The evolution of decision rules in

complex environments

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12 Abandon the urge to simplify everything, to look for formulas and easy answers, 13 and begin to think multidimensionally ... appreciate the fact that life is complex. 14 —M. Scott Peck [1] 15 16 Models and experiments on adaptive decision-making typically consider highly 17 simplified environments that bear little resemblance to the complex, heterogeneous 18 world in which animals (including humans) have evolved. These studies reveal an array 19 of so-called cognitive biases and puzzling features of behaviour that seem irrational in 20 the specific situation presented to the decision-maker. Here we review an emerging 21 body of work that highlights spatiotemporal heterogeneity and autocorrelation as key 22 properties of most real-world environments that may help us understand why these 23 biases evolved. Ecologically rational decision rules adapted to such environments can 24 lead to apparently maladaptive behaviour in artificial experimental settings. We 25 encourage researchers to consider environments with greater complexity to understand 26 better how evolution has shaped our cognitive systems. 27 28 The origins of irrational behaviour 29 Patterns of decision-making in humans reveal some striking deviations from economically 30 rational expectations [2–4]. These include distorted beliefs about external events [5,6], 31 inconsistent preferences that are altered by past experience [7] and current context [8], and 32 apparent violations of the axioms of rational choice theory [9,10]. Such deviations may be 33 caused by cognitive biases [11] (see **Glossary**); here we focus on the behavioural outcomes 34 (outcome biases [12]), since we make no assumptions about the underlying psychological or 35 physiological mechanisms. Mounting evidence suggests that analogous biases exist in other 36 organisms. For example, slime moulds violate regularity [13], domestic dogs show negative 37 contrast effects [14] and honeybees behave pessimistically when agitated [15]. Far from 38 being uniquely human quirks, our biases appear to have deep evolutionary roots. This 39 observation seems difficult to reconcile with the fundamental biological concept of natural 40 selection as an optimising process. Why would evolution produce such apparently irrational 41 behaviour? 42 One possible answer is that in many situations the costs of deviating from the optimal, 43 fitness-maximising decision are negligible, and/or that constraints in the mechanisms 44 underlying decision-making prevent natural selection from reaching this optimum. Studies on 45 noisy information processing [16] and polygenic mutation–selection balance [17] have

argued for the importance of constraints. Here we summarise an emerging line of research that suggests an alternative explanation: that many surprising features of behaviour, which may at first appear irrational, can in fact be understood as the result of ecologically rational decision rules adapted to exploit environments that vary in space and time. The approach we describe is an extension of standard techniques [18] used in behavioural and evolutionary ecology to investigate the adaptive significance of animal behaviour. This approach does not assume that all behaviour is adaptive or that constraints are unimportant, but instead seeks to identify how natural selection shapes the decision rules underlying behaviour [19,20]. The implications of this work for understanding cognitive systems have been largely overlooked, because theoretical models and laboratory experiments alike have traditionally focused on highly simplified situations that fail to capture some of the important complexities of the environments in which organisms have evolved.

The limitations of simple models

Simple mathematical models are of great value in behavioural and evolutionary ecology, where the techniques of game theory and optimisation are used to predict the endpoints of natural selection [21]. This approach has revealed some important general principles of how organisms (including humans) should choose between different options, from food items to potential mates to the age at first reproduction. Most evolutionary models of decision-making consider a highly simplified environment in which the availability of different options is known to the organism and does not change over time. This is of course an unrealistic assumption. In most natural environments, the availability of different options fluctuates in time and space and the fluctuations are often unpredictable.

That mathematical models simplify and abstract the phenomena they aim to represent is not in itself a problem; indeed, this is precisely what models are designed to do, since a model that was as complex as the real world would be of little use. But there is a danger of over-simplification [22] ('Einstein's razor' [23]): if we simplify things too much, we may fail to capture crucial features of natural environments that are needed to understand the behaviour.

The power of simple experiments

Similarly, laboratory experiments place individuals in artificial situations that are far simpler than most situations encountered in the natural world. In many of the standard laboratory protocols routinely used in behavioural ecology and experimental psychology, subjects are

trained and tested using a small number of behavioural options, with straightforward relationships between the available stimuli, the subject's actions and the resulting consequences [24–27]. In these artificial situations, the experimenter has created a deliberately simplified version of the types of problems the animal might encounter in its natural environment; the aim is to isolate the key variables needed to understand the behaviour. Just as with the simplified models discussed earlier, there is a risk that such laboratory settings may not reflect the statistical structure of the environment to which the animal is adapted, making it seem as though the animal is making errors [4]. However, if we recognise this problem, deviations from rational behaviour in simplified laboratory set-ups can be illuminating, as they may reveal unexpected biases that arise from rules adapted to the natural environment.

Irrational behaviour from ecologically rational rules

Natural selection will tend to produce decision rules which, while not optimal, perform well in the kinds of situations the individual normally encounters [19,20,28,29]; that is, they should be ecologically rational [30]. The statistical properties of environments, including the distribution of resources and how that changes over time, favour particular decision rules. For example, noisy miners (a type of bird) change their foraging strategy depending on the resource they are exploiting: they use movement-based rules when searching for invertebrates, which are cryptic and highly mobile, but switch to using spatial memory when searching for nectar, which is found only in fixed, conspicuous locations (flowers) and is quickly depleted [31]. The ecological and evolutionary context is critical; animals follow decision rules that are adapted to the statistical properties of the resource types commonly encountered during their evolutionary history. In novel experimental contexts lacking this structure, such ecologically rational rules may lead to biased or irrational behaviour.

When seeking to understand how natural selection has shaped decision rules, it can be instructive to use a form of reverse engineering. This process starts with the identification of some bias that is not accounted for by current theory. The next step is to consider which particular aspects of environmental complexity need to be included in the models in order to predict that bias. The aim is to identify the minimal amount of real-world complexity that is sufficient to account for observed behaviour, forming a basis for novel predictions that can be used to test the proposed explanation. Models developed in the past few years illustrate the power of this approach and highlight spatiotemporal heterogeneity and autocorrelation as two important factors affecting the psychology of humans and other animals (**Figure 1**).

Incorporating these factors into standard models can explain a number of biases, listed in **Table 1**, that appear irrational in more simplified environments.

