Reference Point Based Multi-Objective Optimization of Reservoir Operation: A Comparison of Three Algorithms

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Abstract

Traditional multi-objective evolutionary algorithms treat each objective equally and search randomly in all solution spaces without using preference information. This might reduce the search efficiency and quality of solutions preferred by decision makers, especially when solving problems with complicated properties or many objectives. Three reference point based algorithms which adopt preference information in optimization progress, e.g., R-NSGA-II, r-NSGA-II and g-NSGA-II, have been shown to be effective in finding more preferred solutions in theoretical test problems. However, more efforts are needed to test their effectiveness in real-world problems. This study conducts a comparison of the above three algorithms with a standard algorithm NSGA-II on a reservoir operation problem to demonstrate their performance in improving the search efficiency and quality of preferred solutions. Under the same calculation times of the objective functions, Pareto optimal solutions of the four algorithms are used in the empirical comparison in terms of the approximation to the preferred solutions. Three performance indicators are then adopted for further comparison. Results show that R-NSGA-II and r-NSGA-II can improve the search efficiency and quality of preferred solutions. The convergence and diversity of their solutions in the concerned region are better than NSGA-II, and the closeness degree to the reference point can be increased by 42.8%, and moreover the number of preferred solutions can be increased by more than 3 times when part of objectives are preferred. By contrast, g-NSGA-II shows worse performance. This study exhibits the performance of three reference point based algorithms and provides insights in algorithm selection for multi-objective reservoir optimization problems.

Keywords: multi-objective optimization, NSGA-II, preference, reservoir operation.

Introduction

Reservoir plays a role in regulating river flows to meet the demands from multiple water users. Its operation and management are affected by the preferences which are related to baseline operating policies, priority of different water demands, water availability and interests of the reservoir (Chou and Wu 2014; Giuliani et al. 2014; Israel and Lund 2008). Taking optimal solution selection as an example, solutions with superiority of domestic water uses are more preferable than those with better performance on irrigation water uses as domestic water demands normally have a higher water supply priority. Solutions with a larger hydropower generation are preferred by power plant operators as these can bring economic benefits. Therefore, it is necessary to take the preference into consideration carefully in the optimization of reservoir operation.

In previous studies, preferences have been considered in several ways in optimizing reservoir operation (Thiele et al. 2009; Fonseca and Fleming 1998). A well-known way is to aggregate different objectives with specified weights into a single one by using aggregating functions, and then the problem can be solved by global optimization methods (Thiele et al. 2009; Barati 2011; Chu et al. 2015). This approach considers the importance of each objective to reflect the relevant preference but it not only has difficulties in deciding the importance properly but also needs a separate run for different sets of weights (Deb and Sundar 2006; Thiele et al. 2009; Chu et al. 2015). To avoid the drawbacks of the single objective optimization, standard multi-objective evolutionary algorithms are applied to provide a set of non-dominated solutions (i.e., Pareto optimal solutions) simultaneously (Tang et al. 2019; Thiele et al. 2009; Giuliani et al. 2014; Fonseca and Fleming 1998). The standard multi-objective evolutionary algorithms treat each objective equally important and search randomly in all solution spaces without applying any preference strategy in their search progress (Zarei et al. 2019; Hosseini 2016; Chu et al. 2015; Barati et al. 2014). As a result, the search efficiency and quality of solutions in the region of interest are low and many Pareto optimal solutions are in uninterested region. There is a possibility that those Pareto optimal solutions which are in the region of interest are not derived especially in the problems with a large number of objectives (Li et al. 2018; Deb and Sundar 2006). To help improve the search efficiency and quality of preferred solutions, incorporating preferences into the search process of multi-objective evolutionary algorithms has gained attention recently (Luo et al. 2015). Additional preference information is used to guide the search toward the preferred part of the Pareto front and more preferred solutions, i.e., solutions in the region of interest, can be provided (Bechikh et al. 2015; Thiele et al. 2009; Deb and Sundar 2006). Many preference based multi-objective evolutionary algorithms have been proposed and they are usually variants of the existing standard evolutionary algorithms (Li et al. 2018; Bechikh et al. 2015; Mohammadi et al. 2012; Said et al. 2010; Molinac et al. 2009; Deb and Sundar 2006). In these preference based multi-objective evolutionary algorithms, preference information is expressed with different methods, such as reference point (Deb and Sundar 2006), reference direction (Deb et al. 2007) and trade-offs (Branke et al. 2001).

Reference point is a natural way to express preference (Mohammadi et al. 2012; Said et al. 2010; Molinac et al. 2009). Deb and Sundar (2006) proposed a modified NSGA-II called R-NSGA-II by modifying a crowding operator based on reference point. Molinac et al. (2009) developed a reference point based optimization algorithm, g-NSGA-II, which replaces Pareto dominance relation with a new variant, g-dominance. Said et al. (2010) extended NSGA-II to r-NSGA-II based on a new variant of Pareto dominance relation, i.e., r-dominance. These reference point based algorithms are applied into benchmark problems in the evolutionary multi-objective optimization community. However, more efforts are needed to demonstrate their effectiveness in real engineering problems, especially in reservoir optimization problems.

