in the model is characterised by the volatility of the costs or revenue. Jorion (2007) defines VaR as the worst loss that will not be exceeded with a given confidence level, based on the estimated distribution of returns. Mathematically, it can be written as $\text{Prob}(\text{Profit/Loss} < -\text{VaR}_\alpha) = \alpha$ (see Figure 1). So,

for a confidence level α of 99% means, on average, there is a 99% chance of the expected loss being lower than VaR. Easy interpretation of the risk position makes VaR a useful risk measure. It keeps managers informed about risk so that they can evaluate and control risk exposure. It is generally calculated for a fixed period of time.

In this context, the hospital would like to minimise the losses with confidence α, i.e., $\text{Prob}(\text{Profit/Loss} < -\text{VaR}_\alpha) = \alpha$. This probability is equivalent to $\text{Prob}(\text{Profit} \geq -\text{VaR}_\alpha) = 1-\alpha$. So, from Corollary 2(c), the VaR_α is given by:

$$-\left[P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}\right](a - bP) \qquad (11)$$

Theorem 3: *The optimal price $P_{\text{VaR}} = \frac{a + (\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})b}{2b}$ minimises the VaR_α(VaR with confidence α) for the hospital and the minimum is given by $-\left(\frac{(a - b\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})^2}{4b}\right)$ where a and b are such that $\frac{a}{b} > \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}$.*

Proof: Rewriting the minimisation problem as a maximisation problem, we have:

$$\underset{P}{\text{Max}} \left\{-\text{VaR}_\alpha\right\}$$

$$\Longleftrightarrow \underset{P}{\text{Max}} \left\{\left[P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}\right](a - bP)\right\}$$

From the equation, we solve the following first-order condition to get the critical point:

$$\frac{\partial(-\text{VaR}_\alpha(P))}{\partial P} = a - 2bP + (\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})b = 0$$

$$\Longleftrightarrow \frac{a + (\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})b}{2b} = P_{\text{VaR}} \qquad (12)$$

As the second derivative $\frac{\partial^2(-\text{VaR}_\alpha(P))}{\partial P^2} = -2b$ is negative, P_{VaR} attains the maximum value of $-\text{VaR}_\alpha(P)$. Substituting the optimal price in Equation (9), we get as $-\text{VaR}_\alpha = \frac{(a - b\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})^2}{4b}$, which can also be written as $\frac{1}{b}(a - bP_{\text{VaR}})^2$.

4.3. Comparing risk minimising and profit maximising price

Theorem 4 compares the two objectives discussed in Theorems 1 and 3. We use the concept of efficient frontier in portfolio analysis. The efficient frontier in portfolio analysis represents the set of portfolios that offers a maximum return for a given level of risk or lowest level of risk for a given level of expected return. Portfolios not on the frontier are sub-optimal as they have low returns for the level of risk or they have a higher risk for a fixed return. In Figure 2, points A, B and C are on the efficient frontier, whereas points D and E are not optimal.

We use the above concept to define a pricing interval we call as efficient pricing interval. It gives the hospital a price range where the expected profit (return) and –VaR (risk) tradeoff occurs. All price levels are dominated by the prices in this interval, i.e., we can always be better off with respect to both objectives from choosing some point within the efficient price interval. At any value outside this interval, we can always find another price which gives higher returns at a lower risk. VaR is the maximum potential loss. So, for easy comparisons with expected profit, we use –VaR. So, we assume a firm will prefer the least value of VaR or highest value of –VaR.

Theorem 4: *The efficient pricing interval is given by $[P^* P_{\mathrm{VaR}}]$.*

Proof: From Equations (4) and (9), we note that the objectives of the two optimisation problems are $\mathrm{Max}_P \left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP)$ and $\mathrm{Max}_P \left\{ (P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) \right\}$ for the profit maximising and risk minimising problems, respectively. We can derive two conclusions by comparing these two objectives. Firstly, $\mathrm{ENP}(P) > -\mathrm{VaR}(P)$ for all values of P, and secondly, $P_{\mathrm{VaR}} > P^*$ (as average cost is less than the worst case cost, i.e., $\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} < \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}$). Let us divide the entire pricing interval $[0, \frac{a}{b}]$ into three intervals $[0, P^*]$, $[P^*, P_{\mathrm{VaR}}]$ and $[P_{\mathrm{VaR}}, \frac{a}{b}]$. We know that $\mathrm{ENP}(P)$ is increasing on interval $[0, P^*]$ and decreasing on interval $[P^*, \frac{a}{b}]$ (P^* being the unique maxima of the concave expected net profit function). Also, $-\mathrm{VaR}(P)$ is increasing on interval $[0, P_{\mathrm{VaR}}]$ and decreasing on interval $[P_{\mathrm{VaR}}, \frac{a}{b}]$ (P_{VaR} being the unique maxima of the concave $-\mathrm{VaR}$ function). We can conclude from above observations that in the interval $[0, P^*]$ and $[P_{\mathrm{VaR}}, \frac{a}{b}]$, both $\mathrm{ENP}(P)$ and $-\mathrm{VaR}(P)$ functions are increasing and decreasing functions, respectively. As we go to left of P^* and right of P_{VaR}, both $-\mathrm{VaR}$ and expected profit decrease. These are the inefficient pricing intervals as there will always exist a price where the expected profit can be increased and risk can be decreased, and it lies in the interval $[P^*, P_{\mathrm{VaR}}]$. Here, $\mathrm{ENP}(P)$ decreases, whereas $-\mathrm{VaR}(P)$ increases, thus giving the interval of the tradeoff between risk and returns, that is, we cannot be better off with respect to both objectives. This interval is what we define as an efficient pricing interval. Figure 3 demonstrates this tradeoff of risk and return using the efficient pricing interval.

