Carbon Efficiency Evaluation: An Analytical Framework Using Fuzzy DEA

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Abstract

Data Envelopment Analysis (DEA) is a powerful analytical technique for measuring the relative efficiency of alternatives based on their inputs and outputs. The alternatives can be in the form of countries who attempt to enhance their productivity and environmental efficiencies concurrently. However, when desirable outputs such as productivity increases, undesirable outputs increase as well (e.g. carbon emissions), thus making the performance evaluation questionable. In addition, traditional environmental efficiency has been typically measured by crisp input and output (desirable and undesirable). However, the input and output data, such as CO_2 emissions, in real-world evaluation problems are often imprecise or ambiguous. This paper proposes a DEA-based framework where the input and output data are characterized by symmetrical and asymmetrical fuzzy numbers. The proposed method allows the environmental evaluation to be assessed at different levels of certainty. The validity of the proposed model has been tested and its usefulness is illustrated using two numerical examples. An application of energy efficiency among 23 European Union (EU) member countries is further presented to show the applicability and efficacy of the proposed approach under asymmetric fuzzy numbers.

Keywords: Energy Efficiency; Data envelopment analysis; Fuzzy expected interval; Fuzzy expected value; Fuzzy ranking approach

1. Introduction

Research on sustainability and environmental efficiency aims to change consumption habits and economic structure at the global level. This necessitates the ability to discern the impact of energy consumption of various economic activities. Given that driving such activities have been greatly dependent on fossil fuels, such resources have a limited supply and would ultimately increase the cost along the supply chain and global trade. To limit such effects in the foreseeable future, researchers use the proxy of energy consumption in the form of CO_2 emissions in dictating sustainability performance. Aside from the European Union or North America, there is also a brewing interest on sustainability development in towards developing countries such as China, India, Taiwan, Middle East and North Africa (Zhou, Chung, & Zhang, 2013; Ramanathan, 2005). This is understandable as downstream activities have been outsourced to more cost competitive countries and markets have been more globalized.

There is a misconception that curbing carbon emissions will result in the productivity reduction of a nation. If this is so, curbing emissions will not benefit developing countries such as India and China who require continuous growth sustained by higher productivity consumption. This seems unfair as the growth enjoyed by developed nations today was an indirect result of the lax carbon emissions control in the past. Hence, the main contention is lesser in curbing emissions but more towards whether those emissions are warranted in terms of efficiency, and whether one can innovate and possess technological progress without having to generate higher levels of carbon emissions. In short, this form of environmental management should be seen more as a stimulus of innovation and not merely a regulatory compliance (Zhu and Sarkis, 2006).

There is also a concern on whether there is a limit on the effects of reducing energy consumption through improving energy efficiency. This argument stems from the notion that being energy efficient would contribute to economic growth that in turn would raise the demand for energy. Thus, it is believed that energy savings by being efficient is only a partial outlook on reducing energy consumption (see Madlener and Alcott, 2009; Recalde and Martin, 2012). Hence, there should be a framework that can holistically account for energy efficiency without impeding on productivity.

This is made more difficult when there is no single measure that could capture sustainable development in its entirety, and their various indicators may address a number of different interpretations on sustainability (Hanley, Moffat, Faichney, & Wilson, 1999). Researchers have often focused on one of the following efficiency measures: environmental, energy or economic perspectives. Nonetheless, the fundamental aspect of efficiency evaluation is to strive for higher outputs, given the same level of inputs (Sarkis and Weinrach, 2001). As such, Data Envelopment Analysis (DEA), first proposed by Charnes, Cooper, & Rhodes (1978), provides a readily available framework for evaluating a set of decision making units (*DMU*s) based on multiple input and output measures. DEA's rapid growth in the past three decades has been excellently documented in Cook and Seiford (2009). A full bibliography on applications of DEA is also reported by Emrouznejad, Praker, & Tavares (2008).

However, it is not so straight forward as outputs in environmental efficiency models make up both desirable and undesirable outputs. For instance, higher GDP (Gross domestic product) index tend to come with higher CO_2 emissions. This means that desirable outputs have to be sacrificed so that inputs can be reallocated for minimization of undesirable outputs (Hernandez-Sancho, Picazo-Tadeo, & Reig-Martinez, 2000).

Despite the challenges related to modeling undesirable and desirable outputs, there is still a large number of DEA applications in environmental performance, especially at the national level (see Zhou, Ang, & Poh, 2008). Färe, Grosskopf, Lovell, & Pasurka (1989) first proposed an environmental assessment model based on a nonparametric DEA framework, which considered both desirable and undesirable outputs together. Since then, many researchers began to provide a variation of one of the following carbon measures: CO_2 emission intensity, CO_2 emissions per capita, carbonization index, energy intensity, and including those in the form of linguistic preferences (see Ang, 1999; Fan et al., 2007; Mielnik and Goldemberg, 1999; Sun, 2005; Tseng, 2011, 2013). Zhang et al. (2008) provided resource and environmental efficiency analysis using the DEA model for 30 provinces in China. They modeled undesirable outputs as inputs in the constant returns to scale technology. Recently, Feng, Chu, Ding, & Liang (2015) considered compensation schemes for carbon allocation with an empirical investigation on OECD countries. In the eco-efficient assessment, Rashidi, Shabani & Saen (2015) modelled energy inputs and extended the slacks-based measure (SBM) and range adjusted measure (RAM) to include non-discretionary and undesirable factors. Mahdiloo et al. (2014) integrated the technical efficiency, ecological efficiency and their newly formed process environmental quality efficiency into a single overall efficiency score through the aid of game theory.

Since traditional DEA models do not account for subjective input and output values, another class of DEA models emerged; that is, fuzzy DEA models. Existing fuzzy DEA models exhibit some shortcomings. We briefly review three of these shortcomings that are inferred from the fuzzy DEA literature.

Most of the proposed methods in existing fuzzy DEA models only cater to crisp efficiency measures (see Saati, Memariani, & Jahanshahloo, 2002 and Lertworasirikul, Fang, Joines, & Nuttle, 2003). In other words, the proposed methods in the literature may not be able to calculate crisp and fuzzy efficiency measures together. Although crisp efficiency measures can provide ease of ranking, fuzzy observations are more informative and realistic in real-world modelling and decision making. It avoids results that are over optimistic and pessimistic.

The second drawback of existing fuzzy DEA models is requires significantly more computational procedure, i.e. the Guo and Tanaka's fuzzy ranking approach (Guo and Tanaka, 2001) needs two linear programming problems to obtain the efficiency value of a given *DMU*, in which the optimal value of the objective function of the primary linear programming problem is used in the secondary linear programing problem. In the possibility approach proposed by Lertworasirikul et al. (2003), all fuzzy constraints are defined with different possibility level. In the case of five levels of possibility, there are 5^{n+2} linear programming problems to be solved. This also means that the model would suffer from complicated computational procedure.

The third drawback in existing fuzzy DEA models is its ability to only cater to either triangular fuzzy numbers (see León, Liern, Ruiz, & Sirvent, 2003) or symmetrical triangular fuzzy numbers (see Guo and Tanaka, 2001).

In this study, we propose a DEA-based framework for evaluating the carbon efficiency in which the input-output data are described by the symmetrical and asymmetrical fuzzy numbers. The proposed model avoids the unnecessary step of converting a set of undesirable outputs into inputs, or the need to form a new variable to capture the undesirable outputs, which are common in environmental models. The proposed model also accounts for inputs and outputs which are imprecise in nature. This has yet to be investigated in the context of carbon efficiency models. This study circumvents the existing drawbacks in the fuzzy DEA literature, while having the ability to provide crisp and fuzzy efficiency measures across different α-levels by only solving only one linear programming problem.

The remainder of the paper is organized as follows: Section 2 provides a brief background on the methodology of DEA and Fuzzy DEA models. Section 3 covers the development of the proposed model. Section 4 illustrates the method with two established numerical examples, which includes model comparisons. Section 5 applies the proposed model to an energy dependency model among 23 EU member countries, and the results statistically validated. Section 6 concludes the study and provides route for future research.

2. Background

In this section, we first recall some basic definitions on fuzzy sets theory (Zimmermann, 1992) and introduce the main concepts needed for the rest of the paper. We then provide a short overview of the conventional DEA models.

2.1. Fuzzy sets

Definition 1. Let X be a classical set of objects, called the universe, whose elements are denoted generically by x. A fuzzy set $\tilde{a}$ in X is a set of ordered pairs:

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) | x \in X\},$$

where $\mu_{\tilde{a}}(x)$ is membership function of x in $\tilde{a}$ that $\mu_{\tilde{a}}: X \to [0,1]$.

Definition 2. The α-level (or α-cut) set of a fuzzy set $\tilde{a}$ is a crisp subset of X and is denoted by:

$$\bar{a}_{(\alpha)} = \{x \in X | \mu_{\tilde{a}}(x) \geq \alpha\}.$$

Definition 3. A fuzzy set $\tilde{a}$ of set X is convex if

$$\mu_{\tilde{a}}(\lambda x_1+(1-\lambda)x_2)\geq min\{\mu_{\tilde{a}}(x_1),\mu_{\tilde{a}}(x_2)\},\ x_1,x_2\in X,\ \lambda\in[0,1].$$

Definition 4. A fuzzy set $\tilde{a}$ is normal if and only if $\sup_x \mu_{\tilde{a}}(x)=1$, that is, the supremum of $\mu_{\tilde{a}}(x)$ over X is unity.

Definition 5. A fuzzy number $\tilde{a}$ is a normal and convex fuzzy set $\tilde{a}$ of the real line $\mathbb{R}$.

Definition 6. A fuzzy number $\tilde{a}=\left(a^l,a^{m_1},a^{m_2},a^u\right)$ is a trapezoidal fuzzy number if

$$\mu_{\tilde{a}}(x)=\begin{cases} \dfrac{x-a^l}{a^{m_1}-a^l}, & a^l<x<a^{m_1}, \\ 1, & a^{m_1}\leq x\leq a^{m_2}, \\ \dfrac{a^u-x}{a^u-a^{m_2}}, & a^{m_2}<x<a^u, \\ 0, & otherwise. \end{cases}$$

Definition 7. The α-level set of a trapezoidal fuzzy number $\tilde{a}=\left(a^l,a^{m_1},a^{m_2},a^u\right)$ can be denoted as an interval, $\left[f^l(\alpha),f^u(\alpha)\right]$, in which $f^l(\alpha)=a^l+\alpha(a^{m_1}-a^l)$ and $f^u(\alpha)=a^u-\alpha(a^u-a^{m_2})$ where $\alpha\in[0,1]$.