This paper aims to study the effectiveness of the incorporation of preference information in multi-objective reservoir optimization by comparing three reference point based algorithms, i.e., R-NSGA-II, r-NSGA-II, and g-NSGA-II on a reservoir operation problem. The original NSGA-II is used as a baseline in comparison. Three performance indicators are adopted to compare the convergence and diversity of solutions in the concerned region, and closeness to the preference point after an empirical comparison. The Nierji Reservoir is taken as a case study to evaluate the performance of the three reference point based algorithms in reservoir operation.

Methodology

Reference Point

Reference point is a vector supplied by a decision maker for expressing preference information. Each of its components represents the desired value at each individual objective. The reference point based multi-objective algorithms apply reference point(s) to guide the optimization search progress to focus on the region of interest (Molinac et al. 2009; Deb and Sundar 2006). A reference point can be set in feasible area or infeasible area as shown in Fig. S1 of supplemental materials (Said et al. 2010; Deb and Sundar 2006).

In order to set a reference point, NSGA-II with a small amount of model simulations can be ran to obtain a set of initial solutions. Afterwards, the reference point can be set with the following steps: (1) store the best value and the worst value of each objective; (2) select an arbitrary solution; (3) adjust the object value of the preferred objectives of the selected solution to an expected value. The expected value is better than the best value of preferred objectives and is not a fixed value. For a minimization optimization problem, the smaller of the objective, the better the solution is. (Liu et al. 2014). Specifically, a reference point in an *M*-objective minimization problem can be set as

$$F_a = (f_1(\mathbf{x}) - a_1, f_2(\mathbf{x}) - a_2, \ldots, f_m(\mathbf{x}) - a_m, \ldots, f_M(\mathbf{x}) - a_M) \quad (1)$$

where $\mathbf{x}$ is one of the initial solutions; $f_m(\mathbf{x})$ is the m-th objective value of solution $\mathbf{x}$. α_m is a preference adjustment value. When the m-th objective is a preferred objective, the adjustment value is positive and larger than the difference between $f_m(x)$ and the best value of the objective. Otherwise, it can be set to be zero or a small positive value.

Reference Point based multi-objective Algorithm

R-NSGA-II

R-NSGA-II, proposed by Deb and Sundar (2006), achieves the preferred solutions by modifying the crowding distance operator of NSGA-II and are validated on benchmark problems with 2 to 10 objectives. The crowding distance is measured by the weighted Euclidean distance shown as formula (2) (Deb and Sundar 2006).

$$d(\mathbf{x}',\mathbf{p}) = \sqrt{\sum_{m=1}^{M} w_m \times ((f_m(\mathbf{x}') - f_m(\mathbf{p})) / (f_m^{\max} - f_m^{\min}))^2} \quad (2)$$

where $\mathbf{x}'$ is a solution vector of each generation population; $\mathbf{p}$ is a reference point vector; M is the number of objectives; w_m is weight of m-th objective; f_m^{max} and f_m^{min} are the maximum and minimum function values of m-th objective in a population.

The basic search steps of R-NSGA-II are similar to NSGA-II: a non-dominated sorting is applied to classify the combined population of the parent and offspring populations into different levels of non-domination. Solutions selected from subsequent non-dominated fronts in the order of their level ranking are kept as candidates (Deb et al. 2002; Deb and Sundar 2006), from which the next generation population are chosen by the crowding distance operator (Deb and Sundar 2006). In

R-NSGA-II, the shorter the modified Euclidean distance between the solution and the reference point, the more likely it is to be preserved for the next generation.

r-NSGA-II

This algorithm, presented by Said et al. (2010), substitutes the Pareto dominance relation of NSGA-II by a *r-dominance* relation. It has been tested on benchmark problems with up to 10 objectives. The *r-dominance* calculates the weighted Euclidean distance between each solution and the reference point first. Then the *r-dominance* relation between two candidates, for instance solution **a** r-dominates solution **b**, can be determined according to the following:

(1) solution **a** dominates solution **b** in the Pareto sense;

(2) $d(\mathbf{a},\mathbf{b},\mathbf{p}) = (d(\mathbf{a},\mathbf{p}) - d(\mathbf{b},\mathbf{p})) / (d_{\mathrm{max}} - d_{\mathrm{min}}) < -a, \qquad a \in [0,1]$

where $d(\mathbf{a},\mathbf{p})$ and $d(\mathbf{b},\mathbf{p})$ are weighted Euclidean distance of solution **a** and solution **b** to the reference point **p** respectively; d_{max} and d_{min} are the maximum and minimum weighted Euclidean distance values; α is the *non-r-dominance* threshold which controls the spread of the Pareto optimal solution near region of preference.

g-NSGA-II

g-NSGA-II couples a *g-dominance* to replace the Pareto dominance relation of NSGA-II, and was applied to 2 two-objective test problems by Molinac et al. (2009). During the non-dominated sorting, a flag setting should be defined firstly for all solutions: a solution is marked with 1 if all objectives of the solution are less than or equal to the corresponding objective values of reference point, or all are greater than or equal to the corresponding objective values of reference point; otherwise, it is flagged with 0. Based on this flag setting, one of the following conditions can be used to determine *g-dominance* relation of two solutions. Take solution **a** and solution **b** as example:

(1) If the flag value of solution **a** is greater than that of solution **b**, solution **a** g-dominates solution **b**;

(2) If the flag value of solution **a** is equal to that of solution **b** and all objectives of solution **a** are less than or equal to that of solutions **b** (at least one is less than relation), solution **a** g-dominates solution **b**.