5. Risk-adjusted pricing decisions

In Theorem 1, we maximised the expected returns of the hospital; and in Theorem 3, we minimised the risk
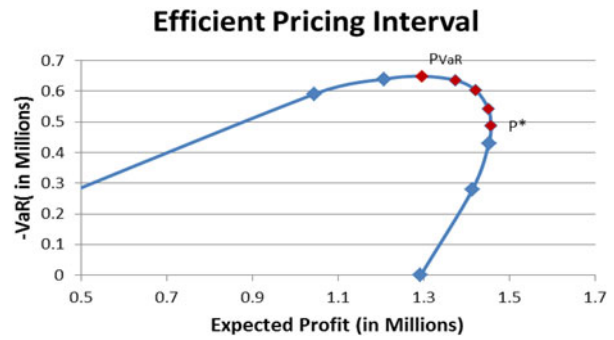

Figure 3. Efficient pricing interval.

given by VaR_α of the hospital. In this section, we use the concept of portfolio optimisation to make risk-adjusted pricing decisions. We have developed thereby two constrained optimisation models to analyse the tradeoff between these conflicting objectives.

5.1. Pricing for return maximising hospital with risk constraint

The hospital wants to maximise the expected returns for a bound on VaR, which can be formulated as the following constrained optimisation problem:

$$\mathrm{Max}_P \left\{ E[NP(P)] \right\}, \text{ subject to : } \mathrm{VaR}_\alpha \leq -\pi$$

$$\Longleftrightarrow (P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP)$$

$$\geq \pi \Big(\text{From Corollary 2(a)} \Big)$$

Theorem 5: *The optimal solution of the problem is $P^* = \frac{a + \left(\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) b}{2b}$, if it satisfies the constraint. Else, the optimal price is given as $P = \frac{(a + kb) \pm \sqrt{(a - kb)^2 - 4b\pi}}{2b}$, where $k = \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}$.*

Proof: The solution to the unconstrained problem is given by Theorem 1. If this solution satisfies the constraint, then it is the optimal solution for the above constrained optimisation as well. Else, we can solve the constrained optimisation problem for the optimal price. The Lagrangian for the constrained optimisation problem is

$$L = \left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP)$$
$$+ \lambda((P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) - \pi)$$

$$\frac{\partial L}{\partial P} = 0 \Rightarrow a - 2bP + \left(\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) b$$
$$+ \lambda \left[a - 2bP + \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)} b \right] = 0$$

$$\lambda((P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) - \pi)$$
$$= 0 \quad \text{(Complementary slackness condition)}$$

$$\left. \begin{array}{l} (P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) \geq \pi \\ \lambda \geq 0 \end{array} \right\}$$

(Feasibility Conditions)
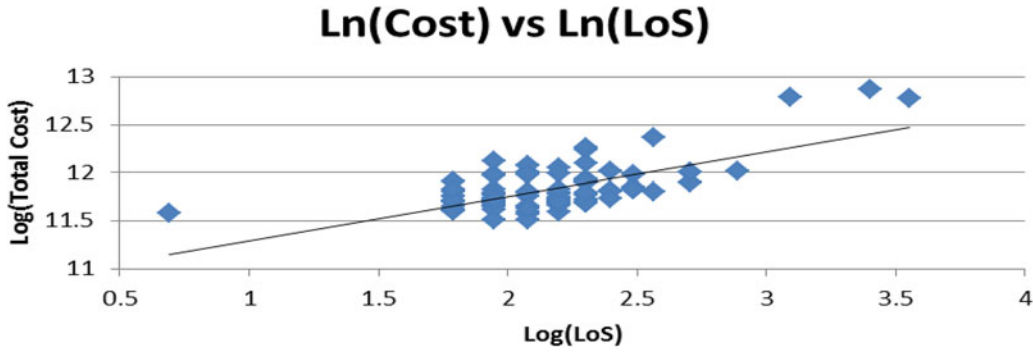
Ln(Cost) vs Ln(LoS)

Figure 4. Log cost vs. log length of stay.

If $\lambda = 0$, then the unconstrained optimal P is the optimal solution, if its feasibility condition (constraint) is satisfied. If $\lambda > 0$, then the constraint is tight, i.e., P is the solution of the following quadratic equation:

$$(P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) = \pi$$
$$\iff bP^2 - (a + b\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})P$$
$$+ a\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)} + \pi = 0$$

The roots of the above equation are given by:

$$P = \frac{(a + kb) \pm \sqrt{(a + kb)^2 - 4b(ak + \pi)}}{2b},$$
$$k = \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)},$$
$$\iff P = \frac{(a + kb) \pm \sqrt{(a - kb)^2 - 4b\pi}}{2b}, \text{ given } \sqrt{(a - kb)^2 - 4b\pi}$$
$$\geq 0 \iff \frac{(a - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})^2}{4b} \geq \pi \text{ (implying that the}$$
bound on VaR is always less than its maximum value). From these roots, choose the one which gives a higher expected profit as both of them give the same VaR.