Spatiotemporal heterogeneity

Conditions in most natural environments are not uniform but vary over time and space. For highly mobile organisms, these two forms of heterogeneity will typically be closely linked; an individual moving through a spatially heterogeneous environment will encounter temporal heterogeneity too. Spatiotemporal heterogeneity has important consequences for behaviour, because in a heterogeneous world an individual's optimal response to current conditions depends on the conditions it expects to encounter in the (near) future [32–35]. The most basic form of heterogeneity we can consider is where the conditions at any one time or place are independent of those at any other time or place (**Box 1**). This is only a crude representation of the heterogeneity in most natural environments (see next section), but it can already account for some interesting biases:

The placebo effect. It is a widely reported (though controversial [36,37]) finding that fake treatments such as sugar pills or sham surgery, known as placebos, can lead to improvement in a patients' health [38]. While health improvement is of course beneficial to the patient, if they are capable of recovering without help it would seem rational to do so immediately, rather than waiting for an external, inert cue. In an environment where conditions change over time, however, a delayed response may be adaptive. If an individual falls sick when conditions are harsh, it may be worth waiting until the environment is perceived to be less challenging, when it will be less costly to mount an immune response. Recent theory [39] has shown that the optimal strategy for recovery depends on the patient's beliefs about current and future conditions, which affects the relative benefits of investing in recovery now rather than later. From this viewpoint, placebos falsely alter the patient's expectations of the costs and benefits of putting effort into recovery, in some cases triggering an immediate response (i.e. a placebo effect). The placebo effect itself is not adaptive, but a generalised response to external cues may be favoured by natural selection if, on average, those cues reliably indicate a change in environmental conditions.

Pessimism. Natural selection should, in general, produce behaviour that is appropriate for the environmental conditions, giving the impression that individuals 'know' what those conditions are even if they cannot perceive them directly. Sometimes, however, humans and other animals consistently behave in a way that does not maximise their short-term gains, but would maximise their short-term gains if conditions were better than they actually are (an

'optimistic' bias) [40,41] or worse than they actually are (a 'pessimistic' bias) [42–44]. Recent theoretical work [45] shows that temporal heterogeneity across generations can select for pessimism: behaviour should be biased towards the response that yields the best results in poor conditions, because it is poor conditions that have the strongest influence on long-term fitness across multiple generations. Other factors, including autocorrelation (see below), may alter the tendency towards optimism or pessimism (**Box 2**).

Spatiotemporal autocorrelation

Environments that are spatiotemporally heterogeneous may also show positive autocorrelation, in that the conditions at a given place and time tend to be similar to those at nearby locations and in the recent past (**Box 1**). One well-known adaptation to spatial autocorrelation is area-restricted search [46], in which successful discovery of an item prompts intensive local searching [47], thereby promoting efficient exploitation of clumped resources [48]. The impact of temporal autocorrelation is less well appreciated, but may be even more important for understanding cognitive adaptations. In environments that change over time, the strength of temporal autocorrelation—and hence the time for which current and future conditions persist—has important consequences for adaptive behaviour [49] and learning [50] and this is reflected in our cognitive systems.

When there is temporal autocorrelation, current conditions not only determine the consequences of current decisions but are also informative of future conditions. This important insight can account for several well-known biases:

The 'hot hand' fallacy. In gambling and sports, there is a widespread but often mistaken belief that players have 'streaks' or 'runs' of success. Basketball players, for example, are perceived to be more likely to shoot successfully if their previous shot hit rather than missed, whereas real data show that the chances of scoring are statistically independent from one shot to the next [51]. This so-called 'hot hand' belief reveals our tendency to see patterns even when none exist [52]. It has been argued that this tendency represents a broad-purpose cognitive adaptation to a world in which most resources are clumped (i.e. positively autocorrelated) in space and time [4,53,54]. Thus the hot-hand fallacy could result from a generalised decision rule that is unable to distinguish sequences of genuinely independent events from autocorrelated sequences. Experimental evidence from computer-based 'foraging' [53] and gambling [54] tasks largely supports this view and suggests that human minds have evolved to expect temporal autocorrelation in the world.

Intransitive and irregular preferences. In an autocorrelated world, the possibility that current behavioural options will persist into the future can affect patterns of choice. Rational choice theory holds that the preference for one option over another should be both transitive and independent of irrelevant alternatives (see Glossary); satisfying the axioms of this theory is both necessary and sufficient to maximise expected benefit [55]. Studies of consumer behaviour [56] and experiments on humans [8–10] and a diverse range of other organisms [13,57–63] have found evidence for context-dependent preferences that appear to violate these axioms of rational choice (though see [64]). However, empirically observed choices are part of a long sequence of choices that individuals make throughout their lives, whereas the axioms refer to one-off choices (which can be choices between alternative decision rules that specify what to do in every possible situation an individual might encounter in its lifetime). In repeated choices, mathematical models [65,66] show that violations of transitivity and regularity can result from decision rules adapted to heterogeneous, autocorrelated environments, in which currently available options provide information about what options will be available in the future (Box 3).

State-dependent valuation learning. An individual's energetic state reflects recent foraging conditions, and can therefore inform it about future conditions in an autocorrelated world. Laboratory studies on birds [67], insects [68] and fish [69] have shown that the value animals place on different options depends on the state they were in when they learnt about those options. When given a choice between two food sources, animals consistently choose the one they previously found to be rewarding when they were hungry, despite the alternative having equal [67] or even higher [70] profitability. Evolutionary simulations [71] have shown that, although this biased valuation appears irrational, it can make sense in certain types of environments that fluctuate slowly between rich and poor conditions. If the best option differs between rich and poor conditions but individuals cannot perceive the conditions directly, state-dependent valuation learning is expected to evolve: food rewards should be more strongly reinforcing when an individual has low energy reserves, which are indicative of poor conditions. Selection favours this bias in the learning rule because making the correct choice under poor conditions is particularly important for fitness [71].

Successive contrast effects. If an individual is uncertain about the temporal pattern of change in conditions, future expectations may also be influenced by conditions experienced in the past. Standard theories of rational choice posit that optimal behaviour is path independent, in that it depends on the current state of the world but not on how that state was reached. If we equate current state with current environmental conditions, this view cannot

account for successive contrast effects, in which an individual's response to current conditions depends on whether conditions were previously better (a negative contrast effect) or worse (a positive contrast effect) [72]. Such sensitivity to change can be understood by recognising that many animals have evolved in an environment where conditions fluctuate over time in an unpredictable way. Assuming the pattern of change is sufficiently stable, the conditions experienced in the past then provide potentially valuable information about the likely pattern of change in the future, which affects optimal behaviour (see **Box 1**). This dependence of optimal behaviour on past experiences can produce positive and negative contrast effects in the artificial situations used in laboratory studies [73]. Similar effects could result from an optimal trade-off between exploration and exploitation in heterogeneous, autocorrelated environments [74].

