Improving discriminating power in DEA models based on deviation variables framework

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ABSTRACT Lack of discriminating power in efficiency values remain a major contention in the literature of data envelopment analysis (DEA). To overcome this problem, a well-known procedure for ranking efficient units; that is, the super-efficiency model was proposed. The method enables an extreme efficient DMU to achieve an efficiency value greater than one by excluding the DMU under evaluation from the reference set of the DEA model. However, infeasibility problems may persist while applying the super-efficiency DEA model under the constant returns-to-scale (CRS), and this problem tends to be compounded under the variable returns-to-scale (VRS). In order to address this drawback sufficiently, we extend the deviation variable form of classical VRS technique and propose a procedure for ranking efficient units based on the deviation variables values framework in both forms – CRS and VRS. With our proposed method, scholars who wish to prescribe theories based on a set of contextual factors need not remove large number of DMUs that are infeasible, thus avoiding problems in generalizability of their findings. We illustrate the performance and validate the efficacy of our proposed method against alternative methods with two established numerical examples.

Keywords: Data envelopment analysis; Infeasibility; Super-efficiency; Discrimination power; Ranking.

1. Introduction

Data envelopment analysis (DEA) was first developed by Charnes, Cooper & Rhods (1978), which assumes a constant returns-to-scale (CRS). It is popularly known as the CCR model and remained the preferred technique for measuring the relative efficiency of decision-making units (DMUs) due to its intuitive ability to prescribe weights from assessments, which depends on multiple inputs and outputs. Banker, Charnes & Cooper (1984) further extended the CCR model by accommodating for variable returns-to-scale (VRS) and more popularly known as the BCC model.

The difference between CRS and VRS is such that the number of efficient DMUs of the former is a subset of the latter (Ahn, Charnes, & Cooper, 1988). This means that one expects a conventional DEA model based on CRS will have lesser number of efficient DMUs as compared to a VRS derived model.

DEA has been one of the fastest growing areas of the Operations Research and Management Science discipline in the past decades (Emrouznejad, Parker, & Tavares, 2008; Hatami-Marbini, Emrouznejad, & Tavana, 2011). However, a major methodological challenge that still persists in the DEA literature is on the lack of discriminating power of the DMUs.

Discriminating power problems can be easily observed when there are close to no DMUs that are inefficient. These DMUs cannot be ranked, and therefore limits the managerial insights or their ability to be complemented by other techniques. For example, if a business analyst is called to investigate a systemic organizational problem where majority of business units are known to have persistent problems and therefore considered to be inefficient. Due to the computation of relative efficiency in DEA models, these business units would be considered as efficient and a value of 1 will be registered as an outcome of the computation. Given the lack of discriminating power of the efficiency scores, the analyst whose interest lies in uncovering the underlying problem may wish to regress the efficiency scores (considered as dependent variable) on some contextual variables. However, this may not be possible without sacrificing data points.

Cross-efficiency evaluation technique was first proposed by Sexton, Silkman, & Hogan (1986) as an attempt at improving discriminating power of DEA. However, non-uniqueness of the DEA optimal input-output weights, i.e., having multiple solutions to optimal weights in DEA decreases the benefit of the cross-efficiency approach. Although recent steps were proposed such as imposing secondary goals to improve variability of cross-efficiency scores, the solution still leaves the non-uniqueness problem looming (see Cook & Zhu, 2014). More details on cross-efficiency evaluation technique can be found in the following literature (e.g. Doyle & Green, 1994; Green, Doyle, & Cook, 1996; Wang & Chin, 2010, 2011).

A procedure for ranking or discriminate efficient units was later proposed by Andersen & Petersen (1993), which is termed as the super-efficiency approach. A detailed discussion of can also be found in the following literature (e.g. Chen, 2005; Chen, Du, & Huo, 2013; Lee, Chu, & Zhu, 2011). However, it is well-known that when applying the super-efficiency approach (Andersen & Petersen, 1993) to the VRS technique, one may end up with an infeasible solution. Although many attempts have been made, the issue still remains a major problem. For instance, Chen (2005) proposed an approach, claiming that both input-oriented and output-oriented super-efficiency models are needed to fully characterize the super-efficiency model of the evaluated DMUs. Soleimani-Damaneh, Jahanshahloo, & Foroughi (2006) refuted these claims by some counterexamples but did not propose an alternative solution. Lee et al. (2011) further extended Chen (2005) and Cook, Liang, Zha, & Zhu (2009)'s work by providing an approach for addressing the infeasibility issue in the super-efficiency DEA models. However, Lee & Zhu (2012) found that the proposed model by Lee et al. (2011) is feasible when the input data are positive but can be infeasible when some of the inputs are zeroes.

