Riding the Wave of Fashion Rental: The Role of Power Structures and Green Advertising

Lina Zhang

l.zhang5@exeter.ac.uk

University of Exeter Business School

University of Exeter

Rennes Drive, Exeter EX4 4PU, United Kingdom

Nottingham Ningbo China Beacons of Excellence Research and Innovation Institute

Taikang East Road, Ningbo 315100, China

Yumeng Zhang*

zhangym67@cneei.com.cn

CNOOC Energy Economics Institute

Dongcheng District, Beijing 100013, China

Anshuman Chutani

Anshuman.Chutani@nottingham.ac.uk

Nottingham University Business School

University of Nottingham

Jubilee Campus, Nottingham NG8 1BB, United Kingdom

* Corresponding author.

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Abstract

In recent years, the rise in the collaborative consumption of fashion products has propelled many fashion brands to tap into rental markets and offer rental services. Although many studies have investigated consumer perceptions and attitudes toward fashion rental services, analytical models that help businesses make more informed supply chain decisions are scarce. Consequently, we develop game-theoretical models to study the fashion market with two firms, i.e., a fashion manufacturer and its rental platform partner. The models consider four green-advertising investment situations: no firm invests, only the fashion manufacturer invests, only the rental platform invests, or both firms invest in green advertising for fashion rental services. We compare pricing and green advertising decisions under different power structures between the two firms across the four green-advertising investment situations. Our results demonstrate when and how the power structure affects the key decision variables. We reveal that it is least profitable for both firms if neither contributes to green advertising, while both firms obtain the highest profits when they both invest in green advertising. In the two situations when only one firm invests in green advertising, we note that it is not necessarily unfavorable for firms to accept the green-advertising cost proactively. Specifically, the Stackelberg follower benefits from investing in green advertising. Furthermore, in these two situations, whether or not the Stackelberg leader covers the green-advertising cost is more important than their identity. Our results and analysis reveal important managerial implications that can assist firms in excelling in the fashion rental market.

Keywords: Fashion rental services, game theory, power structure, green advertising, rental service charge

1 Introduction

The fashion industry has been heavily criticized in the sustainable production and consumption discourse due to its detrimental environmental impacts. It accounts for approximately 10% of global carbon emissions and 20% of wastewater (Ro, 2020) and is responsible for over 92 million tons of textile waste globally each year (Niinimäki *et al.*, 2020). Vigorous and urgent actions are called for to mitigate these disastrous environmental impacts. Owing to their sustainable and budget-friendly credentials, fashion rental services have emerged as a refreshingly guilt-free model, offering appreciable value to consumers and allowing them to save money, avoid waste, be free of commitment, and stay fashionable responsibly (Mintel, 2020). A survey conducted by the marketing research firm Mintel regarding circular retail options shows that 60% of men and 57% of women aged 18–34 would consider renting (Mintel, 2020). Another study has revealed that 17% of millennials reported renting clothing or accessories in the United States based on a survey of approximately 15,000 consumers (Campbell, 2018). Encouraging signs of increased rental volumes and stocks listed by brands and private lenders in renowned rental platforms such as My Wardrobe HQ and By Rotation in the UK and YCloset in China underline the potential of the fashion rental service model (Conlon, 2020; Lo, 2020).

Many fashion manufacturers have partnered with rental platforms to tap into the rental market. Consumers can rent pieces from a wide range of fashion brands such as American Eagle, AllSaints, Vince, Rebecca Taylor, Coach, and Hugo Boss on popular rental platforms such as Rent the Runway and CaaStle (Bowers, 2019). Despite concerns regarding how rental platforms may cannibalize the traditional fashion market, research has shown that working with rental platforms benefits fashion manufacturers as market demand increases. For example, consumers who initially might not buy fashion products may choose to rent them from rental platforms (Feng *et al.*, 2020). Rental services operate with either a subscription model (e.g., offering six pieces of women's clothing for an $88 monthly subscription to Nuuly) or one-time rentals (Repeat.ganni.com, 2021). Specifically, one-time rentals accounted for two-thirds of the revenue of Rent the Runway, with subscriptions making up the rest (Safdar and Kapner, 2017). Motivated by such emerging practices, our paper aims to investigate the fashion rental market with a fashion manufacturer and a rental platform.

With the increasing consumer demand for renting, fashion manufacturers may need to reconsider their pricing strategies for consumers to purchase and the wholesale price when partnering with rental platforms. While rental services compete with new product sales for market shares, they also provide another revenue stream for fashion manufacturers under the expanded fashion rental businesses (Feng *et al.*, 2020). Meanwhile, rental platforms should carefully design their service charges to ensure affordability and attract consumers without compromising profitability. Previous research has examined such pricing strategies under specific single power structures between a fashion manufacturer and a rental platform, such as a balanced power structure (Yuan and Shen, 2019) or an unbalanced power structure (Feng *et al.*, 2020). However, less is known about how different power structures between a fashion manufacturer and a rental platform can influence their pricing decisions. This limitation implies that no light has yet been cast on how fashion manufacturers and rental platforms should assess and select partner firms with different powers.

In addition, firms advertise the eco-friendly feature of rental services to entice more consumers and increase their sales. For example, fashion manufacturers such as H&M and rental platforms such as My Wardrobe HQ have advertised how renting can reduce carbon emissions (Conlon, 2020). Both fashion manufacturers and rental platforms may engage in such green-advertising activities. However, the level of effort these firms should exert is a critical managerial question that has not been sufficiently investigated in the literature, and that is what we aim to address in this paper.

Motivated by such managerial questions in the fashion rental market and by the gaps in the relevant literature, we consider pricing and green-advertising decisions where the manufacturer determines the retail price and the unit wholesale price charged to the rental platform, and the rental platform determines the unit rental service charge. We consider different model setups to include various supply chain power structures and different green-advertising investment arrangements between firms. Specifically, we develop 12 game-theoretic models to investigate the firms' optimal policies under three different power setups (Manufacturer Stackelberg, Rental Platform Stackelberg, and Vertical Nash) and four different green-advertising investment arrangements (no advertising, advertising by manufacturer only, advertising by rental platform only, and advertising by both). We aim to address the following research questions: i) *What are the optimal pricing and green advertising policies of firms under these* *different model setups?* ii) *What is the impact of the channel power structure on firms' policies, and what are the subsequent payoffs?* iii) *How do pricing policies, green advertising levels, and firm profits depend on which firm is undertaking green advertising? Should fashion manufacturers and rental platforms proactively invest in green advertising?*

Our results reveal that power structures influence selling, wholesale, and rental service prices differently. In addition, we find that investing in green advertising benefits both firms. We also discuss when and how the combination of power structures and green-advertising investment situations jointly influence the firms' optimal strategies for fashion rental services and their profits. Relevant and important managerial insights have been presented accordingly.

The rest of the paper is organized as follows. Section 2 reviews the related literature, and Section 3 presents the model assumptions and formulations. The main model results are analyzed and discussed in Section 4, followed by the conclusions and managerial implications in Section 5.

2 Related work

This study is closely related to two streams of literature, i.e., the sharing economy and supply chain power structures. The sharing economy has reshaped many aspects of our economy and society and has been widely examined in the literature over the last decade (Matzler, Veider, and Kathan, 2015; Brydges *et al.*, 2021). For example, the rationality of adopting a mix of leasing and selling durable products (e.g., automobiles) rather than solely leasing or selling has been examined by several researchers (e.g., Desai and Purohit, 1998; Desai and Purohit, 1999; Bhaskaran and Gilbert, 2005). Recently, innovative sharing business models such as Airbnb for room sharing and Uber for car sharing have received extensive attention (Tseng and Chan, 2019). For example, Roma, Panniello, and Lo Nigro (2019) argue that the impacts of the sharing economy (exemplified by Airbnb) on the pricing strategy of incumbent hotels depend on the type of hotels (low/medium-end or high-end), the accommodation period (weekend or weekdays), and the type of consumers. Pricing in the sharing economy has also been examined using a hedonic pricing model on the rates of Airbnb listings in five large Canadian metropolitan areas (Gibbs *et al.*, 2018). Kalathil *et al.* (2019) explore the opportunities of sharing electricity storage in a smart grid and develop game-theoretical models to analyze the pricing and investment decisions for electricity storage sharing. Guo, Li, and Zeng (2019) study the impact of the entry of ride-hailing platforms such as Didi Chuxing and Uber and platform competition on new car purchases and provide insights on pricing strategies and subsidy allocation for the ride-hailing market.

In comparison, fashion rental services, a segment of the sharing economy, have received relatively less research attention. In one related study, the importance of prolonging the life of garments has been highlighted for fully unlocking the environmental benefits of fashion rental services (Zamani, Sandin, and Peters, 2017). Some exploratory and empirical studies have been conducted to understand consumers' perspectives on fashion rental (Mukendi and Henninger, 2020) and how they derive diverse consumption values from the fashion rental services depending on their contamination concerns (Baek and Oh, 2021). Using a systematic literature review approach, Jain *et al.* (2022) identify the main enablers and inhibitors of the consumer adoption of fashion rental consumption and discuss the opportunities and challenges in the fashion rental business. To investigate fashion rental platform operations, Liu *et al.* (2022) examine and compare two fashion sharing platforms, Rent the Runway in the United States and YCloset in China, in terms of their platform operations, such as the range of products and brands available, the rental service charge, logistics operations, and technologies.

Analytical models focusing on policies and practices regarding sharing business in the fashion industry are somewhat scarce. Examining the peer-to-peer collaborative consumption (P2P-CC) of fashion products, Choi and He (2019) underline the benefits of offering P2P-CC to both fashion brands and consumers and reveal that a sharing platform should adopt a revenue-sharing pricing scheme rather than a fixed service charge to accrue more profits. Yuan and Shen (2019) study strategic consumer behaviors regarding product returns and highlight that the presence of fashion rental services could attract strategic consumers who move from purchasing and returning fashion clothes to renting them. Altug and Ceryan (2021) examine the dynamic inventory allocation for rentals and sales by a retailer that prioritizes renting while selectively satisfying some accidental sales demand and explore the impact of market characteristics and prices on the optimal dynamic inventory allocation policy.

The business performance of an organization depends not only on its operating strategies but also on its position or bargaining power in the market (Luo *et al.*, 2017). The impact of power relationships between competitors in a vertical or horizontal supply chain on various supply chain decisions (e.g., retail and wholesale prices, ordering quantities) has been extensively researched (e.g., Bernstein and Federgruen, 2005; Cachon and Kök, 2010; Wu, Chen, and Hsieh, 2012; Shi, Zhang, and Ru, 2013; Chen, Wang, and Jiang, 2016; Wei and Zhao, 2016; Chen, Wang, and Chan, 2017; Zhang and Hezarkhani, 2021). Cachon and Kök (2010) examine the supply chain coordination issues when there is horizontal competition among multiple manufacturers who compete for their common retailer's business. To unveil the effects of retail competition and consumer returns on green product development in the fashion industry, Guo, Choi, and Shen (2020) examine the optimal greenness level and retail price of a fashion product (determined by retailers) and its wholesale price (determined by the manufacturer) in a fashion supply chain with one manufacturer and two rival retailers. Their work underlines the demand-expanding effects of environmental fashion products (captured by the greenness level) and highlights that the optimal greenness level of the fashion products increases in the level of retail price competition. Considering vertical competition between supply chain members (e.g., suppliers versus retailers), a comparative analysis of certain policies (e.g., greening policies in Ghosh and Shah, 2012; pricing, sales effort, and collection effort decisions in Gao *et al.*, 2016) could be conducted across different supply chain structures where either the manufacturer or the retailer is the Stackelberg leader and also where they engage in a Nash game with balanced power. Chen, Wang, and Jiang (2016) investigate the increasingly popular dual-channel phenomenon where the retailer operates both physical retail stores (offline) and virtual online shops (online) and address the pricing decisions for both online and offline channels with vertical competition between the manufacturer and the retailer.

Some researchers have also considered vertical competition between supply chain members and horizontal competition at the same echelon of a supply chain simultaneously. For example, competitive pricing decisions and equilibrium quantities have been derived and compared among six power structures in a triadic setting with two rival retailers and a common supplier in Wu, Chen, and Hsieh (2012). Similarly, Chen, Wang, and Gong (2020) compare pricing decisions and supply chain performance across five different power structures with two retailers and their common manufacturer. Wei and Zhao (2016) examine the pricing decisions of substitutable products distributed by one retailer but produced by duopolistic manufacturers, respectively, with both the manufacturing cost and consumer demand as fuzzy variables. Finally, the pricing policies of two competing manufacturers with differentiated brands sharing a common retailer have been examined in Luo *et al.* (2017) under seven game models with different vertical and horizontal power structures.