Optimism. Temporal autocorrelation across generations may also be important. If there is spatial heterogeneity in environmental conditions and those conditions persist over multiple generations (i.e. temporal autocorrelation is sufficiently high), optimistic behaviour is favoured [45] (cf. pessimism when temporal autocorrelation is weak; see previous section). Alternatively, uncertainty about an external, autocorrelated mortality risk can favour optimism [75] (**Box 2**). Such cognitive biases may appear irrational, but they arise from a strategy that maximises fitness over a longer timescale [76].

As these examples illustrate, some apparently maladaptive behaviours observed in artificial laboratory situations can be seen as ecologically rational if we recognise that organisms are adapted to stochastically fluctuating conditions that are autocorrelated in time and space. By interacting with this rich statistical structure, organisms have evolved to exploit their natural environments efficiently using a range of simple decision rules that need not require complex computation [77,78]. It is important to recognise that such rules may lead to outcome biases in environments that lack this statistical structure. For example, standard laboratory procedures for demonstrating successive contrast effects eliminate any correlation between past and future conditions; an ecologically rational decision rule adapted to exploit this correlation will produce apparently irrational behaviour [73]. Similarly, in tests of context-dependent choice the current options do not predict which options will be available in the future, but the animal may be responding as if they do [65,66] (see **Box 3**).

From 'just-so' stories to predictions and empirical tests

In the approach we have outlined, the aim is to build evolutionary models with the minimal amount of real-world complexity to account for observed patterns of decision-making. But

identifying one potential adaptive explanation does not rule out the existence of other explanations that may account for the observed bias equally well. To move beyond adaptive storytelling, models should generate testable predictions as well as explanations. In particular, evolutionary models of biases in decision-making should identify which factors affect the magnitude of the bias, and therefore the organisms and circumstances in which the bias should be most pronounced.

Although the evolutionary roots of many biases appear to run deep, there is evidence of considerable variation among species. For example, studies have found evidence of successive contrast effects in honeybees, bumblebees, starlings and a variety of mammals, but not in goldfish, toads, pond turtles, chickens or pigeons [79]. This variation could reflect phylogenetic inertia [80] in the underlying neuroendocrine mechanisms that constrain behaviour [81] or ecological differences between species that select for different decision rules [82]. A general expectation of the theories we have reviewed here is that many biases will be most pronounced in species adapted to strongly fluctuating environments, where the fluctuations have a big impact on optimal behaviour. We might therefore expect some biases to be stronger in animals reliant on tightly clumped, ephemeral food sources (e.g. specialist frugivores and nectarivores) than those adapted to stable, widely available resources (e.g. grazing herbivores). To test such broad-scale, comparative predictions, we need quantitative data on variation in biases across species (controlling for selective reporting [83]) and detailed information on the spatiotemporal structure of natural environments (including social dynamics, for which 'reality mining' techniques [84] hold great promise). Differences in feeding ecology have been proposed to explain variation in impulsive behaviour across primates [85]; a more in-depth approach using detailed ecological data might help in understanding the taxonomic distribution of other behaviours that at first appear irrational.

Another exciting possibility is to test the evolutionary predictions experimentally, by manipulating the pattern of environmental change. Taking the simplest case of two environmental states (e.g. high versus low food availability), exposing different experimental groups to different transition probabilities (see **Box 1**) could potentially generate different biases in decision-making, providing that the study organism can adapt behaviourally to the pattern of change. Many of the examples we have discussed involve adaptation over an evolutionary rather than behavioural timescale, but even then it might be possible to test hypotheses using experimental evolution in *Drosophila*, nematodes or other organisms with a short generation time. We hope researchers using these systems will take up this challenge.

Conclusion The evolutionary explanations we have highlighted here represent only one of a number of possible approaches to understanding biases in decision-making; it is important to compare this framework with alternative approaches based on genetic [17] or cognitive [16] constraints. Nonetheless, we believe that insights from evolutionary studies can make an important contribution to this issue by considering how organisms adapt to richer environments. The simple models and experiments routinely used to study decision-making may misrepresent key features of the environment of selection, leading to incorrect predictions and regular reports of seemingly irrational behaviour. The real world can be complex, variable and autocorrelated, and we should expect cognitive and perceptual systems to have evolved to exploit its statistical structure. By considering environments with sufficient richness we can generate novel, testable explanations for many puzzling behavioural and psychological phenomena, which can be meaningfully tested even in simplified laboratory settings. Much exciting work lies ahead (**Box 4**). A better understanding of the statistical structure of real-world environments may help us to understand the workings of the mind [86–88]. Acknowledgements We thank Louise Barrett, Gaurav Malhotra, Liz Paul, the editors and three anonymous reviewers for valuable feedback. This work was funded by the European Research Council (Advanced Grant 250209 to A.I.H.) and the Engineering and Physical Sciences Research Council (grant number EP/I032622/1 to Iain D. Gilchrist).

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305 Glossary 306 **Autocorrelation:** an association across space or time in the state of the environment. Positive 307 autocorrelation (which is our focus here) implies that environmental conditions tend to 308 be more similar between locations and times that are close together, rather than far 309 apart. 310 **Cognitive bias:** a consistent deviation from an accurate perception or judgement of the 311 world. Note that this is a psychological phenomenon that may or may not lead to 312 irrational behaviour. 313 **Contrast effect:** a change in the perceptual, physiological or behavioural response to a given 314 stimulus caused by simultaneous or recent exposure to other stimuli in the same 315 dimension. Here we consider successive contrast effects, in which the response to 316 current conditions is enhanced by previous exposure to worse conditions (a positive 317 contrast effect) or diminished by previous exposure to better conditions (a negative 318 contrast effect). For example, honeybees trained to expect a 50% sucrose solution are 319 more likely to abandon that reward source when it only delivers a 20% solution, 320 compared to honeybees trained with a 20% solution all along [89]. 321 **Decision rule:** a description (without specifying the underlying neural mechanisms) of the 322 relationship between an internal or external stimulus and the choices an individual will 323 make. 324 Ecological rationality: the fit between a particular decision rule and the statistical structure 325 of the environment in which it evolved. 326 **Environmental heterogeneity:** variability in (external) environmental conditions over space 327 (spatial heterogeneity) and/or time (temporal heterogeneity). 328 **Independence of irrelevant alternatives (IIA)**: a principle of rational choice stating that if 329 an individual prefers an option A when given the choice between A and B, then it will 330 also prefer A when given the choice between A, B and a less attractive (i.e. irrelevant) 331 option C. 332 **Irrational behaviour:** acting in a way that is not optimal. In the context of evolutionary 333 theory, rationality—sometimes called biological rationality (B-rationality), to 334 distinguish it from economic rationality (E-rationality) [90–92]—does not imply 335 conscious consideration of different options, but merely behaving in a way that 336 maximizes expected benefit.