Remark 1. Let F(P) denote the set of all trapezoidal fuzzy numbers.

Remark 2. We denote a^l and a^u as the lower and upper bound; respectively, of the fuzzy number $\tilde{a}=(a^l,a^{m_1},a^{m_2},a^u)$. When $a^{m_1}-a^l=a^u-a^{m_2}$ for the trapezoidal fuzzy number $\tilde{a}=(a^l,a^{m_1},a^{m_2},a^u)$, we obtain a symmetrical trapezoidal fuzzy number. If we assume that $a^m=a^{m_1}=a^{m_2}$ in the fuzzy number $\tilde{a}$, we have a triangular fuzzy number as $\tilde{a}=(a^l,a^m,a^u)$.

Definition 8 (Heilpern, 1992)**.** The expected interval (EI) and the expected value (EV) of a fuzzy number $\tilde{a}$ are defined as follows:

$$EI(\tilde{a})=\left[E_1^a,E_2^a\right];\quad EV\left(\tilde{a}\right)=\frac{E_1^a+E_2^a}{2},$$

where $E_1^a=\int_0^1 f^l\left(\alpha\right)d\alpha$ and $E_2^a=\int_0^1 f^u\left(\alpha\right)d\alpha$.

According to definitions 6 and 7, if we assume that $\tilde{a}=(a^l,a^{m_1},a^{m_2},a^u)$ is a trapezoidal fuzzy number then

$$EI\left(\tilde{a}\right)=\left[\frac{a^l+a^{m_1}}{2},\frac{a^{m_2}+a^u}{2}\right];\quad EV\left(\tilde{a}\right)=\frac{a^l+a^{m_1}+a^{m_2}+a^u}{4}.$$

If we further assume that $\tilde{a}=\left(a^l,a^m,a^u\right)$ is a triangular fuzzy number then

$$EI(\tilde{a})=\left[\frac{a^l+a^m}{2},\frac{a^m+a^u}{2}\right]; \quad EV(\tilde{a})=\frac{a^l+2a^m+a^u}{4}.$$

Proposition 1 (Jiménez, Arenas, Bilbao, & Rodríguez, 2007)**.** Let $\tilde{a}$ and $\tilde{b}$ be fuzzy numbers. Then for any non-negative numbers ζ and η, we have

$$EI(\zeta\tilde{a}+\eta\tilde{b})=\zeta EI(\tilde{a})+\eta EI(\tilde{b}), \; EV(\zeta\tilde{a}+\eta\tilde{b})=\zeta EV(\tilde{a})+\eta EV(\tilde{b}).$$

Definition 9 (Jiménez, 1996)**.** Given two fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in the relation $\tilde{a}\geq\tilde{b}$ is defined as follow:

$$\mu_M(\tilde{a},\tilde{b})=\begin{cases} 0 & if \quad E_2^a-E_1^b<0, \\ \dfrac{E_2^a-E_1^b}{E_2^a-E_1^b-(E_1^a-E_2^b)} & if \quad 0\in\left[E_1^a-E_2^b,E_2^a-E_1^b\right], \\ 1 & if \quad E_1^a-E_2^b>0. \end{cases}$$

According to definition 8, $\left[E_1^a,E_2^a\right]$ and $\left[E_1^b,E_2^b\right]$ are the expected intervals of $\tilde{a}$ and $\tilde{b}$. The notation $\mu_M(\tilde{a},\tilde{b})=0.5$ indicates indifference between $\tilde{a}$ and $\tilde{b}$. Moreover, we may have the ordering relation $\mu_M(\tilde{a},\tilde{b})\geq\alpha$ which signifies that $\tilde{a}$ is bigger than, or equal to $\tilde{b}$ at least by a degree of α such that $\tilde{a}\geq_\alpha\tilde{b}$.

2.2. Classical DEA models

Consider the relative efficiency of n *DMU*s which use m inputs (x_{ij}, $i=1,\ldots,m$, $j=1,\ldots,n$) to produce s outputs (y_{rj}, $r=1,\ldots,s$, $j=1,\ldots,n$). The well-known CCR model for measuring the relative efficiency scores of *DMU*s is formulated as the following linear program (LP) problem (Charnes et al., 1978):

$$\begin{aligned} \max \quad & \theta_o=\sum_{r=1}^{s}u_r y_{ro}, \\ \text{s.t.} \quad & \sum_{i=1}^{m}v_i x_{io}=1, \\ & \sum_{i=1}^{m}v_i x_{ij}\geq\sum_{r=1}^{s}u_r y_{rj}, \quad j=1,\ldots,n, \\ & u_r\geq0, \quad r=1,\ldots,s, \\ & v_i\geq0, \quad i=1,\ldots,m, \end{aligned} \tag{1}$$

where $u_r(r=1,\ldots,s)$ and $v_i(i=1,\ldots,m)$ are the output and input weights assigned to the r^{th} output and the i^{th} input; respectively, and DMU_o refers to the *DMU* under evaluation.

Definition 10. DMU_o is CCR-efficient if $\theta_o^* = 1$ and there exists at least one optimal input and output weight (e.g., u^* & v^*), with $u^* > 0$ & $v^* > 0$. Otherwise, DMU_o is CCR-inefficient.

3. Proposed Fuzzy DEA model

When fuzzy input and fuzzy output data are used instead of crisp data, the m-input-s-output data are expressed as $\tilde{x}_{ij} \in F(\mathbb{R})$, $i = 1,...,m$, $j = 1,...,n$; $\tilde{y}_{rj} \in F(\mathbb{R})$, $r = 1,...,s$, $j = 1,...,n$, and the CCR model (1) can be naturally extended to the following fuzzy DEA model.

$$
\begin{aligned}
\max \quad & \theta_o = \sum_{r=1}^{s} u_r \tilde{y}_{ro}, \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \tilde{x}_{io} = 1, \\
& \sum_{i=1}^{m} v_i \tilde{x}_{ij} \geq \sum_{r=1}^{s} u_r \tilde{y}_{rj}, \quad j = 1,...,n, \\
& u_r \geq 0, \quad r = 1,...,s, \\
& v_i \geq 0, \quad i = 1,...,m,
\end{aligned} \tag{2}
$$

where $u_r \left(r = 1,\ldots,s\right)$ and $v_i \left(i = 1,\ldots,m\right)$ are defined as model (1). The optimal value of $\tilde{\theta}_o$ is applied to clarify the fuzzy efficiency measure of DMU_o.

Remark 3. The interpretation of above fuzzy DEA model (2) is in the same manner as its corresponding DEA model (1). Similar to the crisp DEA model, the constraints $\sum_i v_i \tilde{x}_{io} = 1$ & $\sum_i v_i \tilde{x}_{ij} \geq \sum_r u_r \tilde{y}_{rj}, \forall j$ are utilized for normalization of the value of the objective function $\sum_r u_r \tilde{y}_{ro}$. However, the constraint $\sum_i v_i \tilde{x}_{io} = 1$ in the above model (2) is interpreted as "$\sum_i v_i \tilde{x}_{io}$ is approximately equal to one" (see Lertworasirikul et al., 2003).

Several approaches have been developed to solve the above fuzzy LP problem in the fuzzy DEA literature (see Hatami-Marbini, Emrouznejad, & Tavana, 2011a; Emrouznejad, Tavana, & Hatami-Marbini, 2014b). The approaches mainly include 1) the defuzzification approach (e.g., Ghasemi, Ignatius, & Davoodi, 2014; Wang and Chin, 2011), 2) the α-level based approach (e.g., Hatami-Marbini, Tavana, Agrell, & Saati, 2013; Puri & Yadav, 2012), 3) the fuzzy ranking approach (e.g., Emrouznejad et al. 2011; Hatami-Marbini, Saati, & Tavana, 2011b), and 4) the possibility approach (e.g., Khodabakhshi, Gholami, & Kheirollahi, 2010; Lertworasirikul et al., 2003). Fuzzy ranking and α-cut approaches are the most popular among the 4 approaches outlined in the fuzzy DEA literature (Hatami-Marbini et al., 2011a; Emrouznejad & Tavana, 2014a). More details on the fuzzy ranking approach for solving fuzzy DEA can be found in the following literature (Bagherzadeh valami, 2009; Guo and Tanaka, 2001; Hatami-Marbini, Tavana, & Ebrahimi 2011c; Soleimani-damaneh, 2009).

In this paper, we adapted the fuzzy ranking method proposed by Jiménez et al. (2007) to solve the fuzzy DEA model (2).

Definition 11. Given the input weights $v_i\left(i=1,\ldots,m\right)$ and the output weights $u_r\left(r=1,\ldots,s\right)$, the feasibility of model (2) in degree α is defined as

$$min_{j=1,\ldots,n}\left\{\mu_M\left(\sum_{i=1}^{m} v_i\tilde{x}_{ij},\sum_{r=1}^{s} u_r\tilde{y}_{rj}\right)\right\}=\alpha \qquad \& \qquad \mu_M\left(\sum_{i=1}^{m} v_i\tilde{x}_{io},1\right)=\frac{1}{2}.$$

According to definition 9, the above expressions can be written, respectively, as follows:

$$\frac{E_2^{p_j}-E_1^{q_j}}{E_2^{p_j}-E_1^{q_j}-(E_1^{p_j}-E_2^{q_j})}\geq\alpha \qquad \& \qquad \frac{E_2^{d}-1}{E_2^{d}-E_1^{d}}=\frac{1}{2},$$

or

$$\tilde{p}_j \geq_\alpha \tilde{q}_j \ \ \& \ \ \frac{E_2^{d}-1}{E_2^{d}-E_1^{d}}=\frac{1}{2},$$

where $\tilde{p}_j=\sum_{i=1}^{m} v_i\tilde{x}_{ij},\tilde{q}_j=\sum_{r=1}^{s} u_r\tilde{y}_{rj},\quad j=1,\ldots,n$ and $\tilde{d}=\sum_{i=1}^{m} v_i\tilde{x}_{io}$.