Our paper is closely related to the literature on the fashion industry, particularly fashion rental services. As an example, Feng *et al.* (2020) address the pricing strategies for a fashion manufacturer and a rental platform. The authors analyze the optimal fashion retail price, wholesale price, and rental service charge under a wholesale contract and an agency contract, respectively, considering a Stackelberg game with the fashion manufacturer as the leader and the rental platform as the follower. Our paper advances the understanding of the fashion industry and differs from previous studies in several aspects. First, alongside the pricing decisions considered in most cases, we also consider another key marketing issue, the level of green advertising (i.e., advertising rentals as a sustainable practice), as a decision variable. The effect of consumer environmental awareness (CEA) on the increasing demand for green products and services has been noted in recent research, pointing out the benefits of advertising the environmental-friendly nature of green products and services. Specifically, green advertising can lead to a goodwill dynamic by enhancing consumers' knowledge and awareness about the benefits of consuming green products or services and eventually induce more sales (De Giovanni, 2014). Green advertising is also vital in product design for the environment, as discussed in Guo, Choi, and Shen (2020). It is, therefore, appropriate and practically relevant to include the level of green advertising in our model, given the value of advertising sustainable products. Second, we investigate optimal decisions under different supply chain power structures and provide a comprehensive view of the impact of firms' different leadership roles in the fashion market. Third, in addition to the leadership role, we also examine firms' role as green advertisers and demonstrate its influence on firms and the fashion supply chain. Finally, we highlight valuable managerial insights by conducting sensitivity analysis of optimal decisions with regard to key model parameters, such as the effects of green-advertising efficacy, renting efforts, and the salvage value of the returned product at the end of the rental period.

3 The model

We consider a fashion market with two firms: a fashion brand manufacturer that sells a fashion product to consumers (i.e., the selling channel) and a rental platform that sources the same product from the fashion manufacturer and rents it to consumers (i.e., the rental channel). Consumers in this fashion market can either buy or rent the same fashion product.

3.1 Consumer demand

Consumers are heterogeneous in their valuation of the fashion product. All consumers are present at the beginning of the period, and the consumer reservation price v is uniformly distributed, where $v \sim U[0,1]$. The consumer's utility for buying the product decreases in the sale price and the purchase effort of the product. Purchase effort E_b captures the hassle consumers confront when purchasing the product; for example, the time needed to travel to and navigate the store for offline channels (or website for online channels) and the level of convenience of delivery or collections. The smaller the E_b, the more convenient the purchase services to consumers. A consumer derives utility $U_b = v - P_b - E_b$ from purchasing the product at sale price P_b with purchase effort E_b. Given P_b and E_b, consumers with a net surplus of $v - P_b - E_b \geq 0$ will buy the fashion product.

The consumer's valuation of the rented product is αv, where $0 < \alpha < 1$ captures the discounted value of the rental product with a limited pre-determined rental period compared to the unrestricted usage of the purchased product (Yuan and Shen, 2019; Feng *et al.*, 2020); in other words, consumers set a lower reservation price to rent a product than to buy it. For example, consumers of Hirestreet, a UK startup rental platform, can rent mid-market occasion wear from Zara and ASOS for ten days at approximately £10, which is considerably cheaper than buying these items (Felsted and Halzack, 2019). E_r is the amount of effort needed to use the rental services, such as the amount of time needed to select rental products and the hassle of returning the product at the end of the rental period. For rental services with no ownership, consumers need to choose the clothes, wear them, and return them at the end of the rental period, then initiate the cycle again for a new rental period (dubbed a "shampoo-rinse-repeat" model for fashion; Bowers, 2019), which implies additional hassle for consumers (Safdar and Kapner, 2017). Therefore, we assume $E_r > E_b$.

Additionally, the environmental-friendly feature of rental services also impacts consumers' utility due to its contribution to sustainable consumption (Martin, 2016). To promote the environmental friendliness of rental services and potentially entice more consumers, green advertising (e.g., advertising the sustainability benefits of rental services) can be carried out (Basiri and Heydari, 2017; Hong and Guo, 2019). We do not consider the degree of environmental effort spent during the product design stage (i.e., design for the environment) or the production stage for treating or removing environmental pollutants. Rather, we focus on firms' investments in green advertising to communicate with consumers and enhance their awareness of sustainable and environmental products or services. For example, firms can advertise their return policy to increase consumers' willingness to return obsolete products (De Giovanni, 2014) or collaborate with micro-celebrities or influencers on social media platforms such as Instagram to promote online fashion renting services (Shrivastava *et al*., 2021). Endorsements from celebrities such as Carrie Johnson and Holly Willoughby, who wore rented clothes from rental platforms including My Wardrobe HQ and Hurr at the G7 summit 2021 in England, was undoubtedly a powerful marketing strategy for those rental brands to advertise a more sustainable and affordable way to stay fashionable (Tyler, 2021). Such efforts to promote the greenness and sustainability of clothing rental services are considered as the green-advertising level and denoted as t. Accordingly, consumers' sensitivity toward the green-advertising level is denoted as k. Therefore, the consumer's utility from renting the product is $U_r = \alpha v - P_r - E_r + kt$ for a rental service charge of P_r.

Following Chiang, Chhajed, and Hess (2003), consumers' buying or renting choice rests on the comparison of utilities in buying or renting the product. Selling and rental services compete for market shares based on consumer choice. If we assume $v_b = P_b + E_b$ and $v_r = (P_r + E_r - kt)/\alpha$, consumers whose valuation satisfies $v > v_b$ (i.e., $U_b > 0$) would consider buying the product and consumers whose valuation satisfies $v > v_r$ (i.e., $U_r > 0$) would consider renting the product. If we assume $v_{br} = (P_b + E_b - P_r - E_r + kt)/(1-\alpha)$, consumers whose valuation satisfies $v > v_{br}$ (i.e., $U_b > U_r$) would prefer buying to renting, while consumers whose valuation satisfies $v < v_{br}$ (i.e., $U_b < U_r$) would prefer renting to buying. Consumers thus choose to purchase the product if $v > max(v_b, v_{br})$ and rent the product if $v_r < v < v_{br}$. When $v_b > v_r$, $v_r < v_b < v_{br}$ holds; when $v_b < v_r$, $v_{br} < v_b < v_r$ holds. We focus on the case where consumers can select from both renting and buying options, i.e., $v_b > v_r > 0$ and $v_{br} < 1$. Hence, the demand functions of selling and renting the fashion product are respectively given as:

$$D_b = 1 - v_{br} = 1 - (P_b + E_b - P_r - E_r + kt)/(1-\alpha); \quad (1)$$

$$D_r = v_{br} - v_r = (P_b + E_b - P_r - E_r + kt)/(1-\alpha) - (P_r + E_r - kt)/\alpha, \quad (2)$$

where $max\{P_b + E_b - 1 + \alpha, 0\} < P_r + E_r - kt < \alpha(P_b + E_b)$.

The unit production cost of the fashion product is assumed to be zero. As noted in previous literature such as Cai (2010), Feng et al. (2020), Hu et al (2021), Li, Zhang and Tayi (2020), and Zhang, He and Zhao (2019), this assumption has been widely applied for analytical brevity without loss of generality. At the end of the rental period, the fashion product is returned to the rental platform with a salvage value of s, as it can be leased out again or sold to a discounter (Yuan and Shen, 2019; Feng *et al.,* 2020). For example, for a high-value product with a high retail price (e.g., luxury clothes) or a high-quality product with better durability, the value of the product at the end of a rental period would be reasonably high (i.e., a high s). Table 1 summarizes the notations used in this study.

Table 1. Notations used in the study

Notation	Meaning
v	Consumers' valuation of the fashion product, $v \sim U[0,1]$
α	The discounted value of the rented product, $0 < \alpha < 1$
P_b	The sale price of the fashion product
P_r	The rental service charge for the fashion product
E_b	The amount of effort required to purchase the product
E_r	The amount of effort required to rent the product
w	The wholesale price charged by the fashion manufacturer
s	The salvage value of the returned product at the end of the rental period
t	The overall level of green advertising
t_m	The level of green advertising determined by the fashion manufacturer in the *BMS*, *BRS*, and *BVN* models
t_r	The level of green advertising determined by the rental platform in the *BMS*, *BRS*, and *BVN* models
k	Consumer sensitivity toward green advertising
β	Cost coefficient of green advertising
ε	Green-advertising efficacy, $\varepsilon = k^2/\beta$
Π_i^k	Profits of firm i ($i \in \{R, M, SC\}$) under model k ($k \epsilon \{NMS, NRS, NVN, MMS, MRS, MVN, RMS, RRS, RVN, BMS, BRS, BVN\}$), where R denotes the rental platform, M denotes the fashion manufacturer, and SC denotes the entire supply chain

3.2 Green-advertising investment arrangements

Naturally, a higher level of green advertising leads to a greater expense. We denote β as the cost coefficient of green advertising and assume that the green-advertising cost is βt^2. The convexity of this cost function with the quadratic form is consistent with the relevant literature (Basiri and Heydari, 2017; Hong and Guo, 2019).

Many fashion brands have been trying to promote their brand images by committing to environmental programs to combat their notorious reputation as detrimental to the environment. For example, the US-based Patagonia, a designer of outdoor clothing and gear, has devoted itself to becoming the cleanest line with multiple sustainable initiatives, such as 1% for the Planet (where 1% of the company's annual sales are donated to environmental groups), the Action works website (which connects individuals to local or regional environmental groups to actively find solutions to the environmental crisis), and the partnership with the gear rental platform Awayco to offer ski gear rental options to consumers (Danigelis, 2019; Patagonia.com, 2022). Similarly, H&M trialed a clothing rental service in 2019 at its Stockholm flagship store to align with its sustainability work (H&M, 2019). Therefore, a fashion manufacturer may pay for green advertising to promote rental services that fulfill its sustainability commitment. In addition to their contribution to sustainable practices, fashion brands can gain financially from the new revenue stream derived from rental services that complement their conventional retailing businesses, as we illustrate mathematically in Section 4.

For fashion rental platforms, it is in their interest to encourage more people to embrace fashion rental services. Therefore, they also have an incentive to invest in green advertising to market the rental service via celebrity endorsement, as exemplified by the abovementioned My Wardrobe HQ and Hurr. Another example is Rent the Runway, which mobilizes ambassadors, including university students, to promote the sustainability of fashion rental services (Watson, 2020).

To this end, we investigate the pricing and green-advertising decisions when the fashion manufacturer and the rental platform incur a green-advertising cost (or invest in green advertising for fashion rental services). The fashion manufacturer or the rental platform can decide whether it would invest in green advertising for fashion rental services. Therefore, four investment situations are considered regarding the two firms' willingness to invest in green advertising (or incur the green-advertising cost): no firm invests, only the fashion manufacturer invests, only the rental platform invests, or both firms invest. If only one firm invests in green advertising, this firm determines the level of green advertising to maximize its individual profits and bears the green-advertising cost. On the contrary, if both firms invest in green advertising, they determine their levels of green advertising separately to maximize their individual profits and bear the corresponding green-advertising cost.

In our paper, we assume that the cost coefficient of advertising β (equivalently the green-advertising efficacy) is the same for the manufacturer and the rental platform. Notably, the key focus of our paper is to analyze how the pricing, advertising, and firms' profits depend on the leadership role (leader, follower, or simultaneous mover) and the position of the firm in the supply chain (upstream or downstream) that carries the responsibility of advertising. Hence, assuming a same β allows us to focus on these different scenarios and their impact. Such assumption, not unprecedented in the literature, facilitates fair comparison among different advertising structures, as suggested by Liu et al. (2014). Furthermore, it could be argued that the effectiveness of promotional activities is very often a function of the medium of advertising (e.g., TV, newspapers, radio, Google advertisements, Facebook, Instagram, etc.) (Danaher, 2021). For example, when a firm deploys social media to appeal to consumers by hiring an advertising agency or influencer, the effectiveness of the advertising efforts hinges primarily on the advertising channel(s) chosen and the capacity of the agency or influencer, rather than the identity of the delegated firm (i.e., the manufacturer or the rental platform). Thus, an example of a scenario where our model could be applied is when the manufacturer or the rental platform seeks to advertise the rental services through a common medium such as Google advertisements or Facebook.

3.3 Power structure

To capture different negotiation and competition scenarios between the fashion manufacturer and the rental platform, we consider three types of power structures: *Manufacturer Stackelberg* (the fashion manufacturer is more powerful as the Stackelberg leader; the rental platform is the follower), *Rental Platform Stackelberg* (the rental platform is more powerful as the Stackelberg leader; the fashion manufacturer is the follower), and *Vertical Nash* (the fashion manufacturer and the rental platform are equally powerful and make decisions simultaneously).