Outcome bias: a pattern of decision-making that apparently deviates from the predictions	
	rational choice theory. Note that this definition makes no assumptions about
	underlying cognitive processes.
	Path independence: a principle of rational choice stating that an individual's decisions
	should only depend on its knowledge about the current state of the world (including
	itself), not on past states.
Rational choice theory: an economic theory giving an axiomatic definition of	
	(economically) rational behaviour.
Regularity: a principle of rational choice stating that the frequency with which an individual	
	chooses option A when given a choice between A, B and C cannot be higher than the
	frequency of choosing A when given a choice between only A and B.
	Transitivity: a principle of rational choice stating that if an individual prefers option A in a
	choice between A and B, and option B in a choice between B and C, then it must prefer
	A in a choice between C and A.

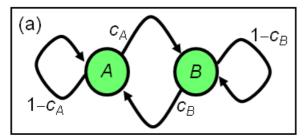
Box 1. Modelling environmental heterogeneity and autocorrelation

Incorporating environmental heterogeneity into models of adaptive behaviour requires the inclusion of an environmental state variable. Often we can capture sufficient complexity with just two environmental states A and B, such as high and low food availability, or safe and dangerous. Next, we characterise stochastic transitions between the environmental states. The simplest case is where the probability of transition (per unit time) between states depends only on the current state (Figure Ia), because then we can write the transition probabilities as single values c_A and c_B (the subscripts indicating the current state), with $c_A + c_B < 1$ representing positive temporal autocorrelation. The length of time the environment stays in state i then follows a geometric distribution with mean $t_i = 1/c_i$. We assume that the individual 'knows' (i.e. is adapted to) these probabilities and can directly perceive the current conditions. We then investigate how environmental heterogeneity affects responses to current conditions, such as predation risk [49]. For a finer gradation of states, this approach can be extended to any number of states n, with an $n \times n$ matrix of transition probabilities. For some systems, such as gradual changes in the food supply, we set all the probabilities of moving between non-adjacent states to zero.

Individuals will often be uncertain about the transition probabilities and we may be interested in how they should respond to this uncertainty. A simple representation considers two possible transition matrices (e.g. fast- or slow-changing conditions). The individual may 'know' the transition probabilities of each matrix, but not which matrix currently applies (Figure Ib). If the environment is temporally autocorrelated, the recent past is informative of the future, so the individual should adjust its behaviour in response to its previous experience of the pattern of change. An optimal decision-maker would learn from past experience using Bayesian updating [93]. We can model this by including a state variable to represent the probability that one particular matrix applies, which can help explain apparently irrational behaviour such as contrast effects [73].

The above assumes that the individual can accurately perceive whether the environmental state is currently *A* or *B*. To explore a situation where the individual knows neither the current conditions nor the transition probabilities with certainty, we can use an additional variable to represent the probability of a given situation. However, note that learning two interdependent probabilities requires three state variables and a very fine grid size; computational limitations may constrain our approach.

We have described the simplest scenario for modelling temporal autocorrelation in a heterogeneous world. Real environments may show more complex patterns of change, but this is a mathematically convenient way to capture some of the statistical structure that could be important for understanding cognitive adaptations.



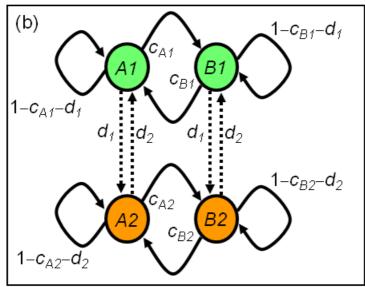


Figure I. Flow diagram showing the dynamics of environmental state in a model of a heterogeneous, temporally autocorrelated world. (a) In the simplest case, there are just two environmental states (here, A and B) and a constant probability c_i of a change from the current state i. (b) If the transition probabilities are uncertain, we can consider two possible situations (here, I and I2) representing different patterns of change between I3 and I3. The environment may switch from situation I4 to the alternative situation with probability I3. The relative magnitude of I4 and I5 I5 I7 reflect our assumptions about the persistence of the two situations (e.g. habitat quality) relative to heterogeneity in current conditions (e.g. food availability). For example, a very small I5 might be used if the pattern of change tends to be stable over the individual's lifetime. More complex scenarios are possible in which I6 also depends on the current environmental state (I6 or I7).

Box 2. The evolution of optimism and pessimism

- Consider an environment composed of a large collection of discrete patches. Individuals mature on a patch, reproduce and die. Some of their offspring disperse to other patches. Patches change over time, independently of one another; in some generations conditions are good, in other generations poor. Whether optimal behaviour appears unduly optimistic or pessimistic that conditions are good depends on the degree of dispersal and autocorrelation [45]:
- (a) When dispersal between patches is low, pessimism is favoured; individuals must behave conservatively in case conditions deteriorate and the whole lineage is wiped out.
- (b) When dispersal rates are higher, dispersal acts as an insurance against a local patch deteriorating, spreading the risk between members of the same lineage, so that individuals no longer need to be conservative. If conditions are positively autocorrelated in time there is a 'multiplier effect' [94], with descendant numbers growing rapidly in a patch over successive generations if conditions are good. Individuals should then take a risk and behave optimistically so as to exploit conditions if these turn out to be good, because behaviour in good conditions has a predominant influence on long-term fitness [45].