Definition 12. $\left(u_r^*,v_i^*\right)$, $i=1,\ldots,m$, $r=1,\ldots,s$, in which $\sum_{i=1}^{m} v_i^*\tilde{x}_{io}=1$ and $\sum_{i=1}^{m} v_i^*\tilde{x}_{ij}\geq\sum_{r=1}^{s} u_r^*\tilde{y}_{rj}$, $j=1,\ldots,n$, is an acceptable optimal solution to the model (2) if

$$\mu_M\left(\sum_{r=1}^{s} u_r^*\tilde{y}_{ro},\sum_{r=1}^{s} u_r\tilde{y}_{ro}\right)\geq\frac{1}{2},$$

for every $\left(u_r,v_i\right)$, $i=1,\ldots,m, r=1,\ldots,s$, in which $\sum_{i=1}^{m} v_i\tilde{x}_{io}=1$ & $\sum_{i=1}^{m} v_i\tilde{x}_{ij}\geq\sum_{r=1}^{s} u_r\tilde{y}_{rj},\forall j$,

or,

$$\sum_{r=1}^{s} u_r^*\tilde{y}_{ro}\geq_{0.5}\sum_{r=1}^{s} u_r\tilde{y}_{ro},$$

for every $\left(u_r,v_i\right)$, $i=1,\ldots,m, r=1,\ldots,s$, in which $\sum_{i=1}^{m} v_i\tilde{x}_{io}=1$ & $\sum_{i=1}^{m} v_i\tilde{x}_{ij}\geq\sum_{r=1}^{s} u_r\tilde{y}_{rj},\forall j$.

Proposition 2. $\left(u_r^*,v_i^*\right)$, $i=1,\ldots,m$, $r=1,\ldots,s$, is an α-acceptable optimal feasible solution of model (2) if it is an optimal feasible solution to the following fuzzy LP:

$$\begin{aligned}
max \quad & \theta_o=\sum_{r=1}^{s} u_r EV\left(\tilde{y}_{ro}\right)\\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i\left(\frac{1}{2}E_1^{x_{io}}+\frac{1}{2}E_2^{x_{io}}\right)=1,\\
& \sum_{i=1}^{m} v_i\left(\left(1-\alpha\right)E_2^{x_{ij}}+\alpha E_1^{x_{ij}}\right)\geq\sum_{r=1}^{s} u_r\left(\alpha E_2^{y_{rj}}+\left(1-\alpha\right)E_1^{y_{rj}}\right),\quad j=1,\ldots,n,\\
& u_r\geq 0,\quad r=1,\ldots,s,\\
& v_i\geq 0,\quad i=1,\ldots,m.
\end{aligned} \tag{3}$$

Proof. Proof is given in Appendix A.

The objective value θ_o in the above model (3) can now exceed one since the inequality constraints of model (3) are provided based on the expected interval (see Proposition 3).

Proposition 3. The efficiency score in the fuzzy LP problem (3) may exceed one for some α-levels.

Proof. Proof is given in Appendix A.

Definition 13. *DMU$_o$* in the above model (3) is efficient at a particular α-level if θ_o^* at the α-level is greater than or equal to one; otherwise it is inefficient at that α-level.

Conforming Model (3) to the definition of expected interval and expected value of fuzzy numbers (definition 8), we have:

$$\begin{aligned}
\max \quad & \theta_o = \sum_{r=1}^{s} u_r \left(y_{ro}^{l} + y_{ro}^{m_1} + y_{ro}^{m_2} + y_{ro}^{u} \right) \Big/ 4 \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \left(x_{io}^{l} + x_{io}^{m_1} + x_{io}^{m_2} + x_{io}^{u} \right) \Big/ 4 = 1, \\
& \sum_{i=1}^{m} v_i \left[(1-\alpha)\left(x_{ij}^{m_2} + x_{ij}^{u} \right) + \alpha \left(x_{ij}^{l} + x_{ij}^{m_1} \right) \right] \geq \\
& \sum_{r=1}^{s} u_r \left[\alpha \left(y_{rj}^{m_2} + y_{rj}^{u} \right) + (1-\alpha)\left(y_{rj}^{l} + y_{rj}^{m_1} \right) \right], \quad j = 1,\dots,n, \\
& u_r \geq 0, \quad r = 1,\dots,s, \\
& v_i \geq 0, \quad i = 1,\dots,m.
\end{aligned} \tag{4}$$

This model is equivalent to a crisp α-parametric programming, while $\alpha \in [0,1]$ is a parameter. For each α-level, we can obtain an optimal solution as the efficiency value of the *DMU* under evaluation.

Definition 14. The fuzzy efficiency of an evaluated *DMU* with the trapezoidal fuzzy input $\tilde{x}_{io}$ = $\left(x_{io}^{l}, x_{io}^{m_1}, x_{io}^{m_2}, x_{io}^{u} \right)$ and output $\tilde{y}_{ro} = \left(y_{ro}^{l}, y_{ro}^{m_1}, y_{ro}^{m_2}, y_{ro}^{u} \right)$ is defined as a trapezoidal fuzzy number as follows:

$$\tilde{\theta}_o^* = \left(l, \mu_1, \mu_2, \upsilon \right), \tag{5}$$

where $\mu_1 = \dfrac{\sum_{r=1}^{s} u_r^* y_{ro}^{m_1}}{\sum_{i=1}^{m} v_i^* x_{io}^{m_2}}$, $\mu_2 = \dfrac{\sum_{r=1}^{s} u_r^* y_{ro}^{m_2}}{\sum_{i=1}^{m} v_i^* x_{io}^{m_1}}$, $l = \mu_1 - \sum_{r=1}^{s} u_r^* \left(y_{ro}^{m_1} - y_{ro}^{l} \right)$, $\upsilon = \mu_2 + \sum_{r=1}^{s} u_r^* \left(y_{ro}^{u} - y_{ro}^{m_2} \right)$, and $u_r^* (r = 1,\dots,s)$ and $v_i^* (i = 1,\dots,m)$ are the obtained output and input weight values from model (4). l, υ and μ_1, μ_2 are the lower bound, upper bound and the mid values of the fuzzy efficiency measure $\tilde{\theta}_o^*$, respectively.

Remark 4. The proposed model (3) and (4) can be extended to consider both desirable and undesirable outputs (see Appendix B).

Definition 15. DMU_o is fuzzy-efficient at a particular α-level if the upper bound of $\tilde{\theta}_o^*$ at that α-level is greater than or equal to one; otherwise it is fuzzy-inefficient at that α-level.

4. Illustration and validation: two numerical examples

To illustrate the proposed method, we consider the following two numerical examples. For simplicity we have chosen these examples where the data structure in the first example is the symmetrical triangular fuzzy numbers, whereas the second example is the asymmetric triangular fuzzy numbers – though any other fuzzy forms could be used.

The first example (see Table 1) with two fuzzy inputs and two fuzzy outputs is taken from Guo and Tanaka (2001). Table 1 provides the data for computation of crisp and fuzzy efficiency measures for Table 2 to Table 5.

Table 1
DMUs with two fuzzy inputs and two fuzzy outputs

DMU	Inputs		Outputs	
	x_1	x_2	y_1	y_2
1	(3.5, 4.0, 4.5)	(1.9, 2.1, 2.3)	(2.4, 2.6, 2.8)	(3.8, 4.1, 4.4)
2	(2.9, 2.9, 2.9)	(1.4, 1.5, 1.6)	(2.2, 2.2, 2.2)	(3.3, 3.5, 3.7)
3	(4.4, 4.9, 5.4)	(2.2, 2.6, 3.0)	(2.7, 3.2, 3.7)	(4.3, 5.1, 5.9)
4	(3.4, 4.1, 4.8)	(2.2, 2.3, 2.4)	(2.5, 2.9, 3.3)	(5.5, 5.7, 5.9)
5	(5.9, 6.5, 7.1)	(3.6, 4.1, 4.6)	(4.4, 5.1, 5.8)	(6.5, 7.4, 8.3)

The results generated from the possibility approach proposed by Lertworasirikul et al. (2003) for different α values are listed in Table 2, while the fuzzy efficiency measures provided by Guo and Tanaka (2001) for different α values are listed in Table 3.

Table 2
The efficiency values by using Lertworasirikul et al.'s model

A	DMU1	Rank	DMU2	Rank	DMU3	Rank	DMU4	Rank	DMU5	Rank
0	1.107	5	1.238	4	1.267	3	1.520	1	1.296	2
0.25	1.032	5	1.173	3	1.149	4	1.386	1	1.226	2
0.5	0.963	5	1.112	3	1.035	4	1.258	1	1.159	2
0.75	0.904	5	1.055	3	0.932	4	1.131	1	1.095	2
1	0.855	5	1.000	1	0.861	4	1.000	1	1.000	1

Table 3
The fuzzy efficiencies by Guo & Tanaka's model

A	DMU1	DMU2	DMU3	DMU4	DMU5
0	(0.66, 0.81, 0.99)	(0.88, 0.89, 1.09)	(0.60, 0.82, 1.12)	(0.71, 0.93, 1.25)	(0.61, 0.79, 1.02)
0.5	(0.75, 0.83, 0.92)	(0.94, 0.97, 1.00)	(0.71, 0.83, 0.97)	(0.85, 0.97, 1.12)	(0.72, 0.82, 0.93)
0.75	(0.80, 0.84, 0.88)	(0.96, 0.99, 1.02)	(0.77, 0.83, 0.90)	(0.92, 0.98, 1.05)	(0.78, 0.83, 0.89)
1	(0.85, 0.85, 0.85)	(1.00, 1.00, 1.00)	(0.86, 0.86, 0.86)	(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)

The crisp efficiencies and fuzzy efficiencies for five *DMU*s' across different α-levels (0, 0.25, 0.5, 0.75, 1) are analysed with model (4) and the results are provided in Table 4 and Table 5.

Table 4
Results of the crisp efficiency measures by proposed method

A	DMU1	Rank	DMU2	Rank	DMU3	Rank	DMU4	Rank	DMU5	Rank

0	0.885	5	1.054	2	0.886	4	1.118	1	1.034	3
0.25	0.862	5	1.027	3	0.869	4	1.063	1	1.034	2
0.5	0.855	5	1.000	1	0.861	4	1.000	1	1.000	1
0.75	0.846	5	0.993	1	0.847	4	0.981	2	0.945	3
1	0.830	4	0.978	1	0.829	5	0.961	2	0.894	3

Note: The results of the proposed model at the 0.5 α-level yield the same outcome as that generated at the α-level=1 in Lertworasirikul et al.'s model.