Performance Indicators

R-Metrics

R-metrics were specifically proposed to evaluate the quality of preferable Pareto optimal solutions of preference based algorithms (Li et al. 2018). R-metrics consist of two indicators, i.e., R-IGD and R-HV, which reveal the convergence and diversity of Pareto optimal solutions in the region of interest simultaneously. They are built on two performance metrics designed for whole Pareto optimal front, Inverted Generational Distance (IGD) metric and Hypervolume (HV) metric and are suitable for partial preferable Pareto optimal solutions (Li et al. 2018). The lower the

R-IGD value or the larger the R-HV value, the better the quality of the preferable Pareto optimal solutions. More details can be found in Li et al. (2018).

Mean Euclidean Distance

Distance of resulting Pareto optimal solutions to the target solutions are usually an indicator adopted for algorithm comparison (Zitzler et al. 2000; Liu et al. 2014). In a reference point based algorithm, solutions with shorter distance to the reference point represent they are more close to region of interest or preference (Liu et al. 2014; Deb and Sundar 2006) and are more likely to be selected. The following equation is applied to assess the mean Euclidean distance value of a set of preferred Pareto optimal solutions to represent closeness degree toward the preference region. The shorter the mean distance of solutions, the better the preference expression of the solutions.

$$\mathrm{Distance} = \sum_{k-1}^{K} d(\mathbf{x_k}, \mathbf{p}) / K = \sum_{k=1}^{K} \sqrt{\sum_{m=1}^{M} ((f_m(\mathbf{x_k}) - f_m(\mathbf{p})) / (f_m^{\max} - f_m^{\min}))^2} / K \tag{3}$$

where K is the number of a set of Pareto optimal solutions; $\mathbf{x_k}$ is the k-th Pareto optimal solution.

Number of Acceptable Alternatives

Reference point based algorithms which employ a biased search are expected to provide more acceptable alternatives (Li et al. 2018). For the calculation of the number of acceptable alternatives, a satisfaction threshold of each preferred objective is given firstly. In this paper, the value of 10% superior ranking order in each objective among the NSGA-II resulting solutions is taken as the satisfaction threshold. Then, a solution, whose value of preferred objective is higher than the satisfaction threshold is regarded as an acceptable alternative. The number of acceptable alternatives can be counted thereafter. This counted indicator, representative of quantity of preferable solutions, shows the searching possibility of alternatives of an algorithm. The bigger the number of acceptable alternatives, the better the corresponding reference point based algorithms.

Case study

Description of the Reservoir

The Nierji Reservoir, located in the main stream of Nen River in northeast of China is taken as a case study. The reservoir with an average annual inflow of 10.65×10^9 m^3 has multiple purposes including hydropower generation, public water supply for domestic and industrial uses, water supply for agricultural use, environmental water requirements downstream and complementing wetland requirements downstream. Its installed capacity ($P_{\max}$) and firm capacity (P_{firm}) are 250 MW and 35MW respectively. According to the design conditions, the reservoir needs to provide annual public water supply of 2.0×10^9 m^3, irrigation demand of 1.65×10^9 m^3 (from the last 10 days of April to the first 10 days of October), and downstream environmental flow of 1.37×10^9 m^3. Additionally, it needs to supply 82×10^6 m^3 per ten days from the last 10 days of August to the last 10 days of September to the wetland downstream. The Nierji Reservoir are operated in accordance with 10 day's operation rule curves which provides operation guidelines for reservoir managers. The basic operation rule curves of the Nierji Reservoir are shown schematically in Fig. S2 of the Supplemental Materials.

The Formulation of Reservoir Operation

The objectives of the reservoir operation include maximizing hydropower generation, minimizing the public water scarcity, minimizing environmental requirements shortage, minimizing the irrigation deficit, and minimizing wetland replenishment shortage. The constraints include the water balance constraint, the water storage limits, the flow limits of hydraulic turbine, the electricity generation capacity constraint, the reliability requirements and the water supply priority constraints. The decision variables are the control points on the reservoir operation rule curves. Considering the word limits, the constraints and the decision variables are shown in the Supplemental Materials. The functions of the objectives are as follows.

Maximize average annual hydropower generation (*Electricity*)

$$\max\ Electricity = \left(\sum_{i=1}^{N}\sum_{j=1}^{J} P_{i,j} \times t_{i,j}\right) / N \tag{4}$$

where $P_{i,j}$ represents the output of hydropower plant during time period j of the i-th simulation year; N is the total number of the simulation years; J is the number of operation periods per year; $t_{i,j}$ represents number of hours in time period j of the i-th simulation year.

Minimize the average public water supply shortage (*Public*)

$$\min\ Public = \sum_{i=1}^{N}\sum_{j=1}^{J} (DP_{i,j} - WP_{i,j}) / N \tag{5}$$

where $DP_{i,j}$ and $WP_{i,j}$ represent public water demands and actual public water supply during time period j of the i-th simulation year respectively.