5.2. Pricing for risk minimising hospital with target returns

A profit making hospital generally has some target to be achieved in terms of profit. They would like to set a minimum profit keeping the risk under control. Similar to the concept of the efficient portfolio, the objective is to minimise the risk (VaR_α) for a given threshold amount of expected profit. So, the problem faced by the hospital can be formulated as:

$$\underset{P}{\text{Min}} \{\text{VaR}(P)\}, \text{ subject to} : E[NP(P)] \geq R$$

Rewriting the problem as maximisation problem,

$$\iff \underset{P}{\text{Max}} \left\{ (P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP) \right\},$$
$$\text{subject to} : \left(P - Ae^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP) \geq R$$

Theorem 6: *The optimal solution of the constrained problem is* $P = \frac{a + \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)} b}{2b}$, *if it satisfies the*

constraint. *Else, the optimal price is given as* $\frac{(a + kb) \pm \sqrt{(a - kb)^2 - 4bR}}{2b}$, *where* $k = \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}}$.

Proof: Solution to the unconstrained problem is obtained from equation (10) as $\frac{a + b\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)}}{2b}$. If this solution satisfies the constraint, then it is the optimal solution for the above constrained optimisation too. Else, we can solve the unconstrained optimisation problem for the optimal price. The Lagrangian for the constrained optimisation problem is:

$$L = (P - \beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})(a - bP)$$
$$+ \lambda \left(\left(\left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right) \right)(a - bP) - R \right)$$
$$\frac{\partial L}{\partial P} = 0 \Rightarrow a - 2bP + (\beta_0 e^{\beta_1 \mu + \beta_1 \sigma \Phi^{-1}(1-\alpha)})b$$
$$+ \lambda \left(a - 2bP + \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} b \right) = 0$$
$$\lambda \left(\left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP) - R \right) = 0$$
(Complementary slackness condition)
$$\left. \begin{array}{c} \left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP) \geq R \\ \lambda \geq 0 \end{array} \right\}$$
(Feasibility Conditions)

If $\lambda = 0$, then the unconstrained optimal P is the optimal solution if its feasibility condition (constraint) is satisfied. If $\lambda > 0$, then the constraint is tight, i.e., P is the solution of the following quadratic equation:

$$\left(P - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)(a - bP) - R = 0$$
$$\iff bP^2 - \left(a + b\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)P + a\beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} + R = 0$$

The roots of the above equation are given by:

$$P = \frac{(a + kb) \pm \sqrt{(a + kb)^2 - 4b(ak + R)}}{2b},$$
$$k = \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}}$$
$$\iff P = \frac{(a + kb) \pm \sqrt{(a - kb)^2 - 4bR}}{2b}, \text{ given } \sqrt{(a - kb)^2 - 4b\pi}$$
$$\geq 0 \iff \frac{\left(a - \beta_0 e^{\beta_1 \mu + \frac{\beta_1^2 \sigma^2}{2}} \right)^2}{4b} \geq \pi \text{ (implying that the}$$

bound on expected profit is always less than its maximum value). From these roots, choose the one which gives lower VaR as both of them give the same expected profit.

6. Numerical illustration of the models

In this section, we have illustrated various models developed in the paper numerically.

6.1. Validating concave function for total cost

We validated the concavity of the cost function using a data set collected from a multi-speciality hospital in India. The data used was collected for cardiac surgery patients. Figure 4 shows the concave trend exhibited by total cost incurred by a patient/consumer as a function of the length of hospitalisation. It is evident that as the length of stay increases, the total cost increases, but at a decreasing rate. To further validate our claim, we ran the power model $\beta_0 X^{\beta_1}$ on the data set and provided the regression results with the dependent variable as log (total cost) and independent variable as log (length of stay in days). We have used the log transformations to linearize the regression model (Table 1). Figure 4 shows the regression model fit.

In the following sections, we demonstrate the insights from the theorems derived in this paper and expected profit, VaR, P^*, P_{VaR}, threshold profit are in monetary terms.

6.2. Effect of cost parameters on expected net profit

Figure 5 shows the graph of net profit for different value of β_1 for $a = 1000$, $b = 0.02$, $\beta_0 = 10000$, $\mu = 2.37$ and $\sigma = 0.2$. It is evident from Figure 5 that cost elasticity of LoS has a concave relationship with expected profit, i.e., profit decreases at an increasing rate. Figure 6 shows the graph of net profit at different value of β_0 for a $= 1000$, b $= 0.02$, $\beta_1 = 0.4$, $\mu = 2.37$ and $\sigma = 0.2$, from which we can conclude that factors other than LoS have a convex relationship with expected profit, i.e., profit decreases at a decreasing rate.