Subsequently, the multiple criteria (or multi-objective) DEA models (Chen, Larbani, & Chang, 2009; Li & Reeves, 1999) were introduced as a means to overcome the discriminating power problem. However, the form of the multiple criteria DEA (MCDEA) model proposed by Li & Reeves (1999) is dependent on the decision maker (DM) conducting interactive programming tasks. By applying the MCDEA model, the approach requires that the three objectives are to be analyzed separately. The aim is to find non-dominated solutions, while allowing the analyst to decide on the most preferred one based on extenuating circumstances. A weighted goal programming approach was further introduced by Bal, Örkcu, & Çelebioğlu (2010) and named as the GPDEA model for solving all 3 objectives of the MCDEA model simultaneously. The GPDEA models sought to convert the MCDEA into its equivalent single objective DEA model, with the intention of improving the discriminating power of efficiency scores. However, Ghasemi, Ignatius, & Emrouznejad (2014) found that there are fundamental flaws associated with the GPDEA models in terms of both weight dispersion and discriminating power, which was later again corroborated by Dos Santos Rubem, De Mello and Meza (2017). More details on the ranking methods can be found in the following literature (Adler, Friedman, & Sinuan-Stern, 2002; Angulo-Meza & Lins, 2002).

Specifically, Ghasemi et al. (2014) proposed a bi-objective weighted MCDEA (BiO-MCDEA) model for solving the MCDEA model and illustrated that the BiO-MCDEA model outperforms the GPDEA method in terms of both weight dispersion and discriminating power. Nevertheless, the BiO-MCDEA model does not provide a full discriminating ability for efficient DMUs in certain cases. We therefore propose a procedure for ranking efficient units based on the deviation variables values framework. We further extend the deviation variable form under VRS technique and the proposed ranking method can be used in both CCR and BCC models without facing infeasibility problems. This solves a major drawback where there can be overwhelmingly large number of DMUs registered to be efficient relative to the total number of DMUs under evaluation (see Cooper, Seiford, & Tone, 2000). In a real world situation, the large number of efficient DMUs would require decision makers to come up with an alternative post-hoc criteria and discriminate the efficient DMUs through some qualitative means. This provides another decision making layer and complexity to fully discriminate DMUs. Our proposed method avoids this need.

The rest of the study is structured as follows. A brief description of the classical DEA models and deviation variable form is given in section 2. Section 3 represents the extension of the deviation variable form under VRS technique and proposes a procedure for ranking efficient units in CCR and BCC models. Section 4 describes the proposed method with two established numerical examples. The performance of our proposed model is compared to other existing methods for performance validation. Section 5 concludes the study.

2. Background

In this section, we provide a short overview of the conventional DEA models and introduce the main concepts needed for the rest of the paper.

2.1. Classical DEA models

Let us consider the case of evaluating the relative efficiency of *n* DMUs which use *m* inputs to produce *s* outputs. The *m*-input-*s*-output data can be expressed as (x_{ij}, $i=1,\ldots,m$, $j=1,\ldots,n$) and (y_{rj}, $r=1,\ldots,s$, $j=1,\ldots,n$). The multiplier form of input-oriented variable returns-to-scale model (Banker, Charnes, & Cooper, 1984) can be formulated as follows:

$$
\begin{aligned}
& max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro} - c_o \\
& s.t. \quad \sum_{i=1}^{m} u_i x_{io} = 1, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} u_i x_{ij} - c_o \le 0, \quad j = 1,\ldots,n, \\
& \qquad u_r \ge 0, \quad r = 1,\ldots,s, \\
& \qquad v_i \ge 0, \quad i = 1,\ldots,m, \\
& \qquad c_o \ \textit{free in sign},
\end{aligned} \tag{1}
$$

where $u_r (r = 1,\ldots,s)$ and $v_i (i = 1,\ldots,m)$ are the input and output weights assigned to input *i* and output *r*, respectively. The input-oriented CCR model (Charnes, Cooper, & Rhodes, 1978) can be easily obtained by assuming $c_o = 0$ in model (1). DMU_o is efficient if the optimal value of the objective function (θ_o^*) is equal to 1, and is considered inefficient if $\theta_o^* < 1$.