Table 5
Results of the fuzzy efficiency measures by proposed model

A	DMU1	DMU2	DMU3	DMU4	DMU5
0	(0.819, 0.885, 0.950)	(1.012, 1.054, 1.096)	(0.747, 0.886, 1.025)	(1.053, 1.118, 1.183)	(0.892, 1.034, 1.176)
0.25	(0.797, 0.862, 0.926)	(0.986, 1.027, 1.068)	(0.733, 0.869, 1.005)	(0.982, 1.063, 1.143)	(0.892, 1.034, 1.176)
0.5	(0.789, 0.855, 0.921)	(0.982, 1.000, 1.018)	(0.726, 0.861, 0.995)	(0.965, 1.000, 1.035)	(0.868, 1.000, 1.132)
0.75	(0.781, 0.846, 0.911)	(0.993, 0.993, 0.993)	(0.715, 0.847, 0.980)	(0.946, 0.981, 1.015)	(0.820, 0.945, 1.071)
1	(0.766, 0.830, 0.893)	(0.978, 0.978, 0.978)	(0.699, 0.829, 0.958)	(0.928, 0.961, 0.995)	(0.774, 0.894, 1.014)

The results can be interpreted and compared in the following way. By comparing Table 2 and Table 4, it can be noted that the efficiency values of the proposed model are less than lertworasirikul et al.'s model. In both models, the efficiency values decrease as α increases, $\alpha \in [0,1]$. From Table 2, *DMU*s 2, 4 and 5 are efficient across all α-levels and *DMU*s 1 and 3 are only efficient at some α-levels. Contrastingly, in Table 4, *DMU*s 2, 4 and 5 are efficient at some α-levels but *DMU*s 1 and 3 are inefficient at all α-levels. It should also be noted that Guo and Tanaka's fuzzy ranking approach shows *DMU*s 1 and 3 to be inefficient at almost all α-levels (see Table 3).

According to definition 15, through the proposed method across α-levels (0 and 0.25) *DMU*s 2, 3, 4, and 5 are fuzzy-efficient, whereas by α = 0.5 *DMU*s 2, 4, and 5 are charactriezed as fuzzy-efficient. While, for α = 0.75, *DMU*s 4 and 5 are fuzzy-efficient and for α = 1, DMU5 is only determined as fuzzy-efficient (see Table 5). Therefore, it can be concluded that for optimistic point of view, the value of α can be specified as 0 or 0.25 and for pessimistic point of view, α can be determined as 0.75 or 1. In other words, by increasing the value of α, the optimistic point of view changes to the pessimistic point of view gradually. Consequently, there is an opportunity for the *DM* to decide on which value of α is the best for the scenario under his or her interpretation. Moreover, it can be noted that DMU5 is the most efficient *DMU*, followed by DMU4, DMU2, and DMU3 respectively.

At α = 1 in Table 2 and Table 3 and α = 0.5 in Table 4, *DMU*s 2, 4 and 5 are efficient for the proposed model. The efficiency values of DMU1 and DMU3 are 0.855 and 0.861 respectively at α = 0.5. While by using Lertworasirikul et al.'s model and Guo and Tanaka's model *DMU*s 2, 4 and 5 are efficient and the efficiency values of DMU1 and DMU3 are 0.855 and 0.861 respectively by α = 1. According to Table 2, Table 3's results, it can be concluded that *DMU*s 1 and 3 are the most inefficient.

In Table 4, *DMU*s 1 and 3 are inefficient across all α values and the remaining *DMU*s (i.e. DMU2, DMU4, and DMU5) are only efficient at the 0, 0.25 and 0.5 α-levels, respectively. With the exception of DMU3 at the 0.5 α-level, it should be emphasised that the solutions up to the 0.5 α-level in Table 4 for the efficient *DMU*s are the same between our proposed model and that of Lertworasirikul et al.'s model (see Table 2). In addition, we find that Lertworasirikul et al.'s model has two additional efficient *DMU*s (DMU1 and DMU3) at the 0 and 0.25 α-levels. This however shows that Lertworasirikul et al.'s model is a close representation to the best-worst case, in which the *DM* is optimistic about the target *DMU* (DMU_o) and pessimistic about the rest of *DMU*s (see Table I in Appendix C for more details). Given that we expect increasing α-levels would increase the "strictness" of the condition to maintain efficiency, we can state the solution provided in our proposed model to be more reasonable and practical, where efficiency values of *DMU*s are not always achievable with an optimistic point of view.

Figure 1 summarizes the performance of three models (proposed model, Lertworasirikul et al.'s model and best-worst case model) according to the results provided in Table 2 and Table 5 as well as Table I in Appendix C. All *DMU*s are characterized as efficient in Lertworasirikul et al.'s model and the best-worst case at α-levels 0 and 0.25. While using the proposed model at these α values, four *DMU*s (DMU2, DMU3, DMU4, and DMU5) are efficient.

At the α = 0.5, both Lertworasirikul et al.'s model and the best-worst model have four efficient *DMU*s (DMU2, DMU3, DMU4, and DMU5); respectively, whereas the proposed model has three efficient *DMU*s (DMU2, DMU4, and DMU5). In addition, at α = 0.75 and α = 1, three *DMU*s (DMU2, DMU4, and DMU5) are efficient for the Lertworasirikul et al.'s model and the best-worst case. On the contrary, using the proposed model, two *DMU*s (DMU4 and DMU5) are efficient at the α = 0.75 and only one *DMU* (DMU5) is efficient at the α = 1.

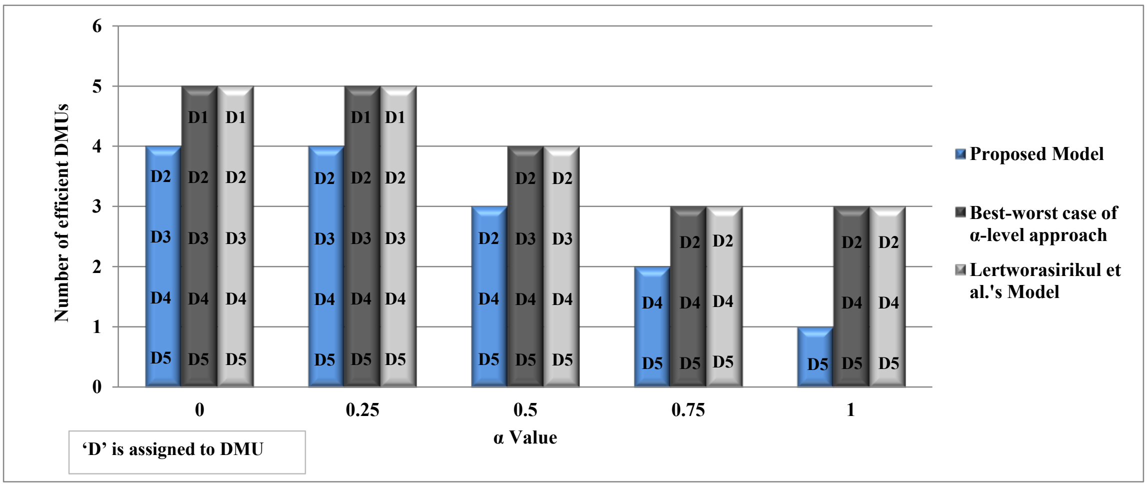

Figure 1. Performance comparison: Proposed Model vs Lertworasirikul et al.'s Model and Best-worst case

From Figure 1, it can be concluded that Lertworasirikul et al.'s model has the same assessment as the best-worst case model of α-level approach which can be observed from Appendix C. Therefore, the results obtained by Lertworasirikul et al.'s model in Table 2 are extremely optimistic in comparison with those obtained by the proposed model in Table 4 and Table 5. The proposed model (4) has the ability to provide both crisp and fuzzy efficiency measures across different α-levels. Although crisp efficiency measures can provide ease of ranking, fuzzy observation are more informative and realistic in real-world modeling and decision making. The proposed method avoids results that are overly optimistic and pessimistic.

Let us continue by considering Table 3, Table 4's results. In Table 3, the fuzzy efficiencies are increased when α increases, whereas the fuzzy efficiencies decreased when α increases in Table 5. The mid values of fuzzy efficiency of the proposed model (other than at $\alpha = 1$) are greater than Guo and Tanaka's model. The mid values of fuzzy efficiency in Table 3 at $\alpha = 1$ are the same as the mid values of fuzzy efficiency at $\alpha = 0.5$ in Table 5. The results of fuzzy efficiencies by Guo and Tanaka's fuzzy ranking approach have been obtained with wider classes of fuzzy numbers in comparison with the results of fuzzy efficiencies by the proposed model. However, *DMU*s 1 and 3 have been evaluated as inefficient by both Guo and Tanaka's model and the proposed model at all α-levels. It should be pointed out that in the proposed model only one linear programming problem is adequate for efficiency assessment whereas the Guo and Tanaka's fuzzy ranking approach needs two linear programming problems for the same evaluation. Furthermore, the data structure in Guo and Tanaka's fuzzy ranking approach is only limited to symmetrical fuzzy triangular numbers.

The possibility approach proposed by Lertworasirikul et al. (2003) also has its setbacks. Given that all fuzzy constraints are defined across different possibility levels, solving a numerical example requires all fuzzy constraints to be satisfied with its respective possibility level. The model needs to be converted into a basic α-level approach, without which it suffers from complicated computational procedure.

The proposed model allows the data structure of fuzzy inputs and fuzzy outputs to be asymmetric and it is able to provide the efficiency values with lesser number of steps to achieve a solution. It has additional advantages over the method of Guo and Tanaka (2001) and Lertworasirikul et al. (2003)'s possibility approach. The proposed model requires only one linear programming problem to obtain the crisp efficiency and fuzzy efficiency values of each DMU.

Next, we consider an example consisting of 10 *DMU*s, with two asymmetric triangular fuzzy inputs and outputs each (see Table 6) adopted from Saati et al. (2002).

The results generated from Saati et al. (2002) proposed model across the α-values are listed in Table 7. In addition, the results of the crisp and fuzzy efficiencies from Model 4 are recorded in Table 8 and Table 9.