Minimize the average environmental requirements shortage (*Environment*)

$$\min\ Environment = \sum_{i=1}^{N}\sum_{j=1}^{J}(DE_{i,j} - WE_{i,j}) / N \tag{6}$$

where $DE_{i,j}$ and $WE_{i,j}$ represent environmental requirements and actual water supply for downstream environment during time period j of the i-th simulation year respectively.

Minimize the average irrigation deficit (*Irrigation*)

$$\min\ Irrigation = \sum_{i=1}^{N}\sum_{j=1}^{J}(DI_{i,j} - WI_{i,j}) / N \tag{7}$$

where $DI_{i,j}$ and $WI_{i,j}$ represents irrigation requirements and actual water for irrigation during time period j of the i-th simulation year respectively.

Minimize the average wetland replenishment shortage (*Wetland*)

$$\min\ Wetland = \sum_{i=1}^{N}\sum_{i=1}^{J}(DW_{i,j} - WW_{i,j}) / N \tag{8}$$

where $DW_{i,j}$ and $WW_{i,j}$ represents wetland requirements downstream and actual water replenishment for wetland during time period j of the i-th simulation year respectively.

Reference Point Setup

As public water demands (domestic and industrial water uses) and environmental requirements have higher priorities than irrigation and wetland requirements, they should be taken into consideration firstly when setting up the reference point. Besides, hydropower generation can bring economic interest and enhance the security of a power grid. It will also be a pursuit in reservoir operation management. In short, public water demands, environmental requirements and hydropower generation are the main considerations in this multi-objective reservoir problem. Therefore, the preferred objectives in reference points could be one or the combination of these relative ones.

Based on the preference analysis, four cases are set firstly: (1) the reference point 1 to show preference for hydropower generation; (2) the reference point 2 to show preference for downstream environment protection; (3) the reference point 3 to show preference for hydropower generation and public water demands; (4) the reference point 5 to show preference for hydropower generation, public water demands, and downstream environment protection. Besides, preference for two low water priority uses, irrigation and wetland requirements is also used as shown in reference point 4. An extreme situation that all objectives are preferred is set as reference point 6. According to the results obtained by NSGA-II with 5000 simulations, values of six reference points are set as Table 1. It is worth mentioning that the objective value of each reference point are not unique.

Table 1. Desired Objective Values of Reference Points

Reference point	(*Electricity*, *Public*, *Environment*, *Irrigation*, *Wetland*) (10^6 kWh, 10^6 m^3, 10^6 m^3, 10^6 m^3, 10^6 m^3)
Reference point 1	(**556**, 25, 10, 60, 18)
Reference point 2	（542, 25, **0**, 60, 18）
Reference point 3	（**556**, **0**, 10, 60, 18）
Reference point 4	（542, 25, 10, **15**, **0**）

Reference point 5	（**556, 0, 0,** 60, 18）
Reference point 6	（**556, 0, 0, 15, 0**）

Items highlighted in bold are preferred objectives in each reference point.

Results and Discussion

This section describes the comparison results of the three reference point based algorithms, i.e., R-NSGA-II, r--NSGA-II, and g-NSGA-II with the standard algorithm NSGA-II. With the ten-day inflow data of a long time series from 1956 to 2013, Pareto optimal solutions of each algorithm are derived under six cases. The parameters for the optimized algorithms are listed in the Supplemental Materials. Considering the randomness of the evolutionary algorithms, each case is run 50 times. The 50 times' solutions of each algorithm in each case are put together to derive the final Pareto optimal solutions through the non-dominated sorting. All Pareto optimal solutions and reference points under six cases are normalized, and 1 represents the best objective value and 0 represents the worst value. For comparison among different cases, each objective applies the same minimum values and the same maximum values in the normalization process, which are determined by all Pareto optimal solutions and reference points under six cases.

Comparison of Pareto Optimal Solutions

Descriptive Statistics

Fig. 1 shows the box plots of each objective values of the Pareto optimal solutions achieved by four algorithms under six different reference point cases. Comparing different sub-figures, it can be seen that the box range of each objective obtained by the reference point preferred algorithms changes when the reference point changes indicating the reference point preferred algorithms play the function for searching different part of optimal Pareto solutions along with different preferences.