2.2. Deviation variable form of CCR model

Li and Reeves (1999) provide a deviation variable form to the CCR model (Charnes, et al., 1978) with three objectives termed as the multi-criteria DEA model. One of the three objectives of the model including its constraints can be expressed in the CCR equivalent:

$$
\begin{aligned}
& \min d_o \text{ or} \left(\max \theta_o = \sum_{r=1}^{s} u_r y_{ro} \right) \\
s.t. \quad & \sum_{i=1}^{m} u_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} u_i x_{ij} + d_j = 0, \quad j = 1,\ldots,n, \\
& u_r \geq 0, \quad r = 1,\ldots,s, \\
& v_i \geq 0, \quad i = 1,\ldots,m, \\
& d_j \geq 0, \quad j = 1,\ldots,n,
\end{aligned} \tag{2}
$$

where u_r & v_i are defined as in model (1), d_o is a deviation variable for DMU$_o$ and d_j is a deviation variable for DMU$_j$. The quantity d_o in the objective function is bounded on an interval [0, 1) and is regarded as a measure of inefficiency. DMU$_o$ is efficient if $d_o = 0$ or, equivalently $\theta_o = 1$, where $\theta_o = 1 - d_o$ is the efficiency measure in a classical DEA.

In comparison with the classical DEA model (1), the input-output weights provided by DEA models (2) is distributed more evenly than those obtained by classical DEA model (1) (see Li & Reeves, 1999). In fact, in a classical DEA problem, if a DMU is efficient, its optimal solution (weight) is almost surely non-unique.

3. Proposed ranking method using the deviation variables

In this section, the deviation variables are used to discriminate and rank efficient DMUs. Similar to model (2), a deviation variable form of BCC model can also be proposed as follows:

$$
\begin{aligned}
& \min d_o \\
s.t. \quad & \sum_{i=1}^{m} u_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} u_i x_{ij} + d_j - c_o = 0, \quad j = 1,\ldots,n, \\
& u_r \geq 0, \quad r = 1,\ldots,s, \\
& v_i \geq 0, \quad i = 1,\ldots,m, \\
& d_j \geq 0, \quad j = 1,\ldots,n, \\
& c_o \text{ free in sign},
\end{aligned} \tag{3}
$$

where u_r, v_i, d_o, & d_j are defined as in model (2). DMU$_o$ is efficient if $d_o = 0$ and the efficiency value of DMU$_o$ is equal to $1 - d_o$.

Our proposed model (3) is formulated as a deviation form of the VRS model. When $c_o = 0$, the model is equivalent to the CRS form. Therefore, the results of our model is very much scale dependent and not scale-invariant. However, one can achieve a full ranking evaluation for all DMUs by using the deviation framework equivalent to the CRS and VRS, as it avoids the infeasibility solutions that may occur through the conventional formulation of CRS or VRS technologies.

In the standard DEA models, inefficient DMUs have scores less than one. However, efficient DMUs are identified by an efficiency score equal to 1, so these DMUs cannot be ranked. One problem that has been discussed frequently in the literature is the lack of discriminating power in DEA applications. To overcome the discrimination power problems, a widely used procedure for ranking efficient units, termed the super-efficiency model was proposed by Andersen and Petersen (1993) hereon referred to as the AP model. The method enables an extreme efficient DMU_o to achieve an efficiency value greater than one by excluding the DMU_o under evaluation from the reference set of the DEA models (see Appendix A). However, by considering the super-efficiency DEA model (AP model) under the BCC technique, the infeasibility of the related linear program is extremely likely to occur.

Although many attempts have been made, the issue still remains a major problem. Our proposed model is able to discriminate and rank efficient DMUs under both CRS & VRS techniques without infeasibility problems. Since DEA hinges on the concept of relative efficiency, attempting to discriminate efficient DMUs with a solution routine that would later discover some DMUs to be infeasible is technically incorrect from a decision-making standpoint. The premise is such that when some DMUs are infeasible, the concept of relative efficiency among the ones that are discriminated informs the decision making process to discard the DMUs that are infeasible and rely on the ranking of the efficient DMUs from the "super-set". However, this set does not comply with the concept of relative efficiency which is a pivotal element in DEA models, thus rendering the concept of 'technology' that makes DEA a practical tool to be without interpretative merit.