In practice, it seems more intuitive to have the fashion manufacturer as the Stackelberg leader, as fashion rental startups such as Hirestreet struggle to bring big brands onto their platforms (Drumm, 2021). However, for new brands or emerging designers, prestigious rental platforms such as Rent the Runway could be well-positioned to decide whether to carry products from these brands and therefore have more power, serving as the Stackelberg leader. For example, apart from offering one-time rentals with a rental price for each product, Rent the Runway also allows consumers to rent a certain number of products (from participating brands) per month for a specified monthly subscription fee (e.g., \$89/month for the free rental of four products). If new brands decide to make their products available for monthly rental products, they need to accommodate the pre-determined rental service charge set by Rent the Runway when making their pricing decisions (e.g., wholesaling and retail prices). In the partnership between the fashion giant H&M's brand COS and YCloset in China, the largest fashion rental platform with 15 million registered users, the companies could potentially be engaged in a business negotiation without a distinctively dominating leader (Reuters, 2019). This scenario presents a reasonable Vertical Nash setting in which the fashion manufacturer and the rental platform make decisions simultaneously. Given the rise of fashion rental services, it is reasonable to argue that high-performing rental platforms will grow more powerful as more consumers opt for sustainable fashion consumption. McKinsey & Company (2019) has noted the possibility of having a dominating "unicorn" player in the rental space. As more fashion brands embrace the rental paradigm, decisions on new collaboration models between fashion brands and rental platforms should be contemplated carefully (McKinsey & Company, 2019). Partnerships between fashion and rental brands with different power structures are the focus of this study, and our inclusion of all the possible game models can shed light on decision-making for a wide range of business scenarios.

3.4 Different game models

Up to this point, we have provided significant evidence to demonstrate two factors that influence the interactions between the fashion manufacturer and the rental platform in practice: first, each firm may or may not invest in the rental channel's green advertising (i.e., four green-advertising investment situations); second, the two firms may have three types of power structures. Therefore, we introduce 12 game models with different power structures and green-advertising investment situations. Table 2 displays the 12 game models, and Figure 1 illustrates the sequence of events under all models. In each game, the fashion manufacturer decides the product sale price P_b and the wholesale price w, while the rental platform decides the rental service charge P_r (equivalently, the rental margin $m = P_r + s - w$). In the BMS, BRS, and BVN models, the green-advertising level t_m is determined by the fashion manufacturer, and the green-advertising level t_r is determined by the rental platform. The green-advertising level t is the overall green-advertising level in the supply chain: t is determined by the fashion manufacturer in the *MMS*, *MRS*, and *MVN* models and is determined by the rental platform in the *RMS*, *RRS*, and *RVN* models; in the *BMS*, *BRS*, and *BVN* models, t represents the sum of the green-advertising levels by the two firms (i.e., $t = t_m + t_r$).

Table 2. 12 game models

	The rental platform does not invest in green advertising	The rental platform invests in green advertising
The fashion manufacturer does not invest in green advertising	• Manufacturer Stackelberg (NMS) • Rental Platform Stackelberg (NRS) • Vertical Nash (NVN)	• Manufacturer Stackelberg (RMS) • Rental Platform Stackelberg (RRS) • Vertical Nash (RVN)
The fashion manufacturer invests in green advertising	• Manufacturer Stackelberg (MMS) • Rental Platform Stackelberg (MRS) • Vertical Nash (MVN)	• Manufacturer Stackelberg (BMS) • Rental Platform Stackelberg (BRS) • Vertical Nash (BVN)

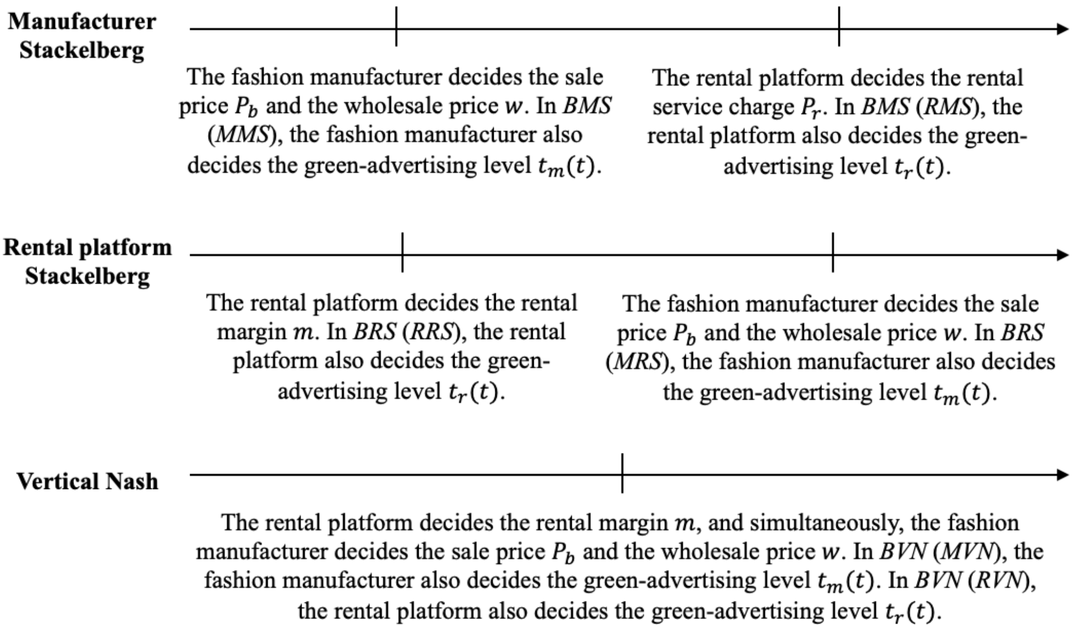

Figure 1. Sequence of events

When neither firm invests in green advertising, the fashion manufacturer and the rental platform participate in games with three different power structures. The first model is *Manufacturer Stackelberg with no investment in green advertising (NMS)*. The fashion manufacturer first decides the product sale price P_b and the wholesale price w, and the rental platform subsequently chooses the rental service charge P_r in response to the fashion manufacturer's decisions. The second model is *Rental Platform Stackelberg with no investment in green advertising (NRS)*, where the rental platform first chooses the rental margin m, and the fashion manufacturer in response decides P_b and w given the rental margin m set by the rental platform. The third model is *Vertical Nash with no investment in green advertising (NVN)*, where the fashion manufacturer makes the decisions for P_b and w, and the rental platform simultaneously decides the rental margin m. In the *NMS*, *NRS*, and *NVN* models, the profit functions of the rental platform (Π_R) and the fashion manufacturer (Π_M) are:

$$\Pi_R(P_r) = (P_r + s - w)D_r; \tag{3}$$

$$\Pi_M(P_b, w) = P_b D_b + w D_r. \tag{4}$$

In the case when only the fashion manufacturer bears the green-advertising costs, the fashion manufacturer and the rental platform participate in games with three different power structures: *Manufacturer Stackelberg with only the fashion manufacturer investing in green advertising (MMS)*, *Rental Platform Stackelberg with only the fashion manufacturer investing in green advertising (MRS)*, and *Vertical Nash with only the fashion manufacturer investing in green advertising (MVN)*, where the fashion manufacturer decides the green-advertising level t, the product sale price P_b, and the wholesale price w. In the *MMS*, *MRS*, and *MVN* models, the profit functions of the rental platform and the fashion manufacturer are:

$$\Pi_R(P_r) = (P_r + s - w)D_r; \tag{5}$$

$$\Pi_M(P_b, w, t) = P_b D_b + w D_r - \beta t^2. \tag{6}$$

When the rental platform bears the green-advertising costs alone, the fashion manufacturer and the rental platform participate in games with three different power structures. Therefore, we introduce three additional game models: *Manufacturer Stackelberg with only the rental platform investing in green advertising (RMS)*, *Rental Platform Stackelberg with only the rental platform investing in green advertising (RRS)*, and *Vertical Nash with only the rental platform investing in green advertising (RVN)*, where the rental platform decides the green-advertising level t and the rental service charge P_r (or the rental margin m). In the *RMS*, *RRS*, and *RVN* models, the profit functions of the rental platform and the fashion manufacturer are:

$$\Pi_R(P_r, t) = (P_r + s - w)D_r - \beta t^2; \tag{7}$$

$$\Pi_M(P_b, w) = P_b D_b + w D_r. \tag{8}$$

Next, we consider the case where both firms contribute to the rental channel's green advertising. Each firm assumes its own green-advertising cost with the same cost efficient β. Hence, the demand functions of selling and renting the fashion product depend on the overall green-advertising level t. As discussed previously, we consider three game models with different power structures: *Manufacturer Stackelberg with both firms investing in green advertising (BMS)*, *Rental Platform Stackelberg with both firms investing in green advertising (BRS)*, and *Vertical Nash with both firms investing in green advertising (BVN)*. In the *BMS*, *BRS*, and *BVN* models, the profit functions of the rental platform and the fashion manufacturer are:

$$\Pi_R(P_r, t_r) = (P_r + s - w)D_r - \beta {t_r}^2; \tag{9}$$

$$\Pi_M(P_b, w, t_m) = P_b D_b + w D_r - \beta {t_m}^2. \tag{10}$$

For all 12 models, the profit function of the entire supply chain is:

$$\Pi_{SC} = \Pi_R + \Pi_M. \tag{11}$$

In our paper, we focus on the scenario where a fashion product is rented only once in the season and discuss its justifications with references to similar approaches in the relevant literature, such as Yuan and Shen (2019). Nevertheless, our models are not limited to the assumption that each fashion product can be rented only once. Hence the results and key insights generated from our models can be extended to a scenario where a fashion product can be rented multiple times. Recall that D_b and D_r denote the consumer demand to buy or rent the fashion product, respectively, as shown in Equations (1) and (2). In other words, D_b (D_r) is the number of consumers who purchase (rent) the product, assuming that each unit can be purchased (rented) only by one consumer in the season. However, the assumption of one rental of each product can be relaxed here. Given the limited life cycle of clothing, hygiene, and perception issues (Feng *et al.*, 2020), we denote the average number of times a fashion product can be rented each season as n and assume that n is exogenous. We also assume that the demand for rentals is spread out uniformly throughout the entire season, thereby avoiding extreme situations where most or all renting consumers ask to rent one product simultaneously (e.g., at the beginning of the season).

Given these fairly plausible and realistic assumptions, the number of fashion products sold by the manufacturer to the rental platform for rentals, denoted as Q_r, can be written as $Q_r = D_r/n$. The quantity that is sold to the consumers will, however, remain D_b. Maintaining consistency with the notation used in the main models, we can re-write the profit functions of the manufacturer and the rental platform in all 12 models. For example, in the *BMS*, *BRS*, and *BVN* models, the profit functions of the rental platform and the fashion manufacturer in Equations (9) and (10) can be re-written as:

$$\Pi_R(P_r, t_r) = P_r D_r - wQ_r + sQ_r - \beta {t_r}^2 = (P_r + \bar{s} - \bar{w})D_r - \beta {t_r}^2; \quad (12)$$

$$\Pi_M(P_b, w, t_m) = P_b D_b + wQ_r - \beta {t_m}^2 = P_b D_b + \bar{w} D_r - \beta {t_m}^2, \quad (13)$$

where $\bar{w} = w/n$ and $\bar{s} = s/n$. Similar modifications can be made to the profit functions in other game models. Our paper focuses on the special case of $n = 1$ to facilitate the elucidation of many meaningful insights. However, even when $n > 1$, we show that the profit functions and hence the optimization problems of the two players in all game structures are essentially the same as in our main models (with $n = 1$) with slight modifications and reinterpretations of $\bar{w}$ and $\bar{s}$ (in place of w and s, respectively), as exemplified in Equations (12) and (13). For example, given the average number of times each product is rented (i.e., n), if we reinterpret w in our original model as w/n (i.e., the average wholesale price per rental for a fashion product), we can extend the results to the case of multiple rentals per fashion product. In other words, when one product can be rented multiple times, we can still use our results to obtain all the managerial insights by interpreting $\bar{w}$ ($\bar{s}$) as an average wholesale price paid by the rental platform to the manufacturer per rental transaction with the consumer (the average salvage value of the returned product at the end of each rental) and w (s) as a one-time wholesale price paid to the manufacturer per fashion product for rental (the salvage value of the returned product at the end of the entire season). For ease of understanding and interpretation without losing generality, we focus on the case of $n = 1$ in the rest of the paper.

4 Analysis

In this section, we provide and compare the optimal strategies in the 12 game models and discuss the impact of the power structure and different green-advertising arrangements. We compare the equilibrium values of the pricing decisions (i.e., sale price, wholesale price, and rental service charge), the level of green advertising, the fashion manufacturer's profits, the rental platform's profits, and the supply chain's profits across all game models. Such comparisons can generate insights into the impacts of the power structure and green-advertising investment arrangements. For ease of readability, all proofs of this paper, such as the equilibrium results for the above 12 game models, are presented in Appendixes A and B.