It can also be optimal to be optimistic about the chances of survival. Imagine an animal that has to survive a given period of *T* days if it is to reproduce. Suppose that the density of predators varied during the evolutionary history of the population, and that there are no cues that provide direct information on the density on a given day. Then the frequency with which different levels of predation occurred in the past specifies the current probability distribution of predation levels. Do we expect anti-predator traits (e.g. cautious behaviour) to evolve so that individuals maximise their expected daily survival given this distribution? It depends [75]:

- (a) If T = 1 or predator density on successive days is independent, then the answer is yes.
- (b) However, if T > 1 and predator density on successive days is positively autocorrelated, then individuals do best to be optimistic about risk. To understand this, consider the extreme case in which T is large and predator density is the same on all days, either always high or always low. If the density is high, the individual will almost certainly die regardless of its anti-predator trait, whereas if it is low the trait value matters. Thus the trait is only really relevant when the density is low, so it should evolve to be optimal given a low density [75]—that is, behaviour should appear optimistic about predation

risk. Weaker autocorrelation in the predator density across successive days will favour a
weaker optimistic bias towards the optimal response for low density.

Box 3. Violations of regularity and transitivity

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A central tenet of studies of decision-making is that in the absence of constraints or costs, decisions should be transitive and regular (see **Glossary**) in sequences of choices (cf. in one-off choices, as required by rational choice theory). In an autocorrelated world, this is not necessarily true.

Foragers often face a choice between options that differ in both the expected rate of energy gain and the risk of predation, which may be positively related. What is the strategy that maximises long-term survival? At high reserves, they should choose options with a low predation risk; at low reserves, to avoid starvation they should choose options with a high probability of energy gain. For intermediate reserve levels, the best option depends not only on the immediate danger but on the longer-term risk of starvation. If options persist into the future, this risk depends on which other options are currently available; options that are not currently chosen may still affect optimal decisions, because they can act as insurance against an energetic shortfall in the future. For example, a dangerous but high-gain option should be avoided when the individual is well-fed, but can be relied on in an emergency if reserves drop to critically low values. In the absence of this insurance option, the individual may be forced to choose riskier foraging options than it would do otherwise, to keep its energy reserves at a safe level. The value of a given option is therefore affected by the presence of other options, which can lead to violations of regularity [65] and transitivity [66] under optimal behaviour. Recent models predict that violations may occur even in cases without state-dependence, where the animal is simply maximising its rate of energy gain [95].

Without autocorrelation, the presence of one option would not affect the value of another. Waksberg *et al.* [96] argued that irregular choice could outcompete rational behaviour in a model with no autocorrelation, but they considered a restricted set of decision rules that did not allow the individual's choice to depend on its current energy reserves [97]. This set does not include the optimal decision rule. In evolutionary models of decision-making that account for heterogeneity, it is important that the best-performing decision rule is optimal over some sufficiently long timescale, otherwise we cannot argue that it would have evolved [76].

Box 4. Outstanding questions

- A major theme of the recent theoretical work discussed here is that in a temporally autocorrelated world, current or past options may be informative about the future. This general principle may shed light on decisions in a range of other situations, such as choice between risky options (i.e. options for which the outcome is variable). Prospect theory is a highly influential descriptive model of human decision-making that captures several interesting features of our attitudes to risk [98], such as our tendency to focus more on changes in state (e.g. wealth) than the states themselves. Could this pattern of decision-making be ecologically rational in an autocorrelated world (see **Box 1**)? If conditions fluctuate over time, organisms may need to take into account the pattern of change to decide whether it is worth gambling on a risky but potentially highly rewarding option.
- How does natural selection shape the mechanisms involved in decision-making? Most models of adaptive decision-making focus on behaviour, ignoring the psychological and physiological mechanisms that produce it. But observed behaviour may be consistently associated with particular psychological and/or physiological states, so to understand decision-making properly we need to model the evolution of these mechanisms explicitly [19]. This can be technically challenging and typically involves computationally intensive methods such as genetic algorithms (e.g. see [99]), but modern computing power is beginning to bring these approaches within reach.
- Studies of the evolution of psychological mechanisms may hold the key to unravelling some of the most enduring mysteries of the human mind, such as why we have emotions and moods. Do affective states enhance or constrain decision-making? One idea is that mood states are an efficient way of summarising recent experiences and can be used to adjust decision thresholds, which might be adaptive in a stochastically changing, autocorrelated environment [100–102] (see **Box 1**). Whether emotions and moods are closely linked to brain mechanisms that promote survival and other fitness components is unclear [103], but this remains a promising direction for future research.
- One of the key challenges of a comparative, evolutionary approach to cognitive biases is how to identify analogous outcome biases in non-human organisms. To allow valid comparisons, behavioural measures need to be both ecologically relevant and applicable to a wide range of taxa. Tests have been devised for impulsive behaviour [104,105] and for optimistic and pessimistic biases [15,106], but what are the behavioural indicators of

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affective states such as anxiety, depression or disappointment? Researchers are beginning to tackle this difficult problem [44,107,108], but much remains to be done.

Table 1. Biases that seem irrational in a simplified world

Description	Why does it seem irrational?
Medicinally inert substances or fake	Individual who is capable of recovery
treatment procedures enhance	without external help should do so
recovery	immediately
Individual behaves as though	Rational decision-maker should base
conditions are better (optimism) or	behaviour on unbiased (Bayesian)
worse (pessimism) than they actually	estimate of current conditions
are	
Misinterpretation of a statistically	In a sequence of trials known to be
independent sequence of successes as	independent (e.g. roulette), estimated
a run of good form	chance of success should not be
	influenced by outcome of previous trial
Individual prefers option A over	Inconsistent with absolute valuation of
option B and option B over option C,	options, which would imply that if A > B
but prefers C over A	and $B > C$ then $A > B > C$
Preference for one option over	Inconsistent with absolute valuation of
another is reversed by presence of a	options, which would imply that ranking
third option	of two options is unaffected by alternative
	options
Individual prefers options they	Rational decision-maker should choose
previously found to be rewarding	whichever option gives greatest benefit,
when in a state of need	irrespective of past states
Response to current conditions	Rational decisions should depend only on
depends on whether conditions in the	current situation; how the decision-maker
past were better or worse	got there is irrelevant
	Medicinally inert substances or fake treatment procedures enhance recovery Individual behaves as though conditions are better (optimism) or worse (pessimism) than they actually are Misinterpretation of a statistically independent sequence of successes as a run of good form Individual prefers option A over option B and option B over option C, but prefers C over A Preference for one option over another is reversed by presence of a third option Individual prefers options they previously found to be rewarding when in a state of need Response to current conditions in the