Comparison between log-normal and Erlang K for optimal profit is shown in Table 2. The following parameter values are used:

Mean LOS	10
Variance LOS	20
K (Erlang)	5
θ (Erlang)	2
μ (log normal)	2.21
σ^2 (log normal)	0.183

Table 1. Regression results for equation $\log Y = \log \beta_0 + \beta_1 \log x$.

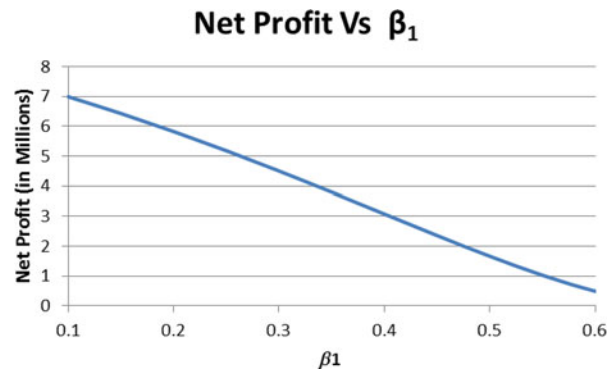|	Estimate	Std. error	t Value	p value	Multiple R^2	Adjusted R^2
(Intercept)	10.829	0.109	99.56	0.000	46.40	45.86
Log (LOS)	0.4633	0.0500	9.26	0.000		

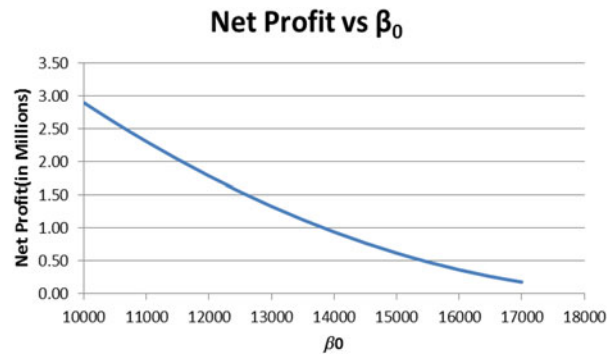**Figure 5.** Expected net profit vs. β_1.

Figure 6. Expected net profit vs. β_0.

6.3. Effect of LOS parameters on expected net profit

Figure 7 shows the graph of net profit at a different value of μ and β_1 for $a = 1000$, $b = 0.02$, $\beta_0 = 10,000$ and $\sigma = 0.2$, from which we can conclude that Expected Profit decreases at an increasing rate with respect to the mean, but the rate increases with cost elasticity of LoS. Figure 8 shows the graph of Net Profit at different value of σ and β_1 for $a = 1000$, $b = 0.02$, $\beta_0 = 10,000$ and $\mu = 1.5$, from which we can conclude that Expected Profit decreases at increasing rate w.r.t standard deviation, but the rate increases with cost elasticity of LoS. (Similar graphs with changing β_0 have no effect on the slope, so they have been omitted.)
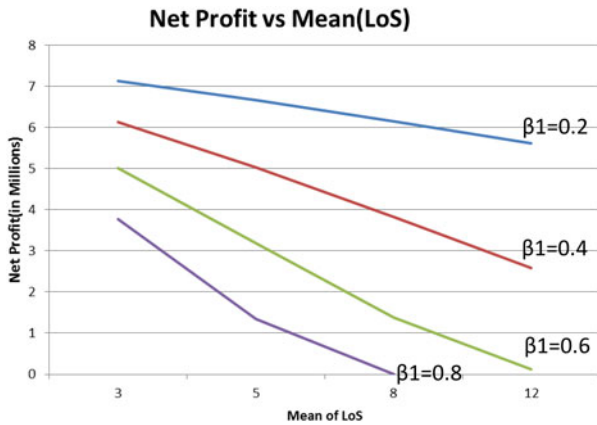
We thereby conclude that cost elasticity is the most important variable as it accentuates the effect of mean and variance of LoS, thus the hospital should exert effort in order to keep a small value of β_1 in order to achieve greater profits.

6.4. Effect of price on the probability of exceeding the threshold profit

Figure 9 demonstrates the results from Theorem 2 and its corollaries. It shows that the probability of

Table 2. Comparison between log-normal and Erlang optimal price.

a (max demand)	b (price sensitivity)	β_0	β_1	Expected cost (Erlang)	P* (for Erlang)	Expected cost (log normal)	P* (log normal)
15,000	0.2	10,000	0.1	12504.02373	43752.01186	12484.65254	43742.33
15,000	0.2	10,000	0.2	15588.79393	45294.39696	15615.20459	45307.6
15,000	0.2	10,000	0.5	30841.64077	52920.82039	30890.85215	52945.43
15,000	0.2	10,000	0.6	38840.23702	56920.11851	38920.65525	56960.33

Figure 7. Expected net profit vs. μ.