To elucidate on this point, let us consider the fact that a remuneration package needs to be allocated based on the performance of the bank branches for that year. Infeasible DMUS would mean that the allocation to these DMUs will not be possible. Even if one disregards these infeasible DMUs and allocate the resources among the rest of the bank branches, the solution of the allocation will not be equitably justified. Banker and Chang (2006) highlighted the poor ranking performance of AP procedure of the super-efficiency model. Contrastingly, they discovered through simulation experiments that the super-efficiency model is able to identify outliers from contaminated data with random noise. Nonetheless, two questions were left unaddressed. First, how does one perform a full ranking when the super-efficiency model is not able to discriminate the efficient DMUs effectively? Second, what does the infeasible solution of certain DMUs mean towards the interpretability of the ranks? In this study, we addressed these gaps by avoiding both infeasibility problems and providing a full ranking procedure for the DMUs.

An effective way to deal with issue is to provide the optimal values of deviation variables for each efficient DMU using the above model (3).

Assume that there are k efficient DMUs and $\left(d_1^*, d_2^*, \ldots, d_n^*\right)_{n_1}$, …, $\left(d_1^*, d_2^*, \ldots, d_n^*\right)_{n_k}$ are the optimal solutions of deviation variables assigned to each efficient DMUn_1, …, DMUn_k respectively. The numerical value of $\frac{d_1^* + d_2^* + \ldots + d_n^*}{n}$ is further provided so as to associate with each efficient DMU, thus allowing the complete ranking of these values from smallest to largest.

Since the efficiency score of DMU$_o$ in the above model is equal to 1- d_o, the smaller value of d_o is equivalent to the larger value of efficiency. On the other hand, the smaller the value of d_o, the less inefficient (and thus the more efficient) for the DMU$_o$. In this case, the objective function of DEA models (2) & (3) minimizes DMUo's inefficiency. That is the reason why we rank the average values of deviation variables from smallest to largest. We use the average of deviation variables as a means to normalize the deviation scores. For ranking purposes, using an average or total sum of deviation will achieve the same ranking outcome.

By solving the LP problem (3) for DMU$_0$, the optimal value of d_o is obtained. If DMU$_0$ is efficient, we also provide the optimal values of d_1 (equivalent to DMU1's inefficiency), d_2 (equivalent to DMU2's inefficiency), …, d_n (equivalent to DMU$_n$'s inefficiency) simultaneously. The value of $d_1 + d_2 + \ldots + d_n$ can therefore be interpreted as the total value of inefficiency associated with the efficient DMU$_0$. Therefore, the sum of the optimal values of deviations can be provided associated with each efficient DMU. Hence, the ranking of these values (or average of all these deviation variables) from smallest to largest can provide the ranking of the efficient DMUs.

It is our aim to achieve the efficiency score for each DMU and the optimal solutions of deviation variables for each efficient using the above model (3) and discriminate the efficient DMUs by ranking the average values from smallest to largest. Thus the proposed method consists of three stages as follows:

Stage 1. First we solve the LP problem (3) for $o = 1, \ldots, n$, to provide the efficiency score of each DMU.

Stage 2. For those DMUs that are efficient, we also provide the optimal solutions of all deviation variables, supposing that DMUn_1, …, DMUn_k are efficient DMUs and $\left(d_1^*, d_2^*, \ldots, d_n^*\right)_{n_1}$, …, $\left(d_1^*, d_2^*, \ldots, d_n^*\right)_{n_k}$ are the optimal solutions of deviation variables associated with these DMUs.

Stage3. The numerical value of $\frac{d_1^* + d_2^* + ... + d_n^*}{n}$ is further provided, which associates with each efficient DMU. Hence, the ranking of these values from smallest to largest can provide the ranking of the efficient DMUs.

With regards to the implication, we would like to stress that we did not alter the non-parametric properties of DEA. For instance, the proposed method is still able to express that if DMU_0 is operating with Y units of outputs with X inputs, then DMU_j should be able to do so if they were to be operating efficiently. Given that in DEA virtual producers can be formed based on composite inputs and composite outputs; our method also retains this feature because the technology frontier is still being preserved.

4. Illustration and validations: two numerical examples

In this section, two numerical examples are presented to describe the proposed models. The purpose is to test out conclusively the performance of our proposed model against similar methods that have been used in two examples.