References

- Peck, M.S. (1993) Further Along the Road Less Travelled: Wisdom For the Journey
- 503 Towards Spiritual Growth, Simon & Schuster Ltd
- 504 2 Gilovich, T. et al., eds (2002) Heuristics and Biases: The Psychology of Intuitive
- 505 *Judgment*, Cambridge University Press
- Baron, J. (2008) *Thinking and Deciding*, 4th edn, Cambridge University Press
- Haselton, M.G. et al. (2009) Adaptive rationality: an evolutionary perspective on
- 508 cognitive bias. *Soc. Cogn.* 27, 733–763
- 509 5 Meissner, K. et al. (2011) The placebo effect: advances from different methodological
- 510 approaches. J. Neurosci. 31, 16117–16124
- Avugos, S. et al. (2013) The "hot hand" reconsidered: a meta-analytic approach.
- 512 *Psychol. Sport Exerc.* 14, 21–27
- Tversky, A. and Griffin, D. (1991) Endowment and contrast in judgments of well-
- being. In Subjective Well-being: An Interdisciplinary Perspective (Strack, F. et al.,
- eds), pp. 101–108, Pergamon Press
- Trueblood, J.S. et al. (2013) Not just for consumers: context effects are fundamental to
- decision making. Psychol. Sci. 24, 901–908
- 518 9 Kalenscher, T. et al. (2010) Neural signatures of intransitive preferences. Front. Hum.
- 519 *Neurosci.* 4, 49
- 520 10 Pettibone, J.C. (2012) Testing the effect of time pressure on asymmetric dominance and
- 521 compromise decoys in choice. *Judg. Decis. Mak.* 513–523
- 522 11 Pronin, E. (2007) Perception and misperception of bias in human judgment. *Trends*
- 523 *Cogn. Sci.* 11, 37–43
- 524 12 Marshall, J.A.R. et al. (2013) On evolutionary explanations of cognitive biases. Trends
- 525 Ecol. Evol. 28, 469–473
- Latty, T. and Beekman, M. (2011) Irrational decision-making in an amoeboid
- organism: transitivity and context-dependent preferences. *Proc. R. Soc. B* 278, 307–312
- 528 14 Bentosela, M. et al. (2009) Incentive contrast in domestic dogs (Canis familiaris). J.
- 529 *Comp. Psychol.* 123, 125–130
- 530 15 Bateson, M. et al. (2011) Agitated honeybees exhibit pessimistic cognitive biases.
- 531 *Curr. Biol.* 21, 1070–1073
- Hilbert, M. (2012) Toward a synthesis of cognitive biases: how noisy information
- processing can bias human decision making. *Psychol. Bull.* 138, 211–237

534 17 Keller, M.C. and Miller, G. (2006) Resolving the paradox of common, harmful, 535 heritable mental disorders: which evolutionary genetic models work best? Behav. Brain 536 Sci. 29, 385–404 537 18 Houston, A.I. and McNamara, J.M. (1999) Models of Adaptive Behaviour: An 538 Approach Based on State, Cambridge University Press 539 19 McNamara, J.M. and Houston, A.I. (2009) Integrating function and mechanism. Trends 540 Ecol. Evol. 24, 670-675 541 20 Fawcett, T.W. et al. (2013) Exposing the behavioral gambit: the evolution of learning 542 and decision rules. Behav. Ecol. 24, 2–11 543 21 McNamara, J.M. and Weissing, F.J. (2010) Evolutionary game theory. In Social 544 Behaviour: Genes, Ecology and Evolution (Székely, T. et al., eds), pp. 88–106, 545 Cambridge University Press 546 Evans, M.R. et al. (2013) Do simple models lead to generality in ecology? Trends Ecol. 22 547 Evol. 28, 578–583 548 Evans, M.R. et al. (2012) Predictive ecology: systems approaches. Phil. Trans. R. Soc. 23 549 *B* 367, 163–169 550 24 Kagel, J.H. et al. (1995) Economic Choice Theory: an Experimental Analysis of Animal 551 Behavior, Cambridge University Press 552 25 Wasserman, E.A. and Zentall, T.R., eds (2006) Comparative Cognition: Experimental 553 Explorations of Animal Intelligence, Oxford University Press 554 26 Shettleworth, S.J. (2010) Cognition, Evolution, and Behavior (2nd edn), Oxford 555 **University Press** 556 Davies, N.B. et al. (2012) An Introduction to Behavioural Ecology (4th edn), Wiley-27 557 Blackwell 558 28 Todd, P.M. and Gigerenzer, G. (2007) Environments that make us smart: ecological 559 rationality. Curr. Dir. Psychol. Sci. 16, 167–171 560 29 Hutchinson, J.M.C. and Gigerenzer, G. (2005) Simple heuristics and rules of thumb: 561 where psychologists and behavioural biologists might meet. Behav. Processes 69, 97-562 124 563 Todd, P.M. et al. (2012) Ecological Rationality: Intelligence in the World, Oxford 30 564 **University Press** 565 31 Sulikowski, D. and Burke, D. (2010) Movement and memory: different cognitive 566 strategies are used to search for resources with different natural distributions. Behav. Ecol. Sociobiol. 65, 621-631