In the analysis, β reflects the cost coefficient of green advertising, while k measures consumer sensitivity toward green advertising. We define green-advertising efficacy ε as $\varepsilon = k^2/\beta$. A larger ε indicates that firms receive a more positive response from consumers for each unit investment in green advertising or incur a lower green-advertising cost to achieve the same positive response from consumers. We also assume that the following two conditions hold in our model: (1) $s > E_r - \alpha E_b$ to ensure the non-negativity of the decision variables (e.g., $s + \alpha E_b - E_r > 0$ for $t^* > 0$)[1], and (2) $[\varepsilon - 2(1-\alpha)\alpha]s < [4(1-\alpha)\alpha - \varepsilon](1-\alpha)$ to guarantee that the rental service charge is lower than the sales price of the fashion product (i.e., $P_b^* > P_r^*$). Similar constraints on model parameters to guarantee the applicability of the proposed analytical models while addressing more practical cases have been imposed in many studies such as Feng et al. (2020), Guo, Choi, and Shen (2020), and Hu et al. (2021). We also make some more assumptions on the model parameters to ensure that the second-order conditions are satisfied, which are discussed in Appendix A at relevant places. In the notation for optimal decision variables, the superscript represents the specific game model it is derived for; for example, Π_R^{BMS} represents the rental platform's profit in the *BMS* model.

[1] If s is too small (i.e., $s \le E_r - \alpha E_b$), then $t^* = 0$. In this case, P_r^* must be significantly lower than P_b^* so that the rental option is not dominated by the purchasing option (i.e., $U_r < U_b$ does not always hold). As P_r^* and s are considerably low, the rental platform's margin ($P_r + s - w$) may render the rental business non-profitable. We therefore avoid such trivial case by focusing on the scenario when $s > E_r - \alpha E_b$.

4.1 Impact of the green-advertising cost

To demonstrate the economic viability of venturing into the rental market, we first compare the optimal profits for the fashion manufacturer and the rental platform in different green-advertising investment situations. The results are displayed in Proposition 1.

Proposition 1. Under each power structure, the following properties hold when $\varepsilon < 8\alpha(1-\alpha)/3$:

(a) $\Pi_R^{BMS} > \Pi_R^{RMS} > \Pi_R^{MMS} > \Pi_R^{NMS}$, $\Pi_R^{BRS} > \Pi_R^{MRS} > \Pi_R^{RRS} > \Pi_R^{NRS}$, and $\Pi_R^{BVN} > \Pi_R^{MVN} > \Pi_R^{RVN} > \Pi_R^{NVN}$.

(b) $\Pi_M^{BMS} > \Pi_M^{RMS} > \Pi_M^{MMS} > \Pi_M^{NMS}$, $\Pi_M^{BRS} > \Pi_M^{MRS} > \Pi_M^{RRS} > \Pi_M^{NRS}$, and $\Pi_M^{BVN} > \Pi_M^{RVN} > \Pi_M^{MVN} > \Pi_M^{NVN}$.

(c) $\Pi_{SC}^{BMS} = \Pi_{SC}^{BRS} > \Pi_{SC}^{MRS} = \Pi_{SC}^{RMS} > \Pi_{SC}^{MMS} = \Pi_{SC}^{RRS} > \Pi_{SC}^{NMS} = \Pi_{SC}^{NRS}$; $\Pi_{SC}^{BVN} > \Pi_{SC}^{MVN} = \Pi_{SC}^{RVN} > \Pi_{SC}^{NVN}$.

The key takeaway from Proposition 1 is that the profits for both firms (and therefore the entire supply chain) are the lowest when no firm invests in green advertising, regardless of the leadership structure. This finding justifies the rationale for either the fashion manufacturer or the rental platform to make some efforts to promote the environmental-friendly features of the rental services, corroborating the practical cases. Additionally, Proposition 1 shows that both firms will perform better financially by jointly investing in green advertising than when only one firm invests, regardless of the leadership structure.

For both firms to advertise, each needs to have capabilities such as marketing talent, advertising channels, and an initial budget to effectively conduct green-advertising activities to promote the rental service. However, this may not necessarily be the case in practice. For example, one motive for a fashion manufacturer to partner with an external third-party rental platform (rather than operating its own) could be to leverage the marketing caliber and reach of the rental partner, as pointed out in the literature on decisions on outsourcing and insourcing (McGovern and Quelch, 2005). Therefore, the fashion manufacturer or a smaller rental platform may not have the marketing capacity or wish to focus on its core operations and, consequently, may not contribute to green advertising. As a result, analyzing the situations when only one firm invests in green advertising is also of importance and value. As the situation with no firm investing in green advertising is least profitable, we focus on the situations when at least one firm invests in green advertising in the rest of the paper.

Corollary 1. When only one firm invests in green advertising, the rental platform, the fashion manufacturer, and the supply chain will earn higher profits if the Stackelberg follower invests in green advertising than if the Stackelberg leader invests in green advertising, i.e., $\Pi_i^{RMS} > \Pi_i^{MMS}$ and $\Pi_i^{MRS} > \Pi_i^{RRS}$, where $i \in \{R, M, SC\}$.

Corollary 1 compares the optimal profits when only one firm invests in green advertising. It shows that the profits for the rental platform and the fashion manufacturer are lower if the Stackelberg leader invests in green advertising. More specifically, for the rental platform (the fashion manufacturer), more profits can be obtained if it invests in green advertising when the fashion manufacturer (the rental platform) is the Stackelberg leader, i.e., $\Pi_R^{RMS} > \Pi_R^{MMS}$ ($\Pi_M^{MRS} > \Pi_M^{RRS}$), as listed in Proposition 1.

Our results shed light on the situation for fashion manufacturers (e.g., H&M and ASOS) and rental platforms (e.g., Rent the Runway and CaaStle) regarding strategic decision-making on whether and under what conditions they should proactively invest in green advertising to promote fashion rental services. A key finding is that bearing the green-advertising cost is not always disadvantageous for a firm. Particularly, if both firms negotiate to decide who will invest in green advertising, both are better off if the Stackelberg follower carries the advertising effort. The supply chain profits are also higher when the Stackelberg follower incurs the green-advertising cost—the Stackelberg leader that incurs the green-advertising costs will be motivated to under-invest in green advertising and consequently fail to cultivate more demand for fashion rental services.

Proposition 2. Comparing the results when at least one firm invests in green advertising, the following properties hold:

(a) $t_m^{BMS} = t_r^{BMS} = t_m^{BRS} = t_r^{BRS} > t^{MRS} = t^{RMS} > t^{MMS} = t^{RRS}$, $t_m^{BVN} = t_r^{BVN} > t^{MVN} = t^{RVN}$.

(b) $w^{BMS} > w^{MMS} > w^{RMS}$, $w^{BRS} > w^{MRS} > w^{RRS}$, and $w^{BVN} > w^{MVN} = w^{RVN}$.

(c) $P_r^{BMS} > \max(P_r^{MMS}, P_r^{RMS})$, $P_r^{BRS} > \max(P_r^{MRS}, P_r^{RRS})$, and $P_r^{BVN} > P_r^{MVN} = P_r^{RVN}$: $P_r^{BMS} = P_r^{BRS} > P_r^{MMS} = P_r^{RRS} \geq P_r^{RMS} = P_r^{MRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $P_r^{BMS} = P_r^{BRS} > P_r^{RMS} = P_r^{MRS} > P_r^{MMS} = P_r^{RRS}$ for $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$.

As illustrated in Proposition 2, under any leadership structure (i.e., Manufacturer Stackelberg, Rental Platform Stackelberg, or Vertical Nash), the level of green advertising, the wholesale price, and the rental service charge are higher when both firms invest in green advertising than when only one firm invests in green advertising. When both firms contribute to green-advertising efforts, the increased efforts render a higher rental service charge to consumers and lead to higher profits for both firms.

Parts (a) and (c) of Proposition 2 also indicate that so long as only the Stackelberg leader (follower) assumes the green-advertising cost, the level of green advertising and the rental service charge remain the same (e.g., $t^{MRS} = t^{RMS}$ and $t^{MMS} = t^{RRS}$), regardless of which firm is the Stackelberg leader (follower). These decision variables remain unchanged when the power structure between the fashion manufacturer and the rental platform is balanced, regardless of which firm incurs the green-advertising cost alone (i.e., the *MVN* and *RVN* models).

Our analysis shows that when only one firm invests in green advertising, the power structure in conjunction with the advertising arrangements influences the firms' optimal strategies in a rental market. When only one firm invests in green advertising, whether the Stackelberg leader covers the green-advertising cost is a vital factor that affects firms' optimal strategies and profits in different game models. For example, if the Stackelberg leader (follower) covers the green-advertising cost, the optimal green-advertising level and sale price remain unchanged regardless of which firm covers the cost. In another case where the power structure is balanced, the optimal green-advertising level and sale price are fixed regardless of which firm bears the green-advertising cost. Whether the Stackelberg leader covers the green-advertising cost also indicates the profit level of each firm: in the case where the

Stackelberg leader does not cover the green-advertising cost, both the fashion manufacturer and the rental platform earn more profits than in the case where the Stackelberg leader covers the green-advertising cost. These results extend the understanding of the competition between the fashion manufacturer and the rental platform by revealing the combinatorial effects of their power structure and green-advertising investment decisions on their optimal strategies.

4.2 Impact of the power structure

Propositions 3–5 provide algebraic comparisons of the optimal values of the key decision variables and profits of the rental platform, the fashion manufacturer, and the supply chain in each green-advertising investment situation.

Proposition 3. When only the fashion manufacturer invests in green advertising, the following properties hold:

(a) $min\{t^{MVN}, t^{MRS}\} > t^{MMS}$: $t^{MVN} \geq t^{MRS} > t^{MMS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $t^{MRS} \geq t^{MVN} > t^{MMS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

(b) $w^{MMS} > w^{MVN}$: $w^{MMS} > w^{MVN} \geq w^{MRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $w^{MMS} \geq w^{MRS} > w^{MVN}$ for $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3]$; $w^{MRS} > w^{MMS} > w^{MVN}$ for $\varepsilon \in (8\alpha(1-\alpha)/3, 4\alpha(1-\alpha))$.

(c) $P_r^{MRS} > P_r^{MVN}$: $P_r^{MMS} \geq P_r^{MRS} \geq P_r^{MVN}$ for $\varepsilon \in (0,2\alpha(1-\alpha)]$; $P_r^{MRS} > P_r^{MVN} > P_r^{MMS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

(d) $\Pi_R^{MRS} > \Pi_R^{MVN} > \Pi_R^{MMS}$.

(e) $\Pi_M^{MMS} > \Pi_M^{MVN}$: $\Pi_M^{MMS} > \Pi_M^{MVN} \geq \Pi_M^{MRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_M^{MMS} \geq \Pi_M^{MRS} > \Pi_M^{MVN}$ for $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3]$; $\Pi_M^{MRS} > \Pi_M^{MMS} > \Pi_M^{MVN}$ for $\varepsilon \in (8\alpha(1-\alpha)/3, 4\alpha(1-\alpha))$.

(f) $min\{\Pi_{SC}^{MRS}, \Pi_{SC}^{MVN}\} > \Pi_{SC}^{MMS}$: $\Pi_{SC}^{MVN} \geq \Pi_{SC}^{MRS} > \Pi_{SC}^{MMS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_{SC}^{MRS} > \Pi_{SC}^{MVN} > \Pi_{SC}^{MMS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

Proposition 3(a) shows that the green-advertising level is the lowest in the *MMS* game, which is a natural consequence when the fashion manufacturer incurs the entire green-advertising cost as the Stackelberg leader in deciding the green-advertising level and the sale and wholesale prices.

Additionally, as the green-advertising efficacy ε increases, both the green-advertising level and the wholesale price grow higher in the *MRS* game compared to other game models, as shown in parts (a) and (b) of Proposition 3.

Interestingly, when the green-advertising efficacy ε is relatively low, consumers in the *MMS* model experience the lowest level of green advertising but pay the highest rental service charge, as shown in parts (a) and (c) of Proposition 3. In this case, consumers' sensitivity toward green advertising is relatively low and/or the cost rate for green advertising is relatively high. Therefore, the fashion manufacturer has an incentive to set a low level of green advertising in the *MMS* model. Meanwhile, the wholesale price is high, as shown in Proposition 3(b), since the fashion manufacturer is interested in maximizing its profits as the Stackelberg leader, which leads to a high rental service charge by the rental platform. In the same spirit, consumers experience the opposite situation in the *MVN* model when ε is relatively low, as they receive the highest level of green advertising and the lowest rental service charge. As such, when the green-advertising efficacy ε is relatively low, more consumers are likely to favor the rental product in the case of a balanced power structure between the two firms (i.e., the *MVN* model) than in the case of an imbalanced power structure (e.g., the *MMS* model).

For relatively high green-advertising efficacy, parts (a) and (c) of Proposition 3 show that the relationships between the rental service charges in different game models mirror the relationships between the green-advertising level: the highest (lowest) rental service charge for the highest (lowest) green-advertising level in the *MRS* (*MMS*) model.

Parts (d) and (f) of Proposition 3 indicate that both the rental platform and the supply chain will realize the lowest profits in the *MMS* model. Being responsible for the green-advertising cost, the fashion manufacturer is inclined to lower the level of green advertising. This result suggests that both the fashion manufacturer and the rental platform will miss out on capturing wider environmentally conscious market for rentals. The loss of potential profits from rental service demand makes the *MMS* model the least profitable market structure for the rental platform and the supply chain. Proposition 3(e) shows that even the manufacturer is worse off in the *MMS* game (than in the *MRS* game) when green-advertising efficiency is relatively high.