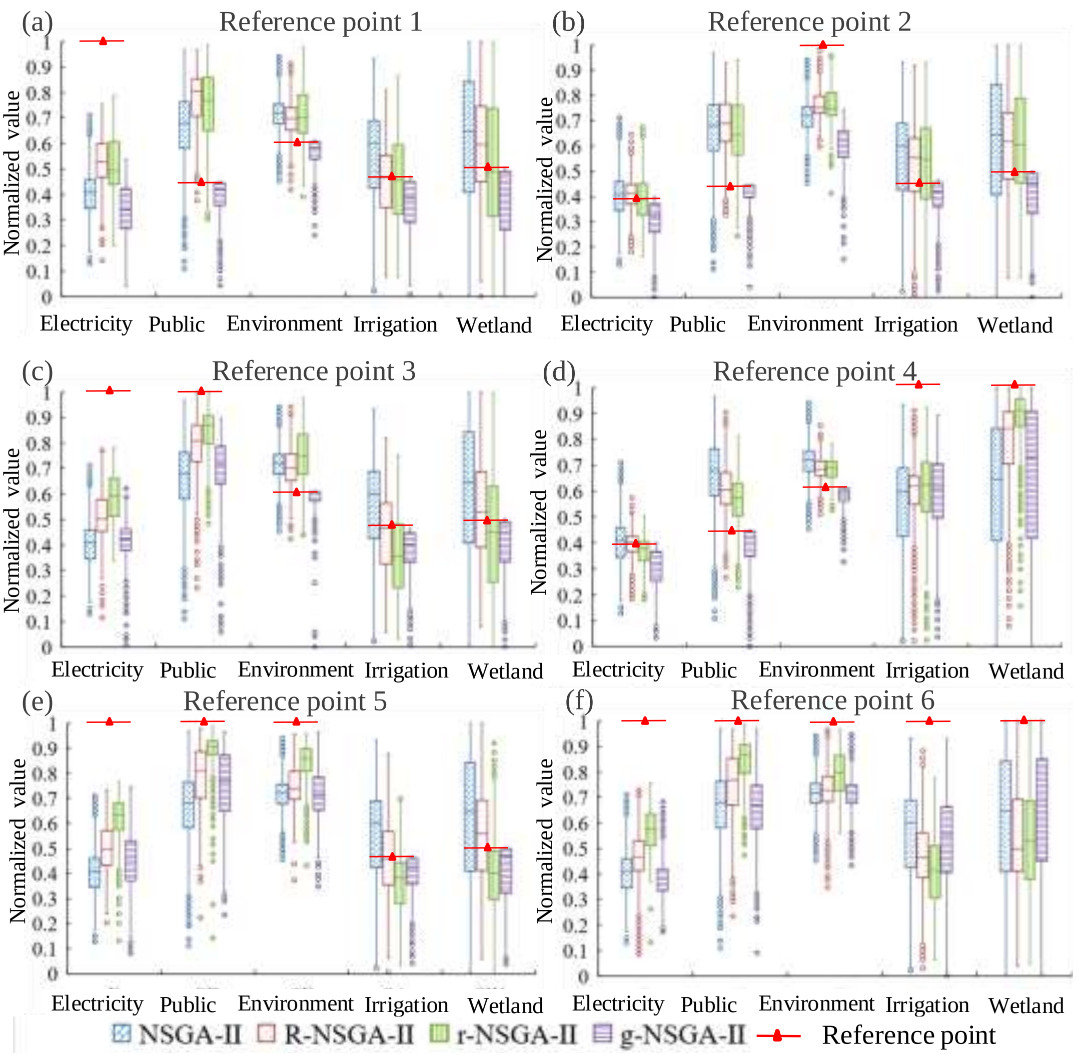

Fig. 1 Pareto optimal solutions from the four algorithms.

Most of the optimal Pareto solutions obtained by R-NSGA-II and r-NSGA-II have good performance on the preferred objectives when part of the objectives are preferred. As shown in Fig. 1, the boxes of the preferred objectives for R-NSGA-II and r-NSGA-II are higher than that of NSGA-II in all reference points except reference point 6, that is, the objective value of the preferred objectives in most of the optimal Pareto solutions obtained by R-NSGA-II and r-NSGA-II are more close to the best value of the preferred objectives and are better than that obtained by NSGA-II. Taking Fig. 1(a) as an example, the upper quartile of the preferred objective (*Electricity*) for NSGA-II with value of 0.46 is almost equal to the lower quartile for R-NSGA-II and r-NSGA-II. This indicates the 75% of the Pareto optimal solutions with high values on *Electricity* in R-NSGA-II and r-NSGA-II do as well as the top 25% of solutions in NSGA-II. Thus, one solution selected from the Pareto optimal solutions of R-NSGA-II or r-NSGA-II has a high possibility being interested in.

When all objectives are considered as preferred objectives, i.e., reference point 6, R-NSGA-II and r-NSGA-II have good performance on some objectives while bad on the others, as shown in Fig. 1(f). Annual hydropower generation (*Electricity*), the average public water supply shortage (*Public*) and the average environmental requirements shortage (*Environment*) are close to the best objective value among most of the Pareto optimal solutions obtained by R-NSGA-II and r-NSGA-II while the average irrigation deficit (*Irrigation*) and the average wetland replenishment shortage (*Wetland*) are opposite. This results from the automatic preference mechanism which searches solutions with better performance in high priority objectives, i.e., *Electricity*, *Public*, and *Environment* when all objectives are preferred. Due to trade-off, these solutions have a worse performance in low priority objectives, i.e., *Irrigation* and *Wetland*.

By contrast, g-NSGA-II cannot supply more Pareto optimal solutions with high values on preferred objectives compared to NSGA-II. In six panels of Fig. 1, most of the solutions of g-NSGA-II are worse than or equal to the standard algorithm NSGA-II in the preferred objectives. This is because g-NSGA-II applies the strict *g-dominance* to approximate the efficient solutions around the area of the most preferred point. The *g-dominance* applies a flag setting of 0 or 1 before non-dominated sorting. Solution with all objectives less than or equal to the reference point, or all objectives greater than or equal to the reference point is marked with 1. Otherwise, it is marked with 0. Solution flagged with 1 dominates solution flagged with 0, and thus has a higher possibility to be retained for the next generation during the search process. However, when many objectives are considered, solutions which can meet the condition of being marked with 1 are few and this makes less solutions to be kept for the next generation. As a result, it is not easy to find more Pareto optimal solutions with high values on preferred objectives.