Figure 8. Expected net profit vs. σ.

exceeding threshold for $a = 1000$, $b = 0.02$, $\beta_0 = 10,000$, $\beta_1 = 0.2$, $\mu = 2.37$ and $\sigma = 0.2$. We observe that the probability of exceeding a threshold has a concave relationship with Price and attains maxima which gives the hospital the upper bound on price if they have a fixed target. Also, this probability varies with the threshold assumed. Here, we have taken three different thresholds (expected profit, a value higher than the expected profit and a value lower than the expected profit).
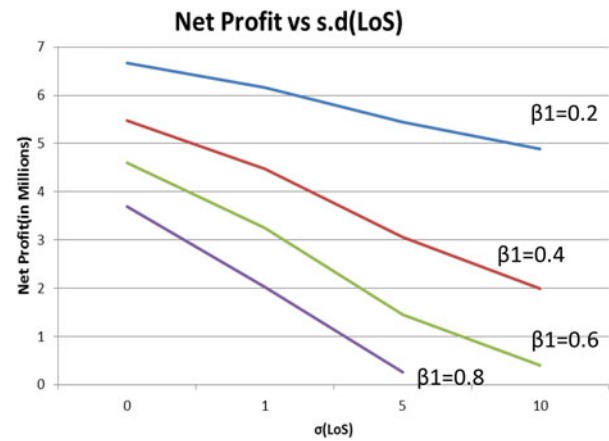
The above graph also demonstrates the inverse relationship between the threshold and the probability. As the threshold decreases, the probability increases, which suggests that if the hospital desires a higher confidence level, then it should set its target threshold profit low.

6.5. Lower bound on price with fixed confidence level

Figure 10 plots threshold profit at different value of price for $a = 1000$, $b = 0.02$, $\beta_0 = 10,000$, $\beta_1 = 0.2$, $\mu = 2.37$, $\sigma = 0.2$ and confidence level $= 95\%$. It shows that for a fixed confidence level, there is a lower bound on a price beyond which the hospital makes positive profits. Also, in this case, the hospital would not accept a price less than Rupees 17,165. It also achieves maxima at VaR minimising price as both of them are equivalent.

6.6. Comparison of return maximising and risk minimising prices

Table 3 shows that the risk minimising price is greater than the return maximising price. For the

values in Table 3, we plot the expected profit and (−VaR) for different values of P given in Table 4 and is shown in Figure 11.

In Figure 11, we observe that the VaR plot always lies beneath the expected profit plot. Also, the optimal price lies between return maximising price and risk minimising price. Outside this price range, we observe from the table and its plot that the hospital would be making a lesser profit with higher risk. The tradeoff lies between these two values. From Table 5, we can note that in the efficient price interval {41,465 44,303}, the hospital can go for a higher profit with higher risk or lower profit with lower risk. Figure 3 plots the value in Table 5 and graphically shows the tradeoff that occurs between the profit maximising and risk minimising prices for different prices.

So, we suggest that the hospital with demand and cost parameters as given in Table 3 should always price the treatment between Rupees 41,465 and 44,303. The exact price depends on the risk averseness.

6.7. Efficient pricing interval with constraints

We know that the constraints/bounds on − VaR and expected profit are below their respective maxima. We can observe from Figure 12 that the constraints either shrink the efficient pricing interval or do not affect it at all. In either case, the optimal price remains within the boundaries of risk minimising and profit maximising price. So, we conclude that the price quoted should be between these bounds depending on the desirability of the hospital for higher profit, higher risk or lower profit, lower risk.
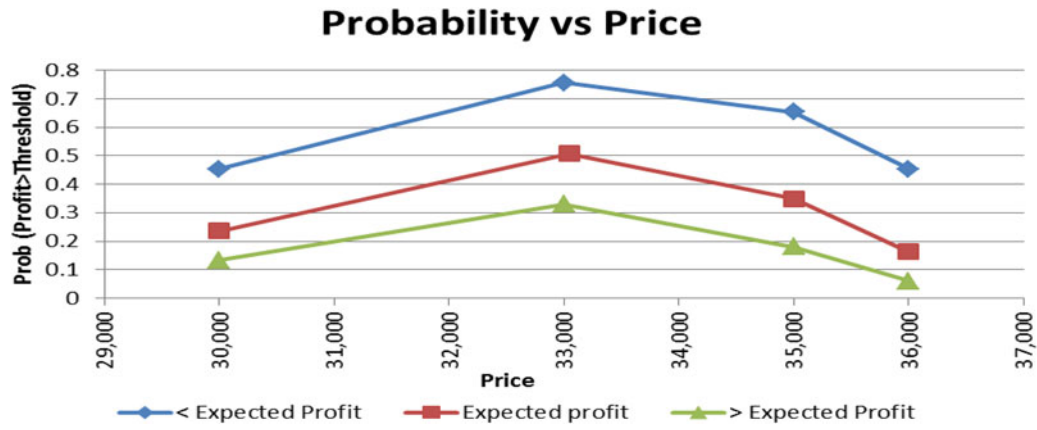
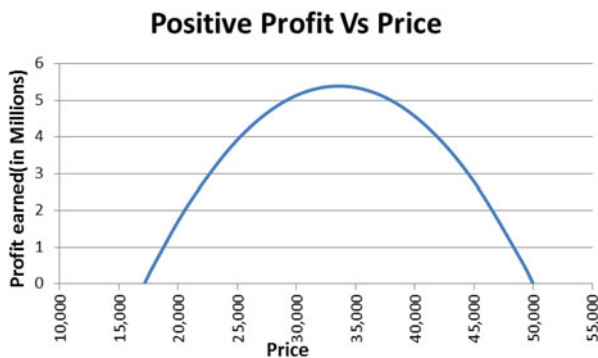**Figure 9.** Probability of exceeding threshold vs. price.
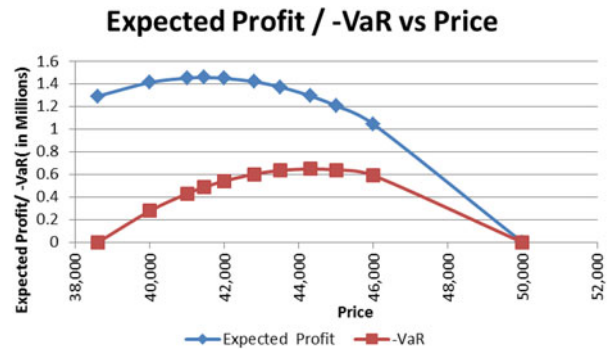