Proposition 4. When only the rental platform invests in green advertising, the following properties hold:

(a) $min\{t^{RVN}, t^{RMS}\} > t^{RRS}$: $t^{RVN} \geq t^{RMS} > t^{RRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $t^{RMS} > t^{RVN} > t^{RRS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

(b) $min\{w^{RVN}, w^{RMS}\} > w^{RRS}$: $w^{RMS} \geq w^{RVN} > w^{RRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $w^{RVN} > w^{RMS} > w^{RRS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

(c) $P_r^{RMS} > P_r^{RVN}$: $P_r^{RRS} \geq P_r^{RMS} \geq P_r^{RVN}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $P_r^{RMS} > P_r^{RVN} > P_r^{RRS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

(d) $\Pi_R^{RRS} > \Pi_R^{RVN}$: $\Pi_R^{RRS} > \Pi_R^{RVN} \geq \Pi_R^{RMS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_R^{RRS} \geq \Pi_R^{RMS} > \Pi_R^{RVN}$ for $\varepsilon \in \left(2\alpha(1-\alpha), \frac{8\alpha(1-\alpha)}{3}\right]$; $\Pi_R^{RMS} > \Pi_R^{RRS} > \Pi_R^{RVN}$ for $\varepsilon \in (\frac{8\alpha(1-\alpha)}{3}, 4\alpha(1-\alpha))$.

(e) $\Pi_M^{RMS} > \Pi_M^{RVN} > \Pi_M^{RRS}$.

(f) $min\{\Pi_{SC}^{RVN}, \Pi_{SC}^{RMS}\} > \Pi_{SC}^{RRS}$: $\Pi_{SC}^{RVN} \geq \Pi_{SC}^{RMS} > \Pi_{SC}^{RRS}$ for $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_{SC}^{RMS} > \Pi_{SC}^{RVN} > \Pi_{SC}^{RRS}$ for $\varepsilon \in (2\alpha(1-\alpha), 4\alpha(1-\alpha))$.

Parts (a) and (b) of Proposition 4 suggest that both the green-advertising level and the wholesale price are the lowest in the *RRS* model. As the Stackelberg leader, the rental platform tends to set a low level of green advertising as it bears the green-advertising costs and carefully determines its rental service charge to induce a lower wholesale price from the fashion manufacturer. When consumers are more sensitive to green advertising and/or the cost rate for green advertising is relatively low, the rental platform will set the green-advertising level highest in the *RMS* model. This situation occurs even though the Stackelberg follower is responsible for assuming the entire green-advertising cost and paying a higher wholesale price than when it is the leader (i.e., the *RRS* model). In this case, the benefits of investing in green advertising to obtain more significant revenues from rental services overtake the relevant costs incurred, and, therefore, more efforts will be spent on green advertising.

Likewise, for relatively low green-advertising efficacy, the *RRS* model seems least attractive to consumers as they are subjected to the highest rental service charge with the lowest level of green advertising. Profits for the fashion manufacturer and the supply chain are the lowest in the *RRS* model.

This finding again can be explained by the motive of the rental platform as the Stackelberg leader to invest in the lowest level of green advertising, which results in less demand for the fashion rental services and essentially lower profits for the fashion manufacturer and the supply chain. Across the three games models, the *RMS* game is the most lucrative game model for the fashion manufacturer, as it has a more significant advantage as the Stackelberg leader. Though it is the least appealing game model for the rental platform given a low green-advertising efficacy ε, the *RMS* game becomes more desirable as ε increases and eventually becomes the preferable game model for the rental platform when ε is sufficiently high, i.e., $\varepsilon \in \left(8\alpha(1-\alpha)/3\,, 4\alpha(1-\alpha)\right)$.

Corollary 2. When only one firm invests in green advertising, the cost-bearing firm will set the lowest level of green advertising when it is the Stackelberg leader. In this case, the profits for the Stackelberg follower and the supply chain are the lowest.

Corollary 3. When only one firm invests in green advertising and consumers are more sensitive to the green advertising and/or the cost rate for the green-advertising cost is relatively low (i.e., a high green-advertising efficacy ε), it is most profitable for the fashion manufacturer, the rental platform, and the supply chain if the Stackelberg follower invests in green advertising compared to other power structures.

It is strategic for the firm bearing the green-advertising cost to lower the level of green advertising for profit maximization when it is the Stackelberg leader. This move aspires to maximize its own profit but makes it least preferable for the other firm and the supply chain. However, our results show that this strategy can be short-sighted for the cost-bearing firm as well when the green-advertising efficacy is sufficiently high. For $\varepsilon \in \left(8\alpha(1-\alpha)/3\,, 4\alpha(1-\alpha)\right)$, the fashion manufacturer, the rental platform, and the supply chain can obtain more profits if the Stackelberg follower assumes the green-advertising cost and thereby decides the green-advertising level. When consumers' sensitivity toward green advertising is relatively high and/or the cost rate for green advertising is relatively low, increasing the level of green advertising leads to more considerable market expansion effects for fashion rental services. If only the Stackelberg follower incurs the green-advertising cost, more green-advertising efforts will be invested, which will lead to increased revenue from the rental services and eventually facilitate higher profits for all firms. Therefore, the firm that bears the green-advertising cost may not be willing to take the role of Stackelberg leader because they can benefit more as a follower when ε is relatively high.

Proposition 5. When both firms invest in green advertising, the following properties hold:

(a) $t_m^{BMS} = t_r^{BMS} = t_m^{BRS} = t_r^{BRS}$. $t_m^{BVN} \geq t_m^{BMS}$ if $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $t_m^{BVN} < t_m^{BMS}$ if $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$.

(b) $w^{BMS} \geq w^{BVN} \geq w^{BRS}$ if $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $w^{BRS} > w^{BMS} > w^{BVN}$ for $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$.

(c) $P_r^{BMS} = P_r^{BRS}$. $P_r^{BMS} = P_r^{BRS} \geq P_r^{BVN}$ if $\varepsilon \in (0, \alpha(1-\alpha)]$or if $\varepsilon \in [2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$; $P_r^{BMS} = P_r^{BRS} < P_r^{VN}$ for $\varepsilon \in (\alpha(1-\alpha), 2\alpha(1-\alpha))$.

(d) $\Pi_R^{BRS} \geq \Pi_R^{BVN}$ and $\Pi_M^{BMS} \geq \Pi_M^{BVN}$. $\Pi_R^{BRS} \geq \Pi_R^{BVN} \geq \Pi_R^{BMS}$ and $\Pi_M^{BMS} \geq \Pi_M^{BVN} \geq \Pi_M^{BRS}$if $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_R^{BMS} > \Pi_R^{BRS} > \Pi_R^{BVN}$ and $\Pi_M^{BRS} > \Pi_M^{BMS} > \Pi_M^{BVN}$if $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$.

(e) $\Pi_{SC}^{BMS} = \Pi_{SC}^{BRS}$. $\Pi_{SC}^{BMS} = \Pi_{SC}^{BRS} \leq \Pi_{SC}^{BVN}$ if $\varepsilon \in (0, 2\alpha(1-\alpha)]$; $\Pi_{SC}^{BMS} = \Pi_{SC}^{BRS} > \Pi_{SC}^{BVN}$ for $\varepsilon \in (2\alpha(1-\alpha), 8\alpha(1-\alpha)/3)$.

Proposition 5(a) shows that in the two Stackelberg games (i.e., *BMS* and *BRS*), the fashion manufacturer and the rental platform spend the same effort on green advertising. In fact, the green-advertising efforts from both firms and the rental charge set by the rental platform remain unchanged in the two Stackelberg games, regardless of who is the Stackelberg leader, as shown in parts (a) and (c) of Proposition 5. Additionally, if ε is low, a Nash game is more beneficial for consumers with a greater level of green advertising but lower rental service charge than in the Stackelberg games. Notably, the nature of these results is consistent with the main model.

Regarding the profits of the two firms, Proposition 5(d) suggests that if $\varepsilon \in (0,2(1-\alpha)\alpha]$, each firm earns the highest profit as the Stackelberg leader and lowest as the Stackelberg follower. However, when $\varepsilon > 2(1-\alpha)\alpha$, a firm can obtain the highest profit as a Stackelberg follower and perform the worst in a Nash game. Proposition 5(e) shows that total supply chain profits in the two Stackelberg games are equal and, therefore, do not depend on which firm assumes the leadership role in a Stackelberg game. Specifically, the total supply chain profits are higher (lower) in the two Stackelberg games than that in the *BVN* game when $\varepsilon > 2(1-\alpha)\alpha$ ($\varepsilon < 2(1-\alpha)\alpha$).

Proposition 6. $P_b = \frac{1-E_b}{2}$ in all 12 game models.

Proposition 6 shows that the fashion manufacturer's competition with the rental platform does not influence its pricing strategy toward consumers. In other words, the fashion manufacturer keeps its sale price unchanged when allowing its products to be rented, regardless of the power structure and the green-advertising investment situation between itself and the rental platform. This finding can be explained as follows. Across different game setups, the fashion manufacturer optimizes its profits by changing its wholesale price, which affects the rental service charge and, therefore, indirectly impacts the consumer demand for the products sold. The consumers then observe a stable sale price, as the fashion retail market is more mature with less frequent pricing alterations over time, corroborating the industrial observation in luxury fashion on the relatively stable retail price (Chiu *et al.*, 2018). This result is also not uncommon in the literature—previous studies (e.g., Chiang, Chhajed, and Hess, 2003) have noted an unchanged retail price across different supply chain structures.

4.3 Sensitivity analysis

In this section, we examine the impacts of three critical model parameters (i.e., the green-advertising efficiency ε, the salvage value s, and the rental service usage effort E_r) on the optimal decision variables and profits. We focus on the nine game models (i.e., *MMS*, *MRS*, *MVN*, *RMS*, *RRS*, *RVN*, *BMS*, *BRS*, *BVN*) in which at least one firm invests in green advertising. Table 3 presents the signs of the first derivatives of optimal decision variables and profits with respect to the parameters, respectively. Here, $sgn[z]$ denotes the sign of the term z: $sgn[z]$ might be $+$, $-$, or 0.

Table 3. Sign of the first derivatives of optimal decisions with respect to ε, s, and E_r.

	$\varepsilon(k^2/\beta)$	s	E_r
t^{j_1}	$+$	$+$	$-$
$t_i^{j_2}$	$+$	$+$	$-$
w^j	$\begin{cases} 0\ for\ j = RMS; \\ +, otherwise \end{cases}$	$+$	$-$
$P_r^{j_1}$	$+$	$sgn[\varepsilon - 2(1-\alpha)\alpha]$	$-$
$P_r^{j_2}$	$+$	$sgn[\varepsilon - (1-\alpha)\alpha]$	$-$
Π_R^{RVN}	$sgn[2(1-\alpha)\alpha - \varepsilon]$	$+$	$-$
$\Pi_R^j\ (j \neq RVN)$	$+$	$+$	$-$
Π_M^{MVN}	$sgn[2(1-\alpha)\alpha - \varepsilon]$	$+$	$-$
$\Pi_M^j\ (j \neq MVN)$	$+$	$+$	$-$
Π_{SC}^j	$+$	$+$	$-$

Note: *i* = *m* (for the fashion manufacturer), *r* (for the rental platform); *j = MMS, MRS, MVN, RMS, RRS, RVN, BMS, BRS, BVN;* j_1 *= MMS, MRS, MVN, RMS, RRS, RVN;* j_2 *= BMS, BRS, BVN.*

As shown in Table 3, the green-advertising efficacy ε has an increasing effect on the level of green advertising and rental service charge under all nine game models. A higher ε implies that consumers are more sensitive toward green advertising and/or a lower cost rate for green advertising, which provides an enormous stimulus for businesses to make more efforts for the green advertising and expand fashion rental services with less expensive investments. Consequently, a higher wholesale price from the manufacturer (except in the *RMS* model) is induced to exploit the increased popularity of fashion rental services. In the *RMS* model, the manufacturer is the Stackelberg leader and does not bear the green-advertising cost. As a result, the optimal wholesale price in the *RMS* model, $w^* = (s + \alpha - E_r)/2$, is the same as in the *NMS* model (see Tables A1 and A3 in Appendix B), where the manufacturer, as the Stackelberg leader unconcerned with the green-advertising investment, makes decisions first and sets the optimal wholesale price that is independent of the green-advertising efficacy. In response to the non-decreasing wholesale price and the increased consumer utility from renting (owing to the enhanced green-advertising level), the rental platform sets a higher rental service charge. Consequently, profits for the fashion manufacturer (except in the *MVN* game), the rental platform (except in the *RVN* game), and the supply chain will monotonously increase as ε increases. Under the Vertical Nash structure, profits for the firm that bears the green-advertising cost increase as ε increases but dwindle after a certain threshold when $\varepsilon > 2\alpha(1-\alpha)$. The benefits from the increased level of green advertising with a growing ε remain more significant than the corresponding green-advertising costs. However, after the threshold when $\varepsilon > 2\alpha(1-\alpha)$, the increasing green-advertising cost grows more considerable, and the cost-bearing firm cannot gain any advantage due to the balanced power structure against its supply chain partner, resulting in shrinking profits for the cost-bearing firm. This finding illustrates the profit pattern in the *MVN* and *RVN* models and suggests that an increasing green-advertising efficacy is not always beneficial.