568 32 McNamara, J.M. et al. (1994) Foraging routines of small birds in winter: a theoretical 569 investigation. J. Avian Biol. 25, 287-302 570 33 Lima, S.L. and Bednekoff, P.A. (1999) Temporal variation in danger drives 571 antipredator behavior: the predation risk allocation hypothesis. Am. Nat. 153, 649–659 572 34 Beauchamp, G. and Ruxton, G.D. (2011) A reassessment of the predation risk 573 allocation hypothesis: a comment on Lima and Bednekoff. Am. Nat. 177, 143–146 574 35 Bednekoff, P.A. and Lima, S.L. (2011) Risk allocation is a general phenomenon: a 575 reply to Beauchamp and Ruxton. Am. Nat. 177, 147-151 576 36 Hróbjartsson, A. and Gøtzsche, P.C. (2004) Is the placebo powerless? Update of a 577 systematic review with 52 new randomized trials comparing placebo with no treatment. 578 J. Intern. Med. 256, 91–100 579 37 Wechsler, M.D. et al. (2011) Active albuterol or placebo, sham acupuncture, or no 580 intervention in asthma. N. Engl. J. Med. 365, 119–126 581 38 Benedetti, F. et al. (2011) How placebos change the patient's brain. 582 Neuropsychopharmacology 36, 339–354 583 39 Trimmer, P.C. et al. (2013) Understanding the placebo effect from an evolutionary 584 perspective. Evol. Hum. Behav. 34, 8–15 585 40 Brydges, N.M. et al. (2011) Environmental enrichment induces optimistic cognitive 586 bias in rats. *Anim. Behav.* 81, 169–175 587 41 Sharot, T. et al. (2012) How dopamine enhances an optimism bias in humans. Curr. 588 Biol. 22, 1477–1481 589 42 Mendl, M.T. et al. (2010) Dogs showing separation-related behaviour exhibit a 590 'pessimistic' cognitive bias. Curr. Biol. 20, R839–R840 591 43 Richter, S.H. et al. (2012) A glass full of optimism: enrichment effects on cognitive 592 bias in a rat model of depression. Cogn. Affect. Behav. Neurosci. 12, 527–542 593 44 Salmeto, A.L. et al. (2011) Cognitive bias in the chick anxiety–depression model. 594 Brain Res. 1373, 124–130 595 45 McNamara, J.M. et al. (2011) Environmental variability can select for optimism or 596 pessimism. Ecol. Lett. 14, 58–62 597 Hills, T.T. et al. (2013) Adaptive Lévy processes and area-restricted search in human 46 598 foraging. PLoS ONE 8, e60488 599 47 Zach, R. and Falls, J.B. (1977) Influence of capturing a prey on subsequent search in

the ovenbird (Aves: Parulidae). Can. J. Zool. 55, 1958-1969

601 48 Krakauer, D.C. and Rodríguez-Gironés, M.A. (1995) Searching and learning in a 602 random environment. J. Theor. Biol. 177, 417-429 603 49 Higginson, A.D. et al. (2012) Generalized optimal risk allocation: foraging and 604 antipredator behavior in a fluctuating environment. Am. Nat. 180, 589-603 605 50 Aoki, K. and Feldman, M.W. (in press) Evolution of learning strategies in temporally 606 and spatially variable environments: a review of theory. Theor. Popul. Biol. (doi: 607 10.1016/j.tpb.2013.10.004) 608 51 Gilovich, T. et al. (1985) The hot hand in basketball: on the misperception of random 609 sequences. Cogn. Psychol. 17, 295–314 610 52 Huber, J. et al. (2010) The hot hand belief and the gambler's fallacy in investment 611 decisions under risk. *Theory Decis.* 68, 445–462 53 612 Wilke, A. and Barrett, H.C. (2009) The hot hand phenomenon as a cognitive adaptation 613 to clumped resources. Evol. Hum. Behav. 30, 161–169 614 54 Scheibehenne, B. et al. (2011) Expectations of clumpy resources influence predictions 615 of sequential events. Evol. Hum. Behav. 32, 326–333 616 55 von Neumann, J. and Morgenstern, O. (1944) Theory of Games and Economic 617 Behavior, Princeton University Press 618 56 Mao, W. and Oppewal, H. (2012) The attraction effect is more pronounced for 619 consumers who rely on intuitive reasoning. Mark. Lett. 23, 339–351 620 57 Freidin, E. and Kacelnik, A. (2011) Rational choice, context dependence, and the value 621 of information in European starlings (Sturnus vulgaris). Science 334, 1000–1002 622 58 Morgan, K.V. et al. (2012) Context-dependent decisions among options varying in a single dimension. Behav. Processes 89, 115-120 623 624 59 Sasaki, T. and Pratt, S.C. (2011) Emergence of group rationality from irrational 625 individuals. Behav. Ecol. 22, 276–281 626 60 Waite, T.A. (2001) Intransitive preferences in hoarding gray jays (*Perisoreus* 627 canadensis). Behav. Ecol. Sociobiol. 50, 116-121 628 61 Shafir, S. et al. (2002) Context-dependent violations of rational choice in honeybees 629 (Apis mellifera) and gray jays (Perisoreus canadensis). Behav. Ecol. Sociobiol. 51, 630 180-187 631 62 Bateson, M. et al. (2002) Irrational choices in hummingbird foraging behaviour. Anim. 632 Behav. 63, 587-596 633 63 Shafir, S. (1994) Intransitivity of preferences in honey bees: support for 'comparative'

evaluation of foraging options. Anim. Behav. 48, 55–67

- 635 64 Monteiro, T. et al. (2013) Starlings uphold principles of economic rationality for delay 636 and probability of reward. Proc. R. Soc. B 280, 20122386 637 65 Trimmer, P. (2013) Optimal behaviour can violate the principle of regularity. *Proc. R.* 638 Soc. B 280, 20130858 639 66 Houston, A.I. et al. (2007) Violations of transitivity under fitness maximization. Biol. 640 Lett. 3, 365–367 641 67 Marsh, B. et al. (2004) Energetic state during learning affects foraging choices in 642 starlings. *Behav. Ecol.* 15, 396–399 643 Pompilio, L. et al. (2006) State-dependent learned valuation drives choice in an 68 644 invertebrate. *Science* 311, 1613–1615 645 69 Aw, J. et al. (2009) State-dependent valuation learning in fish: banded tetras prefer 646 stimuli associated with greater past deprivation. Behav. Processes 81, 333–336 647 70 Pompilio, L. and Kacelnik, A. (2005) State-dependent learning and suboptimal choice: 648 when starlings prefer long over short delays to food. Anim. Behav. 70, 571–578 649 71 McNamara, J.M. et al. (2012) The ecological rationality of state-dependent valuation. 650 Psychol. Rev. 119, 114-119 651 Mitchell, E.N. et al. (2012) Evaluation of an operant successive negative contrast task 72 652 as a method to study affective state in rodents. Behav. Brain Res. 234, 155–160 653 73 McNamara J.M. et al. (2013) An adaptive response to uncertainty generates positive 654 and negative contrast effects. Science 340, 1084-1086 655 74 Freidin, E. et al. (2009) Successive negative contrast in a bird: starlings' behaviour 656 after unpredictable negative changes in food quality. Anim. Behav. 77, 857–865 657 75 McNamara, J.M. et al. (2012) It is optimal to be optimistic about survival. Biol. Lett. 8, 658 516-519 659 76 Houston, A.I. et al. (2012) Is optimism optimal? Functional causes of apparent 660 behavioural biases. Behav. Processes 89, 172–178 661 77 Hutto, D.D. (2013) Psychology unified: from folk psychology to radical enactivism. 662 Rev. Gen. Psychol. 17, 147-178
- 666 Psychol. 22, 170–187

Human Minds, Princeton University Press

Barrett, L. (2011) Beyond the Brain: How Body and Environment Shape Animal and

Papini, M.R. (2009) Role of opioid receptors in incentive contrast. Int. J. Comp.