In addition, the boxes range of *Electricity*, *Public* and *Environment* of solutions by the reference point based algorithms in six sub-figures has a more obvious change than *Irrigation* and *Wetland*, that is, *Electricity*, *Public* and *Environment* are more sensitive. The reason is that *Public* and *Environment* have higher water supply priority in this reservoir problem and preference expressed on them gets a well implement for reference point based algorithms.

Best solutions identification

This part focuses on identifying solutions with the best values on preferred objectives for further compassion. Pareto optimal solutions of each algorithm are conducted a non-dominated sorting procedure in terms of preferred objectives first. The solutions of each algorithm kept after the procedure are shown in Fig. 2. All of them are merged as a recombinant set and a non-dominated sorting procedure conducted again to identify solutions with best values on preferred objectives then. These solutions are named as Re-sorted Pareto optimal solutions and marked with filled dots. They are the best solutions in terms of the preferred objectives.

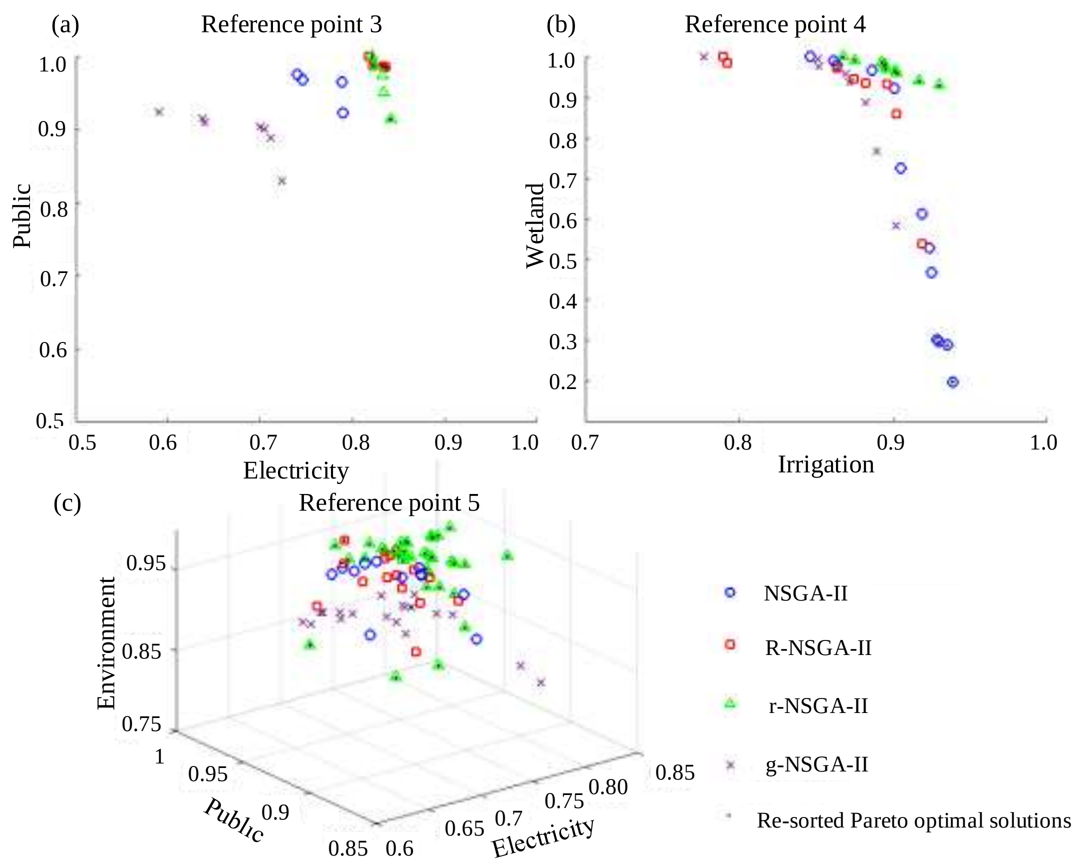

Fig. 2 Best solutions in terms of preferred objectives for reference points 3, 4 and 5.

It is clear that R-NSGA-II and r-NSGA-II show superiority in finding best solutions in terms of part of specific objectives. Fig. 2 shows that the best solutions come from R-NSGA-II and r-NSGA-II in reference points 3 and 5. The best solutions in reference points 1 and 2 are also from r-NSGA-II and R-NSGA-II respectively and this can be seen from Figs. 1(a) and (b). These solutions dominate other solutions in terms of the preferred objectives and this demonstrates that R-NSGA-II and r-NSGA-II can get solutions with the best values of the preferred objective. This reveals the preference strategy of the two reference point based algorithms play the function of guiding the search space to the region of interest. Therefore, the quality of preferred solutions is improved.

When the preferred objectives are *Irrigation* and *Wetland*, most of the best solutions in term of these two objectives come from r-NSGA-II and some of them are from NSGA-II. This is because objectives *Irrigation* and *Wetland* have lower priority in this reservoir problem. Although they are set as preferred objectives, the lower priority makes them the last objectives to be satisfied. As a result, the reference point algorithms do not show absolute advantage in finding solutions with best values on *Irrigation* and *Wetland*. The best solutions of reference point 6 are not demonstrated here as they come from four different algorithms. The performance of four algorithms cannot be well evaluated with this method. Other ways are needed for deep comparison of the algorithms and thus three performance indicators are adopted for further comparison.