Based on Table 3, key decision variables, except the rental service charge, and profits for the rental platform, the fashion manufacturer, and the supply chain, increase in s regardless of the game models. A higher s implies a higher product value and/or a less worn-out garment that has not been rented for many periods, which motivates the firm(s) to invest more (i.e., a higher t) to induce more demand for rental services and maximize profits. Meanwhile, a higher wholesale price w will be imposed on the rental platform for a higher s as the fashion manufacturer inclines to take advantage of the higher margin of the rental platform who can earn more salvage value at the end of the rental period. However, the impact of s on the rental service charge is not linear.

When green-advertising efficiency is relatively high (e.g., $\varepsilon > (1-\alpha)\alpha$ in the *BMS*, *BRS*, and *BVN* game models), the rental service will appeal to consumers who are more sensitive to its sustainability feature owing to a higher t (in response to the higher s). Therefore, the rental platform can set a higher rental service charge to obtain maximum profits. However, when the green-advertising efficiency is relatively low, the compensation effect from the increasing green-advertising level is not significant. In this case, as s increases, the rental platform should decrease the rental service charge to appeal to consumers while ensuring an optimal margin ($P_r + s - w$), i.e., P_r decreases in s for a low ε. As s increases, the higher wholesale price for the manufacturer and the optimal retail margin (increases in s and decreases in P_r) lead to growing profits for both the manufacturer and the rental platform and, therefore, the entire supply chain.

Table 3 also suggests that key decision variables and profits for the rental platform, the fashion manufacturer, and the supply chain decrease in E_r. When the fashion rental services are associated with a lot of hassle (i.e., a higher E_r), the course of action to be taken to make the rental service less unattractive is to set a lower rental service charge rather than to increase the level of green advertising.

In general, newly rolled-out services could entail specific levels of effort for consumers but will grow more convenient and user-friendly with fewer hassles as a wider consumer base embraces the service. Our results suggest that rental platforms should focus more on lowering their service charges rather than boosting their green advertising when in the infancy stage. As fashion rental services become more convenient for consumers with less hassle and effort required, all involved firms will benefit.

5 Concluding remarks

The state of fashion retailing has changed with the growth of fashion rental services, driven by the shift to a circular economy. To embrace this trend, many fashion manufacturers have partnered with rental platforms to rent their products to consumers while preserving their retailing services. The emerging challenges from such business practices include reconsidering the sale price, wholesale price, the rental service charge, and the level of green advertising to attract more eco-friendly consumers. Such decisions become more complicated under different power structures between the fashion manufacturer and the rental platform. Though pertinent, these decisions have not been examined in the extant literature. To fill this gap, our paper develops game-theoretical models to investigate these pricing and marketing decisions under different power structures and green-advertising investment situations.

Our results demonstrate the critical impacts of the green-advertising efficacy on key decision variables. Every firm has its cost coefficient of green advertising, and every consumer exhibits a level of sensitivity toward green advertising. Our analysis reveals that the combinatorial rather than individual effects of these two factors influence firms' decision-making by forming a green-advertising efficacy parameter.

We also highlight that the green-advertising investment situation and the power structure between the fashion manufacturer and the rental platform have diverse (and even no) influences on different decision variables and profits. By examining four green-advertising investment situations (i.e., no firm invests, only the fashion manufacturer invests, only the rental platform invests, and both firms invest in green advertising), we show that both firms obtain the lowest profits when neither invests in green advertising. This finding highlights the importance of investing in the promotion and advertising of fashion rental services as a green and sustainable option for consumers to stay fashionable. Contrarily, when both firms invest in green advertising, they will be rewarded with the highest profits. Additionally, we show that the level of green advertising and the rental service charge vary across different power structures under different green-advertising investment situations, whereas the sale price remains unchanged. Therefore, competition and cooperation between the fashion manufacturer and the rental platform mainly influence the rental services (e.g., the service charge), and consumers perceive a stable retailing market.

Additionally, we present some interesting insights and note that proactively undertaking the green-advertising cost can be beneficial rather than unfavorable for firms under certain circumstances in the case when only a single firm invests in green advertising. Although it is disadvantageous to the cost-bearing firm when the power structure is balanced, our results reveal that a firm should proactively invest in green advertising to maximize its profits when it is the Stackelberg follower. Furthermore, we show that whether the Stackelberg leader covers the green-advertising cost is a critical factor influencing the firms' interactions in the fashion market. For example, when the Stackelberg leader covers the green-advertising cost, both firms earn fewer profits than when the Stackelberg leader refrains from covering the cost, regardless of which firm covers the green-advertising cost.

Though value-adding, this paper also presents some limitations that could serve as future research topics. For example, it would be interesting to extend our study and consider a more complex multiple-rental model with dynamic pricing and green advertising decisions for each rental, as consumers may have different valuations of buying or renting the product across different rental periods. A further extension can be the investigation of the optimal rental periods for the fashion product, considering critical factors such as the fashion product's key properties (e.g., durability) and consumers' perceptions of the rental span of the product. Additionally, this paper focuses on vertical competition, and it would also be illuminating to investigate the optimal decisions under competitive market environments by considering the horizontal competition between multiple fashion manufacturers or rental platforms.

Appendix A

We illustrate the logic of deriving the optimal solutions for the decision variables and profits by providing detailed calculation steps in the *MMS*, *MRS*, *MVN* games, which cover all the three leadership structures. Tables A1-A3 summarize the results for other game models, which can be easily concluded using the same logic. For ease of presentation, we denote $A = s + \alpha E_b - E_r$, $B = \frac{\alpha - s - E_r}{2}$ and $C = \frac{(1-E_b)^2}{4}$.

The *MMS* game

In this case, we solve the rental platform's profit function first.

$$\Pi_R(P_r) = (P_r + s - w)D_r = (P_r + s - w)\left(\frac{P_b + E_b - P_r - E_r + kt}{1-\alpha} - \frac{P_r + E_r - kt}{\alpha}\right).$$

We get $\frac{\partial \Pi_R(P_r)}{\partial P_r} = \frac{s - kt - w - \alpha E_b + E_r - \alpha P_b + 2P_r}{(-1+\alpha)\alpha}$ and $\frac{\partial^2 \Pi_R(P_r)}{\partial P_r^2} = \frac{2}{(-1+\alpha)\alpha} < 0$, which implies that $\Pi_R(P_r)$ is concave in P_r. Let $\frac{\partial \Pi_R(P_r)}{\partial P_r} = 0$, we get $P_r(P_b, w, t) = \frac{-s + kt + w + \alpha E_b - E_r + \alpha P_b}{2}$.

The fashion manufacturer's profit function is

$$\Pi_M(P_b, w, t) = P_b\left(1 - \frac{P_b + E_b - P_r - E_r + kt}{1-\alpha}\right) + w\left(\frac{P_b + E_b - P_r - E_r + kt}{1-\alpha} - \frac{P_r + E_r - kt}{\alpha}\right) - \beta t^2.$$

Substituting $P_r(P_b, w, t)$ into the above equation and then we get $\frac{\partial \Pi_M(P_b,w,t)}{\partial P_b} = \frac{2 - s - kt + 2w - 2\alpha + (\alpha-2)E_b + E_r - 4P_b + 2\alpha P_b}{2(1-\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b^2} = -\frac{2-\alpha}{1-\alpha}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b \partial w} = \frac{1}{1-\alpha}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b \partial t} = -\frac{k}{2(1-\alpha)}$, $\frac{\partial \Pi_M(P_b,w,t)}{\partial w} = \frac{s + kt - 2w + \alpha E_b - E_r + 2\alpha P_b}{2\alpha(1-\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial w^2} = -\frac{1}{\alpha(1-\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial w \partial t} = \frac{k}{2\alpha(1-\alpha)}$, $\frac{\partial \Pi_M(P_b,w,t)}{\partial t} = \frac{1}{2}\left[\frac{k(c-w)}{\alpha(-1+\alpha)} - 4t\beta + \frac{k(-c+P_b)}{-1+\alpha}\right]$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial t^2} = -2\beta$. For $k^2 < 8(1-\alpha)\alpha\beta$,

$$\begin{vmatrix} -\frac{2-\alpha}{1-\alpha} & \frac{1}{1-\alpha} & -\frac{k}{2(1-\alpha)} \\ \frac{1}{1-\alpha} & -\frac{1}{\alpha(1-\alpha)} & \frac{k}{2\alpha(1-\alpha)} \\ -\frac{k}{2(1-\alpha)} & \frac{k}{2\alpha(1-\alpha)} & -2\beta \end{vmatrix} = \frac{k^2 - 8(1-\alpha)\alpha\beta}{2\alpha^2(1-\alpha)^2} < 0, \quad \begin{vmatrix} -\frac{1}{\alpha(1-\alpha)} & \frac{k}{2\alpha(1-\alpha)} \\ \frac{k}{2\alpha(1-\alpha)} & -2\beta \end{vmatrix} = \frac{8(1-\alpha)\alpha\beta - k^2}{4\alpha^2(1-\alpha)^2} > 0,$$

$$\begin{vmatrix} -\frac{2-\alpha}{1-\alpha} & -\frac{k}{2(1-\alpha)} \\ -\frac{k}{2(1-\alpha)} & -2\beta \end{vmatrix} = \frac{8(2-\alpha)(1-\alpha)\beta - k^2}{4(1-\alpha)^2} > 0, \quad \begin{vmatrix} -\frac{2-\alpha}{1-\alpha} & \frac{1}{1-\alpha} \\ \frac{1}{1-\alpha} & -\frac{1}{\alpha(1-\alpha)} \end{vmatrix} = \frac{2}{\alpha(1-\alpha)} > 0$$

and $\Pi_M(P_b, w, t)$ is joint concave in P_b, w, and t. Let $\frac{\partial \Pi_M(P_b,w,t)}{\partial P_b} = \frac{\partial \Pi_M(P_b,w,t)}{\partial w} = \frac{\partial \Pi_M(P_b,w,t)}{\partial t} = 0$, we get:

$$P_b^* = \frac{1-E_b}{2},\ w^* = \frac{s+\alpha-E_r}{2} + \frac{k^2(s+\alpha E_b - E_r)}{2[8(1-\alpha)\alpha\beta - k^2]},\ t^* = \frac{k(s+\alpha E_b - E_r)}{8(1-\alpha)\alpha\beta - k^2}.$$

$P_r^* = \frac{\alpha-s-E_r}{2} + \frac{[4(1-\alpha)\alpha\beta+k^2](s+\alpha E_b-E_r)}{2[8(1-\alpha)\alpha\beta-k^2]}$. $\Pi_R^* = \frac{4(1-\alpha)\alpha\beta^2(s+\alpha E_b-E_r)^2}{[8(1-\alpha)\alpha\beta-k^2]^2}$, $\Pi_M^* = \frac{(1-E_b)^2}{4} + \frac{\beta[s+\alpha E_b-E_r]^2}{8(1-\alpha)\alpha\beta-k^2}$, $\Pi_{SC}^* = \frac{(1-E_b)^2}{4} + \frac{\beta[12(1-\alpha)\alpha\beta-k^2](s+\alpha E_b-E_r)^2}{[8(1-\alpha)\alpha\beta-k^2]^2}$. Note that $P_b^* - P_r^* = \frac{[8(1-\alpha)\alpha\beta-k^2](1-\alpha)-2[k^2-2(1-\alpha)\alpha\beta]s+12(1-\alpha)\alpha\beta E_r-\{[8(1-\alpha)\alpha\beta-k^2]+\alpha[4(1-\alpha)\alpha\beta+k^2]\}E_b}{2[8(1-\alpha)\alpha\beta-k^2]}$.With $E_r > E_b$, $0 < \alpha < 1$, and if $[k^2 - 2(1-\alpha)\alpha\beta]s < [4(1-\alpha)\alpha\beta - k^2](1-\alpha)$, $P_b^* - P_r^* > 0$ holds in the *MMS* game.