4.1. The validity of the proposed method as a simple and useful alternative for discriminating efficient DMUs

The first example is taken from Wong & Beasley (1990) whose input and output data are given in Table 1. The data consists of three inputs and three outputs, which are defined as follows:

x_1: number of academic staff
x_2: academic staff salaries in thousands of pounds
x_3: support staff salaries in thousands of pounds
y_1: number of undergraduate students
y_2: number of postgraduate students
y_3: number of research papers

Table 1
The dataset of seven departments in a university

DMU	Inputs			Outputs		
	x_1	x_2	x_3	y_1	y_2	y_3
1	12	400	20	60	35	17
2	19	750	70	139	41	40
3	42	1500	70	225	68	75
4	15	600	100	90	12	17
5	45	2000	250	253	145	130
6	19	730	50	132	45	45
7	41	2350	600	305	159	97

From Table1, let us compute the efficiency scores using model (3) and provide the values of deviation variables that are associated with those DMUs that are efficient. The results are listed in Table 2. First, by setting c_o = 0 in model 3 (CRS technique), the deviation variable form of CCR model determines that DMUs 1, 2, 3, 5, 6, & 7 are efficient (Table 2). Second, the $d_1^*, d_2^*, \ldots, d_n^*$ can also be obtained as a set of optimal deviation variable for each DMU. We therefore provide the set of optimal deviation variables associate with each efficient DMU (see Table 2). Third, by considering model (3) under VRS technique, the deviation variable form of BCC model determines that with the exception of DMU 4, the remaining DMUs are all efficient. We further achieve the set of optimal deviation variables associate with each efficient DMU (see Table 2). Fourth, by ranking the values of $\sum_j d_j^*/n$ from smallest to largest, we can provide the complete ranking of the efficient DMUs (see Table 2).

Table 2
Proposed method based on the seven department evaluation dataset

DMU	CCR form			BCC form		
	Eff. values	$\sum_j d_j^*/n$	Rank	Eff. Values	$\sum_j d_j^*/n$	Rank
1	1	0.218	5	1	0.218	6
2	1	0.145	4	1	0.145	5
3	1	0.805	6	1	0.056	3
4	0.820	–	7	0.977	–	7
5	1	0.049	2	1	0.041	2
6	1	0.126	3	1	0.126	4
7	1	0.045	1	1	0.034	1

The results generated from the DEA-CCR model (Charnes, et al., 1978), DEA-BCC model (Banker, et al., 1984), and the super-efficiency model (Andersen & Petersen, 1993) under both DEA-CCR & DEA-BCC techniques are listed in Table 3.

Table 3
Classical DEA-CCR & DEA-BCC results based on evaluating the seven department dataset

DMU	CCR form			BCC form	
	Eff. values	Super-Eff. values	Rank	Eff. values	Super-Eff. values
1	1	1.830	1	1	2.500
2	1	1.049	6	1	1.058
3	1	1.198	4	1	2.910
4	0.820	–	7	0.977	–
5	1	1.220	2	1	infeasible
6	1	1.191	5	1	1.229
7	1	1.266	3	1	infeasible

From Table 3, DMU 4 is inefficient and the efficiency value of this DMU under CCR model and BCC model is equal to 0.820 & 0.977 respectively. With the exception of DMU 4, the remaining DMUs (i.e. DMUs 1, 2, 3, 5, 6, & 7) are all efficient. For all the efficient DMUs, the CCR super-efficiency DEA technique returns scores for efficient DMUs 1 – 3 & DMUs 5 – 7 as it was designed to perform. However, the resulting BCC super-efficiency DEA model are infeasible for DMU 5 and DMU 7 (see Table 3). It is a major drawback of the super-efficiency technique, where one may obtain infeasible solutions for efficient DMUs; particularly, under the BCC model.

If one were to compare the efficiency results in Table 2 with the efficiency and super-efficiency values reported in Table 3, DMU 4 is inefficient in the DEA-CCR & DEA-BCC models, and the proposed model. However, our proposed model would be able to discriminate and rank efficient DMUs under both CRS & VRS techniques. Contrastingly, the resulting BCC super-efficiency DEA model could not rank DMU 5 and DMU7. This is the ability of the proposed method against the super-efficiency technique; particularly, under BCC model.