Table 6

DMUs with two fuzzy inputs and two fuzzy outputs

DMU	Inputs		Outputs	
	x_1	x_2	y_1	y_2
1	(6, 7, 8)	(29, 30, 32)	(35.5, 38, 41)	(409, 411, 416)
2	(5.5, 6, 6.5)	(33, 35, 36.5)	(39, 40, 43)	(478, 480, 484)
3	(7.5, 9, 10.5)	(43, 45, 48)	(32, 35, 38)	(297, 299, 301)
4	(7, 8, 10)	(37.5, 39, 42)	(28, 31, 31)	(347, 352, 360)
5	(9, 11, 12)	(43, 44, 45)	(33, 35, 38)	(406, 411, 415)
6	(10, 10, 10)	(53, 55, 57.5)	(36, 38, 40)	(282, 286, 289)
7	(10, 12, 14)	(107, 110, 113)	(34.5, 36, 38)	(396, 400, 405)
8	(9, 13, 16)	(95, 100, 101)	(37, 41, 46)	(387, 393, 402)
9	(12, 14, 105)	(120, 125, 131)	(24, 27, 38)	(400, 404, 406)
10	(5, 8, 10)	(35, 38, 39)	(48, 50, 51)	(470, 470, 470)

In Table 8, the crisp efficiencies decrease as α increases. DMU10 is efficient at all α-levels, whereas *DMU*s 1 and 2 are efficient only at some α-levels. The remaining DMUs are inefficient at all α values. In Saati et al. (2002)'s method, DMU10 is the most efficient, followed by *DMU*s 1 and 2, while the rest of the *DMU*s are inefficient (see Table 7). In Table 9, the fuzzy efficiencies also decrease as α increases for the proposed method.

Table 7

The efficiency values by Saati et al.'s method

A	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
0	1.000	1.000	0.844	0.761	0.780	0.692	0.633	0.852	0.528	1.000
0.25	1.000	1.000	0.780	0.729	0.746	0.664	0.580	0.727	0.458	1.000
0.5	1.000	1.000	0.727	0.707	0.716	0.637	0.633	0.852	0.528	1.000
0.75	1.000	1.000	0.672	0.683	0.697	0.609	0.489	0.543	0.383	1.000
1	1.000	1.000	0.613	0.658	0.681	0.581	0.450	0.473	0.361	1.000

Table 8

Results of the efficiency measures by proposed method

A	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10
0	1.075	1.068	0.682	0.673	0.701	0.619	0.476	0.512	0.383	1.161
0.25	1.037	1.034	0.662	0.665	0.690	0.603	0.461	0.495	0.375	1.124
0.5	1.000	1.000	0.641	0.652	0.679	0.586	0.446	0.479	0.366	1.008
0.75	0.984	0.986	0.621	0.642	0.669	0.569	0.431	0.464	0.358	1.005
1	0.968	0.972	0.600	0.632	0.659	0.552	0.417	0.448	0.350	1.001

A comparison between Table 7 and Table 8 shows that DMU1, DMU2, and DMU10 are efficient across all α-levels (0, 0.25, 0.5, 0.75, 1) for Saati's model; whereas, DMU10 is efficient across all α-levels (0, 0.25, 0.5, 0.75, 1) for the proposed model. Thus, in the proposed model, DMU10 is the most efficient *DMU*, while in Saati et al.'s model; DMU1, DMU2, and DMU10 are equally the most efficient *DMU*s. Saati et al.'s method can be interpreted as the best-worst case of the basic α-level approach, in which the *DM* is optimistic about the target *DMU* (DMU_o) and pessimistic about the remaining *DMU*s (see Appendix C). We further investigate and compare the efficiency scores between Table 7 and Table 8 through the Wilcoxon rank-sum test, which is a non-parametric test for paired observations. The findings indicate that Saati et al.'s model tends to have significantly higher efficiency scores than the proposed model for every α-level (see Appendix D). Alternatively, one can note that the result (0.678) appearing at a lower α-level for the proposed model (α = 0) appears at a higher level for Saati et al.'s model (α = 0.75). This reconfirms the notion that Saati et al.'s model tends to be optimistic in nature. However, in a real application, it would not be possible to always provide the efficiency values of *DMU*s with the best-worst case.

In addition, if one were to observe Saati et al.'s results, increasing the possibilistic level does not render changes in the number of efficient *DMU*s. This implies that there is no relationship between the uncertainty-certainty level and model results, which further indicates lack of discriminatory power. Contrastingly, the proposed model provides three efficient *DMU*s at the 0, 0.25 and 0.5 α-levels but only one efficient *DMU* at the 0.75 and 1 α-level, respectively.

Since the fuzzy efficiency measures are more informative, the results of fuzzy efficiencies provided by the proposed model in Table 9 can be described in following way. Three DMUs; DMU1, DMU2, and DMU10 are fuzzy-efficient across all α-levels (0, 0.25, 0.5). At α = 0.75, DMU1 and DMU10 are fuzzy-efficient and at α = 1, only DMU10 is fuzzy-efficient. It can be observed that by increasing the value of α, the optimistic point of view changes to the pessimistic point of view gradually. Hence, this is a good opportunity for the *DM* to decide on which value of α is the best for the scenario under his or her interpretation.

Table 9
Results of the fuzzy efficiency measures by proposed model

DMU | α	0	0.25	0.5	0.75	1
DMU1	(1.007, 1.078, 1.162)	(0.972, 1.040, 1.122)	(0.993, 1.006, 1.027)	(0.978, 0.990, 1.010)	(0.963, 0.974, 0.993)
DMU2	(1.028, 1.055, 1.134)	(0.995, 1.021, 1.097)	(0.991, 0.995, 1.004)	(0.977, 0.981, 0.990)	(0.963, 0.967, 0.975)
DMU3	(0.627, 0.685, 0.744)	(0.608, 0.665, 0.721)	(0.589, 0.644, 0.699)	(0.570, 0.623, 0.676)	(0.551, 0.602, 0.653)
DMU4	(0.662, 0.681, 0.693)	(0.652, 0.669, 0.682)	(0.643, 0.659, 0.672)	(0.633, 0.648, 0.661)	(0.628, 0.637, 0.651)
DMU5	(0.687, 0.701, 0.716)	(0.677, 0.690, 0.704)	(0.667, 0.679, 0.692)	(0.657, 0.668, 0.681)	(0.651, 0.659, 0.665)
DMU6	(0.588, 0.621, 0.653)	(0.572, 0.604, 0.635)	(0.556, 0.587, 0.618)	(0.540, 0.570, 0.600)	(0.524, 0.553, 0.582)
DMU7	(0.455, 0.475, 0.501)	(0.440, 0.459, 0.485)	(0.426, 0.444, 0.469)	(0.412, 0.430, 0.454)	(0.398, 0.416, 0.439)
DMU8	(0.449, 0.499, 0.561)	(0.435, 0.483, 0.543)	(0.421, 0.467, 0.525)	(0.407, 0.452, 0.508)	(0.394, 0.437, 0.491)
DMU9	(0.373, 0.377, 0.378)	(0.365, 0.368, 0.370)	(0.357, 0.360, 0.362)	(0.349, 0.352, 0.354)	(0.341, 0.344, 0.346)
DMU10	(1.100, 1.146, 1.170)	(1.065, 1.110, 1.133)	(1.030, 1.074, 1.096)	(0.995, 1.037, 1.058)	(0.959, 1.000, 1.020)

5. Carbon efficiency of 23 European Union countries

We further illustrate our proposed model with a dataset comprising 2 inputs and 3 outputs of 23 European Union (EU) member countries (except Bulgaria, Luxembourg, Malta and Romania) (See Appendix E). Data were based on the EU Emissions Trading Scheme of more than 10,000 installations that generate an excess of 20^{MW} per installation within the country. This is believed to capture about half of the CO_2 emissions within EU.

In our proposed energy dependency model, we are able to assess the efficiency of European Union (EU) member countries in terms of their ability to rely more on clean energy, i.e. produce less industrial green house gasses (GHgs). In the EU electronic trading scheme (EU ETS), a 'cap-and-trade' system is in place to control GHgs emitted by installations ranging from factories to power plants. The idea behind the EU ETS is to reduce the total emissions by providing an incentive (penalty) on carbon used below (above) the capital allowance. In each year, an installation will be provided carbon allowances, which will later be audited (verified) in the following year. The excess of unused carbon may be traded, thus providing a motivation for combatting climate change. This remains the motivation for the analysis at the installation plant level.

On the country level, given that all EU member countries participate in the EU ETS trading scheme, we expect that the relative efficiency of these countries to come from their respective intra-country green initiatives. A country that is more efficient will indirectly point towards a greater control over its sustainability measures. This is implied externally.

Although the inputs can be seen as the resources in the production of output, it would be easier to articulate for the intermediation efficiency approach rather than a production approach. An EU country can be seen as an intermediary who facilitates clean energy by matching the controls set by the EU commission and the installation operators. The former can be seen as the depositor of carbon credits and the latter the entity that utilizes the carbon credits.

Thus, taking into account inputs x_1 and x_2, we can draw a parallel between our model and the financial and total expenses used as inputs in the banking industry; loosely speaking. Since x_1 is the total carbon allowances for all the installations within the country, it can be considered as a financial expense because it can be perceived as an administrative cost for a country in managing the installations within its border. In other words, the total free allocation of carbon credits across all installations can be taken as the deposit placed by the regulator, i.e. EU commission for each member country. Input x_2 is the gross inland energy consumption, which is simply the total expenditure in fuel terms.

With regards to the outputs, y_3 can be interpreted in a similar manner as how one would for gross loan portfolio in banking terms. However, the 'loan portfolio' in our case captures the portion of renewable energy as a result of the total conventional fuels consumed. In addition, y_1 and y_2 (both undesirable outputs), which are electricity consumed, and the emission usage as verified by the operator, respectively.

Given that higher carbon emissions are associated with higher productivity, curbing carbon emissions will result in productivity reduction, and this will have an unfair advantage for larger sized countries. Hence, our model (named as the energy dependency model) avoids this problem as the choice of inputs is based on a set of resources that generate carbon emissions and the output will be the extent of those resources in limiting the carbon effects. In addition, we have scaled our measures by taking into account the population size of the respective countries. This treatment of scaling has its dual purpose of overcoming lack of discriminatory power among DMUs which may arise from mixing volume and percentage-based measures (see Dyson et al., 2001). The operational definition of the 2 inputs and 3 outputs are provided in Table 10.