The *MRS* game

In this case, we solve the fashion manufacturer's profit function first:

$$\Pi_M(P_b, w, t) = P_b\left[1 - \frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha}\right] + w\left[\frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha} - \frac{(w+m-s)+E_r-kt}{\alpha}\right] - \beta t^2.$$

We get $\frac{\partial \Pi_M(P_b,w,t)}{\partial P_b} = \frac{-1-m+s+kt-2w+\alpha+E_b-E_r+2P_b}{-1+\alpha}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b^2} = -\frac{2}{1-\alpha}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b \partial w} = \frac{2}{1-\alpha}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial P_b \partial t} = -\frac{k}{1-\alpha}$, $\frac{\partial \Pi_M(P_b,w,t)}{\partial w} = \frac{-m+s+kt-2w+\alpha E_b-E_r+2\alpha P_b}{\alpha(1-\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial w^2} = -\frac{2}{\alpha(1-\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial w \partial t} = \frac{k}{\alpha(1-\alpha)}$, $\frac{\partial \Pi_M(P_b,w,t)}{\partial t} = \frac{-kw+2t\alpha\beta-2t\alpha^2\beta+k\alpha P_b}{\alpha(-1+\alpha)}$, $\frac{\partial^2 \Pi_M(P_b,w,t)}{\partial t^2} = -2\beta$. For $k^2 < 4(1-\alpha)\alpha\beta$, $\begin{vmatrix} -\frac{2}{1-\alpha} & \frac{2}{1-\alpha} & -\frac{k}{1-\alpha} \\ \frac{2}{1-\alpha} & -\frac{2}{\alpha(1-\alpha)} & \frac{k}{\alpha(1-\alpha)} \\ -\frac{k}{1-\alpha} & \frac{k}{\alpha(1-\alpha)} & -2\beta \end{vmatrix} = \frac{2[k^2-4(1-\alpha)\alpha\beta]}{\alpha^2(1-\alpha)^2} < 0$, $\begin{vmatrix} -\frac{2}{\alpha(1-\alpha)} & \frac{k}{\alpha(1-\alpha)} \\ \frac{k}{\alpha(1-\alpha)} & -2\beta \end{vmatrix} = \frac{4(1-\alpha)\alpha\beta-k^2}{\alpha^2(1-\alpha)^2} > 0$, $\begin{vmatrix} -\frac{2}{1-\alpha} & -\frac{k}{1-\alpha} \\ -\frac{k}{1-\alpha} & -2\beta \end{vmatrix} = \frac{4(1-\alpha)\beta-k^2}{(1-\alpha)^2} > 0$, $\begin{vmatrix} -\frac{2}{1-\alpha} & \frac{2}{1-\alpha} \\ \frac{2}{1-\alpha} & -\frac{2}{\alpha(1-\alpha)} \end{vmatrix} = \frac{4}{\alpha(1-\alpha)} > 0$ and $\Pi_M(P_b, w, t)$ is joint concave in P_b, w, and t. Let $\frac{\partial \Pi_M(P_b,w,t)}{\partial P_b} = \frac{\partial \Pi_M(P_b,w,t)}{\partial w} = \frac{\partial \Pi_M(P_b,w,t)}{\partial t} = 0$, we get $P_b^* = \frac{1-E_b}{2}$, $w(m) = \frac{-k^2\alpha+4\alpha\beta(-m+s)+4\alpha^2\beta(1+m-s+\alpha)+k^2\alpha E_b-4(1-\alpha)\alpha\beta E_r}{2[4(1-\alpha)\alpha\beta-k^2]}$, $t(m) = \frac{k(-m+s+\alpha E_b-E_r)}{4(1-\alpha)\alpha\beta-k^2}$.

The rental platform's profit function is

$$\Pi_R(m) = mD_r = m\left[\frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha} - \frac{(w+m-s)+E_r-kt}{\alpha}\right].$$

Substituting $w(m)$ and $t(m)$ into the above equation and we get $\frac{\partial \Pi_R(m)}{\partial m} = \frac{2\beta(-2m+s+\alpha E_b-E_r)}{4(1-\alpha)\alpha\beta-k^2}$ and $\frac{\partial^2 \Pi_R(P_r)}{\partial m^2} = -\frac{4\beta}{4(1-\alpha)\alpha\beta-k^2}$, which shows that $\partial \Pi_R(m)$ is concave in m for $k^2 < 4(1-\alpha)\alpha\beta$,. Let $\frac{\partial \Pi_R(m)}{\partial m} = 0$ and we get $m^* = \frac{s+\alpha E_b-E_r}{2}$. Therefore, $w^* = \frac{s+\alpha-E_r}{2} + \frac{[k^2-2(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)}{2[4(1-\alpha)\alpha\beta-k^2]}$, $t^* =$

$\frac{k(s+\alpha E_b-E_r)}{2[4(1-\alpha)\alpha\beta-k^2]}$, $P_r^*=\frac{-s+\alpha-E_r}{2}+\frac{(1-\alpha)\alpha\beta(s+\alpha E_b-E_r)}{4(1-\alpha)\alpha\beta-k^2}$, $\Pi_R^*=\frac{\beta(s+\alpha E_b-E_r)^2}{2[4(1-\alpha)\alpha\beta-k^2]}$, $\Pi_M^*=\frac{(1-E_b)^2}{4}+\frac{\beta(s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2]}$, $\Pi_{SC}^*=\frac{(1-E_b)^2}{4}+\frac{3\beta(s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2]}$. Clearly, $P_b^*-P_r^*=\frac{[4(1-\alpha)\alpha\beta-k^2](1-\alpha)-[k^2-2(1-\alpha)\alpha\beta]s+[6(1-\alpha)\alpha\beta-k^2]E_r-\{[4(1-\alpha)\alpha\beta-k^2]+2(1-\alpha)\alpha^2\beta]\}E_b}{2[4(1-\alpha)\alpha\beta-k^2]}$ .With $E_r>E_b$, $0<\alpha<1$, and if $[k^2-2(1-\alpha)\alpha\beta]s<[4(1-\alpha)\alpha\beta-k^2](1-\alpha)$, $P_b^*-P_r^*>0$ holds in the *MRS* game.

The *MVN* game

The rental platform's profit function is

$$\Pi_R(m)=mD_r=m\left[\frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha}-\frac{(w+m-s)+E_r-kt}{\alpha}\right].$$

We get $\frac{\partial\Pi_R(m)}{\partial m}=\frac{2m-s-kt+w-\alpha E_b+E_r-\alpha P_b}{(-1+\alpha)\alpha}$ and $\frac{\partial^2\Pi_R(m)}{\partial m^2}=-\frac{2}{(1-\alpha)\alpha}<0$, which means that $\Pi_R(m)$ is concave in m.

The fashion manufacturer's profit function is

$$\Pi_M(P_b,w,t)=P_b\left[1-\frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha}\right]+w\left[\frac{P_b+E_b-(w+m-s)-E_r+kt}{1-\alpha}-\frac{(w+m-s)+E_r-kt}{\alpha}\right]-\beta t^2.$$

We get $\frac{\partial\Pi_M(P_b,w,t)}{\partial P_b}=\frac{-1-m+s+kt-2w+\alpha+E_b-E_r+2P_b}{-1+\alpha}$, $\frac{\partial\Pi_M(P_b,w,t)}{\partial w}=\frac{-m+s+kt-2w+\alpha E_b-E_r+2\alpha P_b}{\alpha(1-\alpha)}$, $\frac{\partial\Pi_M(P_b,w,t)}{\partial t}=\frac{-kw+2t\alpha\beta-2t\alpha^2\beta+k\alpha P_b}{\alpha(-1+\alpha)}$. The concavity for the fashion manufacturer's profit function has been checked in the MRS game model and therefore is omitted here to avoid repetition.

Let $\frac{\partial\Pi_R(m)}{\partial m}=\frac{\partial\Pi_M(P_b,w,t)}{\partial P_b}=\frac{\partial\Pi_M(P_b,w,t)}{\partial w}=\frac{\partial\Pi_M(P_b,w,t)}{\partial t}=0$, we get $P_b^*=\frac{1-E_b}{2}$, $m^*=\frac{2(1-\alpha)\alpha\beta(s+\alpha E_b-E_r)}{6(1-\alpha)\alpha\beta-k^2}$, $w^*=\frac{s+\alpha-E_r}{2}+\frac{[k^2-2(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)}{2[6(1-\alpha)\alpha\beta-k^2]}$, $t^*=\frac{k(s+\alpha E_b-E_r)}{6(1-\alpha)\alpha\beta-k^2}$. Therefore, $P_r^*=\frac{\alpha-s-E_r}{2}+\frac{[k^2+2(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)}{2[6(1-\alpha)\alpha\beta-k^2]}$, $\Pi_R^*=\frac{4(1-\alpha)\alpha\beta^2(s+\alpha E_b-E_r)^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$, $\Pi_M^*=\frac{(1-E_b)^2}{4}+\frac{\beta[4(1-\alpha)\alpha\beta-k^2](s+\alpha E_b-E_r)^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$, $\Pi_{SC}^*=\frac{(1-E_b)^2}{4}+\frac{\beta[8(1-\alpha)\alpha\beta-k^2](s+\alpha E_b-E_r)^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$.

As a result, when $[k^2-2(1-\alpha)\alpha\beta]s<[4(1-\alpha)\alpha\beta-k^2](1-\alpha)$, $P_b^*>P_r^*$ hold for the *MMS* and *MRS* games. We show in Proposition 3 and Appendix B that P_r^* is the lower in the *MVN* game than that in the *MRS* games. Hence, $P_b^*-P_r^*>0$ holds in the *MVN* game under the same condition.

Results for other game models

Table A1. Equilibrium solutions for the RMS, RRS, and RVN games

	RMS	*RRS*	*RVN*
P_b^*	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$
t^*	$\frac{kA}{2[4(1-\alpha)\alpha\beta-k^2]}$	$\frac{kA}{8(1-\alpha)\alpha\beta-k^2}$	$\frac{kA}{6(1-\alpha)\alpha\beta-k^2}$
w^*	$\frac{s+\alpha-E_r}{2}$	$\frac{s+\alpha-E_r}{2}-\frac{[4(1-\alpha)\alpha\beta-k^2]A}{2[8(1-\alpha)\alpha\beta-k^2]}$	$\frac{s+\alpha-E_r}{2}+\frac{(k^2-2(1-\alpha)\alpha\beta)A}{2[6(1-\alpha)\alpha\beta-k^2]}$
P_r^*	$B+\frac{(1-\alpha)\alpha\beta A}{4(1-\alpha)\alpha\beta-k^2}$	$B+\frac{[4(1-\alpha)\alpha\beta+k^2]A}{2[8(1-\alpha)\alpha\beta-k^2]}$	$B+\frac{[2(1-\alpha)\alpha\beta+k^2]A}{2[6(1-\alpha)\alpha\beta-k^2]}$
Π_R^*	$\frac{\beta A^2}{4[4(1-\alpha)\alpha\beta-k^2]}$	$\frac{\beta A^2}{8(1-\alpha)\alpha\beta-k^2}$	$\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$
Π_M^*	$C+\frac{\beta A^2}{2[4(1-\alpha)\alpha\beta-k^2]}$	$C+\frac{4(1-\alpha)\alpha\beta^2A^2}{[8(1-\alpha)\alpha\beta-k^2]^2}$	$C+\frac{4(1-\alpha)\alpha\beta^2A^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$
Π_{SC}^*	$C+\frac{3\beta A^2}{4[4(1-\alpha)\alpha\beta-k^2]}$	$C+\frac{\beta[12(1-\alpha)\alpha\beta-k^2]A^2}{[8(1-\alpha)\alpha\beta-k^2]^2}$	$C+\frac{\beta[8(1-\alpha)\alpha\beta-k^2]A^2}{[6(1-\alpha)\alpha\beta-k^2]^2}$

Table A2. Equilibrium solutions for the BMS, BRS, and BVN games

	BMS	*BRS*	*BVN*
P_b^*	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$
t_m^* (t_r^*)	$\frac{kA}{8(1-\alpha)\alpha\beta-3k^2}$	$\frac{kA}{8(1-\alpha)\alpha\beta-3k^2}$	$\frac{kA}{2[3(1-\alpha)\alpha\beta-k^2]}$
w^*	$\frac{s+\alpha-E_r}{2}+\frac{k^2A}{2[8(1-\alpha)\alpha\beta-k^2]}$	$\frac{s+\alpha-E_r}{2}+\frac{[3k^2-4(1-\alpha)\alpha\beta]A}{2[8(1-\alpha)\alpha\beta-3k^2]}$	$\frac{s+\alpha-E_r}{2}+\frac{[k^2-(1-\alpha)\alpha\beta]A}{2[3(1-\alpha)\alpha\beta-k^2]}$
P_r^*	$B+\frac{[4(1-\alpha)\alpha\beta+k^2]A}{2[8(1-\alpha)\alpha\beta-3k^2]}$	$B+\frac{[4(1-\alpha)\alpha\beta+k^2]A}{2[8(1-\alpha)\alpha\beta-3k^2]}$	$B+\frac{[(1-\alpha)\alpha\beta+k^2]A}{2[3(1-\alpha)\alpha\beta-k^2]}$
Π_R^*	$\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{[8(1-\alpha)\alpha\beta-3k^2]^2}$	$\frac{\beta A^2}{8(1-\alpha)\alpha\beta-3k^2}$	$\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{4[3(1-\alpha)\alpha\beta-k^2]^2}$
Π_M^*	$C+\frac{\beta A^2}{8(1-\alpha)\alpha\beta-3k^2}$	$C+\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{[8(1-\alpha)\alpha\beta-3k^2]^2}$	$C+\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{4[3(1-\alpha)\alpha\beta-k^2]^2}$
Π_{SC}^*	$C+\frac{4\beta[3(1-\alpha)\alpha\beta-k^2]A^2}{[8(1-\alpha)\alpha\beta-3k^2]^2}$	$C+\frac{4\beta[3(1-\alpha)\alpha\beta-k^2]A^2}{[8(1-\alpha)\alpha\beta-3k^2]^2}$	$C+\frac{\beta[4(1-\alpha)\alpha\beta-k^2]A^2}{2[3(1-\alpha)\alpha\beta-k^2]^2}$