4.2. The advantage of the proposed method vs. super-efficiency, MCDEA, BiO-MCDEA, Cross-efficiency evaluation techniques under constant & variable returns-to-scale

The second example is taken from Amirteimoori, Kordrostami, & Nasrollahian (2017), with the input and output data reproduced in Table 4. The data consists of 6 DMUs with 2 inputs and 3 outputs.

Table 4
Dataset for five DMUs

DMU	Inputs		Outputs		
	x_1	x_2	y_1	y_2	y_3
1	32	54	6	0	27
2	37	45	0	14	22
3	24	65	0	0	17
4	39	0	0	12	0
5	0	71	0	9	30

The results of our analysis are presented in Table 5. The 3-step procedure to our analysis is as follows:

First, by using the deviation variable form model (3) under CCR and BCC techniques, model (3) determines that DMUs 1, 2, 4, & 5 are efficient in both CCR and BCC forms. Second, we provide the set of optimal deviation variables ($d_1^*, d_2^*, ..., d_n^*$) associated with each efficient DMU under CCR and BCC form. Third, by ranking the numerical values of $\sum_j d_j^*/n$ assigned to each efficient DMU from smallest to largest, the ranking of efficient DMUs can also be provided (see Table 5).

Table 4
Proposed method results based on the second example dataset

DMU	CCR form			BCC form		
	Eff. values	$\sum_j d_j^*/n$	Rank	Eff. values	$\sum_j d_j^*/n$	Rank
1	1	0.124	3	1	0.123	3

2	1	0.142	2	1	0.140	2
3	0.555	–	5	0.652	–	5
4	1	0.312	1	1	0.312	1
5	1	0.108	4	1	0.107	4

The results generated from the DEA-CCR model (Charnes, et al., 1978), DEA-BCC model (Banker, et al., 1984), and the super-efficiency models (Andersen & Petersen, 1993) for both DEA-CCR & DEA-BCC are shown in Table 6.

Similar to the deviation variable form model, DMUs 1, 2, 4, & 5 are efficient in both CCR and BCC form for the 5 DMU assessment (compare Table 5 and Table 6). The super efficiency values were generated from the AP model for those efficient DMUs for both the CCR and BCC forms (see Table 6).

However, for DMU 1, DMU 4, & DMU 5 and for DMU 1, DMU 2, DMU 4 & DMU 5, the resulting CCR & BCC super-efficiency DEA models produce infeasible results respectively after applying the super-efficiency technique (see Table 6). This is a major drawback of the super-efficiency technique, where one may obtain infeasible solutions for efficient DMUs, especially under the VRS form.

Table 5
Classical DEA-CCR & DEA-BCC results based on the second example dataset

DMU	CCR form		BCC form	
	Eff. values	Super-Eff. values	Eff. values	Super-Eff. values
1	1	infeasible	1	infeasible
2	1	1.095	1	infeasible
3	0.555	–	0.652	–
4	1	infeasible	1	infeasible
5	1	infeasible	1	infeasible

When we compare the results of the efficiency scores in the CCR and BCC forms (Table 6) against the results of our proposed model in Table 5, we found the same set of efficient DMUs. However, an attempt to rank the efficient DMUs by the super-efficiency approach revealed 3 infeasible solutions for DMU 1, DMU 4, & DMU 5 in the CCR form, and 4 infeasible solutions DMU 1, DMU2, DMU 4, & DMU 5 in the BCC form (see Table 6). This highlights the drawback of using the AP super-efficiency ranking method for both CRS and VRS techniques.

We further applied two techniques in the literature that were formulated for the purpose of improving discrimination power, which are the MCDEA model (Li & Reeves, 1999) and the BiO-MCDEA model (Ghasemi, Ignatius, & Emrouznejad, 2014). The efficiency values of Table 7 shows that the MCDEA and BiO-MCDEA models are not able to improve the discrimination power for datasets that have inputs or outputs with zeroes or infinitesimal values.