Comparison of Performance Indicators

R-Metrics

The R-metrics values which reveal the convergence and diversity of preferred Pareto optimal solutions are listed in Table 2. It is clear that the values of g-NSGAII for reference points 1, 2, 3 and 4 are null in the table indicates that the solutions obtained by g-NSGA-II are dominated by other algorithms. This implies that the solutions obtained by g-NSGA-II have not converged to the optimal Pareto front. In other words, g-NSGA-II has difficulty in driving solutions towards to optimal Pareto front. Moreover, though the values of g-NSGAII for reference points 5 and 6 are not null, the R-IGD and R-HV values are worse than that of NSGA-II. All the null values and the worse values indicates g-NSGAII does not improve the convergence and diversity of Pareto optimal solutions in the region of interest. This reveals g-NSGAII do not play the function of reference point for this reservoir problem and fails to guide the optimization search progress for focusing on the region of interest.

Table 2. R-Metric Value of Four Algorithms for Different Reference Point Cases

R-Metric	Algorithm	Reference Point 1	Reference Point 2	Reference Point 3	Reference Point 4	Reference Point 5	Reference Point 6
R-IGD	NSGA-II	0.712	0.458	0.478	0.171	0.247	**0.170***
	R-NSGA-II	**0.649***	0.420	0.471	**0.169***	0.191	0.175
	r-NSGA-II	0.686	**0.409***	**0.406***	0.172	**0.163***	0.208
	g-NSGA-II	/	/	/	/	0.202	0.274
R-HV	NSGA-II	18.370	27.228	18.370	24.205	16.758	12.045
	R-NSGA-II	**20.099***	**29.264***	18.657	**25.070***	18.662	**12.691***
	r-NSGA-II	18.797	28.408	**21.002***	24.498	**20.329***	11.520
	g-NSGA-II	/	/	/	/	18.263	10.588

Items highlighted in bold and * represent the best value. / represents all solutions obtained by the corresponding algorithm are dominated by the other counterparts and no useful solution can be used for R-metric computation.

In contrast, the Pareto optimal solutions obtained by R-NSGA-II and r-NSGA-II can improve convergence and diversity of Pareto optimal solutions in the region of interest. As shown in the table 2, R-NSGA-II and r-NSGA-II have better values on R-IGD and R-HV than NSGA-II under cases where part of objectives are preferred. Especially in reference point 5, the R-IGD values of R-NSGA-II and r-NSGA-II decrease by 22.7% and 34.0%, while the R-HV values increase by 11.4% and 21.3% respectively. The reason is that the essence of the two algorithms is to use the Euclidian distance to the reference point to determine the area of interest and the solutions in this area is more likely to be retained. This way of preserving solutions for the next generation is easy and can be conducted effectively during the search process. It gradually guides the search toward the interesting parts of the Pareto optimal region, and improves the search efficiency and quality of the preferred solutions. Besides, the superiority of R-NSGA-II and r-NSGA-II under different cases are different. r-NSGA-II obtains the best R-IGD and R-HV values in reference points 3 and 5, and the value is more than 10% beyond that for R-NSGA-II. R-NSGA-II obtains the best R-IGD and R-HV values in reference points 1 and 4, and the improvement rate compared to r-NSGA-II is less than 7% in reference points 1, and 3% in reference point 4.

The advantage of R-NSGA-II and r-NSGA-II in convergence and diversity of the preferable Pareto optimal solutions is equal to NSGA-II when all objectives are preferred. It can be seen from the result that the best R-HV values is from R-NSGA-II and the best R-IGD values is from the standard algorithm NSGA-II under reference point 6. The reason is that the objectives are comparative, which means improving some advantage objectives will inevitably decrease the others, and it is impossible to improve all objectives when all objectives are preferred. As shown in Fig. 1(f), the values of *Electricity*, *Public* and *Environment* in R-NSGA-II and r-NSGA-II closer to the best objective value while *Irrigation* and *Wetland* are opposite.

Mean Euclidean Distance

Fig. 3 demonstrates the mean Euclidean distance value of the Pareto optimal solutions to the reference point for different algorithms. This indicator reveals the closeness degree toward the preference region which represented by the reference point. As can be seen, the curves for R-NSGA-II and r-NSGA-II are obviously below to that of NSGA-II under the first five cases, showing that solutions provided by the two algorithms are closer to reference point than that of NSGA-II. This indicates the solutions' closeness degree of R-NSGA-II and r-NSGA-II to the preference region is significantly improved compared with that of NSGA-II. The maximum increment is up to 42.8% among all the reference point cases. For reference point 6 where all objective are preferred, the mean Euclidean distance of R-NSGA-II and r-NSGA-II is slightly smaller than or almost equal to that of NSGA-II. This is the result of the trade-off among all objectives which makes some objectives with good performance and the others with bad when all objectives are preferred. As for g-NSGA-II, the closeness degree has no obvious increment demonstrated by the mean Euclidean distance value which is almost equal to NSHA-II.