663

664

665

78

- Hansen, T.F. and Orzack, S.H. (2005) Assessing current adaptation and phylogenetic
- inertia as explanations of trait evolution: the need for controlled comparisons. *Evolution*
- 59, 2063–2072
- 81 Papini, M.R. (2003) Comparative psychology of surprising nonreward. *Brain Behav*.
- 671 Evol. 62, 83–95
- 672 82 Townsend-Mehler, J.M and Dyer, F.C. (2012) An integrated look at decision-making in
- bees as they abandon a depleted food source. *Behav. Ecol. Sociobiol.* 66, 275–286
- Pautasso, M. (2010) Worsening file-drawer problem in the abstracts of natural, medical
- and social science databases. *Scientometrics* 85, 193–202
- 676 84 Krause, J. et al. (2013) Reality mining of animal social systems. Trends Ecol. Evol. 28,
- 677 541–551
- 85 Stevens, J.R. and Stephens, D.W. (2009) The adaptive nature of impulsivity. In
- 679 Impulsivity: The Behavioral and Neurological Science of Discounting (Madden, G.J.
- and Bickel, W.K., eds), pp. 361–387, APA Press
- 681 86 Anderson, J.R. (1991) Is human cognition adaptive? Behav. Brain Sci. 14, 471–485
- 682 87 Shepard, R.N. (2001) Perceptual-cognitive universals as reflections of the world.
- 683 Behav. Brain Sci. 24, 581–601
- 684 88 Todd, P.M. and Gigerenzer, G. (2001) Shepard's mirrors or Simon's scissors? *Behav*.
- 685 Brain Sci. 24, 704–705
- 686 89 Couvillon, P.A. and Bitterman, M.E. (1984) The overlearning–extinction effect and
- successive negative contrast in honeybees (*Apis mellifera*). J. Comp. Psychol. 98, 100–
- 688 109
- 689 90 Kacelnik, A. (2006) Meanings of rationality. In Rational Animals? (Hurley, S. and
- Nudds, M., eds), pp. 87–106, Oxford University Press
- 691 91 Houston, A.I. et al. (2007) Do we expect natural selection to produce rational
- 692 behaviour? *Phil. Trans. R. Soc. B* 362, 1531–1543
- 693 92 Bateson, M. (2010) Rational choice behavior: definitions and evidence. In
- 694 Encyclopedia of Animal Behavior, vol. 3 (Breed, M.D. and Moore, J., eds), pp. 13–19,
- 695 Academic Press
- 696 93 Trimmer, P.C. et al. (2011) Decision-making under uncertainty: biases and Bayesians.
- 697 Anim. Cogn. 14, 465–476
- 698 94 McNamara, J.M. and Dall, S.R.X. (2011) The evolution of unconditional strategies via
- the 'multiplier effect'. *Ecol. Lett.* 14, 237–243

- 700 95 McNamara, J.M. et al. (in press). Natural selection can favour 'irrational' behaviour. 701 Biol. Lett. 702 96 Waksberg, A.J. et al. (2009) Can irrational behaviour maximise fitness? Behav. Ecol. 703 Sociobiol. 63, 461–471 704 97 Houston, A.I. (2012) Natural selection and rational decisions. In Evolution and 705 Rationality: Decisions, Cooperation and Strategic Behaviour (Okasha, S. and Binmore, 706 K., eds), pp. 50–66, Cambridge University Press 707 98 Wakker, P.P. (2010) Prospect Theory: For Risk and Ambiguity, Cambridge University 708 Press 709 99 Giske, J. et al. (2013) Effects of the emotion system on adaptive behavior. Am. Nat. 710 182, 689–703 711 100 Mendl, M. et al. (2010) An integrative and functional framework for the study of 712 animal emotion and mood. Proc. R. Soc. B 277, 2895–2904 713 101 Nettle, D. and Bateson, M. (2012) The evolutionary origins of mood and its disorders. 714 Curr. Biol. 22, R712-R721 715 102 Trimmer, P.C. et al. (2013) On the evolution and optimality of mood states. Behav. Sci. 716 3, 501–521 717 103 LeDoux, J. (2012) Rethinking the emotional brain. Neuron 73, 653–676 718 104 Stevens, J.R. et al. (2005) The ecology and evolution of patience in two New World 719 monkeys. Biol. Lett. 1, 223–226 720 105 Rosati, A.G. et al. (2007) The evolutionary origins of human patience: temporal 721 preferences in chimpanzees, bonobos, and human adults. Curr. Biol. 17, 1663–1668
- 722 106 Harding, E.J. et al. (2004) Cognitive bias and affective state. Nature 427, 312
- 723 107 Brilot, B.O. *et al.* (2009) Can we use starlings' aversion to eyespots as the basis for a novel 'cognitive bias' task? *Appl. Anim. Behav. Sci.* 118, 182–190
- 725 108 Paul, E.S. *et al.* (2005) Measuring emotional processes in animals: the utility of a cognitive approach. *Neurosci. Biobehav. Rev.* 29, 469–491

Figure 1. Incorporating spatiotemporal heterogeneity and autocorrelation into standard evolutionary models can account for a number of cognitive biases and puzzling features of behaviour. The Venn diagram indicates which combination of factors can produce particular outcomes, with the phenomena discussed in this paper shown in bold type. In a heterogeneous world the environmental conditions change over time or space (e.g. between states *A* and *B*), with positive autocorrelation implying that conditions are more likely to stay the same (thicker arrows) than change (see also **Box 1**). Some of the adaptive explanations we discuss are extensions of standard state-dependent models of behaviour [18] (shown in plain font). Some are based on uncertainty about current conditions and/or the pattern of environmental change [93]. Possible directions for future work are shown in italics.