Table 10
Model Variables and Operational Definition

	Variables	**Definition**
Inputs	Allocated Carbon Allowances (x_1)	It is an allowance distributed each year for free to installations according to the national allocation plan, measured in tonnes of carbon dioxide equivalent.
	Gross Inland energy consumption (GIC) (x_2)	GIC is the quantity of energy, expressed in oil equivalents, consumed within the borders of a country. It is calculated as total domestic energy production plus energy imports and changes in stocks minus energy exports.
Outputs	Electricity consumed from renewable sources (y_1)	Percentage of gross electricity consumed from renewable sources (2006 to 2009).
	Verified emissions (y_2)	The total annual emissions per emitting installation
	Share of renewable energy in fuel consumption of transport (y_3)	The degree to which conventional fuels have been substituted by biofuels in transportation

Note: The simpler energy dependency model using only crisp data can be found in Ghasemi, Joshua, & Emrouznejad (2014).

Input variables (x_1 and x_2) and output variables (y_1 and y_2) are estimated as the asymmetrical fuzzy triangular form for the period 2005-2008 and 2006-2009 respectively, whereas output variable y_3 is a crisp number and taken for the year 2009. We provide a 1 year lag between the inputs and outputs period to account for the necessary time gap needed for realizing the effect. The reason behind computing separate standard deviations for the left and right side is to show that the proposed method is robust not only to symmetrical fuzzy triangular numbers, but also to asymmetrical fuzzy triangular numbers. For analysis with fuzzy symmetrical dataset, interested readers are referred to Guo and Tanaka's (2001) method.

The results of our analysis at the different α-levels (0.25, 0.5 and 0.75) are provided in Table 11. The fuzzy efficiency and crisp efficiency measures for each value of α are obtained. The 3-step procedure of our analysis is as follows: First, our proposed model (4) at α = 0.25 determines that the countries Germany, Latvia, and Sweden are efficient in terms of energy dependency. The efficiency values of these countries are 1.030, 1.045 and 1.012 respectively and the fuzzy efficiency measures of these countries are (1.043, 1.057, 1.126), (1.044, 1.066, 1.206) and (1.004, 1.036, 1.055) respectively (see Table 11). Second, using the proposed model (4) at α = 0.5 countries Latvia and Sweden are still efficient, whereas the efficiency score of these countries is 1 and the fuzzy efficiency of these countries are (1.000, 1.021, 1.154) and (0.997, 1.019, 1.034) respectively (see Table 11). Third, at the α-level 0.75, our proposed model reveals the efficiency values of Germany, Latvia, and Sweden to be 0.951, 0.994, and 0.988 respectively and the fuzzy efficiency measures of these countries to be (0.983, 0.996, 1.060), (0.993, 0.993, 0.993) and (1.005, 1.007, 1.012) respectively. Through the evaluation at different values of α (0.25, 0.5, and 0.75), it can be concluded that Sweden is the most efficient *DMU*, followed by Latvia and Germany respectively.

Table 11

Fuzzy and crisp efficiency measures of 23 European Union member countries

DMU	Efficiency values for α = 0.25			Efficiency values for α = 0.50			Efficiency values for α = 0.75		
	fuzzy measures	crisp values	Rank	fuzzy measures	crisp values	Rank	fuzzy measures	crisp values	Rank
Austria	(0.747, 0.775, 0.824)	0.780	9	(0.729, 0.756, 0.804)	0.760	8	(0.711, 0.738, 0.784)	0.742	8
Belgium	(0.148, 0.151, 0.165)	0.148	22	(0.144, 0.147, 0.161)	0.144	22	(0.141, 0.144, 0.157)	0.141	22
Cyprus	(0.122, 0.122, 0.123)	0.122	23	(0.121, 0.121, 0.122)	0.121	23	(0.120, 0.121, 0.121)	0.120	23
Czech Republic	(0.248, 0.253, 0.261)	0.250	18	(0.246, 0.251, 0.259)	0.248	18	(0.244, 0.249, 0.257)	0.246	18
Denmark	(0.363, 0.398, 0.406)	0.391	13	(0.354, 0.389, 0.396)	0.382	13	(0.346, 0.380, 0.387)	0.373	13
Estonia	(0.332, 0.336, 0.340)	0.333	15	(0.330, 0.334, 0.338)	0.331	15	(0.328, 0.332, 0.336)	0.329	15
Finland	(0.298, 0.299, 0.301)	0.301	16	(0.296, 0.297, 0.299)	0.299	16	(0.294, 0.295, 0.297)	0.297	16
France	(0.864, 0.865, 0.911)	0.845	6	(0.844, 0.845, 0.888)	0.824	6	(0.823, 0.824, 0.867)	0.805	7
Germany	(1.043, 1.057, 1.126)	1.030	2	(1.013, 1.026, 1.093)	0.980	3	(0.983, 0.996, 1.060)	0.951	2
Greece	(0.355, 0.360, 0.372)	0.357	14	(0.351, 0.357, 0.369)	0.354	14	(0.349, 0.354, 0.365)	0.351	14
Hungary	(0.237, 0.239, 0.249)	0.238	19	(0.235, 0.237, 0.247)	0.236	19	(0.233, 0.235, 0.245)	0.234	19
Ireland	(0.175, 0.184, 0.210)	0.187	21	(0.171, 0.180, 0.206)	0.182	21	(0.167, 0.176, 0.201)	0.178	21
Italy	(0.891, 0.901, 1.022)	0.911	5	(0.862, 0.872, 0.988)	0.881	5	(0.834, 0.843, 0.956)	0.852	5
Latvia	(1.044, 1.066, 1.206)	1.045	1	(1.000, 1.021, 1.154)	1.000	1	(0.993, 0.993, 0.993)	0.994	1
Lithuania	(0.396, 0.397, 0.398)	0.397	11	(0.394, 0.395, 0.396)	0.395	11	(0.391, 0.392, 0.393)	0.392	11
Netherlands	(0.236, 0.244, 0.256)	0.238	20	(0.228, 0.236, 0.247)	0.230	20	(0.220, 0.228, 0.239)	0.222	20
Poland	(0.825, 0.825, 0.853)	0.829	7	(0.817, 0.817, 0.844)	0.821	7	(0.809, 0.809, 0.836)	0.813	6
Portugal	(0.658, 0.672, 0.771)	0.695	10	(0.642, 0.657, 0.753)	0.678	10	(0.663, 0.667, 0.671)	0.668	10
Slovakia	(0.286, 0.288, 0.318)	0.292	17	(0.279, 0.281, 0.311)	0.287	17	(0.273, 0.275, 0.304)	0.279	17
Slovenia	(0.364, 0.386, 0.457)	0.394	12	(0.355, 0.376, 0.446)	0.385	12	(0.347, 0.367, 0.435)	0.375	12
Spain	(0.735, 0.765, 0.867)	0.765	8	(0.710, 0.740, 0.837)	0.739	9	(0.686, 0.714, 0.808)	0.714	9
Sweden	(1.004, 1.036, 1.055)	1.012	3	(0.997, 1.019, 1.034)	1.000	1	(1.005, 1.007, 1.012)	0.988	3
United Kingdom	(0.919, 0.933, 1.020)	0.924	4	(0.892, 0.905, 0.990)	0.897	4	(0.865, 0.878, 0.960)	0.870	4

Since the dataset in this section consists of both desirable and undesirable outputs and the data structure in Saati et al.'s method is only limited to desirable outputs, it is not appropriate to test Saati et al.'s (2002) method for the current example. In addition, unlike Saati et al.'s method, the proposed model is able to provide both crisp and fuzzy efficiency measures for each *DMU* by solving not more than one LP problem.

The fuzzy efficiency results (see Table 11) generated by the proposed model are more informative than the crisp values. For instance, in an optimistic point of view, the number of efficient *DMU*s can be increased to five *DMU*s at $\alpha = 0$, indicating that fuzzy efficiency values of Italy and United Kingdom are (0.891, 0.901, 1.022) and (0.919, 0.933, 1.020) respectively, which are considered to be fuzzy-efficient. At $\alpha = 0.75$, there is no efficient *DMU*s using the crisp solution, but the fuzzy efficiency measure reveals that Germany and Sweden are fuzzy-efficient ((0.983, 0.996, 1.060) & (1.005, 1.007, 1.012)). Therefore, the *DM* based on his post-hoc information and subjective judgment may choose one of the few solutions: 1. $\alpha = 0.75$ for two efficient *DMU*s (Germany & Sweden), or 2. $\alpha = 0.5$ for three efficient *DMU*s (Germany, Latvia and Sweden), and 3. $\alpha = 0$ for five efficient *DMU*s (Germany, Italy, Latvia, Sweden and United Kingdom).

6. Conclusion

In this paper, we have proposed a fuzzy DEA model for evaluating the carbon efficiency values and ranking of *DMU*s with fuzzy input and fuzzy output that may not necessarily be symmetrical. Drawing on the concept of the expected interval and the expected value of a fuzzy number, a fuzzy ranking approach that requires only solving one linear programming problem was proposed. Two numerical examples, one with symmetric and another with asymmetric triangular fuzzy numbers were used to demonstrate the applicability of the approach under both symmetrical and asymmetrical fuzzy numbers. A comparison with alternative approaches indicates the benefits of the proposed model, such as discriminant power and anticipation of a more realistic outcome at increasing levels of α value. A third example on an energy dependency case confirms the applicability of our proposed method under asymmetrical fuzzy numbers, especially in the case of carbon monitoring and control. Based on the results of the examples, it can be concluded that the proposed method performs better than the other methods in terms of ease in formulation and requiring lesser number of steps to achieve a solution.

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Appendix A.

Proof of Proposition 2. From definition 11, it can be concluded that (u_r, v_i), $i=1,\ldots,m$, $r=1,\ldots,s$, is α-feasible for model (2) if:

$$\frac{E_2^d - 1}{E_2^d - E_1^d} = \frac{1}{2},\ \sum_{i=1}^m v_i \tilde{x}_{ij} \geq_\alpha \sum_{r=1}^s u_r \tilde{y}_{rj},\quad j=1,\ldots,n,\ \text{where } \tilde{d} = \sum_{i=1}^m v_i \tilde{x}_{io},$$

or,

$$\frac{E_2^d - 1}{E_2^d - E_1^d} = \frac{1}{2},\ \frac{E_2^{p_j} - E_1^{q_j}}{E_2^{p_j} - E_1^{q_j} - (E_1^{p_j} - E_2^{q_j})} \geq \alpha,$$

where $\tilde{d} = \sum_{i=1}^m v_i \tilde{x}_{io}$, $\tilde{p}_j = \sum_{i=1}^m v_i \tilde{x}_{ij}, \tilde{q}_j = \sum_{r=1}^s u_r \tilde{y}_{rj}$, $j=1,\ldots,n$.