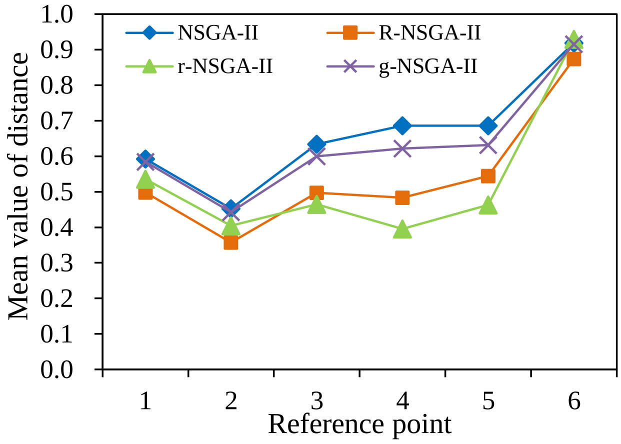

Fig. 3 Mean distance value of Pareto optimal solutions under four algorithms for different reference point cases.

Numbers of Acceptable Alternatives

Table 3 shows the acceptable alternative numbers provided by each algorithm. R-NSGA-II and r-NSGA-II obtain more superior solutions than NSGA-II when part of objectives are preferred. The number of acceptable solutions provided by R-NSGA-II algorithm is three times as many as that provided by NSGA-II algorithm under reference point 1. The acceptable alternatives provided by r-NSGA-II in reference point 3 and reference point 5 are increased by more than 3 times compared with NSGA-II. Even in reference point 4 where the two low water supply priority objectives, i.e., *Irrigation* and *Wetland*, are set as preferred objectives, r-NSGA-II provides more acceptable alternatives than NSGA-II. On the contrary, g-NSGA-II obtain less superior solutions than NSGA-II generally. The number of acceptable alternatives searched by g-NSGA-II is less than 10% of that obtained by NSGA-II in the first five cases. These support more evidence for that R-NSGA-II and r-NSGA-II are more effective than g-NSGA-II as the preference point based algorithms for solving this reservoir operation problem. The numbers of acceptable alternatives of three preference point based algorithms are all zero in reference point 6 where all objectives are preferred. The reason about trade-off described above makes that no one solution owns all objectives better than NSGA-II.

Table 3. Numbers of Acceptable Alternatives Obtained by Four Algorithms for Different Reference Point

Numbers of acceptable alternatives	NSGA-II	R-NSGA-II	r-NSGA-II	g-NSGA-II
Reference point 1	312	**1423**#	**599**#	31
Reference point 2	312	**577**#	**327**#	0
Reference point 3	202	**814**#	**949**#	11
Reference point 4	29	9	**69**#	32
Reference point 5	135	**364**#	**821**#	114
Reference point 6	0	0	0	0

Items highlighted in bold and # denote that the indicator values are the better than that of NSGA-II.

Conclusions

In this paper, a comparison of three reference point based algorithms, i.e., R-NSGA-II, r-NSGA-II and g-NSGA-II with a standard algorithm NSGA-II was conducted on a five-objective reservoir operation problem. The comparison revealed the effectiveness of the incorporation of preference information. Six different reference point settings on the basis of water supply priorities and interests from water users were considered. The four multi-objective evolutionary algorithms were used in empirical comparison in terms of the approximation to the solutions preferred by the decision maker. The convergence and diversity of the Pareto optimal solutions in the region of interest, closeness to the reference point and capacity to search superior preferred alternatives were revealed by three performance indicators for further comparison. The results can be summarized as follows:

- R-NSGA-II and r-NSGA-II both can effectively improve the search efficiency and quality of preferred solutions by applying the reference point to guide the search space to the region of interest. When part of objectives are preferred, they are effective in generating a larger proportion of Pareto optimal solutions with superior performance on preferred objectives and they find the best solution in terms of the preferred objectives. The convergence and diversity of their Pareto optimal solutions in the region of interest are better than the standard algorithm NSGA-II. The increment of closeness degree to reference point can be up to 42.8% to the maximum extent and the number of the preferred solutions can be increased by more than 3 times compared with NSGA-II. When all objectives are preferred, R-NSGA-II and r-NSGA-II do not show superiority as a result of trade-off among all the objectives.

- g-NSGA-II shows worse performance in finding preferred Pareto optimal solutions. The flag setting of 0 or 1 before non-dominated sorting makes it difficult to drive the solutions towards the Pareto optimal when many objectives are considered and affects the search efficiency and quality of preferred solutions. The convergence and diversity of the solutions in the concerned region are inferior to NSGA-II, and the number of effective solutions is less than 10% of NSGA-II in most cases, and moreover the overall closeness of the solutions to the reference point is approximately equal to NSGA-II.

The utilization of three reference point based algorithms in this study shows the way to express preference through reference point(s). The comparison of reference point based algorithms with the standard algorithm demonstrates the value of preference information and reveals the effectiveness of R-NSGA-II and r-NSGA-II in reservoir operation problems. It provides an insight in selecting high performing multi-objective evolutionary algorithms for reservoir operation problems. However, the effectiveness of R-NSGA-II and r-NSGA-II is demonstrated by a single reservoir in this paper, while the reservoir systems in real-world are often complex with reservoirs interconnected. The advantages of the reference point based algorithms are higher in a more complex problem. Therefore, future work should focus on extending the application and comparison of the algorithms to the more complex reservoir systems to explore the potential of these reference point based algorithms.

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