Figure 10. Threshold profit vs. price.

Figure 11. Expected profit and − VaR vs. price.

Table 3. Values used for calculations in Table 4.

a	B	μ	σ	β_0	β_1
1000	0.2	2.37	0.2	10,000	0.5

Table 4. Expected profit and value-at-risk at price P.

P	Expected net profit	−VaR
Lower bound		
38,607	1,293,190	0
40,000	1,413,674	278,600
41,000	1,452,307	430,740
Profit maximizing $P*$		
41,465	1,456,646	487,953
42,000	1,450,939	542,880
42,800	1,421,045	603,792
43,500	1,373,888	636,090
Risk minimizing P_{VaR}		
44,303	1,295,597	649,002
45,000	1,206,837	639,300
46,000	1,045,470	591,440
Max. Price		
50,000	0	0

Table 5. Tradeoff between the bounds.

Price	Net profit	−VaR
41,000	1,452,307	430,740
41,465	1,456,646	487,953
42,000	1,450,939	542,880
42,800	1,421,045	603,792
44,303	1,295,597	649,002
45,000	1,206,837	639,300

with a given price and compared it for different thresholds. We showed that a hospital could vary its target profit around the maximum expected profit for different value risks. We also suggested an upper bound on price which will minimise the risk and a lower bound on price which gives positive profits with a fixed probability. After studying profit and VaR separately, we have developed two models which take both into account and help in making risk-adjusted pricing decisions. We found out that the optimal price would always lie between the risk minimising and profit maximising prices.

The primary contribution of this paper is that we discuss the tradeoff of risk and returns in a healthcare setting. The results presented in the paper provide different bounds on price, which is a useful guide for hospital managers when faced with the problem of the price for treatment under cost uncertainty. The models developed in the paper will help healthcare service providers in price negotiations with insurance companies, organisations' health benefits for their employees and government.

7. Conclusion and implications

In this paper, we analysed an emerging pricing practice across healthcare service providers. We discussed flat rate pricing using models which incorporate the uncertainty arising from severity and length of stay. We find the optimal price for a risk-neutral hospital maximising its returns/expected profit and a risk-averse hospital minimising its VaR with, i.e., VaR_α. We analysed the risk defined as the probability of not obtaining target profit associated
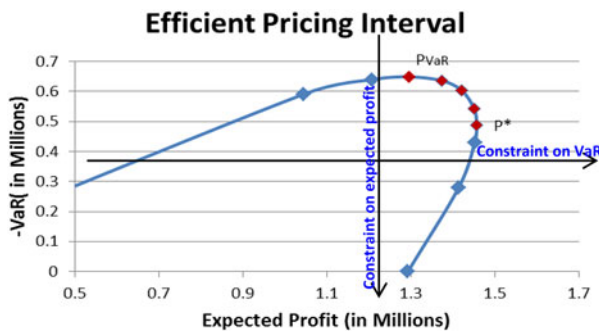
Figure 12. Efficient pricing interval with constraints.

7.1. Importance of work and implications

Several countries are now encouraging price transparency by reporting healthcare prices to the public. In the US, more than 30 states are now reporting prices of different hospitals to the consumers (Sinaiko & Rosenthal, 2011). One of the earliest examples of the fixed price model being reported in the US has been in the state of New Hampshire, which launched the *HealthCost* website to report a fixed pricing structure of about thirty common healthcare services (Tu & Lauer, 2009). From the governmental perspective too, the US government is taking steps to provide complete price information to the patients. For example, the *Centre for Medicare and Medicaid Services (CMS)* in the US has made available the hospital-specific charges for more than 3,000 US hospitals through their website (CMS.gov, 2019). They pay the hospitals flat fees per case; these Medicare hospital payments contribute to about 31% of the hospitals' net revenues (Reinhardt, 2006). In the UK, the NHS Foundation Trusts and private healthcare providers such as BMI Healthcare, TRUSTPLUS, Nova Healthcare and Spire Healthcare provide fixed price packages for treatments.