Table A3. Equilibrium solutions for the NMS, NRS, and NVN games

	NMS	NRS	NVN
P_b^*	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$	$\frac{1-E_b}{2}$
w^*	$\frac{s+\alpha-E_r}{2}$	$\frac{s+2\alpha-\alpha E_b-E_r}{4}$	$\frac{2s+3\alpha-\alpha E_b-2E_r}{6}$
P_r^*	$\frac{2\alpha+\alpha E_b-s-3E_r}{4}$	$\frac{2\alpha+\alpha E_b-s-3E_r}{4}$	$\frac{3\alpha+\alpha E_b-2s-4E_r}{6}$
Π_R^*	$\frac{A^2}{16(1-\alpha)\alpha}$	$\frac{A^2}{8(1-\alpha)\alpha}$	$\frac{A^2}{9(1-\alpha)\alpha}$
Π_M^*	$C+\frac{A^2}{8(1-\alpha)\alpha}$	$C+\frac{A^2}{16(1-\alpha)\alpha}$	$C+\frac{A^2}{9(1-\alpha)\alpha}$
Π_{SC}^*	$C+\frac{3A^2}{16(1-\alpha)\alpha}$	$C+\frac{3A^2}{16(1-\alpha)\alpha}$	$C+\frac{2A^2}{9(1-\alpha)\alpha}$

Appendix B

Proof for Proposition 1

(a) $\Pi_R^{BMS}-\Pi_R^{RMS}=\frac{\beta k^2[16(1-\alpha)\alpha\beta-5k^2](s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][8(1-\alpha)\alpha\beta-3k^2]^2}>0,$

$$\Pi_R^{RMS}-\Pi_R^{MMS}=\frac{k^4\beta(s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][8(1-\alpha)\alpha\beta-3k^2]^2}>0,$$

$$\Pi_R^{MMS} - \Pi_R^{NMS} = \frac{k^2[16(1-\alpha)\alpha\beta - k^2](s+\alpha E_b - E_r)^2}{16(1-\alpha)\alpha[8(1-\alpha)\alpha\beta - k^2]^2} > 0,$$

$$\Pi_R^{BRS} - \Pi_R^{MRS} = \frac{k^2\beta(s+\alpha E_b - E_r)^2}{2[4(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - 3k^2]} > 0,$$

$$\Pi_R^{MRS} - \Pi_R^{RRS} = \frac{k^2\beta(s+\alpha E_b - E_r)^2}{2[4(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - 3k^2]} > 0,$$

$$\Pi_R^{RRS} - \Pi_R^{NRS} = \frac{k^2(s+\alpha E_b - E_r)^2}{8(1-\alpha)\alpha[8(1-\alpha)\alpha\beta - k^2]} > 0,$$

$$\Pi_R^{BVN} - \Pi_R^{MVN} = \frac{k^2\beta[12(1-\alpha)^2\alpha^2\beta^2 - k^4](s+\alpha E_b - E_r)^2}{4[3(1-\alpha)\alpha\beta - k^2]^2[6(1-\alpha)\alpha\beta - k^2]^2} > 0,$$

$$\Pi_R^{MVN} - \Pi_R^{RVN} = \frac{k^2\beta(s+\alpha E_b - E_r)^2}{[6(1-\alpha)\alpha\beta - k^2]^2} > 0,$$

$$\Pi_R^{RVN} - \Pi_R^{NVN} = \frac{k^2[3(1-\alpha)\alpha\beta - k^2](s+\alpha E_b - E_r)^2}{9(1-\alpha)\alpha[6(1-\alpha)\alpha\beta - k^2]^2} > 0.$$

The deviation for other parts in Proposition 1 and Proposition 2 can be easily derived using the same method.

Proof for Proposition 3

(a) $t^{MMS} - t^{MRS} = -\frac{k^3(s+\alpha E_b - E_r)}{2[4(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]} < 0,$

$t^{MMS} - t^{MVN} = -\frac{2k(1-\alpha)\alpha\beta(s+\alpha E_b - E_r)}{[6(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]} < 0,$

$t^{MRS} - t^{MVN} = \frac{k[k^2 - 2(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{2[4(1-\alpha)\alpha\beta - k^2][6(1-\alpha)\alpha\beta - k^2]}$. If $k^2 < 2(1-\alpha)\alpha\beta$, $t^{MMS} < t^{MRS} < t^{MVN}$; if $2(1-\alpha)\alpha\beta < k^2 < 4(1-\alpha)\alpha\beta$, $t^{MMS} < t^{MVN} < t^{MRS}$.

(b) $w^{MMS} - w^{MRS} = \frac{(-1+\alpha)\alpha\beta[3k^2 - 8(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{[4(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]},$

$w^{MMS} - w^{MVN} = \frac{2(-1+\alpha)\alpha\beta[k^2 - 4(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{[6(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]} > 0,$

$w^{MRS} - w^{MVN} = \frac{(1-\alpha)\alpha\beta[k^2 - 2(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{[4(1-\alpha)\alpha\beta - k^2][6(1-\alpha)\alpha\beta - k^2]}$. If $k^2 < 2(1-\alpha)\alpha\beta$, $w^{MRS} < w^{MVN} < w^{MMS}$; if $2(1-\alpha)\alpha\beta < k^2 < 8(1-\alpha)\alpha\beta/3$, $w^{MVN} < w^{MRS} < w^{MMS}$; if $8(1-\alpha)\alpha\beta/3 < k^2 < 4(1-\alpha)\alpha\beta$, $w^{MVN} < w^{MMS} < w^{MRS}$.

(c) $P_r^{MMS} - P_r^{MRS} = -\frac{k^2[k^2 - 2(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{2[4(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]},$

$P_r^{MMS} - P_r^{MVN} = \frac{2(-1+\alpha)\alpha\beta[k^2 - 2(1-\alpha)\alpha\beta](s+\alpha E_b - E_r)}{[6(1-\alpha)\alpha\beta - k^2][8(1-\alpha)\alpha\beta - k^2]},$

$P_r^{MRS} - P_r^{MVN} = \frac{[k^2-2(1-\alpha)\alpha\beta]^2(s+\alpha E_b-E_r)}{2[4(1-\alpha)\alpha\beta-k^2][6(1-\alpha)\alpha\beta-k^2]} > 0$. If $k^2 < 2(1-\alpha)\alpha\beta$, $P_r^{MVN} < P_r^{MRS} < P_r^{MMS}$; if $2(1-\alpha)\alpha\beta < k^2 < 4(1-\alpha)\alpha\beta$, $P_r^{MMS} < P_r^{MVN} < P_r^{MRS}$.

(d) $\Pi_R^{MMS} - \Pi_R^{MRS} = -\frac{\beta\{[k^2-4(1-\alpha)\alpha\beta]^2+16\alpha^2\beta^2\}(s+\alpha E_b-E_r)^2}{2[4(1-\alpha)\alpha\beta-k^2][8(1-\alpha)\alpha\beta-k^2]^2} < 0$,

$\Pi_R^{MMS} - \Pi_R^{MVN} = \frac{16(1-\alpha)^2\alpha^2\beta^3[k^2-7(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{[6(1-\alpha)\alpha\beta-k^2]^2[8(1-\alpha)\alpha\beta-k^2]^2} < 0$,

$\Pi_R^{MRS} - \Pi_R^{MVN} = \frac{\beta[k^2-2(1-\alpha)\alpha\beta]^2(s+\alpha E_b-E_r)^2}{2[4(1-\alpha)\alpha\beta-k^2][6(1-\alpha)\alpha\beta-k^2]^2} > 0$. Thus, $\Pi_R^{MMS} < \Pi_R^{MVN} < \Pi_R^{MRS}$.

(e) $\Pi_M^{MMS} - \Pi_M^{MRS} = -\frac{\beta[3k^2-8(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][8(1-\alpha)\alpha\beta-k^2]}$,

$\Pi_M^{MMS} - \Pi_M^{MVN} = \frac{4(1-\alpha)^2\alpha^2\beta^3(s+\alpha E_b-E_r)^2}{[8(1-\alpha)\alpha\beta-k^2][6(1-\alpha)\alpha\beta-k^2]^2} > 0$,

$\Pi_M^{MRS} - \Pi_M^{MVN} = -\frac{\beta[k^2-2(1-\alpha)\alpha\beta][3k^2-14(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][6(1-\alpha)\alpha\beta-k^2]^2}$. If $k^2 < 2(1-\alpha)\alpha\beta$, $\Pi_M^{MRS} < \Pi_M^{MVN} < \Pi_M^{MMS}$; if $2(1-\alpha)\alpha\beta < k^2 < 8(1-\alpha)\alpha\beta/3$, $\Pi_M^{MVN} < \Pi_M^{MRS} < \Pi_M^{MMS}$; if $8(1-\alpha)\alpha\beta/3 < k^2 < 4(1-\alpha)\alpha\beta$, $\Pi_M^{MVN} < \Pi_M^{MMS} < \Pi_M^{MRS}$.

(f) $\Pi_{SC}^{MMS} - \Pi_{SC}^{MRS} = \frac{k^2\beta[k^2-16(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][8(1-\alpha)\alpha\beta-k^2]^2} < 0$,

$\Pi_{SC}^{MMS} - \Pi_{SC}^{MVN} = \frac{4(1-\alpha)^2\alpha^2\beta^3[3k^2-20(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{[6(1-\alpha)\alpha\beta-k^2]^2[8(1-\alpha)\alpha\beta-k^2]^2} < 0$,

$\Pi_{SC}^{MRS} - \Pi_{SC}^{MVN} = -\frac{\beta[k^2-2(1-\alpha)\alpha\beta][k^2-10(1-\alpha)\alpha\beta](s+\alpha E_b-E_r)^2}{4[4(1-\alpha)\alpha\beta-k^2][6(1-\alpha)\alpha\beta-k^2]^2} < 0$. If $k^2 < 2(1-\alpha)\alpha\beta$, $\Pi_{SC}^{MMS} < \Pi_{SC}^{MRS} < \Pi_{SC}^{MVN}$; if $2(1-\alpha)\alpha\beta < k^2 < 4(1-\alpha)\alpha\beta$, $\Pi_{SC}^{MMS} < \Pi_{SC}^{MVN} < \Pi_{SC}^{MRS}$.

The deviation for Proposition 4 and Proposition 5 can be easily derived using the same method.

Proof for Table 3

We illustrate the derivatives of P_r with respect to ε in Table A4 and the derivatives of other decision variables with respect to the model parameters can be easily concluded following the same step. For ease of presentation, we denote $A = \frac{\alpha-s-E_r}{2}$ and $B = s + \alpha E_b - E_r$.

Table A4. The first derivatives of the p_r with respect to ε

Game models	Optimal solution P_r^*	$\frac{\partial P_r^*}{\partial \varepsilon}$

MMS/RRS	$A+\frac{[4(1-\alpha)\alpha+\varepsilon]B}{2[8(1-\alpha)\alpha-\varepsilon]}$	$\frac{6(1-\alpha)\alpha B}{[8(1-\alpha)\alpha-\varepsilon]^2}>0$
MRS/RMS	$A+\frac{(1-\alpha)\alpha B}{4(1-\alpha)\alpha-\varepsilon}$	$\frac{(1-\alpha)\alpha B}{[4(1-\alpha)\alpha-\varepsilon]^2}>0$
MVN/RVN	$A+\frac{[\varepsilon+2(1-\alpha)\alpha]B}{2[6(1-\alpha)\alpha-\varepsilon]}$	$\frac{4(1-\alpha)\alpha B}{[6(1-\alpha)\alpha-\varepsilon]^2}>0$
BMS/BRS	$A+\frac{[4(1-\alpha)\alpha+\varepsilon]B}{2[8(1-\alpha)\alpha-3\varepsilon]}$	$\frac{10(1-\alpha)\alpha B}{[8(1-\alpha)\alpha-3\varepsilon]^2}>0$
BVN	$A+\frac{[(1-\alpha)\alpha+\varepsilon]B}{2[3(1-\alpha)\alpha-\varepsilon]}$	$\frac{2(1-\alpha)\alpha B}{[3(1-\alpha)\alpha-\varepsilon]^2}>0$

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