Table 7
Minsum of MCDEA and BiO-MCDEA models results based on the second example dataset

DMU	minsum of MCDEA results		BiO-MCDEA results	
	Eff. values	Rank	Eff. values	Rank
1	1	1	1	1
2	1	1	1	1
3	0.549	5	0.549	5
4	1	1	1	1
5	1	1	1	1

In addition, we would like to emphasize that our method should not be confused with the cross-efficiency method. Our proposed method provides unique solutions for the weights and in fact, it is able to provide a better discrimination power. In addition, our method is a self-evaluation method, which still preserves the concept of relative efficiency. The cross-efficiency method, on the other hand, is a peer evaluation method that does not preserve the interpretation of the frontier in deriving the weights. The proposed method provides average of deviations for each efficient DMU. This contrasts the cross-efficiency evaluation technique, where the same set of optimal weights for the DMU under evaluation is applied across other DMUs to form the weighted DMUs the cross efficiency scores. Hence, cross-efficiency method has a major drawback in terms of non-uniqueness of weights (Doyle and Green, 1994).

Our proposed method provides unique solutions for the weights and is able to provide full discrimination of efficient DMUs. The input-output weights provided by DEA models (2) & (3) are distributed more evenly than those obtained by classical DEA model (1) (see Appendix B). In other words, we provide the deviation variables for each efficient DMU using DEA model (3), in which the input-output weights are distributed more evenly and this is the ability of the proposed model vs. DEA cross-efficiency evaluation, thus avoiding non-uniqueness of the DEA optimal input-output weights or multiple optimal weights.

To the best of our knowledge, we have not come across a method that provides a full ranking procedure which can still retain the DEA "technology" for both CRS and VRS, while avoiding non-uniqueness of weights and infeasibility problems. In terms of the disadvantage (or under development), we believe that more effort could be done on the forefront of weight restrictions and deviational variables for future research.

5. Conclusion

In this paper, we have extended the deviation variable form under the BCC model and proposed a procedure for ranking efficient units based on the deviation variable values framework under CRS and VRS techniques in order to improve the discriminating power properties of the efficiency scores. A threestep procedure was suggested to provide a ranking method and discriminate efficient DMUs in both CCR and BCC forms. The first numerical example used in this paper is to demonstrate the performance of the proposed method over the super-efficiency technique (AP model). In terms of a real application, the second example was further used to describe the efficacy of the proposed approach. Based on the results of the examples, it can be concluded that the proposed method outperforms the super-efficiency techniques in terms of discriminating power. We further stress that this has larger methodological implications and would further lend utility to DEA as a complementary method that aids predictive analysis. The inability to discriminate efficiency scores has prevented management scholars and empirical researchers to regress these scores to contextual factors for prescribing theories. To negotiate around the issue, most scholars that seek to have explanatory variables predicting the efficiency outcomes would need to sacrifice DMUs that possess infeasible solutions from their analysis. Given that in a real case study where they may be a large number of infeasible solutions, removing these DMUs from further analysis will have severe effects to the degradation of theory under study. This is because the removal of infeasible solutions would remove DMUs may invalidate an initial sampling plan that was designed to preserve the integrity of the solution. Hence, the generalization of the contextual factors to an underpinning theory may be erroneous given that a large number of infeasible solutions were dropped from further analysis. We suggest future research to pursue this line of inquiry into the methods that combine a multiple-regression approach and DEA.

Appendix A

The super-efficiency technique under DEA model (1) can be expressed as

$$
\begin{aligned}
& max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro} - c_o \\
& s.t. \quad \sum_{i=1}^{m} u_i x_{io} = 1, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} u_i x_{ij} - c_o \leq 0, \quad j = 1,\dots,n,\ j \neq o, \\
& \qquad u_r \geq 0, \quad r = 1,\dots,s, \\
& \qquad v_i \geq 0, \quad i = 1,\dots,m, \\
& \qquad c_o \ \textit{free in sign.}
\end{aligned}
$$

Appendix B

Model (3) Input-Output weights results based on second example dataset					
DMU	Input weights		Output weights		
	v_1	v_2	u_1	u_2	u_3
1	0.0039	0.0162	0.0001	0.0043	0.0370
2	0.0045	0.0185	0.0001	0.0049	0.0424

3	0.0167	0.0092	0.0632	0.0001	0.0001
4	0.0196	0.0106	0.1988	0.0833	0.0001
5	0.0034	0.0141	0.0001	0.0037	0.0322

Classical DEA model Input-Output weights results based on second example dataset

DMU	Input weights		Output weights		
	v_1	v_2	u_1	u_2	u_3
1	0	0.0185	0	0	0.0370
2	0	0.0217	0	0.0020	0.0442
3	0.0167	0.0092	0	0.	0.
4	0.0256	0.0106	0	0.0833	0.
5	0.0044	0.0141	0	0.	0.0333

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