The fixed price model is also helping the government to keep healthcare costs under control, but on the flip side, from the healthcare service provider's perspective, it might not be the ideal pricing strategy. Despite this, there are many hospitals which are looking to offer fixed prices for common medical treatments due to increased competition (Richman et al., 2008). Taking the example of India, with the increase in competition among the private healthcare providers, the hospitals are advertising treatment procedures and are stating the price upfront to the consumers. This increasing transparency in pricing has been beneficial to the consumers as it keeps the prices in control with an assurance of high service quality. Government policies are also forcing the healthcare service providers to quote a flat rate for treatments. Several state governments in India sign flat-rate contracts (or policies like *Rashtriya Swasthya Bima Yojna*) with hospitals for providing healthcare services to the underprivileged section of the society.

Adoption of flat-rate pricing is not just due to the growing pressure by policymakers for price

transparency and increased competition; it also benefits the healthcare service providers, especially in view of "healthcare tourism," wherein the fixed pricing model becomes an integral component to draw consumers/customers not only from one's state but also neighbouring states. Package prices are quite common in the international healthcare market place (Herrick, 2007). According to the report "Medical Value Travel in India" published by KPMG and FICCI, the healthcare tourism market has been estimated to grow at an annual rate of 15% to reach about USD 158.2 billion by 2017. The boom in healthcare tourism has led to the emergence of companies that act as providers of medical tourism. These companies link patients to hospitals in different countries and provide fixed healthcare packages which include the cost of surgery, hospitalisation as well as travel logistics, visa, and transportation and charge a fixed price (Hall, 2013). The leading medical tourism companies include *Med Retreat* – a US-based medical tourism agency, *Planet Hospital* – a medical tourism portal in US, *Indiaheals*, and *Advatech*; the latter two entities are both India-based companies. Healthcare tourism is also popular in many South Asian countries such as Thailand, Bangkok, Malaysia and Singapore. A core element of healthcare tourism is that hospitals provide transparent pricing for a wide range of common surgical procedures with their fixed price packages.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Navonil Mustafee 🆔 http://orcid.org/0000-0002-2204-8924

References

Atienza, N. (2005). *Fitting the 'variable length of hospital stay' with mixtures from different distribution families.* Spain: University of Seville.

Atienza, N., García-Heras, J., Muñoz-Pichardo, J. M., & Villa, R. (2008). An application of mixture distributions in modelization of length of hospital stay. *Statistics in Medicine*, 27(9), 1403–1420. doi:10.1002/sim.3029

Ayer, T., Ayvaci, M. U. S., Karaca, Z., & Vlachy, J. (2019). The impact of health information exchanges on emergency department length of stay. *Production and Operations Management*, 28(3), 740–758. doi:10.1111/poms.12953

CMS.gov (2019). Medicare provider utilization and payment data. Retrieved from https://www.cms.gov/Research-Statistics-Data-and-Systems/Statistics-Trends-and-Reports/Medicare-Provider-Charge-Data/index.html

Demir, E., Lebcir, R., & Adeyemi, S. (2014). Modelling length of stay and patient flows: methodological case studies from the UK neonatal care services. *Journal of the*

Operational Research Society, 65(4), 532–545. doi:10.1057/jors.2013.51

Dittman, D. A., & Morey, R. C. (1981). Optimal pricing of hospital services. *Inquiry*, 18(4), 311–321.

Faddy, M., Graves, N., & Pettitt, A. (2009). Modeling length of stay in hospital and other right skewed data: comparison of phase-type, gamma and log-normal distributions. *Value in Health*, 12(2), 309–314. doi:10.1111/j.1524-4733.2008.00421.x

Galetsi, P., & Katsaliaki, K. (2019). A review of the literature on big data analytics in healthcare. *Journal of the Operational Research Society*. doi:10.1080/01605682.2019.1630328

Garg, C. C., & Karan, A. K. (2009). Reducing out-of-pocket expenditures to reduce poverty: a disaggregated analysis at rural-urban and state level in India. *Health Policy and Planning*, 24(2), 116–128. doi:10.1093/heapol/czn046

Gillespie, J., McClean, S., Garg, L., Barton, M., Scotney, B., & Fullerton, K. (2016). A multi-phase DES modelling framework for patient-centred care. *Journal of the Operational Research Society*, 67(10), 1239–1249. doi:10.1057/jors.2015.114

Gutacker, N., Siciliani, L., & Cookson, R. (2016). Waiting time prioritisation: Evidence from England. *Social Science & Medicine*, 159, 140–151. doi:10.1016/j.socscimed.2016.05.007

Hall, C. M (Ed.). (2013). *Medical tourism: The ethics, regulation, and marketing of health mobility* (Vol. 33). Oxon, UK: Routledge.

Hellervik, A., & Rodgers, G. J. (2007). A power law distribution in patients' lengths of stay in hospital. *Physica A: Statistical Mechanics and Its Applications*, 379(1), 235–240. doi:10.1016/j.physa.2006.12.045

Herrick, D. M. (2007). *Medical tourism: Global competition in health care* (NCPA Policy Report No. 304). Retrieved from http://www.heal-wheel-india.com/white-pappers/Medical-Tourism-Global-Competition-in-Health-Care-NCPA-Report.pdf

Herweg, F., & Mierendorff, K. (2013). Uncertain demand, consumer loss aversion, and flat-rate tariffs. *Journal of the European Economic Association*, 11(2), 399–432. doi:10.1111/jeea.12004

Jorion, P. (2007). *Value at risk: The new benchmark for managing financial risk* (Vol. 2). New York, NY: McGraw-Hill.