The above expressions can also be written as follows:

$$\frac{1}{2}E_1^d + \frac{1}{2}E_2^d = 1,\ (1-\alpha)E_2^{p_j} + \alpha E_1^{p_j} \geq \alpha E_2^{q_j} + (1-\alpha)E_1^{q_j},$$

where $\tilde{d} = \sum_{i=1}^m v_i \tilde{x}_{io}$, $\tilde{p}_j = \sum_{i=1}^m v_i \tilde{x}_{ij}, \tilde{q}_j = \sum_{r=1}^s u_r \tilde{y}_{rj}$, $j=1,\ldots,n$.

According to Proposition **1** and definitions 6 and 8, the above expressions can be transformed as follows:

$$\sum_{i=1}^m v_i \left(\frac{1}{2}E_1^{x_{io}} + \frac{1}{2}E_2^{x_{io}}\right) = 1 \text{ and}$$

$$\sum_{i=1}^m v_i \left((1-\alpha)E_2^{x_{ij}} + \alpha E_1^{x_{ij}}\right) \geq \sum_{r=1}^s u_r \left(\alpha E_2^{y_{rj}} + (1-\alpha)E_1^{y_{rj}}\right),\quad j=1,\ldots,n.$$

In conformity with definition 12, where $\sum_{i=1}^m v_i^* \tilde{x}_{io} = 1$ and $\sum_{i=1}^m v_i^* \tilde{x}_{ij} \geq \sum_{r=1}^s u_r^* \tilde{y}_{rj}$, $j=1,\ldots,n$, (u_r^*, v_i^*), $i=1,\ldots,m$, $r=1,\ldots,s$, is an acceptable optimal solution for model (2) if

$$\mu_M\left(\sum_{r=1}^s u_r^* \tilde{y}_{ro}, \sum_{r=1}^s u_r \tilde{y}_{ro}\right) \geq \frac{1}{2}\quad \forall u_r, v_i \left| \sum_{i=1}^m v_i x_{io} = 1, \sum_{i=1}^m v_i x_{ij} \geq \sum_{r=1}^s u_r y_{ro},\ \forall i,j,r. \right.$$

By using definition 9, it can be transformed as follows:

$$\frac{E_2^{d_1} - E_1^{d_2}}{E_2^{d_1} - E_1^{d_2} - (E_1^{d_1} - E_2^{d_2})} \geq \frac{1}{2},\ \text{where } \tilde{d}_1 = \sum_{r=1}^s u_r^* \tilde{y}_{ro}, \tilde{d}_2 = \sum_{r=1}^s u_r \tilde{y}_{ro},$$

or,

$$\frac{E_1^{d_1} + E_2^{d_1}}{2} \geq \frac{E_1^{d_2} + E_2^{d_2}}{2},\ \text{where } \tilde{d}_1 = \sum_{r=1}^s u_r^* \tilde{y}_{ro}, \tilde{d}_2 = \sum_{r=1}^s u_r \tilde{y}_{ro}.$$

From definitions 6 & 8 and Proposition **1**, the above expression is as follows:

$$\sum_{r=1}^{s} u_r^* EV(\tilde{y}_{ro}) \geq \sum_{r=1}^{s} u_r EV(\tilde{y}_{ro}),$$

which proves Proposition **2**.

Proof of Proposition 3. Let $\alpha = 1/3$ and $j = o$ in the inequality constraint of model (3). Thus, the following inequality holds:

$$\sum_{i=1}^{m} v_i \left(2E_2^{x_{io}} + E_1^{x_{io}}\right) \geq \sum_{r=1}^{s} u_r \left(E_2^{y_{ro}} + 2E_1^{y_{ro}}\right). \tag{I}$$

Since $\sum_{i=1}^{m} v_i \left(\frac{1}{2}E_1^{x_{io}} + \frac{1}{2}E_2^{x_{io}}\right) = 1$ and $\theta_o = \sum_{r=1}^{s} u_r EV(\tilde{y}_{ro}) = \sum_{r=1}^{s} u_r \left(\frac{1}{2}E_1^{y_{ro}} + \frac{1}{2}E_2^{y_{ro}}\right)$ in the above model (3), the inequality (I) can be written as

$$\frac{1}{2}\sum_{i=1}^{m} v_i E_2^{x_{io}} + 1 \geq \theta_o + \frac{1}{2}\sum_{r=1}^{s} u_r E_1^{y_{ro}},$$

or,

$$\theta_o \leq 1 + \frac{1}{2}\left(\sum_{i=1}^{m} v_i E_2^{x_{io}} - \sum_{r=1}^{s} u_r E_1^{y_{ro}}\right). \tag{II}$$

If $\sum_i v_i E_2^{x_{io}} \geq \sum_r u_r E_1^{y_{ro}}$ in inequality (II), and because the objective function is being maximized in model (3), the optimal value of θ_o might be greater than one. Hence, the efficiency value in the fuzzy LP problem (3) can exceed one at some α-levels and it proves Proposition **3**.

Appendix B.

The proposed model (3) can be extended to consider both desirable and undesirable outputs as follows:

$$\begin{aligned}
max \quad & \theta_o' = \sum_{r_1=1}^{s_1} u_{r_1} EV\left(\tilde{y}_{r_1o}\right)_D + \sum_{r_2=1}^{s_2} u_{r_2}' E_1^{(y_{r_2o})_{UD}} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \left(\frac{1}{2}E_1^{x_{io}} + \frac{1}{2}E_2^{x_{io}}\right) = 1, \\
& \sum_{i=1}^{m} v_i \left((1-\alpha)E_2^{x_{ij}} + \alpha E_1^{x_{ij}}\right) \geq \sum_{r_1=1}^{s_1} u_{r_1}\left(\alpha E_2^{(y_{r_1j})_D} + (1-\alpha)E_1^{(y_{r_1j})_D}\right) \\
& \qquad + \sum_{r_2=1}^{s_2} u_{r_2}'\left(\alpha E_2^{(y_{r_2j})_{UD}} + (1-\alpha)E_1^{(y_{r_2j})_{UD}}\right), \quad j = 1,\dots,n, \\
& u_{r_1}, u_{r_2}' \geq 0, \quad r_1 = 1,\dots,s_1,\ r_2 = 1,\dots,s_2 \\
& v_i \geq 0, \quad i = 1,\dots,m.
\end{aligned}$$

where in the objective function the smallest output from the output intervals (the lower bounds of the expected intervals) at any given α-cut are chosen for those outputs that are undesirable. Subscripts 'D' and 'UD' are assigned to indicate desirable and undesirable outputs respectively.

In the same manner, DMU_o in the above model is efficient at a particular α-level if $\theta_o^{'*}$ at the α-level is greater than or equal to one; otherwise it is inefficient at that α-level. According to definition 14, the fuzzy efficiency of an evaluated DMU with the trapezoidal fuzzy input $\tilde{x}_{io} = \left(x_{io}^{l}, x_{io}^{m_1}, x_{io}^{m_2}, x_{io}^{u}\right)$ and output $(\tilde{y}_{ro})_D = \left(y_{ro}^{l}, y_{ro}^{m_1}, y_{ro}^{m_2}, y_{ro}^{u}\right)_D$, $(\tilde{y}_{ro})_{UD} = \left(y_{ro}^{l}, y_{ro}^{m_1}, y_{ro}^{m_2}, y_{ro}^{u}\right)_{UD}$ is defined as a trapezoidal fuzzy number as follows:

$$\tilde{\theta}_o^{'*} = (l, \mu_1, \mu_2, \upsilon),$$

Where $\mu_1 = \dfrac{\sum_{r_1=1}^{s_1} u_{r_1}^{*}(y_{ro}^{m_1})_D + \sum_{r_2=1}^{s_2} u_{r_2}^{'*}(y_{ro}^{m_1})_{UD}}{\sum_{i=1}^{m} v_i^{*} x_{io}^{m_2}}$, $\mu_2 = \dfrac{\sum_{r_1=1}^{s_1} u_{r_1}^{*}(y_{ro}^{m_2})_D + \sum_{r_2=1}^{s_2} u_{r_2}^{'*}(y_{ro}^{m_1})_{UD}}{\sum_{i=1}^{m} v_i^{*} x_{io}^{m_1}}$,

$l = \mu_1 - \sum_{r_1=1}^{s_1} u_{r_1}^{*}\left(y_{ro}^{m_1} - y_{ro}^{l}\right)_D - \sum_{r_2=1}^{s_2} u_{r_2}^{'*}\left(y_{ro}^{m_1} - y_{ro}^{l}\right)_{UD}$, $\upsilon = \mu_2 + \sum_{r_2=1}^{s_2} u_{r_2}^{*}\left(y_{ro}^{u} - y_{ro}^{m_2}\right)_D$, and $u_{r_1}^{*}(r = 1,\ldots,s_1)$, $u_{r_2}^{'*}(r = 1,\ldots,s_2)$ and $v_i^{*}(i = 1,\ldots,m)$ are the obtained output and input weight values from the above fuzzy LP problem. l, υ and μ_1, μ_2 are the lower bound, upper bound and the mid values of the fuzzy efficiency measure $\tilde{\theta}_o^{'*}$, respectively.

Appendix C.

Best-worst case scenario of the α-level based approach

In this case, the decision maker is optimistic about the DMU under evaluation (DMU_o), while pessimistic about the rest of the DMUs in the evaluation set (see Lertworasirikul, 2002). The smallest inputs and the largest outputs from the input and output intervals at any given α-cut are chosen for DMU_o, whereas largest inputs and the smallest outputs from these intervals are utilized for the remaining DMUs, in which input-output dataset are considered as fuzzy numbers. In this manner, therefore, DMU_o would be able to provide the largest possible efficiency value when compared with those obtained by all other approaches. The method is also termed as the best-worst case scenario.