Kumar, A., Costa, A. M., Fackrell, M., & Taylor, P. G. (2018). A sequential stochastic mixed integer programming model for tactical master surgery scheduling. *European Journal of Operational Research*, 270(2), 734–746. doi:10.1016/j.ejor.2018.04.007

Lim, A., & Tongkumchum, P. (2009). Methods for analyzing hospital length of stay with application to inpatients dying in southern Thailand. *Global Journal of Health Science*, 1(1), 27–38. doi:10.5539/gjhs.v1n1p27

Marazzi, A., Paccaud, F., Ruffieux, C., & Beguin, C. (1998). Fitting the distributions of length of stay by parametric models. *Medical Care*, 36(6), 915–927.

Margolis, S. E. (1983). The pricing of physicians' services. *The Journal of Human Resources*, 18(4), 592–608. doi:10.2307/145446

Marshall, A. H., McClean, S. I., Shapcott, C. M., & Millard, P. H. (2002). Modelling patient duration of stay to facilitate resource management of geriatric hospitals. *Health Care Management Science*, 5(4), 313–319. doi:10.1023/A:1020394525938

Marshall, A. H., & McClean, S. I. (2003). Conditional phase-type distributions for modelling patient length of stay in hospital. *International Transactions in Operational Research*, 10(6), 565–576. doi:10.1111/1475-3995.00428

Marshall, A. H., & McClean, S. I. (2004). Using Coxian phase-type distributions to identify patient characteristics for duration of stay in hospital. *Health Care Management Science*, 7(4), 285–289. doi:10.1007/s10729-004-7537-z

Mitchell, B. M. (1978). Optimal pricing of local telephone service. *The American Economic Review*, 68(4), 517–537. https://www.jstor.org/stable/1808923

Monitor. (2014). Written evidence submitted by Monitor (EXP0053) to the UK Parliament – 24/10/2014. http://data.parliament.uk/writtenevidence/committeeevidence.svc/evidencedocument/health-and-social-care-committee/public-expenditure-on-health-and-social-care/written/14757.html.

Newhouse, J. P. (1970). A model of physician pricing. *Southern Economic Journal*, 37(2), 174–183. doi:10.2307/1056127

Odlyzko, A. (1999). *Paris metro pricing for the internet*. Paper presented at the 1st ACM conference on Electronic commerce, ACM, Denver, Colorado. New York, NY, pp. 140–147.

Polverejan, E., Gardiner, J. C., Bradley, C. J., Holmes Rovner, M., & Rovner, D. (2003). Estimating mean hospital cost as a function of length of stay and patient characteristics. *Health Economics*, 12(11), 935–947. doi:10.1002/hec.774

Reinhardt, U. E. (2006). The pricing of US hospital services: Chaos behind a veil of secrecy. *Health Affairs*, 25(1), 57–69. doi:10.1377/hlthaff.25.1.57

Richman, B. D., Udayakumar, K., Mitchell, W., & Schulman, K. A. (2008). Lessons from India in organizational innovation: A tale of two heart hospitals. *Health Affairs*, 27(5), 1260–1270. doi:10.1377/hlthaff.27.5.1260

Ringer, J. S., & Hosek, S. D. (2000). *The elasticity of demand for health care: A review of the literature and its application to the military health system*. Santa Monica, CA: RAND Corporation.

Sinaiko, A. D., & Rosenthal, M. B. (2011). Increased price transparency in health care-challenges and potential effects. *New England Journal of Medicine*, 364(10), 891–894. doi:10.1056/NEJMp1100041

Sonnessa, M., Tànfani, E., & Testi, A. (2017). An agent-based simulation model to evaluate alternative co-payment scenarios for contributing to health systems financing. *Journal of the Operational Research Society*, 68(5), 591–604. doi:10.1057/s41274-016-0022-5

Sundararajan, A. (2004). Nonlinear pricing of information goods. *Management Science*, 50(12), 1660–1673. doi:10.1287/mnsc.1040.0291

Train, K. E. (1991). *Optimal regulation: The economic theory of natural monopoly*. Cambridge, MA: The MIT Press.

Tu, H. T., & Lauer, J. R. (2009). Impact of health care price transparency on price variation: The New Hampshire experience. *Issue Brief – Centre for Studying Health System Change*, 128, 1–4.

Vasilakis, C., & Marshall, A. H. (2005). Modelling nationwide hospital length of stay: Opening the black box. *Journal of the Operational Research Society*, 56(7), 862–869. doi:10.1057/palgrave.jors.2601872

Ward, B. W., Clarke, T. C., Nugent, C. N., & Schiller, J. S. (2016). *Early release of selected estimates based on data from the 2015 National Health Interview Survey*. National Center for Health Statistics. Retrieved from https://www.cdc.gov/nchs/nhis/releases/released201605.htm