Assume that data of inputs and outputs are uncertain and are defined as in the fuzzy DEA model (1). The fuzzy DEA model (1) based on the best-worst case of the basic α-level approach can be transformed into the following LP problem.

$$\begin{aligned}
max \quad & \theta_o = \sum_{r=1}^{s} u_r \left[(1-\alpha) y_{ro}^{u} + \alpha y_{ro}^{m_2} \right] \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \left[(1-\alpha) x_{io}^{l} + \alpha x_{io}^{m_1} \right] = 1, \\
& \sum_{r=1}^{s} u_r \left[(1-\alpha) y_{io}^{u} + \alpha y_{io}^{m_2} \right] - \sum_{i=1}^{m} v_i \left[\left(1-\alpha\right) x_{io}^{l} + \alpha x_{io}^{m_1} \right] \le 0, \quad \text{for } DMU_o,
\end{aligned} \qquad (i)$$

$$\sum_{r=1}^{s} u_r \left[(1-\alpha) y_{ij}^{l} + \alpha y_{ij}^{m_1} \right] - \sum_{i=1}^{m} v_i \left[\left(1-\alpha\right) x_{ij}^{u} + \alpha x_{ij}^{m_2} \right] \le 0, \quad j = 1,...,n, j \ne o,$$
$$u_r \ge 0, \quad r = 1,...,s,$$
$$v_i \ge 0, \quad i = 1,...,m.$$

The results of the efficiency values for five *DMU*s across different α-levels provided by the above-mentioned model (*i*) are recorded in Table I.

Table I

Results of the best-worst case of basic α-level approach

α	DMU1	DMU2	DMU3	DMU4	DMU5
0	1.000	1.000	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000
0.5	0.963	1.000	1.000	1.000	1.000
0.75	0.904	1.000	0.932	1.000	1.000
1	0.855	1.000	0.861	1.000	1.000

Note: Results are based on data from Table 1 and it is termed as the best-worst case scenario.

Results of the efficiency values by Saati et al. model (2002)

Consider Saati et al.'s (2002) model:

$$max\ \theta_o = \sum_{r=1}^{s} u_r \tilde{y}_{ro}$$
$$s.t. \quad \sum_{i=1}^{m} v_i \tilde{x}_{io} = 1, \qquad (ii\text{-}1)$$
$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0, \quad \forall j, \qquad (ii\text{-}2)$$
$$\left(1-\alpha\right) x_{ij}^{l} + \alpha x_{ij}^{m} \le \tilde{x}_{ij} \le \alpha x_{ij}^{m} + \left(1-\alpha\right) x_{ij}^{u}, \quad \forall i, j, \qquad (ii\text{-}3)$$
$$\left(1-\alpha\right) y_{rj}^{l} + \alpha y_{rj}^{m} \le \tilde{y}_{rj} \le \alpha y_{rj}^{m} + \left(1-\alpha\right) y_{rj}^{u}, \quad \forall r, j, \qquad (ii\text{-}4)$$
$$u_r, v_i \ge 0, \quad \forall i, r,$$

where the inputs and outputs of DMUs take the triangular fuzzy forms,

$$\tilde{x}_{ij} = \left(x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u}\right), \tilde{y}_{rj} = \left(y_{rj}^{l}, \mathrm{y}_{rj}^{m}, \mathrm{y}_{rj}^{u}\right).$$

By considering the constraints (*ii-3*) and (*ii-4*) in the above LP problem (*ii*), for $j = o$, the value of $\tilde{x}_{io}$ and $\tilde{y}_{ro}$ are left to vary between the lower and upper bound of the interval. However, given the objective function is to ensure the largest possible output while maintaining a fixed level of input, the model favors the upper bound of $\tilde{y}_{ro}$ at any given α-cut for *DMUo*. Since the model naturally will select the lower bound of the inputs and the upper bound of outputs prior to generating a feasible region for the *DMUo*, one could claim that this is an overly optimistic case. In this case, the decision maker is relatively more optimistic about the *DMU* under evaluation (*DMUo*) than the rest of the *DMU*s in the evaluation set.

This is further validated by comparing Table I (Best-Worst case model) and Table II (Saati's et al,'s model), where it is not surprising to find that the results are identical for the efficient DMUs.

The results are provided in Table II as follows:

Table II

Results of Saati et al.'s model (2002) by the basic α-level approach

α	DMU1	DMU2	DMU3	DMU4	DMU5
0	1.00	1.00	1.00	1.00	1.00
0.25	1.00	1.00	1.00	1.00	1.00
0.5	0.96	1.00	1.00	1.00	1.00
0.75	0.90	1.00	0.93	1.00	1.00
1	0.85	1.00	0.86	1.00	1.00

Note: Results are based on data from Table 1. The analysis revealed that Saati's model generated similar results as the best-worst case of the basic α-level approach.

Appendix D.

Wilcoxon Rank-Sum Test for Efficiency Scores at various α-level

	α-level				
	α = 0	α = 0.25	α = 0.50	α = 0.75	α = 1
[1]Saati et al.'s model	0.812	.738	0.722	0.678	0.636
[1]Proposed model	0.678	.664	0.647	0.632	0.616
Z score	-1.58	-1.58	-2.38**	-2.701**	-2.701**

Note: ** p<0.01, * p<0.05, [1]Median values are provided throughout α-levels

Appendix E.

Dataset of 23 European Union (EU) member countries (except Bulgaria, Luxembourg, Malta and Romania)

Countries	Inputs		Outputs		
	x_1	x_2	y_1	y_2	y_3
Austria	(3852.831, 3859.261, 4088.326)	(4.105, 4.130, 4.143)	(59.038, 61.363, 64.980)	(30431149.7, 30877788.0, 34257611.3)	29.7
Belgium	(5482.183, 5570.069, 5930.748)	(5.501, 5.567, 5.719)	(3.960, 4.359, 5.391)	(50909839.5, 52309906.0, 58854242.2)	4.6
Cyprus	(5129.346, 5168.322, 5931.206)	(2.503, 2.544, 2.615)	(0.080, 0.105, 0.241)	(5301632.4, 5398429.0, 5550466.2)	4.6
Czech Republic	(9118.167, 9143.108, 9958.099)	(4.384, 4.445, 4.545)	(5.278, 5.400, 6.535)	(78434586.7, 81411193.0, 86089153.4)	8.5
Denmark	(4127.390, 5368.883, 5891.529)	(3.575, 3.742, 3.828)	(24.109, 26.276, 26.757)	(25514734.8, 28903322.8, 29669980.7)	19.9
Estonia	(11868.697, 12644.922, 17730.956)	(4.195, 4.263, 4.361)	(2.744, 2.770, 5.642)	(10543759.6, 12824533.3, 15100014.2)	22.8
Finland	(8042.764, 8073.851, 9178.899)	(6.531, 6.934, 7.151)	(25.214, 26.613, 30.189)	(37933246.2, 39404099.0, 40729047.4)	30.3
France	(2339.298, 2355.266, 2595.119)	(4.396, 4.450, 4.468)	(12.655, 13.210, 13.641)	(121944012.9, 122187749.8, 131358717.5)	12.3
Germany	(5663.155, 5680.577, 6609.058)	(4.148, 4.173, 4.201)	(12.144, 14.079, 15.187)	(460132421.3, 466547204.3, 497945234.4)	9.8
Greece	(6152.616, 6167.129, 6649.658)	(2.807, 2.812, 2.821)	(6.221, 9.788, 12.606)	(67103547.9, 69049406.8, 73428038.9)	8.2
Hungary	(2866.826, 2871.584, 3226.158)	(2.698, 2.709, 2.716)	(4.447, 5.026, 6.174)	(24797230.9, 25517632.8, 28949847.7)	7.7
Ireland	(4509.643, 4562.434, 4636.275)	(3.664, 3.671, 3.719)	(10.202, 10.817, 12.493)	(19812387.8, 20137099.0, 22376151.9)	5.0
Italy	(3377.458, 3528.422, 3617.762)	(3.082, 3.114, 3.162)	(15.417, 16.020, 19.090)	(214118419.7, 214849610.5, 240163387.0)	8.9
Latvia	(1646.243, 1649.356, 1984.917)	(1.967, 2.018, 2.063)	(40.230, 41.122, 46.793)	(2690969.9, 2755652.5, 2934636.1)	34.3
Lithuania	(2494.862, 3088.397, 3667.146)	(2.594, 2.622, 2.635)	(3.890, 4.590, 5.196)	(5735129.9, 6101529.3, 6325666.5)	17.0
Netherlands	(5102.297, 5121.683, 5551.620)	(4.949, 5.065, 5.164)	(7.060, 7.440, 8.455)	(78037242.6, 80278953.5, 82032494.9)	4.1
Poland	(5981.237, 5981.847, 6661.263)	(2.449, 2.534, 2.564)	(3.632, 4.112, 5.195)	(203627617.2, 203629078.8, 212774211.0)	8.9
Portugal	(3326.457, 3334.406, 3766.393)	(2.349, 2.469, 2.544)	(28.999, 29.543, 34.383)	(29314495.8, 30625933.5, 31806468.8)	24.5
Slovakia	(5690.809, 5693.558, 5904.246)	(3.399, 3.424, 3.485)	(16.570, 16.609, 18.308)	(23522217.2, 24247998.0, 26893635.9)	10.3
Slovenia	(4100.878, 4265.796, 4285.218)	(3.693, 3.697, 3.836)	(26.492, 28.110, 33.534)	(8558502.4, 8704486.0, 9265279.7)	16.9
Spain	(3548.038, 3683.311, 3805.272)	(3.230, 3.260, 3.356)	(19.523, 20.841, 24.492)	(161830548.8, 166673212.0, 185429526.3)	13.3
Sweden	(2417.824, 2421.166, 2594.181)	(5.417, 5.544, 5.597)	(49.794, 52.610, 53.601)	(18519799.2, 19119676.8, 20949676.2)	47.3

United Kingdom	(3452.942, 3467.132, 3500.572)	(3.671, 3.724, 3.764)	(5.063, 5.356, 6.250)	(247352097.1, 251186691.3, 274600586.4)	2.9

Note: Data from x_1, y_2 are gathered from Carbonmarketdata.com, whereas European commission's Eurostat are the sources for variables x_2, y_1 and y_3. The data has been scaled for the population size of each country gathered from the United Nations Department of Economic and Social Affairs. Intelligent Insights International provides a compilation of sources to validate the above variables.

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